Introduction to Strings, Dualities and D-branes

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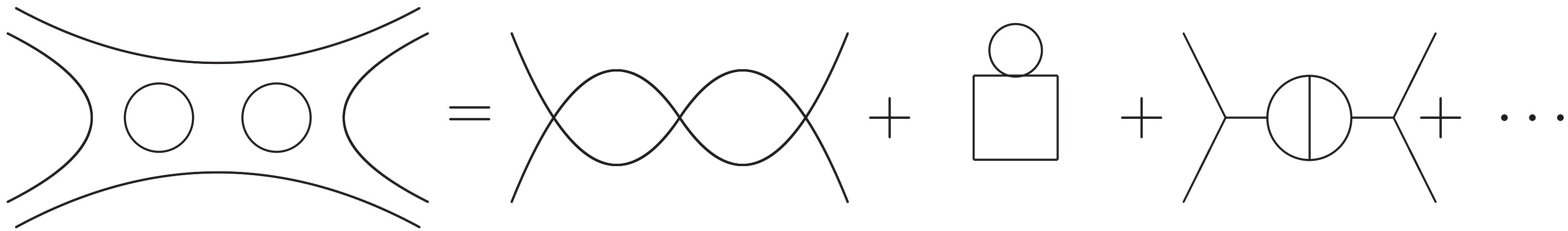
Content

- Particles. forces and strings
  - universal sectors of closed and open string
  - SUGRA and dualities
  - T-duality and quantum geometry
- D-branes
  - E-dyn. and electric-magnetic duality
  - black p-branes and D-branes
- Compactification:
  - Calabi–Yau, mirror symmetry and generalized geometry
- Toric Calabi–Yau constructions
  - classifictiaon and applications

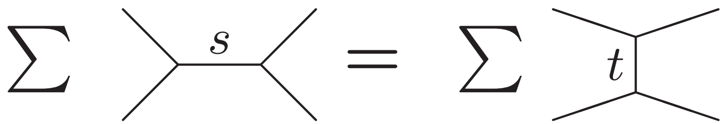
Particles, forces and strings

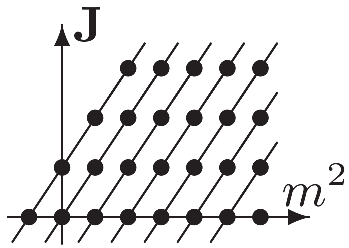
- the “fundamental” string commercial: unify particles = oscillation modes & interactions = unique geometrical



but elementary? ... just cut it into pieces ⇒ point particle constituents

- historically: s/t/u duality of scattering amplituedes





Regge trajectories

- quantum dualities may be the real core or string theory, I presume
- apropos history: invented for strong interactions

  30++ years: AdS/CFT → strong coupling expansions in gauge theory (dual gravity)

Geometry & weakly coupled strings

most general renormalizable & conformally invariant action:

$$\frac{1}{4\pi\alpha'}\sqrt{|g|}g^{mn}\partial_m X^\mu \partial_n X^\nu G_{\mu\nu}(X) - \frac{1}{4\pi\alpha'}\varepsilon^{mn}\partial_m X^\mu \partial_n X^\nu B_{\mu\nu}(X) + \frac{1}{4\pi}\sqrt{|g|}\phi(X)R^{(2)}$$

- 10 almost free bosons X^μ = space-time coordinates + fermions
- Kaluza-Klein (hidden dimensions): # light modes ↔ Betti numbers
  + additions: winding states become light for singular geometries
- universal:
  - graviton $G_{\mu\nu}$: massless spin-two & general coord. inv.
    Einstein-Hilbert action plus $\mathcal{O}(R^2)$ corrections
  - B-field (non-symmetric metric): topological limit →
    deformation quantization / non-commutative geometry
  - dilaton: VEV $\langle\phi\rangle$ = coupling constant
- gravitino(s) (type II), gauge fields (heterotic, type I)

consistent motion

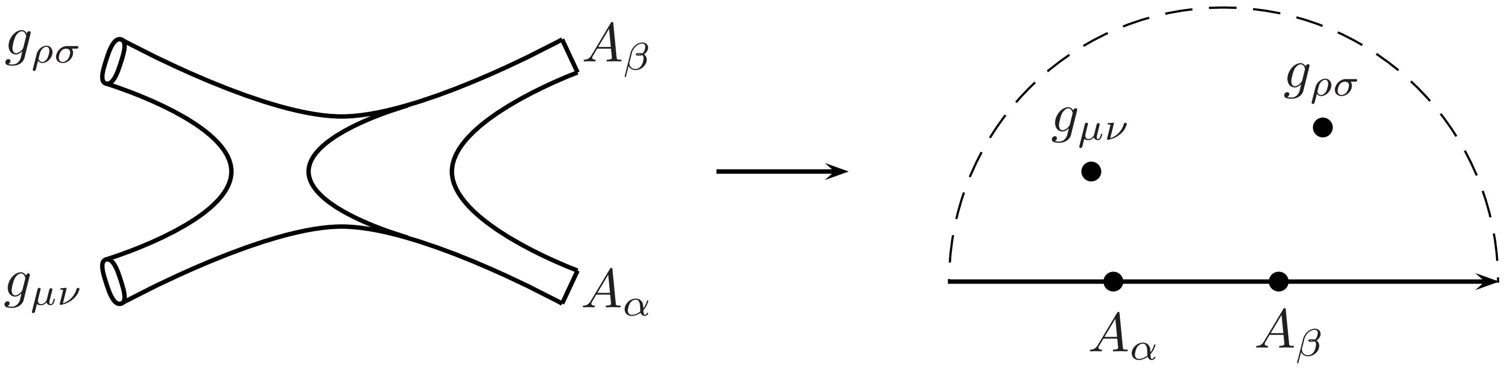
⇒ effective action

$$\frac{1}{2\kappa_0^2}\int d^D X\sqrt{-G}e^{-2\phi}(R + 4\partial_\mu\phi\partial^\mu\phi - \tfrac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho}) + \ldots$$

Open strings, gauge invariance & electrodynamics

Closed string (conformal gauge): $\mathcal{L} = \frac{1}{2\pi\alpha'}\partial X^\mu \bar{\partial} X^\nu \Big(g_{\mu\nu}(X) + B_{\mu\nu}(X) \Big)$

String diagram for photon–graviton scattering:



- Gauge invariance: $\delta_\Lambda \left(\int_\Sigma X^* B \right) = 0$ for $\delta_\Lambda B = d\Lambda$ (Stockes' theorem)

  But: open strings → boundary term $\delta_\Lambda \left(\int_\Sigma B \right) = \int_{\partial\Sigma} \Lambda$

- Introduce compensator field ($\sim$ surface charge in E-dyn.):

$$S_A = \int_{\partial\Sigma} A = \int_{\partial\Sigma} ds \partial_s X^\mu \, A_\mu(X) = \int_\Sigma F \qquad \text{with} \quad F = dA$$

Gauge invariance: $\delta B = d\Lambda, \qquad \delta A = -\frac{1}{2\pi\alpha'}\Lambda + d\lambda \quad \Rightarrow$

Invariant field strengths: $H = dB = d\mathcal{F}$, $\mathcal{F} = B + 2\pi\alpha' F$.

– D-brane effective action . . . $\mathcal{L}_{BI} \sim \sqrt{|\, g_{\mu\nu} + \mathcal{F}_{\mu\nu}\,|}$ . . . gauge invariant

– Superstrings: $A \;\longrightarrow$ superpartner gaugino λ

Boundary conditions and noncommutativity

- **strings $\longrightarrow$ operators $X^\mu(\sigma,\tau) \longrightarrow$ NC operator product**

$$S = \frac{1}{2\pi\alpha'}\int_{\mathbb{H}} \mathrm{d}^2 z\, \partial X^\mu \bar{\partial} X^\nu \Big(g_{\mu\nu}(X) + \mathcal{F}_{\mu\nu}(X) \Big), \qquad \mathcal{F} = B + 2\pi\alpha'\, dA$$

- **Surface terms . . . fluctuations about background value $X \to X + \xi$:**

$$\begin{aligned} &\partial(\delta X^\mu \bar{\partial} X^\nu (g_{\mu\nu} + \mathcal{F}_{\mu\nu})) + \bar{\partial}(\partial X^\mu \delta X^\nu (g_{\mu\nu} + \mathcal{F}_{\mu\nu})) \\ &= \partial(\delta X^\mu \bar{\partial} X^\nu (g_{\mu\nu} + \mathcal{F}_{\mu\nu})) + \bar{\partial}(\partial X^\nu \delta X^\mu (g_{\mu\nu} - \mathcal{F}_{\mu\nu}) \end{aligned}$$

- **Boundary conditions:** $\int dy \partial() = -\int dy \bar{\partial}()$

$$g_{\mu\nu}(\partial - \bar{\partial})X^\nu - \mathcal{F}_{\mu\nu}(\partial + \bar{\partial})X^\nu = g_{\mu\nu}\partial_{\textbf{tang.}}X^\nu - \mathcal{F}_{\mu\nu}\partial_{\textbf{normal}}X^\nu = 0$$

Propagator:

$$\langle \zeta^\mu(u,\bar u)\,\zeta^\nu(w,\bar w)\rangle_{\mathcal F} = -\alpha'\Big\{ g^{\mu\nu}\big(\ln|u-w| - \ln|u-\bar w|\big) + G^{\mu\nu}\ln|u-\bar w|^2 - \Theta^{\mu\nu}\ln\Big(\frac{\bar w - u}{\bar u - w}\Big)\Big\}$$

– define $(g_{\mu\nu}+\mathcal{F}_{\mu\nu})^{-1} = G^{(\mu\nu)} + \Theta^{[\mu\nu]}$

– $g^{\mu\nu}$-term $\longrightarrow 0$ at boundary $\partial\Sigma \Rightarrow G^{\mu\nu} =$ "open string metric"

– for $G^{\mu\nu} \longrightarrow 0$ propagator $\longrightarrow$ antisym. step function at $\partial\Sigma$

$\Rightarrow$ non-commutative non-singular operator product $\sim$ Moyal

– Proposal: define non-commutative product

$$f(x)\circ g(x) = \frac{1}{\sqrt{|g+\mathcal{F}|}} \int \mathcal{D}\xi\; e^{-S[X=x+\xi]}\; f(X(0))\; g(X(1))$$

- includes quantum corrections; regularized by putting $u=0$, $w=1$
- off-shell A_∞ administrating non-associativity
- BI measure comes up in sum over graphs

Consistency: dimension
Stability: no tachyon $\Rightarrow$ **Maximal 10D supergravities** +15y/'84

(effective low energy action)

- IIA: Two gravitinos of opposite chirality (non-chiral)

$$S_{Bose}^{IIA} = S_{NS} - \frac{1}{4\kappa_{10}^2}\int dV\ (|F_2|^2 + |\widetilde{F}_4|^2) - \frac{1}{4\kappa_{10}^2}\int B_2 \wedge F_4 \wedge F_4$$

$\int dV \ := \int d^{10}X\sqrt{-G}, \qquad |F_p|^2 = \frac{1}{p!}F_{\mu_1\ldots\mu_p}F^{\mu_1\ldots\mu_p}, \qquad \tilde{F}_4 = dC_3 - C_1 \wedge dB_2$

universal part $S_{NS} = \int dV\ e^{-2\phi}(R + 4(\partial\phi)^2 - \frac{1}{2}H^2)$ "Neveu–Schwarz"

Einstein's unification: extra dimensions (Kaluza–Klein)
& asymmetric metric $\hat{=}$ torsion $\tilde{g}_{\mu\nu} = g_{\mu\nu} + B_{\mu\nu}$

but: $B_{\mu\nu}$ is (generalized) gauge field ... directly related to strings!

Chern-Simons terms $\longleftrightarrow$ Anomalies

SUGRA gives structure to hidden dimensions constrains topology

M-theory: the good, the ugly, and the bad

- **Trademark of SUGRA:** simple idea

  unique but ugly action: many many pages of fermion interactions

  simple & surprising implications

- The good: **11D SUGRA** is the **most beautiful SUGRA**

  particles: **graviton** $g_{\mu\nu}$, **gravitino** ψ^{μ}_{α} and **3-form gauge** field $A_{\mu\nu\rho}$
  'completely' unique (non coupling constant) ... IIA = 11D / circle:

  $R_{11d} \sim g_{11d}$: strong coupling limit grows Lorentz invariant effective 11th dimension

- The bad: Membrane quantization failed (no mass gap!)

  but: 11th dimension can be unserstood in terms of D-branes!
  This phenomenon was called "M-theory", in the hope that it may be more fundamental than stings. But so far this hope did not materialze, hence strings are again called "strings" and not "string/M-theory" ;-)

Signatures of 11 dimensions:

11-dimensional Lorentz invariance organizes BPS states into multiplets, whose counting is related to (statistical) black hole entropy
Counting yields integers, which can be translated into new Number theoretical properties of Donalson–Thomas invariants of symplectic manifolds (Donaldson theory) ... [Gopakumar–Vafa 2000]

The type II-B story $\ni$ AdS/CFT = holography

- $S_{Bose}^{IIB} = S_{NS} - \frac{1}{4\kappa_{10}^2} \int dV \ (|F_1|^2 + |\tilde{F}_3|^2 + \frac{1}{2}|\tilde{F}_5|^2) - \frac{1}{4\kappa_{10}^2} \int C_4 \wedge H_3 \wedge F_3$
  + all the ugly fermion terms

  NS-fields: e^{ϕ}, B_2, graviton

  RR-fields: C_0, C_2, C_4^+

- One **observes** exact classical $SL(2,\mathbb{R})$-invariance:

  $$\tau = C_0 + ie^{-\phi} \rightarrow \frac{a\tau + b}{c\tau + d}, \qquad \begin{pmatrix} F_3 \\ H_3 \end{pmatrix} \rightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} F_3 \\ H_3 \end{pmatrix}$$

  instantons corrections / θ–angle = axion= C_0

  Peccei–Quinn story is universal !

  Exact $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z})$ symmetry after quantization:

  strong coupling $\leftrightarrow$ weak coupling ... explained by D-branes

T-duality

- Down to 4 dimensions: Kalazu–Klein on circle
- Kaluza–Klein oszillation modes: $E_O \sim n^2/R^2$
- Stringy winding modes: $E_W \sim 4\pi^2\ R^2 w^2$

$E_{total} = E_O + E_W$ invariant under n $\leftrightarrow$ w

R $\leftrightarrow 2\pi\alpha'/$R

Theorem: String scattering amplitudes in effective low energy theory in lower dimension are invariant to all orders in perturbation theory (there is also a QFT argument implying non-perturbative invariance)

Bosonization & a 1st glance at quantum geometry

The following compactificatoins are **exactly quantum equivalent:**

- n **periodic free bosons** X**:**
  compactification of root lattice of $SO(n)$
- Non-geometrical "CFT substructure": $2n$ **free fermions**
- WZW-Model ('87): **Interacting bosons** with WZ-coupling on group manifold of $SO(2n)$ (dimension $\sim n^2$!!!)
- strongly curved spacetime $\Rightarrow$ critical dimension is strongly reduced by quantum corrections to general relativity (exactly computable!)

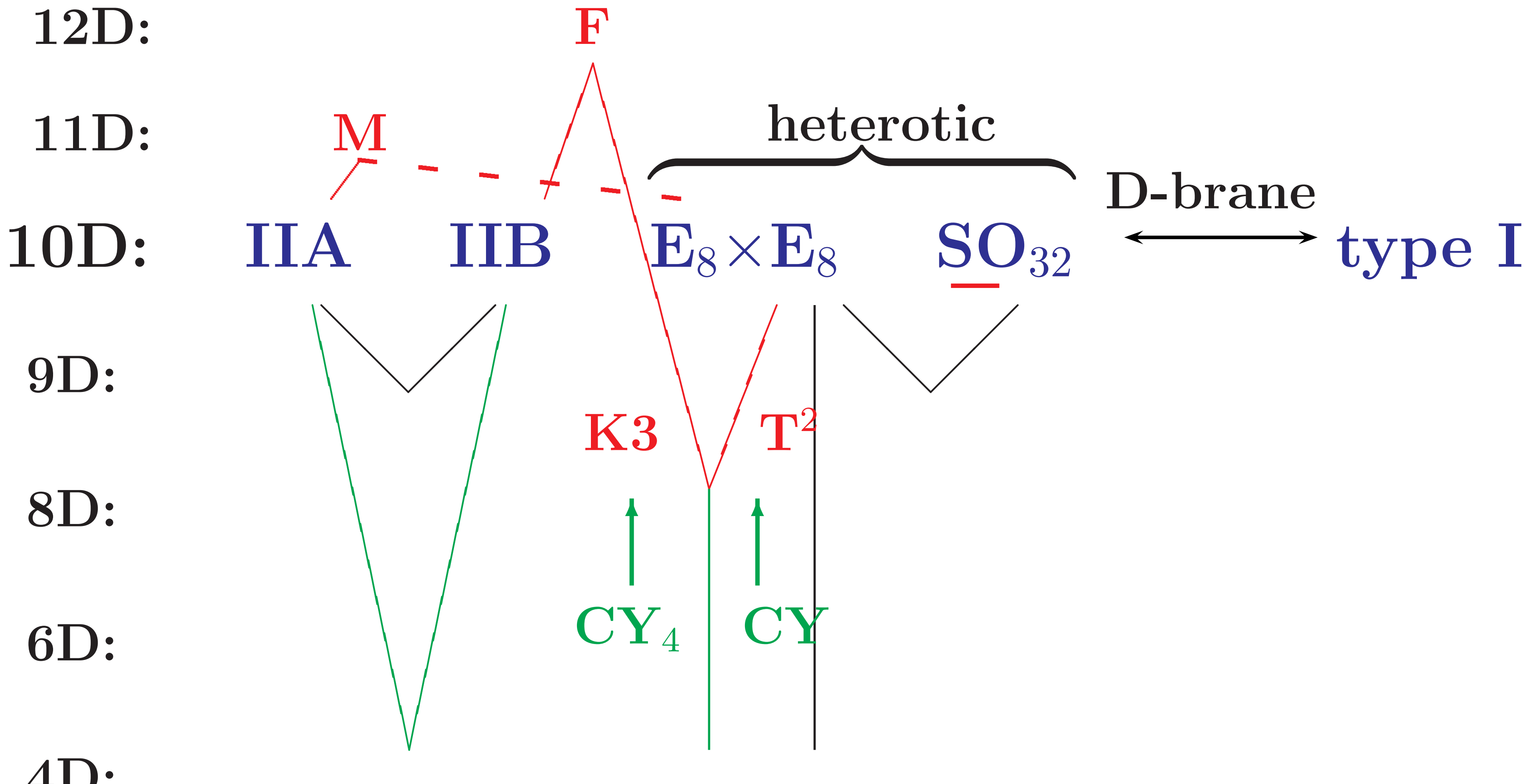
Problem: **exactly solvable** = stuck at **isolated points** in configuration space of strings $\Rightarrow$ loose full power of geometry:

- Way out: geometrical compactification
  (relatively) large hidden dimensions $\gg 10^{-35}m$
  $\Rightarrow$ can trust Einstein equations

The web of string dualities

"geometrical compactification":

$\mathcal{M}_{10} = M_4 \times K$



D-branes: What about RR-fields

so far we ignored $\approx 3/4$ of the fields!

SUGRA people (P. Townsend et al.; ... A. Stromonger):

- **solitonic black branes** (AKA D-branes)
- Some electrodynamics:
- antisymmetric tensor fields $A_{\mu_1 \dots \mu_p} \mapsto$ p-form $A^{(p)}$
- differential d: $A \mapsto dA \;.\;.\;.\; \partial_{[\mu_0} A \mu_1 \dots \mu_{p]}$ Edyn: $curl = d$
- Hodge dual $*$: $A \mapsto *A \;.\;.\;.\; \varepsilon_{\mu_1 \dots \mu_d} A^{\mu_1 \dots \mu_p}$ Edyn: $div = *d*$
- $d^2 = 0 \leftrightarrow$ div rot = rot grad = 0, $F \longrightarrow *F \Leftrightarrow E \leftrightarrow B$
- field strength $F = dA$, gauge invariance $A' = A + d\Lambda$
- physics in $d > 4$: antisymmetry + gauge invariance $\Rightarrow$ in 4d spin ≤ 1
- origin: two gravitini $\longrightarrow$ bi-spinor $\psi \otimes \psi$
  Clebsch–Gordan for $SO(d) \Rightarrow$ even/odd forms

Integration: p-forms over p-dimensional objects:

$\int_V F$ is invariant under deformation if $dF = 0$

(homogeneous Maxwell equation)

$$Q_E = \int_{S^2} *F, \qquad Q_M = \int_{S^2} F$$

S^2 = sphere enclosing point particle.

Black branes: Take YM instanton, add 1 time and 5 space dimensions

p-form gauge field: enclosed by $10 - p - 1$ dimensional sphere,

electric (p-1)-brane, magnetic (10-p-3)-brane.

e.g.: Maxwell force: $\int_V A \;\rightarrow\; \int A ds,\; \int B d^2\Sigma, \ldots\; B_2 \rightarrow$ strings

- **Polchinski '89:** T-duality for open strings
  need winding $\Rightarrow$ fix string ends Dirichlet boundary conditions

- **"fix string ends at D-brane"**

Polchinski: 6 years later: identity **D-brane = black brane**

- test: compute RR-charge from open string scattering amplitude (PRL '95)

  closed strings can 'fall' into black branes and end behind their horizon

- implication : compute strong coupling expansionof spectrum and interactions of collective modes = photons and photinos of solition (black hole) in type II (closed) strings with weakly coupled dual type I (open) string scattering

- New type of matter: $S_{effective}$ = Dirac-Born-Infeld + Wess-Zumino

  $$S_{DBI} = -\mu_p \int_{\mathcal{V}_{p+1}} \mathrm{Tr}(e^{-\phi}\sqrt{-\det(G_{ab}+B_{ab}+2\pi\alpha' F_{ab}})$$

  Fermions: anomalies

  $$S_{WZ} = \mu_p \int_{\mathcal{V}_{p+1}} \sum_i C^{(i)} \wedge e^{2\pi\alpha' F + B} \sqrt{\frac{\hat{\mathcal{A}}(4\pi^2\alpha' R_T)}{\hat{\mathcal{A}}(4\pi^2\alpha' R_N)}}$$

  $\hat{\mathcal{A}}$=Dirac genus

  conserved D-brane charges are classified by K-theory(coherent sheaves)

  they can change dimensions and/or decay in curved space

- by S-dual weakly coupled open string:

  RR-charge $\mu_p^2 = \frac{\pi}{\kappa_{10}^2}(4\pi^2\alpha')^{3-p} = 2^{2\phi}\tau_p^2, \ . \ . \ . \ \tau_p$=brane tension

  BPS-branes/extremal BH: no force $\mathcal{A}_{NS-NS} = -\mathcal{A}_{R-R} \rightarrow$ many checks!

- gauge fields are localized on branes (weak gravity)

  bifundamental matter brane intersections (cf. standard model!)

- stack of N branes: large N QCD / decouple gravity: **AdS/CFT**
- Grand Unification / brane worlds dilute gravity $\rightarrow$ reduce M_{Pl}

  different phenomenological scenarios:

  GUT vs. "large volume"$=10^{11}GeV$: Inflaton=size of hidden dim. decide at LHC?

- IIA: $\tau_p \sim g^{-1}\alpha'^{-(p+1)/2} \rightarrow$ mass $\tau_p^{1/(p+1)} \sim g^{-1/(p+1)}\alpha'^{-1/2}$

  for $g \rightarrow \infty$ D0-brane dominats $\tau_0 = \frac{1}{g\sqrt{\alpha'}}$, $\Rightarrow 11^{th}$ dimension, $R_{10} = g\sqrt{\alpha'}$

  M-theory: low energy limit is 11-dimensional SUGRA

- IIB: $SL(2,\mathbb{Z})$ duality $\leftrightarrow$ solitonic strings

The CY story and mirror symmetry

- T-duality: IIA on circle $\Leftrightarrow$ IIB on circle with inverse radius (non-chiral)
- Supersymmetric compactification: $D\eta = 0$ "Killing spinor eq."
- complex structure $J_\mu^\nu = \eta^\dagger(\gamma_\mu\gamma^\nu - \delta_\mu^\nu)\eta$ ... after some Fierzing: $J^2 = -\mathbb{1}$
- 3-form$= \eta^\dagger\gamma_{[\mu}\gamma_\nu\gamma_{\rho]}\eta \Rightarrow$ vanishing 1st Chern class
- Calabi (conjecture) – Yau (proof): SUSY $\Rightarrow$ Einstein equations
- Mirror symmetry $\equiv$ **T-duality in curved space**
  $\longrightarrow$ infinite instanton sum $\leftrightarrow$ classical geometry (with different topology!)
- 3x complex geometry: • world sheet (conformal invariance), SUSY $\Rightarrow$
  - hidden dimensione = Calabi–Yau geometry
  - parameter space = Kähler space of light scalar fields (SUSY pheno)

  instanton counting = holomorphic curves on the quintic [Candelas et al. '90]
- Seiberg–Witten '94 electric-magnetic duality in $N = 2$ SYM
  Holomorphicity, Riemann Hilbert: auxiliary Riemann surface explicit tests!
  $\subseteq CY$ ... $\rightarrow$ "geometrical engineering" (Klemm, Vafa, ...)
  First few terms tested by explicit instanton calculations

Fluxes, RR-fields, and generalized geometries

"Traditionally" 1/2 of the fields had been set to 0 !

- 1995 [Polchinski]: D-branes = sources for RR fields
- D-branes and fluxes required for moduli stabilization
- Killing spinor [SUSY] $\nabla\eta = 0$ but $\nabla = D^{LC} + Tor(B, C)$

  $\rightarrow$ generalized complex structure $\mathcal{J}^2 = -\mathbb{1}$ on $T \oplus T^*$ [Hitchin '02]

$$\begin{pmatrix} -J & -\omega^{-1} \\ \omega & J^t \end{pmatrix}$$

- constructed in terms of pure spinors $\Leftrightarrow$ differential forms

$$J = \eta^\dagger \Gamma_\mu{}^\nu \eta \text{ and } \mathcal{O}_{\mu\nu\rho} = \eta^\dagger \Gamma_{\mu\nu\rho} \eta$$

Why 4 dimensions:

till 90s: hope for some non-perturbative reason for uniqueness of SM,

but:

- *2nd string revolution* '95: (some!) non-perturbative results

  Witten's talk: String theory in arbitrary dimensions → non-perturb. consistent

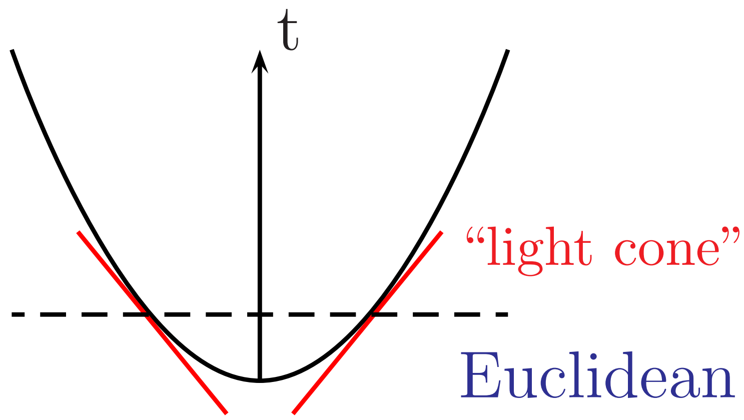
- Assuming SUSY well below the Planck scale (hierarchie):

$$W_{eff} \sim e^{-\text{Vol}}\Big(\, . \, . \, . \, . \, . \, \Big) \quad \xrightarrow{\text{Vol} \to \infty} \quad 0$$

  – cosmological constant $\langle \mathbf{W} \rangle > \mathbf{0} \quad \Rightarrow$ METAstable

  → Strings "want" to live in 10d

Speculations:



- Hartle–Hawking: “tunneling from nothing”

  boundary condition: no boundary

  – flat space suppressed (Vol$\rightarrow \infty$)

  – Linde: assumes cosmological principle !

  Torus is flat, finite volume, but non-isotropic!

  kind of spontaneous symmetry breaking

- Later: effective QFT; dynamical choice of space-time and forces?

  “local cosmological principle” by inflation

  but part of the “multiverse”?

- Brandenberger–Vafa ’89: efficient string annihilation in $d \leq 4$

  – in $d \leq 4$ / $d > 4$ two 2-worldsheets (2d) generically meet / miss

- dynamical choice of space-time?

  10^{500} flux vacue (on single CY) misleading → mostly bumps in mountain range

- stability and probability of inflation–cascades important!

- → antropic ? don't care: in any case

  – discrete values (Dirac-Zwanziger & flux quantization)

  – unique theory or choice of 'vacua' = solutions of EoM = consistent backgrounds?

  – in (non-unified) QFT: continuum of parameters

- so what about phenomenology:

- we have but a few small roads into the jungle of "Quantum Geometry"

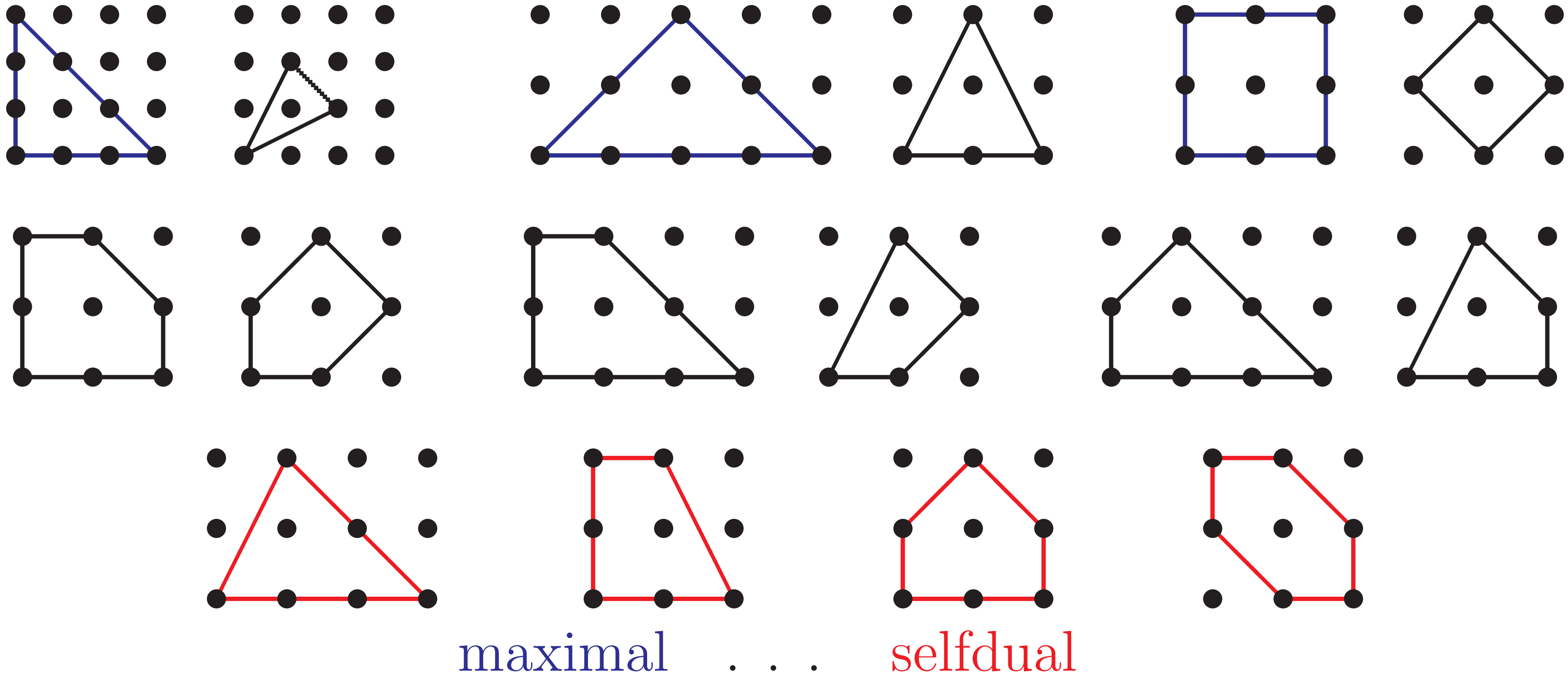
  premature or not?

- LHC: consistent with SUSY GUT → take the heterotic road

  GUT-success mere coincidence → type II / brane worlds

- how can we hope to ever come close? ... hierarchy/SUSY & small gauge couplings

Calabi–Yau [Batyrev]: Reflexivity



- 16 hypersurface tori (Calabi–Yau 1-folds), CY 3-folds: $\dim(\Delta) = 3+\text{codimension} \geq 4$
- 5 Fano 2-folds (smooth, $c_1 > 0$)
- 1 Fano hypersurface: $\mathbb{P}^1$ ("hyperplane" in $\mathbb{P}^1 \times \mathbb{P}^1$)

Reflexivity & mirror symmetry

N lattice: $v_i \in \Delta^\circ$ ... homogeneous coordinates z_i,
'toric' (T-invariant) divisors $D_i : \{z_i = 0\}$ (e.g. $\mathbb{P}^n : D_i \sim H$)

M lattice: $m \in \Delta \to$ Monomials $\prod z_i^{\langle m, v_i \rangle + 1}$
'+1' $\Rightarrow$ sections of a line bundle (Cartier divisor).

Batyrev '93: generic hypersurface is CY $\Leftrightarrow$ Δ reflexive, mirror symmetry: $\Delta \longleftrightarrow \Delta^\circ$

formula for Betti numbers: count points $l(\theta)$ on dual faces $\theta \subset \Delta$ and $\theta^\circ \subset \Delta^\circ$

$$h_{11}(X_\Delta) = h_{2,1}(X_{\Delta^\circ}) = l(\Delta^\circ) - 1 - \dim \Delta \quad - \sum_{\mathrm{codim}(\theta^\circ)=1} l^*(\theta^\circ) + \sum_{\mathrm{codim}(\theta^\circ)=2} l^*(\theta^\circ) l^*(\theta)$$

Maximal coherent triangulation: CY is regular for 3-folds / generic singularities for 4-folds

GLSM (Witten 1993): $U(1)^N$ SYM with $L = L_{kin} + L_W + L_{gauge} + L_{D,\theta}$
D-term $\Leftrightarrow$ moment map $D = -\sum q_i |z^2| - r$
$\forall U(1)$: $r_j =$ Kähler parameters, charges $q_i =$ 'weights'
Complex structure: coefficients in polynomials

Strominger–Yau–Zaslov: SLAG fibration with T^3 fibers $\to$ MS = T-duality
$\Delta \subset M$ = image of $\mathbb{P}_\Delta$ under moment map (symplectic reduction)
$\to$ duality of face lattice

Classification results

- general Algorithm: hep-th/9505120 [M.K., H. Skarke]

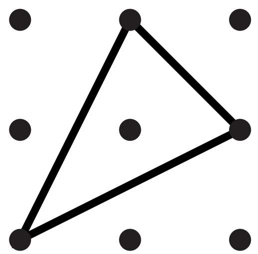
- **maximal** objects are **Newton polytopes** $\sum q_i n_i = d$,
  $n_i \geq 0$, $d = \sum q_i \Rightarrow \vec{n} = \vec{1} \in \Delta$ is the only possible interior point
  $\vec{1} \in \Delta^0 \;\Rightarrow$ **finitely many weights** $\vec{q}$'s

- any reflexive $\Delta \;\subset\; \Delta_{max}$ comes from **combined weights**
  - simplex decomposition of 'minimal' $\Delta^* \rightarrow$ baricentric coord.
  - $1x + 1y + 1z = 3$, $1x + 1y + 2z = 4$, $\substack{1x+1y+0u+0v=2 \\ 0x+0y+1u+1v=2}$
  - the polytope may live on a **sublattice** (finitely many)

maximal Δ $\rightarrow$ enumerate all reflexive subpolytopes on sublattices

$\updownarrow$

minimal Δ^*

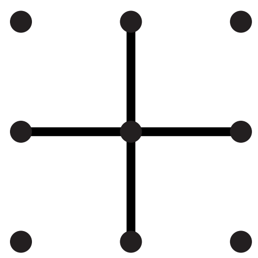


simlpex “3”

barycentric coordinates q_i

$$q_i = n_i/d$$

$$\sum n_i \vec{v}_i = 0 \;\rightarrow\; d = \sum n_i$$



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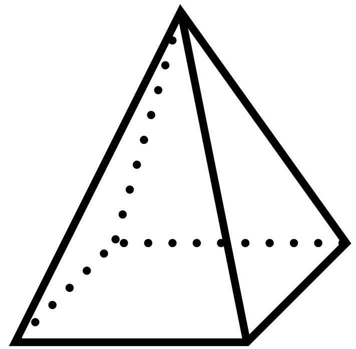


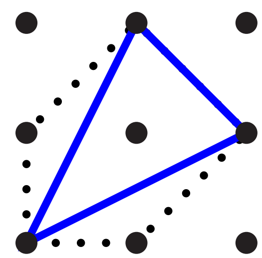
$\underbrace{3+3}$

combined weight systems (CWS)

1	1	0	0	1
0	0	1	1	1

Weight vector $\rightarrow$ Newton polytope Δ_q $\leftrightarrow$ Δ_q^*

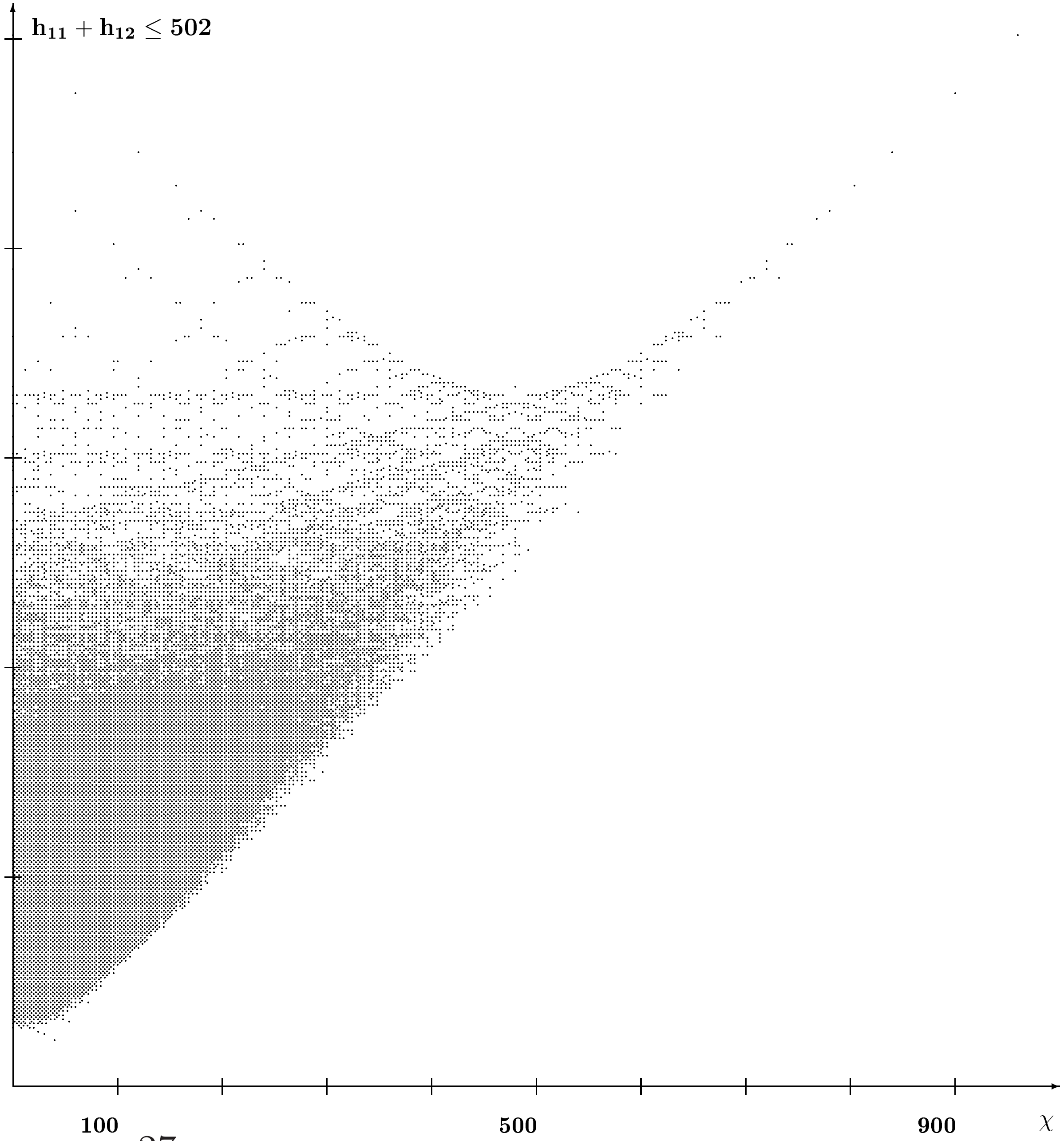
$$\vec{q} = (\tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}) \;\rightarrow\; \Delta_q = \left\langle \begin{pmatrix}3\\0\\0\end{pmatrix}, \begin{pmatrix}0\\3\\0\end{pmatrix}, \begin{pmatrix}0\\0\\3\end{pmatrix} \right\rangle - \begin{pmatrix}1\\1\\1\end{pmatrix} \;\leftrightarrow$$



Lemma: In each dimension there is only a finite number of weights $(d, \vec{n})$ such that Δ_q has an interior lattice point.

4 dimensions: [hep-th/0002240]

- 184.026 weights, 308+25+7 maximal reflexive polyhedra
- 473.800.776 reflexive polyhedra
- 30.108 pairs of Hodge numbers
- 4.5 GB database
  (internet search mask)
- test: mirror symmetry !



Conifold transitions to non-toric Calabi–Yau varieties

V. Batyrev & M.K. (in preparation)

- Toric CICYs are *numerous and easy to work with!*
- Combinatorial *mirror symmetry!* [Batyrev-Borisov]: tools for computing quantum cohomology = Gromov-Witten = instanton sums
- But **how generic are they?**
- Reid's phantasy = Candelas: Other worlds around the corner (1990)

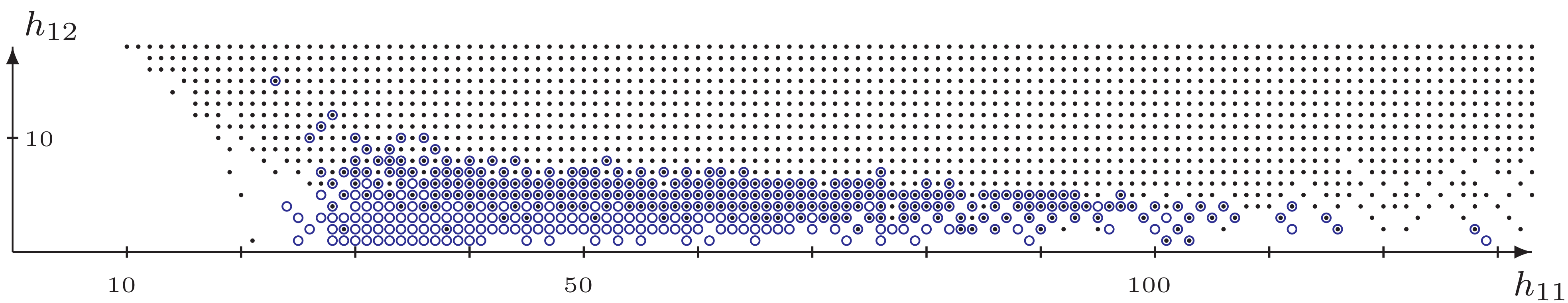
  Moduli spaces of (all?) Calabi–Yau spaces are connected by singular transitions: singular geometry, but smooth physics: Black hole condensation (Strominger 1995)

V.Batyrev, M.K.: 4d reflexive *conifold* polytopes with ∃ smoothing deformation

$h_{11} = 1$: 8871 CYs with h12 = 21,23-51,53,55,59,61,65,73,76,79,89,101,103,129
210 smooth: h12 = 25,28-41,45,47,51,53,55,59,61,65,73,76,79,89,101,103,129

$h_{11} = 2$: 43080 CYs with h12 = 22,24-80,82-90,96,100,102,103,111,112,116,128
3470 smooth: h12 = 26,28-60,62-68,70,72,74,76,77,78,80,82-84,86,88,90,96,100,102,112,116,128

$h_{11} = 3$: . . .



Picard number $h_{11} = 1$: 210 smooth CYs with 69 different topologies
intersection numbers (topological) vs. instanton numbers (symplectic)

Picard Fuchs operators: $\theta = t\,\frac{d}{dt}$

$$\theta^4 + \frac{2}{29}\ t\ \theta(24\theta^3 - 198\theta^2 - 128\theta - 29) - \frac{4}{841}\ t^2\ (44284\theta^4 + 1729544\theta^3 + 2485899\theta^2 + 1720577\theta + 47096)$$
$$-\frac{4}{841}\ t^3\ (525708\theta^4 + 24147772\theta^3 + 44476432\theta^2 + 38390499\theta + 1275594)$$
$$-\frac{8}{841}\ t^4\ (1415624\theta^4 + 7911004\theta^3 + 17395449\theta^2 + 17396359\theta + 6496262)$$
$$-\frac{16}{841}\ t^5\ (\theta+1)(2152040\theta^3 + 12186636\theta^2 + 24179373\theta + 16560506)$$
$$-\frac{32}{841}\ t^6\ (\theta+1)(\theta+2)(1912256\theta^2 + 9108540\theta + 11349571)$$
$$-\frac{10496}{841}\ t^7\ (\theta+1)(\theta+2)(\theta+3)(5671\theta + 16301) - \frac{24529152}{841}\ t^8\ (\theta+1)(\theta+2)(\theta+3)(\theta+4)$$

Torsion in (co)homology V. Batyrev & M.K. [math.AG/0505432]

- **Mirror symmetry** exchanges h_{21} **complex structure** and h_{11} **Kähler** moduli
- What about integral cohomology?
- Universal coefficient theorem

$$\mathrm{tor}(H_i(X,\mathbb{Z})) \cong \mathrm{tor}(H^{i+1}(X,\mathbb{Z}))^*$$

- Poincaré duality:

$$\mathrm{tor}(H_i(X,\mathbb{Z})) \cong \mathrm{tor}(H^{2d-i}(X,\mathbb{Z}))$$

- 3-folds $\Rightarrow$ two independent torsion groups:
  $\mathrm{tor}\, H_1(X,\mathbb{Z}) \cong \mathrm{tor}\, H^2(X,\mathbb{Z})^*$ (related to fundamental group)
  $\mathrm{tor}\, H_2(X,\mathbb{Z}) \cong \mathrm{tor}\, H^3(X,\mathbb{Z})^*$ (topological Brauer group)
- **conjecture:** exchanged under mirror symmetry
- verified for all 473 800 776 toric Calabi–Yau hypersurfaces: 16+16 cases with torsion

Torsion curves for the "Heterotic standard model"

with V. Braun, B. Ovrut and E. Scheidegger

A (3,3) paramerter example with $\pi_1 = \mathbb{Z}_3 \times \mathbb{Z}_3$

- Schoen: Fiber product of two elliptic fibers over $\mathbb{P}^1$
- $\mathbb{Z}_3$ phase (toric) $\times$ $\mathbb{Z}_3$ permutation (non-toric)
- Direct curve counting in A model limited to base-degree 1
- (permutation extension of) Batyrev–Borisov mirror
  $\Rightarrow$ B-model (MS) calculation of instantons sum
- **Surprise: Self-mirror** $\Rightarrow \mathbb{Z}_3 \times \mathbb{Z}_3$ torsion curves
- tools for computation of torsion curves (spectral sequences)
- Application: torsion curves cannot be holomorphic $\rightarrow$ SUSY breaking
  moduli stabilization (vs. Beasley–Witten): single curve in homology class!