Photon Magnetic Moment and Vacuum Magnetization in a Superstrong Magnetic Field

Selym Villalba Chávez

Institut für Physik-Theorie, Karl-Franzens Universität Graz, Austria.

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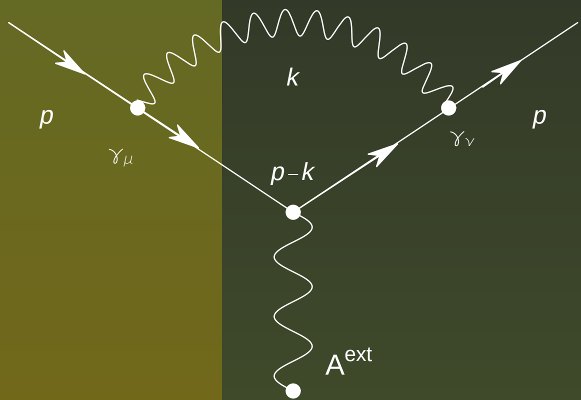
Introduction

Electron Magnetic Moment

Julian Schwinger 1951



$$\sim \frac{\alpha}{2\pi}\frac{e_0}{4m_0}\sigma^{\mu\nu}F^{ext}_{\mu\nu}$$

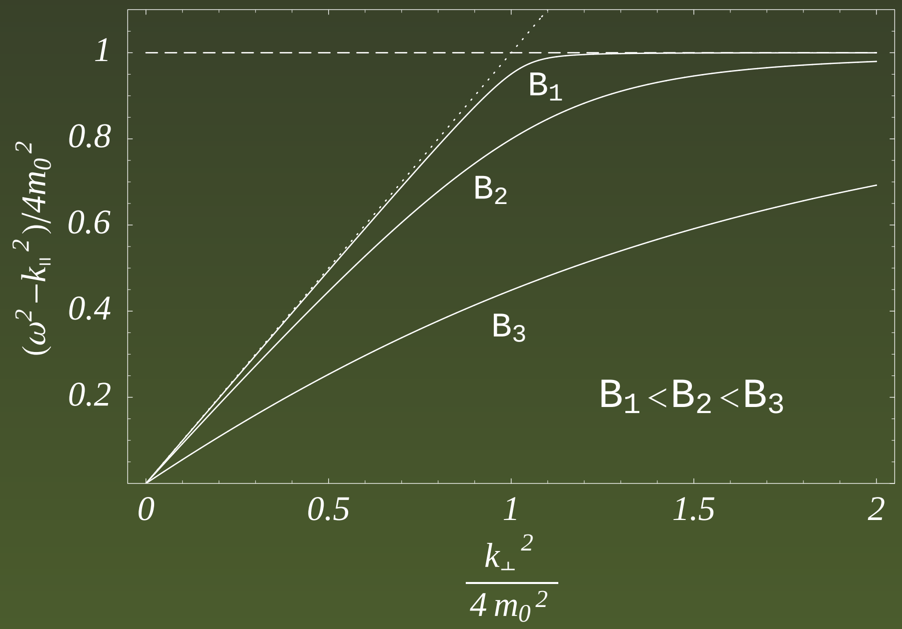
- $\delta\mu$ is defined at low energies as the coefficient of the linear term in the $\mathbf{B}-$expansion .
- The electron magnetic moment is given by

$$\vec{\mu} = \frac{e_0}{m_0}\mathbf{S}\left(1+\frac{\alpha}{2\pi}\right) \text{ with } \mathbf{S} = \frac{1}{2}\vec{\sigma}.$$

- It is a consequence of QED radiative corrections.

Motivation: Photon Dispersion Law

By considering the propagation of electromagnetic radiation, Shabad (1971) showed the drastic departure of the photon dispersion law from the light cone curve near the energy thresholds for free pair creation.

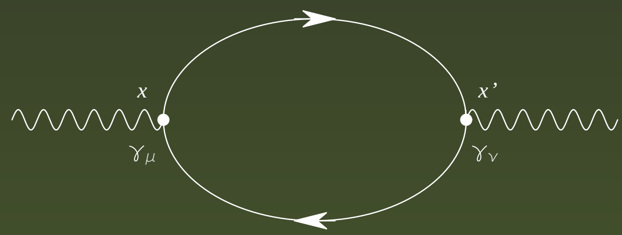


Vacuum Polarization.

The Schwinger-Dyson Equation

$$\partial^{\nu}\mathfrak{F}_{\mu\nu}(x) + \int d^4x'\Pi_{\mu\nu}(x,x'|\mathrm{B})\mathcal{A}^{\nu}(x') = 0$$

$$\Pi_{\mu\nu}(x,x'|\mathrm{B}) =$$



- The expression is the covariant form of the Maxwell equations in a neutral polarized medium due to the external field action.

Tensorial Structure of $\Pi_{\mu\nu}$

In momentum space

$$\left[k^2\eta_{\mu\nu} - \Pi_{\mu\nu}(k|\mathrm{B})\right]\mathcal{A}^\nu(k) = 0$$

The QED-symmetries allow to express

$$\Pi_{\mu\nu}(k|\mathrm{B}) = \sum_{i=1}^{4} \varkappa^{(i)} \frac{\mathrm{a}_\mu^{(i)}\mathrm{a}_\nu^{(i)}}{\left(\mathrm{a}_\nu^{(i)}\right)^2}$$

- $\mathrm{a}_\mu^{(1)} = (k^2F_{\mu\nu}F^{\nu\sigma}k_\sigma - k_\mu kF^2k)/(2\Delta)^{1/2}$.
- $\mathrm{a}_\mu^{(2)} = F^*_{\mu\nu}k^\nu/(2\Delta)^{1/2}$.
- $\mathrm{a}_\mu^{(3)} = F_{\mu\nu}k^\nu/(2\Delta)^{1/2}$ and $\mathrm{a}_\mu^{(4)} = k_\mu$.

Dispersion Equations

In this context

$$\mathfrak{D}_{\mu\nu} = \sum_{i=1}^{3} \frac{1}{\mathrm{k}^2 - \varkappa^{(i)}} \frac{\mathrm{a}_\mu^{(\mathrm{i})} \mathrm{a}_\nu^{(\mathrm{i})}}{\left(\mathrm{a}^{(\mathrm{i})}\right)^2} - \frac{\zeta}{\mathrm{k}^2} \frac{\mathrm{k}_\mu \mathrm{k}_\nu}{\mathrm{k}^2}$$

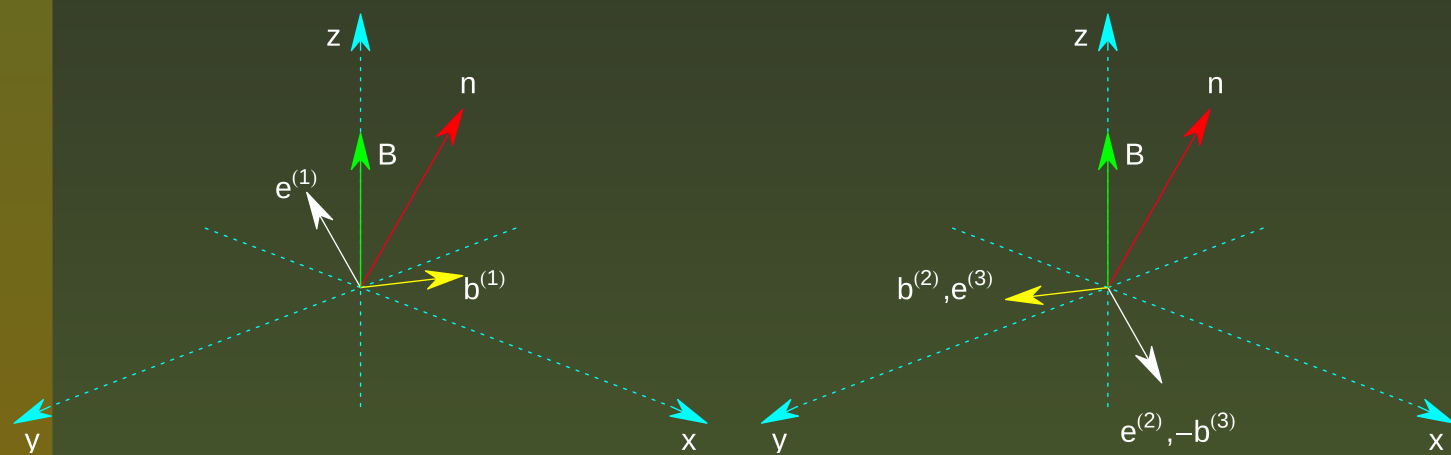
- The photon dispersion equations

$$k^2 = \varkappa^{(i)} \left(z_2, z_1, 2\Delta\right), \quad i = 1, 2, 3.$$

- Here $z_1 = kF^{*2}k/(2\Delta)$ and $z_2 = -kF^2k/(2\Delta)$.
- $\Pi_{\mu\nu}$ satisfies the gauge condition $\varkappa^{(i)}(0, 0, 2\Delta) = 0$.

Electric and Magnetic Fields

By considering $\mathrm{e}^{(i)} = \frac{\partial}{\partial x_0}\mathbf{a}^{(i)} - \frac{\partial}{\partial \mathbf{x}}\mathrm{a}_0^{(i)}$ and $\mathrm{b}^{(i)} = \nabla \times \mathbf{a}^{(i)}$



$$\mathbf{e}^{(1)} = -\mathbf{n}_\perp \omega, \;\; \mathbf{b}^{(1)} = \mathbf{n}_\perp \times \mathbf{k}_\parallel,$$

$$\mathbf{e}_\perp^{(2)} = \mathbf{k}_\perp k_\parallel, \;\; \mathbf{e}_\parallel^{(2)} = \mathbf{n}_\parallel (k_\parallel^2 - \omega^2), \;\; \mathbf{b}^{(2)} = -\omega(\mathbf{k}_\perp \times \mathbf{n}_\parallel)$$

$$\mathbf{e}^{(3)} = -\omega(\mathbf{n}_\perp \times \mathbf{n}_\parallel), \;\; \mathbf{b}_\perp^{(3)} = -\mathbf{n}_\perp k_\parallel; \;\; \mathbf{b}_\parallel^{(3)} = \mathbf{n}_\parallel k_\perp.$$

Photon Magnetic Moment

Infrared Domains

Photon

$$\varkappa_2 = -\frac{\alpha}{3\pi}\frac{\mathrm{e}}{\mathrm{m}^2}\mathrm{F}^*_{\mu\nu}\mathrm{k}^\mu \mathrm{a}^{\nu(2)},$$

Electron

$$\varepsilon = \frac{\mathrm{e}_0}{2\mathrm{m}_0^2}\mathrm{F}^*_{\mu\nu}\mathrm{p}^\mu \mathbf{S}^\nu$$

$$\mathrm{S}^\mu = \gamma^5\left(\gamma^\mu - \frac{\Pi^\mu}{\mathrm{m}}\right) \quad \text{with} \quad \Pi_\mu = \mathrm{p}_\mu + \frac{\mathrm{i}}{2}\mathrm{eF}_{\mu\nu}\mathrm{x}^\nu$$

- In the Photon case the electron spin operator is replaced by the polarization corresponding to the second mode.
- $\mathrm{a}^{\mu(2)}$ carries out the virtual spin polarization effects of $e^\pm$ pair.

IR-Dispersion Law

Solving the dispersion equation

$$\omega \simeq |\mathbf{k}| - \underbrace{\frac{1}{2}\frac{\alpha \mathrm{b}}{3\pi|\mathbf{k}|}\frac{\mathrm{kF^2k}}{2\Delta}}_{\mathrm{U}} \quad \text{with} \quad \mathrm{b} = \frac{\mathrm{B}}{\mathrm{B_c}}$$

Here $\mathrm{B_c} = 4{,}42 \cdot 10^{13}\mathrm{G}$ is the critical field.

- The second one involves the dipole moment contribution of the virtual $e^{\pm}$ pair.
- The latter is linear in the external field.

Photon Magnetic Moment

The interacting term can be written as

$$U = -\underbrace{\mathfrak{g}\frac{e}{2m}\left[\mathbf{n}_{\|}\sin\phi - \mathbf{n}_{\perp}\cos\phi\right]}_{\vec{\mu}_\gamma}\cdot\mathbf{B}$$

- $\mathfrak{g} = \frac{\alpha}{3\pi}\frac{k_\perp}{m_0}$ is a sort of Landé factor
- ϕ is the angle between the wave vector and the external field.

In presence of **B** the classical spin of the electromagnetic field is given by

$$\mathbf{s}_\gamma = \begin{cases} 0 & \text{for} \quad k_\perp = 0 \\ \mathbf{n}_{\|}\times\mathbf{n}_{\perp} & \text{for} \quad k_\perp \neq 0 \end{cases}$$

Properties

Having this into account

$$\vec{\mu}^{\gamma} = \mathfrak{g}\frac{\mathrm{e}}{2\mathrm{m}_0}\overbrace{\mathbf{n}\times\mathbf{s}_{\gamma}}^{\text{spin}}$$

- $\mu_{\gamma} \longrightarrow 0$ when $k_{\perp} \longrightarrow 0$.
- $\vec{\mu}_{\gamma} \cdot \mathbf{n} = 0$.
- $\mathbf{n} \times \mathbf{s}_{\gamma}$ arises due to vortices of the virtual $e^{\pm}$ pair.
- The transversal component is projected out by the scalar product $\vec{\mu}_{\gamma} \cdot \mathbf{B}$.

Precession

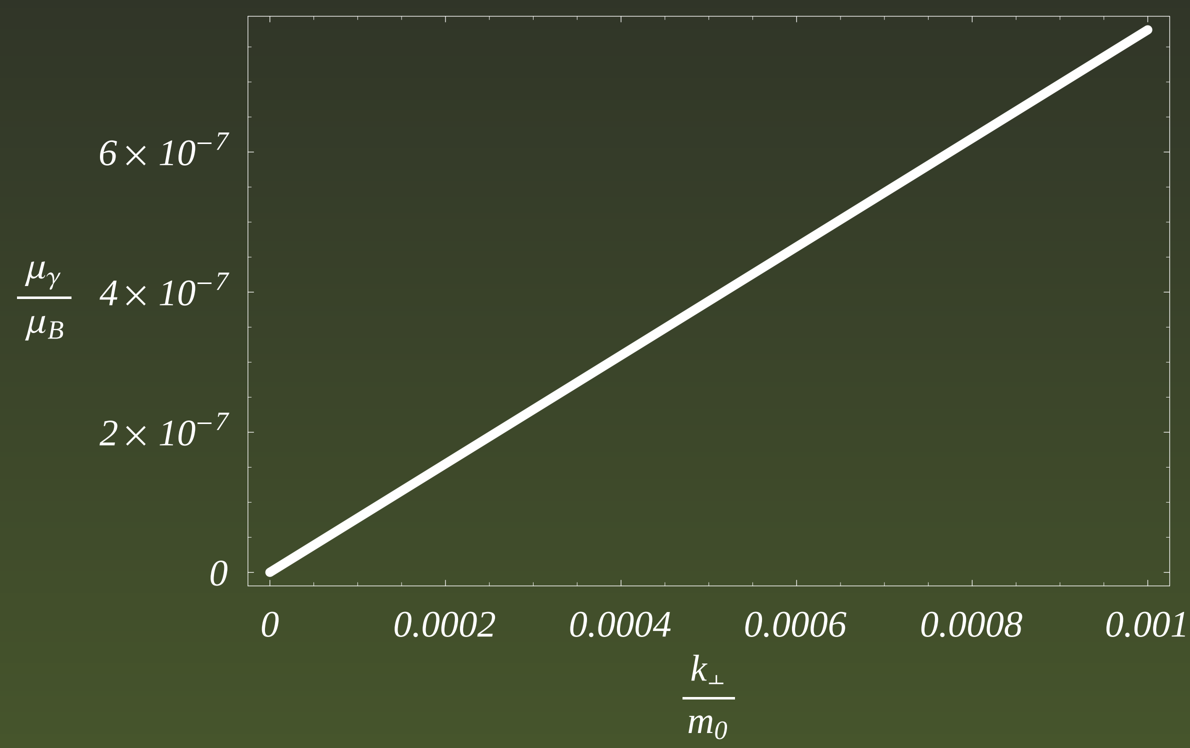
However the external field does exert a torque on the magnetic dipole which tends to line up $\vec{\mu}^{\gamma}$ with **B**.

$$\vec{\tau} = \vec{\mu}^{\gamma} \times \mathbf{B} = \mathfrak{g}\frac{\mathrm{e}}{\mathrm{m}}\cos\phi|\mathbf{B}|\mathbf{s}_{\gamma} = \mathbf{s}_{\gamma}\frac{\mathrm{d}\varphi}{\mathrm{dt}} \ \text{ with } \ 0 \le \varphi \le 2\pi.$$

From the above equation follows

$$\mathtt{w}_{\mathrm{prec.}} \equiv \frac{\mathrm{d}\varphi}{\mathrm{dt}} = \mathfrak{g}\frac{\mathrm{e}}{\mathrm{m}}|\mathbf{B}|\cos\phi.$$

Numerical Values

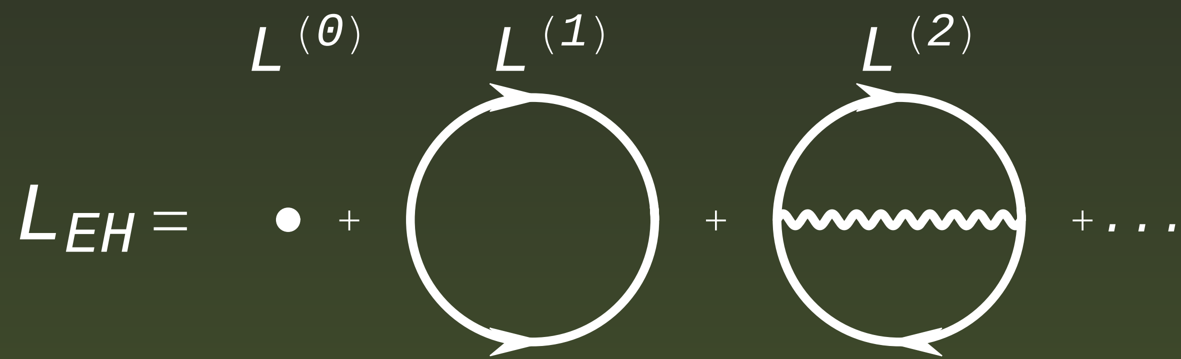


- The Bohr magneton $\mu_{\mathrm{B}} = \mathrm{e}/(2\mathrm{m}_0) = 9{,}274 \cdot 10^{-21}\mathrm{erg}/\mathrm{G}.$

Vacuum Magnetization

Diagrammatic Interpretation

Up to two loops the Euler-Heisenberg Lagrangian is represented by



- The first loop gives the contribution of the virtual free electron-positron pairs created and annihilated spontaneously in vacuum and interacting with the external field. The radiative corrections emerge from two-loops due to exchange of the virtual photons.

Two-loop Decomposition

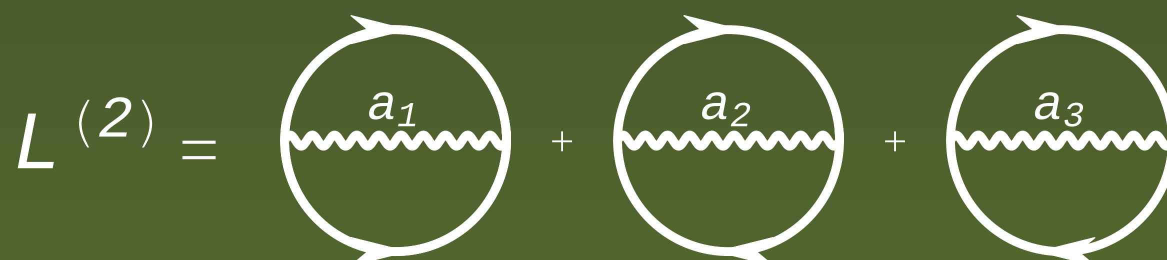
We can express

$$\mathfrak{L}^{(2)} = \frac{i}{2}\int \frac{\mathrm{d}^4\mathrm{k}}{(2\pi)^4}\Pi_{\mu\nu}(\mathrm{k})\mathfrak{D}_{\nu\mu}(\mathrm{k})$$

By considering the diagonal structure of $\Pi_{\mu\nu}$ we can write $\mathfrak{L}^{(2)} = \sum_{i=1}^{3} \mathfrak{L}_i^{(2)}$ with

$$\mathfrak{L}_i^{(2)} = \frac{i}{2}\int \frac{\mathrm{d}^4\mathrm{k}}{(2\pi)^4}\frac{\varkappa_i}{\mathrm{k}^2 - \varkappa_\mathrm{i}}.$$

which implies



Supercritical limit

For $\mathrm{B} \gg \mathrm{B_c}$ with $\mathrm{B_c} = \mathrm{m}^2/\mathrm{e} = 4{,}42 \cdot 10^{13}\mathrm{G}$ we have that

$$\mathfrak{L}^{(1)}_{\mathrm{R}}(\mathrm{b}) \simeq \frac{\mathrm{m}^4\mathrm{b}^2}{24\pi^2}\left\{\ln\left(\frac{\mathrm{b}}{\gamma\pi}\right) + \frac{6}{\pi^2}\zeta'(2)\right\}$$

where $\mathrm{b} = \mathrm{B}/\mathrm{B_c}$. The two-loop contribution read

$$\begin{aligned}
\mathfrak{L}^{(2)}_{1\mathrm{R}} &\approx \frac{\alpha\mathrm{m}^4\mathrm{b}^2}{16\pi^3}\mathcal{N}_1, \\
\mathfrak{L}^{(2)}_{2\mathrm{R}} &\approx \frac{\alpha\mathrm{m}^4\mathrm{b}^2}{32\pi^3}\left[\mathcal{N}_2\ln\left(\frac{\mathrm{b}}{\gamma\pi}\right) - \frac{1}{3}\ln^2\left(\frac{\mathrm{b}}{\gamma\pi}\right)\right], \\
\mathfrak{L}^{(3)}_{3\mathrm{R}} &\approx \frac{\alpha\mathrm{m}^4\mathrm{b}^2}{32\pi^3}\left[\mathcal{N}_3\ln\left(\frac{\mathrm{b}}{\gamma\pi}\right) + \frac{1}{3}\ln^2\left(\frac{\mathrm{b}}{\gamma\pi}\right)\right],
\end{aligned}$$

where $\mathcal{N}_1 = 0{,}09$, $\mathcal{N}_2 = \frac{1}{3} - \frac{4\zeta'(2)}{\pi^2}$ and $\mathcal{N}_3 = \frac{2}{3} + \frac{4\zeta'(2)}{\pi^2}$.

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Vacuum Energy

The effective potential of $\mathfrak{L}_{\rm EH}$ defines

$$\mathcal{E}_{\rm vac} = -\mathfrak{L}_{\rm R}^{(1)} - \sum_{i=1}^{3} \mathfrak{L}_{i\rm R}^{(2)} + \ldots$$

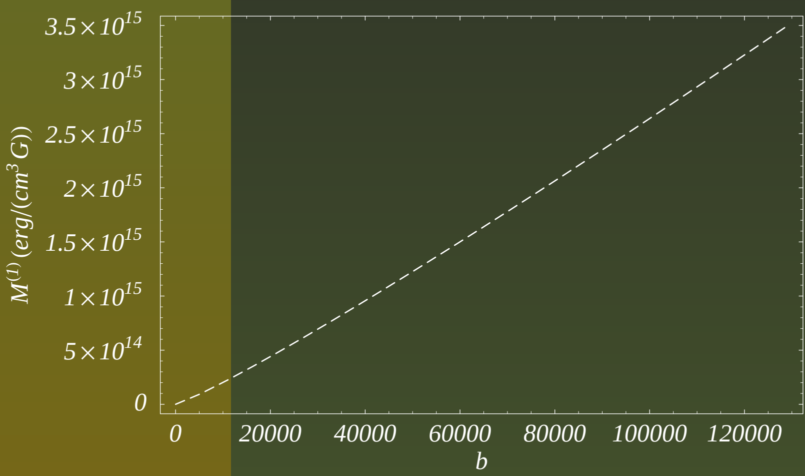
- The vacuum acquires a non trivial magnetization

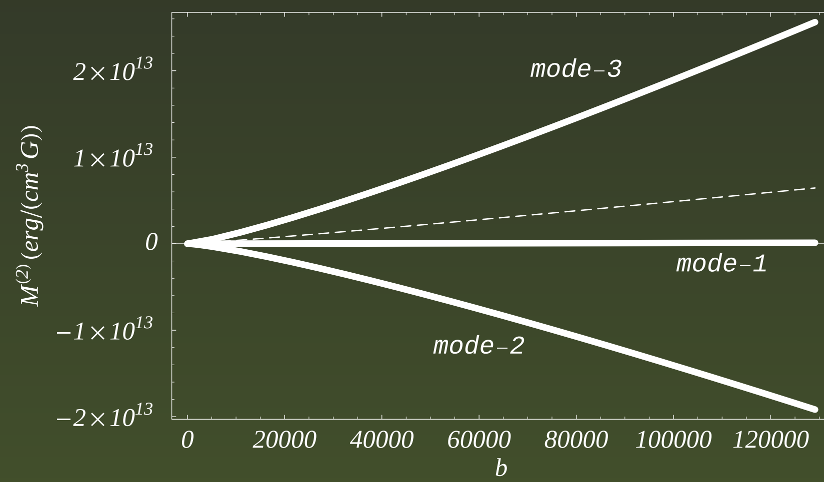
$$\mathrm{M}_{\rm vac} = -\frac{\partial \mathcal{E}_{\rm vac}}{\partial \mathbf{B}} = \mathrm{M}^{(1)} + \sum_{i=1}^{3} \mathrm{M}_i^{(2)}.$$

where

$$\mathrm{M}^{(1)} = \frac{\partial \mathfrak{L}_{\rm R}^{(1)}}{\partial \mathrm{B}} \approx \frac{\mathrm{m}^4 \mathrm{b}}{24\pi^2 \mathrm{B}_c} \left[2 \ln \left(\frac{\mathrm{b}}{\gamma \pi} \right) + 1 + \frac{12\zeta'(2)}{\pi^2} \right].$$

Numerical evaluation





The complete two-loop contribution

$$M^{(2)} \approx \frac{\alpha m^4 b}{32\pi^3 B_c}\left[1 + 2\ln\left(\frac{b}{\gamma\pi}\right)\right] > 0.$$

Conclusions

Conclusions

1. An infrared photon propagating in an strong magnetized vacuum presents a nonzero vector anomalous magnetic moment perpendicular to the wave vector which precesses around **B**.
2. This magnitude is due to the interaction between virtual electron positron quantum pairs with the external magnetic field.
3. The vacuum polarization tensor modifies the zero-point energy of the vacuum and therefore generate a nontrivial magnetization.

The end