Hadron Spectroscopy in Lattice Quantumchromodynamics

Georg Engel

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Karl–Franzens–Universität Graz

Advisor: Christian B. Lang

Co-workers: M. Limmer and D. Mohler

Contents

Lattice QCD Introduction

Strong Interaction - QCD

Candidate theory for strong interaction (well established):
Quantumchromodynamics (QCD)

- Quarks (Fermions): up, down, strange, charm, bottom and top represented by the field $\psi^{(f)}(x)$
- Gluons (Bosons):
  represented by the field $A_\mu(x)$ ($U_\mu(x)$ on the lattice)

- Quarks and gluons confined to hadrons.
- All hadrons should be predicted by QCD.
- This talk shows prediction of QCD for some hadron masses.

Continuum Theory

Contents

QCD in functional integral representation:
All possible field configurations contribute to expectation values with a weight factor $e^{-S(\psi,\overline{\psi},A)}$.

$$
\begin{aligned}
\langle O(\psi,\overline{\psi},A)\rangle &= \langle\langle O(\psi,\overline{\psi},A)\rangle_F\rangle_G \\
&= \frac{\int \mathcal{D}[A]\mathcal{D}[\overline{\psi},\psi]\, e^{-S(\psi,\overline{\psi},A)} O(\psi,\overline{\psi},A)}{\int \mathcal{D}[A]\mathcal{D}[\overline{\psi},\psi]\, e^{-S(\psi,\overline{\psi},A)}}
\end{aligned}
$$

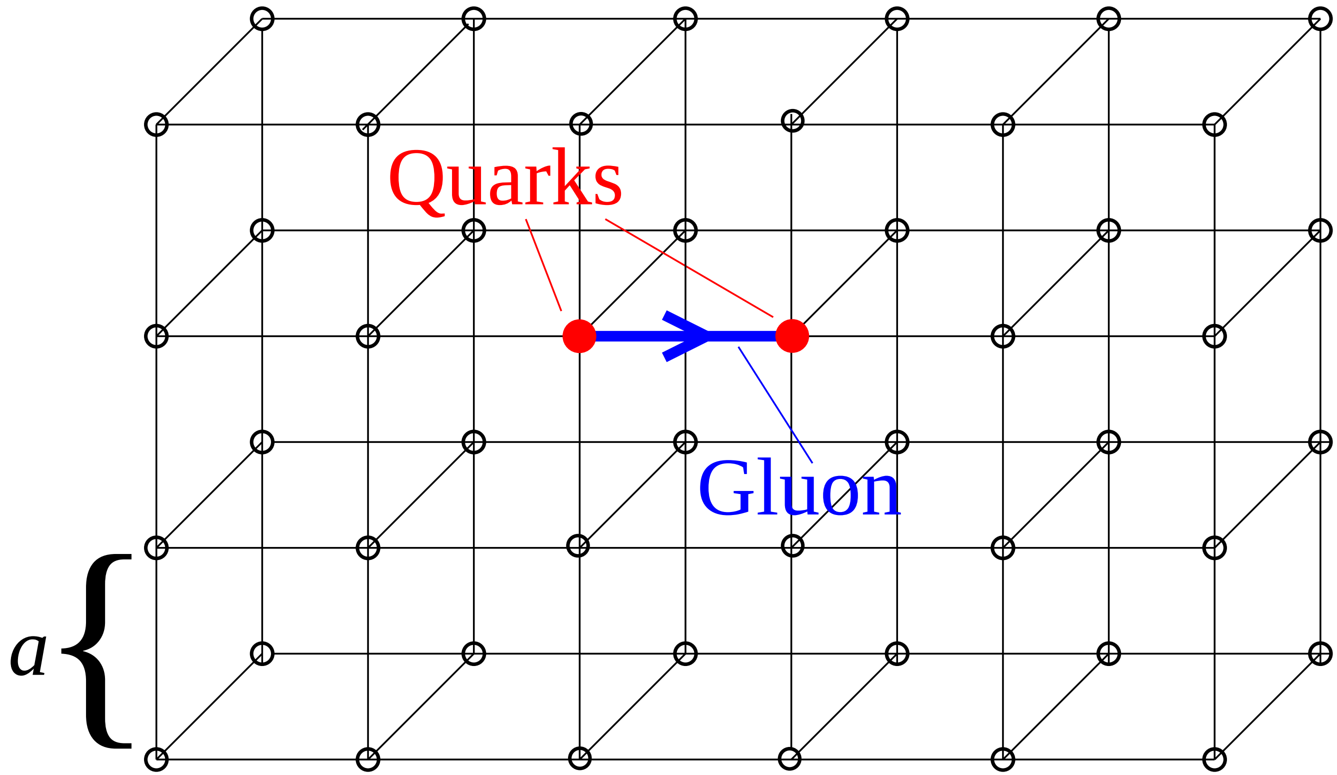
$$
\begin{aligned}
S &= \int d^4x \sum_{f=1}^{N_f} \overline{\psi}^{(f)}(x)\left[\gamma_\mu D_\mu + m^{(f)}\right]\psi^{(f)}(x) \\
&\quad + \frac{1}{2g^2}\,\mathrm{Tr}\left[F_{\mu\nu}(x)F_{\mu\nu}(x)\right]
\end{aligned}
$$

Regularization: The Lattice

Contents

- One possible regularization:
  Introduce **momentum ultraviolet-cutoff**, equivalent to a minimum distance ($\longrightarrow$ spacetime grid).
- If we continue to require **local gauge symmetry**, we obtain **Lattice QCD**.
- Lattice spacing $a \longrightarrow 0$ (critical point)
- Discretization of integration and differentiation:
  $\int dx \longrightarrow \Sigma_x \ , \quad \partial_\mu \longrightarrow \frac{1}{a}\left(\delta_{n+\hat{\mu},m} - \delta_{n,m}\right)$

Treatment on the Lattice

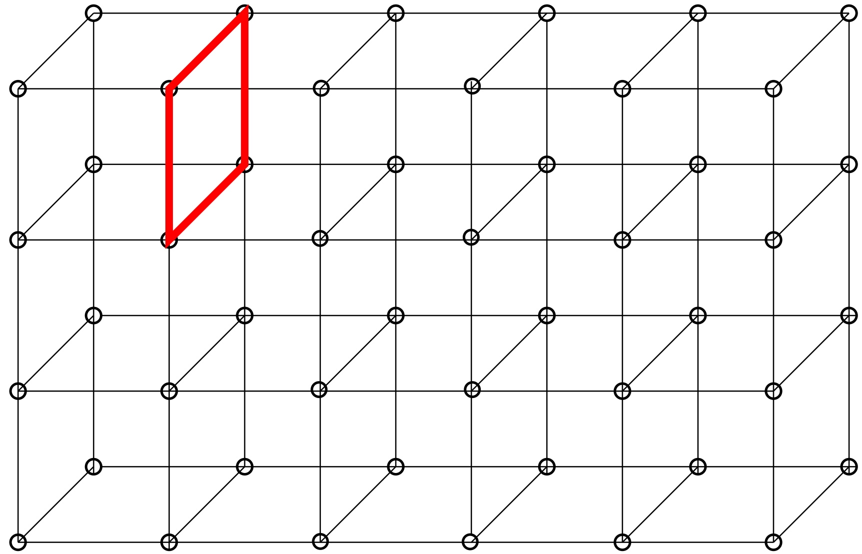


Quarks $\sim \overline{\psi}(n), \psi(n)$

Gluons $\sim$ Links $\sim$ Gauge Transporter $\sim U_\mu(n)$

Lattice Gauge Action

Wilson gauge action: Plaquettes



$$U_{\mu\nu}(n) = U_\mu(n)U_\nu(n+\hat{\mu})U_\mu(n+\hat{\nu})^\dagger U_\nu(n)^\dagger$$

$$S_g = \frac{\beta}{3}\sum_n \sum_{\mu<\nu} \mathrm{Re}\,\mathrm{Tr}\left[1 - U_{\mu\nu}(n)\right]$$

Lattice Fermion Action

- Fermionic action $S_F = a^4 \sum_f \overline{\psi}^{(f)}(n)\, D^{(f)}(n,m)\, \psi^{(f)}(m)$
- Naive Dirac operator shows unwanted **doublers**: 16 instead of 1 fermion.
- Removing of doublers: **Wilson Dirac operator** D_W

$$D_W^{(f)}(n,m) = \left(m^{(f)} + \frac{4}{a}\right) \delta_{n,m} - \frac{1}{2a} \sum_{\mu=\pm 1}^{\pm 4} \left(1-\gamma_\mu\right) U_\mu(n)\, \delta_{n+\hat{\mu},m}$$

- Construct operator with **quantum numbers of a physical state**.
- Example: Pion interpolator

$$O_\pi \quad = \quad \overline{u}\, \gamma_5\, d$$

- Extract **hadron masses** from the exponential behavior of the correlation function of the operator.

$$\begin{aligned} \langle O(n_t)\bar{O}(0)\rangle &= \frac{1}{Z}\int \mathcal{D}[U]\mathcal{D}[\overline{\psi},\psi]\, e^{-S(\psi,\overline{\psi},U)}\, O(n_t)\bar{O}(0) \\ &= \sum_n A_n e^{-tE_n} \end{aligned}$$

Writing out the pion correlator yields:

$$\begin{aligned}\langle O_\pi(n_t)\bar{O}_\pi(0)\rangle &= \frac{1}{Z}\int \mathcal{D}[U]\mathcal{D}[\overline{\psi},\psi]e^{-S_G(U)} \\ &\quad e^{-S_F(\psi,\overline{\psi},U)}O_\pi(n_t)\bar{O}_\pi(0) \\ &= \frac{1}{Z}\int \mathcal{D}[U]e^{-S_G(U)}det[D(U)] \\ &\quad tr\left[D_u^{-1}(U;0,n_t)\gamma_5 D_d^{-1}(U;n_t,0)\gamma_5\right]\end{aligned}$$

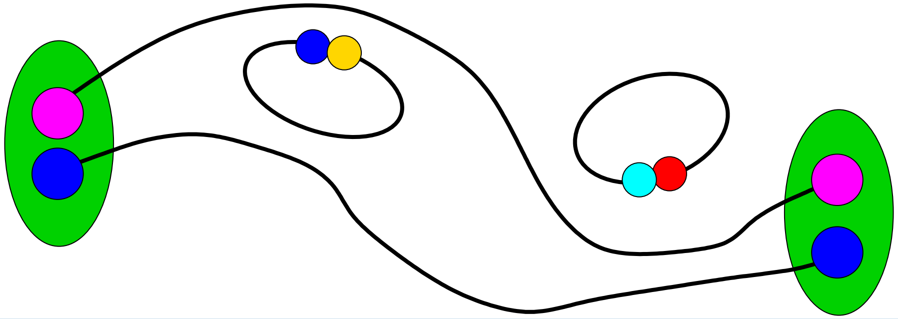
- Fermions integrated out analytically.
- Monte Carlo simulation using importance sampling with $e^{-S_G(U)}det[D(U)]$

Hadron Spectroscopy - Details

Writing out the pion correlator yields:

$$
\begin{aligned}
\langle O_\pi(n_t)\bar{O}_\pi(0)\rangle &= \frac{1}{Z}\int \mathcal{D}[U]\mathcal{D}[\overline{\psi},\psi]e^{-S_G(U)} \\
&\quad e^{-S_F(\psi,\overline{\psi},U)}O_\pi(n_t)\bar{O}_\pi(0) \\
&= \frac{1}{Z}\int \mathcal{D}[U]e^{-S_G(U)}det[D(U)] \\
&\quad tr\left[D_u^{-1}(U;0,n_t)\gamma_5 D_d^{-1}(U;n_t,0)\gamma_5\right]
\end{aligned}
$$

- Fermions integrated out analytically.
- Monte Carlo simulation using importance sampling with $e^{-S_G(U)}det[D(U)]$



Variational Method

- Construction of operators with physical quantum numbers not unique.
- Freedom of choice e.g. in dirac structure and space distribution.
- Example: Pion operator

$$O_1 = \overline{u}\,\gamma_5\,d$$
$$O_2 = \overline{u}\,\gamma_5\gamma_t\,d$$

- Variational method: a **matrix of correlators**, built from several operators with same quantum numbers.

Variational Method II

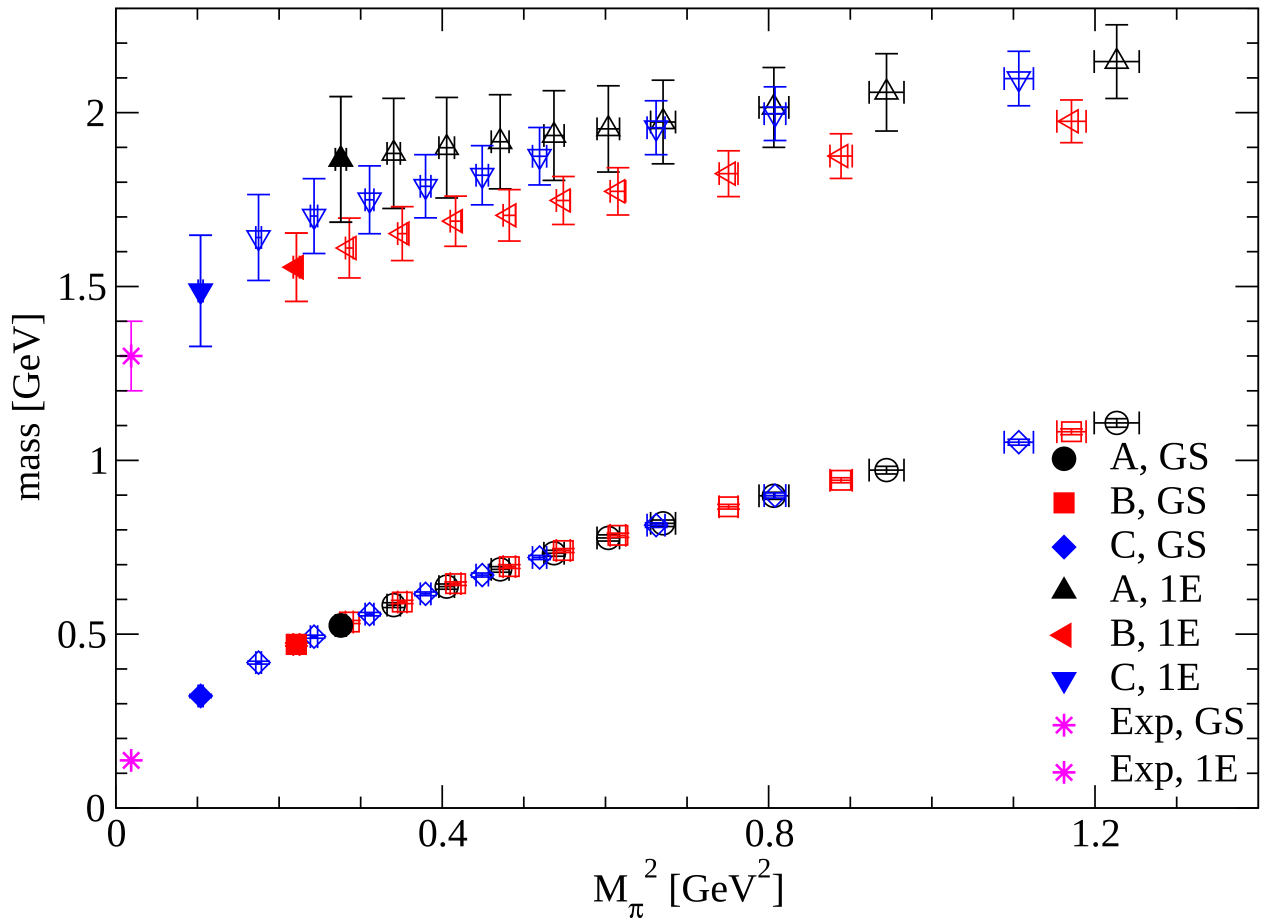
- Consider the generalized eigenvalue problem of the matrix of correlators

$$
\begin{aligned}
C_{ij}(t) &\equiv \sum_n \langle 0|O_i|n\rangle\langle n|O_j^\dagger|0\rangle \\
C(t)\,\vec{v}_k &= \lambda_k(t)\,C(t_0)\,\vec{v}_k \\
\lambda_k(t,t_0) &\propto e^{-t\,m_k}\left(1+\mathcal{O}(e^{-t\,\Delta m_k})\right)
\end{aligned}
$$

- Each **eigenvalue** is related only to a **single mass** at large time separations.
- Corresponding eigenvectors characterize the states.
- A good basis of different interpolators is crucial.

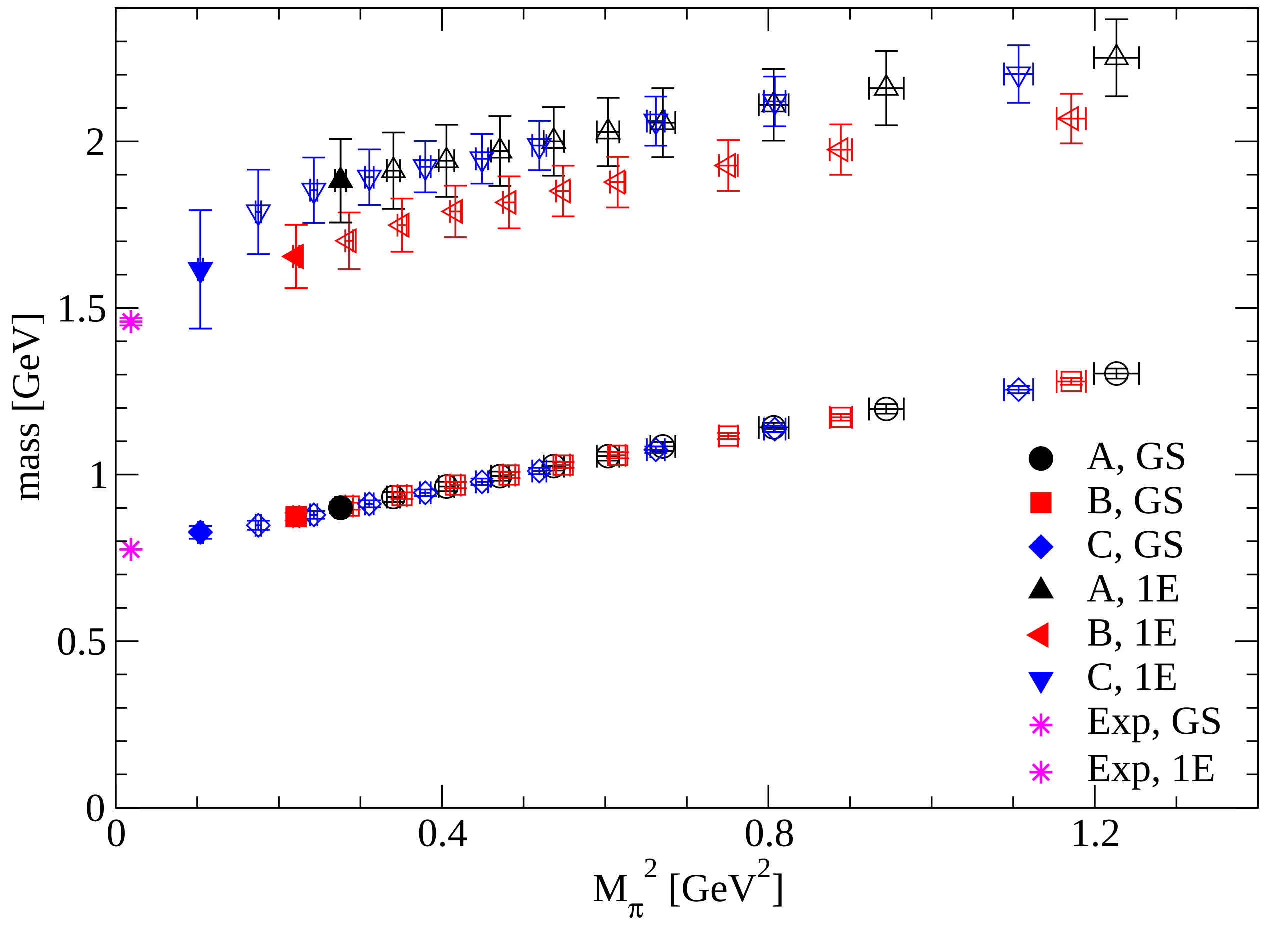
Results Hadron Spectroscopy

Results: Pion (0^{-+})



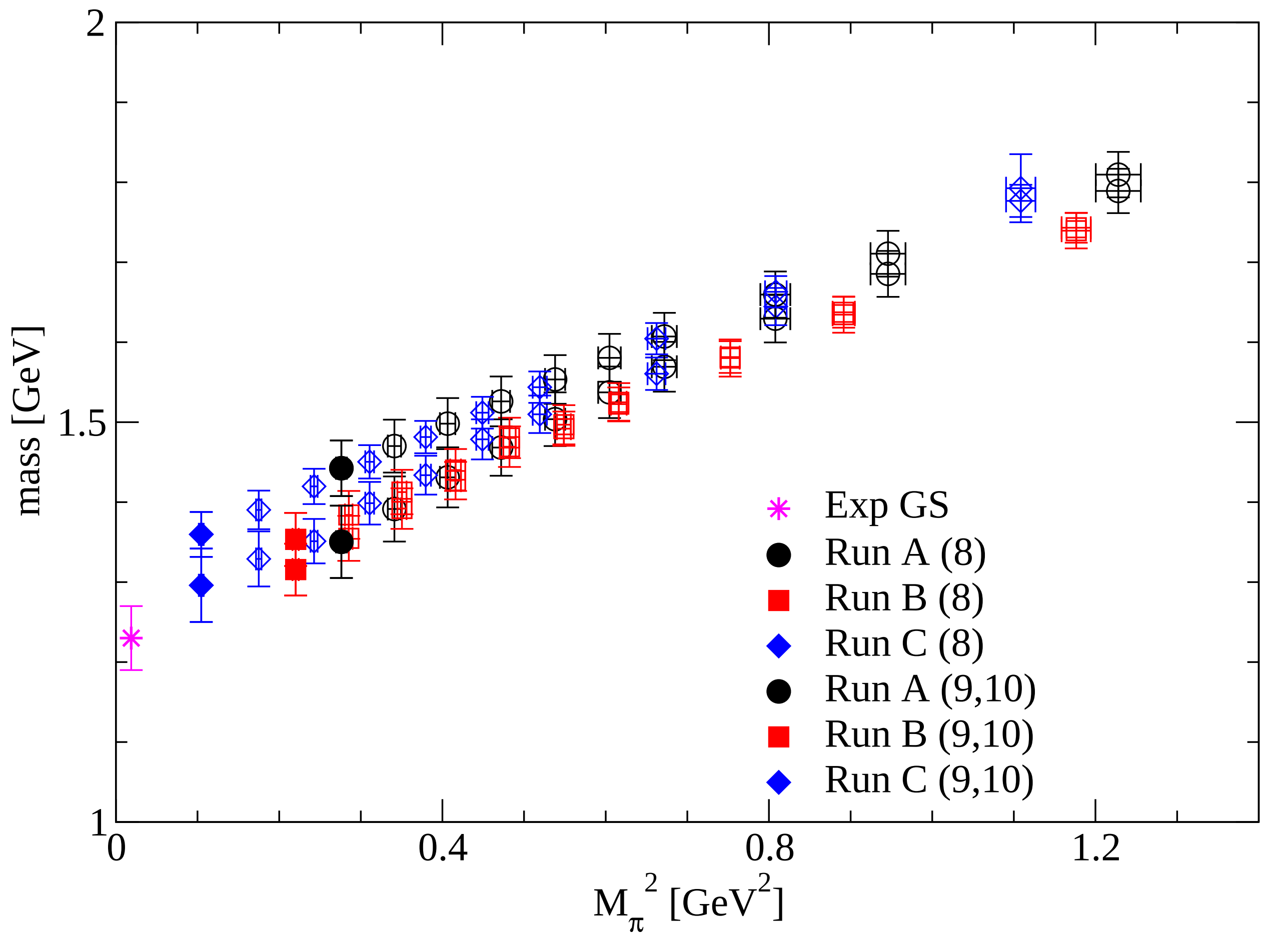
Ground state shown vs. itself. A signal of the first excited state seen.

Results: Rho (1^{--})



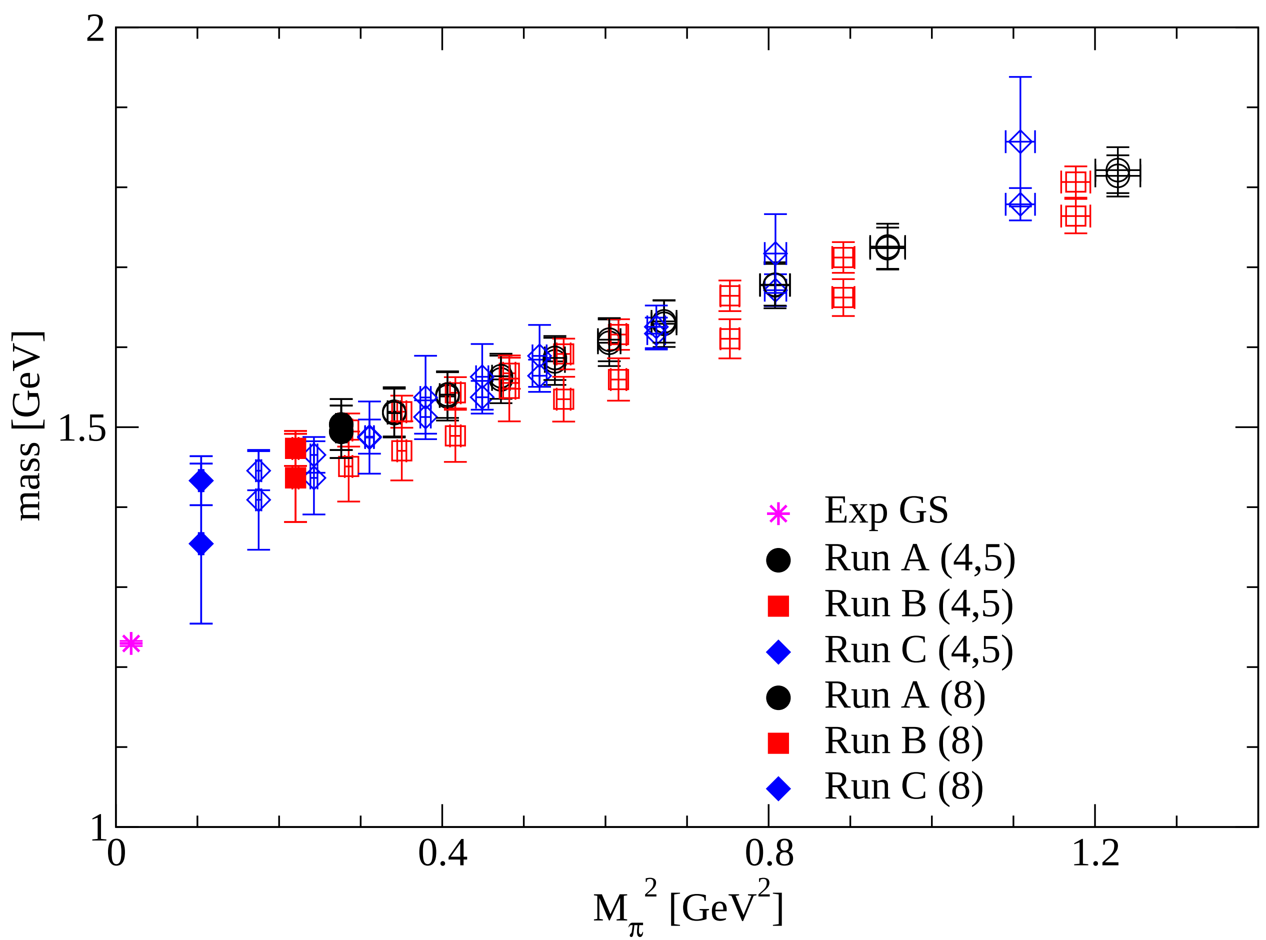
Ground state with very small error bar. First excited state signal found.

Results: a_1 (1^{++})



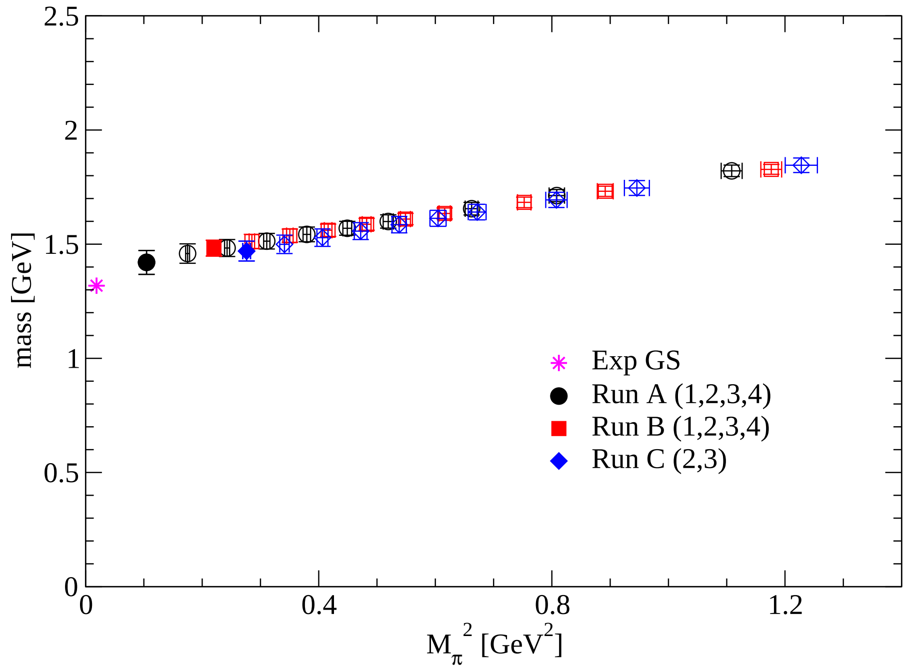
Systematic uncertainty of choosing the interpolators, is expected to shrink when using higher statistics.

Results: b_1 (1^{+-})

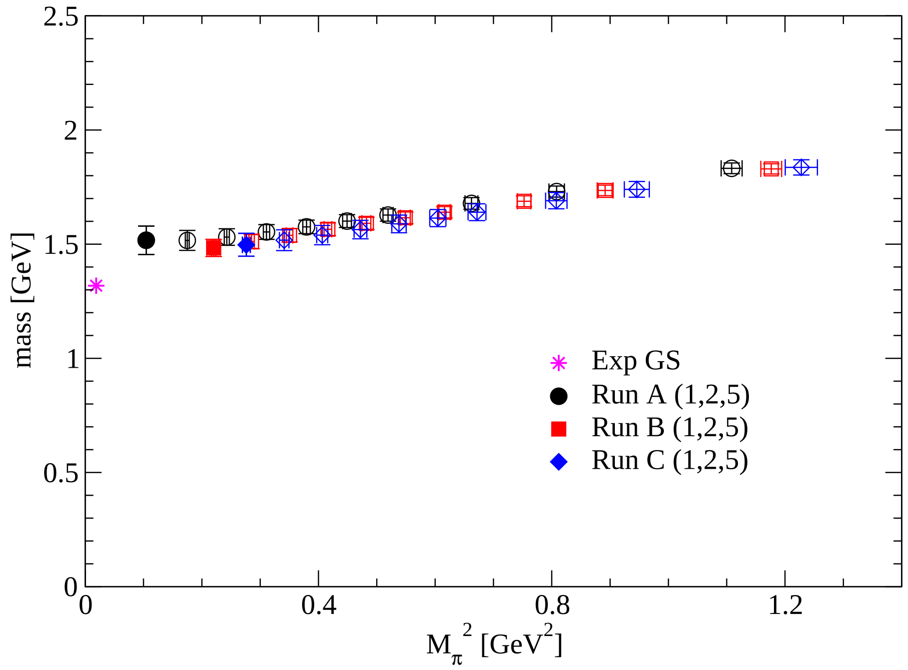


Again, higher statistics may remove the systematic uncertainty.

Results: a_2 (2^{++})



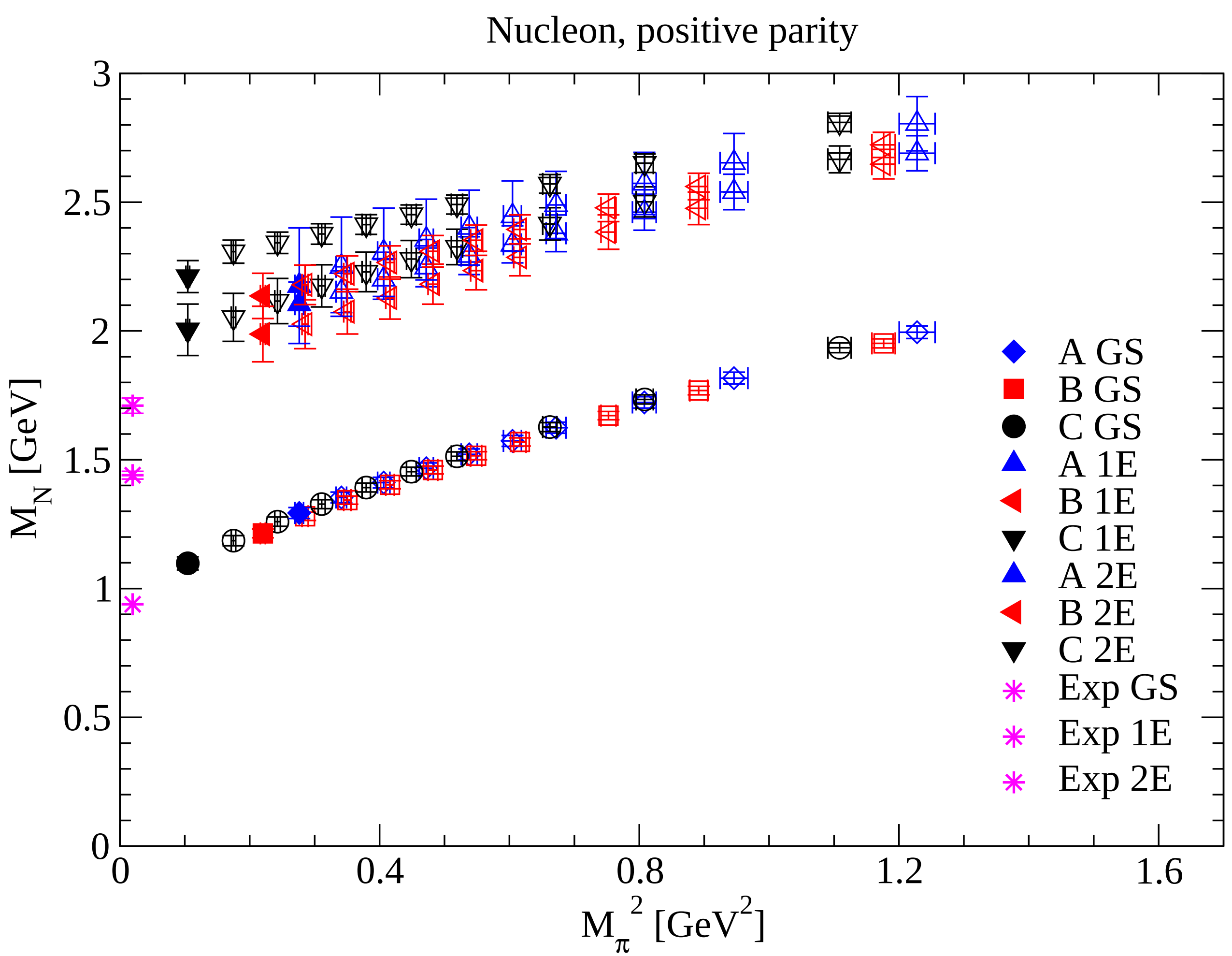
(a) Representation $\mathbf{T}_2$



(b) Representation $\mathbf{E}$

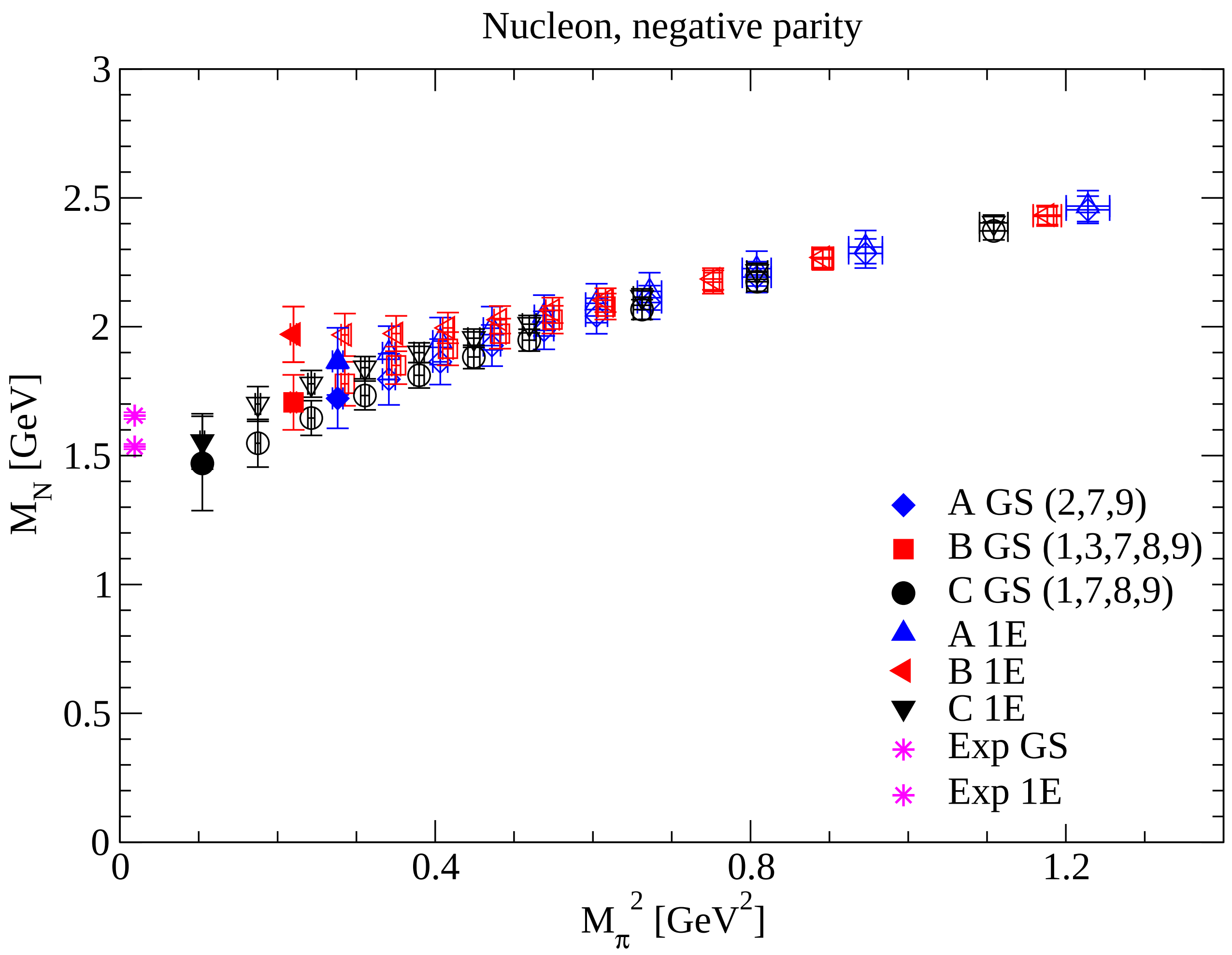
Reasonable agreement between different lattice spin representations found.

Results: Nucleon pos. parity $1/2(1/2^+)$



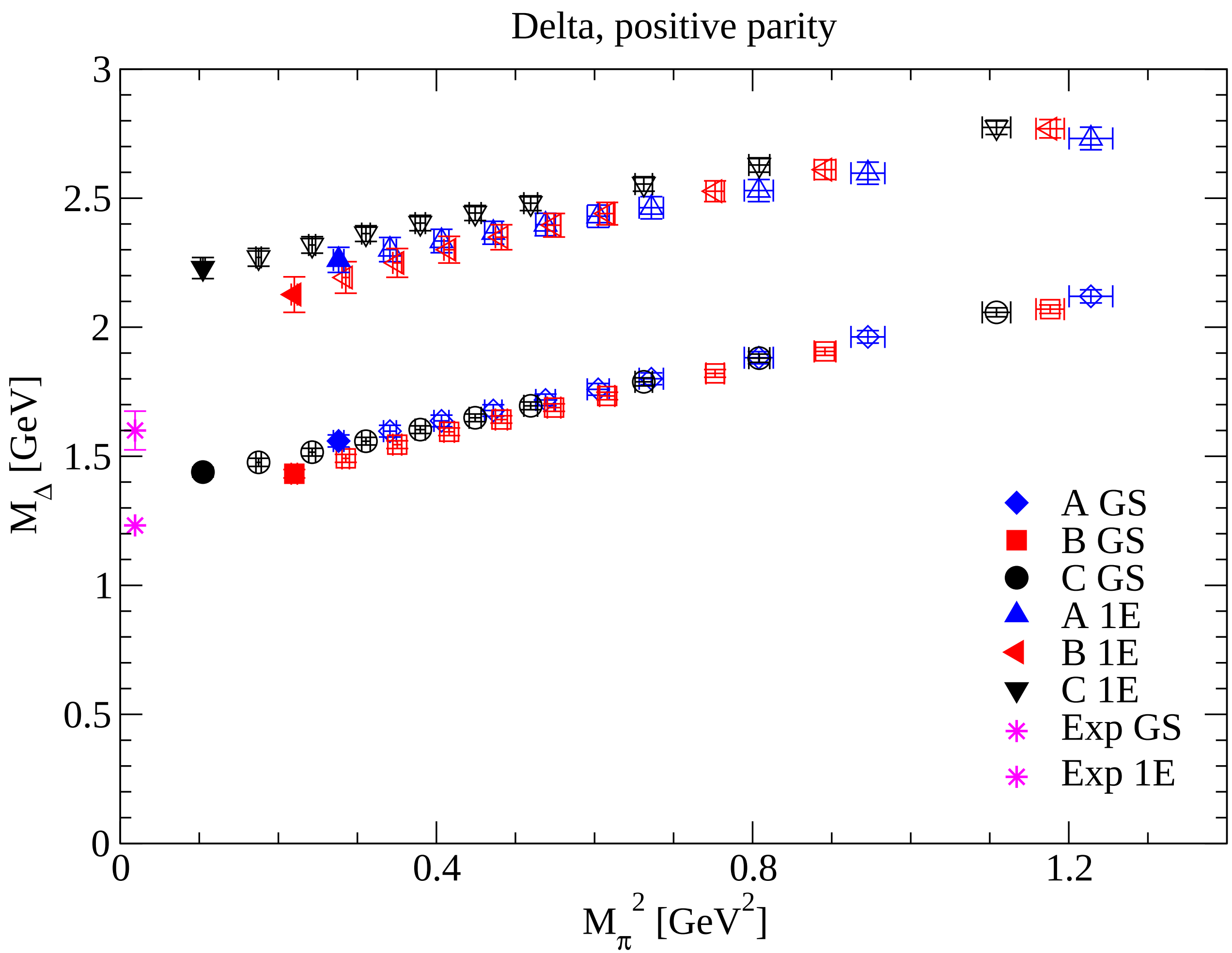
Ground state comes out nicely, excited states are too high.
Identification of Roper would need smaller pion masses.

Results: Nucleon neg. parity $1/2(1/2^-)$



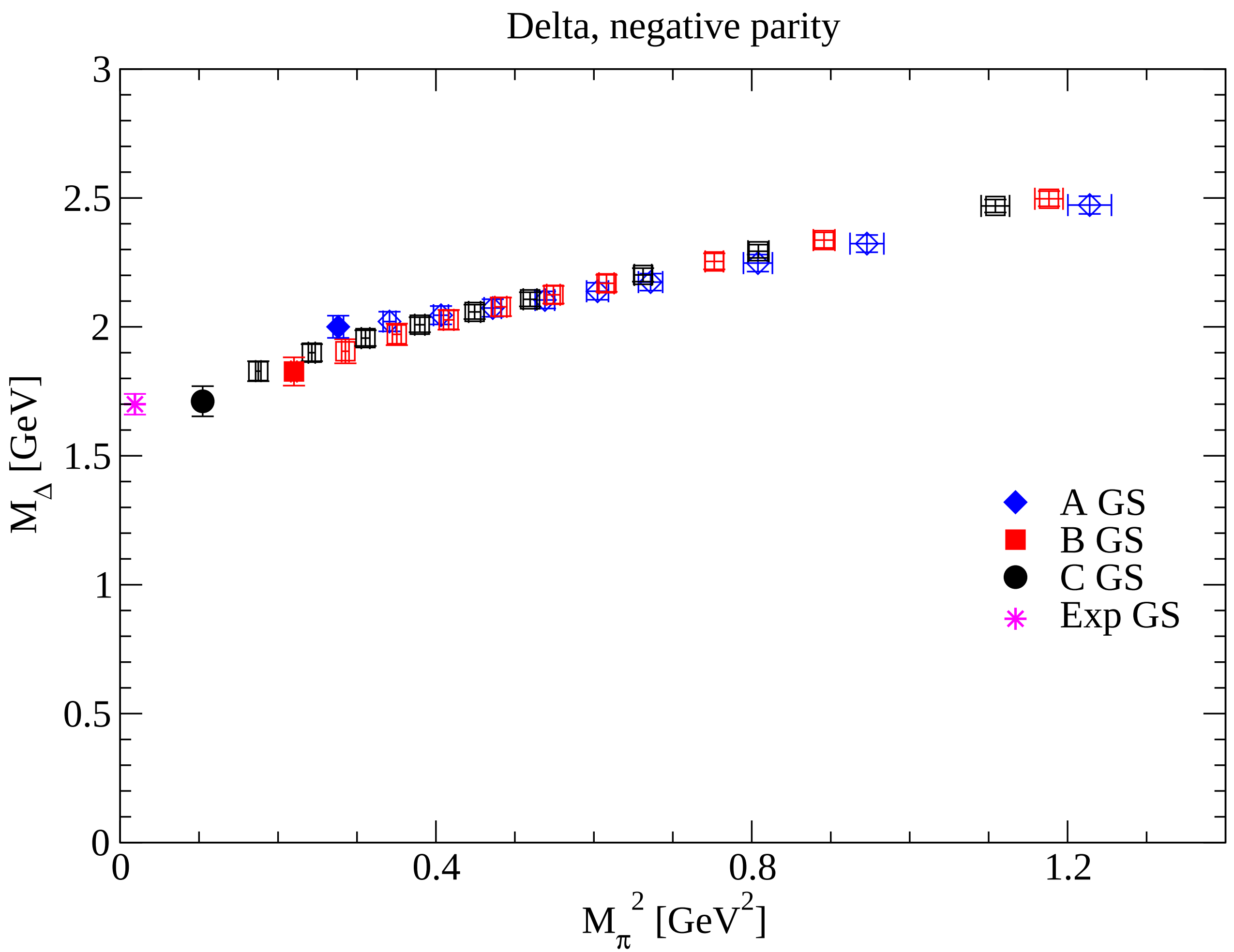
Ground and first excited state are very close together, which means a difficulty in resolving them.

Results: Delta pos. parity $3/2(3/2^+)$



Ground and first excited state clearly seen, but too high, maybe due to finite volume effects.

Results: Delta neg. parity $3/2(3/2^-)$



Ground state comes out nicely.

Conclusion and Outlook

Conclusion

- Lattice QCD is used for ab-initio calculation of hadron masses.
- Ground states of several hadrons predicted with high accuracy.
- Excited states of pion, rho, nucleon and delta found.
- Agreement between different lattice spin representations in the a_2 $(\mathbf{2}^{++})$ channel verified.
- Results of strange hadrons will be published soon.
- In future we plan to calculate additional phase points and to perform the chiral extrapolation to the physical pion mass.

Thank You!!