Beyond Mean Field in the Polyakov Quark Meson Model

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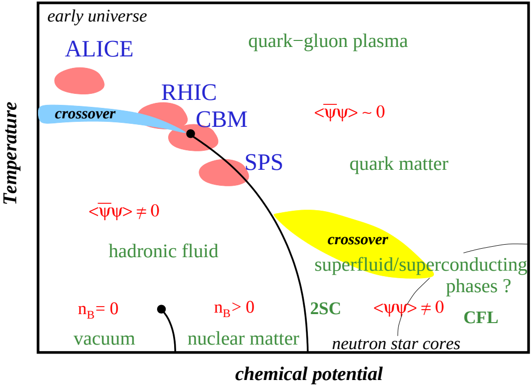
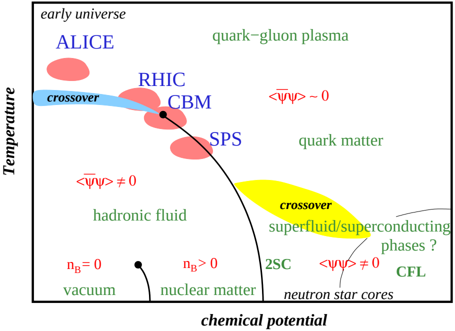
Overview

1. Motivation
2. Effective Model
3. Analytic Results
4. Numerical Results
5. Summary & Outlook



Chiral Symmetry

$$SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_{L+R}$$



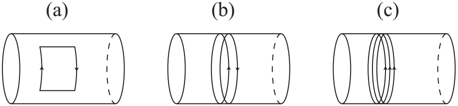
- well-defined for $m_q \to 0$
- order parameter:
  chiral condensate $\langle \bar{\psi}\psi \rangle$

$$\langle \bar{\psi}\psi \rangle \begin{cases} = 0 & \text{symmetric} \\ \neq 0 & \text{broken} \end{cases}$$

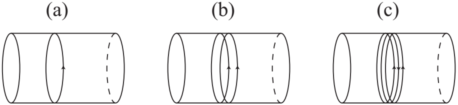
Center Symmetry $Z(N_c)$

$\Phi = e^{-\beta F_q}$ with $\beta = 1/T$, $F_q \ldots$ free energy of a static colour source

center symmetry respected:



center symmetry broken:



- well-defined for $m_q \to \infty$
- order parameter:
  Polyakov Loop Φ

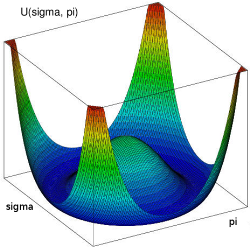
$$\Phi \begin{cases} = 0 & \text{confined phase} \\ \neq 0 & \text{deconfined phase} \end{cases}$$

[illustration: Fukushima '02]

Effective Models

Properties

- effective model for QCD
  - same universality class
  - same critical physics
- renormalizable
- chiral aspects
  - explicit & spontaneous symmetry breaking
  - e.g. Nambu–Jona-Lasinio (NJL), Quark-Meson (QM) models



- confinement
  - gluonic effects via Polyakov Loop
  - e.g. PNJL, **PQM model**

Polyakov-Quark-Meson Model (PQM)

$$\mathcal{L}_{PQM} = \bar{\psi}\left(\not{D} + ig(\sigma + i\gamma_5\vec{\tau}\vec{\pi})\right)\psi + U(\sigma,\pi) + \mathcal{U}(\Phi,\bar{\Phi})$$

- $(\sigma, \vec{\pi})\ldots$ $O(4)$-representation of the meson fields ($N_f = 2$)
- $g\ldots$ Yukawa coupling
- $D^\mu = \partial^\mu - (iA^\mu - \mu)\delta^4_\mu$
- $U(\sigma,\vec{\pi})\ldots$ meson potential

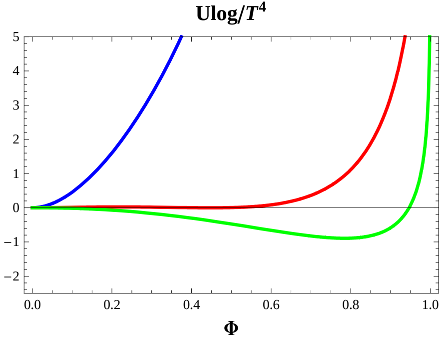
$$U(\sigma,\vec{\pi}) = \frac{1}{2}m^2(\sigma^2 + \vec{\pi}^2) + \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2)^2 - c\sigma$$

Polyakov Loop Potential

[Roessner, Ratti, Weise '06]

Logarithmic

$$\frac{\mathcal{U}(\Phi,\bar{\Phi})}{T^4} = -\frac{a(T)}{2}\Phi\bar{\Phi} + b(T)\log[1-6\Phi\bar{\Phi}-3(\Phi\bar{\Phi})^2+4(\Phi^3+\bar{\Phi}^3)]$$



$\mu = 0$

- blue ... $0.5\,T_0$
- red ... T_0
- green ... $1.5\,T_0$

$a(T) = a_0 + a_1 \frac{T_0}{T} + a_2 \left(\frac{T_0}{T}\right)^2$

$b(T) = b_3 \left(\frac{T_0}{T}\right)^3$

$a_{0,1,2}$, b_3 fitted to lattice data

Mean Field Approximation

Grand Potential

$$\Omega(\langle\sigma\rangle, \langle\Phi\rangle, \langle\bar{\Phi}\rangle) = \Omega_{\bar{q}q}(\langle\sigma\rangle, \langle\Phi\rangle, \langle\bar{\Phi}\rangle) + U(\langle\sigma\rangle) + \mathcal{U}(\langle\Phi\rangle, \langle\bar{\Phi}\rangle)$$

$$\begin{aligned}\Omega_{\bar{q}q} &= -2TN_f \int \frac{d^3p}{(2\pi)^3} \\ &\quad \Big\{\log\left[1 + 3\langle\bar{\Phi}\rangle e^{-\beta(E_q+\mu)} + 3\langle\Phi\rangle e^{-2\beta(E_q+\mu)} + e^{-3\beta(E_q+\mu)}\right] \\ &\quad +\log\left[1 + 3\langle\Phi\rangle e^{-\beta(E_q-\mu)} + 3\langle\bar{\Phi}\rangle e^{-2\beta(E_q-\mu)} + e^{-3\beta(E_q-\mu)}\right]\Big\}\end{aligned}$$

with $E_q = \sqrt{p^2 + m_q^2}$ where $m_q = g\langle\sigma\rangle$

Mean Field Approximation

Grand Potential

$$\Omega(\langle\sigma\rangle, \langle\Phi\rangle, \langle\bar{\Phi}\rangle) = \Omega_{\bar{q}q}(\langle\sigma\rangle, \langle\Phi\rangle, \langle\bar{\Phi}\rangle) + U(\langle\sigma\rangle) + \mathcal{U}(\langle\Phi\rangle, \langle\bar{\Phi}\rangle)$$

$$\Omega_{\bar{q}q} = -2TN_f \int \frac{d^3p}{(2\pi)^3}$$

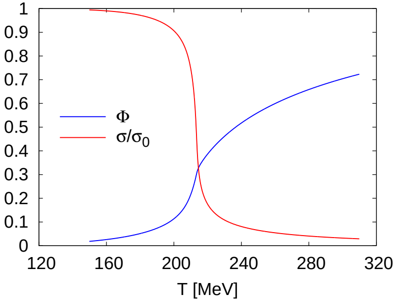
$$3\left\{\log\left[1 + e^{-\beta(E_q+\mu)}\right] + \log\left[1 + e^{-\beta(E_q-\mu)}\right]\right\}$$

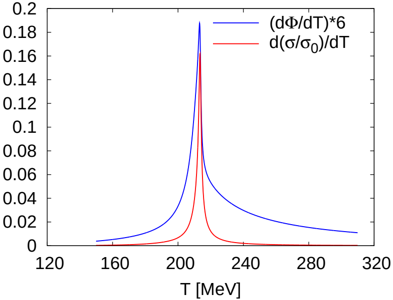
for $\Phi, \bar{\Phi} \to 1$

Equations of Motion

$$\left.\frac{d\Omega}{d\sigma}\right|_{\min} = \left.\frac{d\Omega}{d\Phi}\right|_{\min} = \left.\frac{d\Omega}{d\bar{\Phi}}\right|_{\min} = 0$$

$[\mu = 0\,\mathrm{MeV}, \;\; T_0 = 270\,\mathrm{MeV}]$





Renormalization Group

[cf. talk by M. Mitter]

$$\partial_k \Omega_k(T,\mu) = \frac{k^4}{12\pi^2}\left[\frac{3}{E_\pi}\coth\left(\frac{E_\pi}{2T}\right) + \frac{1}{E_\sigma}\coth\left(\frac{E_\sigma}{2T}\right) - \frac{2\nu_q}{E_q}\left\{1 - N_q(T,\mu;\Phi,\bar{\Phi}) - N_{\bar{q}}(T,\mu;\Phi,\bar{\Phi})\right\}\right]$$

$$N_q(T,\mu;\Phi,\bar{\Phi}) = \frac{1 + 2\bar{\Phi} e^{(E_q-\mu)/T} + \Phi e^{2(E_q-\mu)/T}}{1 + 3\bar{\Phi} e^{(E_q-\mu)/T} + 3\Phi e^{2(E_q-\mu)/T} + e^{3(E_q-\mu)/T}}$$

$$N_{\bar{q}}(T,\mu;\Phi,\bar{\Phi}) \equiv N_q(T,-\mu;\bar{\Phi},\Phi)$$

and $E_\pi = \sqrt{k^2 + 2\Omega_k'}$, $E_\sigma = \sqrt{k^2 + 2\Omega_k' + 4\sigma^2\Omega_k''}$, $\nu_q = 2N_c N_f = 12$

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$\Phi, \bar{\Phi} \to 1$:

$$N_q(T,\mu;1,1) = \frac{1}{1+\exp((E_q-\mu)/T)}$$

$$N_{\bar{q}}(T,\mu;1,1) = \frac{1}{1+\exp((E_q+\mu)/T)}$$

Solution of the Flow Equation

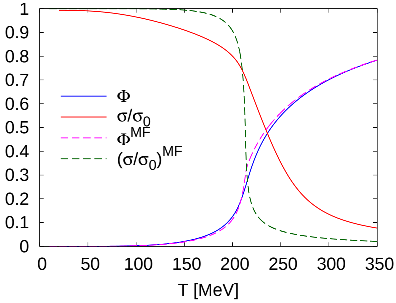
Expansion of the Potential

$$\Omega_k(\sigma) = \sum_{j=0}^{N} a_{2j,k}(\sigma^2 - \sigma_{0,k}^2)^j - c\sigma$$

N ... truncation order

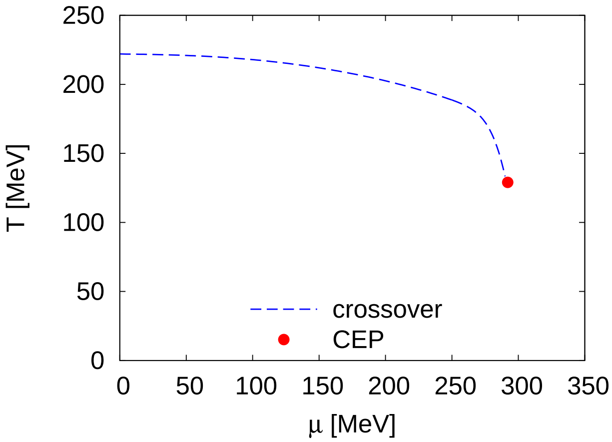
$\longrightarrow$ **finite set of coupled differential equations**

$[\mu = 0\,\text{MeV}, \;\; T_0 = 270\,\text{MeV}]$



Towards the Phase Diagram

[preliminary]



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  - chiral and confinement properties
- mean field analysis
- include fluctuations via renormalization group
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Outlook

- adopt grid numerics → first order phase transition in RG
- include Polyakov loop fluctuations via RG