

Scheme Dependence of RG Flows

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Overview

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 - Concepts
 - Exact RG (ERG)
 - Proper-Time RG (PTRG)
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Motivation

- Quantum Chromodynamics - strong interaction
- “running coupling” - scale dependency
- QCD phase diagramm

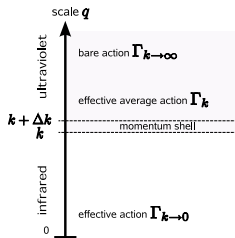
Motivation

- Quantum Chromodynamics - strong interaction
- “running coupling” - scale dependency
- QCD phase diagramm

- functional methods:
 - **renormalization group**
 - Dyson-Schwinger equations
- lattice simulations
- effective theories

Integration of Momentum Shells

- generating functional
 $\Gamma \propto \log \int \mathcal{D}\phi e^{S[\phi]}$
- introduce scale dependency
 $S \rightarrow S_k$
- integrate over a momentum shell
 \Rightarrow scale dependent couplings



Effective average Action

- generating functional of 1PI n -point correlation functions
- functional differential equation for Γ_k - FRG
- coupled system of ordinary differential equations for eff. potential U_k

Effective average Action

- suppose Γ_Λ known at ultraviolet (UV) scale Λ
- RG flow give dependency on the scale k
- flow $\partial_k \Gamma_k$ and Γ_Λ
 \Rightarrow initial value problem
- $\Gamma_{k=0} = \Gamma$

- different flow equations
- comparison of
 - Exact (Wetterich) RG
 - Proper-Time RG

bare action $\Gamma_{k=\Lambda} \equiv -S$



effective action $\Gamma_{k \rightarrow 0}$

Exact Renormalization Group

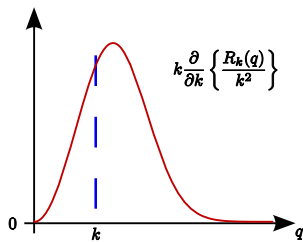
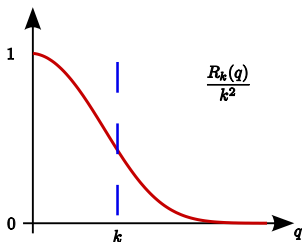
Flow Equation for Effective average Action

[Wetterich 1993]

$$\partial_t \Gamma_k = \frac{1}{2} \text{Tr} \left\{ \left(\Gamma_k^{(2)} + R_k \right)^{-1} \partial_t R_k \right\}, \quad \partial_t = k \partial_k$$

$$\partial_t \Gamma_k = \frac{1}{2} \text{Tr} \left\{ \text{Diagram} \right\}$$

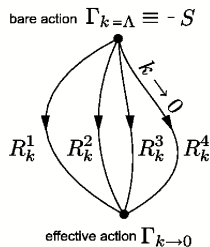
The diagram shows a dashed circle with a solid black dot on the left and a solid white circle with a cross on the right, connected by a dashed line.



Regulators - Scheme Dependency

Different Choices for the Regulator possible

- controls details of momentum shell integration
- regulators \leftrightarrow different trajectories in "Theory Space"
- physics independent of regulator
- test approximations



Optimized Regulator

[Litim 2000]

- stability of flow
- \exists optimized regulator $R_k(q^2) = (k^2 - q^2)\Theta(k^2 - q^2)$

Scheme dependency of the PTRG

PTRG flow for different regulators (m)

[Liao 1996]

$$\partial_t \Gamma_{k,m} = \text{Tr} \left\{ \left(\left(\Gamma_k^{(2)} + k^2 \right)^{-1} k^2 \right)^m \right\}$$

- based on loop expansion of effective action
- contains information beyond one-loop order via “RG-improvement”
- simple structure and simple connection between different regulators
- effective UV cutoff

“Controlling” the Approximation in the PTRG

- generalized PTRG
- compare to ERGE
- missing piece in PTRG $\propto \partial_t \Gamma_k^{(2)}$

Exact Relation

$$\text{ERG} = \text{PTRG} + \text{Addition from gPTRG} \propto \partial_t \Gamma_k^{(2)}$$

“Controlling” the Approximation in the PTRG

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Exact Relation

$$\text{ERG} = \text{PTRG} + \text{Addition from gPTRG} \propto \partial_t \Gamma_k^{(2)}$$

Good Approximation?

$$\text{ERG} \approx \text{PTRG}$$

Expansion in Derivatives and Taylor-Expansion

Two-Flavour Quark-Meson Model

as Ansatz for $\Gamma_{\Lambda=1200\text{MeV}}$

[Schaefer, Pirner 1999]

$$\Gamma_{k=\Lambda} = \int d^4x \left\{ \frac{1}{2}(\partial^\mu \vec{\Phi}) \cdot (\partial^\mu \vec{\Phi}) + \frac{m^2}{2}(\vec{\Phi} \cdot \vec{\Phi}) + \frac{\lambda}{4}(\vec{\Phi} \cdot \vec{\Phi})^2 - c\sigma \right\}$$

$$- \int d^4x \left\{ \bar{q} (\not{\partial} + ig(\sigma + i\underline{\gamma}\vec{\tau} \cdot \vec{\pi})) q \right\}$$

$$\vec{\Phi} = (\sigma, \vec{\pi})$$

Expansion in Derivatives and Taylor-Expansion

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$$\vec{\Phi} = (\sigma, \vec{\pi})$$

Effektive average Potential

- $U_k(\phi) = \frac{\Gamma_k[\phi]}{V_4}$
- Ansatz: $U_k(\phi) = \sum_{i=0}^n a_{2i}(k) (\phi^2 - \phi_0^2(k))^i - c\phi$

β -Function for $\rho_0 = \phi_0^2$

PTRG for $m = 2$

[Papp et al. 1999]

$$\partial_t \rho_{0,k} \propto k^4 \left\{ \frac{12a_{4,k} + 24a_{6,k}\rho_{0,k}}{\left(k^2 + \left(\frac{c}{\sqrt{\rho_{0,k}}} + 8a_{4,k}\rho_{0,k}\right)\right)} + 3 \frac{4a_{4,k}}{\left(k^2 + \left(\frac{c}{\sqrt{\rho_{0,k}}}\right)\right)} - 4N_f N_c \frac{g^2}{\left(k^2 + g^2\rho_{0,k}\right)} \right\}$$

$$\partial_t \rho_0(k) \propto \text{diagram 1} + \text{diagram 2} + \text{diagram 3} - \text{diagram 4}$$

The diagrams represent the graphical interpretation of the terms in the beta function:

- Diagram 1:** A loop with a red dot on top and a red square on the bottom. The bottom line is solid, and the top line is dashed.
- Diagram 2:** A loop with a red dot on top and a red square on the bottom. The bottom line is solid, and the top line is dashed. There are 'x' marks on the bottom line.
- Diagram 3:** A loop with a black dot on top and a black square on the bottom. The bottom line is solid, and the top line is dashed.
- Diagram 4:** A loop with a black dot on top and a black dot on the bottom. The bottom line is solid, and the top line is dashed.

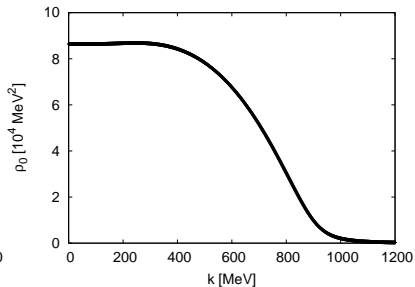
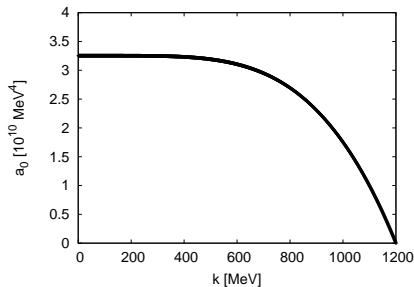
β -Functions in graphs

PTRG for $m = 2$

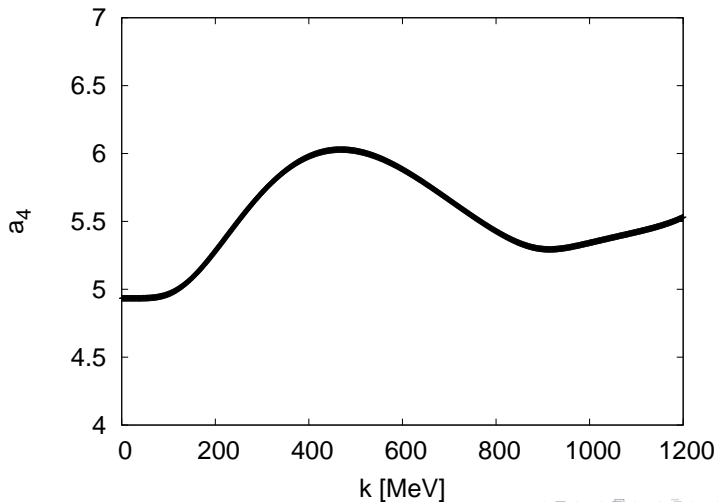
$$\begin{aligned}
 \partial_t a_0(k) &\propto \text{[Red dot in red dashed circle]} + 3 \text{[Black dot in black dashed circle]} - 12N_f \text{[Black dot in black solid circle]} \\
 \partial_t \rho_0(k) &\propto \text{[Red dot on top of red dashed circle with red square below]} + \text{[Red dot on top of red dashed circle with red square below and crosses]} + \text{[Black dot on top of black dashed circle with black square below]} - \text{[Black dot on top of black dashed circle with black square below]} \\
 \partial_t a_4(k) &\propto \text{[Black dot on top of black dashed circle with black square below]} - \text{[Black dot on top of black dashed circle with black square below and crosses]} + \text{[Red dot on top of red dashed circle with red square below]} + \text{[Red dot on top of red dashed circle with red square below and crosses]} \\
 &+ \text{[Red dot on top of red dashed circle with red square below and crosses]} + \text{[Red dot on top of red dashed circle with red square below and crosses]} - \text{[Red dot on top of red dashed circle with red square below and crosses]} + \text{[Black dot on top of black dashed circle with black square below and crosses]}
 \end{aligned}$$

Numerical Results

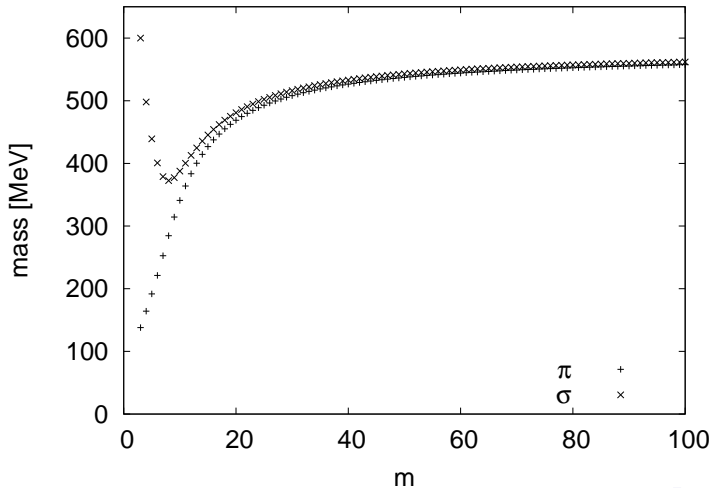
- $\Lambda = 1200\text{MeV}$
- Taylor-expansion up to ϕ^{12}
- optimized regulator
- $m_\pi = 138\text{MeV}$
- $m_\sigma = 600\text{MeV}$
- $f_\pi = 93\text{MeV}$



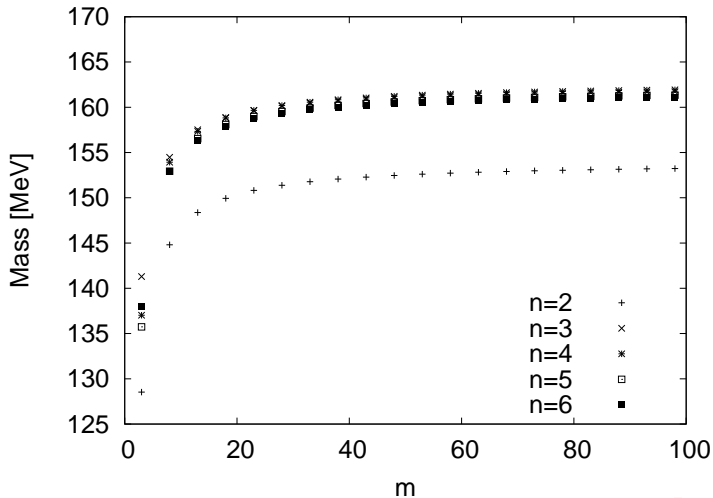
Numerical Results



Without taking the effective UV cutoff into account



Different Truncation Orders



Summary and Outlook

- different RG flows
 - graphical interpretation of the flow equations
 - different PTRG regulators
 - effect of truncation order
 - $n = 3$ good approximation
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- $T, \mu \neq 0$
 - three quark flavors