Scheme Dependence of RG Flows

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Overview

Motivation

- Quantum Chromodynamics - strong interaction
- “running coupling” - scale dependency
- QCD phase diagramm

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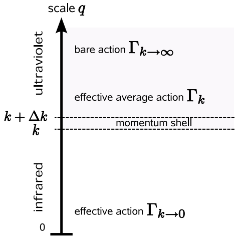
- functional methods:
    - **renormalization group**
    - Dyson-Schwinger equations
- lattice simulations
- effective theories

Integration of Momentum Shells

- generating functional
  $\Gamma \propto \log \int \mathcal{D}\phi \; e^{S[\phi]}$
- introduce scale dependency
  $S \longrightarrow S_k$
- integrate over a momentum shell
  $\Rightarrow$ scale dependent couplings



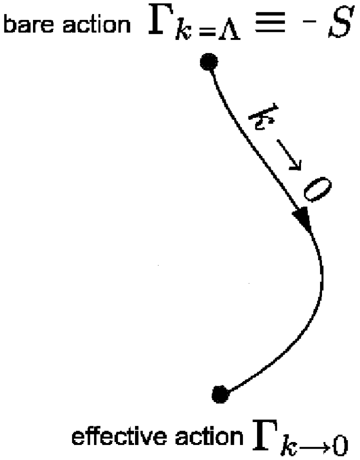
Effective average Action

- generating functional of 1PI n-point correlation functions
- functional differential equation for Γ_k - FRG
- coupled system of ordinary differential equations for eff. potential U_k

Effective average Action

- suppose Γ_Λ known at ultraviolet (UV) scale Λ
- RG flow give dependency on the scale k
- flow $\partial_k \Gamma_k$ and Γ_Λ $\Rightarrow$ initial value problem
- $\Gamma_{k=0} = \Gamma$

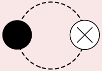
- different flow equations
- comparison of
  - Exact (Wetterich) RG
  - Proper-Time RG

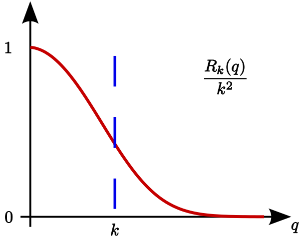


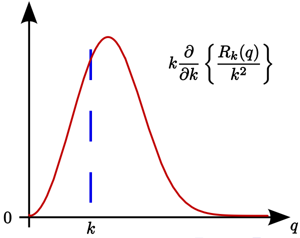
Exact Renormalization Group

Flow Equation for Effective average Action [Wetterich 1993]

$$\partial_t \Gamma_k = \frac{1}{2} Tr \left\{ \left(\Gamma_k^{(2)} + R_k \right)^{-1} \partial_t R_k \right\}, \qquad \partial_t = k\partial_k$$

$$\partial_t \Gamma_k = \frac{1}{2}$$ 

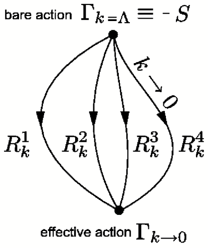




Regulators - Scheme Dependency

Different Choices for the Regulator possible

- controls details of momentum shell integration
- regulators $\leftrightarrow$ different trajectories in "Theory Space"
- physics independent of regulator
- test approximations



Optimized Regulator

[Litim 2000]

- stability of flow
- $\exists$ optimized regulator $R_k(q^2) = (k^2 - q^2)\Theta(k^2 - q^2)$

Scheme dependency of the PTRG

PTRG flow for different regulators (m) [Liao 1996]

$$\partial_t \Gamma_{k,m} = Tr\left\{\left(\left(\Gamma_k^{(2)} + k^2\right)^{-1} k^2\right)^m\right\}$$

- based on loop expansion of effective action
- contains information beyond one-loop order via "RG-improvement"
- simple structure and simple connection between different regulators
- effective UV cutoff

"Controlling" the Approxmation in the PTRG

- generalized PTRG
- compare to ERGE
- missing piece in PTRG $\propto \partial_t \Gamma_k^{(2)}$

Exact Relation

ERG $=$ PTRG $+$ Addition from gPTRG $\propto \partial_t \Gamma_k^{(2)}$

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Good Approximation?

ERG $\approx$ PTRG

Expansion in Derivatives and Taylor-Expansion

Two-Flavour Quark-Meson Model as Ansatz for $\Gamma_{\Lambda=1200\text{MeV}}$

[Schaefer, Pirner 1999]

$$\Gamma_{k=\Lambda} = \int d^4x \, \left\{ \tfrac{1}{2}(\partial^\mu \vec{\Phi}) \cdot (\partial^\mu \vec{\Phi}) + \tfrac{m^2}{2}(\vec{\Phi}\cdot\vec{\Phi}) + \tfrac{\lambda}{4}(\vec{\Phi}\cdot\vec{\Phi})^2 - c\sigma \right\}$$

$$- \int d^4x \, \left\{ \bar{q} \left(\partial\!\!\!/ + ig(\sigma + i\underline{\gamma}\vec{\tau}\cdot\vec{\pi}) \right) q \right\}$$

$$\vec{\Phi} = (\sigma, \vec{\pi})$$

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Effektive average Potential

- $U_k(\phi) = \frac{\Gamma_k[\phi]}{V_4}$
- Ansatz: $U_k(\phi) = \sum_{i=0}^{n} a_{2i}(k) \left(\phi^2 - \phi_0^2(k) \right)^i - c\phi$

β-Function for $\rho_0 = \phi_0^2$

PTRG for $m = 2$ [Papp et al. 1999]

$$\partial_t \rho_{0,k} \propto k^4 \left\{ \frac{12 a_{4,k} + 24 a_{6,k} \rho_{0,k}}{\left(k^2 + \left(\frac{c}{\sqrt{\rho_{0,k}}} + 8 a_{4,k} \rho_{0,k}\right)\right)} + 3 \frac{4 a_{4,k}}{\left(k^2 + \left(\frac{c}{\sqrt{\rho_{0,k}}}\right)\right)} - 4 N_f N_c \frac{g^2}{(k^2 + g^2 \rho_{0,k})} \right\}$$

$\partial_t \rho_0(k) \propto$ 

β-Functions in graphs

PTRG for $m = 2$

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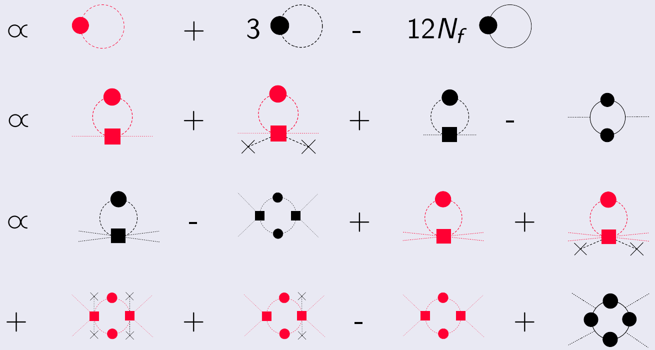
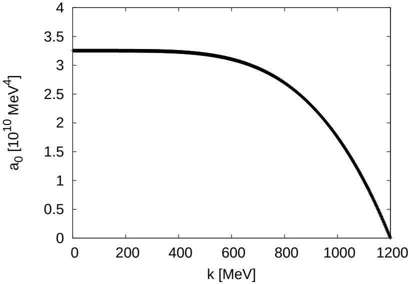
$\partial_t a_4(k) \propto$

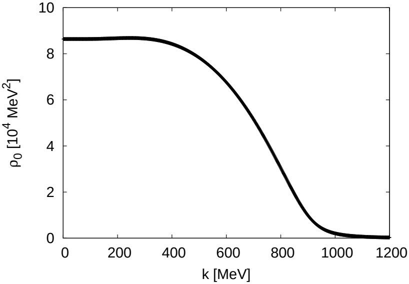


Numerical Results

- $\Lambda = 1200\text{MeV}$
- Taylor-expansion up to ϕ^{12}
- optimized regulator

- $m_\pi = 138\text{MeV}$
- $m_\sigma = 600\text{MeV}$
- $f_\pi = 93\text{MeV}$





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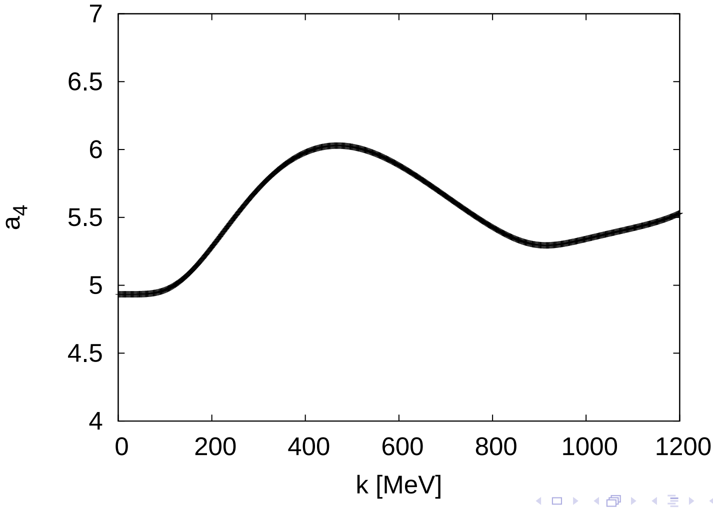
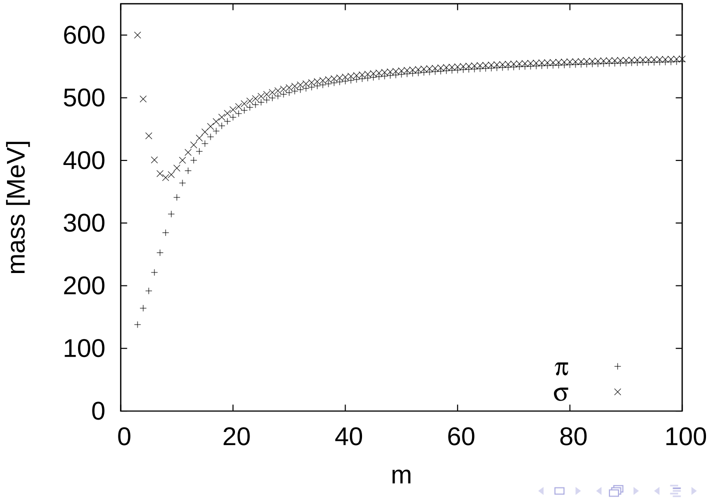
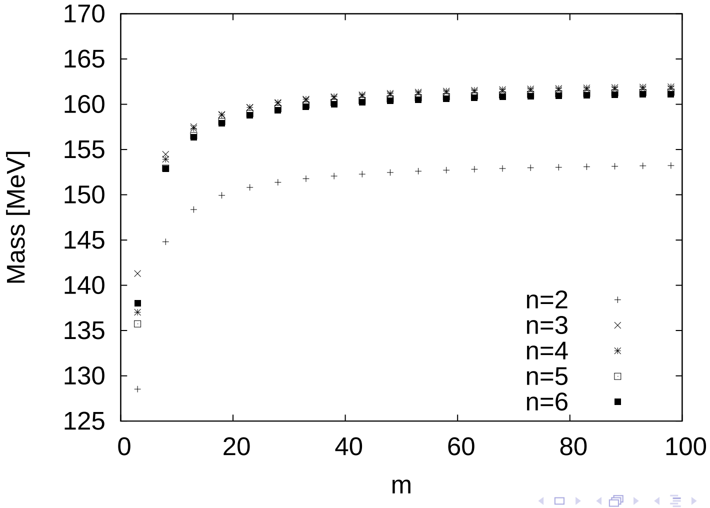


Without taking the effective UV cutoff into account



Different Truncation Orders



Summary and Outlook

- different RG flows
- graphical interpretation of the flow equations
- different PTRG regulators
- effect of truncation order
- $n = 3$ good approximation

- $T, \mu \neq 0$
- three quark flavors