

PhD Project: Baryon properties in a covariant Faddeev approach

Off-Topic: Quantum and Modified Classical Gravity

Helios Sanchis Alepuz¹

in collaboration with: Reinhard Alkofer¹

Diana Nicmorus¹

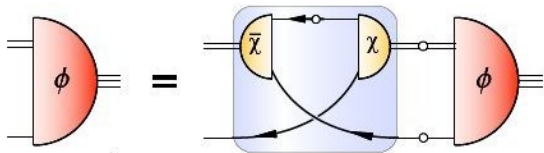
¹ University of Graz

Hallstatt Workshop
Hallstatt



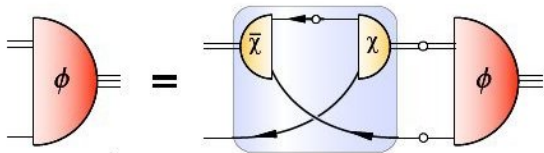
Sketch of the Thesis project

**Goal: Baryon
properties solving
BSE for the
quark-diquark model**



Sketch of the Thesis project

Goal: Baryon properties solving BSE for the quark-diquark model



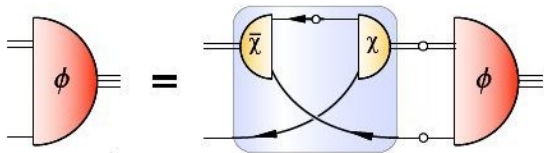
$$\Phi(p, P) = \int_k K(p, k, P) S(k_q) \Phi(k, P) D(k_d)$$

Ingredients:

- ▶ Quark Propagator S : Calculated **solving** it's DS equation.
- ▶ Faddeev approximation: Assumes 2-quark correlations to be dominating in the nucleon.
- ▶ Diquark propagator D and amplitude χ (on-shell and off-shell)
- ▶ Two-particle interaction kernel K

Sketch of the Thesis project

Goal: Baryon properties solving BSE for the quark-diquark model



$$\Phi(p, P) = \int_k K(p, k, P) S(k_q) \Phi(k, P] D(k_d)$$

Nucleon and Delta masses calculated with this approach:

D. Nicmorus et al. Delta-baryon mass in a covariant Faddeev approach. arxiv:0812.1665.

G. Eichmann et al. Meson and nucleon properties from Dyson-Schwinger QCD. PoS CONFINEMENT8:077 (2008)

Quark Propagator

Solve DSE for quark propagator

$$\text{---}\bigcirc\text{---}^{-1} = \text{---}^{-1} + \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---}$$

$$S^{-1}(p) = Z_2(\not{p} + m_{bare}) - \int^{\Lambda} K(p, q)S(q)$$

Rainbow-ladder truncation:

Vector part of quark-gluon vertex γ_{μ} , free gluon propagator $D_{free}^{\mu\nu}$, effective interaction $\alpha(k^2)$

Quark Propagator

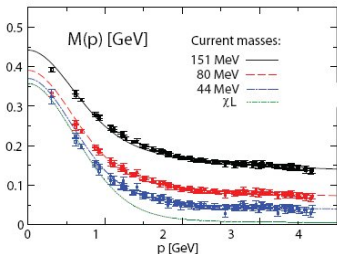
Solve DSE for quark propagator

$$\text{---} \circ \text{---}^{-1} = \text{---}^{-1} + \text{---} \circ \text{---} \text{---} \text{---}^{-1}$$

$$S^{-1}(p) = Z_2(\not{p} + m_{bare}) - \int_q^\Lambda K(p, q) S(q)$$

Rainbow-ladder truncation:

Vector part of quark-gluon vertex γ_μ , free gluon propagator $D_{free}^{\mu\nu}$, effective interaction $\alpha(k^2)$





Off-Topic: Quantum and Modified Classical Gravity

Approaches to Quantum Gravity

Quantum Field Theory in Curved Spacetime

- ▶ Hawking Radiation of Black Holes
- ▶ Entropy of Black Holes (Bekenstein-Hawking formula)
- ▶ Particle Creation in Primordial Universe
- ▶ and more...

Approaches to Quantum Gravity

Quantum Field Theory in Curved Spacetime

Quantum Field Theory of Gravity

- ▶ Gravity is described as an ordinary spin-2 quantum field (graviton) in Minkowski spacetime
- ▶ Requires supersymmetry to avoid divergences

Approaches to Quantum Gravity

Quantum Field Theory in Curved Spacetime

Quantum Field Theory of Gravity

String Theory, M/Theory, etc.

- ▶ Matter as well as gravitons are now excitations of a string or a brane, etc.
- ▶ It's also formulated in Minkowski spacetime
- ▶ Requires extra dimensions to be properly formulated

Approaches to Quantum Gravity

Quantum Field Theory in Curved Spacetime

Quantum Field Theory of Gravity

String Theory, M-Theory, etc.

LOOP QUANTUM GRAVITY

or Canonical Quantum gravity (we apply canonical quantization rules on GR)

Canonical Formalism

How do we quantize canonically?

Canonical Formalism

Steps:

1. Choose canonical variables x and write the Lagrangian using them: $\mathcal{L}(x, \dot{x}, \dots)$
2. Now we define our canonical momentum by: $p = \frac{\partial \mathcal{L}}{\partial \dot{x}}$
3. And the Hamiltonian is given by: $\mathcal{H} = \dot{x}p - \mathcal{L}$
4. The CLASSICAL evolution is given by the Poisson brackets:

$$\dot{x} = \{x, \mathcal{H}\} \quad \dot{p} = \{p, \mathcal{H}\}$$

At the QUANTUM level we use the Dirac prescription:

$$\dot{x} = i[x, \mathcal{H}] \quad , \quad \dot{p} = i[p, \mathcal{H}] \quad [x, p] = i\hbar$$

5. If our variables are fields ($\phi(x)$ and $\pi(x)$): $[\phi(x), \pi(y)] = i\hbar\delta(x - y)$

But...we need Time!!

In the Hamiltonian (thus, in the canonical) formalism we need a time variable to define momenta and evolution.

But...we need Time!!

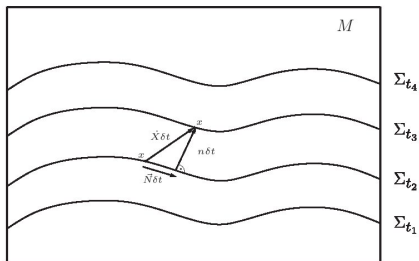
In the Hamiltonian (thus, in the canonical) formalism we need a time variable to define momenta and evolution.

But... General Relativity wasn't that thing where time and space are merged into SPACETIME ??

But...we need Time!!

How to break spacetime:

We define an **arbitrary** (and unspecified) "time function" t and a vector t^μ such that $t^\mu \partial_\mu t = 1$, and "foliate" our spacetime into spatial 3-surfaces of constant t



Canonical Formulation

- ▶ Once this splitting is done, the canonical variable is the 3-metric $h(x)$ on the 3-surfaces, and the corresponding canonical momenta.
- ▶ The arbitrariness of the time function leads (non-trivially) to the striking result $\mathcal{H} = 0!$

Canonical Formulation

- ▶ Once this splitting is done, the canonical variable is the 3-metric $h(x)$ on the 3-surfaces, and the corresponding canonical momenta.
- ▶ The arbitrariness of the time function leads (non-trivially) to the striking result $\mathcal{H} = 0!$

Problem of time in General Relativity¹ (NO time evolution ?)

$$\dot{\phi} = \{\phi, H\} = 0 \quad \text{The same at quantum level}$$

¹ Julian Barbour. The Nature of Time. Winner of FQXi Essay Contest. www.platonica.com

Canonical Formulation

- ▶ Once this splitting is done, the canonical variable is the 3-metric $h(x)$ on the 3-surfaces, and the corresponding canonical momenta.
- ▶ The arbitrariness of the time function leads (non-trivially) to the striking result $\mathcal{H} = 0!$

Problem of time in General Relativity¹ (NO time evolution ?)

$$\dot{\phi} = \{\phi, H\} = 0 \quad \text{The same at quantum level}$$

"Old" Quantum Gravity (1967-)

Imposing this constraint over physical states:

$$\boxed{\mathcal{H}\Psi = 0} \quad \text{Wheeler-deWitt equation}$$

Extremely difficult to solve and to interpret.

¹ Julian Barbour. The Nature of Time. Winner of FQXi Essay Contest. www.platonian.com

Loop Quantum Gravity

- ▶ With Wheeler-deWitt equation, the canonical approach to Quantum Gravity found an almost NO-GO.
- ▶ After decades of no advances, Ashtekar[†] proposed a new set of variables (Loop variables) in which the equations were simpler (but not simple). Loop Quantum Gravity is born.

[†] A. Ashtekar. New variables for classical and quantum gravity. Phys. Rev. Lett. 57 (1986) 22447.

A. Ashtekar. New Hamiltonian formulation of general relativity. Phys. Rev. D36(1987) 1587602.

Loop Quantum Gravity

Main "predictions" of LQG:

- ▶ Spacetime is discrete at Planck scales; this might be even **observable!** (Fermi Gamma-ray telescope)
- ▶ Replaces the Big Bang singularity by a Big Bounce
- ▶ Gives a microscopical derivation of Black Hole entropy (after defining the Barbero-Immirzi parameter)
- ▶ No need for extra dimensions (but not forbidden)

↓ T. Thiemann. Modern Canonical Quantum General Relativity. (Cambridge University Press, Cambridge, 2007)

Loop Quantum Gravity II

Some problems of LQG:

- ▶ Thiemann proposal to solve the Problem of Time is controversial
- ▶ The Discrete \leftrightarrow Continuum step is not clear at the moment.
- ▶ There is an unknown parameter (the Barbero-Immirzi parameter), which is fixed "by hand" to give physical results.
- ▶ What are the observables? What is the BI-parameter? What is...? \leftarrow
Interpretation problems

Modified Classical Gravity



Modified Gravity

- ▶ General Relativity is the **simplest** metric theory: $\mathcal{L} = \int_V R(g, \nabla)$

Modified Gravity

- ▶ General Relativity is the **simplest** metric theory: $\mathcal{L} = \int_V R(g, \nabla)$
- ▶ We can modify the gravity lagrangian: $\mathcal{L} = \int_V f(R(g, \nabla))$

Two ways of doing this:

- ▶ Metric formalism: $g^{\mu\nu}$ is the only variable of the theory. Covariant derivatives are given by the metric. \rightarrow Almost ruled out
- ▶ Palatini formalism: Metric and connection (derivative), are independent.

Modified Gravity

- ▶ General Relativity is the **simplest** metric theory: $\mathcal{L} = \int_V R(g, \nabla)$
- ▶ We can modify the gravity lagrangian: $\mathcal{L} = \int_V f(R(g, \nabla))$
Two ways of doing this:
 - ▶ Metric formalism: $g^{\mu\nu}$ is the only variable of the theory. Covariant derivatives are given by the metric. \rightarrow Almost ruled out
 - ▶ Palatini formalism: Metric and connection (derivative), are independent.

Nice features of Palatini $f(R)$'s:

- ▶ In vacuum (no matter) they are equivalent to GR with **cosmological constant** \Rightarrow No need for Dark Energy
- ▶ When matter is included, the dynamics changes drastically \Rightarrow New effects will appear

Modified Gravity II

Relation with LQG

One can mimic the Big Bounce of LQG using a particular[↓] $f(R) \Rightarrow f(R)$ as effective theories of quantum gravity.

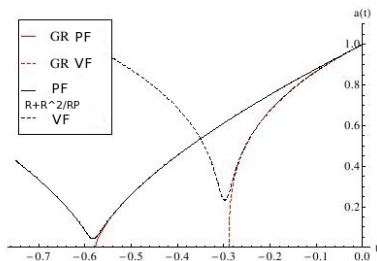
[↓] G.J. Olmo and P. Singh. Effective Action for Loop Quantum Cosmology a la Palatini. JCAP 0901:030 (2009)

Modified Gravity II

Relation with LQG \Rightarrow Mimic Big Bounce of LQG[↓]

Is that $f(R)$ unique?

NO. Big Bounces are a very general feature of $f(R)$'s in Palatini formalism[♭].



[↓] G.J. Olmo and P. Singh. Effective Action for Loop Quantum Cosmology a la Palatini. JCAP 0901:030 (2009)

[♭] C. Barragan, G.J. Olmo and H. Sanchis Alepuz. Bouncing Cosmologies in Palatini $f(R)$ theories. Phys.Rev. D80 024013 (2009)

Final words

- ▶ If $f(R)$'s are less singular than Einstein's General Relativity, one might consider to use them as a starting point for Canonical Quantization[‡].
- ▶ Can we "design" a $f(R)$ such that the Quantum equations are simplified?
- ▶ But problems appear:
 - ▶ The Hamiltonian and equations are far more complicated
 - ▶ There are extra constraints besides the Hamiltonian constraint
 - ▶ Are the Ashtekar variables still valid (or useful)?

[‡]G. Olmo and H. Sanchis Alepuz. Work in progress.

Summarizing...

In summary:

- ▶ LQG is a, mathematically rigorous, non-perturbative approach to quantum gravity, which preserves the geometrical philosophy of Einstein's General Relativity.
- ▶ It is very difficult to solve and to interpret, but very promising.
- ▶ Modified theories of gravity are nice effective models for LQG, and also nice by themselves.

THANKS!