Bethe-Salpeter-Equation Studies of Meson Properties

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Outline

Preliminaries

Four forces in nature:

- Gravitation
- Electromagnetic interaction
- Weak interaction
- Strong interaction

Strong force found in atomic nuclei - binds protons, neutrons and their constituents together

Described by Quantum Chromodynamics (QCD):
theory of (anti)quarks and gluons

Quantum Chromodynamics

Properties:

- Quarks, antiquarks: massive fermions (analogy: electrons)
  6 flavors: up, down, strange, charm, bottom, top
- Gluons: massless bosons (analogy: photons)
- all carry color: red, green, blue
  (anticolor: cyan, magenta, yellow)

All particles observed so far (in a detector) are white
Combinations of:

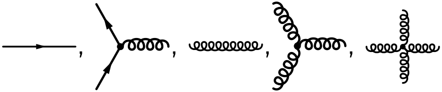
3 colors: r-g-b (or c-m-y)	color-anticolor: r-c, g-m, b-y
$\Downarrow$	$\Downarrow$
Baryons (3 quarks)	Mesons (quark-antiquark)
proton, neutron, ...	pion, rho, ...

Quantum Chromodynamics

- Relativistic quantum field theory
- Theory characterized by Lagrangian:

$$\mathcal{L}_{QCD}[\psi,\bar{\psi},A] = \bar{\psi}(x)(\gamma_\mu(\partial_\mu + iA_\mu(x)) + m)\psi(x) + \frac{1}{4g^2}F_{\mu\nu}F^{\mu\nu}$$

Diagrams contained in $\mathcal{L}_{QCD}$:



- Quantization: (Euclidean) path integral

$$\langle \hat{O} \rangle = \frac{1}{Z}\int \mathcal{D}[\psi,\bar{\psi},A]e^{-\int d^4x \mathcal{L}[\psi,\bar{\psi},A]}\hat{O}[\psi,\bar{\psi},A]$$

Methods

Lattice QCD

- Discretize $\mathcal{L}[\psi, \bar{\psi}, A]$
- Compute $\langle \hat{O} \rangle$ directly with Monte-Carlo integration (talks by J. Danzer, M. Limmer, D. Mohler)

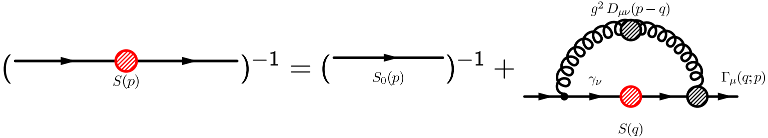
Dyson-Schwinger and Bethe-Salpeter equations (DSEs and BSE)

- Consider Green functions: describe propagation and interaction of fields
- They are solutions of an infinite system of coupled self-consistent equations (Dyson-Schwinger equations) (talk by M. Q. Huber)
- Bound states: described by (homogeneous) Bethe-Salpeter equation

Dressed Propagators and Vertices

- In Lagrangian: ‘bare’ interactions vertices and propagators
- Solutions of DSEs and BSE: ‘dressed’ quantities
- In diagrams denoted by 
- E. g., dressed quark propagator describes all possible ways a quark can propagate
- Includes closed loops, possible interactions with other particles, ...

Quark Gap Equation

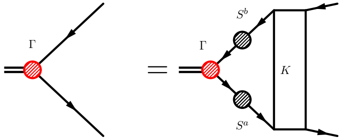
- DSE describing propagation of a quark
- Solution: (inverse) dressed quark propagator
  $S^{-1}(p) = i\,\gamma \cdot p\, A(p^2) + B(p^2)$
- 

$$S^{-1}(p) = S_0^{-1}(p) + \frac{4}{3}\int \frac{d^4q}{(2\pi)^4}\, g^2\, D_{\mu\nu}(p-q)\,\gamma_\nu\, S(q)\,\Gamma_\mu(q;p)$$

- Will be needed later on: mesons are bound states of quarks, antiquarks and gluons!

Bethe-Salpeter Equation

- Consider quark-antiquark scattering
- Project on a bound state of mass $P^2 = -M^2$



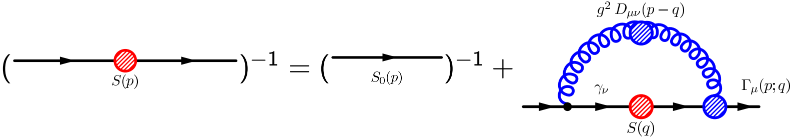
$$\Gamma(p;P) = \int_q S^a(q_+)\,\Gamma(q;P)\,S^b(q_-)\,K(q,p;P)$$

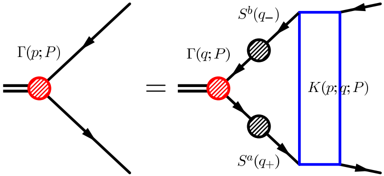
- $\Gamma(p;P)$ Bethe-Salpeter Amplitude (BSA)
- S^a, S^b dressed quark propagators
- $K(q,p;P)$ quark-antiquark scattering kernel

Rainbow-Ladder Truncation

- Solution of the gap equation is possible for any gluon propagator and quark-gluon vertex
- But: quark-antiquark scattering kernel is not known!
- In addition to DSEs: relations between Green functions of QCD that reflect the symmetries of the theory (Slavnov-Taylor and Ward-Takahashi identities)
- Here: chiral symmetry is important (see later) therefore we want to satisfy the axial-vector Ward-Takahashi identity
- $\Rightarrow$ Restrictions on the scattering kernel!
- Simplest way to implement this: rainbow-ladder truncation

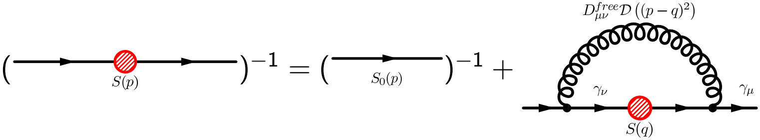
Rainbow-Ladder Truncation

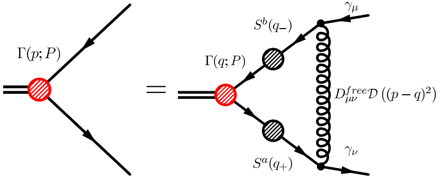




[1] [Maris, Tandy 1999]

Rainbow-Ladder Truncation





Effective quark-gluon interaction[1]
$\mathcal{D}(p^2) = D\,\frac{2\pi^2}{\omega^6} p^2 e^{-p^2/\omega^2} + \mathcal{F}_{UV}(p^2)$

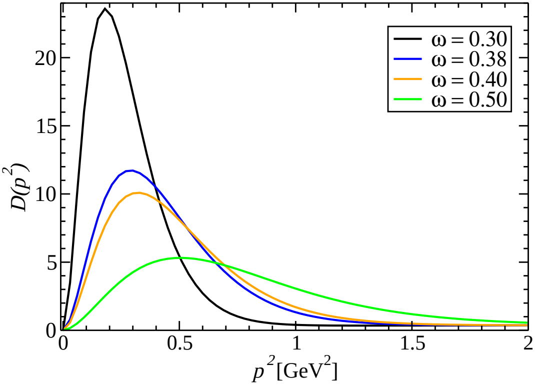
[1] [Maris, Tandy 1999]

Effective Interaction

- Modelled to describe ‘running’ coupling of QCD
- Low momenta (infrared): Gaussian (width ω, strength D)
- High momenta (ultra-violet): $\mathcal{F}_{UV}$ reflects correct perturbative behaviour
- Parameters are fitted to describe observables (pion properties and chiral condensate)
- Other definitions are possible!

Effective Interaction

$\mathcal{D}(p^2)$ for some values of ω ($\omega \cdot D$ is kept constant)



Introduction: Quantum Chromodynamics

Continuum QCD: Dyson-Schwinger and Bethe-Salpeter Equations

Quarks and the Chiral Phase Transition

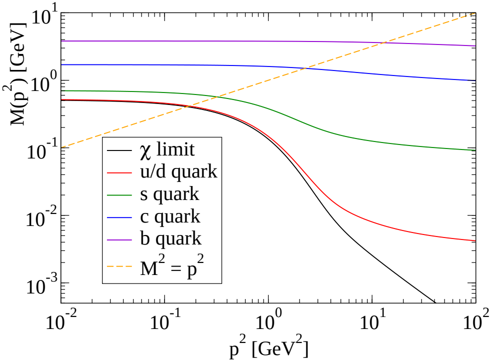
Mesons and the BSE

Summary and Outlook

Quark Masses

Solve gap equation for the quark propagator by iteration: obtain momentum dependent quark mass function

$$M(p^2) = B(p^2)/A(p^2)$$



Start calculation with $m_q = 0$
$\Rightarrow M(p^2) \neq 0$

Dynamical breaking of chiral symmetry!

Dynamical Chiral Symmetry Breaking

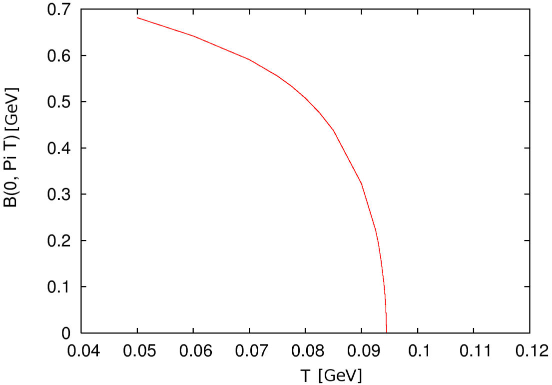
- Chiral Symmetry: symmetry of massless QCD
- Quark masses are small $\Rightarrow$ Remnants of symmetry should be visible in nature!
- Experiments show no symmetry in physical states (e.g., masses of nucleon and parity partner)
- $\Rightarrow$ Symmetry is broken dynamically!
- Affects masses:
  Small quark masses ($m_u \sim 5\mathrm{MeV}$) $\Leftrightarrow$ Large proton mass ($m_p \sim 1\mathrm{GeV}$)
  Pions are almost massless $\Leftrightarrow$ Goldstone bosons corresponding to broken symmetry!

Chiral Phase Transition

- Dynamically broken symmetry at zero temperature is restored at higher temperature
- Increasing temperature: observe chiral phase transition
- Study by extending formalism to finite temperature (gap equation remains valid):
- Modify momenta: $p_\mu \rightarrow (\mathbf{p}, \omega_k)$
- Introduce discrete energies (Matsubara frequencies) $\omega_k = (2k+1)\pi T,\ k \in \mathbb{Z}$

Chiral Phase Transition

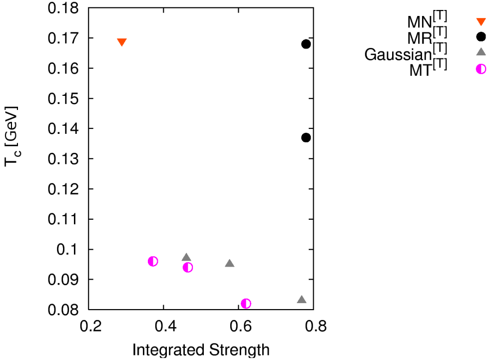
Solve gap equation for various temperatures:



Transition temperature $T_c = 0.094$ GeV

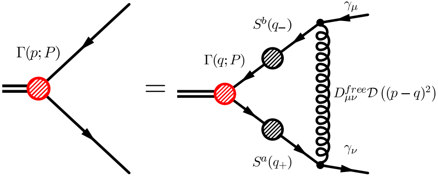
Chiral Phase Transition

- The phase transition temperature depends on the effective quark-gluon interaction
- Different *Ansätze* - different T_c
- No simple relation between T_c and model parameters



Quantum Numbers

- Homogeneous BSE:



- Valid for all possible mesonic bound states
- Different particles - different quantum numbers
- Encoded in the structure of the BS-amplitude $\Gamma(p; P)$

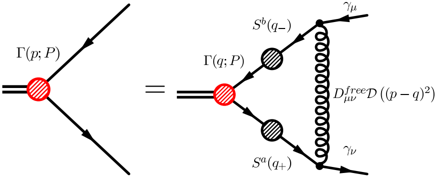
Quantum Numbers

- Decomposition:

$$\Gamma(p;P) = \sum_i T_i(\gamma, p, P) F_i(p^2, P \cdot p, P^2)$$

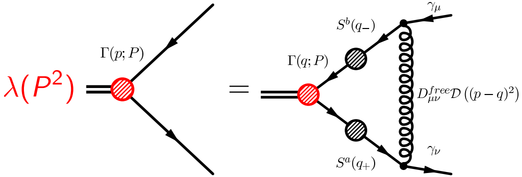
- $T_i(\gamma, p, P)$ encode Dirac (spinor) structure of particle
- $F_i(p^2, P \cdot p, P^2)$ scalar functions, no spinor structure
- Different particles have different spinor structure (and open Lorentz indices corresponding to their spin):
  Pseudoscalars (e.g., pion): 4 components
  Vector particles (e.g., rho): 8 components

Solution Method



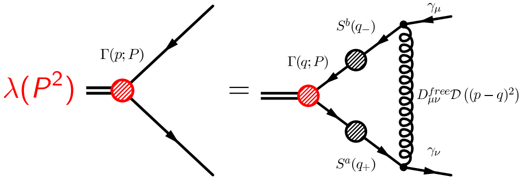
- Only valid if $P^2 = -M^2$ (M mass of bound state)

Solution Method



- Only valid if $P^2 = -M^2$ (M mass of bound state)
- Introduce an eigenvalue $\lambda(P^2)$
- Recover the original equation if $\lambda(P^2 = -M^2) = 1$
  $\Rightarrow$ Solution!

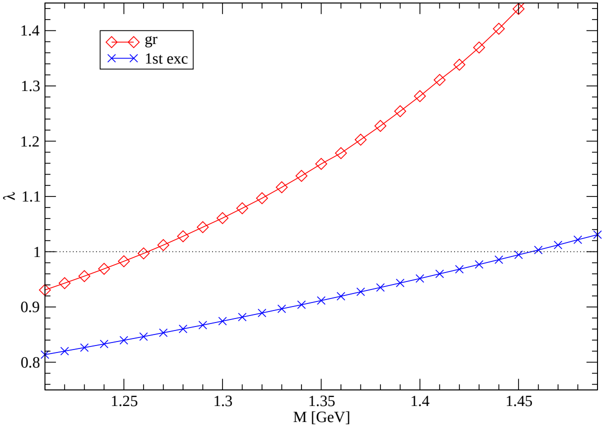
Solution Method



- Strategy: 'scan' values of P^2
- Compute the largest few eigenvalues
- $\lambda_0(P^2) = 1 \Rightarrow P^2 = -M_0^2$ mass of the ground state
- $\lambda_1(P^2) = 1 \Rightarrow P^2 = -M_1^2$ mass of the first excited state
- ...

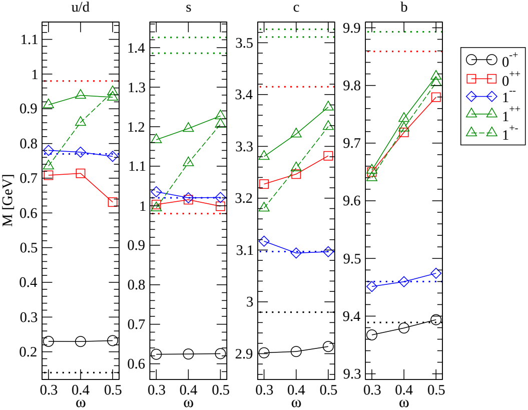
Solution Method

Illustration: curves for ground and first excited state



Meson Masses

- Procedure applicable from $m_q = 0$ to $m_q = m_b$ and beyond



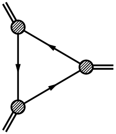
Observe dependence on ω: "spatially extended" states feel the IR properties of the effective interaction

Summary

- Dyson-Schwinger and Bethe-Salpeter equations allow nonperturbative studies of continuum QCD
- For numerical studies: work in symmetry preserving truncation (e.g., rainbow-ladder)
- Finite temperature:
  study the chiral phase transition
- Zero temperature:
  compute meson properties, e.g. masses

Outlook

- Use amplitudes $\Gamma(p;P)$ to compute hadronic decays:



$\Rightarrow$ obtain decay widths

- Improve finite temperature calculation
- Study exotic quantum numbers
- Improve truncation beyond rainbow-ladder
  $\Rightarrow$ reliable study of excited states
- Treat resonances in the BSE framework