Hadron Spectroscopy with Dynamical Chirally Improved Fermions

Markus Limmer

Karl–Franzens–Universität Graz

Advisor: Christian B. Lang

Co-workers: G. Engel and D. Mohler

October 22, 2008

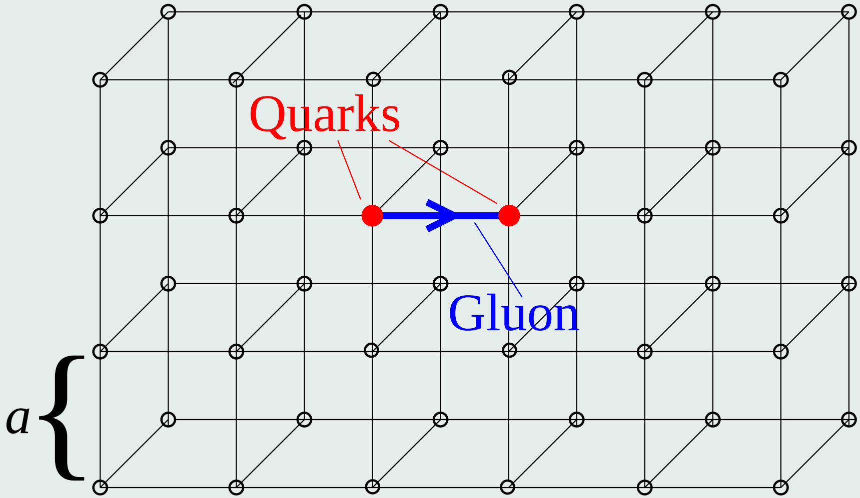
Quantum Chromo Dynamics: Continuum and lattice

- QCD in the continuum described by the (Euclidean) *Lagrangian*

$$\mathcal{L}[\psi,\overline{\psi},A] \;=\; \sum_{f=1}^{N_f} \overline{\psi}^{(f)}\left[\gamma_\mu(\partial_\mu + iA_\mu) + m^{(f)}\right]\psi^{(f)} + \frac{1}{2g^2}\,\mathrm{Tr}\left[F_{\mu\nu}\,F_{\mu\nu}\right]$$

- We want to non-perturbatively regulate this $\mathcal{L}$ $\Rightarrow$ *Lattice QCD*
- Gauge invariance $\Rightarrow$ introduce gauge links $U_\mu(n)$
- Transform derivatives
- Turn integrals into (discrete, finite) sums

The lattice with quarks and gluons



Quarks $\sim \overline{\psi}, \psi$ Gluons $\sim U_\mu$

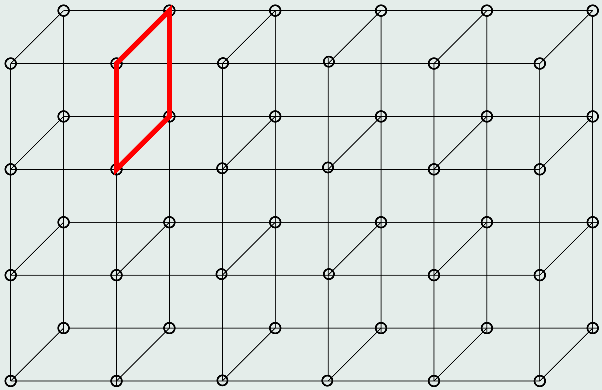
First formulation by Wilson in 1974

- Need gauge invariant object $\Rightarrow$ trace over closed loop of gauge links
- Smallest possible closed loop: *Plaquette* $U_{\mu\nu}$
- *Wilson gauge action* $S_g \sim \sum_n \sum_{\mu<\nu} \text{Re Tr}\left[1 - U_{\mu\nu}(n)\right]$
- With naive discretization: *"Doubling problem"* in the fermionic part $\Rightarrow$ add *"Wilson term"*
- Fermionic action $S_f = a^4 \sum_f \overline{\psi}^{(f)} D_W^{(f)} \psi^{(f)}$
- *Wilson Dirac matrix* D_W

$$D_W^{(f)}(n,m) = \left(m^{(f)} + \frac{4}{a}\right) \delta_{n,m} - \frac{1}{2a} \sum_{\mu=\pm1}^{\pm4} \left(1 - \gamma_\mu\right) U_\mu(n)\, \delta_{n+\hat{\mu},m}$$

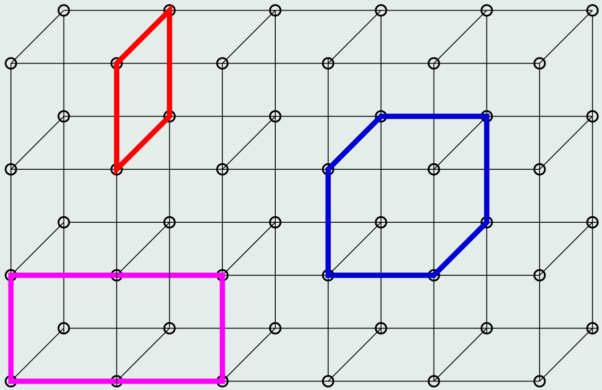
Improving the gauge action

- Wilson gauge action has $\mathcal{O}(a^2)$ errors
- Enhancement by adding other closed loops of gauge links
- Wilson gauge action: Plaquettes



Improving the gauge action

- Wilson gauge action has $\mathcal{O}(a^2)$ errors
- Enhancement by adding other closed loops of gauge links
- Lüscher-Weisz gauge action: Plaquettes, rectangulars, parallelograms



Chiral symmetry

- Continuum version of χ-*symmetry*:

$$\{D, \gamma_5\} = D\,\gamma_5 + \gamma_5\,D = 0$$

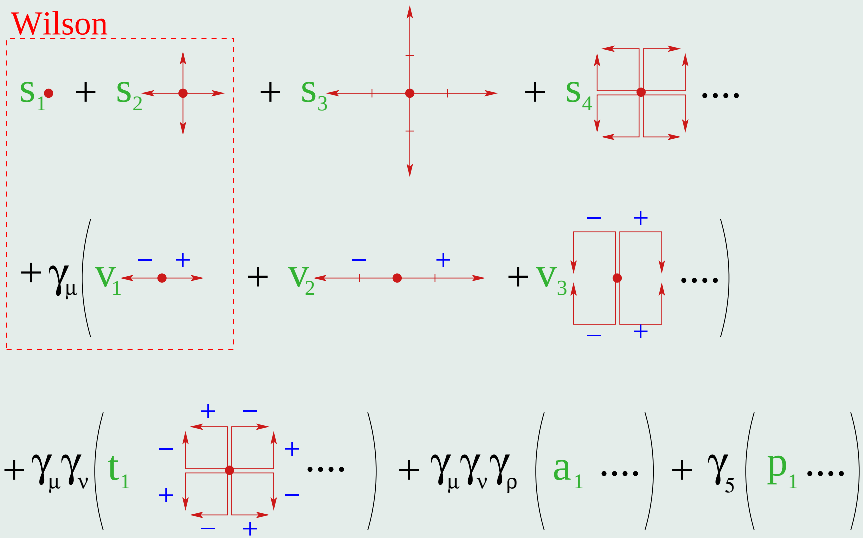
- The lattice version of χ-symmetry, i.e., the *Ginsparg-Wilson relation*:

$$\{D, \gamma_5\} = D\,\gamma_5 + \gamma_5\,D = a\,D\,\gamma_5\,D$$

- Unfortunately most Dirac operators violate the GW-relation
- Exact GW type: Overlap
- Non GW type: Wilson (improved), Staggered, Twisted mass
- GW-type: Domain Wall, Fixed Point, *Chirally Improved*

The Chirally Improved Dirac operator

- Make the most general ansatz for D: $D_{\mathsf{CI}}(n,m) = \sum_{i=1}^{16} c_{nm}^{(i)}(U)\ \Gamma_i$
- Plug this D_{CI} into GW-relation, truncate lengths of paths (4 in our case) and compare coefficients
- Set of coupled quadratic equations, which can be solved!



Quenched vs. dynamical simulation

- We are interested in

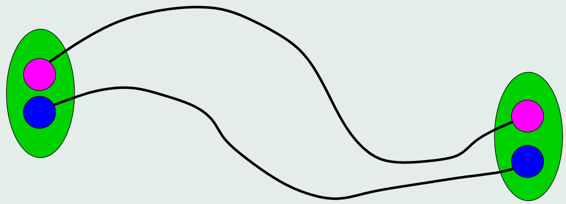
$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}[\overline{\psi}, \psi] \mathcal{D}[U] \; \exp\left[- S_g \right] \exp\left[- \overline{\psi} \, (D + m) \, \psi \right] \; O$$

- Integrating out the fermions $\overline{\psi}, \psi$

$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}[U] \; \exp\left[- S_g \right] \det(D + m) \; O$$

- Two possibilities to do the simulation:

quenched $\rightsquigarrow$ set $\det(D + m)$ to 1



Quenched vs. dynamical simulation

- We are interested in

$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}[\overline{\psi}, \psi] \mathcal{D}[U] \; \exp\left[- S_g \right] \exp\left[- \overline{\psi} \, (D + m) \, \psi \right] \; O$$

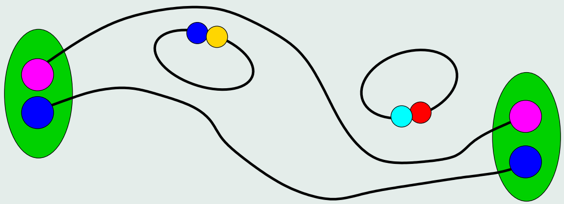
- Integrating out the fermions $\overline{\psi}, \psi$

$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}[U] \; \exp\left[- S_g \right] \det(D + m) \; O$$

- Two possibilities to do the simulation:

quenched $\rightsquigarrow$ set $\det(D + m)$ to 1

dynamical $\rightsquigarrow$ use $\det(D + m)$ as it is



Hybrid Monte Carlo

- Determinant implemented via *pseudo fermion fields ϕ*

$$\int \mathcal{D}[\overline{\psi},\psi]\; e^{\left[-\overline{\psi}\,(DD^\dagger)\,\psi\right]} = \det\left[DD^\dagger\right] = \int \mathcal{D}[\phi^\dagger,\phi]\; e^{\left[-\phi^\dagger\,(DD^\dagger)^{-1}\,\phi\right]}$$

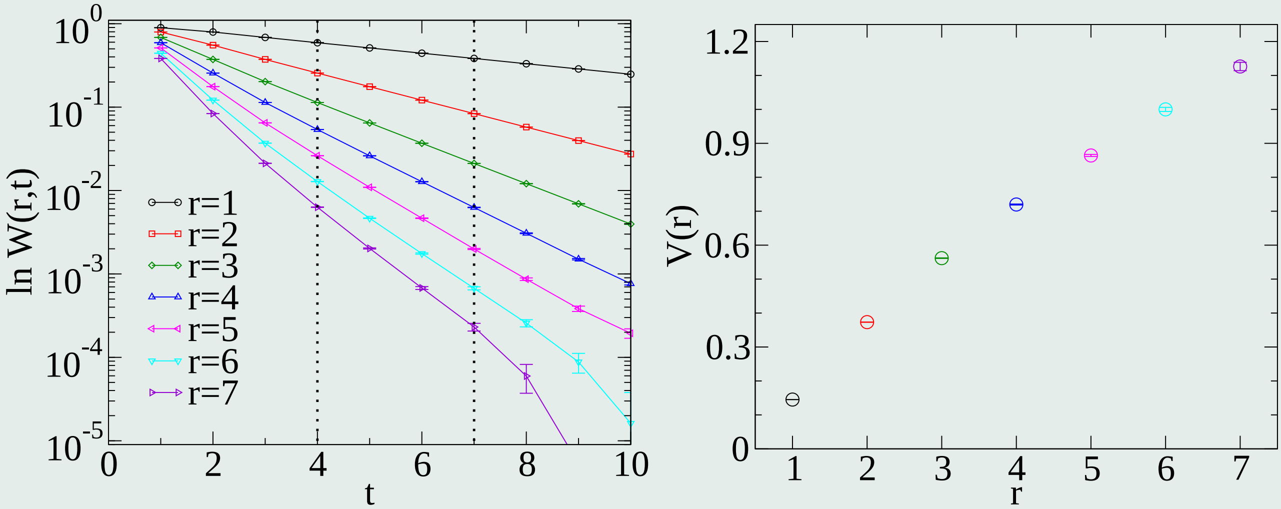
- The simulation is done with *Hybrid Monte Carlo*: Introduce "momenta" P_μ conjugate to links $U_\mu \Rightarrow S_P = 1/2\sum_{x,\mu} P_\mu(x)^2$

1. Update the P_μ using Gaussian random noise
2. Update the fields ϕ using Gaussian random noise ϱ via $\phi = D^\dagger\varrho$
3. Evolve system $\{U, P\}$ according to Hamiltonian E.O.M. (leapfrog algorithm or Omelyan integration)
4. End up with an accept/reject step to correct for numerical errors (finite δ_t)

How to set the scale

- *Smoothing* of the configurations (Hypercubic smearing)
- Extract the *static quark potential* from Wilson W loops via

$$W(r,t) \sim \exp\big(-t\,V(r)\big)$$

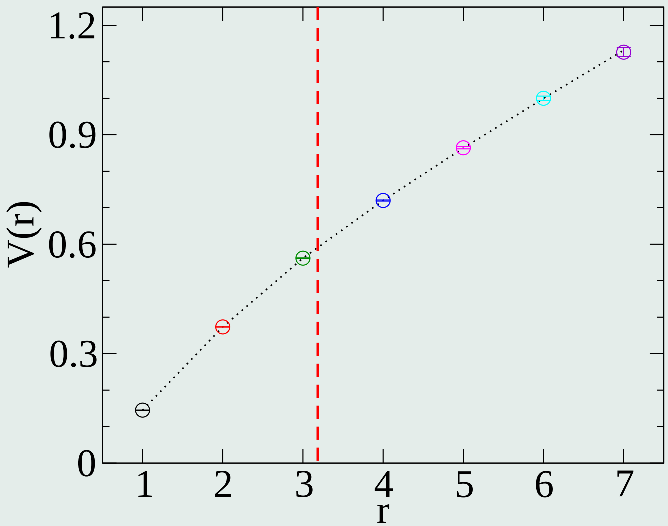


How to set the scale

- Lattice spacing a determined from the potential $V(r)$

$$V(r) = A + \frac{B}{r} + \sigma\, r + C\, \Delta V(r) \quad , \quad \Delta V(r) = \left[\frac{1}{\boldsymbol{r}}\right] - \frac{1}{r}$$

- Lattice spacing a via



$\Longrightarrow$ $V'(r_0) r_0^2 = 1.65$ corresponds to $r_0^{(\text{exp})} \simeq 0.5$ fm

$$\Downarrow$$

$$a = r_0^{(\text{exp})} \sqrt{\frac{\sigma}{1.65 + B}}$$

A tool for studying hadron masses

- The masses are measured with *correlators* of the form

$$C(\vec{p},t) = \langle \mathcal{O}(\vec{p},t)\mathcal{O}^\dagger(\vec{p},0)\rangle \ , \quad \mathcal{O}(\vec{p},t) = \sum_{\vec{x}} \tilde{\mathcal{O}}(\vec{x},t)e^{-i\vec{x}\cdot\vec{p}}$$

- "Design" *interpolating field operators* to get correct *quantum numbers* (and of course symmetries)

- Projection to fixed spatial momentum $\vec{p}$ (often zero)

$$E(\vec{p}) \approx \sqrt{m^2 + \vec{p}^2}$$

Interpolating field operators

- Example for a *meson interpolator*

$$\mathcal{O}_M = \overline{u}_{\text{smear}}\ \Gamma\ d_{\text{smear}}$$

- We analyze 5 different types of isovector mesons

state	J^{PC}	Γ	particle
scalar	0^{++}	$\mathbf{1}, \gamma_t$	a_0
pseudoscalar	0^{+-}	$\gamma_5, \gamma_t\gamma_5$	π
vector	1^{--}	$\gamma_i, \gamma_t\gamma_i$	ρ
axialvector	1^{++}	$\gamma_i\gamma_5$	a_1
pseudovector	1^{+-}	$\gamma_i\gamma_j$	b_1

Interpolating field operators

- We analyze 2 different *baryons* with positive and negative parity
- The *nucleon*

$$O_N^{(j)} = \epsilon_{abc}\,\Gamma_1^{(j)}\,u_{a,\text{smear}}\left(u_{b,\text{smear}}^T\,\Gamma_2^{(j)}\,d_{c,\text{smear}} - d_{b,\text{smear}}^T\,\Gamma_2^{(j)}\,u_{c,\text{smear}}\right)$$

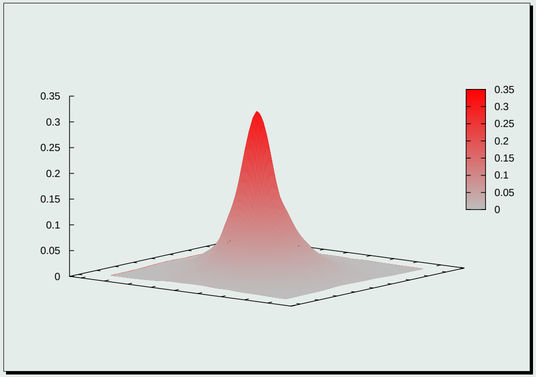
j	$\Gamma_1^{(j)}$	$\Gamma_2^{(j)}$
1	$\mathbb{1}$	$C\gamma_5$
2	γ_5	C
3	i	$C\gamma_t\gamma_5$

- The *delta*

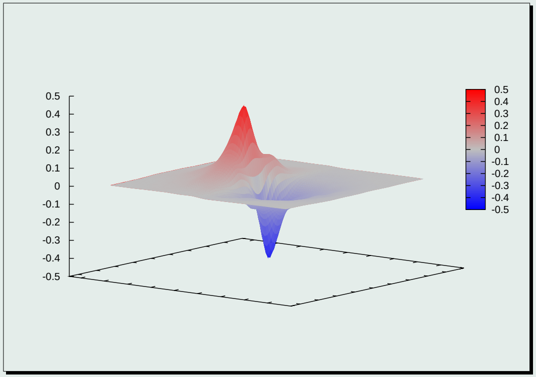
$$O_\Delta^{(k)} = \epsilon_{abc}\,u_{a,\text{smear}}\left(u_{b,\text{smear}}^T\,C\,\gamma_k\,u_{c,\text{smear}}\right)\,, \quad k = 1,2,3$$

Quark sources

- Correlators are built from *quark propagators*
- Quark propagators are calculated from *quark sources*
- Two possibilities: *point source* $\sim$ δ-peak ☹
  smeared source $\sim$ Gaussian ☺
- We use 3 types of smeared sources (sinks): narrow, wide, derivative



Narrow or wide



Derivative

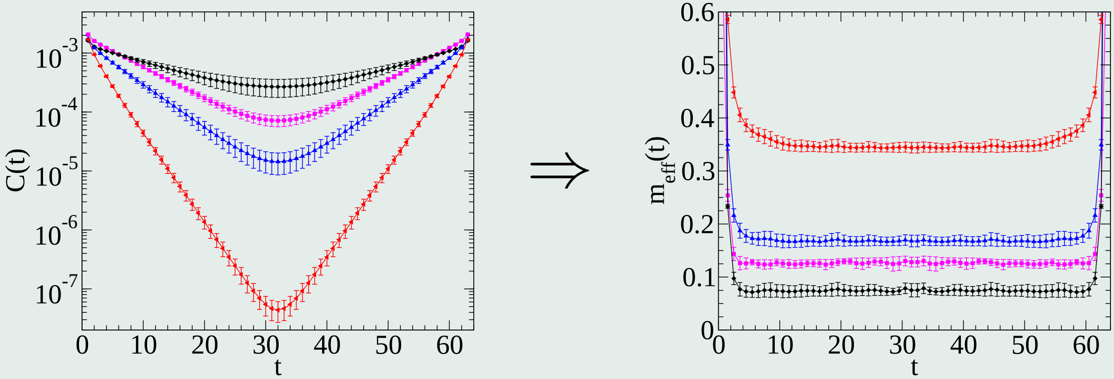
Extraction of masses

- Basic definition of a *correlator*

$$C(t) \equiv \sum_n \langle 0|\mathcal{O}|n\rangle\langle n|\mathcal{O}^\dagger|0\rangle \;\rightsquigarrow\; c_0\, e^{-t\,m_0} + c_1\, e^{-t\,m_1} + c_2\, e^{-t\,m_2} + \ldots$$

- First idea for ground states: Calculate *"effective mass"*

$$m_{\text{eff}}\left(t+\frac{1}{2}\right) = \ln\left|\frac{C(t)}{C(t+1)}\right|$$



Extraction of masses

- More sophisticated: Use a matrix of correlations C_{ij}

$$C_{ij}(t) = \sum_n \langle 0|\mathcal{O}_i|n\rangle\langle n|\mathcal{O}_j^\dagger|0\rangle$$

- Solve the *generalized eigenvalue problem*

$$C(t)\,\boldsymbol{v} = \lambda(t)\,C(t_0)\,\boldsymbol{v}$$

- Each eigenvalue corresponds to one single mass

$$\lambda_k(t) \sim e^{-t\,m_k}\left[1 + \mathcal{O}(e^{-t\,\Delta m_k})\right]$$

- Eigenvectors are *“fingerprints”* of the states

Simulation details

- *Two* mass degenerate *light quarks*
- Improving the speed of the code by several means (e.g., mass preconditioning or a mixed-precision inverter)
- Three ensembles of lattice size $16^3 \times 32$

ensemble	β_{LW}	m_{bare}	#confs	a [fm]	m_π [MeV]	m_{AWI} [MeV]
A	4.70	-0.05	100	0.151(2)	526(7)	43.0(4)
B	4.65	-0.06	200	0.150(1)	469(4)	34.1(2)
C	4.58	-0.077	200	0.144(1)	318(5)	15.3(3)

Partially quenched data points

- We have *one fully dynamical* point ($m_{\text{val}} = m_{\text{sea}}$) ...
- ... and *nine partially quenched* ($m_{\text{val}} > m_{\text{sea}}$) points
- Because we want to learn from heavy quark physics ...
- ... and we want to go from heavy to light quark masses (chiral extrapolation)
- Dynamical point: Full symbol ● ■ ◆
- Partially quenched points: Open symbols ○ □ ◇

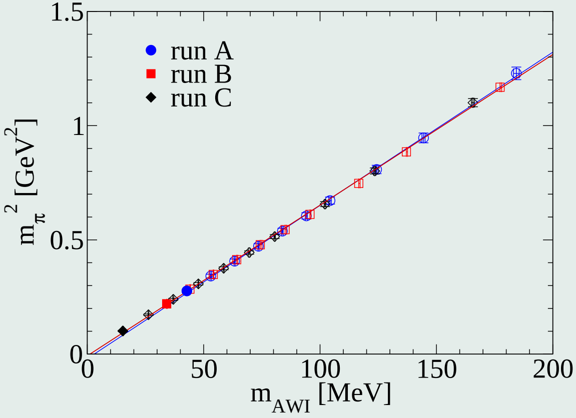
The AWI-mass (PCAC-mass)

- Quark mass from the *axial Ward identity*

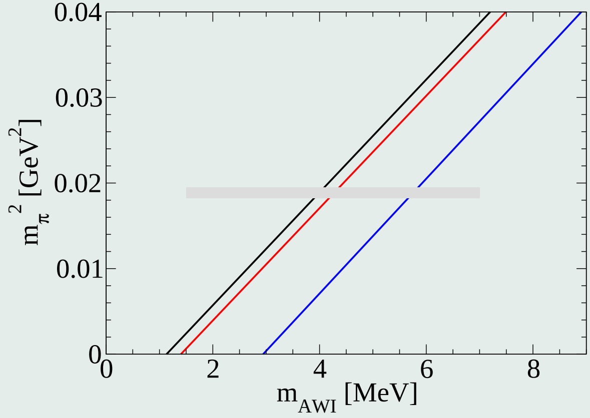
$$m_{\text{AWI}} = \frac{1}{2} \frac{\langle\, \partial_t A_4(t) P(0)\, \rangle}{\langle\, P(t) P(0)\, \rangle}$$

- *Gell-Mann-Oakes-Renner* relation: Connection of m_π and quark mass

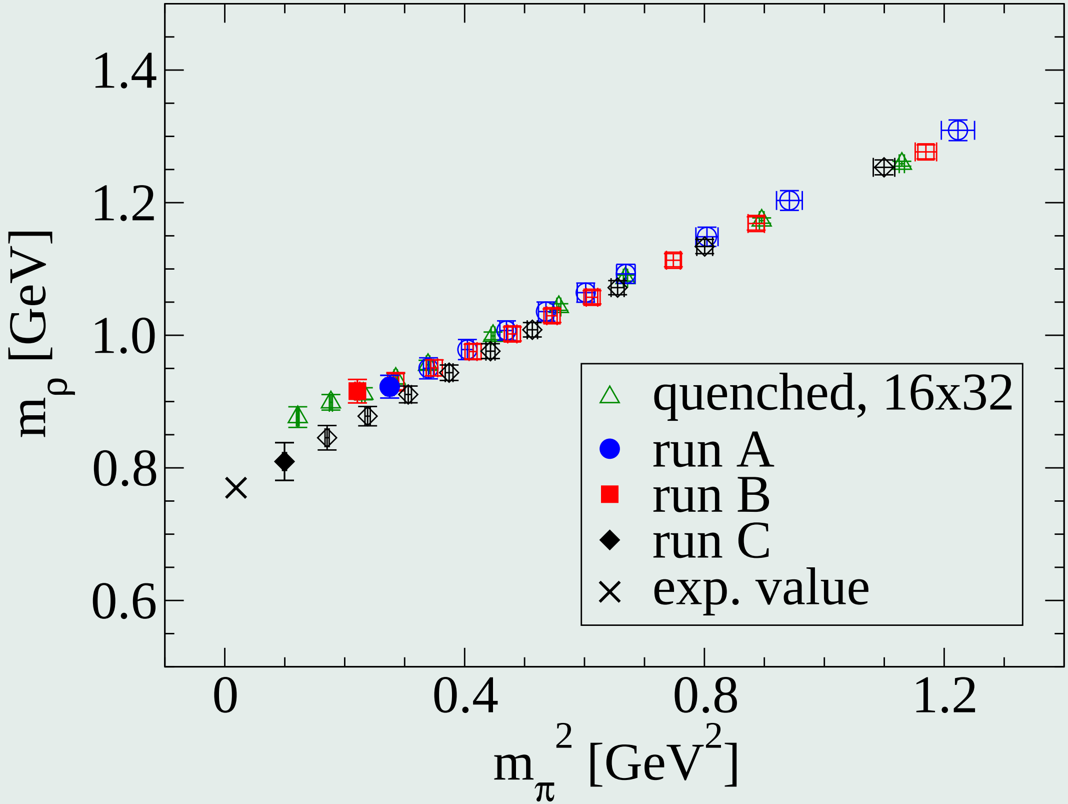
$$f_\pi^2\, m_\pi^2 = -2\, m_{\text{quark}}\, \Sigma$$



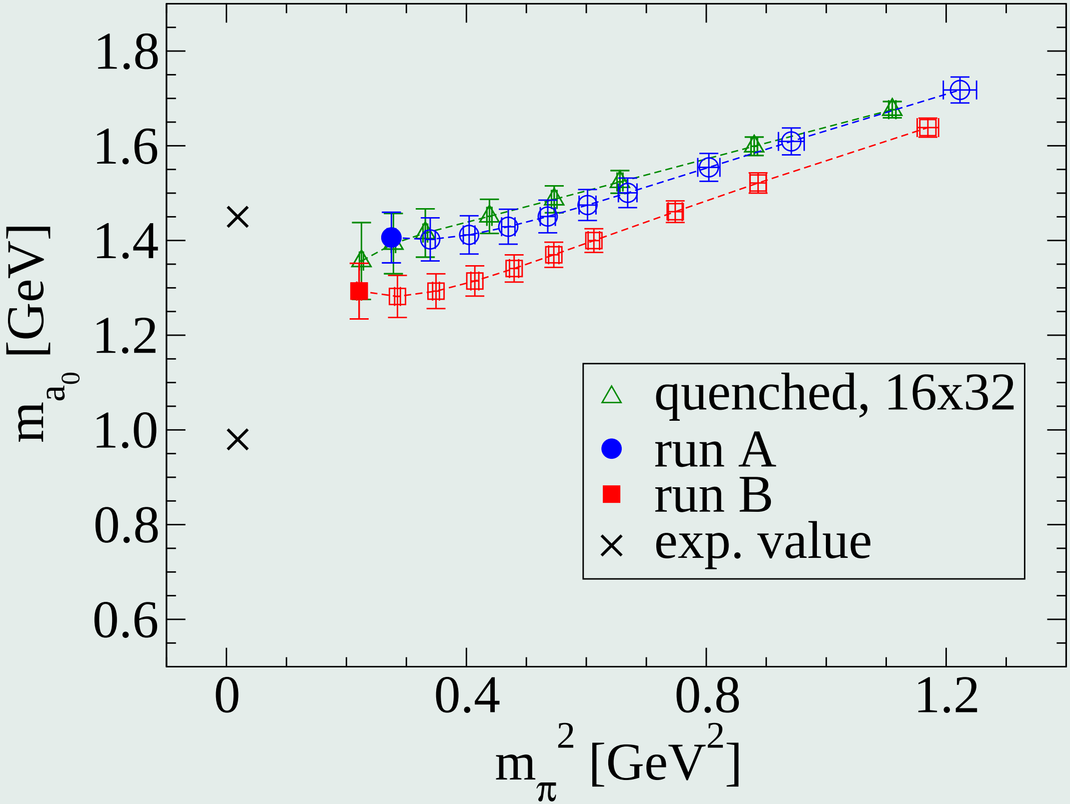
$\Rightarrow$



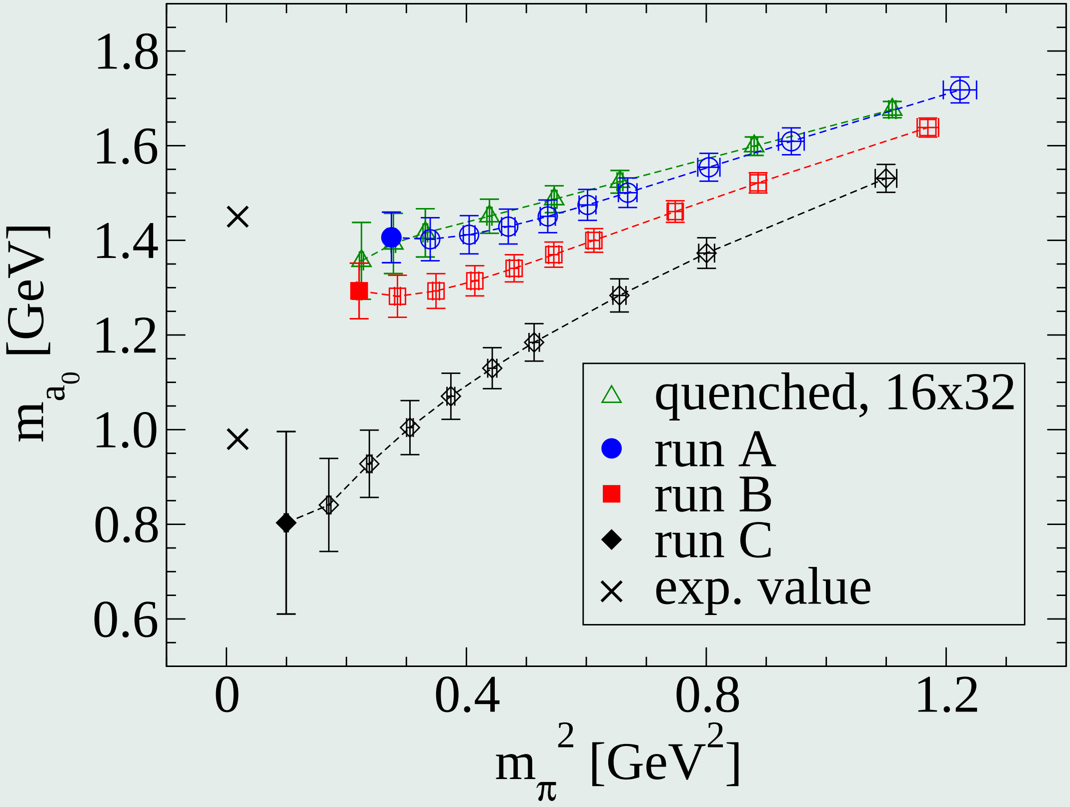
The vector mesons ρ



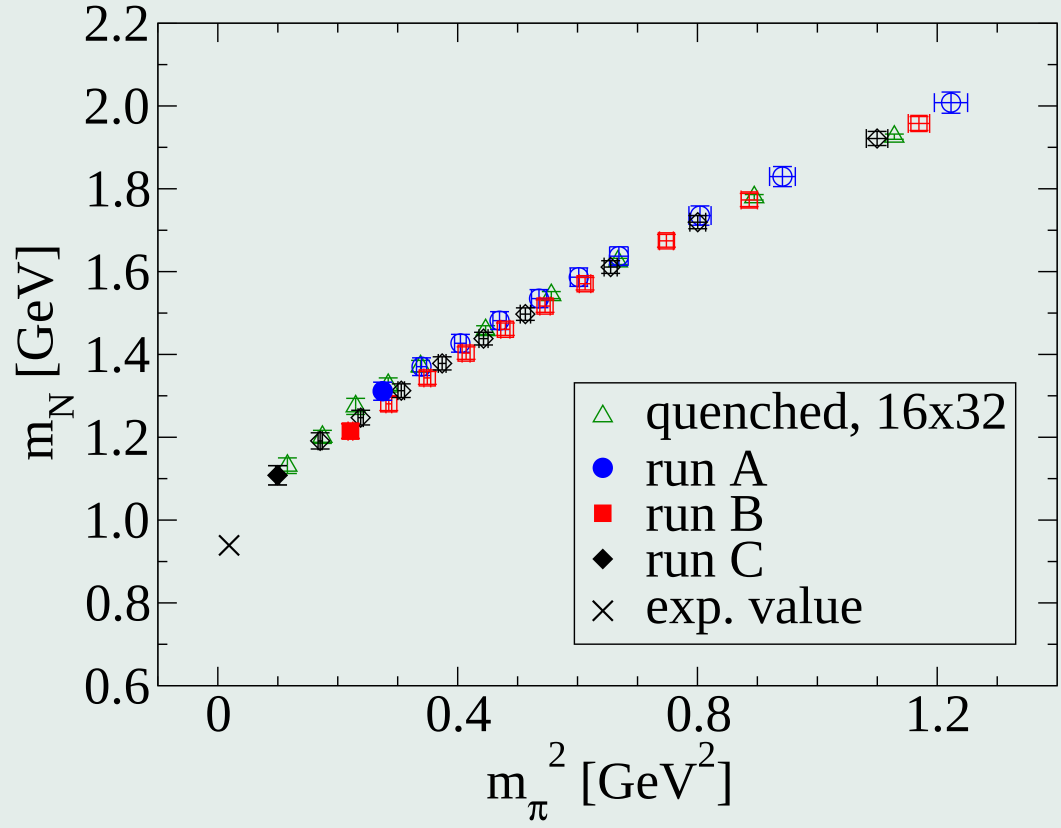
The scalar mesons a_0



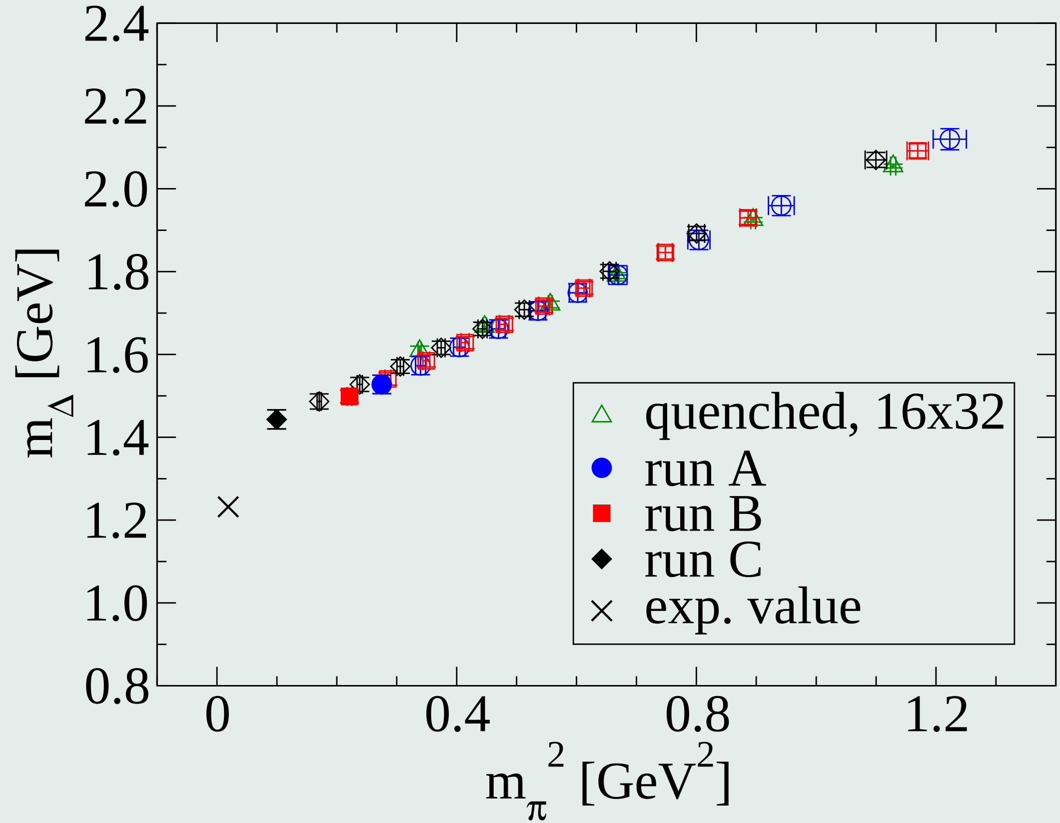
The scalar mesons a_0



The positive parity nucleon



The positive parity delta



Summary

- We introduced the lattice as a non-perturbative regulator of QCD
- First formulation of Wilson in 1974
- Improvments of gauge and fermion action
- We ran a 2-flavor simulation using dynamical CI quarks
- Three ensembles on $16^3 \times 32$ lattices are available
- We tried to enhance spectroscopy results with better quarks sources
- Usage of variational method rather than multi parameter fits to correlators
- AWI-mass lies in the physical region
- Ground states masses come out reliably

Thank you!