Excited hadrons from lattice QCD

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Outline

Standard model

	Fermions		
quarks	u	c	t
	d	s	b
leptons	e^-	μ^-	τ^-
	ν_e	ν_μ	ν_τ

Bosons	
electromagn. force	γ
weak force	W^+, W^-, Z
strong force	gluons

- In the following: Theory of strong interaction (QCD)
- More specifically: Hadron spectroscopy

  Mesons: $\bar{q}q$ states in the quark model
  Baryons: qqq states in the quark model

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Mesons: $\bar{q}q$ states in the quark model
Baryons: qqq states in the quark model

QCD - the theory of the strong interaction

- *Asymptotic freedom:* Interaction arbitrarily weak at small distances $\rightarrow$ Perturbation theory succesful at large energies/ small distance.
- *Confinement:* We do not observe free quarks or gluons in nature

QCD action

$$S_{QCD}[\psi,\bar{\psi},A] = \sum_{f=1}^{N_f} \int d^4x \bar{\psi}^{(f)}(x) \left(\not{D} + m^{(f)} \right) \psi^{(f)} + \frac{1}{2g^2} Tr[F_{\mu\nu}(x) F_{\mu\nu}(x)]$$

$$F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) + i[A_\mu(x), A_\nu(x)]$$

$$\not{D} = \gamma_\mu D_\mu \quad \text{with} \quad D_\mu = \partial_\mu + iA_\mu(x)$$

Why Lattice QCD?

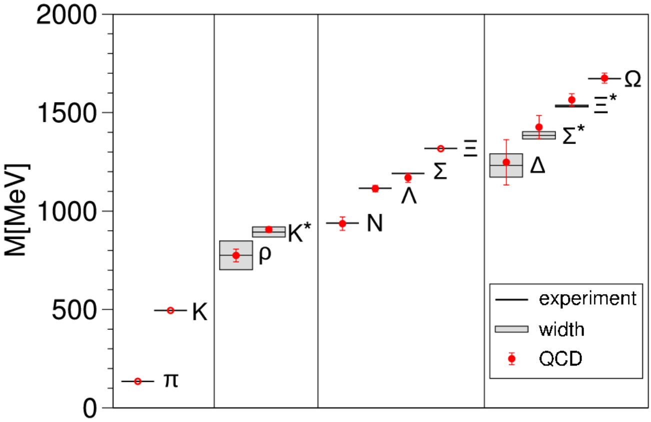
Some Problems can not be attacked with perturbation theory:

- Chiral symmetry breaking
  - Explicit: Non-zero quark masses
  - Spontaneous: The pion is a Goldstone boson
- Confinement and the low energy properties of hadrons
  - Hadron masses
  - Low energy parameters (decay constants, current quark masses, LEC of Chiral Perturbation Theory)
  - Form factors, matrix elements, structure functions

$\Rightarrow$ We need non-perturbative methods!
Lattice QCD is an ab-initio calculation for non-perturbative properties!

Motivation: ground state spectrum

Recent (impressive) results: Postdiction of the ground state spectrum



BMW-collaboration 2008

Euclidean correlators

- Euclidean correlator of two Hilbert-space operators $\hat{O}_1$ and $\hat{O}_2$.

$$\left\langle \hat{O}_2(t)\hat{O}_1(0) \right\rangle_T = \frac{1}{Z_T} \mathrm{tr} \left(e^{-T\hat{H}} e^{t\hat{H}} \hat{O}_2 e^{-t\hat{H}} \hat{O}_1 \right)$$

$$T \to \infty : \qquad \sum_n e^{-t\Delta E_n} < 0|\hat{O}_2|n >< n|\hat{O}_1|0 >$$

$$\text{with} \quad \Delta E_n = E_n - E_0$$

- Can also be expressed as a Euclidean path integral

$$\left\langle \hat{O}_2(t)\hat{O}_1(0) \right\rangle_T = \frac{1}{Z_T} \int \mathcal{D}[\psi, \bar{\psi}, U] e^{-S_E} O_2[\psi, \bar{\psi}, U] O_1[\psi, \bar{\psi}, U],$$

$$Z = \int \mathcal{D}[\psi, \bar{\psi}, U] e^{-S_E}.$$

- No field operators appear on the right.
- "Simple" integral over the classical Euclidean action
- Can be evaluated with an (importance sampling) Markov chain Monte Carlo

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Lattice CQD

Regularization of QCD by a 4-d Euclidean space-time lattice. (Kenneth Wilson 1974)

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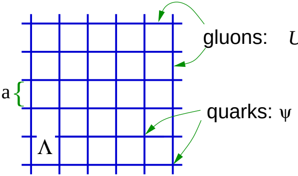

- Lattice $\Lambda = \{\vec{n} = (n_1, n_2, n_3, n_4) | n_i \in \{0, 1, \ldots, L_i - 1\}\}$
- $\vec{x} \quad \rightarrow \quad a\vec{n}$ with lattice spacing a
- $\int d^4x \ldots \quad \rightarrow \quad a^4 \sum_{n \in \Lambda} \cdots$
- $D_\mu \psi(\vec{x})$ suitable covariant lattice derivative

The Wilson gauge action

- Required: gauge invariance of S_G
- Gauge invariant quantity: Trace over any closed loop $\mathcal{L}$

$$L[U] = \mathrm{tr}\left[\prod_{(n,\mu)\in\mathcal{L}} U_\mu(n)\right]$$

- Simplest possible such loop: plaquette

$$U_{\mu\nu}(n) = U_\mu(n)U_\nu(n+\hat{\mu}) \cdot U_{-\mu}(n+\hat{\mu}+\hat{\nu})U_{-\nu}(n+\hat{\nu})$$

[scale=0.3]plaquette.eps

- Wilson gauge action: sum over all plaquettes

$$S_G[U] = \frac{2}{g^2}\sum_{n\in\Lambda}\sum_{\mu<\nu} ReTr(1 - U_{\mu\nu}(n))$$

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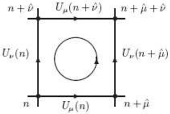

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The Wilson fermion action

- Naive fermion action contains doublers
  Quark propagator has additional unphysical poles
- The doublers can be removed by adding a Wilson term to the lattice Dirac operator

$$D^{(f)}(n|m)^{\alpha\beta}_{ab} = -\frac{1}{2a}\sum_{\mu=\pm 1}^{\pm 4}(1-\gamma_\mu)_{\alpha\beta}U_\mu(n)_{ab}\delta_{n+\hat{\mu},m} + \left(m^{(f)} + \frac{4}{a}\right)\delta_{\alpha\beta}\delta_{ab}\delta_{mn}$$

with $\gamma_{-\mu} = -\gamma_\mu$

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Chiral symmetry on the lattice

- Continuum chiral symmetry $\{D, \gamma_5\} = 0$
- Lattice implementations of above equation contain doublers (Nielsen-Ninomiya theorem)
- Ginsparg-Wilson relation:

$$D\gamma_5 + \gamma_5 D = aD\gamma_5 D$$

$\rightarrow$ Lattice version of chiral symmetry

- Ginsparg-Wilson fermions: Fermions with full chiral symmetry.
- Construction of an approximate solution $\rightarrow$ talk by M. Limmer 2 weeks ago.

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Monte-Carlo simulation of fermions

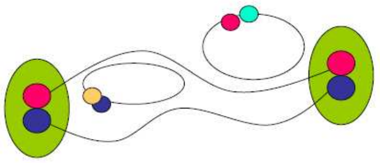
"Full QCD":

$$C(t) \propto \int \mathcal{D}[U]\, \mathcal{D}[\psi,\bar\psi]\, \mathrm{e}^{-S_G[U]-\bar\psi D[U]\psi}\, N(t)\bar N(0)$$

$$= \int \mathcal{D}[U]\, \mathrm{e}^{-S_G[U]}\, (\det D_u \det D_d \ldots)$$

$$\times \left[D_u^{-1} D_d^{-1} \ldots + \ldots\right]$$

- Set $\det D \equiv 1$ (no dynamical fermion vacuum, i.e. no sea quarks)
- Gauge field vacuum is fully dynamical (Monte Carlo)
- Consider only the valence quarks
- Hadron correlation functions are built from the quark propagators



Monte-Carlo simulation of fermions

Quenched approximation:

$$C(t) \propto \int \mathcal{D}[U]\, \mathcal{D}[\psi,\bar{\psi}]\, \mathrm{e}^{-S_G[U]-\bar{\psi}\,D[U]\,\psi}\, N(t)\bar{N}(0)$$
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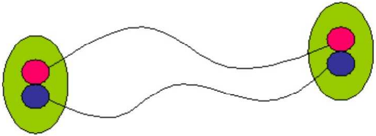
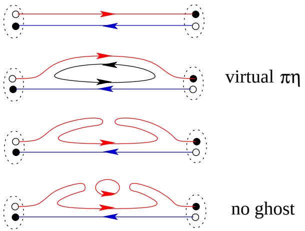


Quenched versus full QCD

- quenched approximation is unphysical; motivated by the lack of computing power
- quenched artifacts (“ghosts”) from hairpin diagrams
  Example (a_0 channel):

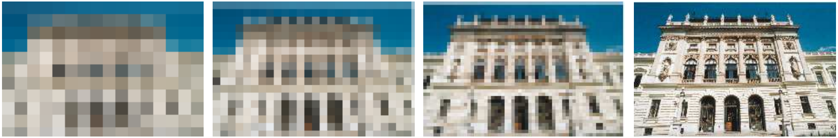


- seems to work surprisingly well for most ground states
- Scattering states are (in some cases) not present

Extrapolations: Continuum limit

Continuum limit: $a(g, m) \to 0$

- Lattice artifacts should become small!
- Different lattice actions should lead to same result in the continuum limit!
- Need simulations at multiple different lattice spacings
- Constructions of *improved actions* helps



Extrapolations: Thermodynamic limit

Thermodynamic limit: $L \to \infty \quad (L \cdot a = const.)$

- Hadron physics in a box of a few fm $\to$ finite volume effects
- Need relatively large boxes for excited states ($am_\pi L \geq 4$)
- Might need even larger boxes for axial charge, ...
- Finite volume effects can be utilized to identify scattering states/resonances!

Extrapolations: Chiral extrapolation

Chiral limit: $m \rightarrow m_0 \quad (M_\pi \rightarrow M_{\pi,exp})$

- Physical u, d quark masses small $\rightarrow$ Simulation very expensive!
- Chiral Perturbation Theory (χPT) $\leftrightarrow$ Lattice QCD
- We want to understand chiral symmetry breaking!
- What about excited states?

The problem with excited states

$$\left\langle \hat{O}_2(t)\hat{O}_1(0)\right\rangle_T \propto \sum_n e^{-tE_n} < 0|\hat{O}_2|n >< n|\hat{O}_1|0 >$$

- The whole tower of states contributes
- Ground state is dominant at large t
- Exited states appear as sub-leading exponentials
- Fit to several exponentials leads to poor results/ is unstable

Methods for disentangling the states

- *Bayesian analysis* (stepwise reduction of exponential with biased estimators) Minimize

$$F = \chi^2 + \lambda\,\phi,$$

  where ϕ is a stabilizing function of the fit parameters (prior).
  Used by Mathur et al. (05,06), Lee et al (03), Juge et al. (06), Zanotti et al. (03), Melnichouk et al.(03)
- Reconstruction of spectral density with *maximum entropy method*
  Used by Sasaki et al. (05)
- *Variational analysis*
  Used by Burch et al. (03-06), Basak et al. (05, 06), ...
- *Evolutionary algorithms*
  Used by Petry et al.(07)

Variational method

Variational method (C.Michael; Lüscher and Wolff)
Matrix of correlators projected to fixed momentum (will assume 0)

$$C(t)_{ij} = \sum_n \mathrm{e}^{-tE_n} \langle 0|O_i|n\rangle \left\langle n|O_j^\dagger|0\right\rangle$$

Solve the generalized eigenvalue problem:

$$C(t)\vec{\psi}^{(k)} = \lambda^{(k)}(t)C(t_0)\vec{\psi}^{(k)}$$
$$\lambda^{(k)}(t) \propto \mathrm{e}^{-tE_k}\left(1+\mathcal{O}\left(\mathrm{e}^{-t\Delta E_k}\right)\right)$$

At large time separation: only a single mass in each eigenvalue.
Eigenvectors can serve as a fingerprint.

Variational method II

- Interesting observation for $t \leq 2t_0$

$$\Delta M_k \equiv M_{N+1} - M_k$$

Blossier et al. [arXiv: 0808.1017]

- Can also determine couplings:

$$C(t) = \sum_{n=1}^{\infty} v_i^{(n)} v_i^{(n)\star} \mathrm{e}^{-tE^{(n)}} \quad \text{with} \quad v_i^{(n)} = \left\langle 0 | O_i | H^{(n)} \right\rangle$$

$$R(t)_i^{(k)} = \frac{|\sum_j C(t)_{ij} \psi_j^{(k)}|^2}{\sum_k \sum_l \psi_k^{(k)\star} C(t)_{kl} \psi_l^{(k)}} \approx v_i^{(k)} v_i^{(k)\star} \mathrm{e}^{-tE^{(k)}}$$

Burch et al. [arXiv: 0809.1103]

Angular momentum (mesons)

- *Reminder:* No unique spin assignment on the lattice.
  Five irreducible representations:

Irrep of O	J	Spinors in irrep
A_1	$0,4,\ldots$	$1,\gamma_t,\gamma_5,\gamma_t\gamma_5$
A_2	$3,6,\ldots$	
E	$2,4,5,\ldots$	
T_1	$1,3,4,5,\ldots$	$\gamma_i,\gamma_t\gamma_i,\ \gamma_5\gamma_i,\gamma_t\gamma_5\gamma_i$
T_2	$2,3,4,5,\ldots$	

- Classification of interpolator basis by representations
- Identify spin by degeneracies in various representations

Approach I: displaced quarks

- Quarks with a relative displacement connected by paths
- Take combinations of such paths belonging to a definite representation with a definite *PC*
- Can use straight, L-shaped, U-shaped or plaquette type of paths for mesons

  C. Michael, Lacock et al., Trinlat; Petry et al.
- Same approach is used by LHPC collaboration for baryons

  Basak et al., LHPC Collaboration , . . .

Approach II: Smeared quarks

- Smearing of quarks from a single point to create extended objects.
- Classify those smearings by representations and PC.
- Charm sector: Liao and Manke, Dudek et al., ...
  Light sector: Burch et al., Gattringer et al., ...

- For both approaches: Classify by operator $\otimes$ path/smearing.
  Example:

$$T_1 \otimes T_1 = A_1 \oplus E \oplus T_1 \oplus T_2$$

Quark sources

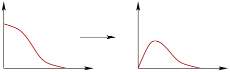
- Jacobi smeared quark sources, e.g., $u_s \equiv (S\,u)_x$

$$S = M\,S_0 \quad \text{with} \quad M = \sum_{n=0}^{N} \kappa^n H^n$$

$$H(\vec{n},\vec{m}\,) = \sum_{j=1}^{3} \left(U_j\,(\vec{n},0)\,\delta\left(\vec{n}+\hat{j},\vec{m}\right) + U_j\left(\vec{n}-\hat{j}\,,0\right)^{\dagger}\delta\left(\vec{n}-\hat{j},\vec{m}\right)\right).$$

- Fewer quark propagators
- Combination allows nodes in the interpolating operators
- Derivative quark sources W_{d_i}:

$$D_i(\vec{x},\vec{y}) = U_i(\vec{x},0)\delta(\vec{x}+\hat{\imath},\vec{y}) - U_i(\vec{x}-\hat{\imath},0)^{\dagger}\delta(\vec{x}-\hat{\imath},\vec{y})\;,$$
$$W_{d_i} = D_i\,S_w\;.$$

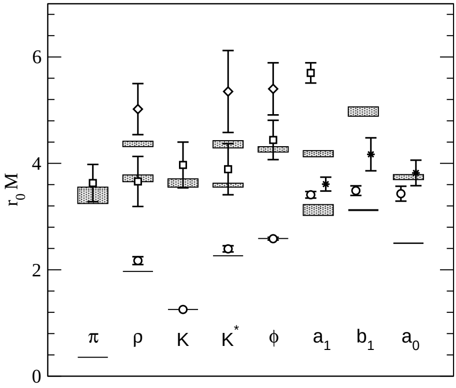


Some things people work on...

- Light mesons (Burch et al., Gattringer et al.; LHP Collaboration; Petry et al., Kentucky group,. . .)
- Baryons (Burch et al., Basak et al., LHP Collaboration; Mathur et al.; Lasscock et al., . . .)
- Charmonium (Liao and Manke, Dudek et al., Bali and Ehmann, . . .)
- Heavy-light mesons (Burch et al.; Sommer at al., . . .)
- Tetraquarks (Kentucky group; Prelovsek et al., . . .)

Mesons from quenched QCD

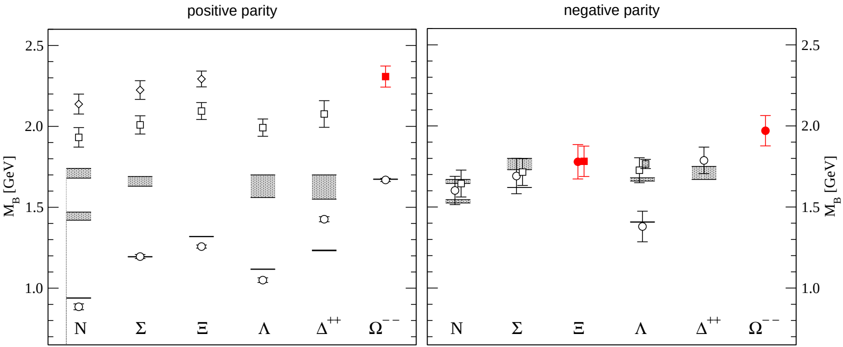
(Naive) chiral extrapolations: Mesons



Burch et al., PRD 73 (2006) 094505

Baryons from quenched QCD

(Naive) chiral extrapolations: Baryons



Burch et al., PRD 74 (2006) 014504

Excited pions I

Mesons with derivative sources: Pion channel

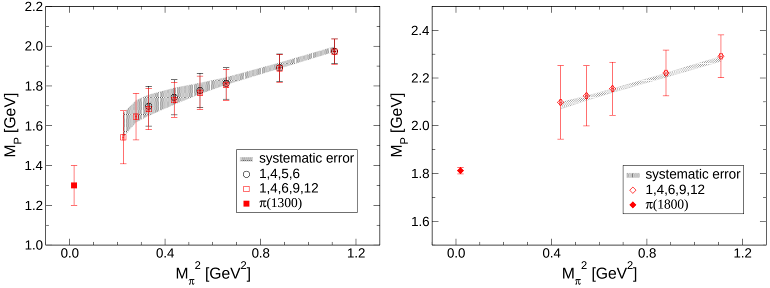


Figure: 1st and 2nd excitation of π

Gattringer et al., PRD 78 (2008) 034501

Excited pions II

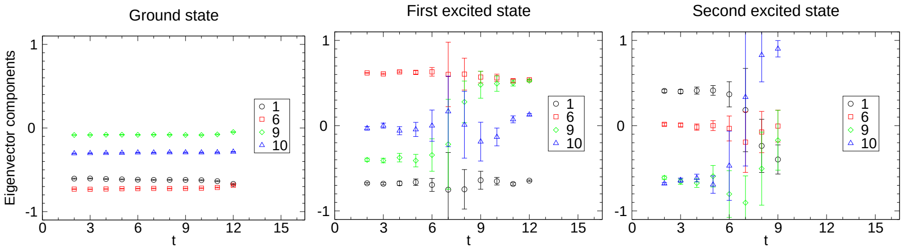
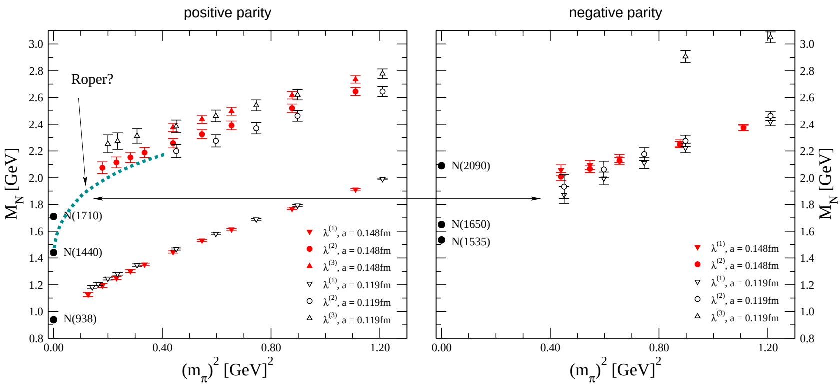


Figure: Eigenvector components for ground state and lowest excitations

Gattringer et al., PRD 78 (2008) 034501

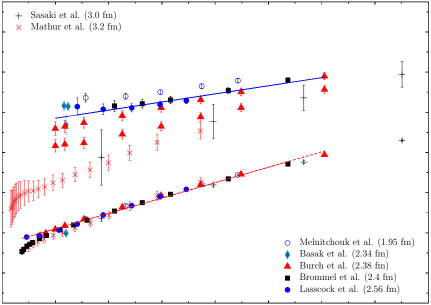
The Roper puzzle I



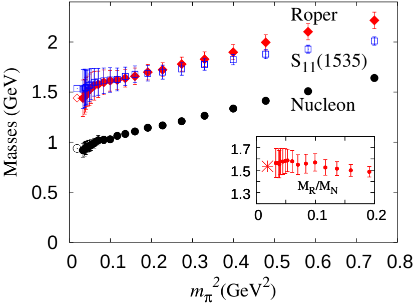
Burch et al., PRD 74 (2006) 014504

Level crossing (+-+-) to (++–)?

The Roper puzzle II



From Lasscock et al., PRD 76 (2007) 054510

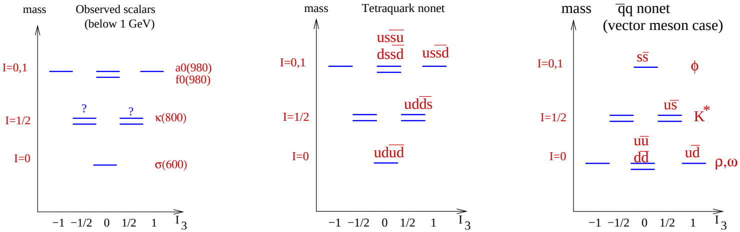


From Mathur et al., Phys. Lett. B 605 137 (2005)

Origin of this discrepancy is still unclear!

The scalar meson puzzle

- Low lying scalars could be tetraquark states



- quark models would place $\bar{q}q$ with $L = 1$ above 1GeV
- $m_\kappa < m_{a_0}$ hard to reconcile with $\bar{u}s$ and $\bar{u}d$
- $a_0(980)$ couples well with $K\bar{K}$

Mesons with derivative sources: Isovector Scalar 1 0^{++}

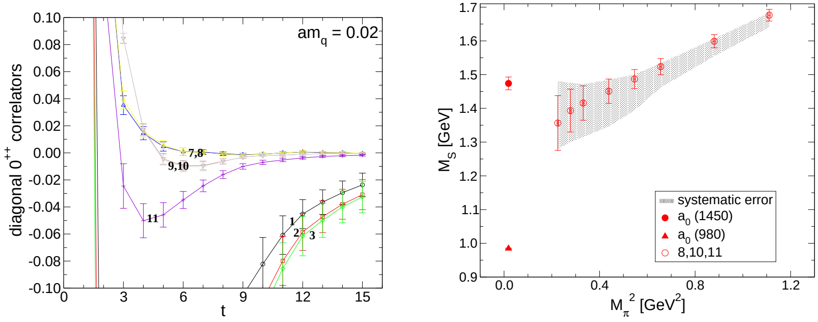


Figure: Diagonal correlators and ground state in the 0^{++} channel

Gattringer et al., PRD 8 (2008) 034501

Other quenched $\bar{q}q$ results compatible with $m_{a_0} \approx 1.45\mathrm{GeV}$
Full QCD?

Full QCD: a_0 revisited

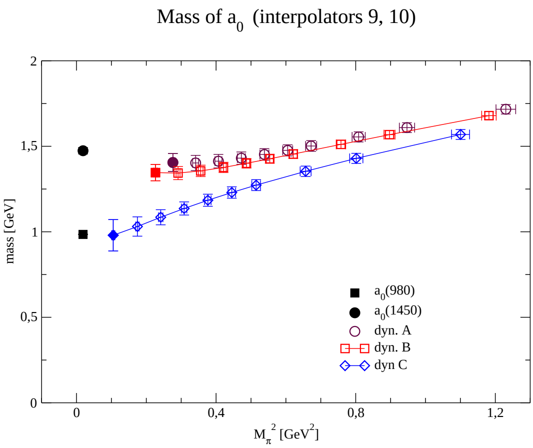


Figure: a_0 from one possible set of interpolators

Do we see a level crossing here ($\pi\eta$-channel?)

The role of scattering states

- Contributions from scattering states:

$$C(t) = \frac{1}{Z}\Big(|\langle 0|\mathcal{O}|P_1 P_2\rangle|^2 e^{-E^{P_1P_2}\, t} + |\langle P_1^\dagger P_2^\dagger|\mathcal{O}|0\rangle|^2 e^{-E^{P_1P_2}(T-t)}$$
$$+ |\langle P_1^\dagger|\mathcal{O}|P_2\rangle|^2 e^{-E_{P_1}(T-t)} e^{-E_{P_2}t} + |\langle P_2^\dagger|\mathcal{O}|P_1\rangle|^2 e^{-E_{P_2}(T-t)} e^{-E_{P_1}t} \Big)$$

  Fit form will get more complicated!
- Multiple volumes useful/needed to identify scattering states (via volume dependence of spectral weights)
- Different choices of boundary conditions might simplify the problem
- Similar terms occur for the negative parity nucleon.

The role of scattering states

- Contributions from scattering states:

$$C(t) = \frac{1}{Z}\Big(|\langle 0|\mathcal{O}|P_1P_2\rangle|^2 e^{-E^{P_1P_2}\ t} + |\langle P_1^\dagger P_2^\dagger|\mathcal{O}|0\rangle|^2 e^{-E^{P_1P_2}(T-t)}$$
$$+ |\langle P_1^\dagger|\mathcal{O}|P_2\rangle|^2 e^{-E_{P_1}(T-t)} e^{-E_{P_2}t} + |\langle P_2^\dagger|\mathcal{O}|P_1\rangle|^2 e^{-E_{P_2}(T-t)} e^{-E_{P_1}t}\Big)$$

  Fit form will get more complicated!
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Tetraquarks in the scalar channel

Tetraquarks with a diquark - anti-diquark structure

$$[qQ]_a \equiv \epsilon_{abc}[q_b^T C\gamma_5 Q_c - Q_b^T C\gamma_5 q_c]$$
$$[\bar{q}\bar{Q}]_a \equiv \epsilon_{abc}[\bar{q}_b C\gamma_5 \bar{Q}_c^T - \bar{Q}_b C\gamma_5 \bar{q}_c^T]\ ,$$

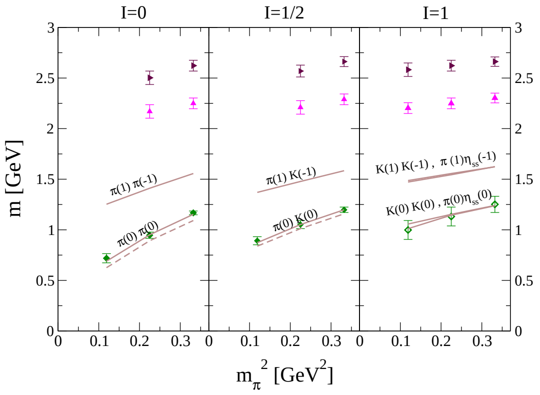
We simulate

$$\mathcal{O}^{I=0} = [ud][\bar{u}\bar{d}]\ , \quad \mathcal{O}^{I=1/2} = [ud][\bar{d}\bar{s}]\ , \quad \mathcal{O}^{I=1} = [us][\bar{d}\bar{s}]$$

with different source/sink smearings and use the variational method.

S.Prelovsek and D.M., arXiv:0810.1759

Scalar tetraquarks - results



Main conclusion: No evidence of low lying scalar tetraquarks for M_π in the range $344 \ldots 576\mathrm{MeV}$

S.Prelovsek and D.M., arXiv:0810.1759

Excited charmonium

Method: Variational method with a large basis of standard and hybrid interpolators

Interesting states:

- 1^{-+} and 2^{+-} exotics
- X(3872) probably 1^{++} ; first excitation
- Z(3920) 2^{++} ; first excitation
- Y(4260) 1^{--} ; 7th+ state in that representation? tetraquark?

So far (on the lattice):

- Quenched study without continuum extrapolation (Dudek et al.)
- Ongoing $N_F = 2$ study on QCDSF lattices (Bali, Ehmann)
- Results from the CLQCD Collaboration claiming identification of the X(3872)
- Some questionable lattice results claiming identification of the Y(4260) as either a hybrid meson or a tetraquark.

Interpolating fields: Charmonium

$$\overrightarrow{D}_i$$

$$\mathbb{B}_i = \epsilon_{ijk} \overrightarrow{D}_j \overrightarrow{D}_k = -\frac{\mathrm{i}}{2} \epsilon_{ijk} F^{jk}$$

$$\mathbb{D}_i = |\epsilon_{ijk}| \overrightarrow{D}_j \overrightarrow{D}_k$$

+ Gaussian and point sources/sinks

- (Anti-)symetrization $\overleftrightarrow{D}_i = \overleftarrow{D}_i - \overrightarrow{D}_i$ needed for non-vanishing momentum
- Hybrid operators can have exotic quantum numbers:

$$0^{--}, 0^{+-}, 1^{-+}, 2^{+-}$$

$$3^{-+} \quad \text{can be reached by} \quad \mathbb{A} = |\epsilon_{ijk}| D_i D_j D_k$$

- Corresponds to paths in T_1^{--}, T_1^{+-}, T_2^{++}, A_2^{--} representations
- Enables us to reach all representations

Summary

- Excited states are difficult on the lattice
- Variational method can serve to reliably extract excited states
- Extrapolations will have to be controlled
- Resonances and scattering states make the situation even more difficult