Hadron Spectroscopy with Dynamical Chirally Improved Fermions

Markus Limmer

Karl–Franzens–Universität Graz

Advisor: Christian B. Lang

Co-workers: G. Engel and D. Mohler

October 2, 2008

What is hadron spectroscopy?

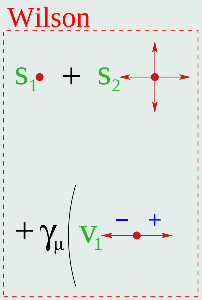
- In general: Study of hadronic properties
- Matrix elements (weak decays) ☺
- Hadronic decays ☹
- Decay constants (e.g., f_π, f_ρ) ☺
- Ground state masses ☺
- First excited state masses 😐
- Second and higher excited state masses ☹

What are dynamical Chirally Improved Fermions?

- Dynamical = Full theory, no quenching!
- Chirally Improved fermions are a truncated solution to the Ginsparg-Wilson relation: $\not{D}\,\gamma_5 + \gamma_5 \not{D} = a\,\not{D}\,\gamma_5\,\not{D}$
- Use the general ansatz $D_{nm} = \sum_{i=1}^{16} c_{nm}^{(i)}(U)\;\Gamma_i$

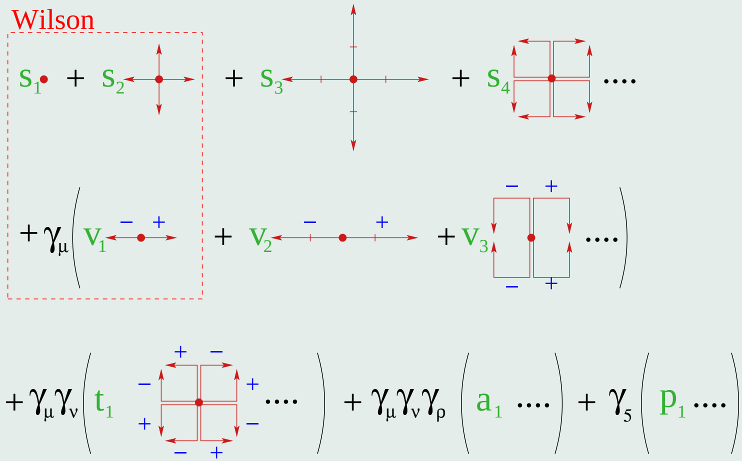
What are dynamical Chirally Improved Fermions?

- Dynamical = Full theory, no quenching!
- Chirally Improved fermions are a truncated solution to the Ginsparg-Wilson relation: $\not{D}\,\gamma_5 + \gamma_5 \not{D} = a\,\not{D}\,\gamma_5\,\not{D}$
- Use the general ansatz $D_{nm} = \sum_{i=1}^{16} c_{nm}^{(i)}(U)\;\Gamma_i$



What are dynamical Chirally Improved Fermions?

- Dynamical = Full theory, no quenching!
- Chirally Improved fermions are a truncated solution to the Ginsparg-Wilson relation: $\not{D}\,\gamma_5 + \gamma_5 \not{D} = a\,\not{D}\,\gamma_5\,\not{D}$
- Use the general ansatz $D_{nm} = \sum_{i=1}^{16} c_{nm}^{(i)}(U)\;\Gamma_i$



Simulation details

- Two mass degenerate light quarks using CI fermions
- Lüscher–Weisz gauge action (incorporating one level of stout smearing)
- Standard Hybrid Monte–Carlo algorithm
- Mass preconditioning via the "Hasenbusch trick"
- Mixed–precision inverter for the conjugate gradient
- Three ensembles of lattice size $16^3 \times 32$

ensemble	β_{LW}	m_0	#confs	a [fm]	m_π [MeV]	m_{AWI} [MeV]
A	4.70	-0.05	100	0.151(2)	526(7)	43.0(4)
B	4.65	-0.06	200	0.150(1)	469(4)	34.1(2)
C	4.58	-0.077	200	0.144(1)	318(5)	15.3(3)

How to fix the scale?

- Smoothing of the configurations (Hypercubic smearing)
- Lattice spacing a determined from static quark potential

$$V_L(r) = A + \frac{B}{r} + \sigma\, r + C\,\Delta V(r) \quad , \quad \Delta V(r) = \left[\frac{1}{\boldsymbol{r}}\right] - \frac{1}{r}$$

(perturbative lattice Coulomb potential $[1/\boldsymbol{r}]$)

- Sommer parameter fixed to $r_0^{(\text{exp})} = 0.48$ fm
- Lattice spacing a via

$$\frac{r_0^{(\text{exp})}}{a} = \sqrt{\frac{1.65 + B}{\sigma}}$$

Let's extract the masses! ... but how?

- Basic definition of a *correlator*

$$C(t) \equiv \sum_n \langle 0|\mathcal{O}|n\rangle\langle n|\mathcal{O}^\dagger|0\rangle \;\rightsquigarrow\; c_0\, e^{-t\, m_0} + c_1\, e^{-t\, m_1} + c_2\, e^{-t\, m_2} + \dots$$

- We use a matrix of correlations: $C_{ij}(t) = \sum_n \langle 0|\mathcal{O}_i|n\rangle\langle n|\mathcal{O}_j^\dagger|0\rangle$
- Solve the generalized eigenvalue problem

$$C(t)\,\boldsymbol{v} = \lambda(t)\,C(t_0)\,\boldsymbol{v}$$

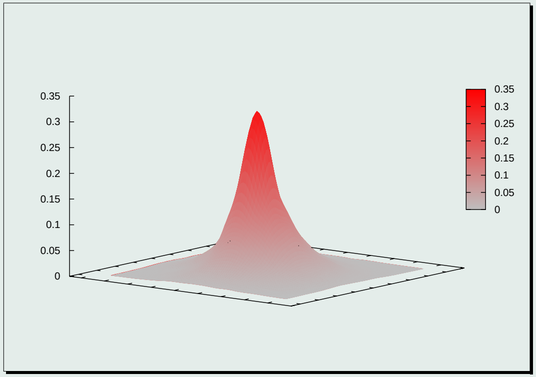
- Each eigenvalue corresponds to one single mass

$$\lambda_k(t) \sim e^{-t\, m_k}\left[1 + \mathcal{O}(e^{-t\,\Delta m_k})\right]$$

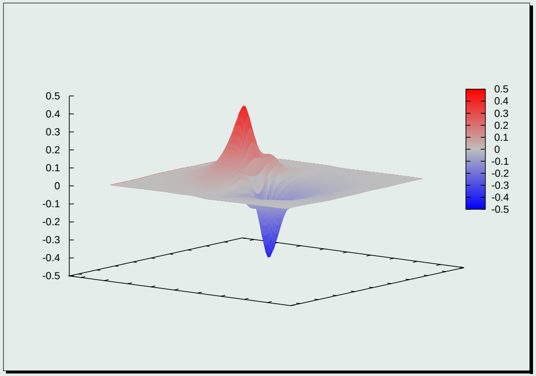
- Eigenvectors are "fingerprints" of the states

Where do the different interpolators come from?

- We let quarks propagate from time 0 to some time t
- Quarks are represented by *sources* (time 0) and *sinks* (time t)
- Two possibilities: *point source (sink)* $\sim$ δ-peak ☹
  smeared source (sink) $\sim$ Gaussian ☺
- We use 3 types of smeared sources (sinks): narrow, wide, derivative



Narrow or wide



Derivative

Results for mesons

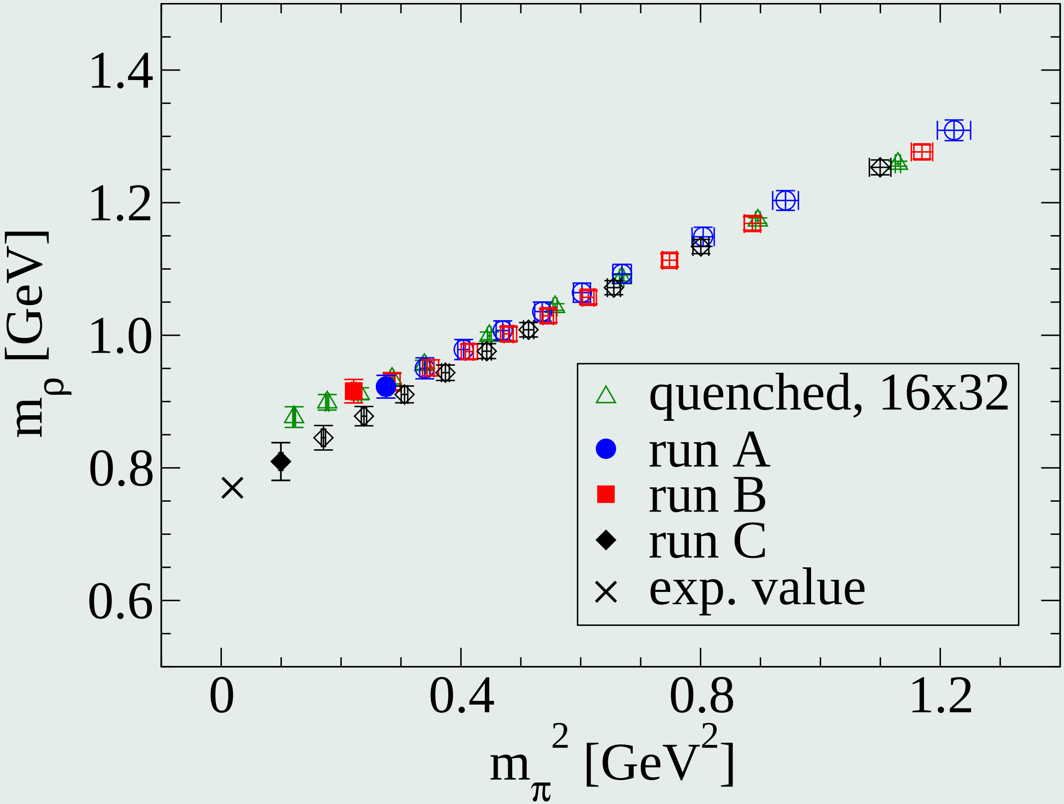
- General form of a meson is

$$O(n) = \overline{\psi}^{(f_1)}(n)\ \Gamma\ \psi^{(f_2)}(n)$$

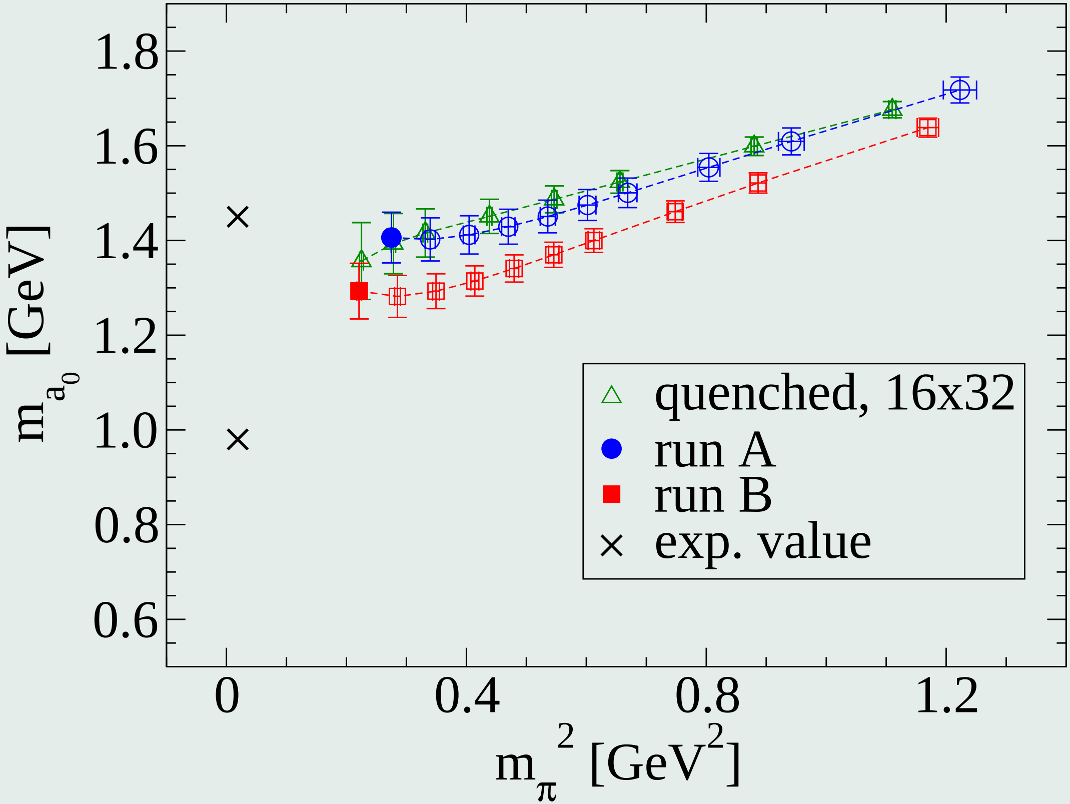
- We analyze 5 different types of isovector mesons

state	J^{PC}	Γ	particle
scalar	0^{++}	$\mathbf{1}, \gamma_t$	a_0
pseudoscalar	0^{+-}	$\gamma_5, \gamma_t\gamma_5$	π
vector	1^{--}	$\gamma_i, \gamma_t\gamma_i$	ρ
axialvector	1^{++}	$\gamma_i\gamma_5$	a_1
pseudovector	1^{+-}	$\gamma_i\gamma_j$	b_1

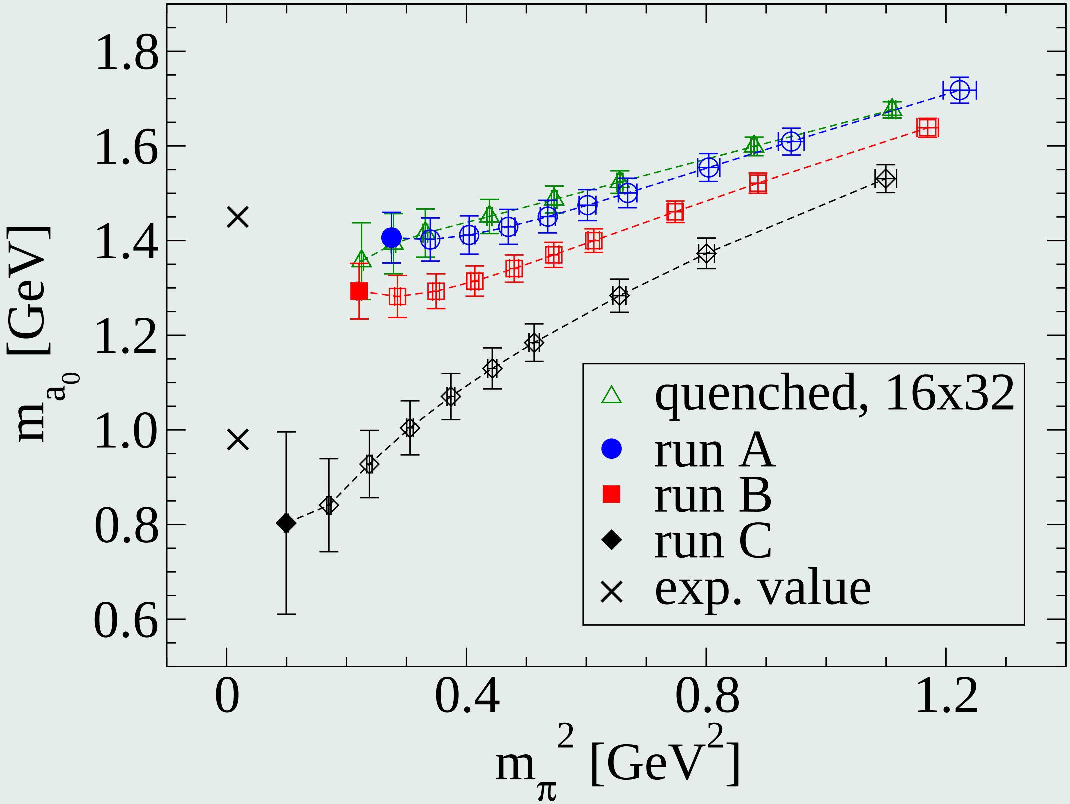
The vector mesons ρ



The scalar mesons a_0



The scalar mesons a_0



Results for baryons

- We analyze 2 different baryons with positive and negative parity
- The nucleon

$$O_N^{(j)} = \epsilon_{abc}\, \Gamma_1^{(j)}\, u_a \left(u_b^T\, \Gamma_2^{(j)}\, d_c - d_b^T\, \Gamma_2^{(j)}\, u_c \right)$$

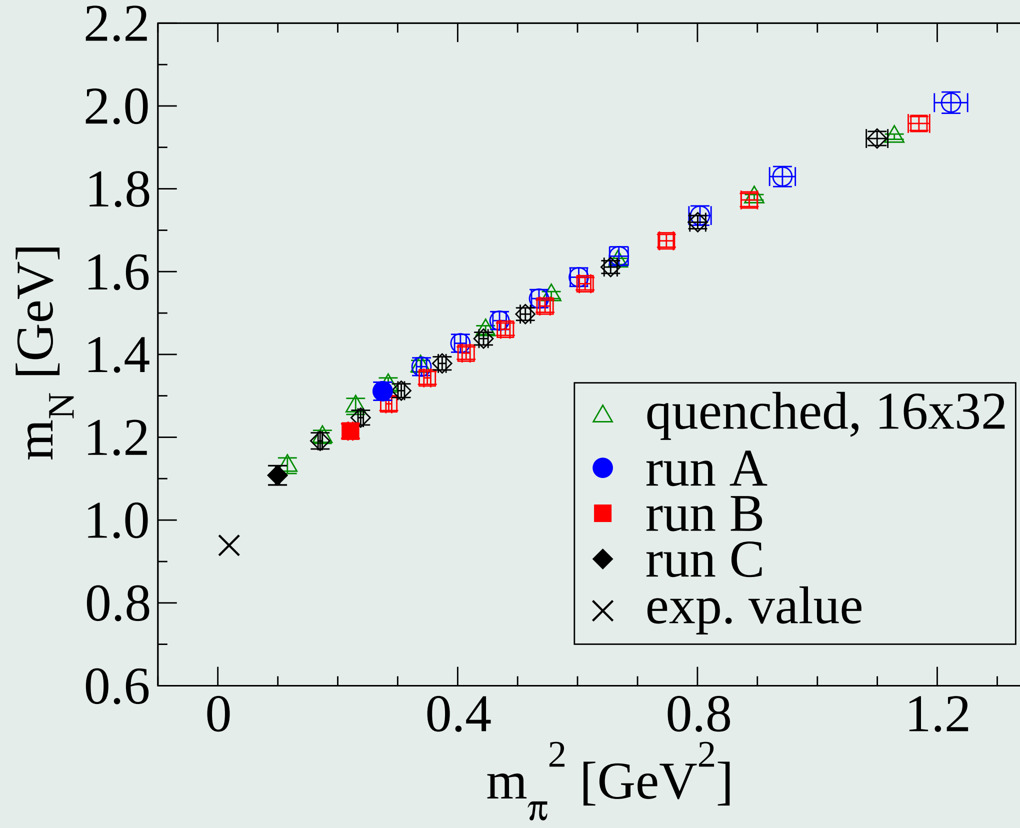
j	$\Gamma_1^{(j)}$	$\Gamma_2^{(j)}$
1	$\mathbf{1}$	$C\gamma_5$
2	γ_5	C
3	i	$C\gamma_t\gamma_5$

- The delta

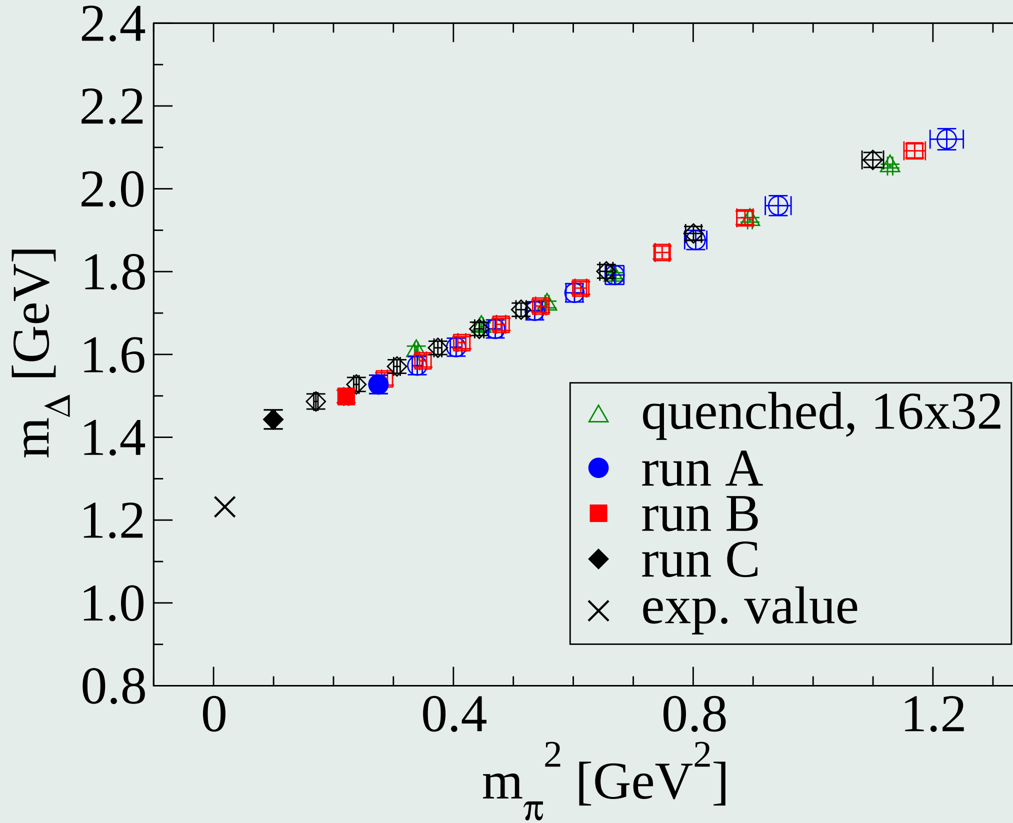
$$O_\Delta^{(k)} = \epsilon_{abc}\, u_a \left(u_b^T\, C\, \gamma_k\, u_c \right) , \quad k = 1, 2, 3$$

- Parity comes from projector $(\mathbf{1} \pm \gamma_t)/2$

The positive parity nucleon



The positive parity delta



Summary & outlook

- We use 2 dynamical Chirally Improved quarks
- Three run sequences on $16^3 \times 32$ lattices are available
- Ground states come out reliably

- Desirable are also excited states $\rightsquigarrow$ different quark smearing and/or link smearing
- What about the a_0? $\rightsquigarrow$ bigger lattice volumes are needed; momentum analysis

Thank you!