



Christian
Winteler

r-Process
Basics

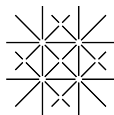
Solving a
Reaction
Network

Dynamic
r-Process
Calculations

Outlook

Numerical Methods for Extended Reaction Networks in r-Process Calculations

Christian Winteler



UNI
BASEL

Department of Physics

Eurograd Workshop Blaubeuren, 2008



Outline

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Dynamic
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- 1 **r-Process Basics**
Why do we need yet another process?
How does it work?
Where does it occur?
- 2 **Solving a Reaction Network**
Basic Structure
Integration Method
- 3 **Dynamic r-Process Calculations**
Current Status
- 4 **Outlook**



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Indications for the r-Process

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r-Process Basics

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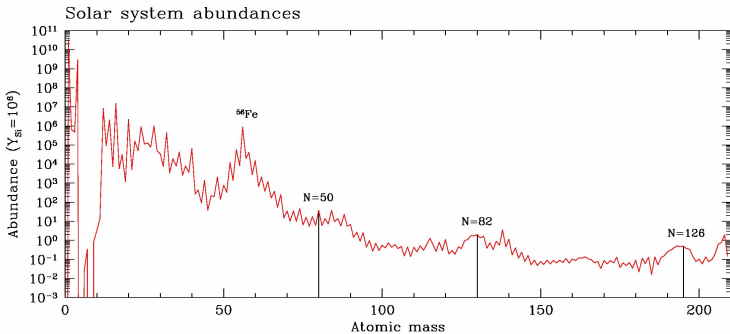
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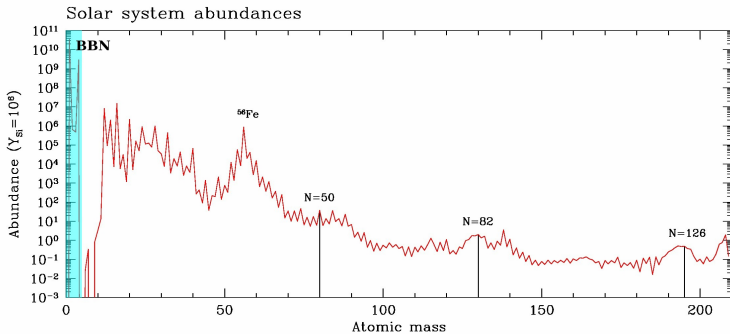
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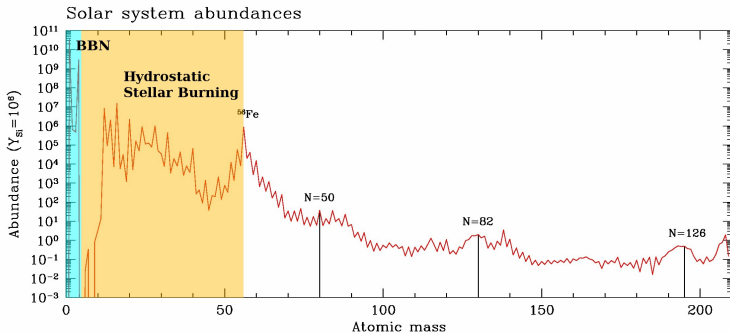
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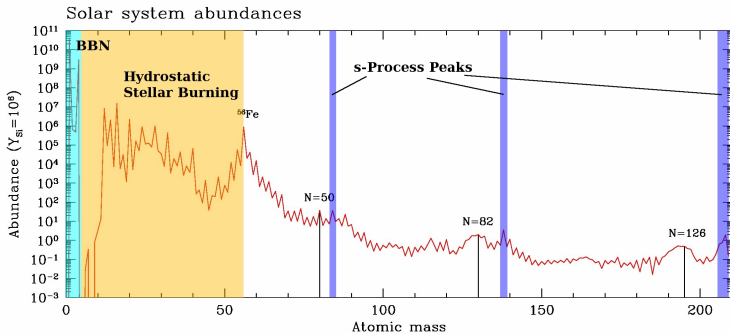
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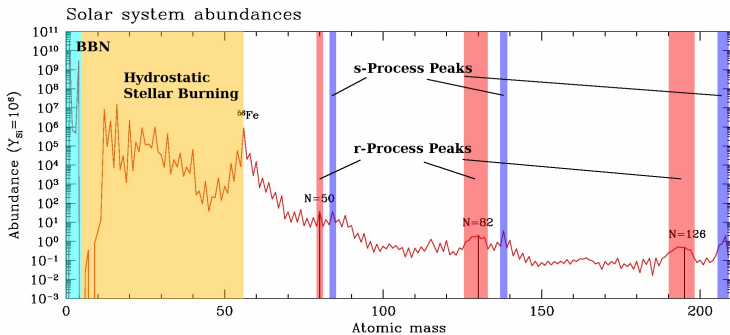
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The r-Process Path

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Dynamic r-Process Calculations

Outlook

- Formation of nuclei heavier than iron through charged particle reactions inhibited due to presence of Coulomb barrier.
- BBFH [57]: Elements heavier than iron can be synthesized by subsequent neutron captures and β^- -decays when iron-group nuclei are exposed to a flux of neutrons.
- $\lambda_{\beta} \gg \lambda_{(n,\gamma)} \Rightarrow$ s-Process
- $\lambda_{(n,\gamma)} \gg \lambda_{\beta} \Rightarrow$ r-Process



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Network

Dynamic
r-Process
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Double-Peak Structure

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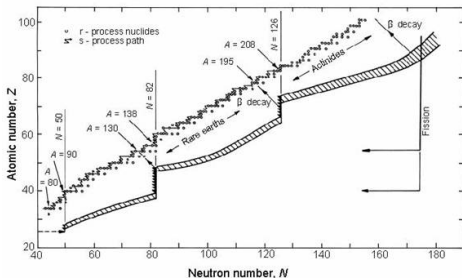
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- r-process reaches magic neutron numbers at lower Z than s-process

⇒ r-peaks at lower A

- Several nuclei with magic neutron numbers
- Mainly β -decays toward stability, but also β -delayed n-emission, spontaneous or β -delayed fission and α -emission ($A > 210$) ⇒ r-peaks smeared out



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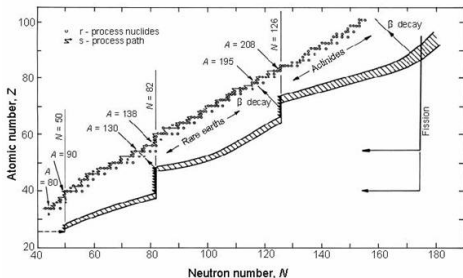
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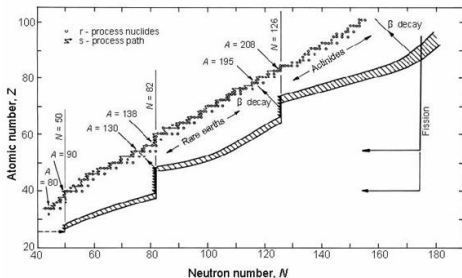
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r-Process Conditions

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Reaction
Network

Dynamic
r-Process
Calculations

Outlook

Classical r-process model

- Large neutron densities $N_n \geq 10^{21} \text{ cm}^{-3}$ or
 - Large neutron-to-seed ratio: $1 < \frac{Y_n}{Y_{\text{seed}}} < 180$
 - Heaviest r-Process elements $A \sim 240$ built from iron-group seeds ($A \sim 60$) $\Rightarrow \sim 180$ neutrons needed
 - Higher abundances at lower $A \Rightarrow$ on average $\frac{Y_n}{Y_{\text{seed}}} \gtrsim 80$
 - Sufficiently high temperatures, but not too high in order to prevent photodisintegrations
- $\Rightarrow T \simeq 1 \text{ GK}$
- Short timescales: \sim seconds
- \Rightarrow Explosive scenarios



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r-Process
Calculations

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r-Process Sites I - Single or Multiple?

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Dynamic r-Process Calculations

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- Observation of extremely metal-poor halo-stars:
 - Very old stars
 - Contributions from only a few r-process events
 - Remarkable agreement with scaled solar r-abundances for $A > 135$ [Truran et al. 02]
- ⇒ **robust mechanism** produces $A > 135$
 - No agreement below $A = 135$
- ⇒ Either different sites or single site with different conditions



r-Process Sites II - Candidates

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Solving a
Reaction
Network

Dynamic
r-Process
Calculations

Outlook

[Cowan et al. 1991 / Qian 2003]

Core-Collapse Supernovae

- Prompt Mechanism
- Neutrino-driven winds
- MHD-jets

Other possible sites

- Inhomogeneous Big Bang
- Neutron-Star Mergers



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Network

Dynamic
r-Process
Calculations

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r-Process site still not identified (yet?)!



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Reaction
Network

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Integration Method

Dynamic
r-Process
Calculations

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The Reaction Network

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r-Process
Basics

Solving a
Reaction
Network

Basic Structure
Integration Method

Dynamic
r-Process
Calculations

Outlook

- Reaction Network (changes in abundances of a nucleus are determined by reactions creating and destroying said nucleus)

$$\dot{Y}_i = \underbrace{\sum_j N_j^i \lambda_j Y_j}_{\text{one-body reactions}} + \underbrace{\sum_{j,k} \frac{N_{j,k}^i}{1 + \delta_{jk}} \rho N_A \langle \sigma v \rangle_{j;k} Y_j Y_k}_{\text{two-particle reactions}}$$

- Neglecting charged-particle reactions, this can be rewritten as

$$\dot{Y}(Z, A) = \underbrace{\sum_{Z', A'} \lambda_{Z', A'} Y(Z', A')}_{\text{one-body reactions}} + \underbrace{\sum_{Z', A'} \rho N_A \langle \sigma v \rangle Y_n Y_{Z', A'}}_{\text{neutron-induced reactions}} \quad (1)$$



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r-Process
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Solving a
Reaction
Network

Basic Structure
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Dynamic
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Implicit Backward Euler-Scheme

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Reaction
Network

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r-Process
Calculations

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- Initial value problem:

$$\dot{\mathbf{y}}(t) = \mathbf{f}(\mathbf{y}(t))$$

- Backward Euler method:

$$\mathbf{y}_{n+1} = \mathbf{y}_n + \mathbf{h} \cdot \mathbf{f}(\mathbf{y}_{n+1})$$

- Applied to our case:

$$\mathbf{y}_{n+1} = \mathbf{y}_n + \tilde{\mathbf{h}} \cdot \underbrace{\frac{d\mathbf{y}_{n+1}}{dt}}_{=: \mathbf{f}(\mathbf{y}_{n+1})} \quad (2)$$



Implicit Backward Euler-Scheme

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Reaction
Network

Basic Structure
Integration Method

Dynamic
r-Process
Calculations

Outlook

- Expanding $\mathbf{f}(\mathbf{y}_{n+1})$ to first order about the known $\mathbf{f}(\mathbf{y}_n)$ yields

$$\underbrace{\mathbf{f}(\mathbf{y}_n + \Delta)}_{=:\mathbf{y}_{n+1}} = \mathbf{f}(\mathbf{y}_n) + \underbrace{\frac{\partial \mathbf{f}(\mathbf{y}_n)}{\partial \mathbf{y}_n}}_{\tilde{\mathbf{J}}} \cdot \Delta$$

- Using this and $\Delta = \mathbf{y}_{n+1} - \mathbf{y}_n$ we can rewrite Eq.[2] as:

$$\left(\frac{1}{h}\tilde{\mathbf{I}} - \tilde{\mathbf{J}}\right)\mathbf{y}_{n+1} = \mathbf{f}(\mathbf{y}_n) + \left(\frac{1}{h}\tilde{\mathbf{I}} - \tilde{\mathbf{J}}\right)\mathbf{y}_n$$

- Solve by Inversion of $\left(\frac{1}{h}\tilde{\mathbf{I}} - \tilde{\mathbf{J}}\right)$ until convergence-criteria are fulfilled

- Full network calculations: $\sim 6000 \times 6000$ matrices



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Basics

Solving a
Reaction
Network

Basic Structure
Integration Method

Dynamic
r-Process
Calculations

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r-Process
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Dynamic r-Process Calculations

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r-Process
Basics

Solving a
Reaction
Network

Dynamic
r-Process
Calculations

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Outlook

- Change in Abundance of nucleus (Z,A):

$$\dot{Y}(Z, A) =$$

- take into account all possible reactions



Dynamic r-Process Calculations

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Solving a
Reaction
Network

Dynamic
r-Process
Calculations

Current Status

Outlook

- Change in Abundance of nucleus (Z,A):

$$\dot{Y}(Z, A) = n_n Y_{Z, A-1} \langle \sigma v \rangle_{Z, A-1}^{(n, \gamma)} - Y_{Z, A} \left(n_n \langle \sigma v \rangle_{Z, A}^{(n, \gamma)} \right)$$

- take into account all possible reactions:

neutron capture reactions



Dynamic r-Process Calculations

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Solving a
Reaction
Network

Dynamic
r-Process
Calculations

Current Status

Outlook

- Change in Abundance of nucleus (Z,A):

$$\begin{aligned} \dot{Y}(Z, A) = & n_n Y_{Z, A-1} \langle \sigma v \rangle_{Z, A-1}^{(n, \gamma)} + Y_{Z, A+1} \lambda_{Z, A+1}^{(\gamma, n)} \\ & - Y_{Z, A} \left(n_n \langle \sigma v \rangle_{Z, A}^{(n, \gamma)} + \lambda_{Z, A}^{(\gamma, n)} \right) \end{aligned}$$

- take into account all possible reactions:
neutron capture reactions, **photodisintegrations**



Dynamic r-Process Calculations

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Dynamic
r-Process
Calculations

Current Status

Outlook

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- take into account all possible reactions:
neutron capture reactions, photodisintegrations, β -decays



Dynamic r-Process Calculations

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Network

Dynamic
r-Process
Calculations

Current Status

Outlook

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- take into account all possible reactions:
neutron capture reactions, photodisintegrations, β -decays,
 β -delayed neutron emission (1n, 2n or 3n)



Current Method

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Solving a
Reaction
Network

Dynamic
r-Process
Calculations
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Outlook

- Network consisting of two coupled parts [Cowan et al. 1983]:
 - Charged particle network produces neutrons (e.g. via $^{13}\text{C}(\alpha, n)$)
 - “r-process code” nuclei from $Z=14-114$
 - Number of Neutrons is transmitted back and forth between the two.
 - Choose timestep such that $n_n = \text{const.}$ over the whole timestep
- ⇒ Set of linear equations for the \dot{Y}
- Solve Network for individual Z-chains



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r-Process
Basics

Solving a
Reaction
Network

Dynamic
r-Process
Calculations
Current Status

Outlook

- For each nucleus (Z,A) along a Z-chain only 3 unknown coefficients $Y_{Z,A-1}$, $Y_{Z,A}$ and $Y_{Z,A+1}$
⇒ Tridiagonal Matrix
- Limit of the approximation:
 - Endphase of r-process n_n rapidly decreases
⇒ timestep very small
- ⇒ Need a more holistic approach
(i.e. full network calculation)



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Network:

- Combine charged particle and r-Process networks (implicit treatment of n_n)
- Integrate Full Network \Rightarrow Inversion of Huge Matrices
 - Take advantage of matrix sparseness (PARDISO [Schenk et al.2000])
- Parallelized code?

Models [Liebendörfer Group]

- 3D-Simulations
- Exploding Supernovae?!



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