Coulomb gauge gluon propagator and Gribov copies in $SU(2)$ lattice gauge theory

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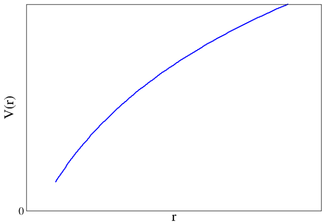
outline

- Motivation
- $SU(2)$ lattice simulation: setup
- Gribov gauge copies
- improved gauge fixing
- Results (both 2+1 & 3+1 dim)
- Renormalisation
- Conclusion

Motivation

- Hadronic matter composed of quarks and gluons
- Theoretical description: Quantum chromodynamics (QCD)
- confinement: only colourless particles observed linearly rising potential between quarks

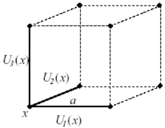
$$V(r) \sim \sigma\, r$$



Primary objects in QFT : Green functions, e.g.

- Gluon Propagator $\rightarrow$ related to confining properties
  - Gribov-Zwanziger criterion (involves gluon propagator)
  - Hamiltonian approach (H. Reinhardt, W. Schleifenbaum) [main : ingredient : gluon propagator]

$U_\mu(x) \in SU(2)$ $\{U_\mu(x)\}$ configuration of all links in the lattice



- Generate lattice $SU(2)$ configuration $\{U_\mu(x)\}$ by Monte Carlo simulation (M. Creutz, *Phys.Rev. D21 (1980) 2308*)

$$\{U_\mu^1(x)\} \to \{U_\mu^2(x)\} \to \{U_\mu^3(x)\} \to ... \to \{U_\mu^N(x)\}.$$

$$dP(\{U_\mu(x)\}) \sim exp[-\beta S(\{U_\mu(x)\})]d\{U_\mu(x)\} \quad where \quad \beta = \frac{4}{g_0^2}$$

$$measure : \quad \langle \mathcal{O} \rangle \simeq \frac{1}{N_{MC}} \sum_{i=1}^{N_{MC}} \mathcal{O}(U_i).$$

$SU(2)$ lattice simulation : Coulomb gauge fixing

Coulomb gauge $\nabla \cdot A = 0$ is implemented into lattice by minimizing functional

$$F_t[g] = \sum_{\mathbf{x}} \sum_{i=1}^{3} \frac{1}{2} tr[1 - U_i^g(\mathbf{x}, t)]$$

where $g(\mathbf{x}, t)$ is gauge transformation

$$U_i^g(\mathbf{x}, t) = g(\mathbf{x} + \hat{\mu}, t) U_i(\mathbf{x}, t) g^\dagger(\mathbf{x}, t). \leftarrow \textit{time fixed}$$

- numerical iterative, many local minima (Gribov copies), search for global minimum
- bad algorithm = bad results
- continuum: $\nabla A^g = 0$, many solution $g(x)$

Gribov gauge copies

- many gauge transformations A^g satisfy $\nabla \cdot A^g = 0$ (Gribov copies)
- many solutions to $\nabla \cdot A^g = 0$ $\equiv$ many local minimun to $F[g]$

improved gauge fixing

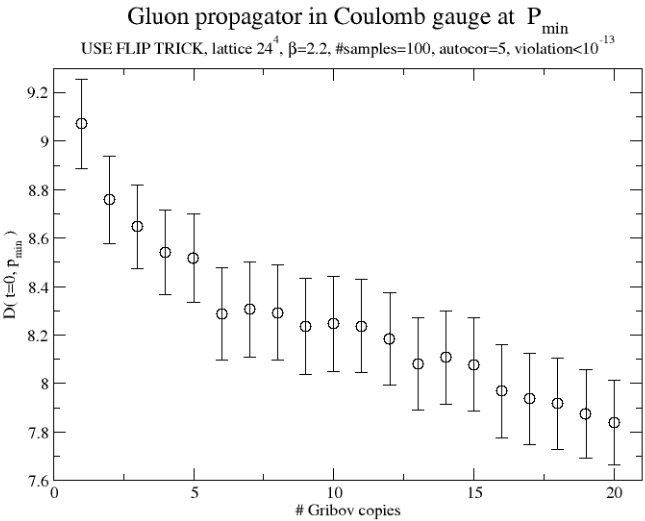
random gauge transformation: $g(x)$

- give many choices of gauge copies
- Coulomb gauge fixing for each copy, by overrelaxation
- best copy is selected from n gauge copies which gives minimum functional $F(g)$

1. run algorithm many times with random starting point and produce best result
2. flip trick
    - standard algorthm : only periodic gauge transformation $g(x)$
    - periodic b.c. allow also for antiperiodic $g(x)$
      $\longrightarrow$ flip sign by hand to get a full picture

on top of standard procedure

improved gauge fixing



Result: gluon propagator

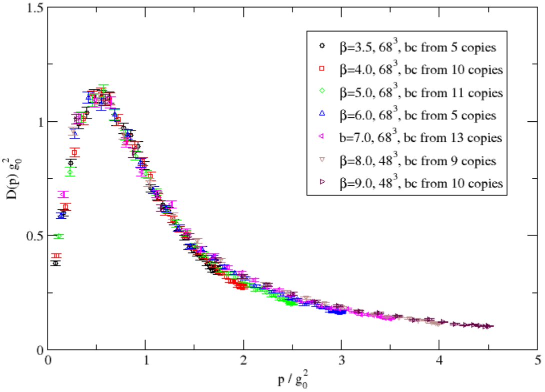
$$\int \frac{dp_0}{2\pi} \langle A_i(-\vec{p}, -p_0) A_i(\vec{p}, p_0) \rangle = D(\vec{p})$$

gluon-two point function of equal-times

- parameters are identical to Langfeld/Moyaerts, PRD,**70**,074507(2004)
- use improved gauge fixing (flip & choose best copy)

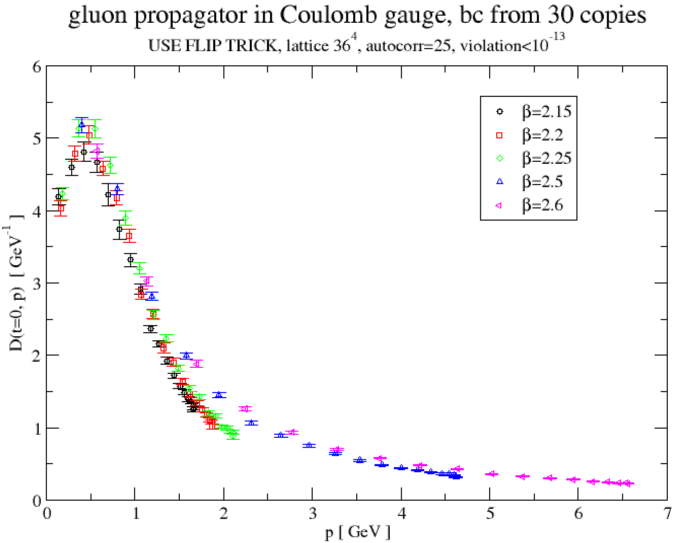
	IR	UV
lattice: Moyerts/Langfeld	$D(\mathbf{p}) \to \text{const}$	$D(\mathbf{p}) \sim \lvert\mathbf{p}\rvert^{3/2}$
Analytic: Reinhardt/Feucher	$D(\mathbf{p}) \to 0$	$D(\mathbf{p}) \sim \lvert\mathbf{p}\rvert$

Result: gluon propagator in 2+1 dimensions



IR : improved g.f. suppresses $D(p)$

Result: gluon propagator in D=3+1



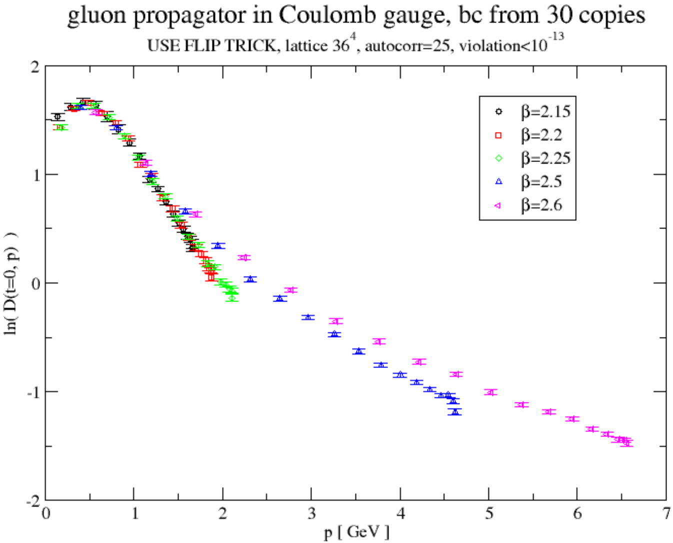
Result: gluon propagator in D=3+1

- IR: improved g.f. supresses $D(p)$
- Problem with renormalisation
- UV: power law fit, $D(p) \sim p^{-\beta}$
  $\beta = 1.57(8)$, but $1/(p \, ln \, p)$ also possible

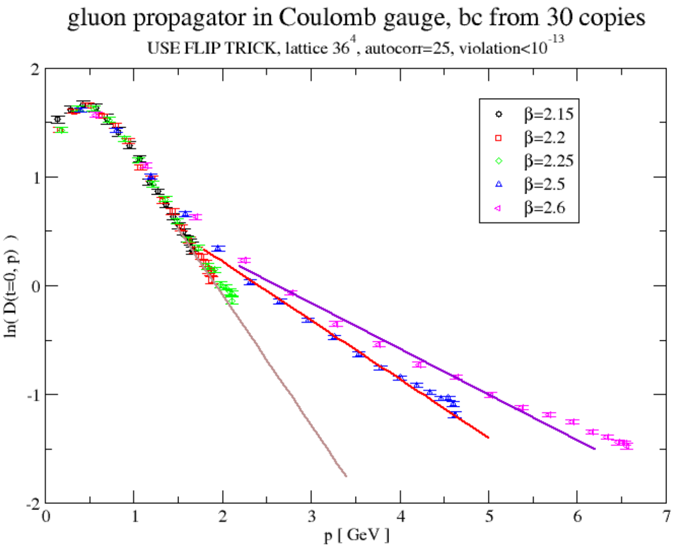
Renormalisation

- lattice : lattice spacing $a(\beta) \to 0$ as $\beta \to \infty$
  β corresponds to cutoff in continuum
- Renormalisation:
  $D(p^2, \beta) \cdot Z(\beta) = D_R(p^2)$ independent of β
- $\Rightarrow$ curves for different β should relate by constant factors

Result: gluon propagator in D=3+1 (renormalised)



Result: gluon propagator in D=3+1 (renormalised)



What goes wrong in Coulomb gauge?

- General (different times) propagators is renormalised,
  but function of p_0^2 , $\vec{p}^2$ separately.
- integrate over p_0 to get equal time propagator
  $\longrightarrow$ integrate over renorm. condition ?
- under investigation currently **? !**

Conclusion

- Gluon propagators are sensitive to quality of gauge fixing
  - IR : Propagator suppressed
- Outlook
  - Renormalisation for equal time propagator
  - ghost propagator
  - Coulomb form factor & Coulomb potential

SU(2) lattice simulation: setup

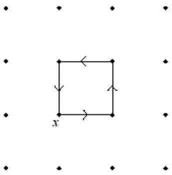
from path integral

$$Z = \int \prod_{x,\mu} [dU_{x,\mu}] exp(-S(U))$$

where $U_{x,\mu} \in SU(N_c)$ and the action

$$S(\{U\}) = \beta \sum_{x} \sum_{\mu<\nu} (1 - \frac{1}{N_c} \mathrm{Re\ Tr\ U_p})$$

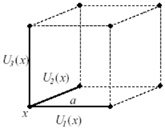
$$U_p \equiv U_\mu(x) U_\nu(x+\hat{\mu}) U_\mu^\dagger(x+\hat{\nu}) U_\nu^\dagger(x)$$



$SU(2)$ lattice simulation : setup

$U_\mu(x) \in SU(2)$ forming lattice and is represented by

$$U_\mu(x) = a_0(x)I + i\vec{a(x)} \cdot \vec{\sigma}$$



- Generate lattice $SU(2)$ configuration $\{U\}$ by Monte Carlo simulation (M. Creutz, *Phys.Rev. D21 (1980) 2308*)

$$\{U_1\} \to \{U_2\} \to \{U_3\} \to \ldots \to \{U_N\}.$$

$$dP(U') \sim exp[-\beta S(U')]dU' \quad where \quad \beta = \frac{4}{e_0^2}$$

$$measure : \quad \mathcal{O} \simeq \frac{1}{N_{MC}} \sum_{i=1}^{N_{MC}} \mathcal{O}(U_i).$$

$SU(2)$ lattice simulation : Coulomb gauge fixing

Coulomb gauge $\nabla \cdot A = 0$ was implemented into lattice by minimizing functional

$$F[U(x,t)] = \sum_{\mathbf{x}} \sum_{\mu=1}^{3} \frac{1}{2} tr[1 - U_\mu^g(\mathbf{x},t)]$$

where

$$U_\mu^g(\mathbf{x},t) = g(\mathbf{x}+\hat{\mu},t) U_\mu(\mathbf{x},t) g^\dagger(\mathbf{x},t).$$

This can be done by iterating overrelaxation

$$g(\mathbf{x},t) = \frac{V^\dagger(\mathbf{x},t)}{\sqrt{det V(\mathbf{x},t)}}$$

$$V(\mathbf{x},t) = \sum_{i=1}^{3} \{U_i(x-k,t) + U_i^\dagger(x,t)\}$$

$SU(2)$ lattice simulation : Gluon propagator

Observable : measure gluon propagator

- equal-time gluon propagator

$$D(p) \sim \int \frac{dp_0}{2\pi} \sum_{i,a} \langle A_i^a(p_0, \mathbf{p}) A_i^a(p_0, -\mathbf{p}) \rangle$$

- lattice equal-time gluon propagator

$$\begin{aligned} \hat{D}(p) &= \frac{1}{3V} \sum_{i=1}^{3} \sum_{a=1}^{3} \langle \hat{A}_i^a(p_0, \mathbf{p}) \hat{A}_i^a(p_0, -\mathbf{p}) \rangle \\ &= \frac{1}{3V} \sum_{i=1}^{3} \sum_{a=1}^{3} \langle |\hat{A}_i^a(p_0, \mathbf{p})|^2 \rangle \end{aligned}$$

Gribov gauge copies

- In Coulomb gauge $\nabla \cdot A = 0$
- gauge transformation $A^g = gAg^{-1} - \frac{i}{g_0}(\nabla g)g^{-1}$
  $\implies \nabla \cdot A^g = 0$ Gribov copies
- many gauge copies A^g satisfy $\nabla \cdot A^g = 0$
  positivity of Faddeev-Popov operator, $-\nabla \cdot D[A^g] \geq 0$
  $\rightarrow$ Gribov region, Ω

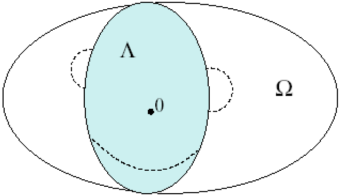
Gribov gauge copies

in Gribov region Ω: existence of several gauge copies inside

- along gauge orbit A^g, many local minima of $F[g]$

$$F[g] = -\frac{1}{3V}\sum_{\mathbf{x}}\sum_{\imath=1}^{3}\frac{1}{2}tr[U_\mu^g(\mathbf{x},t)]$$

- absolute minima of $\rightarrow$ fundamental modular region (FMR), Λ



Gribov gauge copies

- many gauge transformations A^g satisfy $\nabla \cdot A^g = 0$ (Gribov copies)
- many solutions to $\nabla \cdot A^g = 0$
  $\equiv$ many local minimun to $F[g]$

improved gauge fixing

$$U_\mu^g(\mathbf{x}, t) = g(\mathbf{x} + \hat{\mu}, t) U_\mu(\mathbf{x}, t) g^\dagger(\mathbf{x}, t)$$

$$\textit{minimize} \quad F[g] = -\frac{1}{3V} \sum_{\mathbf{x}} \sum_{\imath=1}^{3} \frac{1}{2} tr[U_\mu^g(\mathbf{x}, t)]$$

- flip trick
- random gauge transformation

improved gauge fixing

$$U^g_\mu(\mathbf{x},t) = g(\mathbf{x}+\hat{\mu},t)U_\mu(\mathbf{x},t)g^\dagger(\mathbf{x},t)$$

$$minimize \quad F[g] = -\frac{1}{3V}\sum_{\mathbf{x}}\sum_{\imath=1}^{3}\frac{1}{2}tr[U^g_\mu(\mathbf{x},t)]$$

flip trick

- $g(x+L\hat{\nu}) = \pm g(x)$
- at fixed x_ν
  flip: $U_\nu(x_\nu, x_\perp) \longrightarrow -U_\nu(x_\nu, x_\perp)$ if $F[g] > 0$

$$U_\mu^g(\mathbf{x},t) = g(\mathbf{x}+\hat{\mu},t)U_\mu(\mathbf{x},t)g^\dagger(\mathbf{x},t)$$

$$minimize \quad F[g] = -\frac{1}{3V}\sum_{\mathbf{x}}\sum_{\imath=1}^{3}\frac{1}{2}tr[U_\mu^g(\mathbf{x},t)]$$

random gauge transformation: $g(x)$

- give many choices of gauge copies
- Coulomb gauge fixing for each copy, by overrelaxation
- best copy is selected from n gauge copies which gives minimum functional $F(g)$

Renomalization

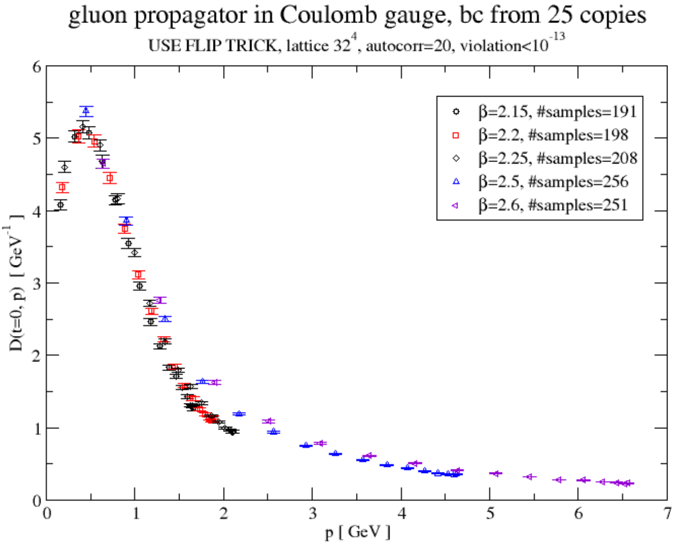
- Lattice simulation $\rightarrow$ *bare gluon propagator,* $D_B(p, \beta)$ *various* $\beta \in [2.15, ..., 2.6]$
- $\Longrightarrow$ *renormalized continuum gluon propagator* $D_R(p^2, \beta)$ *by*

$$Z(\beta, p^2, \mu) D_B(p^2, \beta) = D_R(p^2, \beta, \mu)$$

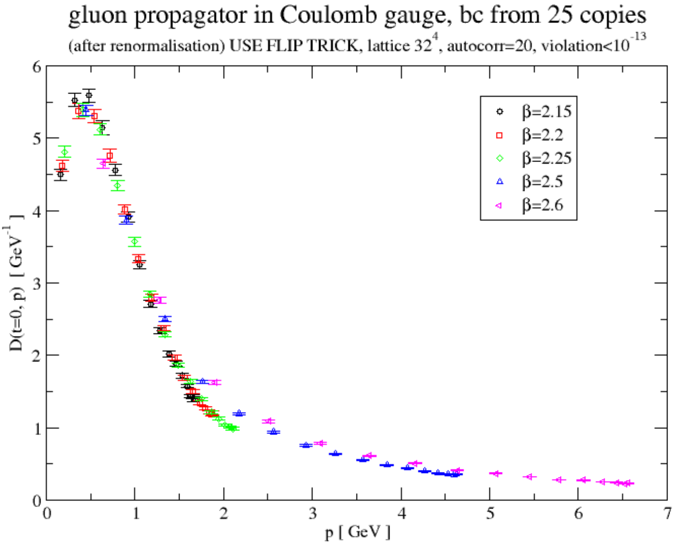
  where $Z(\beta, p^2, \mu)$ *is renormalised constant*
- *larger* $\beta \rightarrow$ *lattice space,* $a \rightarrow 0$
- *at some chosen scale* μ

$$D_R(p^2 = \mu^2) = \frac{1}{\mu^2}$$

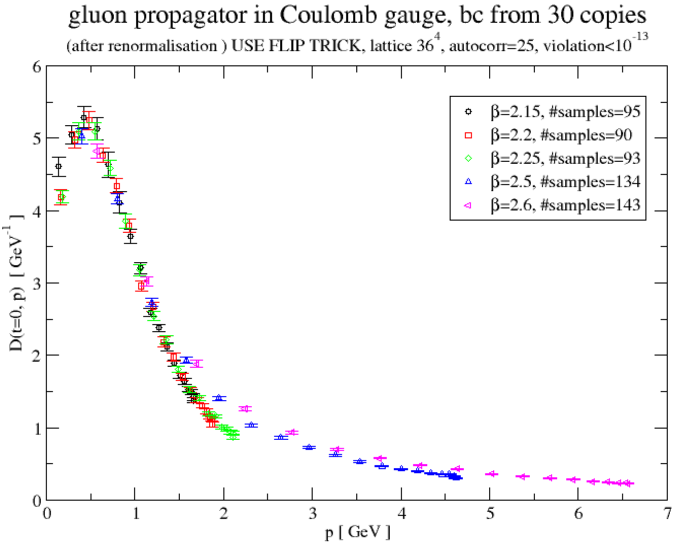
Result: gluon propagator in D=3+1



Result: gluon propagator in D=3+1 (renormalised)



Result: gluon propagator in D=3+1 (renormalised)



Renormalisation

Landau gauge

$$Z(\beta, p^2, \mu) D_B(p^2, \beta) = D_R(p^2, \beta, \mu)$$

$$D_R(p^2 = \mu^2) = \frac{1}{\mu^2}$$

$$Z(\beta) D_B(p^2, \beta) = D_R(p^2)$$

Coulomb gauge

$$Z(\beta, \vec{p}^2, p_0^2, \mu) D_B(p^2, \beta) = D_R(\vec{p}^2, p_0^2, \beta, \mu)$$

$$D_R(\vec{p}^2 + p_0^2 = \mu^2) = \frac{1}{\mu^2} f(p_0^2, \mu)$$

$$Z(\beta, \lambda = \frac{p_0^2}{\mu^2})$$

Result: gluon propagator in D=3+1 (renormalised)