Charmed Hadron Production within the Generalized Parton Picture

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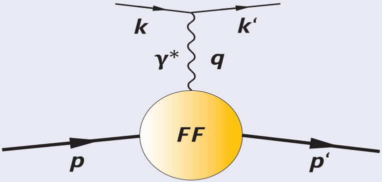
Elastic Electron-Proton Scattering

ES



Amplitude

$$\mathcal{T} \sim \frac{1}{q^2} j^{el}_{\mu} J^{\mu}_{p}, \qquad (1)$$

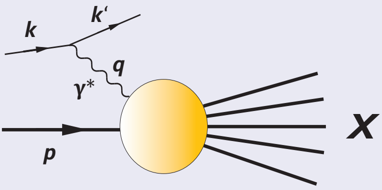
with the (known) electron current j^{el}_{μ} and the a priori unknown proton current J^{μ}_{p}.

$\Rightarrow J^{\mu}_{p} = e\bar{u}\left(p', s'\right)[?]\, u(p)\, e^{i\left(p'-p\right)\cdot x}$, where we have to find [?].

$\Rightarrow [?] = \left[F_1\left(q^2\right)\gamma^{\mu} + \frac{\kappa}{2M}F_2\left(q^2\right) i\sigma^{\mu\nu}q_{\nu}\right]$, with the two FFs, $F_1\left(q^2\right)$ and $F_2\left(q^2\right)$.

Deep Inelastic Scattering (DIS): Parton Distribution Fct.s (PDFs)

DIS



Kinematics

c.m. energy sqrd.:
$s = (p+k)^2$

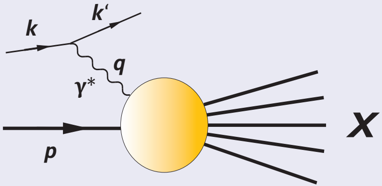
momentumtransf.:
$Q^2 = -q^2 = -\left(k - k'\right)^2$

enrgy loss:
$\nu = (p \cdot q)/M = E - E'$

scaling variable:
$x_B = Q^2/(2p \cdot q)$

recoil mass sqrt.:
$M_X^2 = (p+q)^2$

DIS



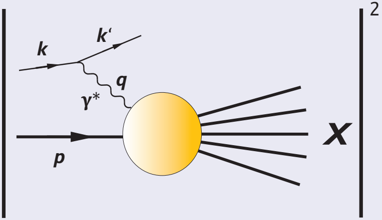
Amplitude

$$\mathcal{T}_X \sim \left(\frac{1}{q^2}\right) j^{el}_{\mu} H^{\mu}, \qquad (2)$$

with the electron current j^{el}_{μ}
and the hadronic current H^{μ}.

⇒ But, we have to go to the cross section level!

DIS



Cross Section

$$d\sigma \sim L^{el}_{\mu\nu} W^{\mu\nu} \qquad (3)$$

with the leptonic tensor $L_{\mu\nu}$ and the hadronic tensor $W^{\mu\nu}$.

Construction of $W^{\mu\nu}$ (Lorentz decomposition, structure functions):

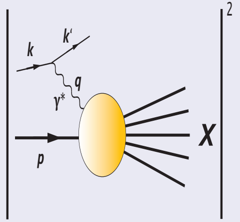
$$W^{\mu\nu} = W_1 \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) + W_2 \frac{1}{M^2} \left(p^\mu - \frac{p \cdot q}{q^2} q^\mu \right) \left(p^\nu - \frac{p \cdot q}{q^2} q^\nu \right), \qquad (4)$$

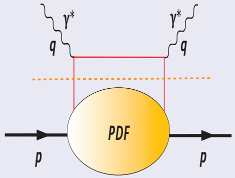
with the structure functions $W_1 \left(\nu, q^2\right)$ and $W_2 \left(\nu, q^2\right)$.

DIS and Optical Theorem

The Optical Theorem relates

$\sigma \quad \sim \quad \mathcal{A}$ of fwd. CS





Thus, $$W_{\mu\nu}(P,q) \sim \mathcal{I}\left[i \int d^4 x e^{iqz} \langle p(P) | \mathcal{T} J^\dagger_\mu(z) J_\nu(0) | p(P) \rangle \right], \quad (5)$$

where the e.m. current $J_\mu(z) = \sum_q e_q \bar{\Psi}_q(z) \gamma_\mu \Psi_q(z)$.

Definition of PDFs

For $Q^2 >> M^2$

Product of currents expanded around values of z on the light-cone orthogonal to P.

- Formally, $z^\mu = \lambda n^\mu$ with $n \cdot P = 1$ and $n^2 = 0$.
- Leading term in expansion, light-cone operator

$$\bar{\Psi}_q(z) \not{n} \Psi_q(0) . \quad (6)$$

Definition of quark PDF

$$q(x, \mu_F) = \frac{1}{2} \sum_{\text{spin}} \int \frac{d\lambda}{2\pi} e^{-i\lambda x} \langle p(P) | \bar{\Psi}_q(z) \not{n} \Psi_q(0) | p(P) \rangle . \quad (7)$$

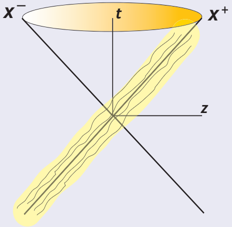
Incoherence and Scale Separation

Both, in ES and in DIS it was assumed, that ...

- resolution scale Q^2 is large enough to probe the internal structure.
- the γ^* interacts with a single quark inside the proton.
- The "deviation" of the proton from beeing a point-like particle was described by functions like FFs, PDFs, etc.

Let's revise these assumption...

The partonic interpretation of the PDFs is strictly valid only in the infinite momentum frame, $\vec{p} >> M$,
where the proton moves very fast. This leads to the picture,



of the fast moving proton. The world lines of the partons are spread out in x^+ and pushed together in x^- direction.

Probability for coherent scattering on an $n-$parton configuration:

$$\mathcal{P}_n \sim \left(\frac{|\delta z_T|^2}{\pi R_N^2} \right)^n \sim \left(\frac{1}{Q^2 \pi R_N^2} \right)^n , \tag{8}$$

with the transverse area of the nucleon πR_N^2 and photon $|\delta z_T|^2 = 1/Q^2$.

⇒ Probability suppressed by nth power of large γ^* virtuality.

⇒ γ^* "sees" only one parton per collision ("Handbag picture").

- Power suppressed corrections go under the name "higher twist".

The character of relevant distances is consequence of the deep virtual kinematics, $Q^2 \to \infty$.

Large virtualities, Q^2, and energies, $\nu \equiv p \cdot q$, probe short-distance and -time structures.

The relevant distances in DIS: Target proton rest frame, γ^* three-mom. in neg. z-direction. Then,

$$q^\mu = \left(\frac{Q^2}{2Mx_B}, 0, 0, -\frac{Q^2}{2Mx_B}\sqrt{1 + 4M^2x_B^2/Q^2} \right) \tag{9}$$

When Q^2 large, LC components of mom. transf. approx.

$$q^- \sim Q^2/\left(Mx_B\right), \quad q^+ \sim Mx_B. \tag{10}$$

Since integrand in PDF definition oscillatory function, $e^{i\left(q^-z^+ + q^+z^-\right)}$, vanishing result unless distances

$$z^- \sim \left(Mx_B\right)/Q^2, \quad z^+ \sim 1/\left(Mx_B\right). \tag{11}$$

Thus,

$$\Rightarrow z^2 \approx 2z^+ z^- \sim 1/Q^2 \to 0, \tag{12}$$

provided transverse separations are small.

DIS probes strong interaction dynamics close to the light-cone

$$z^2 \approx 0$$

Thus, approx. neglect dependence of all coord. components, except for z^-.

We see...

Hard partonic subprocess

occupies very small space-time volume.

Soft hadron dynamics

much larger space-time scales.
(typical hadronic scale $\sim$ 1 GeV.)

Quite likely, the two scales are uncorrelated and will not interfere.

Quite likely, the two scales are uncorrelated and will not interfere. This results in the QM incoherence property of physics at different scales ,...

...the Factorization

of, e.g., the DIS structure functions

$$W_i\left(x_B, Q^2\right) = \hat{W}_i\left(x_B/x, Q^2/\mu_F^2\right) \otimes q\left(x, \mu_F^2\right) , \qquad (13)$$

where $q\left(x, \mu^2\right)$ is a quark distribution function and $\hat{W}_i\left(x_B/x, Q^2/\mu_F^2\right)$ is a perturbatively calculable short-distance quark-photon c.s.

Factorization and Evolution

QCD corrections: E.g. real gluon emission.

Basic idea of factorization:

Contribution of region of phase space of the quark propagator momentum l,

- where $|l^2| = \mathcal{O}\left(Q^2\right)$ is large $\rightarrow$ short distance fluctuation belonging to hard partonic c.s., contributes at order $\alpha_S\left(Q^2\right)$ to $\hat{W}$.
- where $|l^2| = \mathcal{O}\left(\Lambda^2_{QCD}\right)$ is small $\rightarrow$ soft long distance fluctuation within the proton, happened long before the scattering, belongs to the PDF $q\left(x\right)$.

To disentangle the two regions...

1. introduce an auxiliary factorization scale μ_F
2. decompose phase space integral over $d|l^2|$ in region below and above μ_F
3. calculate partonic structure function, now depending on μ_F/Q and α_S, order by order in pQCD
4. use PDFs at the chosen factorization scale μ_F/Q

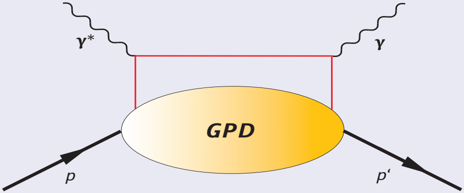
The evolution of PDFs under a change of μ_F is governed by DGLAP equations.

Deeply Virtual Compton Scattering (DVCS)

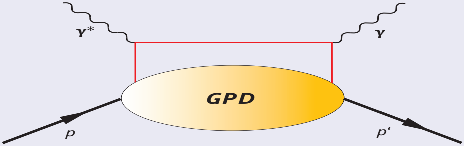
Lets directly generalize the situation from DIS to DVCS,

$$p\,(p)\,\gamma^*\,(q) \to p\left(p^{'}\right)\gamma\left(q^{'}\right), \quad \left(p \neq p^{'}\right). \qquad (14)$$

DVCS



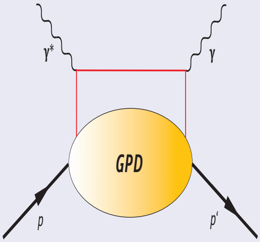
DVCS



We have

- an exclusive process,
- the same operators as in DIS,
- but sandwiched between proton states of non-forward kinematics.
- optical theorem → DIS cross section

DVCS



Kinematics

photon virtualities:

$$Q^2 = -q^2, \quad \left(q^{'}\right)^2 = 0$$

scaling variable:

$$x_B = Q^2 / \left(2p \cdot q\right)$$

averaged proton momentum:

$$P = \frac{1}{2}\left(p + p^{'}\right)$$

momentum transf.:

$$\Delta = p^{'} - p, \quad \Delta^2 = t$$

Due to non-vanishing mom. transf. ⇒ partonic momentum fractions in the initial and final state proton different, too. We parametrize

$$\text{incoming parton mom.:} \, (x+\zeta)\, P = \frac{x+\zeta}{1+\zeta} p \tag{15}$$

$$\text{outgoing parton mom.:} \, (x-\zeta)\, P = \frac{x-\zeta}{1-\zeta} p', \tag{16}$$

where x is now the averaged momentum fraction w.r.t. to P and ζ is the deviation from the averaged mom. fraction, called "skewedness".

Factorization of DVCS

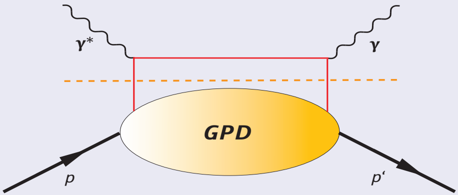
Again we apply factorization to split the

- perturbative short-distance physics
- from soft long-distance physics.

We handle the

- former in terms of partonic hard scattering amplitudes,
- the latter by introducing Generalized Parton Distributions (GPD).

Factorization of DVCS



$$\mathcal{A}\left[\gamma^* p \to \gamma p\right] = [\text{partonic amplitude}] \bigotimes [\text{GPD}] \qquad (17)$$

Notice: Factorization takes place on the amplitude level!

Definition of quark GPDs

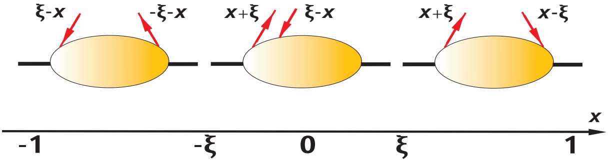
$H^q(x,\zeta,t|\mu_F)$ and $E^q(x,\zeta,t|\mu_F)$ from

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p\left(p^{'},s^{'}\right)|\bar{\Psi}_q\left(-\frac{z}{2}\right) \not{n}\, \Psi_q\left(\frac{z}{2}\right)|p(p,s)\rangle|_{\mu_F} =$$
$$H^q(x,\zeta,t|\mu_F)\,\bar{u}\left(p^{'},s^{'}\right)\not{n}\,u(p,s) + \quad (18)$$
$$E^q(x,\zeta,t|\mu_F)\,\bar{u}\left(p^{'},s^{'}\right)\frac{i\sigma^{\beta\alpha}\Delta_\alpha n_\beta}{2M}u(p,s),$$

where the light-like separation between the quark fields again is def. by $z^\mu = \lambda n^\mu$, with $n \cdot P = 1$, $n^2 = 0$ and $n \cdot p = 1 - \zeta$, $n \cdot p^{'} = 1 + \zeta$.

Parton Model Interpretation

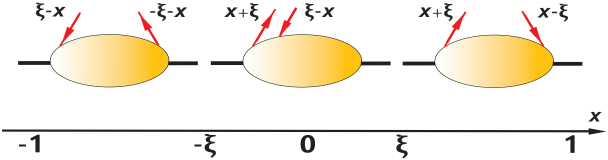
Depending on the sign of $x \pm \zeta$, the parton is a quark or an antiquark. The support properties of GPDs w.r.t. to x lead to three regions.



These are,

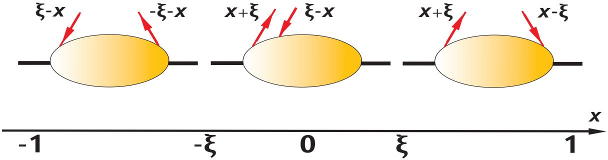
- $-1 < x < -\zeta$,
- $-\zeta < x < \zeta$,
- $\zeta < x < 1$.

The support properties of GPDs w.r. to x lead to three regions.



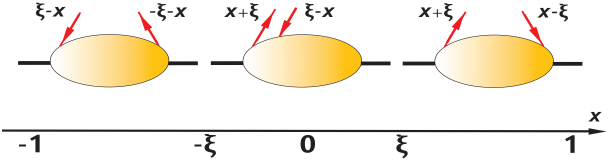
- $-1 < x < -\zeta$
  - anti-quark emission with long. mom. $(\zeta - x)\,P$
  - anti-quark absorption with long. mom. $(-\zeta - x)\,P$

The support properties of GPDs w.r. to x lead to three regions.



- $-\zeta < x < \zeta$
  - quark emission with long. mom. $(x + \zeta)\,P$
  - anti-quark emission with long. mom. $(\zeta - x)\,P$

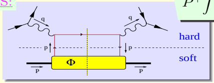
The support properties of GPDs w.r. to x lead to three regions.



- $\zeta < x < 1$
  - quark emission with long. mom. $(x + \zeta)\,P$
  - quark absorption with long. mom. $(x - \zeta)\,P$

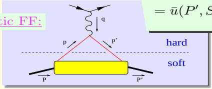
Limiting Cases for GPDs

DIS:



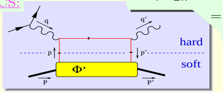
$$P^+ \int \frac{\mathrm{d}z^-}{2\pi}\, e^{i\,xP^+z^-} \langle P,S|\,\overline{\psi}(\tfrac{-\bar z}{2})\,\gamma^+\,\psi(\tfrac{\bar z}{2})\,|P,S\rangle$$
$$= \bar u(P,S)\,\gamma^+\,u(P,S)\;f_1(x)$$

elastic FF:



$$\langle P',S'|\,\overline{\psi}(0)\,\gamma^+\,\psi(0)\,|P,S\rangle$$
$$= \bar u(P',S')\left\{\gamma^+\,F_1(t) + \frac{i\sigma^{+\nu}\Delta_\nu}{2M}\,F_2(t)\right\}u(P,S)$$

DVCS:



$$\bar P^+ \int \frac{\mathrm{d}z^-}{2\pi}\, e^{i\,\bar x\bar P^+ z^-} \langle P',S'|\,\overline{\psi}(\tfrac{-\bar z}{2})\,\gamma^+\,\psi(\tfrac{\bar z}{2})\,|P,S\rangle$$
$$= \bar u(P',S')\left\{\gamma^+\,H(x,\xi,t) + \frac{i\,\sigma^{+\nu}\Delta_\nu}{2M}\,E(x,\xi,t)\right\}u(P,S)$$

Forward Limit: GPDs $\rightarrow$ PDFs

In the forward limit, i.e. $p = p^{'}$, $s = s^{'}$ and $\zeta = 0$, the expression defining GPDs becomes:

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p\left(p^{'}, s^{'}\right) | \bar{\Psi}_q \left(-\frac{z}{2}\right) \not{n}\, \Psi_q \left(\frac{z}{2}\right) | p\left(p, s\right)\rangle|_{\mu_F} \rightarrow$$
$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p\left(p, s\right) | \bar{\Psi}_q \left(-\frac{z}{2}\right) \not{n}\, \Psi_q \left(\frac{z}{2}\right) | p\left(p, s\right)\rangle|_{\mu_F} = \qquad (19)$$
$$H^q\left(x, \zeta = 0, t = 0 | \mu_F\right) \bar{u}\left(p, s\right) \not{n}\, u\left(p, s\right) =$$
$$q\left(x | \mu_F\right) \bar{u}\left(p, s\right) \not{n}\, u\left(p, s\right)$$

Forward Limit: GPDs $\rightarrow$ PDFs

In forward limit $p = p^{'}$, $s = s^{'}$ and $\zeta = 0 \Rightarrow$ GPDs reduce to the usual PDFs. Thus

$$H^q\left(x, \zeta = 0, t = 0 | \mu_F\right) = \begin{cases} q\left(x, \mu_F\right), & \text{for} \quad x > 0 \\ -\bar{q}\left(-x, \mu_F\right), & \text{for} \quad x < 0. \end{cases} \tag{20}$$

Formally we obtain

$$E^q\left(x, \zeta = 0, t = 0 | \mu_F\right) = \begin{cases} e^q\left(x, \mu_F\right), & \text{for} \quad x > 0 \\ -\bar{e}^q\left(-x, \mu_F\right), & \text{for} \quad x < 0. \end{cases} \tag{21}$$

but these PDFs are not probable in DIS ($\Delta \rightarrow 0$)!!

Local Limit: GPDs $\rightarrow$ FFs

The x^0-Mellin-Moment renders the quark field operators local:

$$\sum_q e_q \int_{-1}^{+1} dx \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p\left(p^{'}, s^{'}\right) | \bar{\Psi}_q \left(-\frac{z}{2}\right) \not{n} \Psi_q \left(\frac{z}{2}\right) | p\left(p, s\right)\rangle =$$

$$\sum_q e_q \int d\lambda \delta\left(\lambda\right) \langle p\left(p^{'}, s^{'}\right) | \bar{\Psi}_q \left(-\frac{z}{2}\right) \not{n} \Psi_q \left(\frac{z}{2}\right) | p\left(p, s\right)\rangle =$$

$$\sum_q e_q n^\mu \langle p\left(p^{'}, s^{'}\right) | \bar{\Psi}_q \left(0\right) \gamma_\mu \Psi_q \left(0\right) | p\left(p, s\right)\rangle =$$

$$n^\mu \left[F_1\left(t\right) \bar{u}\left(p^{'}, s^{'}\right) \gamma_\mu u\left(p, s\right) + F_2\left(t\right) \bar{u}\left(p^{'}, s^{'}\right) \frac{i\sigma_{\mu\nu}\Delta^\nu}{2M} u\left(p, s\right)\right] \tag{22}$$

Local Limit: GPDs → FFs

Thus

$$F_1\left(t\right) = \sum_q e_q F_1^q\left(t\right) = \sum_q e_q \int_{-1}^{+1} dx \quad H^q\left(x,\zeta,t\right), \qquad (23)$$

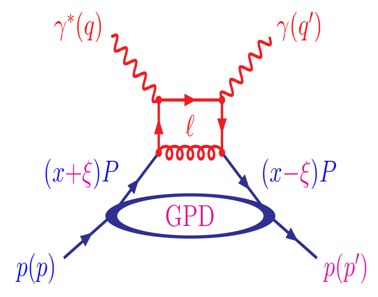
and

$$F_2\left(t\right) = \sum_q e_q F_2^q\left(t\right) = \sum_q e_q \int_{-1}^{+1} dx \quad E^q\left(x,\zeta,t\right). \qquad (24)$$

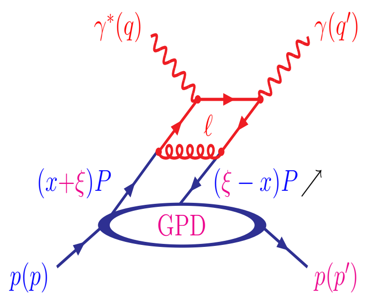
Factorization and Evolution

QCD correction included similar as in DIS. However, for evolution equation distinguish different support regions w.r.t. x!

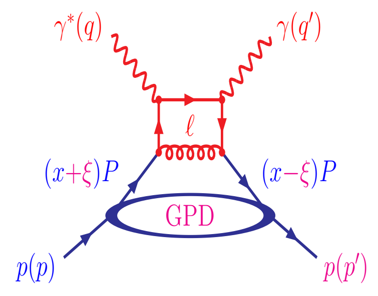
$|x| > \zeta$



$|x| < \zeta$



$|x| > \zeta$



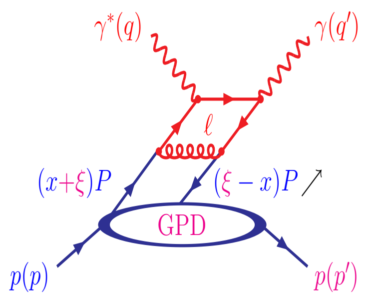
DGLAP region

Describes parton splitting,

$$q\left(x+\zeta\right) \rightarrow q\left(x^{'}+\zeta\right) \quad g\left(l\right), \tag{25}$$

as in the case of PDFs.

$|x| < \zeta$



ERBL region

Describes redistribution of partonic momenta,

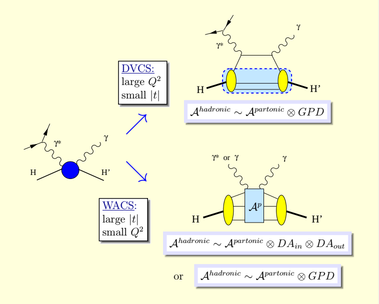
$$q\,(x+\zeta) \quad \bar{q}\,(\zeta - x) \rightarrow q\left(x' + \zeta\right) \quad \bar{q}\left(\zeta - x'\right), \tag{26}$$

as in the case of Distribution Amplitudes (DAs) for hard exclusive processes.

Wide Angle Compton Scattering (WACS)

The Idea of GPDs can be used also in other kinematical regions than in the deeply virtual one:
E.g. in WACS, where Q^2 is small and Mandelstam $|t|$ is the large scale.

DVCS and WACS



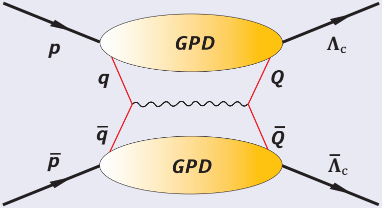
Charmed Hadron Production

Of course, GPDs can be used for describing other processes than CS. We, e.g., investigate the

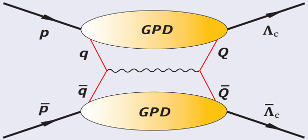
charmed hadron production

$$p \quad \bar{p} \quad \longrightarrow \quad \Lambda_c \quad \bar{\Lambda}_c, \tag{27}$$

where through a proton anti-proton anihilation the heavy baryon and anti-baryon is produced.



- Large hadron mass provides us with a hard scale.
- This results in large Mandelstam variables (see e.g. WACS).
- Apply factorization of hard and soft physics.
- BUT: Here we have a flavor change!



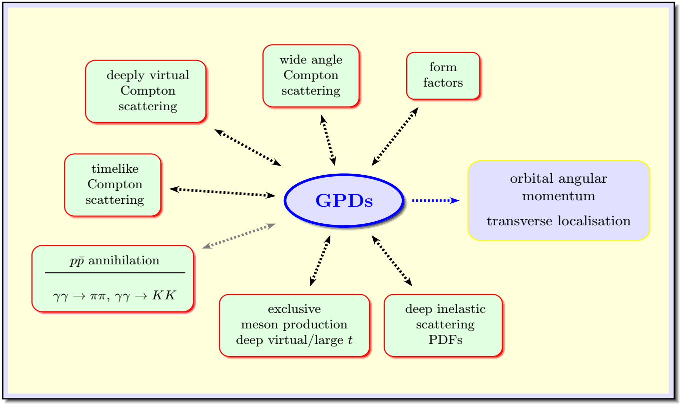
Hard process

- $q \quad \bar{q} \quad \longrightarrow \quad Q \quad \bar{Q}$
- described by perturbatively calculable Feynman diag.

Soft process

- Long distance effects of (anti)proton-(anti)lambda formation
- parametrized by GPDs.

Summary


deeply virtual Compton scattering
wide angle Compton scattering
form factors
timelike Compton scattering
GPDs
orbital angular momentum
transverse localisation
$p\bar{p}$ annihilation
$\gamma\gamma \to \pi\pi$, $\gamma\gamma \to KK$
exclusive meson production deep virtual/large t
deep inelastic scattering PDFs