Dynamical Fermions in Lattice QCD

Markus Limmer

Karl–Franzens–Universität Graz – Institut für Physik

Fachbereich Theoretische Physik

In collaboration with:

C. Gattringer, C. B. Lang, D. Mohler

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Chiral symmetry on the lattice

Chiral symmetry on the lattice

- We want to non–perturbatively regulate (Euclidean) QCD Lagrangian

$$\mathcal{L} = \frac{1}{2g^2}\,\mathrm{Tr}\left[F_{\mu\nu}\,F_{\mu\nu}\right] + \overline{\psi}\,(\not{D} + m)\,\psi$$

$\Rightarrow$ lattice formulation

- Continuum version of chiral symmetry:

$$\not{D}\,\gamma_5 + \gamma_5\,\not{D} = 0$$

- But: Chiral symmetry is violated by most lattice Dirac operators
- Lattice version of chiral symmetry: *Ginsparg–Wilson relation*

$$\not{D}\,\gamma_5 + \gamma_5\not{D} = a\,\not{D}\,\gamma_5\,\not{D}$$

Chiral symmetry on the lattice

- GW–exact operators (unfortunately very expensive): Overlap, Fixed Point
- GW–type operators: Domain Wall, **Chirally Improved**
- Make the most general ansatz for D

$$D(x,y) = \sum_{\alpha=1}^{16} \Gamma_\alpha \sum_{p \in \mathcal{P}^\alpha_{x,y}} c^\alpha_p \langle l_1, l_2, \ldots, l_{|p|} \rangle$$

- Certain constraints for coefficients to obtain "lattice symmetries", i.e., translation and rotation invariance, C, P, γ_5–hermiticity
- Plug this ansatz into GW–relation $\Rightarrow$ system of infinitely many quadratic equations for infinitely many coefficients
- Introduce a cutoff (restrict to paths of maximal length 3) and solve the system

Implementation of the “dynamics”

Implementation of the "dynamics"

- We are interested in

$$\langle A \rangle = \frac{1}{Z} \int \mathcal{D}[\,\overline{\psi}, \psi, U\,]\; e^{-S_g}\; e^{-\overline{\psi}\,(\not{D}\,+m)\,\psi}\; A$$

- Integrating out the fermions

$$\langle A \rangle = \frac{1}{Z} \int \mathcal{D}\,[U]\; e^{-S_g}\; \det(\not{D} + m)\; A$$

- The "easy" way: quenched simulation

$$\langle A \rangle = \frac{1}{Z} \int \mathcal{D}\,[U]\; e^{-S_g}\; 1\; A$$

- A much "smarter" way: dynamical simulation

$$\langle A \rangle = \frac{1}{Z} \int \mathcal{D}\,[U]\; e^{-S_g}\; \det(\not{D} + m)\; A$$

Implementation of the "dynamics"

- Determinant implemented via *pseudo fermion fields*

$$\det\left[DD^\dagger\right] \propto \int \mathcal{D}\left[\phi_R, \phi_I\right]\; e^{-\phi^\dagger\,(DD^\dagger)^{-1}\,\phi}$$

- The simulation is done with *Hybrid Monte Carlo*: Introduce "momenta" P_μ conjugate to links $U_\mu \Rightarrow$ represents a classical system with Hamiltonian $H = S_g + P^2/2$
- The evolution is then done as follows:

1. Generate the fields ϕ using Gaussian random noise ϱ via $\phi = D^\dagger \varrho$
2. Create the P_μ using Gaussian random noise
3. Evolve system $\{U, P\}$ according to the classical Hamiltonian E.O.M. (leapfrog algorithm), treating ϕ as an external background field
4. End up with an accept/reject step to correct for numerical errors (finite δ_t)

Our simulation

Our simulation

- We used *Lüscher–Weisz gauge action*: $S_g \sim \left(S_{plaq} + S_{rect} + S_{twist} \right)$
- *Stout smearing* of the gauge fields
- Up to now 2 run sequences (A,B) and a third one in progress (C), all on $16^3 \times 32$ lattices

	Run A	**Run B**	**Run C**
β	4.650	4.700	4.519
m	-0.060	-0.050	-0.094
a [fm]	$0.1592(6)$	$0.1602(7)$	0.16
m_{AWI} [MeV]	$31.5(2)$	$47.9(5)$	15
m_π [MeV]	$445(5)$	$488(5)$	300
$\#$ confs	100	53	100

- First 100 configurations skipped for thermalization, then every 5$^{\text{th}}$ used for analysis

Hadron masses

Hadron masses

- Set up several interpolators with
  - different quark source shapes (narrow, wide, derivative source)
  - different interpolating fields (e.g., χ_1, χ_2, χ_3 for the nucleon)
- Compute correlation matrix

$$C_{ij} = \left\langle\, \overline{O}_i(t)\, O_j(0)\, \right\rangle$$

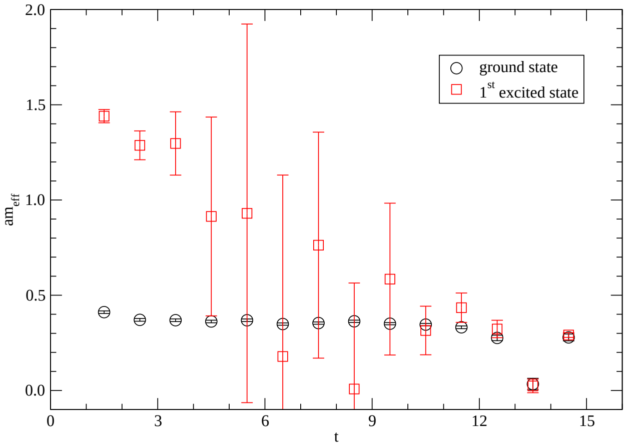
- Solve generalized eigenvalue problem

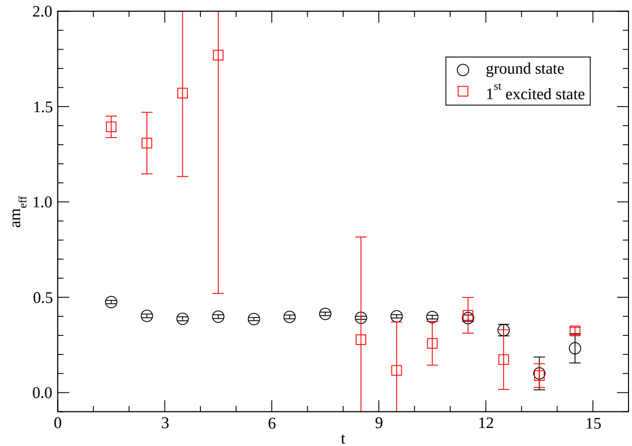
$$\lambda_k(t) \propto e^{-m_k\, t} \left[1 + \mathcal{O}(e^{-\Delta E_k\, t})\right]$$

- States can be "identified" via eigenvectors

Hadron masses

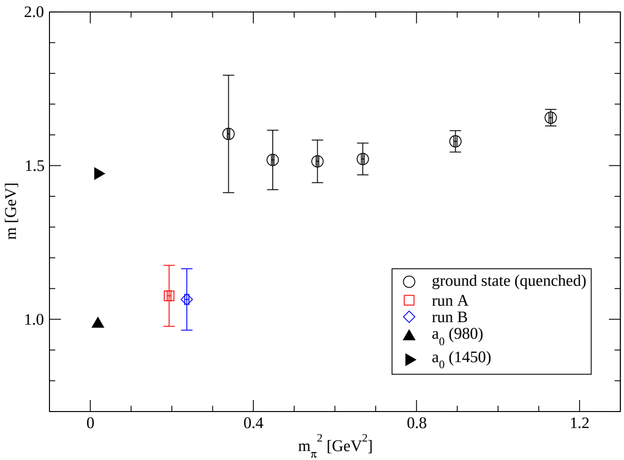
Effective masses for the pion, run A (l.h.s.) & run B (r.h.s.)

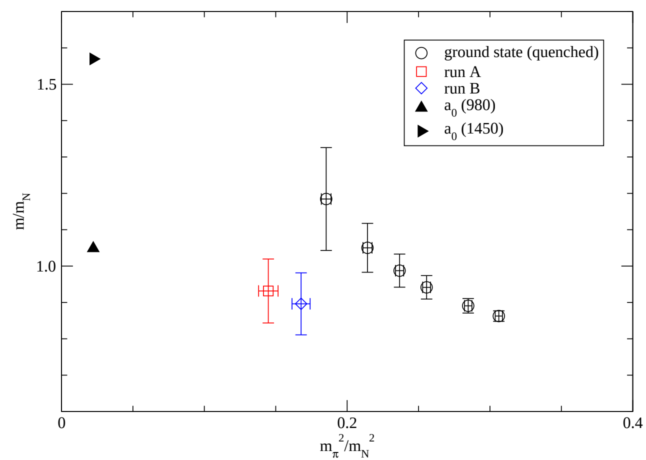




Hadron masses

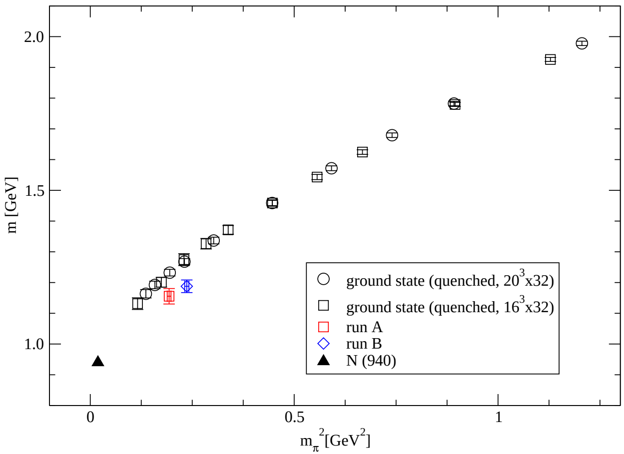
Scalar meson a_0

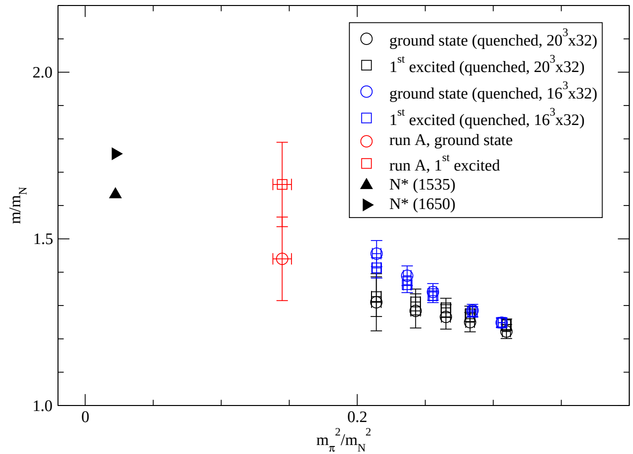




Hadron masses

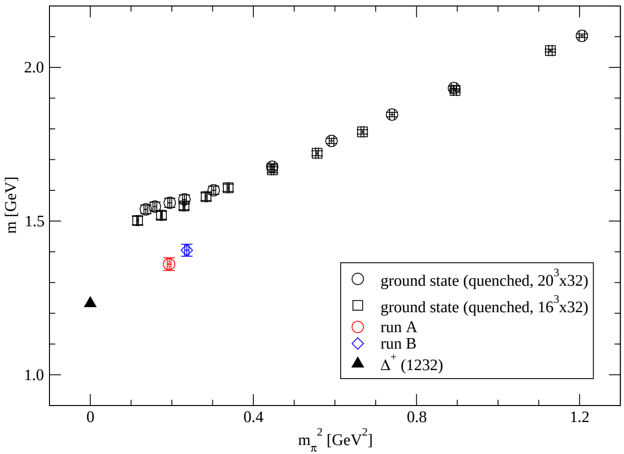
Nucleon, pos. parity (l.h.s.) & neg. parity (r.h.s.)

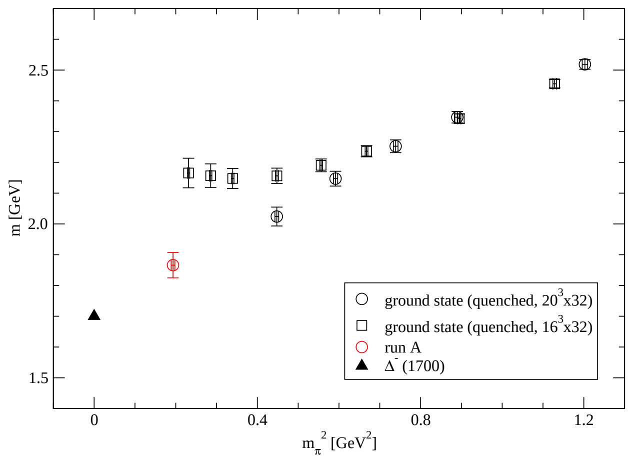




Hadron masses

Delta, pos. parity (l.h.s.) & neg. parity (r.h.s.)





Summary & Outlook

- We could "approximate" chiral symmetry with the CI operator
- Full dynamical simulation of 2 mass degenerate quark flavors
- Significant improvement compared to quenched data: results are much closer to physical value
- Very first results, analysis is going on
- Hopefully I can tell you more next year ...

Thank You!