The relativistic self-energy in nuclear dynamics

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Outline

Relativity in nuclear systems?

Relevance of relativity:

$k_F/M \simeq 1/4 \rightarrow$ velocity $v \simeq 1/4c$
$\rightarrow$ moderate corrections from relativistic kinematics

But:

- Relativistic dynamics
  RMF, Hadronic many-body theory (DBHF), QCD sum rules

  $\rightarrow \quad \Sigma_s \simeq -350$ MeV, $\Sigma_0 \simeq +300$ MeV

- Cancellation in mean field potential $U_{s.p.} \simeq \Sigma_0 + \Sigma_s \simeq -50$ MeV
- Large spin-orbit force $U_{S.O.} \propto (\Sigma_0 - \Sigma_s)\vec{L} \cdot \vec{S} \simeq +750$ MeV
- Effective nucleon mass in nuclear matter $M^* = M + \Sigma_s$

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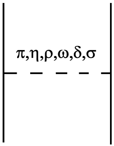
Relativistic NN potentials



- **Bonn A**

  field theoretical relativistic one-boson-exchange potential,

  various scales and spin-isospin structure is associated with meson exhange,

  long range part due to OPE, short/intermediate range = heavy mesons

- **CD Bonn**

  charge independence breaking due to pion mass splitting,

  readjustment of certain parameters in each partial wave (phenomenological high-precision NN potential with 43 free parameters)

Non-relativistic NN potentials

- Nijm 93 and Nijmegen I/II

  long range part due to OPE, approximate OBE amplitudes
- Argonne v_{18}

  long range part due to OPE, intermediate and short range parametrized via operators O_α and strength functions V_α
- Idaho potential

  Chiral effective field theory, N^3LO, D. Entem and R. Machleidt, (29 free model parameters)
- V_{lowk}

  Derivation of an effective low-momentum potential V_{lowk} from modern NN potentials (out-integration of high-momentum modes, $\Lambda \simeq 2fm^{-1}$, and use of renormalization group methods)

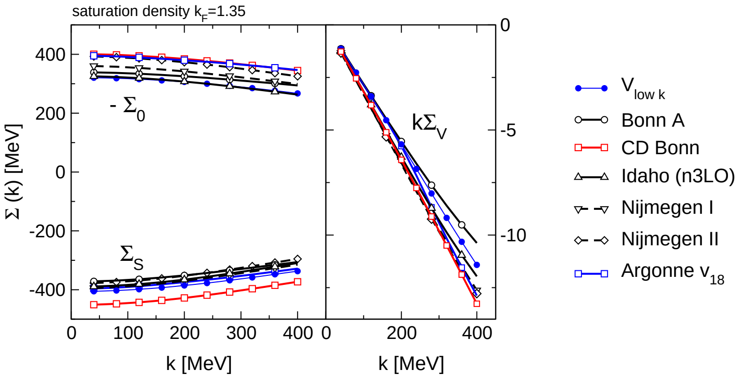
Nucleon self-energy in Hartree-Fock approximation

$$\Sigma = -i \int_F \left(\mathrm{Tr}[GV] - GV \right)$$

- $|LSJ\rangle \rightarrow$ partial wave helicity basis $\rightarrow$ plane wave helicity basis $\rightarrow$ **covariant operator basis**
- translational and rotational invariance, parity conservation, time reversal invariance

$$\Sigma(k, k_\mathrm{F}) = \Sigma_\mathrm{s}(k, k_\mathrm{F}) - \gamma_0\, \Sigma_0(k, k_\mathrm{F}) + \boldsymbol{\gamma} \cdot \mathbf{k}\, \Sigma_\mathrm{v}(k, k_\mathrm{F})$$

Large scalar/vector fields



→ Mapping of NN potentials on covariant operator basis

→ large scalar/vector fields → universal feature of NN interaction

O.P., Fuchs, van Dalen, PRC 73 (2006) 014003

Role of contact terms

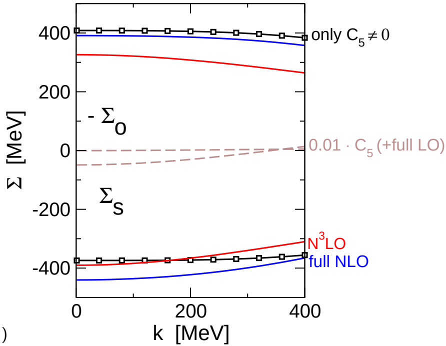
LO

$$V = -\frac{g_A^2}{4f_\pi^2}\,\frac{\vec{\sigma}_1\cdot\vec{k}\,\vec{\sigma}_2\cdot\vec{k}}{q^2+m_\pi^2}, \qquad V = C_S + C_T\,\vec{\sigma}_1\cdot\vec{\sigma}_2$$

NLO

leading order 2π exchange

$$V = \ldots + C_5(-i\vec{S}\cdot(\vec{q}\times\vec{q}\,')) + \ldots + C_7(..)$$



SO force (NLO contact terms) → large scalar/vector fields
→ Nucleon mass $M^* = M + \Sigma_S$ → short-distance physics

O.P., C. Fuchs, PRC 74 (2006) 034325

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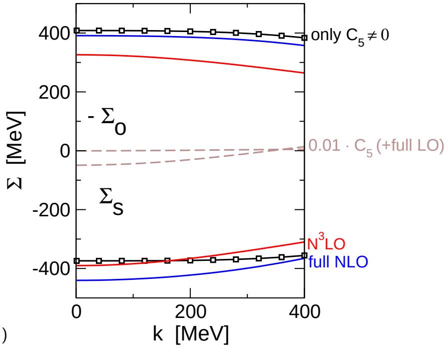

SO force (NLO contact terms) $\rightarrow$ large scalar/vector fields
$\rightarrow$ Nucleon mass $M^* = M + \Sigma_S$ $\rightarrow$ short-distance physics

O.P., C. Fuchs, PRC 74 (2006) 034325

Relativity in nuclear systems?

What's known

Finite nuclei (RMF)

large scalar/vector fields $\Longrightarrow$ SO force

What's new

NN-scattering

large scalar/vector fields $\Longleftarrow$ SO force

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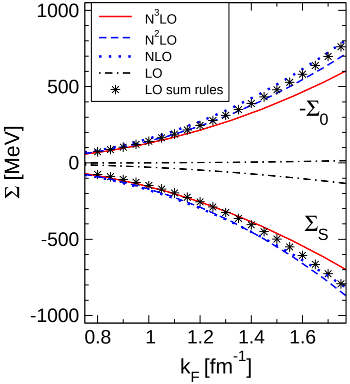
Connection to QCD sum rules

$$\Sigma_s = -\frac{8\pi^2}{\Lambda_B^2}[\langle\bar{q}q\rangle_\rho - \langle\bar{q}q\rangle_0] = -\frac{8\pi^2}{\Lambda_B^2}\frac{\sigma_N}{m_u + m_d}\rho_S = -\frac{\sigma_N M}{m_\pi^2 f_\pi^2}\rho_S$$

$$-\Sigma_0 = -\frac{64\pi^2}{3\Lambda_B^2}\langle\bar{q}\gamma_0 q\rangle_\rho = -\frac{32\pi^2}{\Lambda_B^2}\rho$$



Joffe formulae

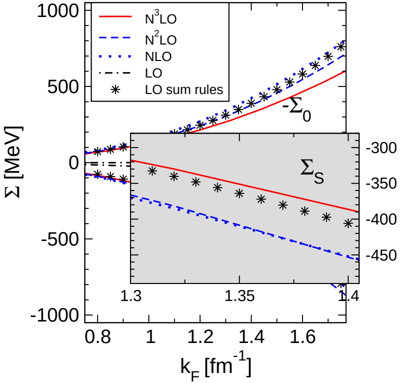
QCD sum rules and chiral EFT fields well comparable at moderate densities (both obtained to leading order in density)

O.P., C. Fuchs, PRC 74 (2006) 034325

Connection to QCD sum rules

$$\Sigma_s = -\frac{8\pi^2}{\Lambda_B^2}[\langle\bar{q}q\rangle_\rho - \langle\bar{q}q\rangle_0]$$
$$= -\frac{8\pi^2}{\Lambda_B^2}\frac{\sigma_N}{m_u + m_d}\rho_S$$
$$= -\frac{\sigma_N M}{m_\pi^2 f_\pi^2}\rho_S$$

$$-\Sigma_0 = -\frac{64\pi^2}{3\Lambda_B^2}\langle\bar{q}\gamma_0 q\rangle_\rho$$
$$= -\frac{32\pi^2}{\Lambda_B^2}\rho$$



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O.P., C. Fuchs, PRC 74 (2006) 034325

Nucleon mass in matter

$$M^* = M + \Sigma_s$$

QCD sum rules

$$\frac{M^*}{M} = 1 - \frac{\sigma_N}{m_\pi^2 f_\pi^2}\rho_B$$

Model independent prediction

$$\frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0} = 1 - \frac{\sigma_N}{m_\pi^2 f_\pi^2}\rho_B$$

$\rightarrow$ consistent comparison of effective nucleon mass and chiral condensate in matter

$$\frac{M^*}{M} \stackrel{?}{=} \frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0}$$

Chiral condensate in nuclear matter

Order parameter of spontaneous chiral symmetry breaking
$\bar{q}q \equiv \frac{1}{2}(\bar{u}u + \bar{d}d), \quad m \equiv \frac{1}{2}(m_u + m_d)$

$$\langle \bar{q}q \rangle_0 = -(225 \pm 25\,\mathrm{MeV})^3$$

From Hellmann-Feynman theorem

$$2m\left(\langle \bar{q}q \rangle_{\rho_B} - \langle \bar{q}q \rangle_0\right) = m\frac{d\mathcal{E}}{dm}$$

Energy density $\qquad \mathcal{E} = M\rho_B + \frac{E}{A}\rho_B$

Equation-of-state $\qquad \mathrm{E/A} = \frac{1}{\rho}\int_F \frac{d^3\mathbf{k}}{2\pi^3}\left[\frac{k^2}{2M} + \frac{1}{2}\,U_{\mathrm{s.p.}}(k, k_{\mathrm{F}})\right]$

Chiral condensate in nuclear matter

$$\frac{\langle \bar{q}q \rangle_{\rho_B}}{\langle \bar{q}q \rangle_0} = 1 - \frac{\rho_B}{m_\pi^2 f_\pi^2} \left[\sigma_N + m \frac{d}{dm} \frac{E}{A} \right]$$

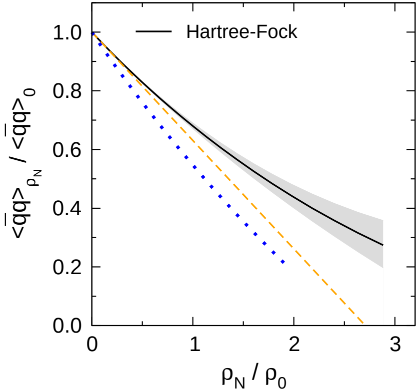
Gell-Mann-Oakes-Renner $\quad 2m\langle \bar{q}q \rangle_0 = -m_\pi^2 f_\pi^2,$

pion-nucleon sigma-term $\quad \sigma_N = m\frac{dM}{dm} =< N|m\bar{q}q|N >$

Comparison of $\frac{M^*}{M}$ and $\frac{\langle \bar{q}q \rangle_{\rho_B}}{\langle \bar{q}q \rangle_0}$ is done with the same chiral EFT interaction and at the same order

Quark mass dependence of the nuclear forces,
E. Epelbaum, W. Glöckle, Ulf-G. Meissner, Eur. Phys. J. A **18**, 499 (2003)

Chiral condensate in nuclear matter at NLO



Leading order approximation

$$\frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0} = 1 - \frac{\sigma_N}{m_\pi^2 f_\pi^2} \rho$$

NLO chiral EFT potential (Hartree-Fock)

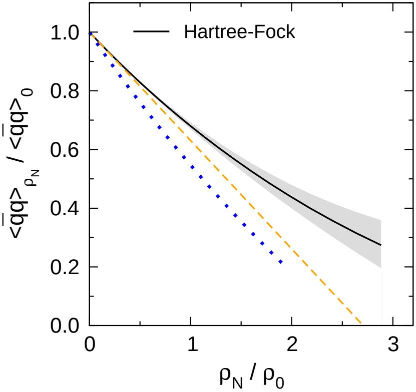
$$\frac{M^*}{M} = 1 + \frac{\Sigma_S}{M}$$

$$\rightarrow \frac{M^*}{M} \neq \frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0}$$

M^* generated by NLO contact interactions

Change of chiral condensate mainly due to virtual low momentum pions

Chiral condensate in nuclear matter at NLO



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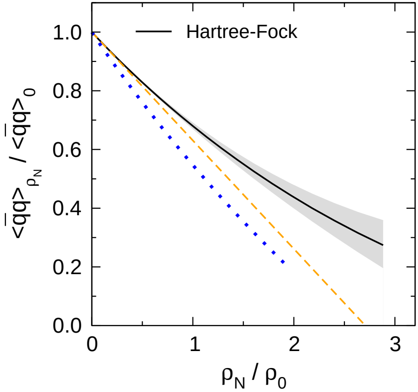

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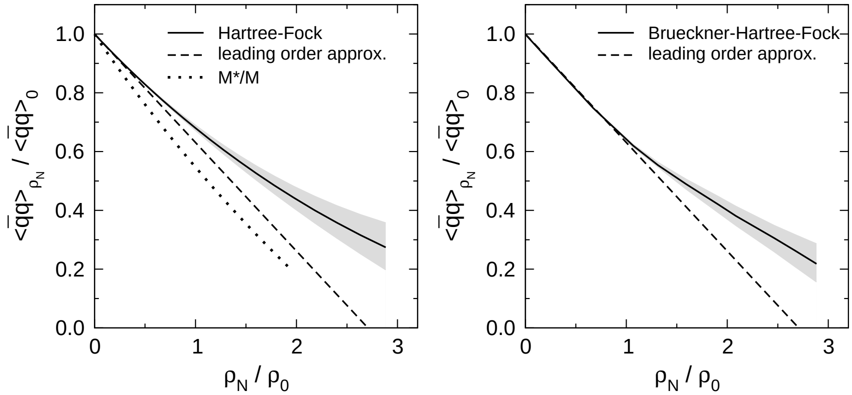
Chiral condensate in nuclear matter



→ short range NN correlations have minor influence on the density dependence of the chiral condensate

Summary & conclusions

- Relativistic self-energy
  $\longrightarrow$ Vacuum structure of *NN* interaction enforces the generation of large scalar/vector fields ($\simeq 300 - 350$ MeV)
- Self-energy from chiral EFT
  NLO contact terms (LEC C_5 connected to SO Force)
  $\longrightarrow$ large scalar/vector fields
- Scalar condensate in matter (NLO)
  Effective nucleon mass does not depend only on the scalar condensate (20%),
  different physical origin

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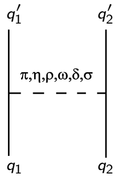
Thank you!

One-boson exchange potentials

Bonn and CD-Bonn potentials

$$V(\mathbf{q}',\mathbf{q}) = \sum_{\alpha=s,ps,v} \bar{V}_\alpha(\mathbf{q}',\mathbf{q})\ \mathcal{F}^2_\alpha(\mathbf{q}',\mathbf{q};\lambda_\alpha)$$



$$-i\bar{V}_\alpha(q',q) = \frac{\bar{u}(-\mathbf{q}')\ \kappa_2^{(\alpha)} u(-\mathbf{q}) P_\alpha\ \bar{u}(\mathbf{q}')\ \kappa_1^{(\alpha)} u(\mathbf{q})}{(q'-q)^2 - m_\alpha^2}, \qquad u_\lambda(\mathbf{q}) = \sqrt{\frac{E+M}{2M}} \begin{pmatrix} 1 \\ \frac{2\lambda|\mathbf{q}|}{E+M} \end{pmatrix} \chi_\lambda$$

Dirac structure $\kappa^{(s)} = g_s \mathbf{1}, \quad \kappa^{(ps)} = g_{ps} \frac{\not{q}' - \not{q}}{2M} i\gamma_5, \quad \kappa^{(v)} = g_v \gamma^\mu + \frac{f_v}{2M} i\sigma^{\mu\nu}$

→ long range=OPE, short/intermediate range = heavy mesons

Non-relativistic potentials

Low energy expansion of OBE potential

$$V(\mathbf{q}', \mathbf{q}) = \sum_{\alpha=1,5} \left[V_\alpha + V'_\alpha \, \tau_1 \cdot \tau_2 \right] O_\alpha$$

$$\begin{aligned} O_1 &= 1, \\ O_2 &= \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2, \\ O_3 &= (\boldsymbol{\sigma}_1 \cdot \mathbf{k})(\boldsymbol{\sigma}_2 \cdot \mathbf{k}), \\ O_4 &= \frac{i}{2}(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \mathbf{n}, \\ O_5 &= (\boldsymbol{\sigma}_1 \cdot \mathbf{n})(\boldsymbol{\sigma}_2 \cdot \mathbf{n}), \end{aligned}$$

$$\begin{aligned} \mathbf{k} &= \mathbf{q}' - \mathbf{q}, \\ \mathbf{P} &= \frac{1}{2}(\mathbf{q}' + \mathbf{q}), \\ \mathbf{n} &= \mathbf{q} \times \mathbf{q}' \equiv \mathbf{P} \times \mathbf{k}, \end{aligned}$$

Nucleon self-energy in Hartree-Fock approximation

$$\Sigma = -i \int_F \left(\mathrm{Tr}[GV] - GV \right)$$

- **Fermi covariants**

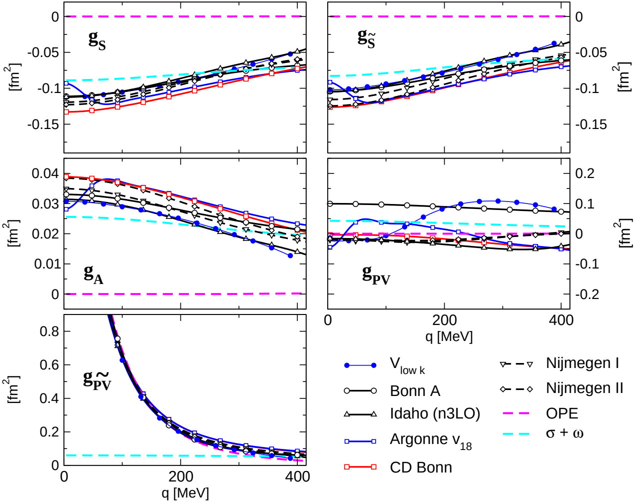
  $$\mathrm{S} = \mathbf{1} \otimes \mathbf{1}, \quad \mathrm{V} = \gamma^\mu \otimes \gamma_\mu, \quad \mathrm{T} = \sigma^{\mu\nu} \otimes \sigma_{\mu\nu}, \quad \mathrm{P} = \gamma_5 \otimes \gamma_5, \quad \mathrm{A} = \gamma_5 \gamma^\mu \otimes \gamma_5 \gamma_\mu$$

- **Pseudovector choice** $\Gamma_m = \{\mathrm{S}, \tilde{\mathrm{S}}, (\mathrm{A} - \tilde{\mathrm{A}}), \mathrm{PV}, \widetilde{\mathrm{PV}}\}$

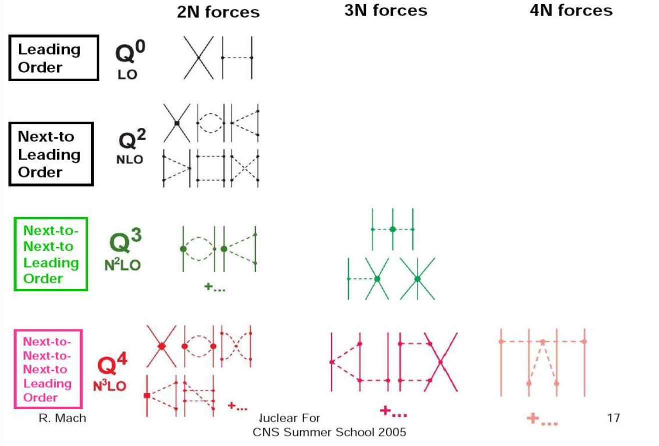
- $|LSJ\rangle \rightarrow$ partial wave helicity basis $\rightarrow$ plane wave helicity basis $\rightarrow$ **Covariant operator basis**

- $$\hat{V}^{\mathrm{I}}(|\mathbf{q}|, \theta) = g^{\mathrm{I}}_{\mathrm{S}}(|\mathbf{q}|, \theta)\, \mathrm{S} - g^{\mathrm{I}}_{\tilde{\mathrm{S}}}(|\mathbf{q}|, \theta)\, \tilde{\mathrm{S}} + g^{\mathrm{I}}_{\mathrm{A}}(|\mathbf{q}|, \theta)\, (\mathrm{A} - \tilde{\mathrm{A}}) + g^{\mathrm{I}}_{\mathrm{PV}}(|\mathbf{q}|, \theta)\, \mathrm{PV} - g^{\mathrm{I}}_{\widetilde{\mathrm{PV}}}(|\mathbf{q}|, \theta)\, \widetilde{\mathrm{PV}}$$

Lorentz invariant amplitudes



Chiral *NN* potential $\mathcal{L} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{NN}$



$\left(\frac{Q}{\Lambda}\right)^{\nu}$

Q is momentum (derivative)

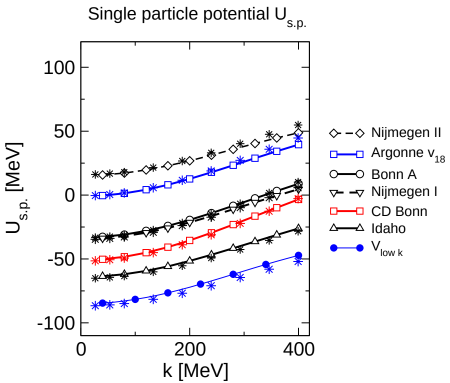
or pion mass m_π

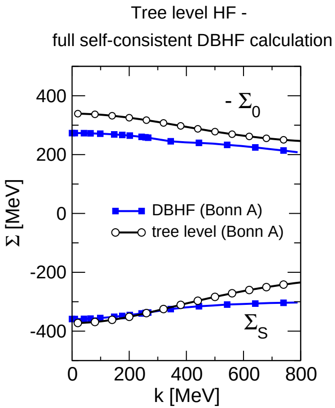
Λ is chiral symmetry

breaking scale

R. Machleidt, Nuclear Forces - Lecture 4, CNS Summer School 2005

Remarks





$$U_{s.p.}(k) = \frac{M}{E_{\mathbf{k}}} \langle \bar{u}(k) | \Sigma | u(k) \rangle$$
$$= M\Sigma_{\rm s}/E_{\mathbf{k}} - \Sigma_0 + \Sigma_{\rm v}\mathbf{k}^2/E_{\mathbf{k}}$$

Chiral condensate in nuclear matter

$$\langle\psi(\lambda)|\frac{d}{d\lambda}H(\lambda)|\psi(\lambda)\rangle = \frac{d}{d\lambda}E(\lambda)$$

$$\langle\psi(\lambda)|\frac{d}{d\lambda}H(\lambda)|\psi(\lambda)\rangle = \frac{d}{d\lambda}\langle\psi(\lambda)|H(\lambda)|\psi(\lambda)\rangle$$

Explicit chiral symmetry breaking $\mathcal{H}_{\rm QCD} = \mathcal{H}_0 + \mathcal{H}_{\rm m}$, $\mathcal{H}_{\rm m} = m_u\,\bar{u}u + m_d\,\bar{d}d + \cdots$

Introduce $\bar{q}q \equiv \frac{1}{2}(\bar{u}u + \bar{d}d)$, $m_q \equiv \frac{1}{2}(m_u + m_d)$, $\delta m_q = m_d - m_u$

$$\mathcal{H}_{\rm m} = 2m_q\,\bar{q}q - \frac{1}{2}\delta m_q(\bar{u}u - \bar{d}d) + \cdots$$

Identify $\lambda \to m_q$ and $H \to \int d^3x\,\mathcal{H}_{\rm QCD}$

$$2m_q\langle\psi(m_q)|\int d^3x\,\bar{q}q|\psi(m_q)\rangle = m_q\frac{d}{dm_q}\langle\psi(m_q)|\int d^3x\,\mathcal{H}_{\rm QCD}|\psi(m_q)\rangle\,.$$

$|\psi(m_q)\rangle = |\rho_N\rangle$ (ground state of nuclear matter at ρ_N)
$|\psi(m_q)\rangle = |0\rangle$ (vacuum state)
Taking the difference of these two cases one obtains

$$2m_q(\langle\bar{q}q\rangle_{\rho_N} - \langle\bar{q}q\rangle_0) = m_q\frac{d}{dm_q}\left(\mathcal{E}(\rho_N) - \mathcal{E}(0)\right)\,.$$

Chiral condensate in nuclear matter

$$2m(\langle\bar{q}q\rangle_\rho - \langle\bar{q}q\rangle_0) = m\frac{d}{dm}\left(M + \frac{E}{A}\right)\rho$$

$$\frac{\langle\bar{q}q\rangle_\rho}{\langle\bar{q}q\rangle_0} = 1 + \frac{m\,\rho}{2m\langle\bar{q}q\rangle_0}\frac{d}{dm}\left(M + \frac{E}{A}\right)$$

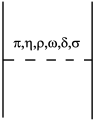
$$\frac{\langle\bar{q}q\rangle_\rho}{\langle\bar{q}q\rangle_0} = 1 - \frac{\rho}{m_\pi^2 f_\pi^2}\left[\sigma_N + m\frac{d}{dm}\frac{E}{A}\right]$$

Gell-Mann-Oakes-Renner (GOR) relation $2m\langle\bar{q}q\rangle_0 = -m_\pi^2 f_\pi^2,$

pion-nucleon sigma-term $\sigma_N = m\frac{dM}{dm} =< N|m\bar{q}q|N >$

Chiral condensate in nuclear matter

DBHF with OBE Potential



$$m\frac{dE}{dm} = \sum_{S,V,\pi,\rho}\left[\frac{\partial E}{\partial m_i}\frac{dm_i}{dm} + \frac{\partial E}{\partial g_i}\frac{dg_i}{dm_i} + \cdots\right] + \sigma_N\frac{\partial E}{\partial M}$$

$$\sigma_S \equiv m\frac{dm_S}{dm} = C_S\sigma_N$$

$0.5 < C_S < 1$

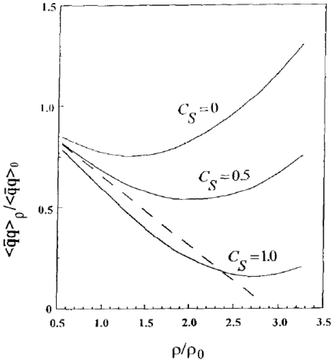


Fig. 3. The ratio $\langle\bar{q}q\rangle_\rho/\langle\bar{q}q\rangle_0$ of the chiral condensate at baryon density ρ with respect to its value at $\rho = 0$. Dashed curve: leading order result with $\bar{\sigma}_N = \sigma_N$. The solid curves correspond to different scalar "sigma terms" as in Fig. 2.

R. Brockmann, W. Weise, Phys. Lett. B 367 (1996)

Chiral condensate in nuclear matter

$$\frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0} = 1 - \frac{\rho}{m_\pi^2 f_\pi^2} \left[\sigma_N + m \frac{d}{dm} \frac{E}{A} \right]$$

$$\frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0} = 1 - \frac{\rho}{m_\pi^2 f_\pi^2} \left[\sigma_N + m \frac{\partial (E/A)}{\partial M} \frac{dM}{dm} + m \frac{\partial (E/A)}{\partial m_\pi} \frac{dm_\pi}{dm} + \cdots \right]$$

$$\frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0} = 1 - \frac{\rho}{\rho^\chi} \left[1 + \frac{\partial (E/A)}{\partial M} + \frac{\partial (E/A)}{\partial m_\pi} \frac{m_\pi}{2\sigma_N} \right]$$

$$2m \langle \bar{q}q \rangle_0 = -m_\pi^2 f_\pi^2 \qquad \sigma_N = m \frac{dM}{dm}, \qquad \frac{dm_\pi}{dm} = \frac{m_\pi}{2m} \quad , \quad \rho^\chi \equiv \frac{m_\pi^2 f_\pi^2}{\sigma_N}$$

Quark mass dependence of the nuclear forces,
E. Epelbaum, W. Glöckle, Ulf-G. Meissner, Eur. Phys. J. A **18**, 499 (2003)