Relating Dyson-Schwinger equations of first and second order formalisms

Author: Selym Villalba Chávez
Supervisor: Reinhard Alkofer

†Institut für Physik-Theorie, Universitätsplatz 5, A−8010, Graz, Austria.
September 12, 2007.

OUTLINE

- Introduction.
- The equivalence in a scalar field theory.
- Extension to Yang-Mills theory.
- Perspectives.

Introduction

The Coulomb Gauge

In the standard path integral formalism

$$\vec{\nabla} \cdot \vec{A} = 0$$

1. The number of dynamical variables is the same number of degree of freedom.
2. Existence of two independent propagation modes.
3. The physical mode is associated to the spatial propagator.

$$\mathfrak{D}_{\mathcal{A}\mathcal{A}} \sim \mathbf{k}^2 \;\; \text{when} \;\; k \longrightarrow 0$$

Confinement

Confinement is explained from the temporal mode.

$$\mathfrak{D}_{\mathcal{A}_0\mathcal{A}_0} = \frac{1}{g^2}\mathcal{V}(|\mathbf{x}|)\delta(t) + \frac{1}{g^2}\Pi(\mathbf{x}, t),$$

- $\mathcal{V}(|\mathbf{x}|)$ color Coulomb potential
- $\Pi(\mathbf{x}, t)$ non-instantaneous vacuum polarization.

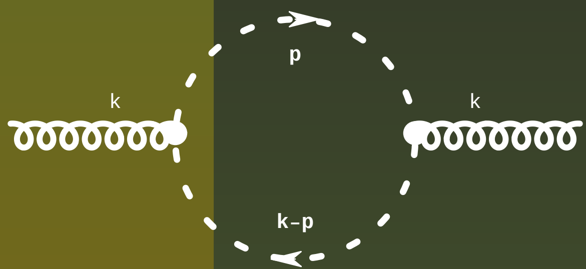
Gribov found

$$\lim_{|\mathbf{x}|\to\infty} \mathcal{V}(|\mathbf{x}|) = \infty$$

Numerical calculations have shown that at long-distance behave as

$$\mathcal{V}(|\mathbf{x}|) \propto |\mathbf{x}|$$

Energy Divergences



$$\sim \int \frac{dp_0}{2\pi} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{(\mathbf{k}-\mathbf{p})^2\mathbf{k}^2} \to \infty$$

- Yang-Mills theory seems not to be regularizable by standard proceedure.
- It has been shown that these divergences are cancelled up to two loop in perturbation theory.
- General proof of this cancellation has not been done.

A possible solution

$$\langle 0_{\rm out}|0_{\rm in}\rangle = \int \mathcal{D}\pi\mathcal{D}\phi \exp\left[\frac{i}{\hbar}\int d^4x\,\{\pi\partial_t\phi - \mathcal{H}(\pi,\phi)\}\right]$$

We can perform the integration of the non-physical degrees of freedom which remove the mentioned problem.

Price:

- The Schwinger-Dyson equations become cumbersome.

Motivation:

The renormalizability of the non-abelian gauge theories in Coulomb gauge constitutes a challenge in theoretical physics.

Our Goal

To Proof the Yang-Mills theory renormalizability in Coulomb gauge.

$\equiv$Equivalence between both formalisms$\equiv$.

- To identify the standart propagators and vertexes in terms of those that appear in the phase-space formalism.

The equivalence in a scalar field theory

The ϕ^4 Theory

Let us consider self-interacting scalar field Lagrangian

$$\mathfrak{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4,$$

whose Hamiltonian is written as

$$\mathcal{H} = \frac{1}{2}\pi^2 + (\nabla\phi)^2 + \frac{1}{2}\phi^2 + \frac{\lambda}{4!}\phi^4$$

so, the action in canonical variables is given by

$$\mathcal{S}_F = \int d^4x \left\{ \pi\partial_t\phi - \frac{1}{2}\pi^2 + \frac{1}{2}\phi\nabla^2\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 \right\}.$$

The 1PI Effective Action

We introduce the generating functional of connected graphs

$$\mathcal{Z}^{\mathcal{H}}[J_i] = \exp\left(i\mathcal{W}^{\mathcal{H}}[J_i]\right)$$

and we build the one particle irreducible effective action

$$\Gamma^{\mathcal{H}}[\pi_c, \phi_c] = \mathcal{W}^{\mathcal{H}}[J_{\pi_c}, J_{\phi_c}] - \int d^4x \left(J_\pi \pi_c + J_\phi \phi_c\right)$$

which satisfy the equations

$$\frac{\delta\Gamma^{\mathcal{H}}}{\delta\pi_c} = -J_\pi \ \text{ and } \ \frac{\delta\Gamma^{\mathcal{H}}}{\delta\phi_c} = -J_\phi$$

The Classical Motion Equations

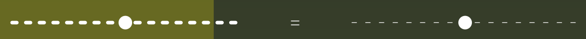
By passing to the Schwinger-Dyson relation we get

$$\frac{\delta\Gamma}{\delta\pi_c} = \partial_t\phi_c - \pi_c \Rightarrow \quad \pi_c = \partial_t\phi_c$$

$$\frac{\delta\Gamma}{\delta\phi_c} = -\partial_t\pi_c + (\nabla^2 - m^2)\phi_c - \frac{\lambda}{3!}\left[\phi_c^3 + \phi_c\tilde{\Delta}_{\phi\phi} + \tilde{\Delta}_{\phi\psi_i}\frac{\delta\tilde{\Delta}_{\phi\phi}}{\delta\psi_i}\right]$$

- $\psi_i = \phi_c, \pi_c$.
- $\tilde{\Delta}$ is a functional depending on the canonical variables.
- It is reduced to the propagators when $\phi_c, \pi_c = 0$.

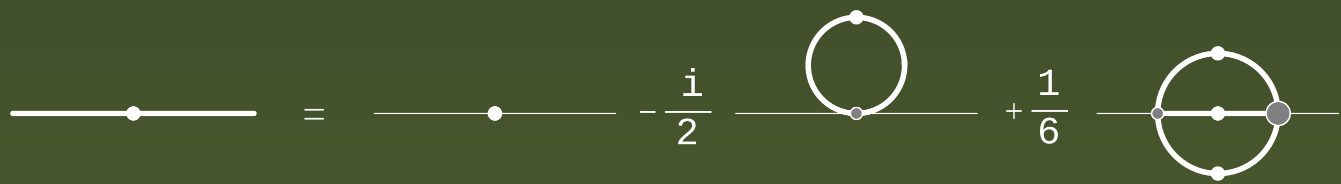
Scwhinger-Dyson Equations

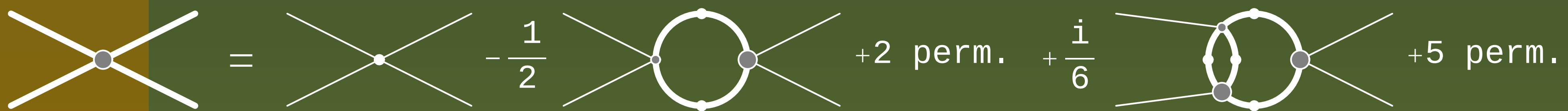


$$\Leftrightarrow \Gamma^{\mathcal{H}(ij)}_{\pi\pi} = -\delta^4(x - x')$$

$$\Leftrightarrow \Gamma^{\mathcal{H}(ij)}_{\pi\phi} = -\partial_t \delta^4(x - x')$$

- There are not dressed vertexes that involve π_c-field.





⋮

The Effective action

$$\Gamma^{(\mathcal{H})}[\pi_c, \phi_c] = \frac{1}{2}\Gamma^{\mathcal{H}(ij)}_{\pi\pi}\pi^i_c\pi^j_c + \Gamma^{\mathcal{H}(ij)}_{\phi\pi}\phi^i_c\pi^j_c + \frac{1}{2}\Gamma^{\mathcal{H}(ij)}_{\phi\phi}\phi^i_c\phi^j_c + \ldots$$

By considering the substitution of

$$\pi^i_c = \Gamma^{\mathcal{H}(ij)}_{\pi\phi}\phi^j_c$$

we get

$$\Gamma^{\mathfrak{L}(ij)}_{\phi\phi} = \frac{1}{2}\left(i\Gamma^{\mathcal{H}(il)}_{\phi\pi}\Delta^{\mathcal{H}(lm)}_{\pi\pi}\Gamma^{\mathcal{H}(mj)}_{\pi\phi} + \Gamma^{\mathcal{H}(ij)}_{\phi\phi}\right)$$

- $\Gamma^{\mathfrak{L}(ij)}_{\phi\phi}$ is the full propagator in the standart path integral formulation.

Consequences

$$\Gamma^{(\mathfrak{L})}[\phi_c] \equiv \Gamma^{(\mathcal{H})}[\pi_c, \phi_c]$$

It means that

$$\mathcal{W}^{(\mathfrak{L})}[j_\phi] = \mathcal{W}^{(\mathcal{H})}[j_\pi, j_\phi] - j_\pi^i \pi_c^i$$

- We have found the explicit relation between the dressed inverse propagators.

Extension to Non – Abelian Gauge theory.

Yang-Mills Theory

In the present case

$$\mathcal{S}_{Y-M} = \mathcal{S}_F + \mathcal{S}_{F-P}.$$

being

$$\mathcal{S}_F = \int d^4x \left\{ -\vec{\Pi}^a \cdot \vec{E}^a - \frac{1}{2} \left(\vec{\Pi}^a \cdot \vec{\Pi}^a + \vec{B}^a \cdot \vec{B}^a \right) \right\},$$

whereas

$$\mathcal{S}_{F-P} = \int d^4x \left[-\frac{1}{2\xi} \left(\vec{\nabla} \cdot \vec{A}^a \right)^2 - \bar{c}^a \vec{\nabla} \cdot \vec{D}^{ab} c^b \right].$$

- $\vec{E}^a = -\partial^0 \vec{A} - \vec{\nabla} A_0^a + g f^{abc} \vec{A}^b A_0^c.$
- $\vec{D}^{ab} = \vec{\nabla} \delta^{ab} - g f^{acb} \vec{A}^c.$

The Interacting Part

An additional vertex appear given by

$$\sim g f^{abc} \vec{\Pi}^a \vec{A}^b A_0^c,$$

Consequences

- Generation loops that involve propagators like $\mathfrak{D}_{\Pi A}$, $\mathfrak{D}_{\Pi A_0}$, and so on.
- The classical version of $\vec{\Pi}_c$ cannot be found as in the scalar field theory.
- The equivalence between both formalisms is not evident.

The Tree Level Elements

Nevertheless, the bare propagators in this theory are given by

$$\Gamma^{0(AA)}_{ij} = \mathcal{S}^{(AA)}_{ij} + i\mathcal{S}^{(A\Pi)}_{il}\Delta^{lm}_{0(\Pi\Pi)}\mathcal{S}^{(\Pi A)}_{mj}$$

$$\Gamma^{0(A_0A_0)}_{ij} = i\mathcal{S}^{(A_0\Pi)}_{il}\Delta^{lm}_{0(\Pi\Pi)}\mathcal{S}^{(\Pi A_0)}_{mj}$$

$$\Gamma^{0(AA_0)}_{ij} = i\mathcal{S}^{(A\Pi)}_{il}\Delta^{lm}_{0(\Pi\Pi)}\mathcal{S}^{(\Pi A_0)}_{mj}$$

We point out that, $\Gamma^{0(c\bar{c})}_{ij}$, $\Gamma^{(AAAA)}_{ijkl}$, $\Gamma^{(AAA)}_{ijk}$ and $\Gamma^{(Ac\bar{c})}_{ijk}$ are the same in both formalism.

However, the remaining vertex is given by

$$\Gamma^{0(AA_0AA_0)}_{ijkl} = i\mathcal{S}^{(\Pi AA_0)}_{mkl}\Delta^{mp}_{0(\Pi\Pi)}\mathcal{S}^{(\Pi AA_0)}_{pij}\,.$$

Perspectives

Perspectives

1. We have found the Schwinger-Dyson equation within the phase space formalism with interpolating gauge.
2. We are implementing a general proceedure to indentify the dressed propagators and vertexes in the standard formalism.
3. To find the Slavnov-Taylor identities
4. To study the Coulomb gauge limit.

The end