

*Double beta decay in deformed nuclei  
Why and how to describe ?*

*Mohamed A.S. Yousef*

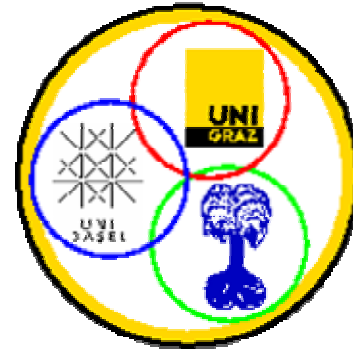
*Collaboration with*

*Prof. A. Faessler and Dr. V. Rodin*

EBERHARD KARLS  
UNIVERSITÄT  
TÜBINGEN



*Todtmoos 2007*



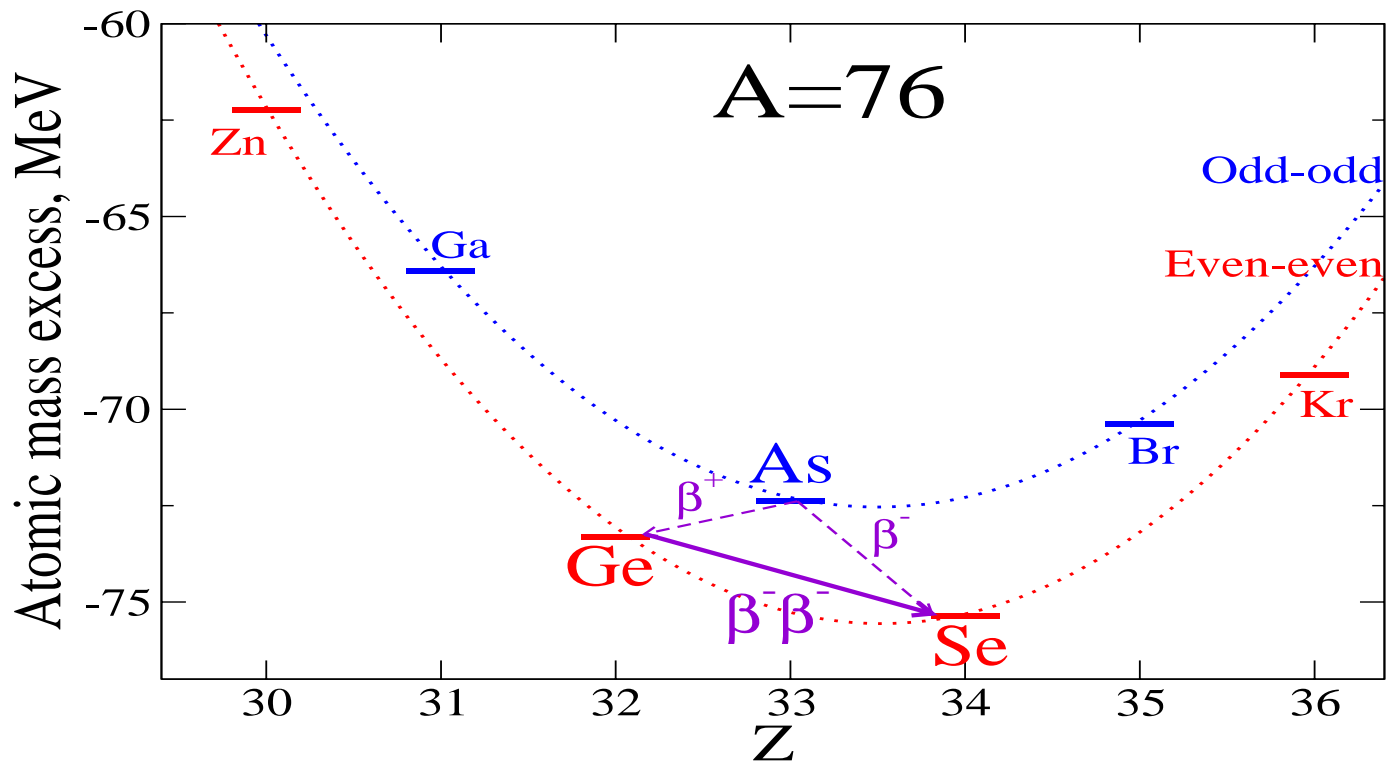
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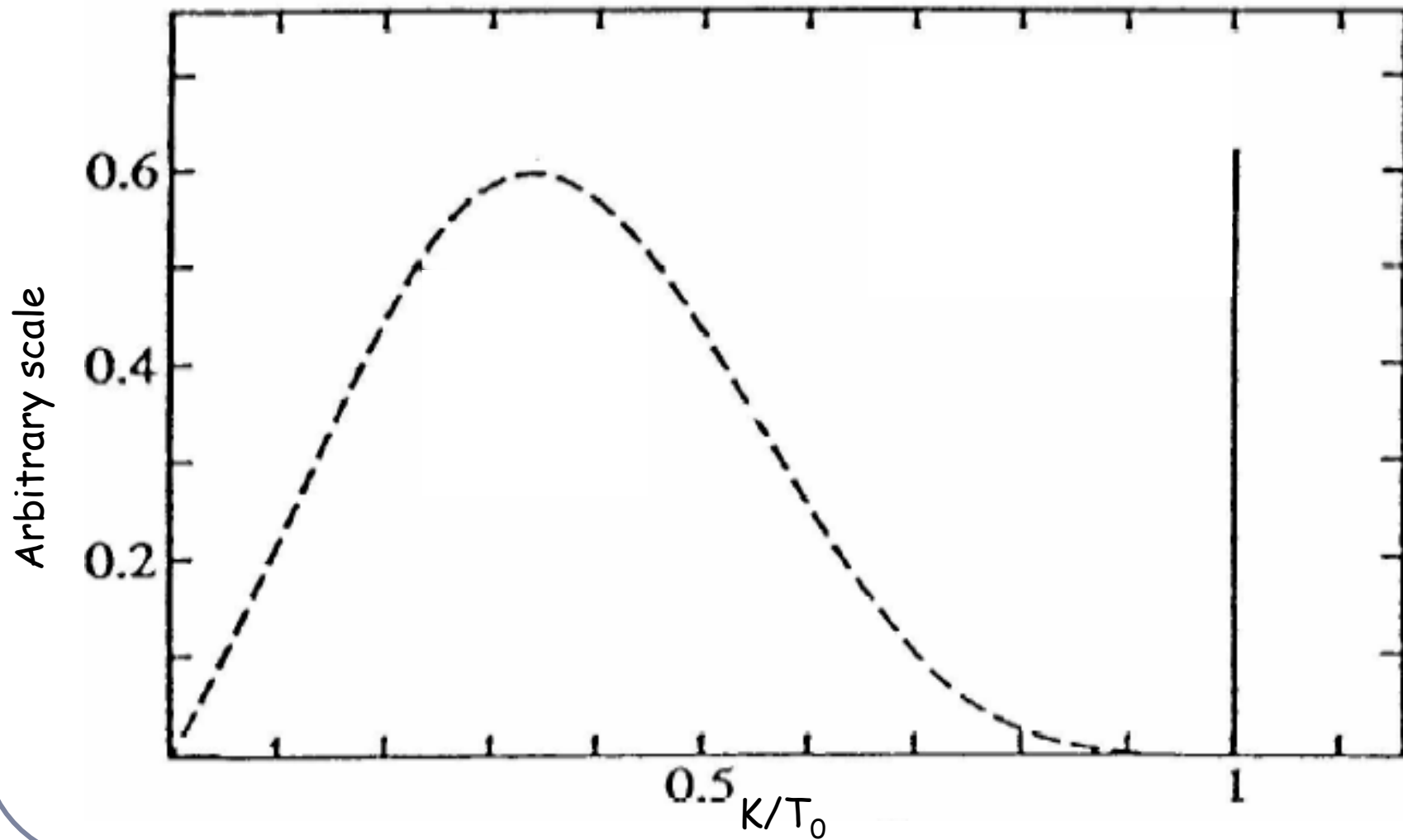
# Introduction

Mass parabola

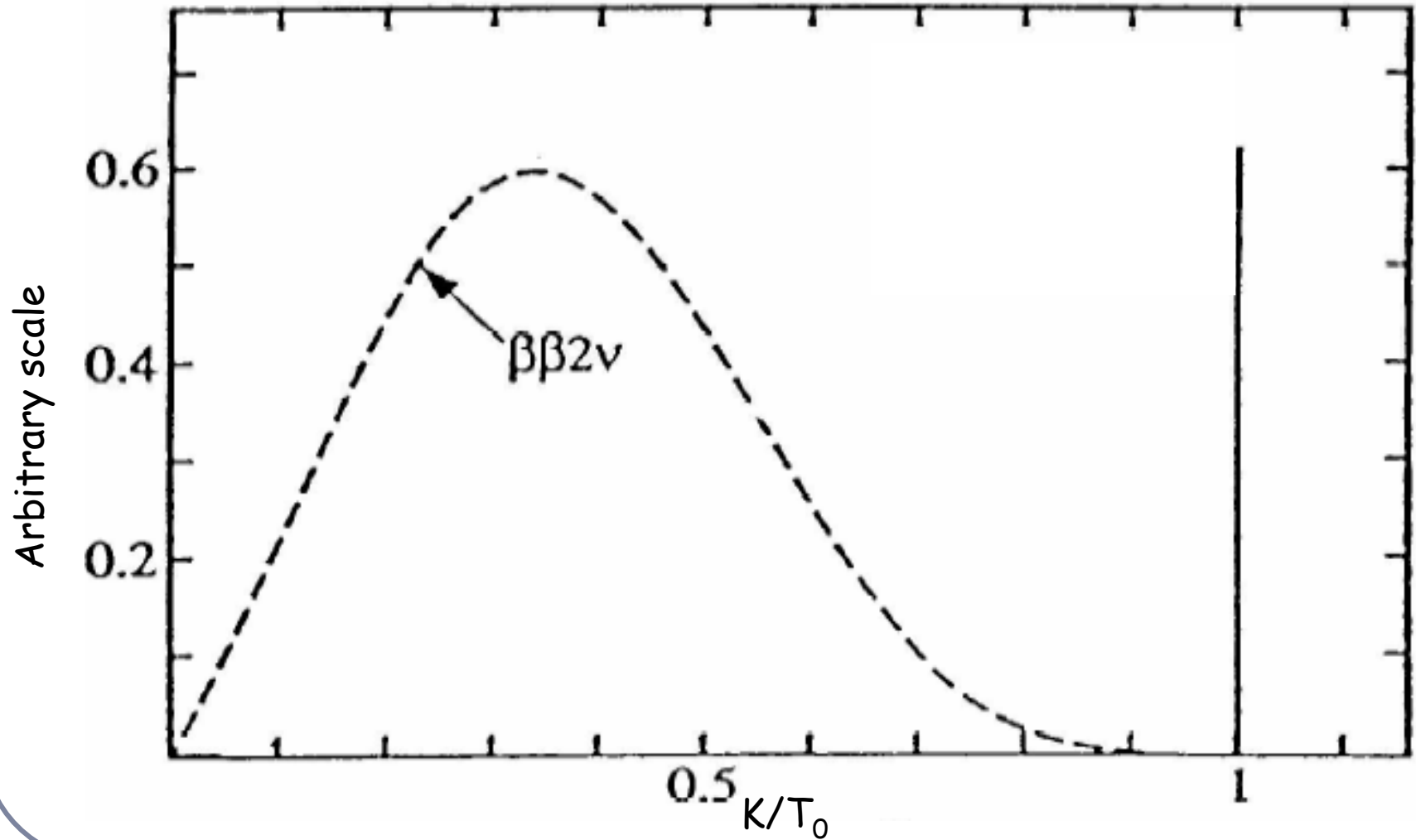


Mass parabolas: Coulomb interaction + Nucleon pairing

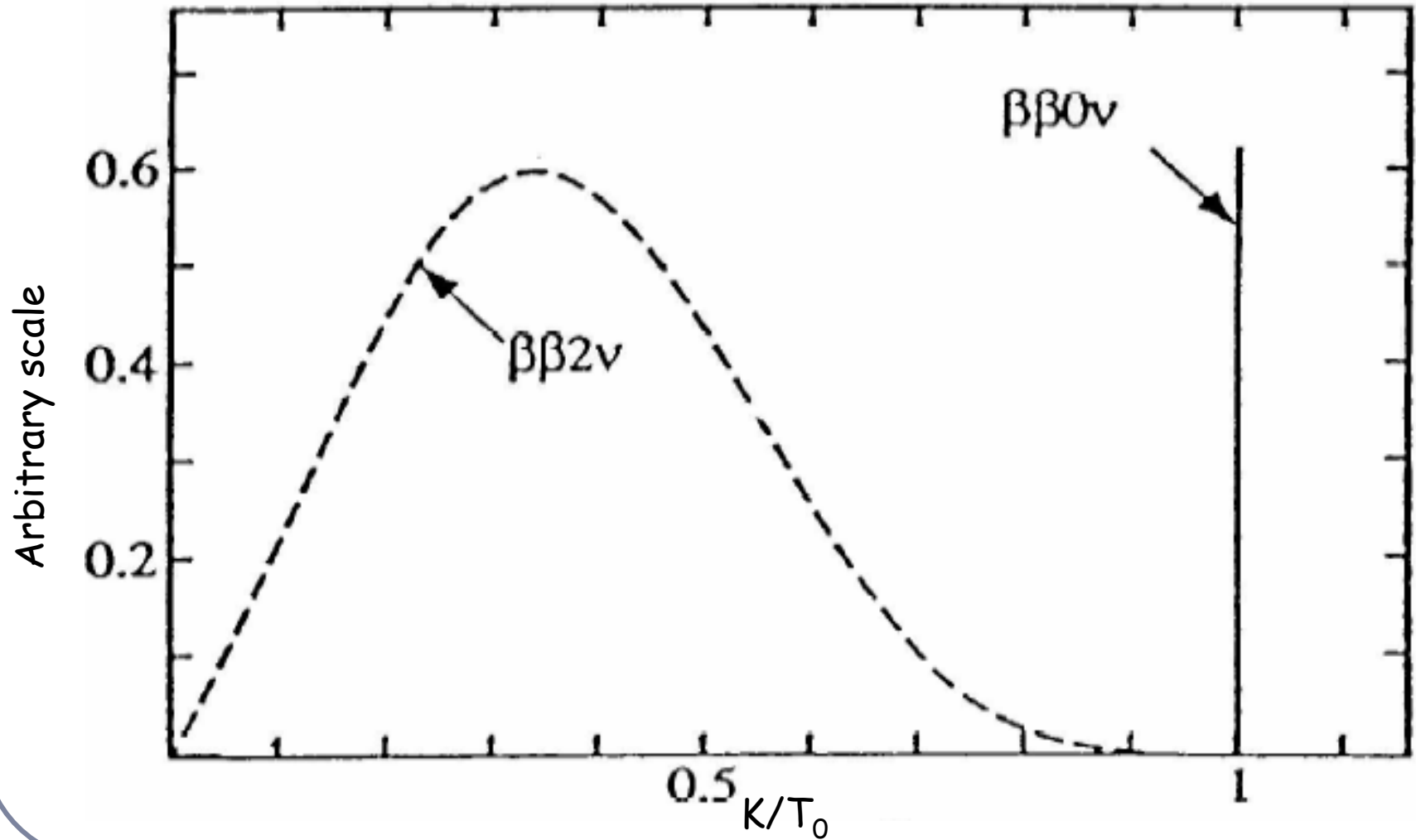
## Two electron energy spectrum



## Two electron energy spectrum

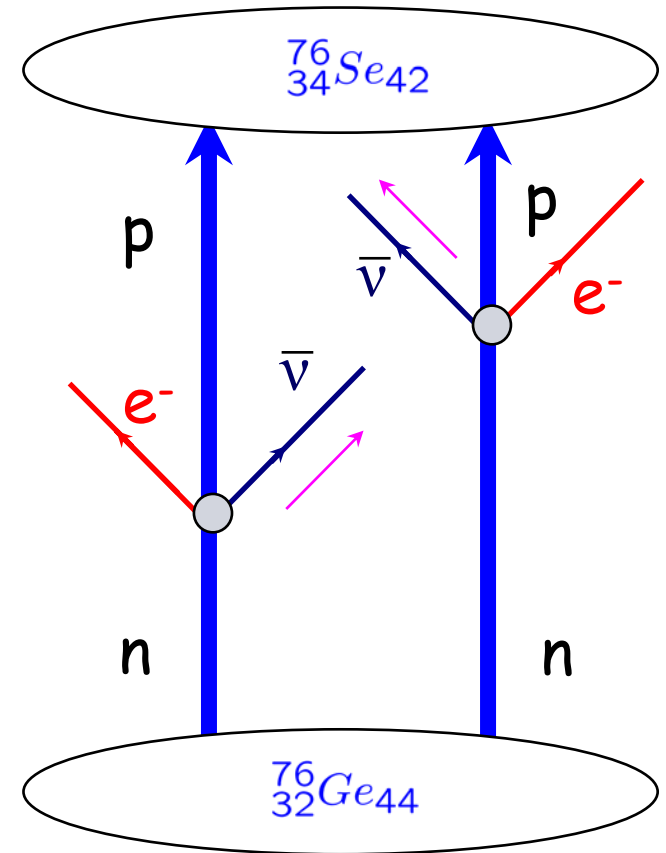


## Two electron energy spectrum



## Two neutrino double beta decay ( $2\nu\beta\beta$ )

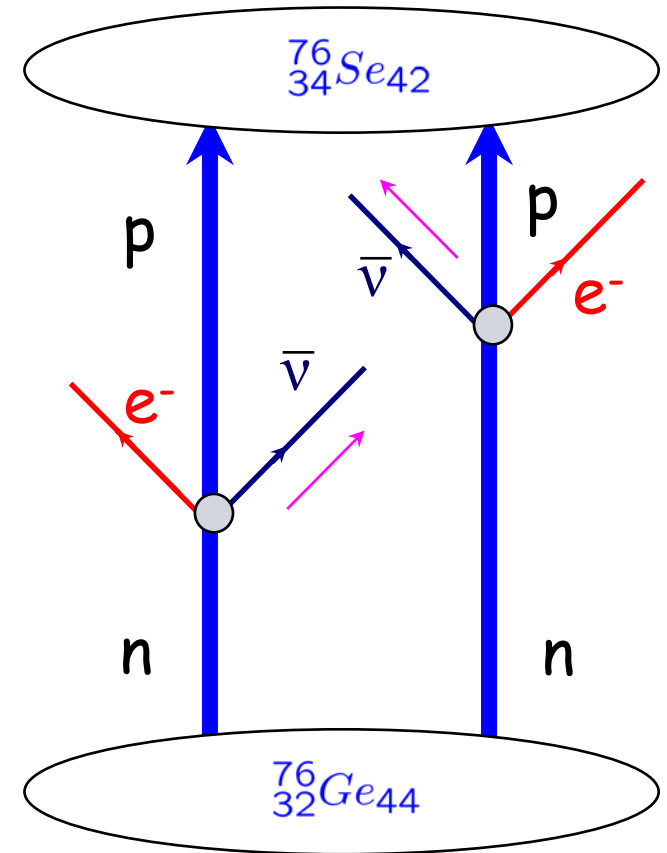
$$(A, Z) \rightarrow (A, Z + 2) + 2e^{-} + 2\bar{\nu}_e$$



## Two neutrino double beta decay ( $2\nu\beta\beta$ )

$$(A, Z) \rightarrow (A, Z + 2) + 2e^{-} + 2\bar{\nu}_e$$

This process fully consistent with **SM** of electroweak interaction.



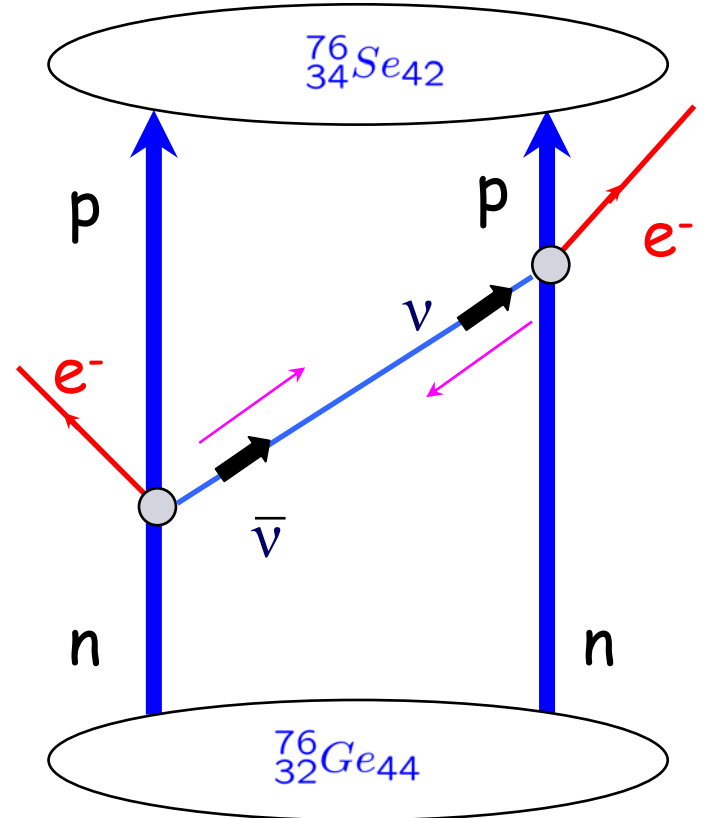
measured  $T_{1/2}^{2\nu}$  (A. Barabash, 2005)

Isotope	$T_{1/2}^{2\nu}$ , in $10^{19}$ y
---------	-----------------------------------

$^{48}\text{Ca}$	$4.2^{+2.1}_{-1.0}$
$^{76}\text{Ge}$	$150 \pm 10$
$^{82}\text{Se}$	$9.2 \pm 0.7$
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$^{128}\text{Te}$	$(2.5 \pm 0.3) \times 10^5$
$^{130}\text{Te}$	$90 \pm 10$
$^{136}\text{Xe}$	$> 81$ (90% CL)
$^{150}\text{Nd}$	$0.78 \pm 0.07$
$^{238}\text{U}$	$200 \pm 60$

# Neutrinoless double beta decay ( $0\nu\beta\beta$ )

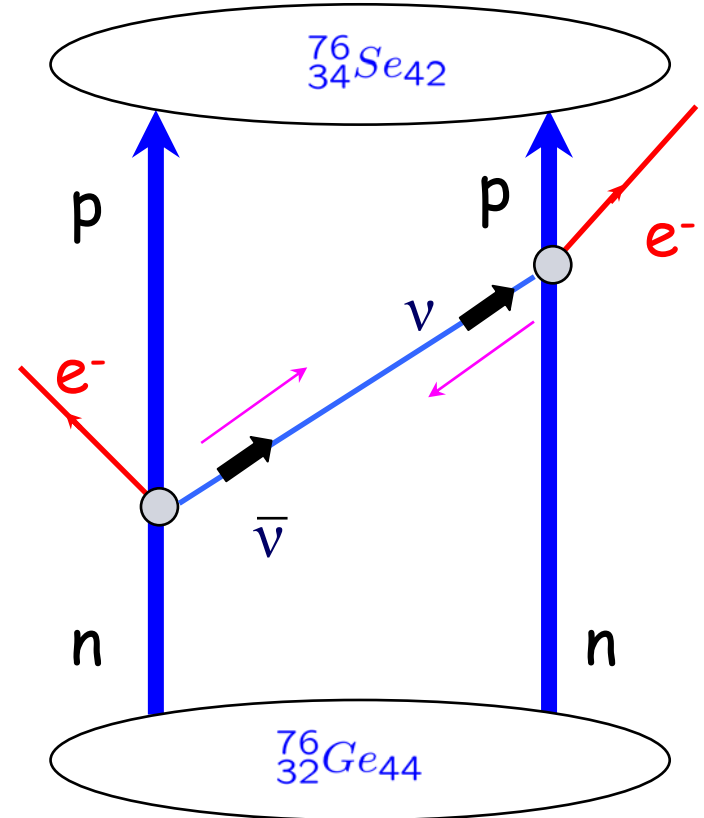
$$(A, Z) \rightarrow (A, Z + 2) + 2 e^{-}$$



## Neutrinoless double beta decay ( $0\nu\beta\beta$ )

$$(A, Z) \rightarrow (A, Z + 2) + 2 e^{-}$$

Forbidden in **SM** electroweak interaction and it may occur if lepton number conservation is not an exact symmetry of nature .

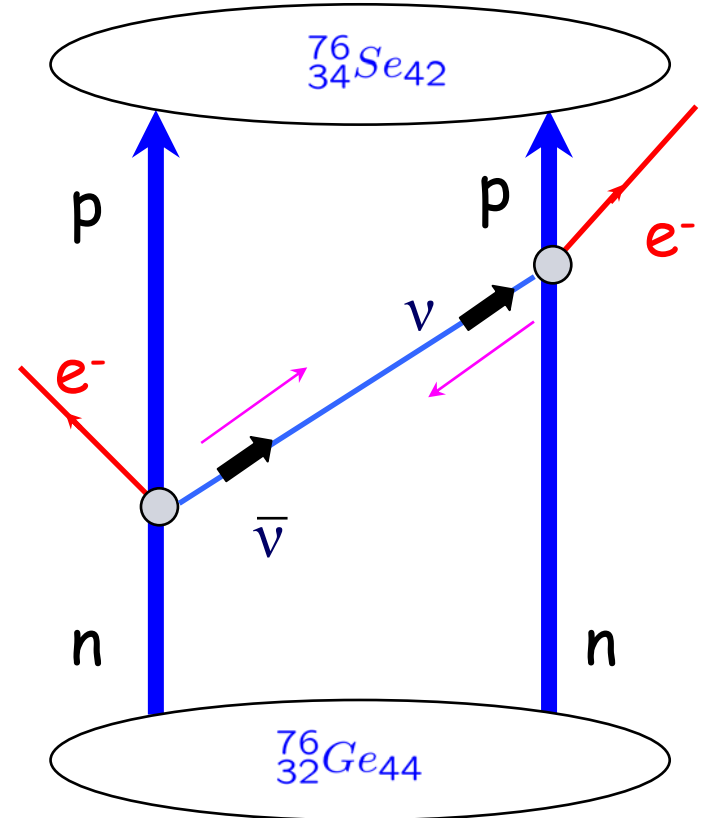


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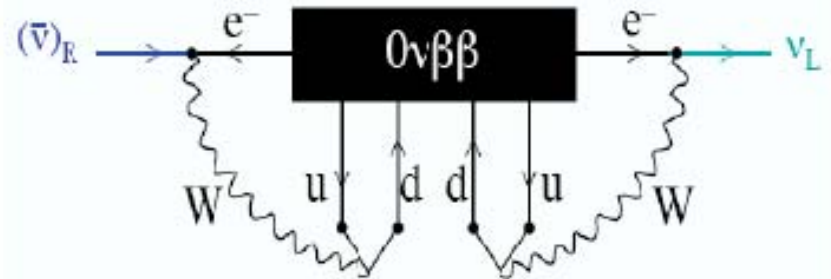
If it is observed  $\longrightarrow$  neutrino is **massive Majorana particle**



# Majorana Mass

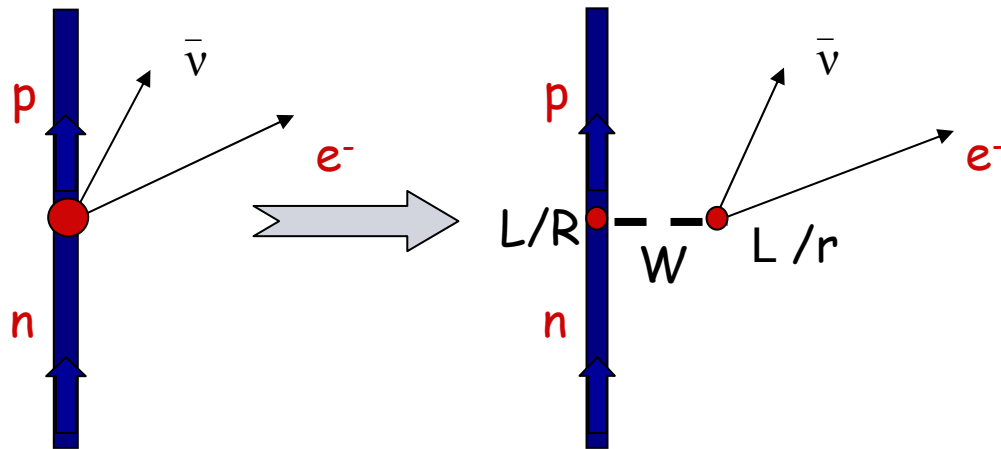
$$\mathcal{L}_M = m_D \bar{\nu} \nu + m_M \bar{\nu} \nu^c + m_M^* \bar{\nu}^c \nu$$

Schechter-Valle  
theorem



# Grand Unification

## Left-right symmetric $SO(10)$ Models



$$W_1(81 \text{ GeV}) = W_L \cos\theta + W_R \sin\theta$$

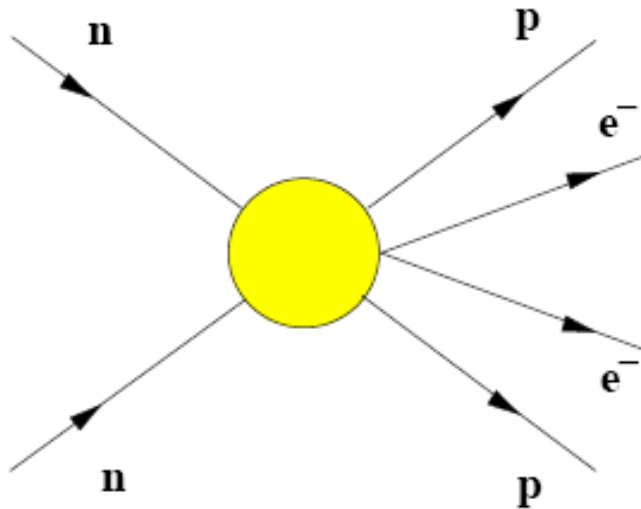
$$W_2(? \text{ GeV}) = W_R \cos\theta - W_L \sin\theta$$

$$j_{l/r}^\mu = \bar{e} \gamma^\mu (1 \mp \gamma_5) \nu, \quad J_{L/R}^\mu = \bar{n} \gamma^\mu (g_V \mp g_A \gamma_5) p$$

$$g_V=1, g_A=1.25$$

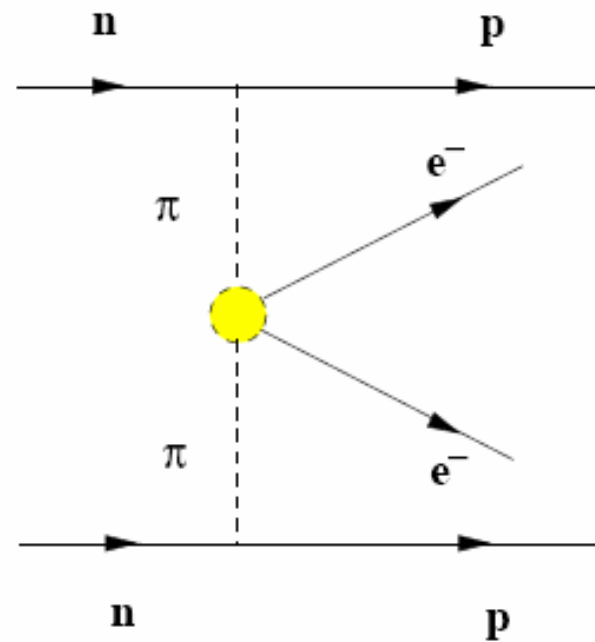
# Heavy particle exchange

Two-nucleon mechanism



Can be neglected

Pion-exchange mechanism



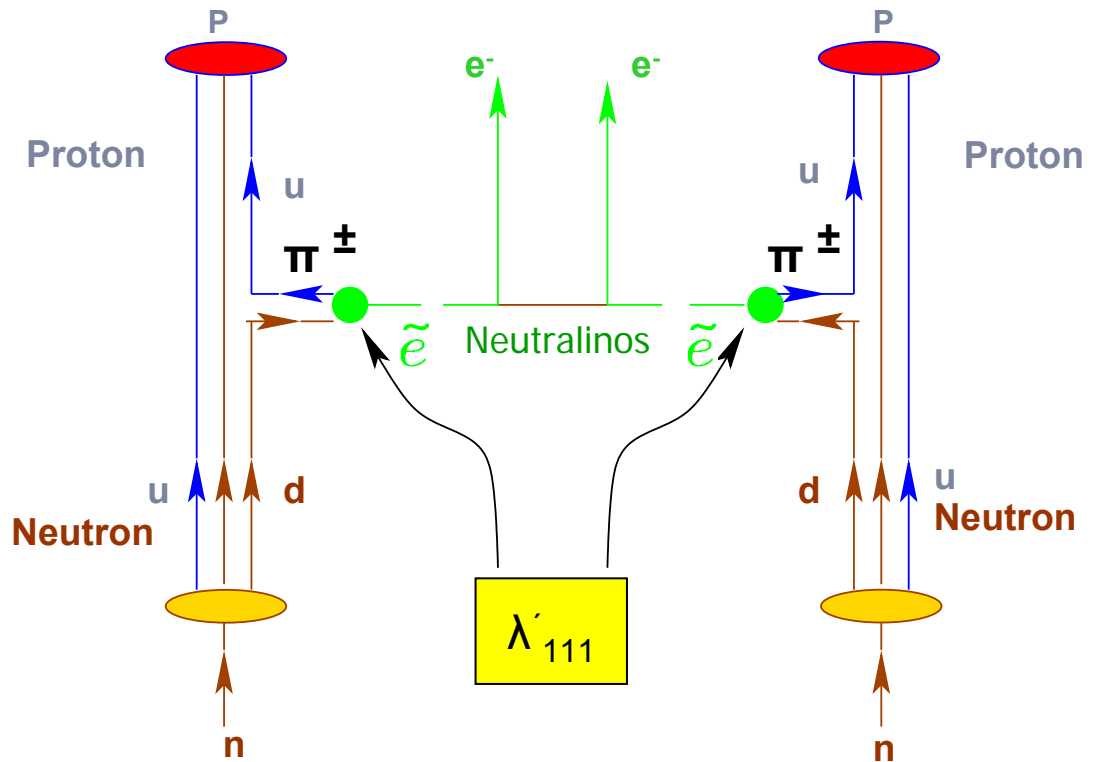
The dominant contribution

# R-parity violating SUSY

Bosons  $\leftrightarrow$  Fermions

$$\tilde{\chi} = \tilde{\gamma}, \tilde{Z}^0, \tilde{h}_1^0, \tilde{h}_2^0$$

Neutralinos

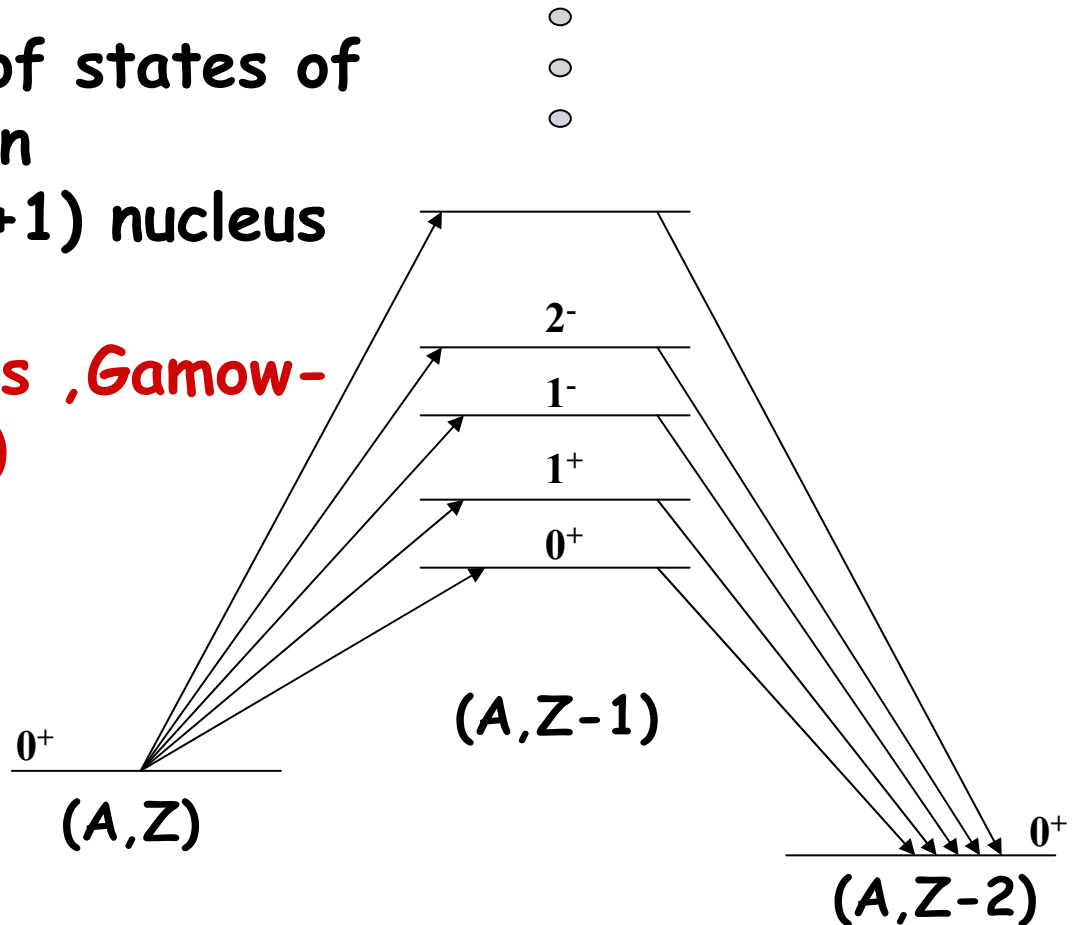


Faessler, Kovalenko, Simkovic, PRL(1997): coupling to pions can increase the SUSY 0-decay probability by a factor 10000!

# Nuclear Dynamics of $\beta\beta$ Decay

Virtual excitation of states of all multipolarities in intermediate  $(A, Z+1)$  nucleus

( $2\nu\beta\beta$  only  $1^+$  states, Gamow-Teller transitions)



## Matrix Elements

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**Frank Avignone:** "Nuclear matrix elements for Double Beta Decay are as important as the data to determine Neutrino Mass"  
(Erice'05)

## Matrix Elements

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The rate of  $2\nu\beta\beta$  decay is

$$1/T_{1/2}^{2\nu} = G_{2\nu} |M_{2\nu}|^2$$

The rate of  $0\nu\beta\beta$  decay is

$$1/T_{1/2}^{0\nu} = G_{0\nu} |M_{0\nu}|^2 \langle m_\nu \rangle^2$$

The leptonic phase space factors  $G$  are accurately calculable.

The fundamental particle physics parameter  $\langle m_\nu \rangle$  can be determined with the same accuracy as the **nuclear matrix elements**

## 2νββ decay : Description of the Gamow-Teller amplitudes

The double Gamow-Teller transition from g.s. to g.s.:

$$M_{GT}^{2\nu} = \sum_{m_i m_f} \frac{\langle 0_f^+ \| \beta^- \| 1_{m_f}^+ \rangle \langle 1_{m_i}^+ \| \beta^- \| 0_i^+ \rangle}{(E^{m_f} + E^{m_i})/2}$$

## 0ν decay :

All multipoles contribute (0<sup>+</sup>, 1<sup>-</sup>, .....), enhanced role of nucleon short range correlations.

$$|M_{0\nu}| \equiv M_{GT}^{0\nu} - \frac{g_V^2}{g_A^2} M_F^{0\nu} = \left\langle f \left| \sum_{lk} H(r_{lk}, \bar{A}) \tau_l^+ \tau_k^+ (\vec{\sigma}_l \cdot \vec{\sigma}_k - \frac{g_V^2}{g_A^2}) \right| i \right\rangle$$

Neutrino Potential

## Why to focus on the nuclear deformation?

Nuclear transition	$\langle M'_{0\nu} \rangle$		$T_{1/2}^{0\nu} (\langle m_{\beta\beta} \rangle = 50\text{meV})$ [yrs]
	RQRPA	QRPA	
$^{76}\text{Ge} \rightarrow ^{76}\text{Se}$	3.92	4.51	$8.60 \times 10^{26}$
$^{100}\text{Mo} \rightarrow ^{100}\text{Ru}$	2.78	3.34	$2.37 \times 10^{26}$
$^{130}\text{Te} \rightarrow ^{130}\text{Xe}$	2.95	3.66	$2.16 \times 10^{26}$
$^{150}\text{Nd} \rightarrow ^{150}\text{Sm}$	4.16	4.74	$2.23 \times 10^{25}$

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# Decay rate dependance:

---

	1. Q-value (MeV)	2. Coulomb Effects * Z
$^{76}\text{Ge} \rightarrow ^{76}\text{Se}$	2.039	32
$^{100}\text{Mo} \rightarrow ^{100}\text{Ru}$	3.034	42
$^{130}\text{Te} \rightarrow ^{130}\text{Xe}$	2.533	52
$^{150}\text{Nd} \rightarrow ^{150}\text{Sm}$	3.367	60

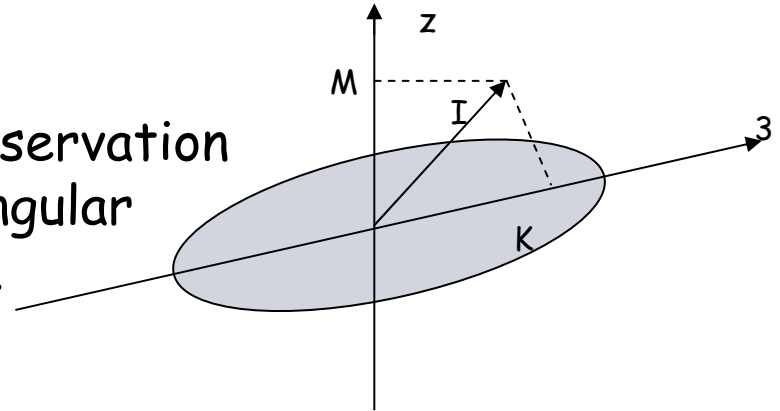
\* Coulomb Effects  $\propto (Z/1-\exp(-cZ))^4$

measured  $T_{1/2}^{2\nu}$  (A. Barabash, 2005)

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# Deformed Nuclei:

Use of deformed single particle basis  
 → the s.p. wave function without conservation of the total angular momentum, only angular momentum projection (K) is conserved.



$$M_{GT}^{2\nu} = \sum_{m_i, m_f} \sum_{K=0, \pm 1} \frac{\langle 0_f^+ \| \beta_K^- \| 1_{m_i}^+(K) \rangle \langle 1_{m_f}^+(K) | 1_{m_i}^+(K) \rangle \langle 1_{m_i}^+(K) \| \beta_K^- \| 0_i^+ \rangle}{(E_K^{m_f} + E_K^{m_i})/2}$$

The **overlap** is necessary since the two sets of intermediate states calculated within the QRPA are not the same.

Where,

$$\beta_K^- = \sum_{pn} \langle p | \tau^+ \sigma_K | n \rangle a_p^+ a_n$$

## Quasi Random Phase Approximation

---

**pn QRPA** : successfully exploited in nuclear physics to describe the properties of excited states of open shell nuclei and to calculate intensities of various nuclear reaction, including double beta decay but Violates PEP by generating too many ground state correlations.

**RQRPA**: Taking into account the quasiparticle occupation numbers in the QRPA ground state, restore the PEP but violates the model independent ISR .

**SCQRPA**: Fixing the number of particles in the ground state, but also violates the model independent ISR .

**FR-QRPA**: Fulfills the model independent ISR.

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# QRPA Formalism (deformed nuclei)

---

QRPA wave functions of GT excitations for even-even nuclei in the laboratory frame

$$|1M(K), m\rangle = \sqrt{\frac{3}{16\pi^2}} \left[ D_{MK}^1(\varphi, \vartheta, \psi) Q_K^{m^+} + (-1)^{1+K} D_{M-K}^1(\varphi, \vartheta, \psi) Q_{-K}^{m^+} \right] |RPA\rangle \quad (K = \pm 1)$$

$$|1M(K), m\rangle = \sqrt{\frac{3}{8\pi^2}} \left[ D_{MK}^1(\varphi, \vartheta, \psi) Q_K^{m^+} \right] |RPA\rangle \quad (K = 0)$$

The intrinsic states are generated by the phonon creation operator

---

$$Q_K^{m\dagger} = \sum_i (X_{i,K}^m A_i^\dagger(K) - Y_{i,K}^m \tilde{A}_i(K))$$

$$A_{pn}^\dagger(K) = \alpha_{p\rho_p}^\dagger \tilde{\alpha}_{n\rho_n}^\dagger$$

$$[A_i, A_j^\dagger] = \delta_{ij} + X$$

The quasi particle creation and annihilation operators can be defined by the Bogolyubov transformation

$$\begin{pmatrix} \alpha_\tau^\dagger \\ \tilde{\alpha}_\tau \end{pmatrix} = \begin{pmatrix} u_\tau & v_\tau \\ -v_\tau & u_\tau \end{pmatrix} \begin{pmatrix} a_\tau^\dagger \\ \tilde{a}_\tau \end{pmatrix}$$

Excitation energy and forward-and backward-going amplitudes -  
by solving **QRPA** matrix equation:

---

$$\begin{pmatrix} \mathcal{A}(K) & \mathcal{B}(K) \\ -\mathcal{B}(K) & -\mathcal{A}(K) \end{pmatrix} \begin{pmatrix} X_K^m \\ Y_K^m \end{pmatrix} = \omega_K^m \begin{pmatrix} X_K^m \\ Y_K^m \end{pmatrix}$$

where

$$\mathcal{A}_{ij}(K) = \langle \text{RPA} | [A_i, [H, A_j^\dagger]] | \text{RPA} \rangle$$

$$\mathcal{B}_{ij}(K) = -\langle \text{RPA} | [A_i, [H, \tilde{A}_j]] | \text{RPA} \rangle$$

# Nuclear Hamiltonian

---

The total Hamiltonian is defined as:  $H = H_o + H_{int}$

$$H_o = \sum_{\tau} \epsilon_{\tau} a_{\tau}^{\dagger} a_{\tau} \quad (\tau = p, n) \quad \text{Single particle part}$$

$$H_{int} = \sum_{pnp'n'} v_{pn p'n'} a_p^{\dagger} a_n^{\dagger} a_{n'} a_{p'} \quad \text{Residual two body interaction}$$

# Single Particle States

---

Single-particle wavefunctions in **deformed** Woods-Saxon potential :

$$\begin{aligned}
 |\tau\rho_\tau\rangle = \sum_{N_d n_z} [ & b_{N_d n_z \Omega_\tau}^{(+)} |(N_d n_z \Lambda_\tau), \Omega_\tau = \Lambda_\tau + 1/2\rangle |\Sigma = 1/2\rangle \\
 & + b_{N_d n_z \Omega_\tau}^{(-)} |(N_d n_z \Lambda_\tau + 1), \Omega_\tau = \Lambda_\tau + 1 - 1/2\rangle |\Sigma = -1/2\rangle ]
 \end{aligned}$$



Deformed harmonic oscillator wave function **Spin wave function**

The selection rules  $\Omega_p - \Omega_n = K$  for  $K = 0, \pm 1$  and  $\pi_p \pi_n = 1$

---


$$\mathcal{A}_{p_1 n_1 p_2 n_2}(\mathbf{K}) = (E_{p_1} + E_{n_1}) \delta_{p_1 p_2} \delta_{n_1 n_2} + g_{pp} [V_{p_2 \tilde{n}_2 p_1 \tilde{n}_1} (u_{p_1} u_{n_1} u_{p_2} u_{n_2}) + V_{\tilde{p}_1 \tilde{n}_1 \tilde{p}_2 n_2} (v_{p_1} v_{n_1} v_{p_2} v_{n_2})] \\ - g_{ph} [V_{p_2 n_1 p_1 n_2} (u_{p_2} v_{n_2} v_{n_1} u_{p_1}) + V_{p_1 n_2 p_2 n_1} (v_{p_1} u_{n_1} v_{p_2} u_{n_2})]$$

$$\mathcal{B}_{p_1 n_1 p_2 n_2}(\mathbf{K}) = -g_{pp} [V_{p_2 \tilde{n}_2 p_1 \tilde{n}_1} (v_{p_1} v_{n_1} u_{p_2} u_{n_2}) + V_{\tilde{p}_1 n_1 \tilde{p}_2 n_2} (u_{p_1} u_{n_1} v_{p_2} v_{n_2})] \\ - g_{ph} [V_{p_2 n_1 p_1 n_2} (v_{p_1} u_{n_1} u_{p_2} v_{n_2}) + V_{p_1 n_2 p_2 n_1} (v_{p_2} u_{n_2} u_{p_1} v_{n_1})]$$

where  $E_{\tau} = \sqrt{\epsilon_{\tau}^2 + \Delta_{\tau}^2}$

# Separable Interaction

---

$$\begin{aligned}
 H_{\text{int}} = & \chi \sum_{K=0, \pm 1} (-1)^K (\beta_{1K}^- \beta_{1-K}^+ + \beta_{1-K}^+ \beta_{1K}^-) \\
 & - \kappa \sum_{K=0, \pm 1} (-1)^K (P_{1K}^- P_{1-K}^+ + P_{1-K}^+ P_{1K}^-)
 \end{aligned}$$

The operators  $\beta^-$  and  $P^-$  are ph and pp components of the spin-isospin  $\tau^+ \sigma$ , namely,

$$\beta_K^- = \sum_{pn} \langle p | \tau^+ \sigma_K | n \rangle a_p^\dagger a_n, \quad \beta_K^+ = (\beta_K^-)^\dagger$$

$$P_K^- = \sum_{pn} \langle p | \tau^+ \sigma_K | n \rangle a_p^\dagger \tilde{a}_n^\dagger, \quad P_K^+ = (P_K^-)^\dagger$$

## Realistic Interaction:

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As a realistic two-body interaction we use the nuclear matter  $G$ -matrix.

$$G = V + V \frac{Q}{W - H_0 + i\varepsilon} G$$

Bethe-Goldstone equation

## The G-matrix in the deformed single particle basis:

---

$$|N_d n_z \Lambda_T\rangle = \sum_{N_o} \sum_{\substack{I \\ \text{overlap integral}}} A_{N_d n_z \Lambda_T}^{N_o I n_r} |N_o I \Lambda_T\rangle$$

Wave function in deformed basis      Wave function in spherical basis

### Single particle state

$$|\tau \rho_T\rangle = \sum_{N_o I j} B_{N_o I j}^{(\tau)} |(N_o I j), \Omega_T\rangle$$

deformed Woods-saxon state

spherical harmonic oscillator state

### Two body wavefunction

$$|p \rho_p n \rho_n\rangle = \sum_{(N_o I j)_p} \sum_{(N_o I j)_n} B_{(N_o I j)_p}^{(p)} B_{(N_o I j)_n}^{(n)} \sum_J C_{j_p \Omega_p j_n \Omega_n}^{JK} |(N_o I j)_p (N_o I j)_n, JK\rangle$$

## The G-matrix in the deformed single particle basis:

---

$$\begin{aligned}
 \langle \widetilde{p\rho_p n \rho_n} | G | \widetilde{p'\rho_{p'} n'\rho_{n'}} \rangle &= \sum_J \sum_{(N_o|j)_p} \sum_{(N_o|j)_n} \sum_{(N_o|j)_{p'}} \sum_{(N_o|j)_{n'}} B_{(N_o|j)_p}^{(p)} B_{(N_o|j)_n}^{(n)} B_{(N_o|j)_{p'}}^{(p')} B_{(N_o|j)_{n'}}^{(n')} \\
 &\times (-1)^{j_n - \Omega_n} (-1)^{j_{n'} - \Omega_{n'}} C_{j_p \Omega_p j_n \Omega_n}^{JK} C_{j_{p'} \Omega_{p'} j_{n'} \Omega_{n'}}^{JK} \\
 &\times \langle (N_o|j)_p (N_o|j)_n, J | G | (N_o|j)_{p'} (N_o|j)_{n'}, J \rangle
 \end{aligned}$$

G-matrix elements in  
 spherical single particle basis  
 Bonn CD potential

# Nuclear deformation

$$\beta = \sqrt{\frac{\pi}{5}} \frac{Q_p}{Zr_c^2}$$

Exp. (nuclear reorientation method)

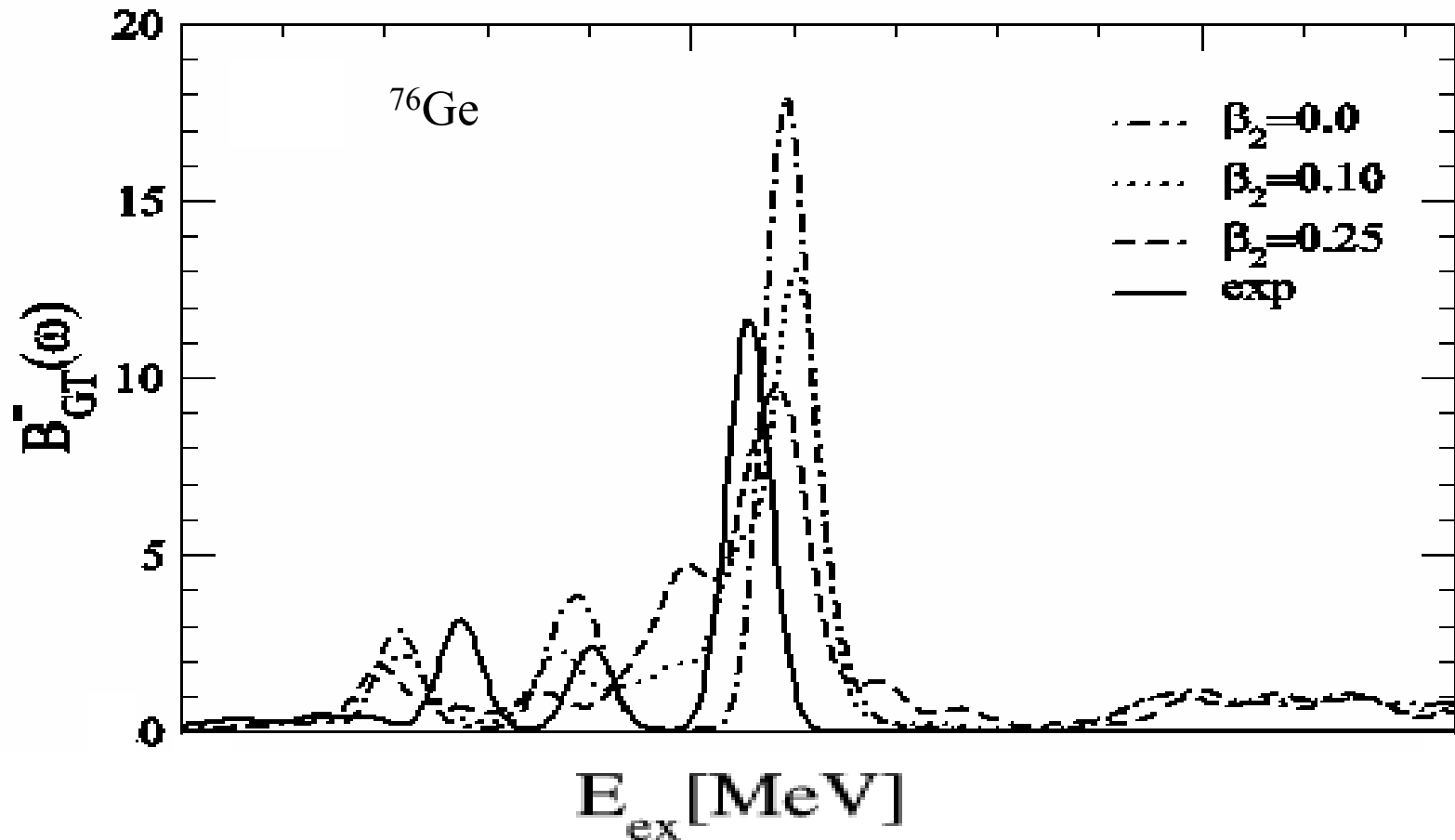
Theor. I (Rel. mean field theory)

Theor. II (Microsc.-Macrosc. Model of Moeller and Nix)

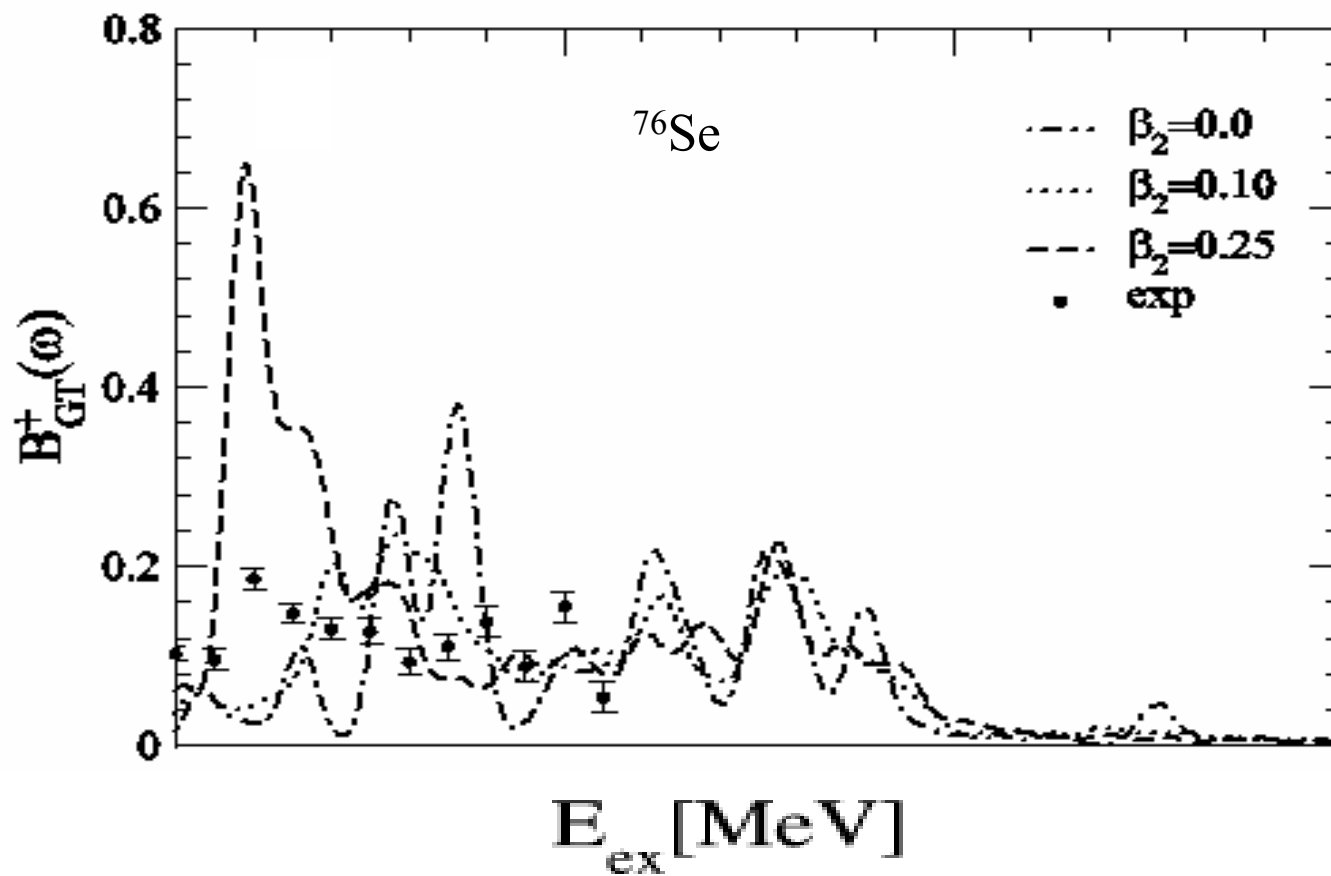
Nucl.	Exp	Theor. I	Theor. II
<sup>48</sup> Ca	0.00	0.00	0.00
<sup>48</sup> Ti	+0.17	-0.01	0.00
<sup>76</sup> Ge	+0.09	0.16	0.14
<sup>76</sup> Se	+0.16	-0.24	-0.24
<sup>82</sup> Se	+0.10	0.13	0.15
<sup>82</sup> Kr		0.12	0.07
<sup>96</sup> Zr		0.22	0.22
<sup>96</sup> Mo	+0.07	0.17	0.08
<sup>100</sup> Mo	+0.14	0.25	0.24
<sup>100</sup> Ru	+0.14	0.19	0.16
<sup>116</sup> Cd	+0.11	-0.26	-0.24
<sup>116</sup> Sn	+0.04	0.00	0.00
<sup>128</sup> Te	+0.01	-0.00	0.00
<sup>128</sup> Xe		0.16	0.14
<sup>130</sup> Te	+0.03	0.03	0.00
<sup>130</sup> Xe		0.13	-0.11
<sup>136</sup> Xe		0.00	0.00
<sup>136</sup> Ba		0.00	0.00
<sup>150</sup> Nd	+0.37	0.22	0.24
<sup>150</sup> Sm	+0.23	0.18	0.21

# Separable Interaction

$\beta^-$  strength

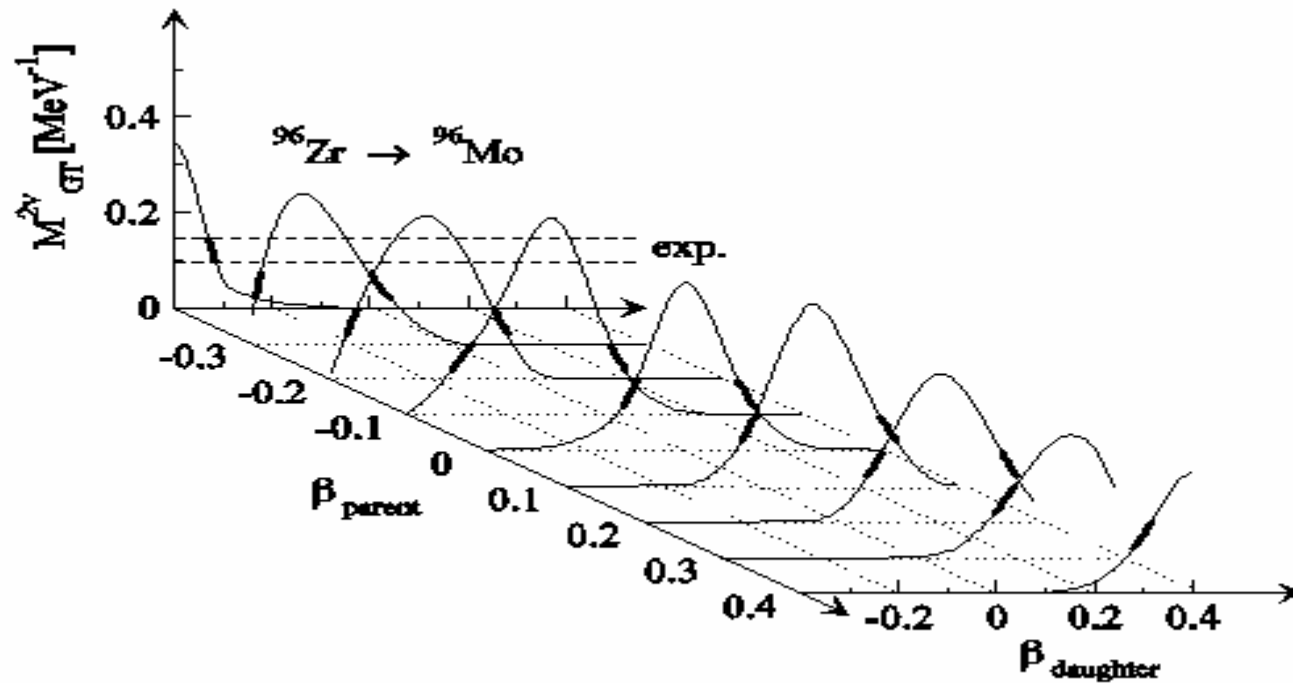


# $\beta^+$ strength



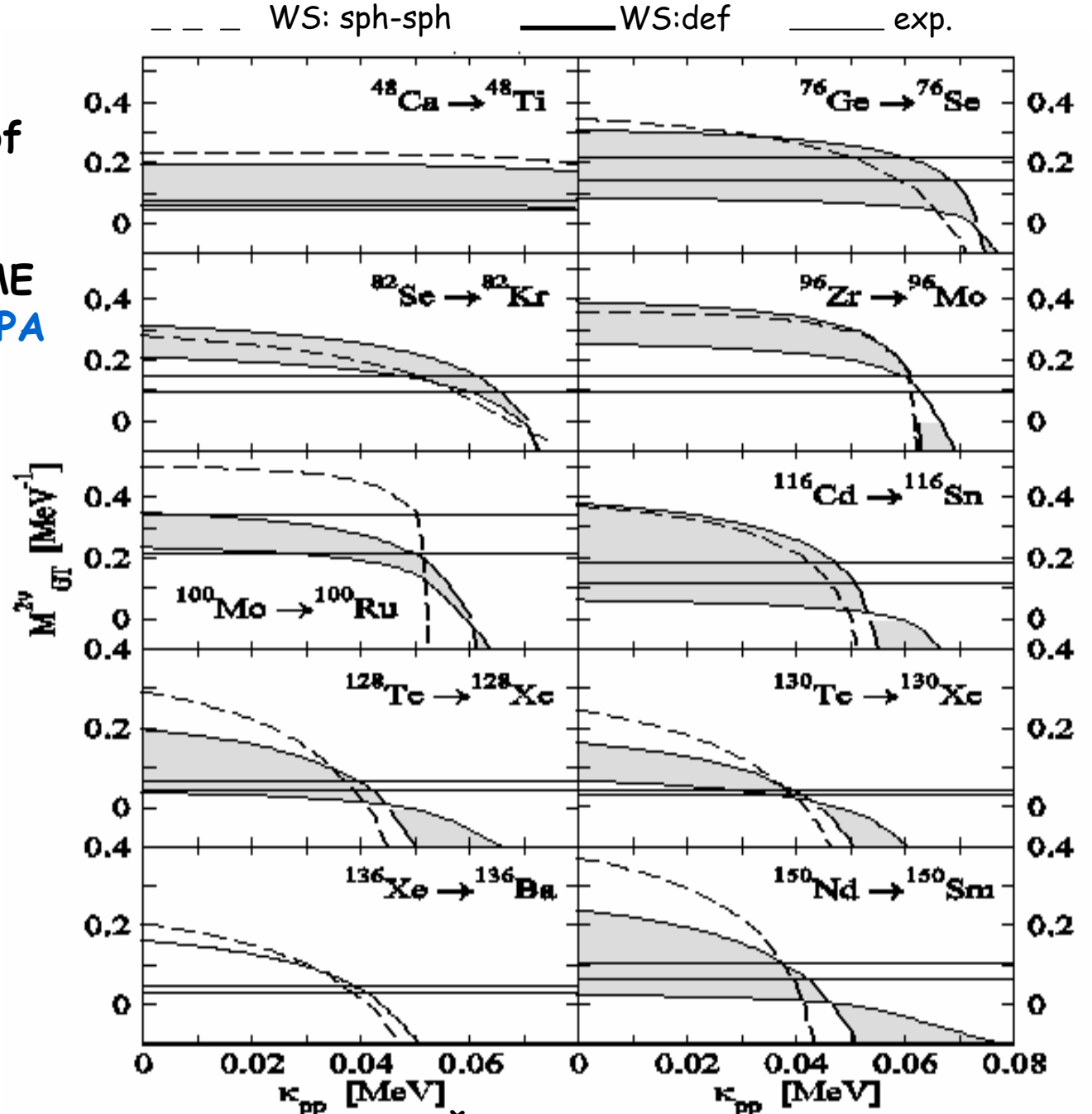
## The suppression of the NME depends on relative deformation of initial and final nuclei

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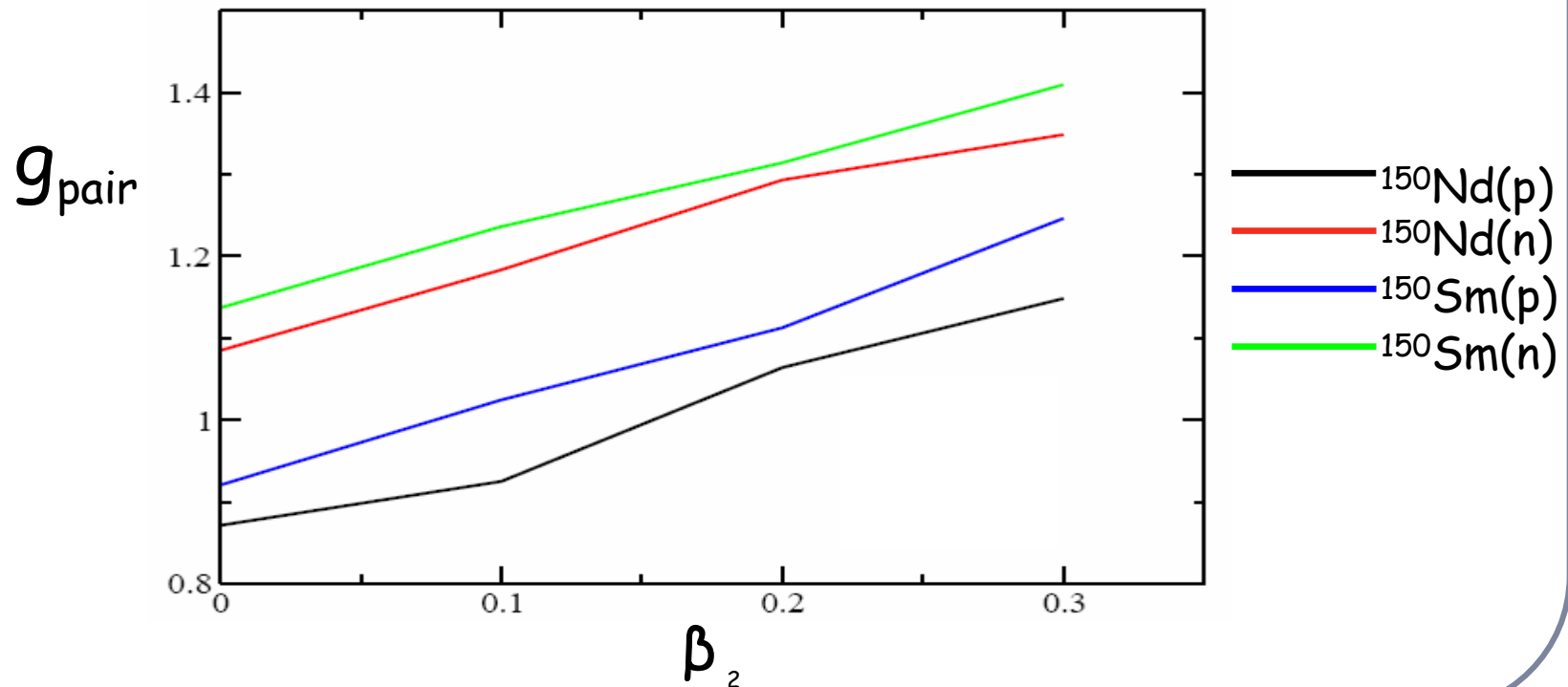
Alvarez, Sarriguren, Moya, Pacearescu, Faessler, Šimkovic,  
Phys. Rev. C 70 (2004) 321

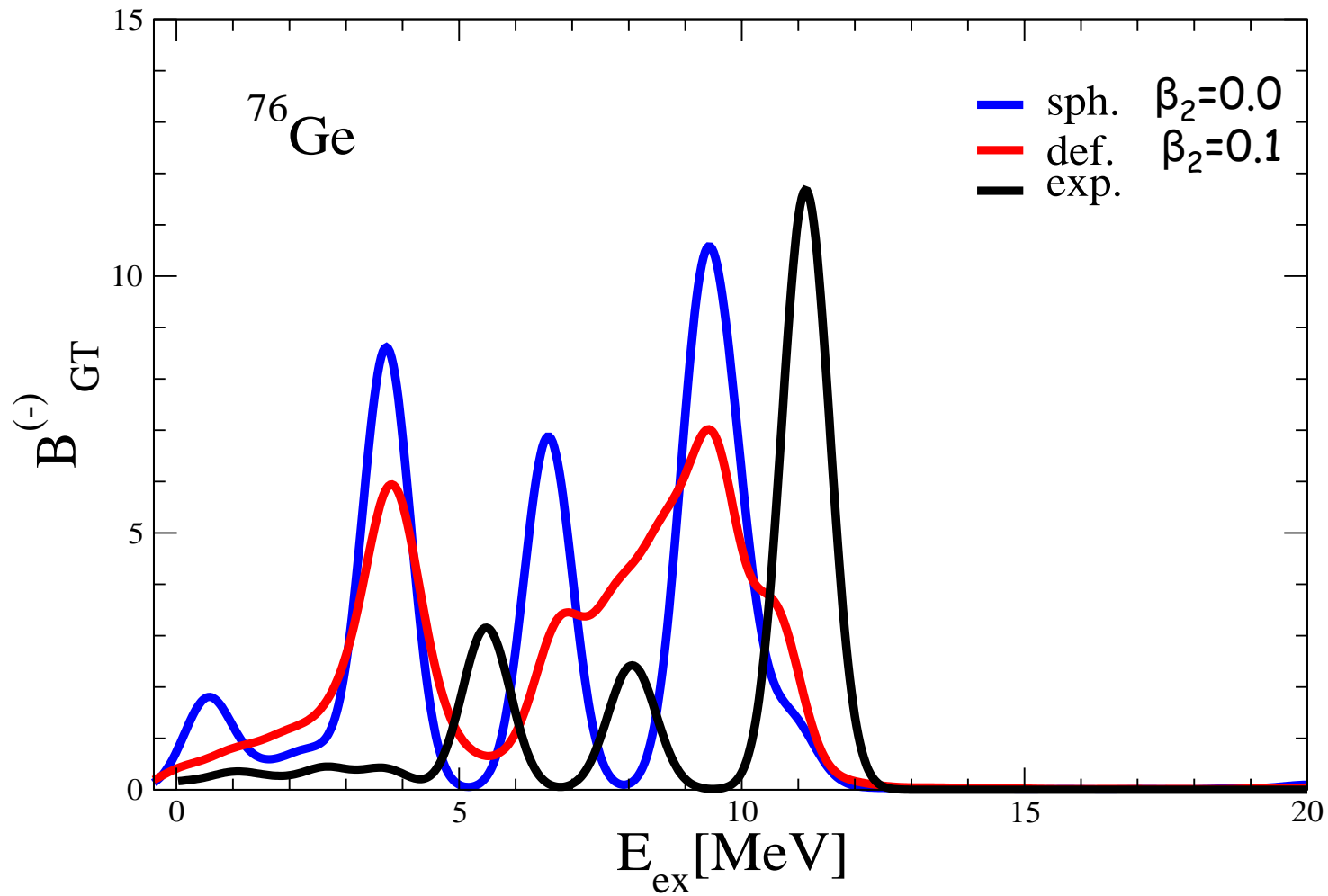
Systematic study of  
the deformation  
effect on  
the  $2\nu\beta\beta$ -decay NME  
within deformed QRPA



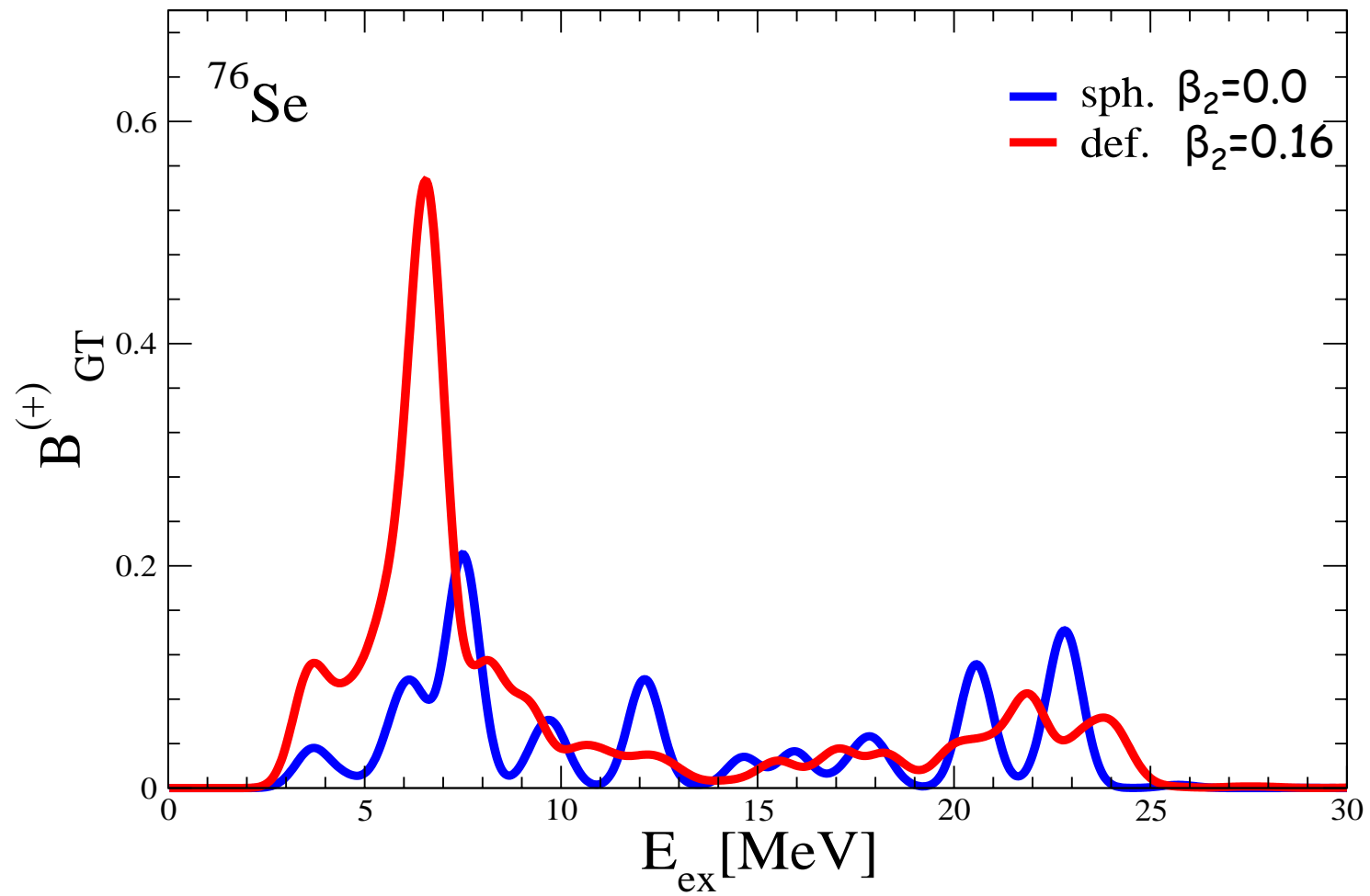
# Realistic Forces

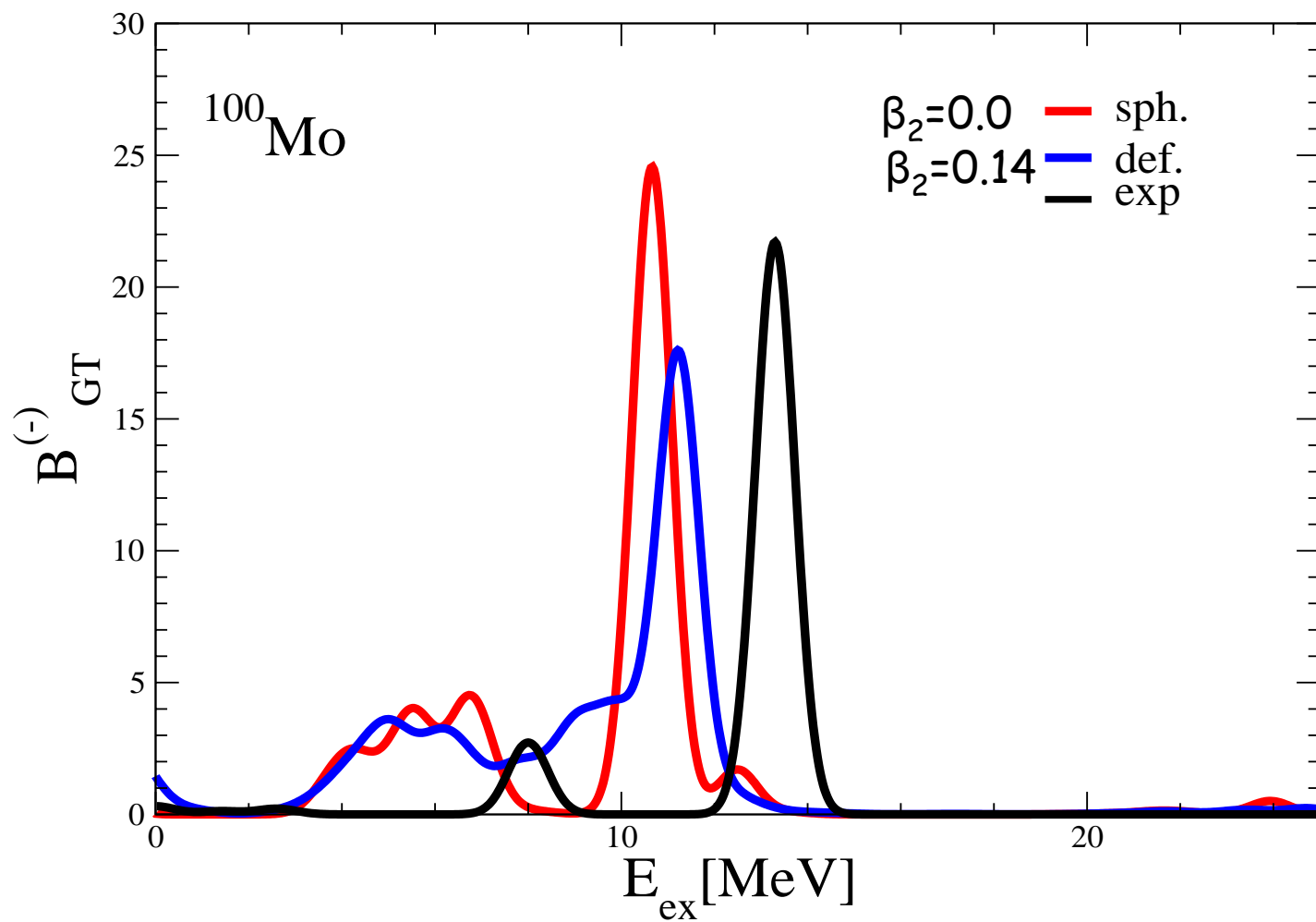
- The BCS equation is solved and the strengths  $g_{\text{pair}}^p$  and  $g_{\text{pair}}^n$  are determined to reproduce the experimental odd-even mass difference for each nucleus



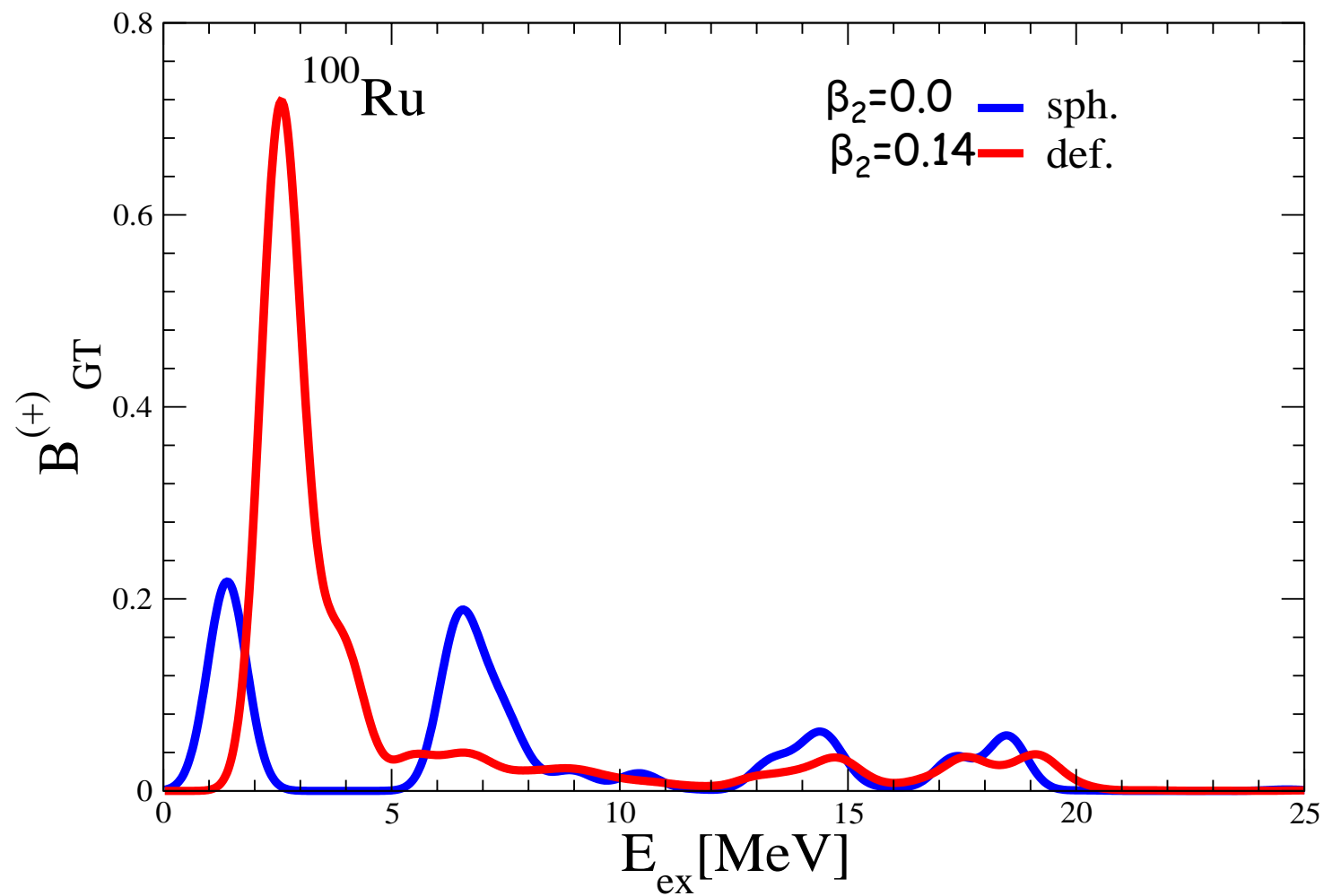


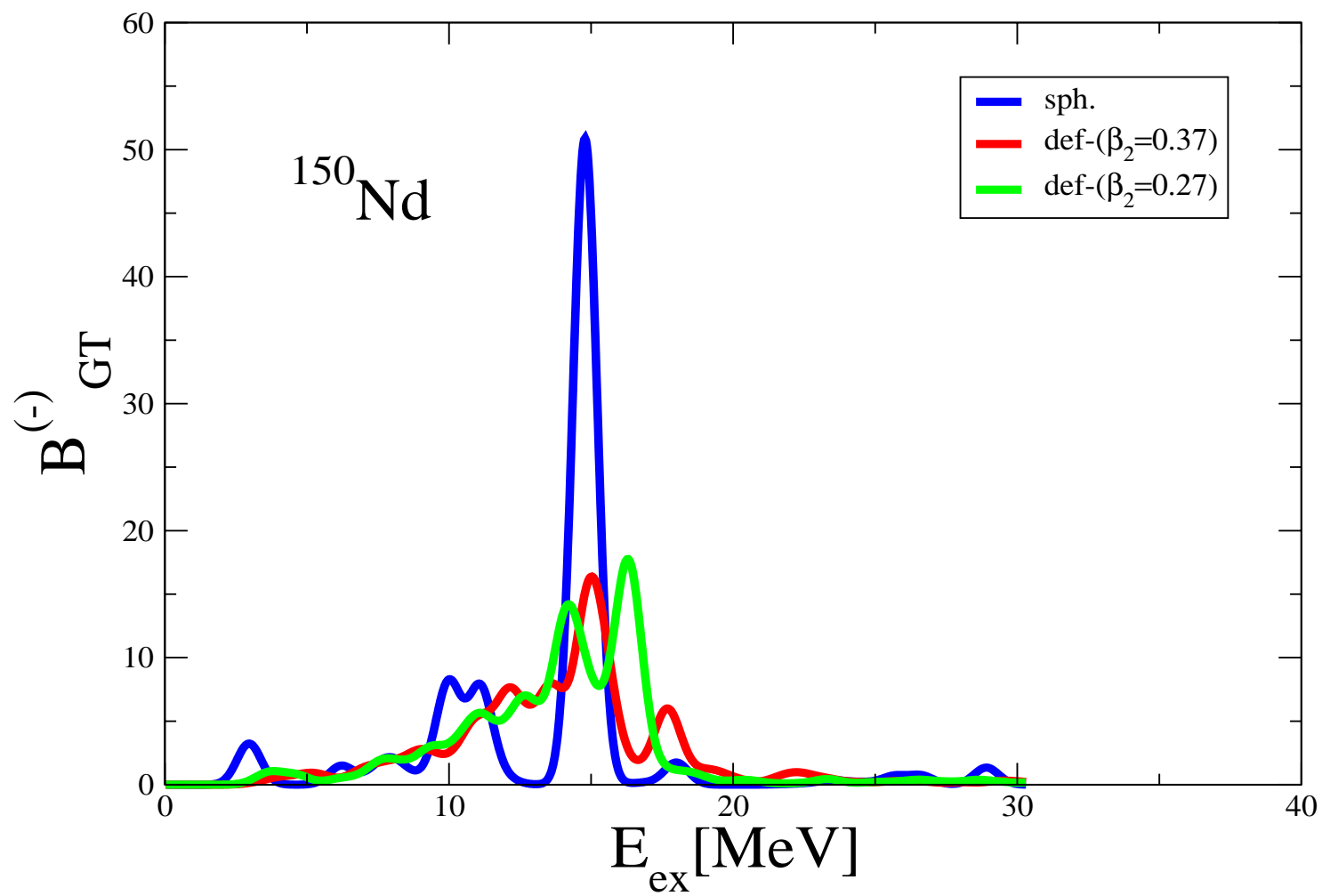
$g_{\text{ph}}=1$



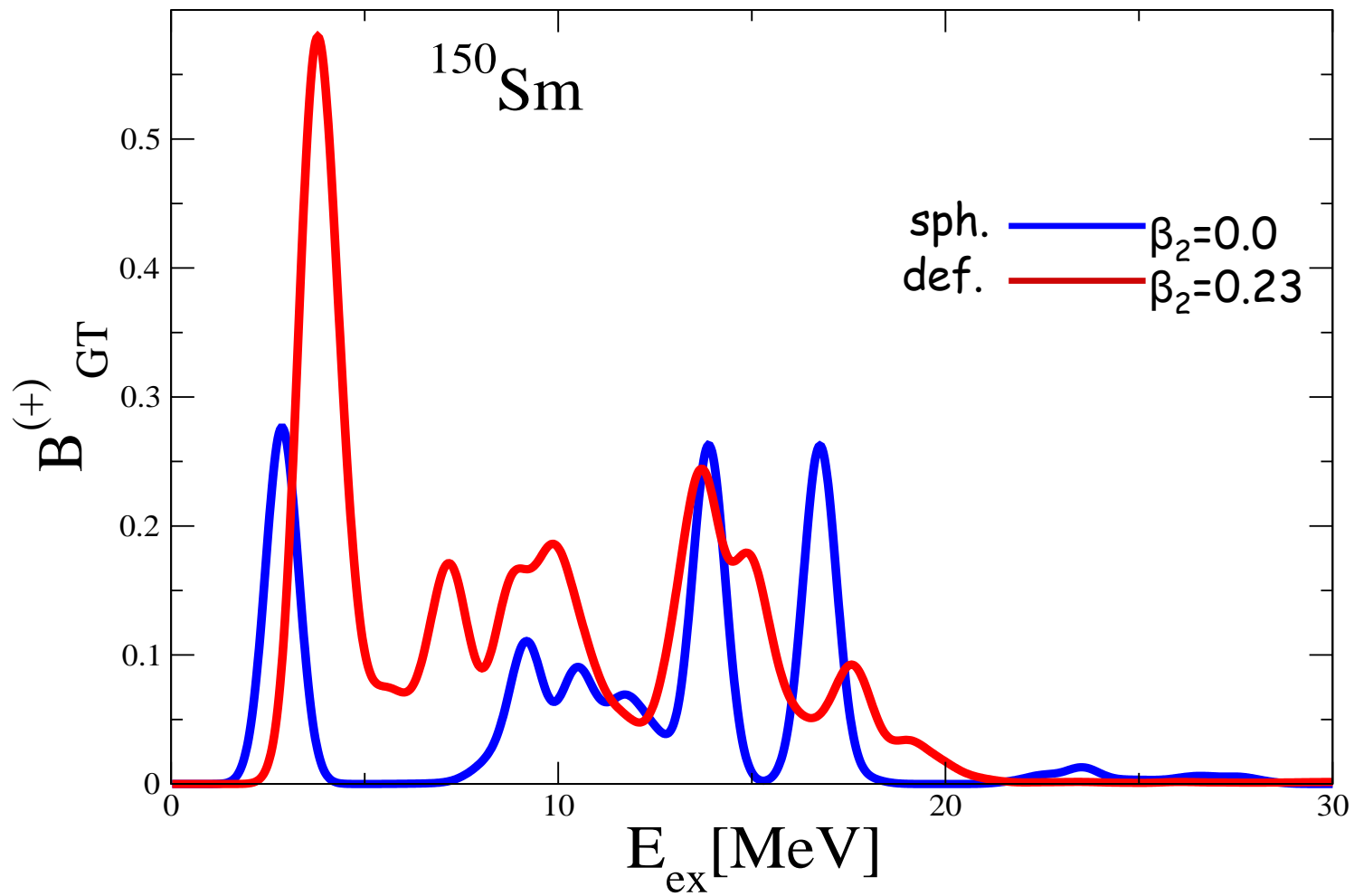


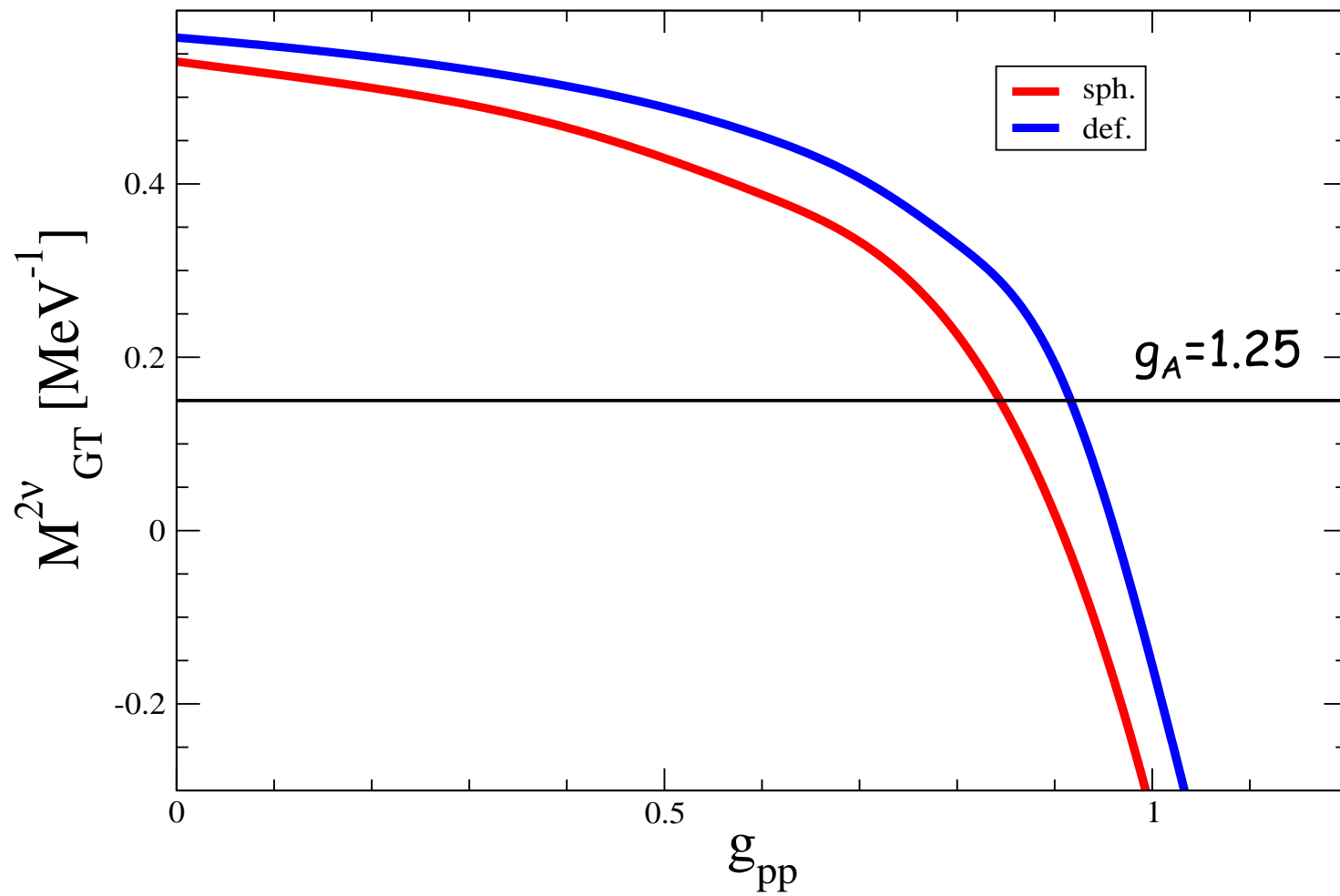
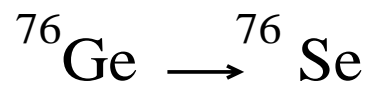
$g_{ph}=1$

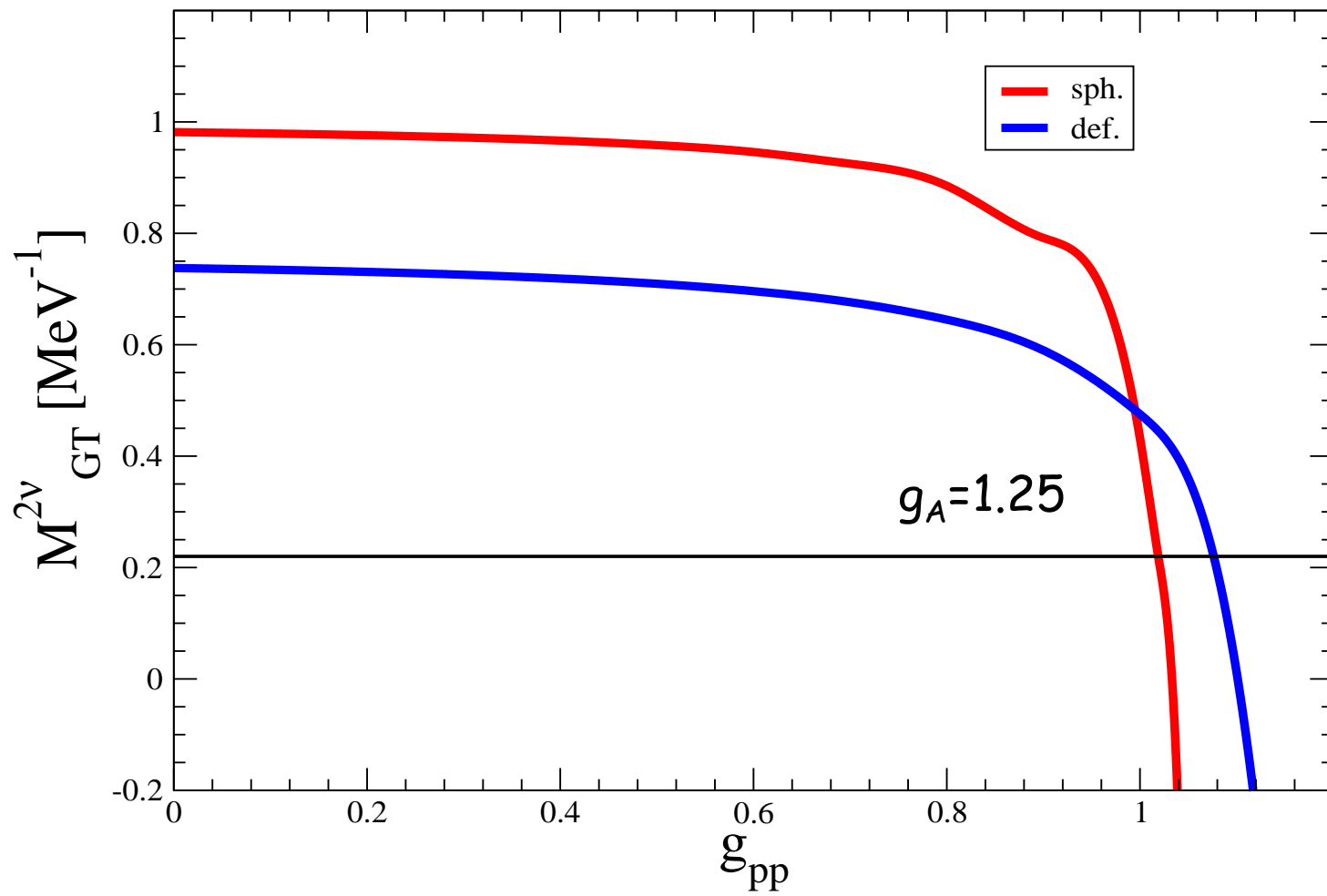
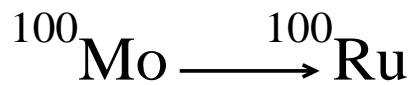




$g_{ph}=1$







# Summary

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- The calculated  $0\nu\beta\beta$  decay half life for  $^{150}\text{Nd}$  assuming it is a spherical nucleus stimulates a lot of interest to study the effect of deformation on the double beta decay .

## For Separable forces:

- $2\nu\beta\beta$  decay matrix elements is obtained for different nuclei within the deformed QRPA.
- The effect of deformation on the  $2\nu\beta\beta$  decay matrix elements is large for a significant difference in deformation of the parent and daughter nuclei and is not related with the increasing amount of the ground state correlations close to a collapse of the QRPA solution.

## Realistic force

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- BCS equations are solved for protons and neutrons. The protons and neutron gaps are determined to reproduce the odd-even mass differences.
- $G$ -matrix in deformed s.p. basis is calculated using that for spherical s.p. basis.
- The QRPA equation is solved using the realistic forces and  $B(GT)$  strengths are calculated for  $^{76}\text{Ge}$ ,  $^{76}\text{Se}$ ,  $^{100}\text{Mo}$ ,  $^{100}\text{Ru}$ ,  $^{150}\text{Nd}$  and  $^{150}\text{Sm}$  nuclei considering deformation.
- $2\nu\beta\beta$  matrix element for  $^{76}\text{Ge}$  and  $^{100}\text{Mo}$  with deformation have been obtained and in progress to calculate Nd.

# Outlook

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- Go Further to study the matrix elements of  $0\nu\beta\beta$  decay within **deformed** QRPA and **realistic** forces.

# Parameters

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- Particle particle strength  $g_{pp}$  .
- Particle hole strength  $g_{ph}$  .
- Spin orbit coupling constant.

