# Non-perturbative pressure of hot QCD

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Schladming 2.2007

[hep-lat/0609015] with Ari Hietanen

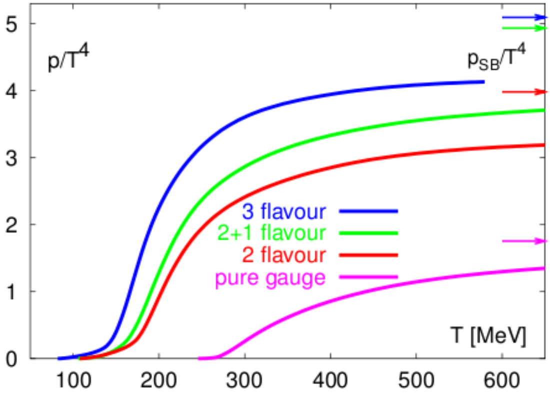
Pressure of the quark-gluon plasma:

- Fundamental thermodynamical property of the system.
- Variety of applications in heavy ion collisions and precision cosmology.

There are three degenerate quarks, $N_{\mathrm{c}} = 3$, in the physical case, but we study $N_{\mathrm{c}}$-dependence because:

- Large-$N_{\mathrm{c}}$ limit simplifies but is similar in phenomenology. Planar diagram theory, Eguchi-Kawai model ...
- Beyond SM: Strings, GUTs, technicolor,...

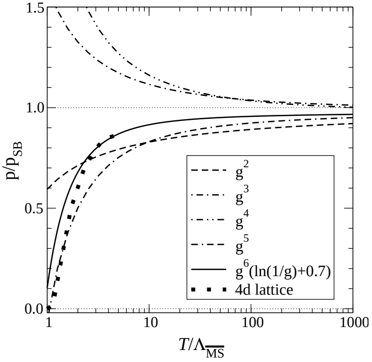
Naively one thinks that the plasma is non-interacting $p \to p_{sb}$, when $T \ggg \Lambda_{QCD}$




Karsch 2001

- Weak coupling expansion of pressure in hot QCD:

$$\frac{p}{p_{SB}} = 1 + g^2 + g^3 + g^4 \ln g + g^4 + g^5 + g^6 \ln g + g^6 + \dots$$



# Dimensional reduction ($T \gtrsim 2T_c$)

- At high $T$ the temporal direction shrinks away:

$$S = \int^{\frac{1}{T}} \mathrm{d}\tau \int \mathrm{d}^3 x \mathcal{L}_{4d}(x,\tau) \approx \frac{1}{T} \int \mathrm{d}^3 x \mathcal{L}_{3d}(x),$$

By *perturbatively* integrating out quarks and temporal modes, a super-renormalizable three dimensional effective gauge theory can be constructed (MQCD).

# Magnetostatic QCD

- MQCD: $S^{\rm 3d} = \int \mathrm{d}^3x \frac{1}{2g_3^2} \sum \mathrm{Tr}[F_{kl}^2]$ $\qquad g_3^2 \sim g^2 T$
- Has only one dynamical scale $\sim 1/N_{\rm c} g_3^2 = \lambda$.
- For the full theory, we need free energy $f_{\overline{\rm MS}}$ in the 3d-theory

$$f_{\overline{\rm MS}} \equiv -\lim_{V\to\infty} \frac{1}{V} \ln \left[ \int \mathcal{D}A_i \exp\left(-S^{\rm 3d}\right) \right]_{\overline{\rm MS}}$$

$$= -g_3^6 \frac{d_A N_{\rm c}^3}{(4\pi)^4} \left[ \left( \frac{43}{12} - \frac{157}{768}\pi^2 \right) \ln \frac{\bar{\mu}}{2N_{\rm c} g_3^2} + B_{\rm G}(N_{\rm c}) \right]$$

- $B_{\rm G}(N_{\rm c})$ is purely non-perturbative and contains all the non-perturbative contributions
  $\longrightarrow$ determine $B_{\rm G}(N_{\rm c})$ numerically using lattice simulations.

$$
\begin{aligned}
\frac{p_{\rm QCD}(T)}{T^4\mu^{-2\epsilon}} &= g^0\left\{\alpha_{\rm E1}\right\} + g^2\left\{\alpha_{\rm E2}\right\} + \frac{g^3}{(4\pi)}\left\{\frac{d_A}{3}\alpha_{\rm E4}^{3/2}\right\} \\
&+ \frac{g^4}{(4\pi)^2}\left\{\alpha_{\rm E3} - d_A C_A\left[\alpha_{\rm E4}\left(\frac{1}{4\epsilon}+\frac{3}{4}+\ln\frac{\bar\mu}{2gT\alpha_{\rm E4}^{1/2}}\right)+\frac{1}{4}\alpha_{\rm E5}\right]\right\} \\
&+ \frac{g^5}{(4\pi)^3}\left\{d_A\alpha_{\rm E4}^{1/2}\left[\frac{1}{2}\alpha_{\rm E6} - C_A^2\left(\frac{89}{24}+\frac{\pi^2}{6}-\frac{11}{6}\ln 2\right)\right]\right\} \\
&+ \frac{g^6}{(4\pi)^4}\Bigg\{\beta_{\rm E1} - \frac{1}{4}d_A\alpha_{\rm E4}\left[(d_A+2)\beta_{\rm E4}+\frac{2d_A-1}{N_c}\beta_{\rm E5}\right] \\
&\qquad -d_A C_A\left[\Gamma_E + \left(\frac{1}{4\epsilon}+\ln\frac{\bar\mu}{2gT\alpha_{\rm E4}^{1/2}}\right)\right] \\
&\qquad +d_A C_A^3\Bigg[\beta_{\rm M} + \alpha_{\rm M}\left(\frac{1}{\epsilon}+8\ln\frac{\bar\mu}{2gT\alpha_{\rm E4}^{1/2}}\right) \\
&\qquad +\alpha_{\rm G}\left(\frac{1}{\epsilon}+8\ln\frac{\bar\mu}{2g^2TC_A}\right)+B_{\rm G}\Bigg]\Bigg\} + \mathcal{O}(g^7)+\mathcal{O}(\epsilon)\,. \qquad (1)
\end{aligned}
$$

- On the lattice:

$$S_a = \beta \sum_{\mathbf{x}} \sum_{k<l}^{3} \left(1 - \frac{1}{N_\mathrm{c}} \mathrm{ReTr}[P_{kl}(\mathbf{x})]\right), \quad \beta = \frac{2Nc}{g_3^2 a}$$

The free energy on lattice

$$f_a \equiv - \lim_{V\to\infty} \frac{1}{V} \ln \left[ \int \mathcal{D}U_k \exp\left(-S_a\right) \right],$$

- In perturbation theory, both $f_{\overline{\mathrm{MS}}}$ and $f_a$ are IR-divergent, but their difference is IR-insensitive $\to$ can be calculated in perturbation theory:

$$f_a - f_{\overline{\mathrm{MS}}} = C_1 \frac{1}{a^3} \left( \ln \frac{1}{a g_3^2} + C_1' \right) + C_2 \frac{g_3^2}{a^2} + C_3 \frac{g_3^4}{a} + C_4 g_3^6 \left( \ln \frac{1}{a\bar{\mu}} + C_4' \right) + \mathcal{O}(a)$$

- $B_{\rm G}$ is solved:

$$8\frac{d_A N_c^6}{(4\pi^2)^4} B_{\rm G} = \lim_{\beta\to\infty} \beta^4 \Big\{ \overbrace{\langle 1 - \frac{1}{N_c}{\rm Tr}[P_{12}]\rangle_a}^{\text{Measure on lattice [hep-lat/0609015]}} - \underbrace{\left[\frac{c_1}{\beta} + \frac{c_2}{\beta^2} + \frac{c_3}{\beta^3} + \frac{c_4}{\beta^4}(\ln\beta + c_4')\right]}_{\text{Get from perturbation theory}} \Big\}$$

- $c_4'$ is known only for $N_{\rm c} = 3$ using stochastic perturbation theory: $c_4'(3) = 7.0(3)$

$$P_{\rm G}(\beta, N_{\rm c}) \equiv \frac{32\pi^4\beta^4}{d_A N_{\rm c}^6} \left\{ \langle 1 - \frac{1}{N_{\rm c}}{\rm Tr}[P]\rangle_a - \left[\frac{c_1}{\beta} + \frac{c_2}{\beta^2} + \frac{c_3}{\beta^3} + \frac{c_4}{\beta^4}\ln\beta\right]\right\},$$

$$B_{\rm G}(N_{\rm c}) - \left(\frac{43}{12} - \frac{157}{768}\pi^2\right) c_4' = P_{\rm G}(\infty, N_{\rm c}).$$

# Lattice computations

Outline of lattice calculations:

- Measure $\langle 1 - \frac{1}{N_c}\mathrm{Tr}[P_{12}]\rangle$ as a function of $\beta$ for $N_\mathrm{c} = 2, 3, 4, 5, 8$
- Subtract ultraviolet divergences.
- Take the continuum limit = extrapolate $\beta \to \infty$
- Find the functional form of non-perturbative input $P_\mathrm{G}(N_\mathrm{c}, \infty)$.

# Significance loss

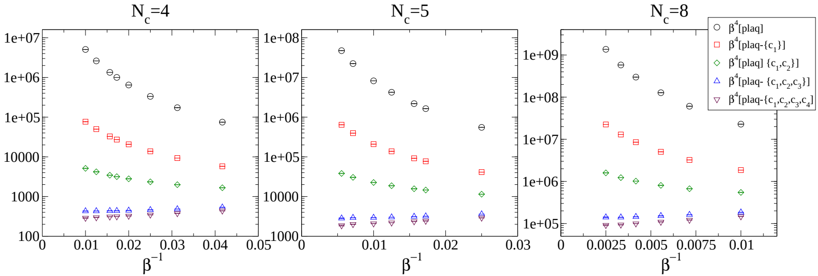



Figure: Massive significance loss due to the subtraction of ultraviolet divergences in the plaquette expectation value. The physics is in the sixth decimal of the plaquette. One point = 64h×32processors.

# Continuum limit

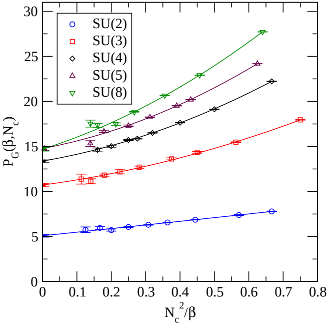
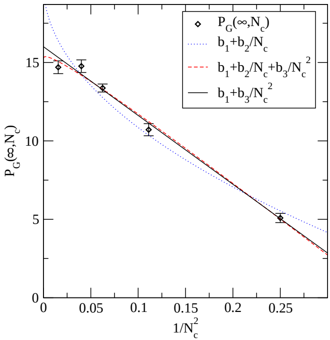



Figure: Continuum extrapolations of $P_{\rm G}(\beta, N_{\rm c})$ for each $N_{\rm c}$.

- We expect systematical errors of order 1-$\sigma$ from the choice of the fitting function.

# $N_c$-dependence




- Term $N_c^{-1}$ gives a very bad description of data or has coefficient consistent with zero within our resolution.

$$P_G(N_c,\infty) = 15.9(2)[1 - 2.8(1)N_c^{-2}] \qquad (= 11.0 \pm 0.3,\ \text{for } N_c = 3)$$

# Summary

- At high temperatures ($T \gtrsim 2T_c$) long range properties of hot QCD can be isolated into a simpler 3d pure gauge theory.
- We have studied the $N_{\rm c}$-dependence of plaquette expectation value and free energy of the effective theory.
  - High precision lattice measurements of plaquette expectation value with $N_{\rm c} = 2, 3, 4, 5, 8$ were performed[1].
  - Non-perturbative input $P_{\rm G} = 15.9(2) - 44(2)N_{\rm c}^{-2}$, seems to be a function $N_{\rm c}^2$.
  - Higher order terms are small and physical case $N_{\rm c} = 3$ is well described by this form.
  - For SU(3), $B_{\rm G}(3) = 0.1 \pm 0.5$

[1] Total of $1.2 \times 10^{17}$flop$\approx 1000s \times 140$Tflop/s were used.

# Finite lattice

There is only one scale in the theory the “glue-ball” correlation length $\sim 1/N_c g_3^2 = \lambda$, which is also the confinement scale.

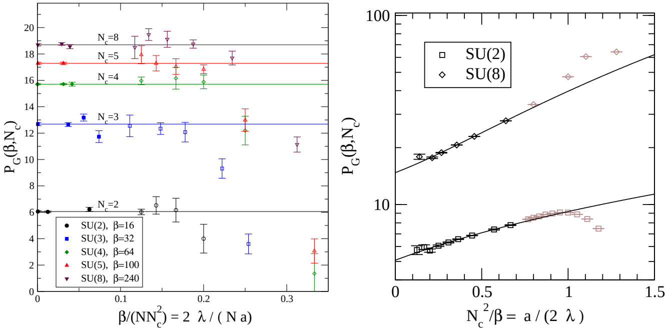



Figure: $P_{\mathrm{G}}(\beta, N_{\mathrm{c}})$ as a function of the physical lattice size and physical lattice spacing. $\longrightarrow N_{\mathrm{c}}^2 < \beta \lesssim N(N_{\mathrm{c}}/3)^2$