Effects Of Anisotropy in (2+1)-dimensional QED

In preparation: JAB, C. S. Fischer, R. Williams
Effects of Anisotropy in QED$_3$ from Dyson–Schwinger equations in a box

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Outline

1. Motivation
2. Technical Aspects
3. Results
4. Summary and Outlook
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Discovery of high-temperature superconductivity in 1986.


- critical temperature > 77 K
- ceramical compounds
- need ‘critical doping’
- non-superconducting phase is insulating anti-ferromagnetic

Look for effective theory to describe phenomenon
Experiments show:

Ding, Norman, Campuzano, PRB 54, R 9678 (1996)

- energy gap function with "d-wave-symmetry"
  → nodal quasiparticles (qp)
- qp: linear energy dispersion relation at the nodes
- vortex-antivortex interactions described by U(1) gauge theory
- qp + gauge fields confined to superconducting plane

⇒ We get a hint to $QED_3$. 

Possible Solution: $QED_3$

Translation of "experimental output" to QED-language?

**Reformulate task:**

→ study order parameter of the transition
→ find critical quantities

Feature: Inherent Anisotropy

Nodes of gap function and inherent anisotropy define the metric-like quantity...

\[ \epsilon_{\vec{k}} = v_f q_1 + O(q^2) \]
\[ \Delta_{\vec{k}} = v_\Delta q_2 + O(q^2) \]

\[
\begin{pmatrix}
  g^{\mu\nu}_1
\end{pmatrix} = \begin{pmatrix}
  1 & (v_F)^2 \\
  (v_\Delta)^2 & 1
\end{pmatrix}
\]

\[
\begin{pmatrix}
  g^{\mu\nu}_2
\end{pmatrix} = \begin{pmatrix}
  1 & (v_\Delta)^2 \\
  (v_F)^2 & 1
\end{pmatrix}
\]

The Anisotropic Lagrangian

\[ \mathcal{L}^{iso} = \sum_{j=1,2} \bar{\Psi}_j \left\{ \sum_{\mu=0}^{2} \gamma_{\mu} \left( \partial_{\mu} + i a_{\mu} \right) \right\} \psi_j. \]

\[ \Downarrow \]

\[ \mathcal{L}^{aniso} = \sum_{j=1,2} \bar{\Psi}_j \left\{ \sum_{\mu=0}^{2} \gamma_{\nu} \sqrt{g^{j}_{\nu\mu}} \left( \partial_{\mu} + i a_{\mu} \right) \right\} \psi_j \]
The Anisotropic Dyson–Schwinger Equations

- Landau gauge

\[ S_{-1}^{-1}(\vec{p}) = S_{0i}^{-1}(\vec{p}) + \epsilon^2 \int \frac{d^3q}{(2\pi)^3} \left\{ \sqrt{g}^{\mu\alpha} \gamma_{\alpha}(\vec{q}) S_{Fi}(\vec{q}) \times \sqrt{g}^{\nu\gamma} \Gamma_{\gamma}(\vec{q}) D_{\mu\nu}(\vec{p} - \vec{q}) \right\} \]
How To Solve The DSEs?

We have a rather complex structure of the equations.

- look for CPU friendly environment
- evaluation on 3 dimensional torus
  

- search for self-consistent solutions

How do we formulate the equations on a torus?
The Torus

- periodic boundary conditions for bosons
- antiperiodic boundary conditions for fermions

⇒ discretized momentum space

\[
\int \frac{d^3 q}{(2\pi)^3} \rightarrow \frac{1}{L^3} \sum_{\text{all momenta}} \ldots
\]

The relevant parameters:
- the box size \( L e^2 \) in coordinate space
- the number of lattice points in momentum space \( N \)
The Task

study dynamical generation of mass

→ Know: depends on number of fermion flavours
→ Effects of anisotropy?
→ Look at $B_{\text{max}}$ depending on $v_f, v_\Delta$

Probe the anisotropic plane for $N_f^{\text{crit}}$. 
Large-$N_f$ Approximation

- expansion in $e^2$ keeping coupling $\alpha = \frac{N_f e^2}{8}$ fixed
- vacuum polarization given by:

$$\Pi^{\mu\nu}(p) = \frac{N_f e^2}{16 v_F v_\Delta |\bar{p}|} \sum_i \left( \bar{p}_i^2 g^{\mu\nu}_i - g^{\mu\alpha}_i p_\alpha g^{\nu\delta}_i p_\delta \right)$$

isotropic limit:

$$\Pi^{\mu\nu}(p^2) = \frac{N_f e^2}{8p} \left( p^2 \delta^{\mu\nu} - p^\mu p^\nu \right)$$
Large-$N_f$ Approximation

The phase diagram in velocity phase for a torus of $40^3$ points and $L e^2 = 600$:

- $N_f^c$ is strongly volume dependent
  
  Goecke, Fischer, Williams, PRB 79, 064513 (2009).

- continuum limit can be obtained by extrapolation

$\Rightarrow$ Increasing $N_f^c$ away from plateau around $v_f = v_\Delta = 1$. 

JAB, Fischer, Williams: in preparation
Improved Photon And Vertex Ansatz

- Anomalous dimension $\kappa$ of fermion vector dressing and vacuum polarization in IR

  \[ \kappa_{\mathrm{IR}} = \frac{e^2 N_f}{16 \sqrt{2} v_F v_\Delta} \left( \sqrt{\frac{p_i^2}{p_i^2 + e^2}} + \frac{1}{p_i^{1+2\kappa}} \frac{e^2}{p_i^2 + e^2} \right) \]

  \[ \kappa_{\mathrm{IR}} = \frac{e^2 N_f}{16 \sqrt{2} v_F v_\Delta} \left( \sqrt{\frac{p_i^2}{p_i^2 + e^2}} + \frac{1}{p_i^{1+2\kappa}} \frac{e^2}{p_i^2 + e^2} \right) \]

  \[ JAB, \text{Fischer, Williams: in preparation.} \]

- Ansatz for vacuum polarization generalized to anisotropic spacetime

  \[ \Pi_i (\vec{p}) = e^2 N_f \left( \frac{1}{\sqrt{p_i^2 + e^2}} + \frac{1}{p_i^{1+2\kappa}} \frac{e^2}{p_i^2 + e^2} \right) \]

  \[ JAB, \text{Fischer, Williams: in preparation.} \]

- Insert minimal Ball-Chiu vertex

  \[ \Gamma_i^\beta (\vec{p}, \vec{q}) = \gamma^\beta \frac{A_i^\beta (\vec{p}) + A_i^\beta (\vec{q})}{2} \]

  \[ Ball, \text{Chiu, PRD 22 2542 (1980).} \]
Improved Photon And Vertex Ansatz

The phase diagram in velocity phase for a torus of $40^3$ points and $Le^2 = 600$:

- $\kappa = 0.0358$ fixed in isotropic limit
- agreement with lattice calculations: Hands, Thomas, PRB 72, 054526 (2005); Thomas, Hands, PRB 75, 134516 (2007)

Decreasing $N_f^c$ as a function of $v_f$ and $v_\Delta$ away from maximum around $v_f = v_\Delta = 0.4$.

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Conclusion

Summary

- \( \text{QED}_3 \) is potential effective low-energy theory for high temperature superconductors
- Changes between isotropic and anisotropic \( \text{QED}_3 \)
- Dyson-Schwinger equations in anisotropic space-time
- Results in large-\( N_f \) approximation
- Improved results within more sophisticated truncation scheme

What is left to do ...

- extrapolation to infinite volume
- solve photon equation explicitly
Thank you for your attention!

Questions??