QCD deconfinement transition via gauge theory/XY spin model duality

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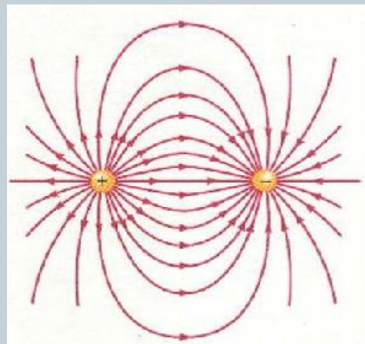
arXiv:1112.6389, 1211.2824, 1308.0027 , 1310.3522

Outline

- Preliminaries
- Motivation for QCD(adj) on $R^3 \times S^1$
- The physics of QCD(adj) on $R^3 \times S^1$
- Confinement phenomena in QCD(adj)
- Deconfinement in QCD(adj) on $R^2 \times S^1{}_L \times S^1{}_\beta$: Coulomb gas, spin XY model, duality and all that
- Future directions

An overview: confinement

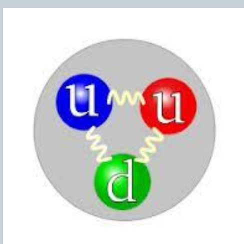
- Confinement is the mechanism for holding quarks inside nucleons

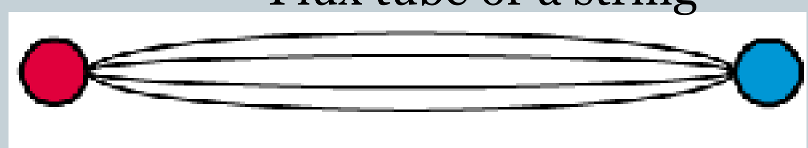


Not confined

$$V = \frac{1}{R}$$

Flux tube or a string



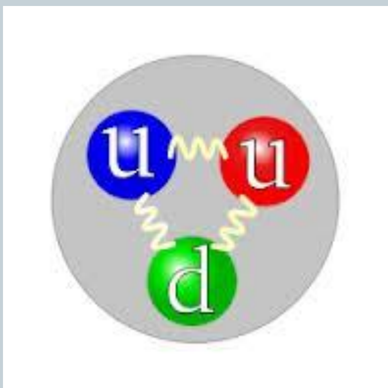


Confined $V = \sigma R$

- This picture has been confirmed through extended computer simulations, comprehensive and expensive

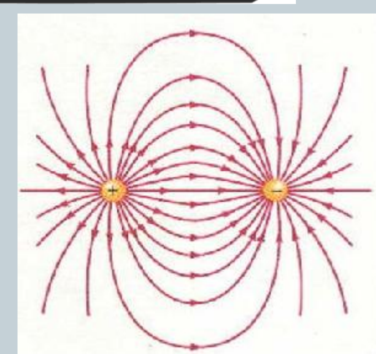
An overview: deconfinement

- Heating up the nuclei



$$V = \sigma R$$

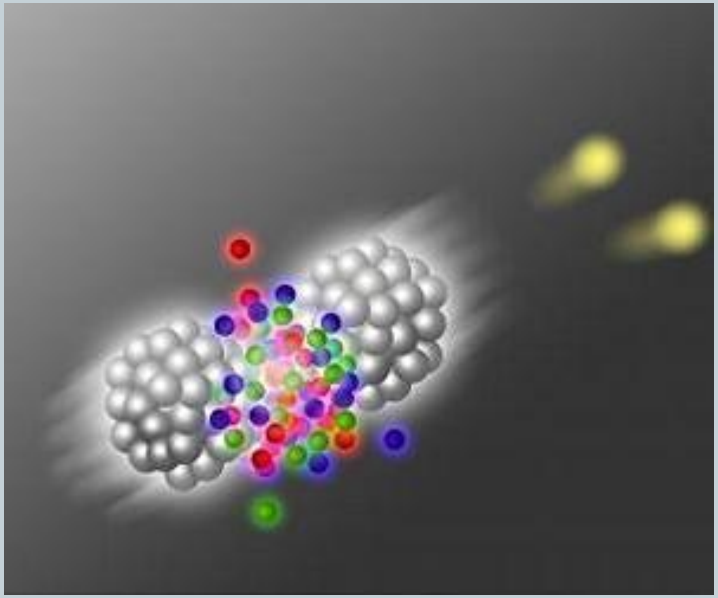
deconfinement

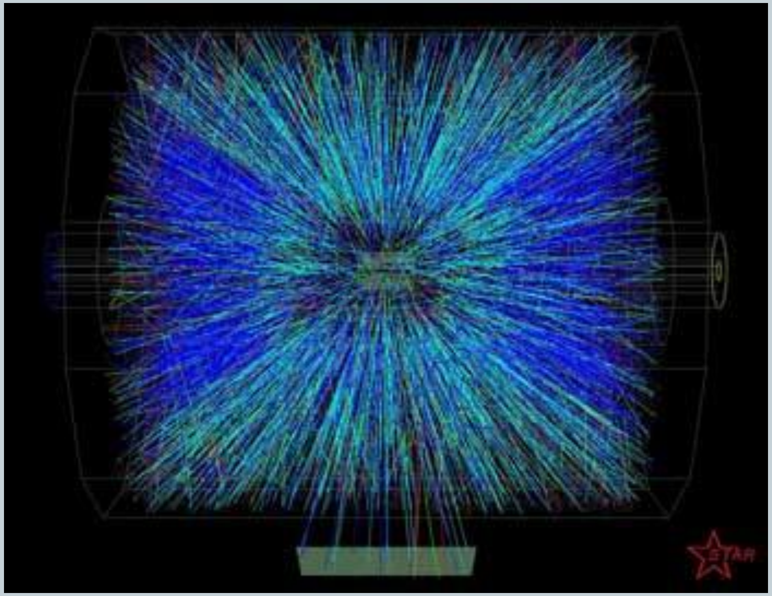


$$V = \frac{1}{R}$$

An overview: deconfinement

- As we increase the temperature, deconfinement happens (phase transition; like water-ice, magnetization etc)
- Quark-gluon plasma: a new state of matter $T = 10^{12} K$





Gauge theories on: $R^{2,1}\times S^1$ Motivation

- QCD is a strongly coupled system, not analytically tractable even at the deconfinement point
- One needs a simpler theory that is under complete control, yet resembles the original theory
- A promising setup is Yang-Mills on $R^{2,1}\times S^1$

Spatial circle

N. Davies, T. Hollowood, V. Khoze, M. Mattis, M. Shifman, M. Unsal, E. Poppitz, M. Anber,T. Sulejmanpasic, G. Dunne, P.Argyres, T. Misumi, M. Nitta, N. Sakai, A.Behtash, T. Schaefer, A. Zhitnitsky, M. Ogilvie, B. Teeple, A. Cherman, D. Dorigoni, L. Yaffe,...

Gauge theories on: $R^{2,1} \times S^1$ Motivation

- Pros:
- Perform reliable semi-classical calculations
- Test the rules of different symmetries (center, topological, chiral, etc.)
- Disentangle different physical phenomena (e.g. confinement & chiral symmetry breaking)
- Mapping to lower-dimensional condensed matter systems (simulations, or using analogue systems to test our guage theories)
- FUN!!!!

Gauge theories on: $R^{2,1} \times S^1$ Motivation

- Cons:
- Not the real world
- Large L limit is not under control

Gauge theories on: $R^{2,1}\times S^1$ Motivation

- Ultimate goal
- Cook up models that contain the same ingredients of realistic theories (adjoint & fundamental fermions, magnetic field, the vacuum angle, etc)
- Compare the results with existing experiments (either real or full 4-D lattice experiments)
- Make predictions and propose further experiments

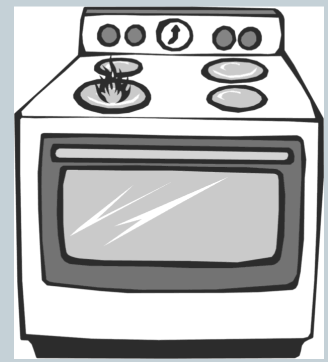
Gauge theories on $R^{2,1} \times S^1$

- Today's meal: deconfining phase transition in SU(N) QCD with adjoint fermions

Adj fermions

QCD

$\langle \mathrm{tr}[\Omega] \rangle$

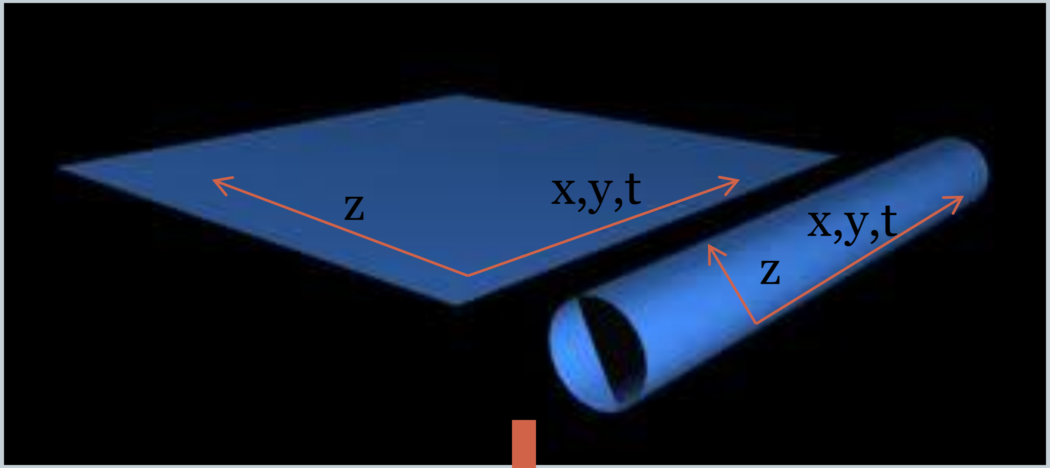


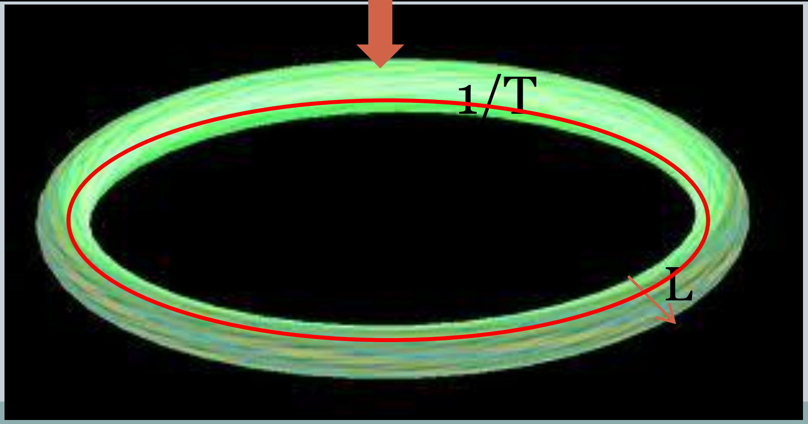
- Lattice experiments for SU(N) model were conducted in 4-D : SU(3) first order transition   Karsch and Lutgemeir 1998
- Supersymmetric SU(2) second order

  Bergner, Giudice, Mnster, Piemonte, and Sandbrink 2104

Gauge theories on $R^{2,1} \times S^1$

- Strategy: compactify 2 dimensions:





QCD(adj) on $R^3 \times S^1$, Formulation

SU(2): $$S = \int_{R^3 \times S^1} \frac{1}{g^2} \mathrm{tr}\left[-\frac{1}{2} F_{\mu\nu} F^{\mu\nu} + 2i\,\mathrm{tr}\, \overline{\lambda_I}\, \overline{\sigma}^{\mu} D_\mu \lambda_I \right]$$

$SU(n_f) \times U(1)$

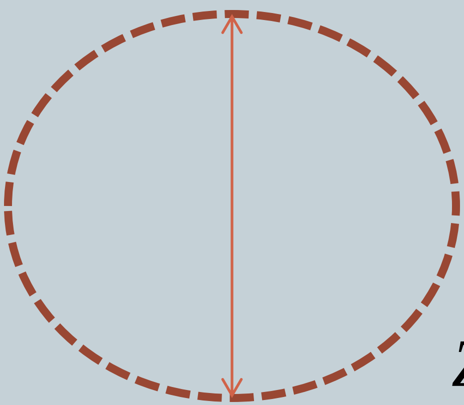
n_f Adjoint fermions with periodic boundary conditions along the S^1 circle

Georgi-Glashow model

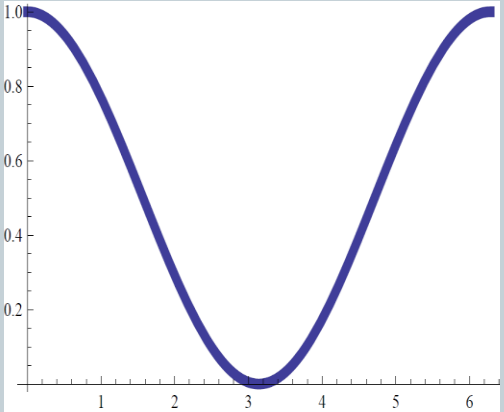
$$\overset{\text{small } L}{\overbrace{\Rightarrow}} \int_{R^3} \frac{L}{g^2} \mathrm{tr}\left[-\frac{1}{2} F_{ij} F^{ij} + \left(D_i A_4\right)^2 - \frac{g^2}{2} V_{eff}(A_4) \right]$$

One-loop effect

Compact scalar

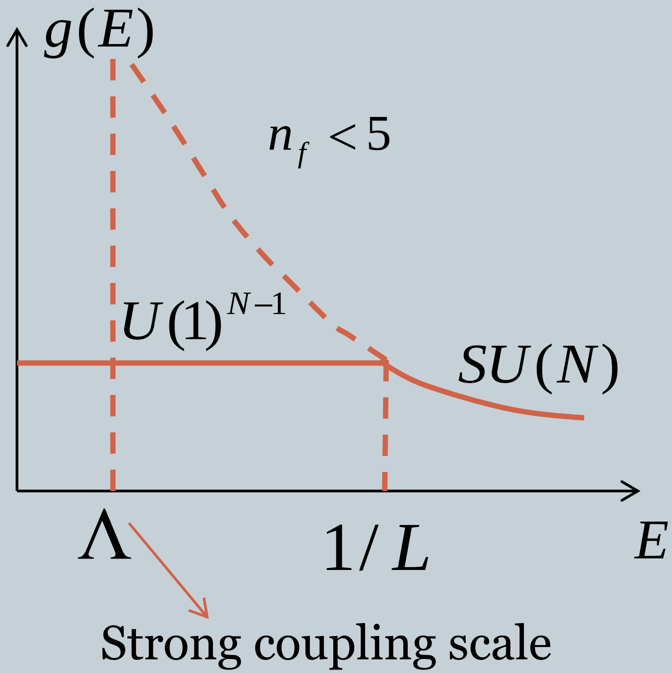


Z_2 symmetry



QCD(adj) on $R^3 \times S^1$, perturbative treatment

- The theory abelianizes $SU(2) \to U(1)$
- at small S^1 the gauge coupling freezes at small value
- The theory is effectively free massless 3-D $\partial_i F_{ij} = 0$



$$S = \int_{R^{1,2}} \frac{1}{2L}\left(\frac{g}{4\pi}\right)^2 (\partial\sigma)^2 + i\bar{\lambda}_I \bar{\tau}_j \partial^j \lambda_I \underbrace{+ O\left(\frac{1}{L}\right)}_{\text{Ws and heavy charged fermions}}$$

dual photon

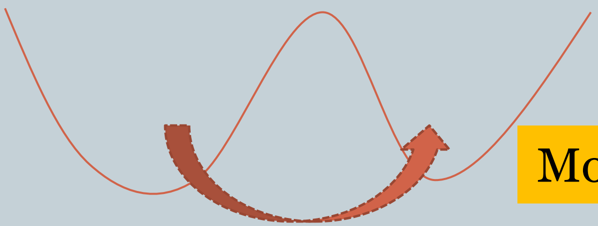
massless fermions

QCD(adj) on $R^3 \times S^1$:monopole-instantons

- More interesting story to tell: non-perturbative effects (Polyakov confinement mechanism)
- Feynman path integral

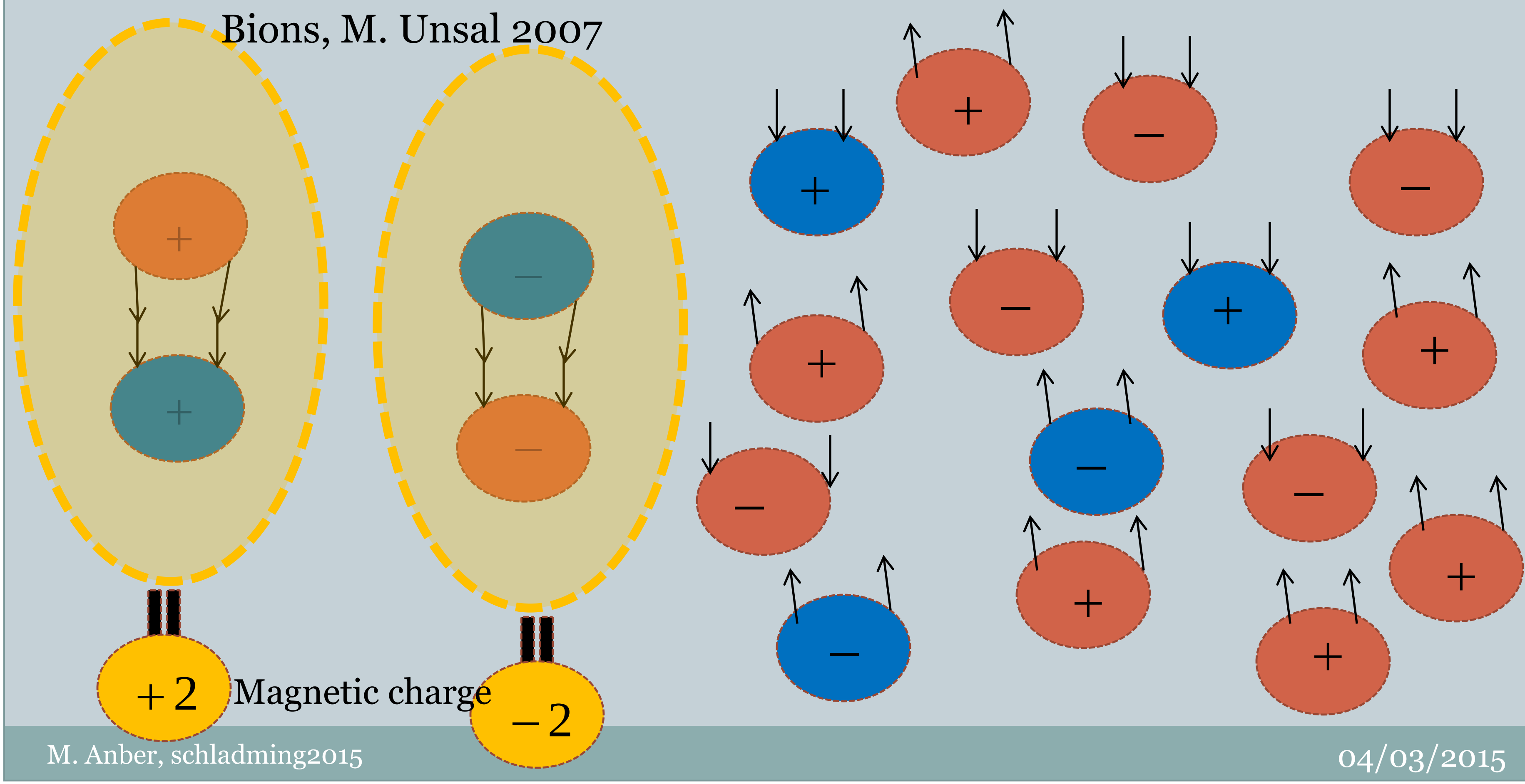
$$Z_{\text{Euclid}} = \sum_{\text{paths}} e^{-S_E}$$

Perturbative +non-perturbative (instantons)



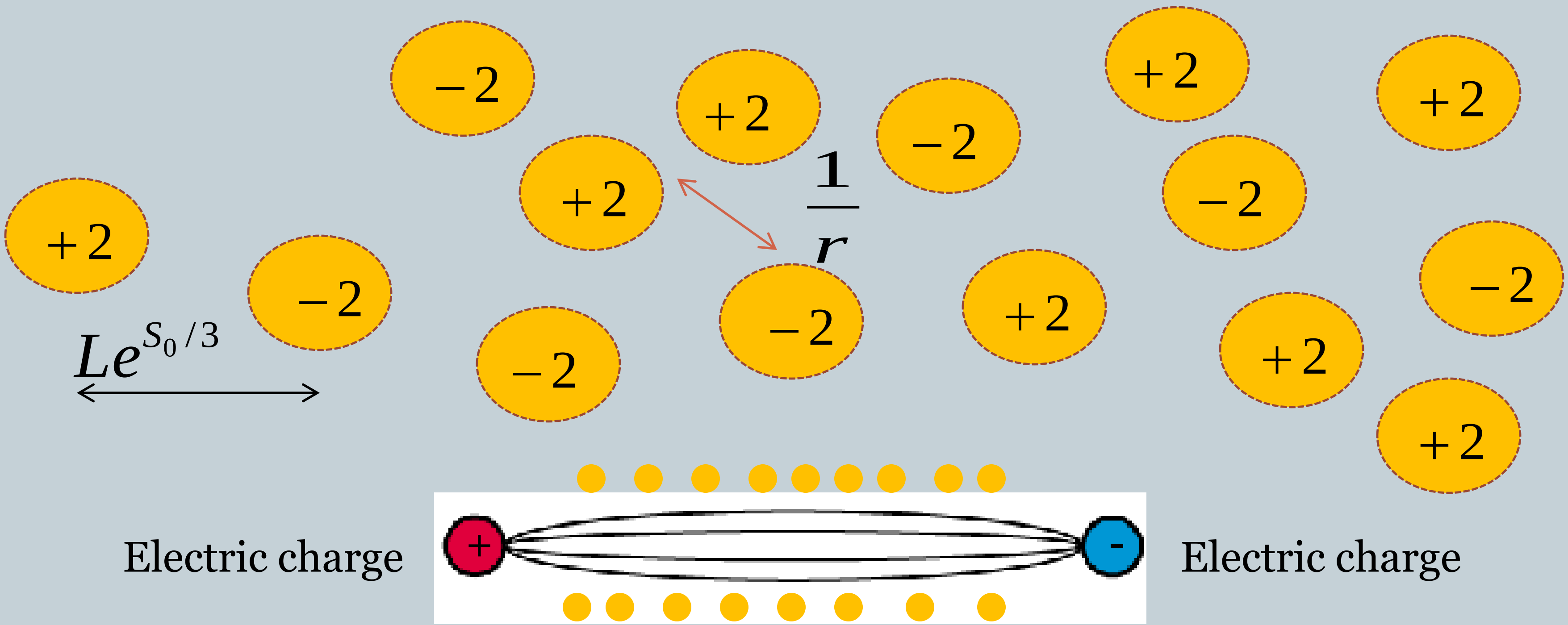
Monopole-instantons

QCD(adj) on $R^3 \times S^1$:bions



QCD(adj) on $R^3 \times S^1$:confinement

Bions proliferate in the vacuum: 3-D Coulomb gas

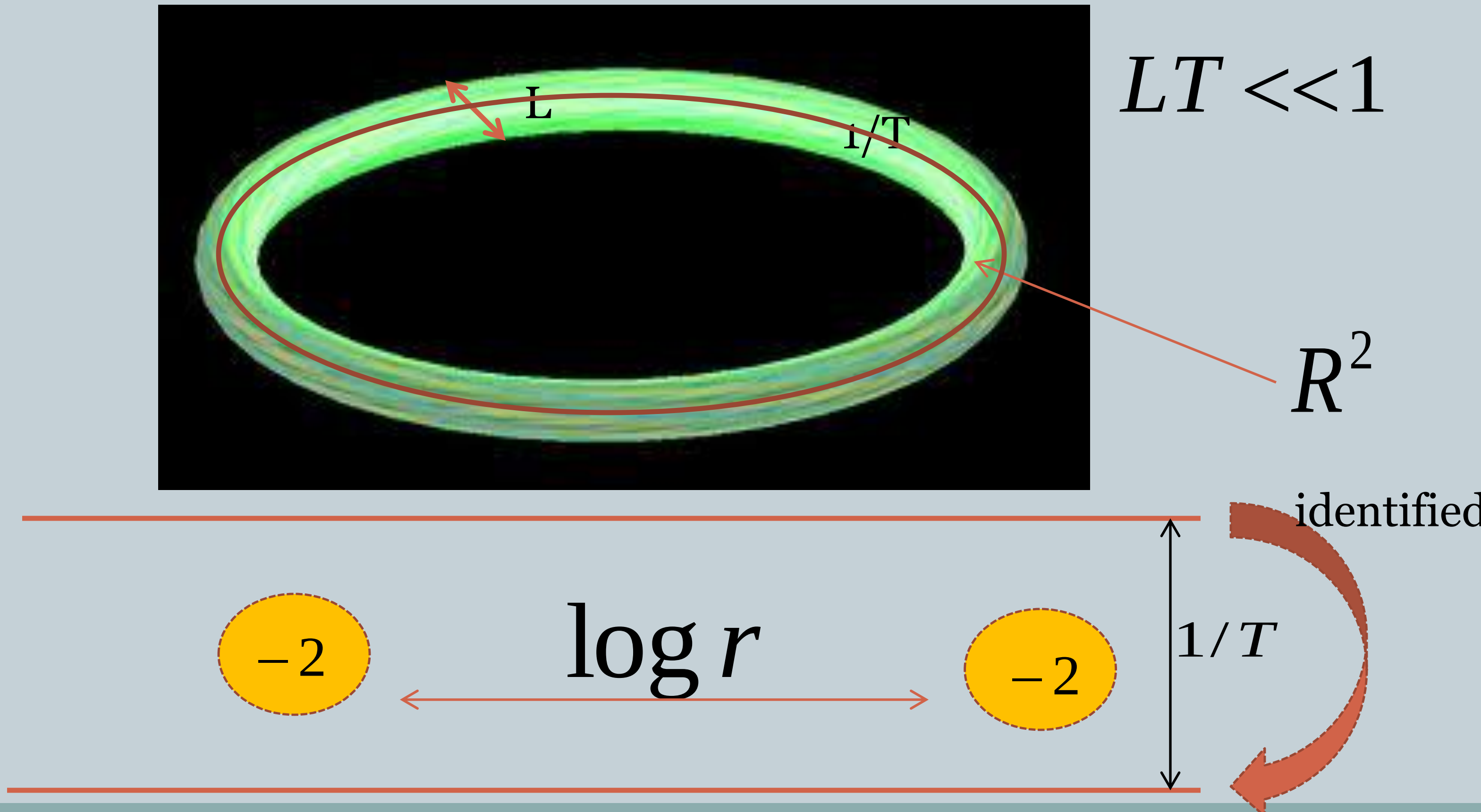


$$S = \int_{R^3} \frac{1}{2L}\left(\frac{g}{4\pi}\right)^2 (\partial\sigma)^2 + ce^{-S_0}\cos(2\sigma) + \text{fermions}$$

QCD(adj) on $R^2 \times S^1{}_L \times S^1{}_\beta$

M.A., E. Poppitz, M. Unsal, arXiv:1112.6389

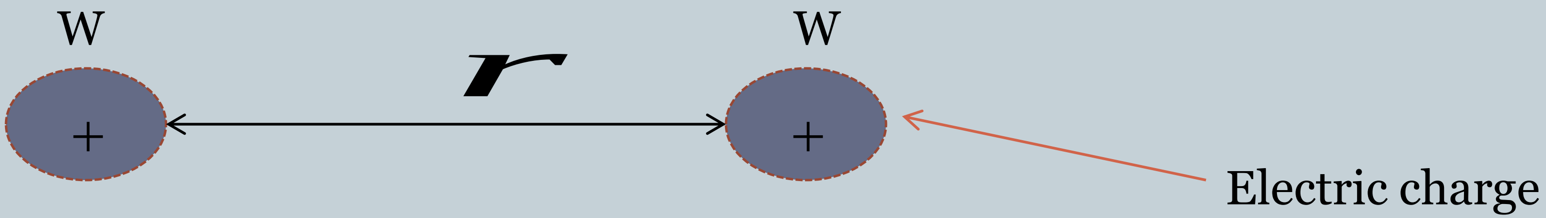
- At finite temperature we compactify the time direction



QCD(adj) on $R^2 \times S^1{}_L \times S^1{}_\beta$

- The story has one more twist!
- At finite temperature, the W's and charged fermions are important

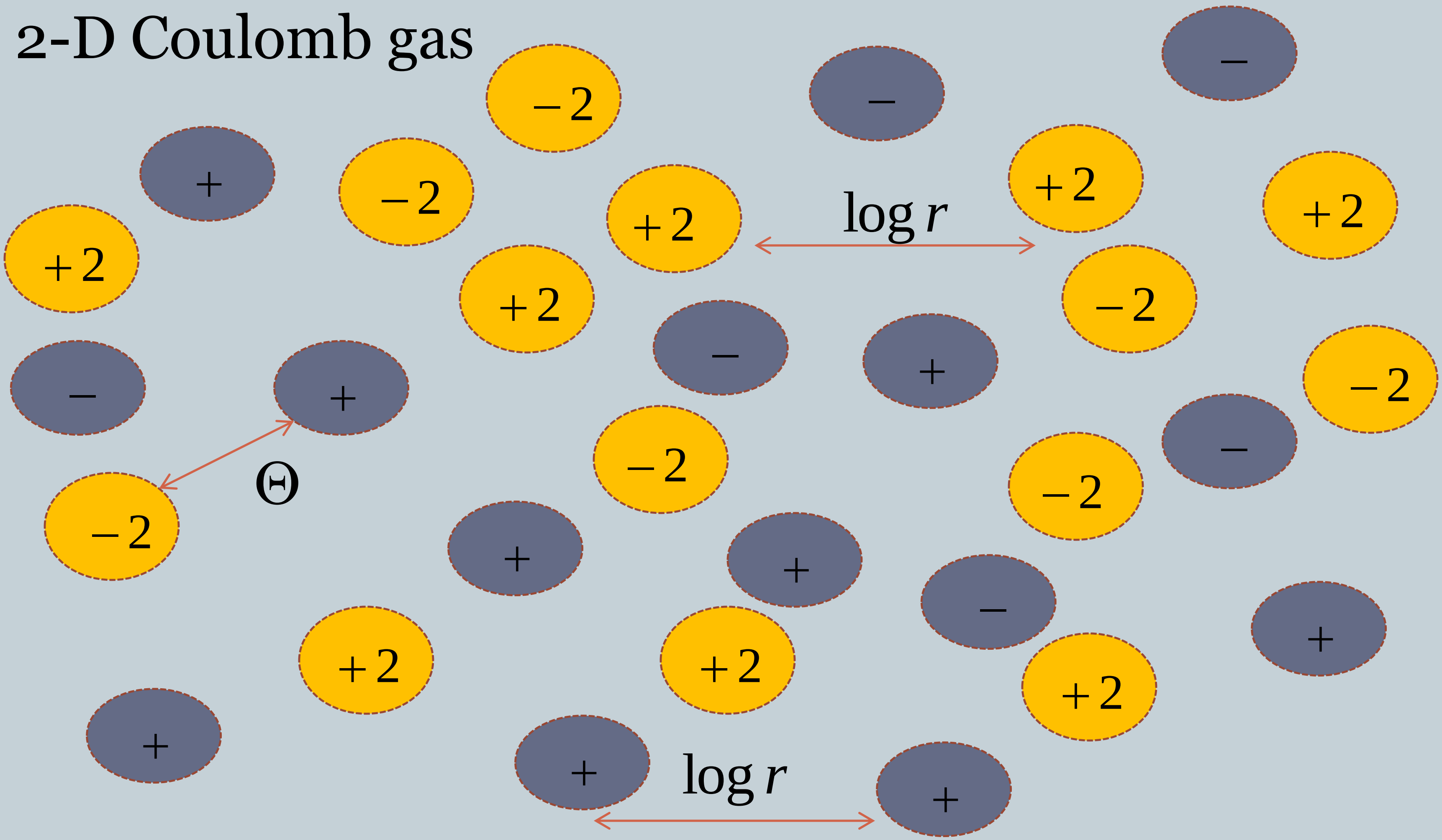
$$\text{density} \propto e^{-m_W/T}$$



$$\frac{g^2}{2\pi LT}\log r$$

QCD(adj) at finite temperature

- 2-D Coulomb gas



QCD(adj) at finite temperature

- The partition function for SU(2)

$$Z=\sum_{q_a,q_A,N_{bion-},N_{bion+},N_{W+},N_{W-}}\frac{\xi_{bion}^{N^i_{bion+}+N^i_{bion-}}}{N_{bion+}!N_{bion-}!}\frac{\xi_W^{N^i_{W+}+N^i_{W-}}}{N_{W+}!N_{W-}!}\prod_{a,i}\int d^2R^i{}_a\prod_{Ai}\int d^2R^i{}_A$$

$$\exp\left[\sum_{i,ja,b,A,B}\begin{array}{l}\frac{32\pi LTq_aq_b}{g^2}\log\left|\vec{R}^i_a-\vec{R}^j_b\right|+\frac{g^2q_Aq_B}{2\pi LT}\log\left|\vec{R}^i_A-\vec{R}^j_B\right|\\+4iq_aq_A\Theta\left(\vec{R}^i_a-\vec{R}^j_A\right)\end{array}\right]$$

e-m duality

$$\xi_{bion}\Longleftrightarrow 2\xi_W,\quad \frac{32\pi LT}{g^2}\Longleftrightarrow\frac{g^2}{2\pi LT}$$

Weakly coupled self-dual point

QCD(adj) at finite temperature

- The partition function for SU(3)

$$Z = \sum_{\vec{Q}_i, N_{bion-}, N_{bion+}, N_{W+}, N_{W-}} \frac{\xi_{bion}^{N^i_{bion+} + N^i_{bion-}}}{N_{bion+}! N_{bion-}!} \frac{\xi_W^{N^i_{W+} + N^i_{W-}}}{N_{W+}! N_{W-}!} \prod_{a,i} \int d^2 R^i{}_a \prod_{Ai} \int d^2 R^i{}_A$$

$$\exp\left[\sum_{i,j a,b,A,B} \underbrace{\frac{8\pi L T}{g^2}}_{4/\kappa} \vec{Q}_i . \vec{Q}_j \log\left|\vec{R}^i_a - \vec{R}^j_b\right| + \underbrace{\frac{g^2}{2\pi L T}}_{\kappa} \vec{\alpha}_i . \vec{\alpha}_j \log\left|\vec{R}^i_A - \vec{R}^j_B\right| + 2i \vec{\alpha}_j . \vec{Q}_i \Theta\left(\vec{R}^i_a - \vec{R}^j_A\right) \right]$$

For $SU(3)$ $\quad \vec{Q}_i . \vec{Q}_j = 3 \vec{\alpha}_i . \vec{\alpha}_j$

duality $\kappa \to \frac{12}{\kappa}, \xi_W \to \xi_{bion}$

Potential sign problem for simulations

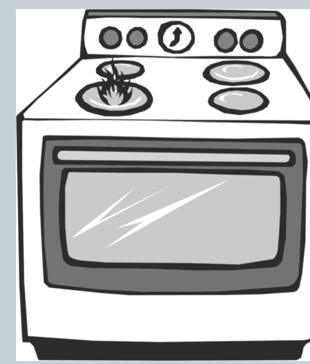
Strong coupling at the self-dual point

Mapping QCD(adj) to spin models

- Next, we construct the spin model that has identical Coloumb gas partition function

- Cook again, but now with "different" looking ingredient

Spin model



$$\left\langle e^{i\vartheta(\vec{x})} e^{-i.\vartheta(\vec{0})} \right\rangle$$

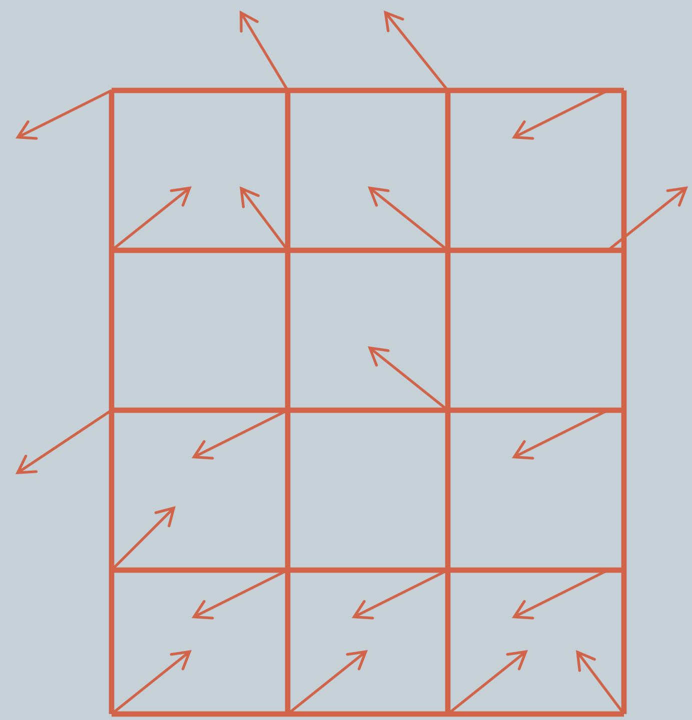
Mapping QCD(adj) to spin models

- This 2D Coulomb gas is **EXACTLY** equivalent to XY-spin models:

$$H = A \underbrace{\sum_{<ij>} \cos(\vartheta_i - \vartheta_j)}_{\vec{S}_i \cdot \vec{S}_j} + B \sum_i \cos(4\vartheta_i)$$

Nearest neighbor interaction

External field



Spin model for SU(2)

- For SU(2)

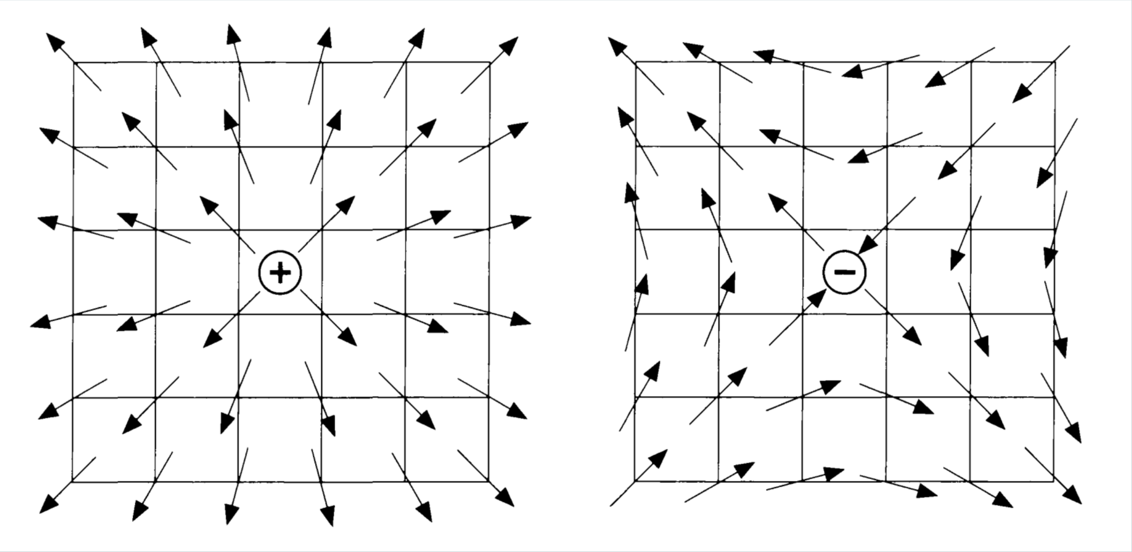
$$Z = \prod_i \int_0^{2\pi} d\vartheta_i e^{\frac{K}{2\pi}\sum_{<ij>}\cos\left(\vartheta_i - \vartheta_j\right) + g\sum_i \cos(4\vartheta_i)}$$

- Fluctuations in $\{\vartheta_i\}$: duals of the photon sourced by magnetic bions

Spin model for SU(2)

- Vortices: describe electric excitations in theory (W-bosons) excited at $T > T_c$

$$\frac{1}{2\pi}\oint d\vec{l}.\nabla\vartheta = \pm 1$$



(Kardar, Statistical Physics of Fields)

Spin model for SU(2)

- Exact $U(1)$ symmetry, $\vartheta \to \vartheta + c$, is broken to $Z_4 : \vartheta \to \vartheta + \frac{2\pi}{4}$
- The Z_4 symmetry is an enhancement for $Z_2^c \times Z_2^{d\chi}$
- Studying the RGEs, we find 2nd order phase transition

Spin model for SU(3)

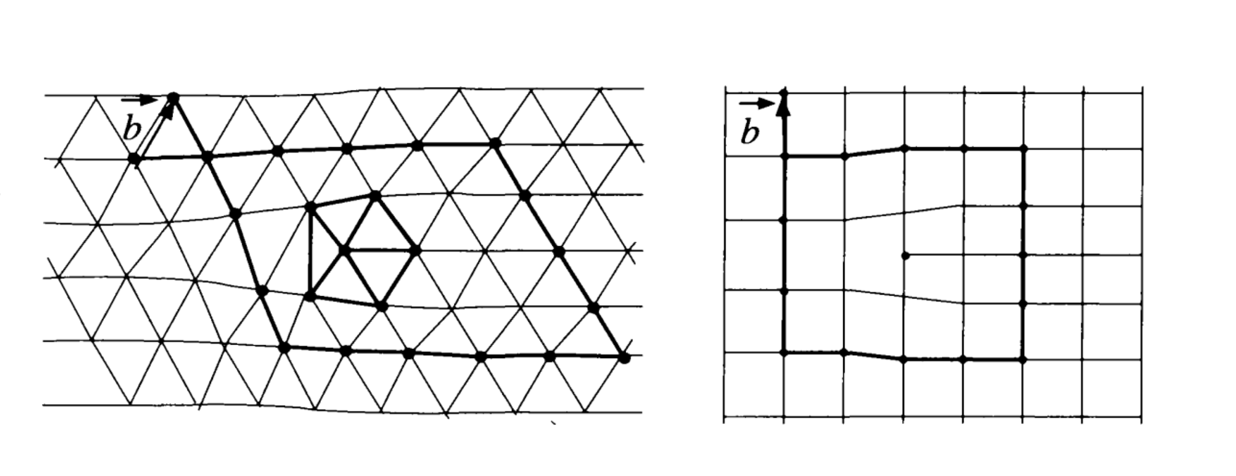
M.A., S. Collier, E. Poppitz, arXiv:1211.2824

- The spin model for SU(3) is

$$-\beta H = \sum_{x;\hat{\mu}=1,2} \sum_{i=1}^{N_c=3} \frac{\kappa}{4\pi} \cos 2\vec{\nu}_i \cdot (\vec{\theta}_{x+\hat{\mu}} - \vec{\theta}_x) + \sum_x \sum_{i=1}^{N_c=3} \tilde{y} \cos 2(\vec{\alpha}_i - \vec{\alpha}_{i-1}) \cdot \vec{\theta}_x$$

- Kinetic term: similar to a model used to describe melting of a 2d crystal on a triangular lattice (Nelson, 1977)

$\vec{\vartheta}$ has two components this corresponds to two photons



(Kardar, Statistical Physics of Fields)

Monte Carlo simulations

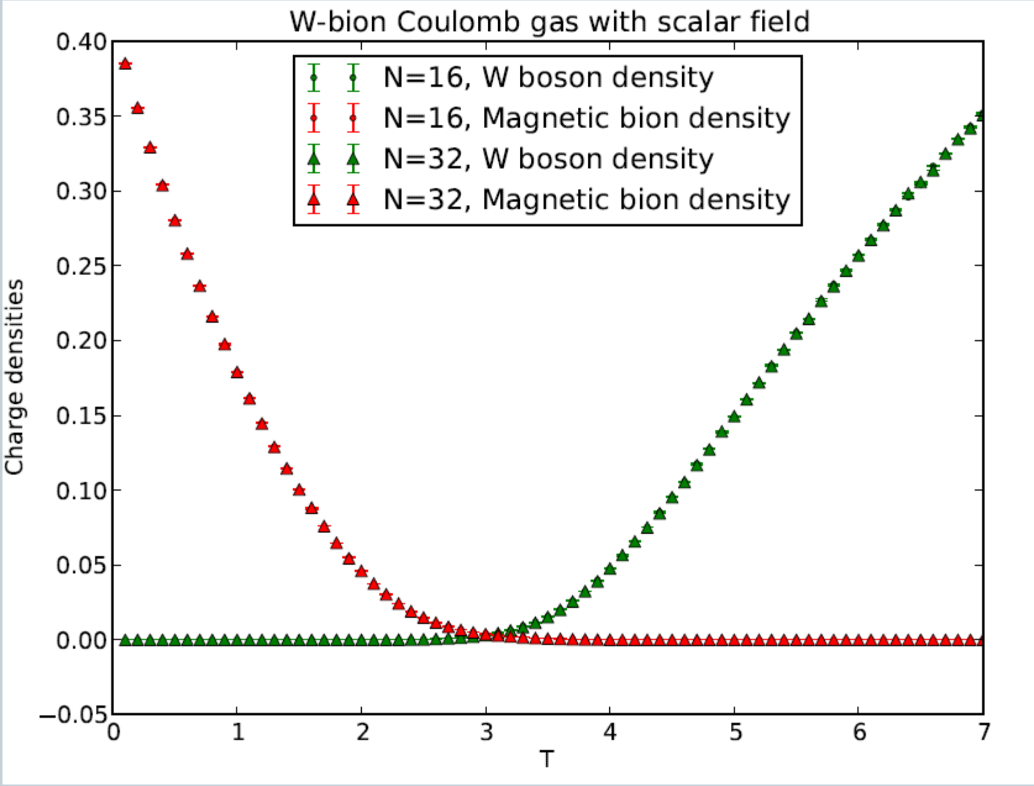
- Monte Carlo simulations is some science and a lot of gambling! (Ulam and Von Neumann)

- Use random numbers to learn about physical systems

- The phase space of the statistical systems can be infinitely large, we need only to sample the system
- We need to consider only “representative samples”

Results for SUR(2)

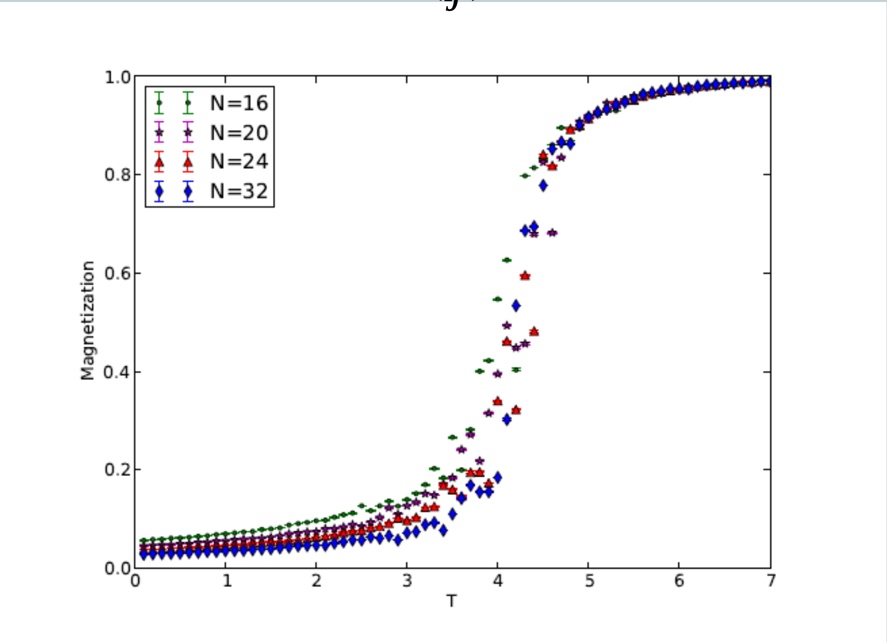
- Coulomb gas simulations



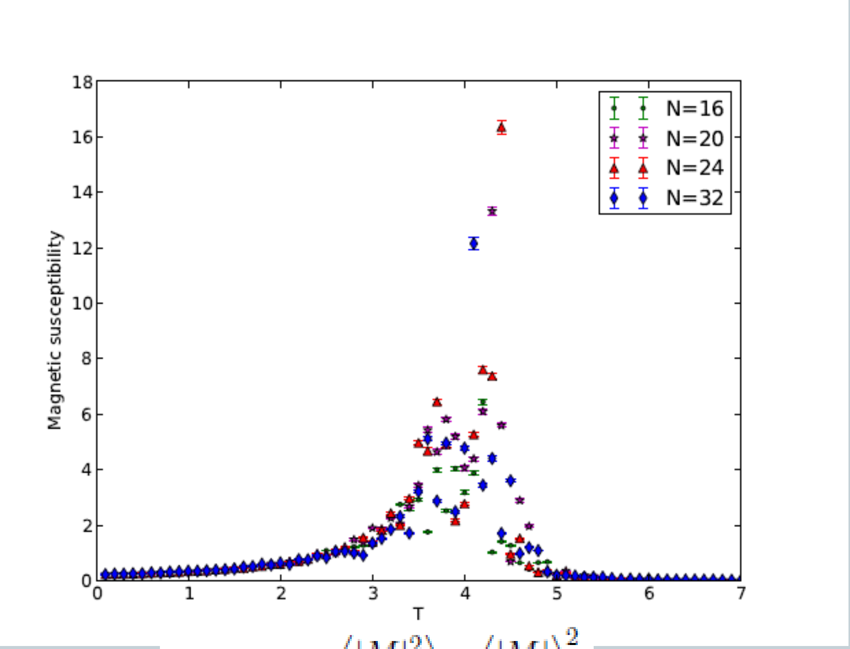
Results for SU(2)

- XY-model simulations

$$H = \frac{-8T}{\pi\kappa}\sum_{<ij>}\cos\left(\vartheta_i - \vartheta_j\right) + M_W T e^{-\frac{M_W}{T}}\sum_i \cos(4\vartheta_i)$$



$$m = \frac{1}{N^2}\langle|\sum_x e^{i\theta_x}|\rangle = \frac{\langle|M|\rangle}{N^2}$$



$$\chi(m) = \frac{\langle|M|^2\rangle - \langle|M|\rangle^2}{N^2}$$

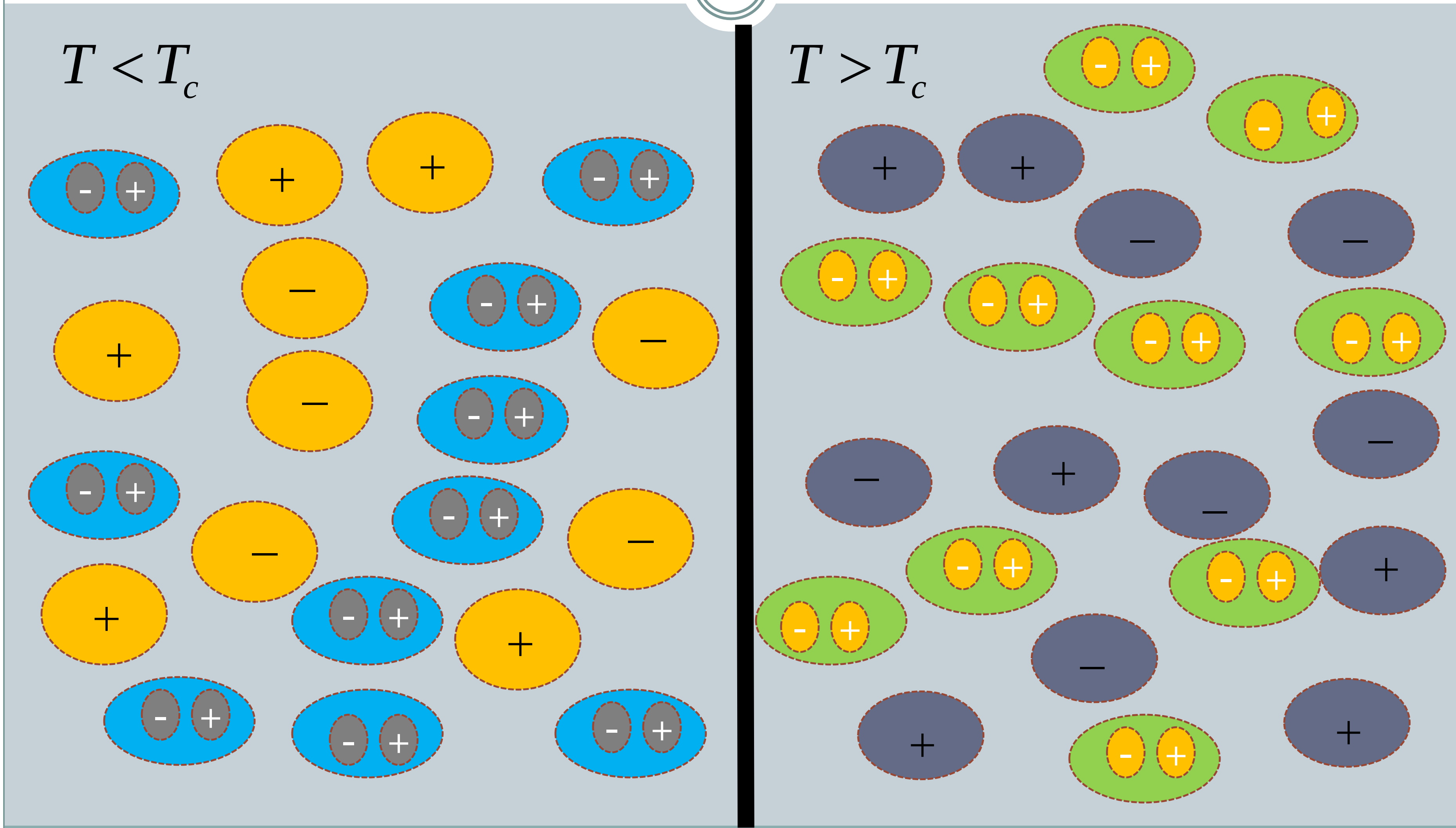
Results for SU(2)

- Vortices (magnetic charges) structure

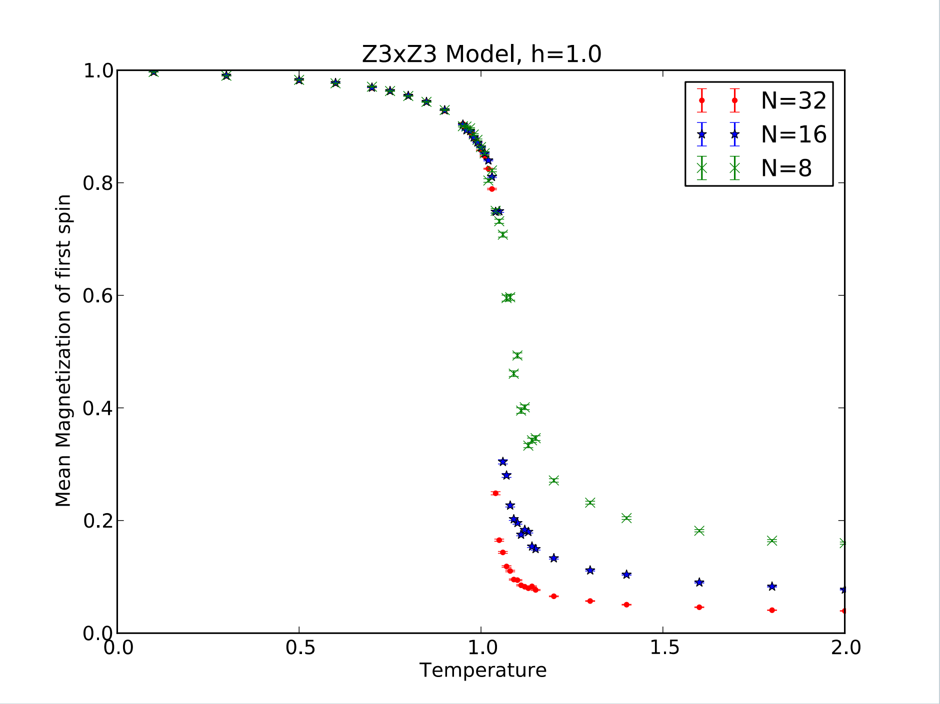
MOVIE

- The transition is second order (compare with SU(3))

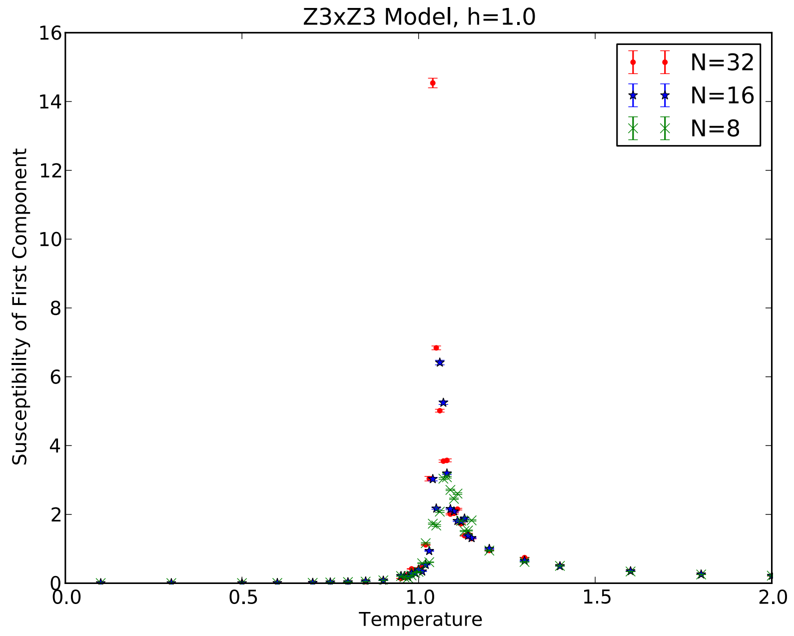
Results for SU(2)

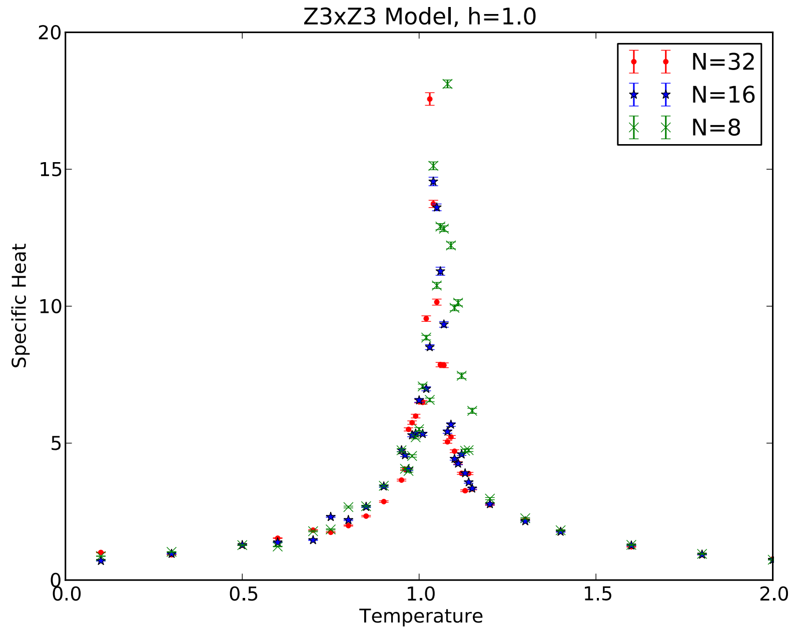


Results for SU(3)



Results for SU(3)

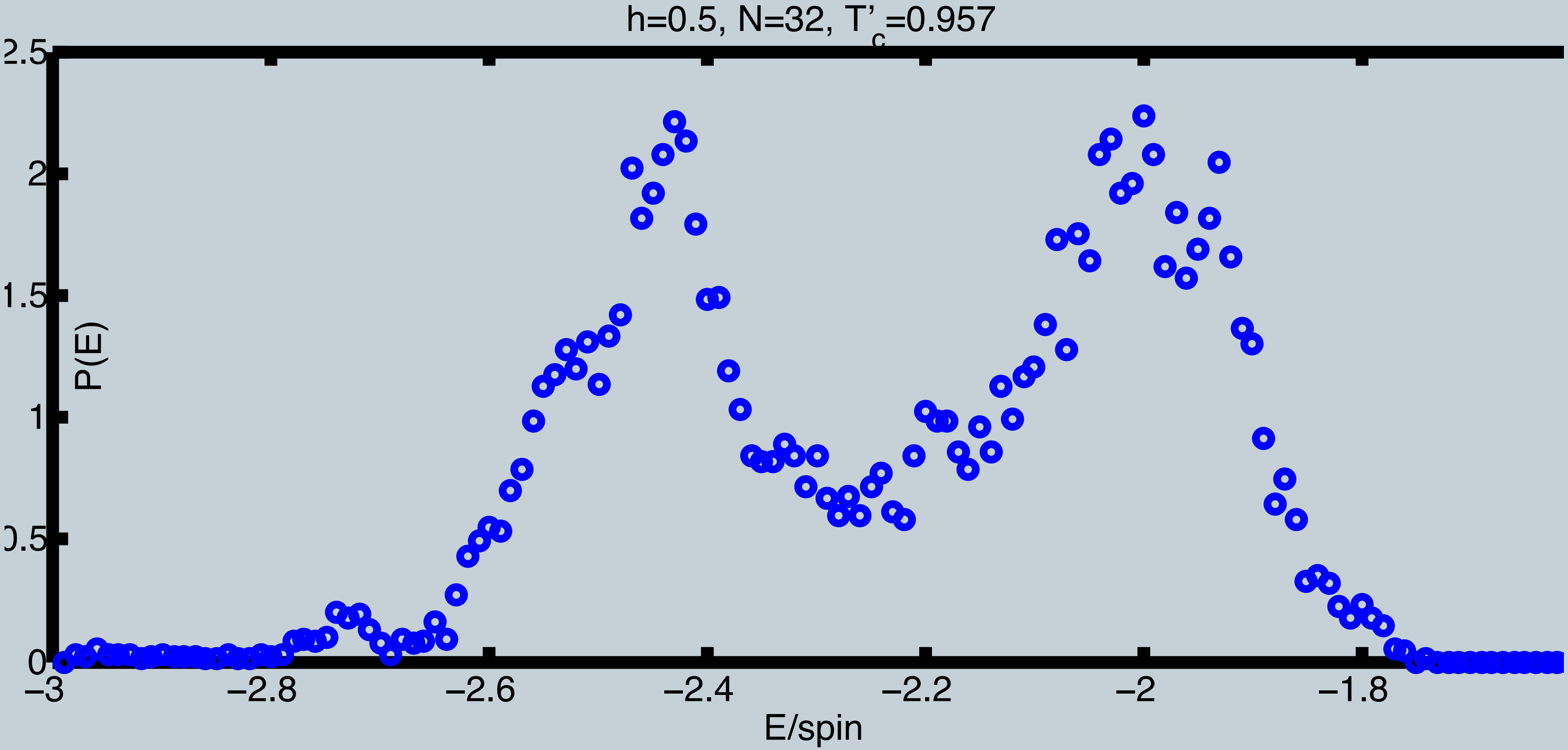




Single transition

Results for SU(3)

- Phase coexistence at the critical temperature:



compare with Karsch and Lutgemeir 1998

Conclusions

- Study of the phase transition in the Coulomb gas and the XY-spin model dual to the simplified QCD
- The phase transition is second order for SU(2)
- The phase transition is first order for SU(3)
- We would like to study other groups
- One would also want to study other effects, like adding fundamental fermions and turning on a background magnetic field
- Work along these lines is in progress