Sarma phase in relativistic and non-relativistic systems

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In Collaboration with
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The Sarma Phase

homogeneous superfluid phase with gapless fermionic excitations

\[ \Delta > 0 \]

- 2 fermion species with spin imbalance

\[ \delta \mu = \frac{\mu_1 - \mu_2}{2} \]

- Dispersion relation [lowest branches]

\[ E_p^{(\pm)} = \sqrt{\varepsilon_p^2 + \Delta^2} \pm \delta \mu \]

[\varepsilon_p \ldots \text{microscopic dispersion relation}; \Delta \ldots \text{pairing gap}]

[Sarma (1963)]

[Boettcher, TKH, Pawlowski, Strodthoff, von Smekal, Wetterich (2014)]
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  \[ \varepsilon_p \ldots \text{microscopic dispersion relation}; \Delta \ldots \text{pairing gap} \]
- Sarma: \( \Delta > 0 \) and lowest branch below zero

[Sarma (1963)]

[Boettcher, TKH, Pawlowski, Strothoff, von Smekal, Wetterich (2014)]
The Sarma Phase

homogeneous superfluid phase with gapless fermionic excitations

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  \[ \varepsilon_p \ldots \text{microscopic dispersion relation; } \Delta \ldots \text{pairing gap} \]

- Sarma: \( \Delta > 0 \) and lowest branch below zero
- non-monotonous behavior of occupation numbers
- \( T > 0 \): Fermi surfaces smeared out
  → no sharp distinction
  → Sarma crossover

[Sarma (1963)]

[Sarma phase in relativistic and non-relativistic systems] Tina K. Herbst (ITP Heidelberg)
Talk Outline

1. Sarma Phase in a Relativistic System
2. A Potential Non-Relativistic Analog
3. Taking a Second Look and a Proposition
4. Conclusions
A Relativistic System:

Quark-Meson Model at Finite Isospin Chemical Potential

[www.gsi.de]
Quark-Meson Model with Isospin Chemical Potential

2 flavors of quarks, $\psi = (u, d)^T$, coupled to mesons, $\sigma, \vec{\pi} = (\pi_0, \pi_+, \pi_-)$

$\mu_q = \mu_B/3$: imbalance between quarks and antiquarks

$\mu_I$: imbalance between up and down quarks
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$|\mu_I| > m_\pi/2$: pions condense in a Bose condensate
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[Kamikado, Strodthoff, von Smekal, Wambach (2013)]

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- fix $\mu_I = m_\pi > m_\pi/2$ (pion condensation possible)
- vary $\mu_q$
- order parameter: $\Delta^2 \sim \pi_+\pi_-$
- mean field approximation: no bosonic fluctuations
- Sarma criterion: $\Delta = \mu_q$
Quark-Meson Model with Isospin Chemical Potential

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- include fluctuations with FRG
  [cf. talks by F. Rennecke, M. Mitter, I. Boettcher, D. Roscher,...]
  $\rightarrow$ strong modifications of the phase structure
  two transition branches at low $T$
  (first and second order)
Quark-Meson Model with Isospin Chemical Potential

- 2 flavors of quarks, $\psi = (u, d)^T$, coupled to mesons, $\sigma, \vec{\pi} = (\pi_0, \pi^+, \pi^-)$
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- Sarma phase down to $T = 0$
  - Well-defined
  - Fairly large
  - Measurable?
- Can we use cold atoms “tool box” to learn more about this phase?
A Potential Non-Relativistic Analog

[Gubbels, Stoof (2012) ]
**Unitary Fermi Gas**

ultracold two-component fermions close to a broad Feshbach resonance

- 2 fermion species, $\psi = (\psi_1, \psi_2)^T$, with chem. potentials $\mu_{1,2}$
- bosonization in particle-particle channel: $\phi \sim \psi_1 \psi_2$ [diatomic molecule; Cooper pair]
- unitary regime: $s$-wave scattering length diverges, $a^{-1} = 0$; strongly coupled
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\[ T/\mu \]

- $\mu > 0$: condensation
- order parameter: $\Delta^2 \sim \phi \phi^*$
- first: mean field approximation
- Sarma criterion: $\Delta = \delta \mu$

Sarma phase in relativistic and non-relativistic systems

Tina K. Herbst (ITP Heidelberg)
**Unitary Fermi Gas**

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very similar!

Sarma phase in relativistic and non-relativistic systems

Tina K. Herbst (ITP Heidelberg)
Including Fluctuations - FRG

- bosonic order-parameter fluctuations included

\[ \Rightarrow \partial_t U_k(\Delta), \quad \partial_t g^2 = \eta \phi g^2 \]

[Feshbach coupling]

- solve flow equation on a grid

[Boettcher, Braun, TKH, Pawlowski, Roscher, Wetterich (2014)]
Including Fluctuations - FRG

- bosonic order-parameter fluctuations included
  \[ \partial_t U_k(\Delta), \partial_t g^2 = \eta_\phi g^2 \]
  \( [g \ldots \text{Feshbach coupling}] \)

- solve flow equation on a grid
  \([\text{Boettcher, Braun, TKH, Pawlowski, Roscher, Wetterich (2014)}]\)

Fluctuations:
- \( T_c \) down
- critical imbalance \( \delta \mu_c(T = 0) \) grows
- agreement with experiment \& Monte Carlo
  \([\text{Ku et al. (2012), Navon et al. (2013)}]\)
  \([\text{Goulko and Wingate (2010)}]\)
Including Fluctuations - FRG

- Bosonic order-parameter fluctuations included
  \[ \partial_t U_k(\Delta), \partial_t g^2 = \eta \phi g^2 \]
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- Solve flow equation on a grid
  [Boettcher, Braun, TKH, Pawlowski, Roscher, Wetterich (2014)]

Differences to the Relativistic System:

- **NO** splitting of transition line
- **NO** Sarma phase at \( T = 0 \)
- Sarma phase **shrinks**
Sarma Phase away from Unitarity

- Estimate possible: relativistic system slightly on BCS-side
- \( \rightarrow \) study full imbalanced BCS-BEC crossover \((a^{-1} \neq 0)\) at \( T = 0 \)
Sarma Phase away from Unitarity

- Estimate possible: relativistic system slightly on **BCS-side**
- \( \rightarrow \) study full imbalanced BCS-BEC crossover \((a^{-1} \neq 0)\) at \( T = 0 \)

MFA:
- Sarma phase at \( T = 0 \) occurs . . .
- \( \ldots \) but on **BEC-side** of the crossover

[cf. Sheehy, Radzihovsky (2006), Parish, Marchetti, Lamacraft, Simons (2007)]
Sarma Phase away from Unitarity

- Estimate possible: relativistic system slightly on **BCS-side**
- → study full imbalanced BCS-BEC crossover \((a^{-1} \neq 0)\) at \(T = 0\)

![Diagram showing the Sarma phase in relativistic and non-relativistic systems.](Diagram.png)

**MFA:**
- Sarma phase at \(T = 0\) occurs . . .
- . . . but on **BEC-side** of the crossover
- Quantum Critical Point
  → if transition of second order, there is always a Sarma phase!

[cf. Sheehy, Radzihovsky (2006), Parish, Marchetti, Lamacraft, Simons (2007)]
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- . . . but on **BEC-side** of the crossover
- Quantum Critical Point
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- Impact of fluctuations ?

[cf. Sheehy, Radzihovsky (2006), Parish, Marchetti, Lamacraft, Simons (2007)]
Sarma Phase away from Unitarity

- Estimate possible: relativistic system slightly on BCS-side
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**FRG:**
- transition line barely changed
- Sarma onset moves right

[MFA: open symbols; FRG: full symbols]
Sarma Phase away from Unitarity

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**FRG:**
- transition line barely changed
- Sarma onset moves *right*
- QCP moves right
- **NO** Sarma on BCS-side!

[MFA: open symbols; FRG: full symbols]
Why are they so different?
A Second Look at the Two Systems

- **Relativistic System**
  - $u_R$, $d_R$
  - $u_L$, $d_L$
  - $\sigma, \pi_0, \pi^+, \pi^-$

- **Non-Relativistic System**
  - 1
  - 2
  - $\phi, \phi^*$

- **Proposition**
  - Rel. system: additional $SU(2)_L \times SU(2)_R$ chiral symmetry
  - D.o.f. do not match!

Sarma phase in relativistic and non-relativistic systems

Tina K. Herbst (ITP Heidelberg)
A Second Look at the Two Systems

- **Relativistic System**
  - \( u_R \), \( d_R \), \( u_L \), \( d_L \)
  - \( \sigma, \pi_0, \pi^+, \pi^- \)

- **Non-Relativistic System**
  - 1
  - 2
  - \( \phi, \phi^* \)

- rel. system: additional \( SU(2)_L \times SU(2)_R \) chiral symmetry
- d.o.f. do not match!
- MF phase structure agrees
  - \( \rightarrow \) discrepancies in fermionic sector subleading
- \( \rightarrow \) large difference beyond MFA
  - \( \rightarrow \) bosons and their fluctuations crucial!

Sarma phase in relativistic and non-relativistic systems

*Tina K. Herbst* (ITP Heidelberg)
A Second Look at the Two Systems

<table>
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<th>rel.</th>
<th>non-rel.</th>
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- rel. system: additional $SU(2)_L \times SU(2)_R$ chiral symmetry
- d.o.f. do not match!
- MF phase structure agrees → discrepancies in fermionic sector subleading
- large difference beyond MFA → bosons and their fluctuations crucial!

A Proposition

non-relativistic system with four fermion species and interactions

$$\hat{H} \sim \lambda \left[ (\psi_1 \psi_2)^\dagger \psi_1 \psi_2 + (\psi_3 \psi_2)^\dagger \psi_3 \psi_2 + (\psi_1 \psi_4)^\dagger \psi_1 \psi_4 + (\psi_3 \psi_4)^\dagger \psi_3 \psi_4 \right]$$

- same $SU(2) \times SU(2)$ symmetry as the rel. system
- phase structure likely similar
- Sarma phase at $T = 0$ possible
Take-home Messages

- rel. and non-rel. systems for BCS-BEC crossover similar on MF-level . . .
- . . . but very different beyond
- bosonic d.o.f. and their fluctuations essential
- rel. system: ’non-trivial’ phase structure at low $T$ (Sarma!)
- non-rel. system:
  - good agreement with experiment and QMC for UFG
  - Sarma phase at low $T$ only on BEC-side
- proposition for a non-rel. system that might be more similar to the rel. one

Stay Tuned & Thanks!
Backup: A Simple Criterion for the Sarma Phase

zero-crossing of the lower branch if

\[ \delta \mu > \min_p \sqrt{\varepsilon_p^2 + \Delta^2} \]

assume:

\[ \min_p \varepsilon_p = 0 \implies \delta \mu_c > \Delta_c \]

[NB: not valid on BEC-side, where \( \mu < 0 \)]
Backup: A Simple Criterion for the Sarma Phase

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[NB: not valid on BEC-side, where \( \mu < 0 \)]

\[ \Delta = \delta \mu \]

\[ E^\pm_p \]

\[ \Delta = \delta \mu \]

\[ \Delta + \delta \mu \]

\[ \Delta - \delta \mu \]

\[ \epsilon_p \min \]

\[ \epsilon_p \max \]

\[ [\text{Boettcher, TKH, Pawlowski, Strodthoff, von Smekal, Wetterich (2014)}] \]

► Second Order Transition:
Condition always fulfilled
Backup: A Simple Criterion for the Sarma Phase

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\[\star\star\star\]

Second Order Transition:
Condition always fulfilled

First Order Transition:
\( \Delta_c \) vs \( \delta \mu_c \) decides

\( \Delta_c < \delta \mu_c \): Sarma phase

[Boettcher, TKH, Pawlowski, Strothoff, von Smekal, Wetterich (2014)]
Backup: A Simple Criterion for the Sarma Phase

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[NB: not valid on BEC-side, where \( \mu < 0 \)]

\[ \Delta = \delta \mu \]

\[ \Delta/\Delta_0 \]

\[ \delta \mu_{c,III} \]

\[ E_\pm \]

\[ \Delta + \delta \mu \]

\[ \Delta - \delta \mu \]

\[ \epsilon_{\min} \]

\[ \epsilon_{\max} \]

\[ \Delta_c \]

\[ \delta \mu_c \]

- **Second Order Transition:**
  Condition always fulfilled

- **First Order Transition:**
  \( \Delta_c \) vs \( \delta \mu_c \) decides
  - \( \Delta_c < \delta \mu_c \): Sarma phase
  - \( \Delta_c > \delta \mu_c \): no Sarma phase

[Boettcher, TKH, Pawlowski, Strodthoff, von Smekal, Wetterich (2014)]
Backup: A Simple Criterion for the Sarma Phase

zero-crossing of the lower branch if

\[ \delta \mu > \min_p \sqrt{\varepsilon_p^2 + \Delta^2} \]

assume: \[ \min_p \varepsilon_p = 0 \implies \delta \mu_c > \Delta_c \]

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<td>( \delta \mu_c, \text{III} )</td>
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<tr>
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</tr>
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- **Second Order Transition:**
  Condition always fulfilled

- **First Order Transition:**
  \( \Delta_c \) vs. \( \delta \mu_c \) decides
  - \( \Delta_c < \delta \mu_c \): Sarma phase
  - \( \Delta_c > \delta \mu_c \): no Sarma phase

[Boettcher, TKH, Pawlowski, Strodthoff, von Smekal, Wetterich (2014)]
Backup: Quark-Meson Model

\[ \mathcal{L}_{QMiso} = \bar{\psi} \left( \frac{\partial}{\partial \tau} + g(\sigma + i\gamma^5\pi\tau) - \gamma_0\mu_q - \gamma_0\tau_3\mu_I \right) \psi \\
+ \frac{1}{2}(\partial_\nu\sigma)^2 + \frac{1}{2}(\partial_\nu\pi_0)^2 + U(\chi, \rho) - c\sigma \\
+ \frac{1}{2}(\partial_\nu + 2\mu_I\delta_\nu^0)\pi_+(\partial_\nu - 2\mu_I\delta_\nu^0)\pi_- , \]

- 2 flavors
- quark (\(\mu_q\)) and isospin (\(\mu_I\)) chemical potentials
- \(SU(2)_L \times SU(2)_R \times U(1)_V\) symmetry
- chiral symmetry breaking: \(\chi \sim \langle \bar{\psi}\psi \rangle\)
- pion condensation: \(\Delta^2 = g^2 \rho = g^2 \pi_+\pi_-\)
Backup: FRG for the BCS-BEC Crossover

\[ \mathcal{L}_{UFG} = \sum_{\sigma=1,2} \psi_{\sigma}^* \left( \partial_{\tau} - \frac{\nabla^2}{2M_\sigma} - \mu_\sigma \right) \psi_\sigma + g \left( \phi^* \psi_1 \psi_2 + \text{h.c.} \right) \]

\[ + \phi^* \left( Z_\phi \partial_{\tau} - A_\phi \frac{\nabla^2}{4M} \right) \phi + \nu_\Lambda \phi^* \phi. \]

- 2 species of fermions, \( \sigma = 1, 2 \)
- bosonization: \( \phi \sim \psi_1 \psi_2 \) (particle-particle channel)
- spin imbalance by different chemical potentials \( \mu_1, \mu_2 \)
- \( \nu_\Lambda \sim a^{-1} \) fine tuned to fix scattering length
- condensation: \( \Delta^2 = g^2 \rho = g^2 \phi^* \phi \)

**Renormalization of the Propagators**

\[ P_{\psi_{\sigma},k}(iq_0, \vec{q}) = iq_0 + q^2 - \mu_\sigma, \]
\[ P_{\phi,k}(iq_0, \vec{q}) = A_{\phi,k} \left( iq_0 + \frac{q^2}{2} \right). \]
Backup: Renormalization and the Sarma Condition

Fluctuations modify chemical potentials and can thus influence the Sarma criterion, \( \Delta = \delta \mu \).

- Fluctuations increase \( \mu \)
- \( \mu_{\sigma, \text{eff}} \approx \mu_{\sigma} + 0.6 \mu_{\overline{\sigma}} \) \[ \mu_{\overline{\sigma}} \ldots \text{chem. pot. of other species} \]
- For imbalance: \( \delta \mu_{\text{eff}} = (\mu_{1, \text{eff}} - \mu_{2, \text{eff}})/2 \approx 0.4 \delta \mu \)
- \( \Rightarrow \) Sarma criterion even less likely fulfilled
- Here: unren. Sarma criterion not fulfilled at \( T = 0 \) \( \Rightarrow \) ren. criterion not fulfilled either