Glueballs and their decay in holographic QCD

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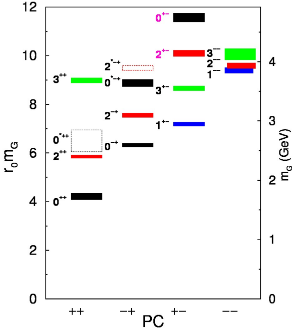
Still elusive: Gluonia (Glueballs)

Spectrum of *bare* glueballs
(prior to mixing with $q\bar{q}$ states)
more or less known from lattice:

$m_{0^{++}} \sim 1.7$ GeV
$m_{2^{++}} \sim 2.4$ GeV

Morningstar & Peardon hep-lat/9901004



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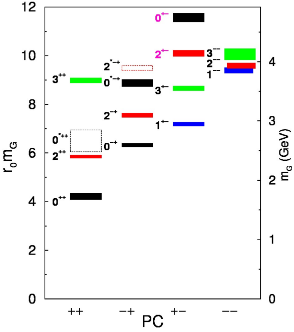

Interactions of glueballs still unclear:

- Are glueballs broad or narrow? ($\Gamma_G \propto 1/N_c^2$)
- Do they mix with $q\bar{q}$ strongly or weakly? (mixing $1/N_c$ suppressed)

$\rightarrow$ no conclusive identification of any glueball in meson spectrum

most discussed lowest 0^{++} candidates:

narrow $f_0(1500)$ or $f_0(1710)$ vs. broad background ("red dragon")

tensor candidates: broad $f_2(1950)$ or very narrow (unconfirmed) $f_J(2220)$

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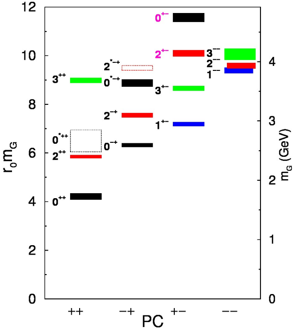

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Gauge/gravity duality a new tool to study glueball properties from first principles

Holographic QCD

Celebrated AdS/CFT duality relates strongly coupled large-N_c supersymmetric Yang-Mills theories to supergravity on anti-de Sitter space in 5 dimensions (AdS$_5 \times S^5$)

Holographic QCD: generalization to nonconformal nonsupersymmetric case
Options:

- **Bottom-up**: breaking of conformal invariance (necessary for confinement) by hand and matching to QCD with holographic dictionary, e.g.
  hard-wall model (Erlich-Katz-Son-Stephanov 2005)
  soft-wall model (Karch-Katz-Son-Stephanov 2006)
- **Top-down**: first-principles constructions from superstring theory with nonconformal D-branes
  - here: **Witten**[1998]-**Sakai-Sugimoto**[2004] **model**

Both approaches surprisingly successful quantitative description of low-energy QCD with minimal set of parameters

WSS model: almost parameter-free (1 coupling at a certain mass scale)!

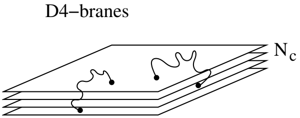
New results on: TU WIEN

- Glueball decay pattern [arXiv:1501.07906, with F. Brünner & D. Parganlija]

Witten model: Holographic nonsupersymmetric QCD

E. Witten, Adv. Theor. Math. Phys. 2, 505 (1998):

Type-IIA string theory with $N_c \to \infty$ $D4$ branes dual to $4+1$-dimensional super-Yang-Mills theory



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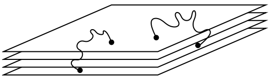

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supersymmetry completely broken by compactification
on "thermal-like" circle $x_4 \equiv x_4 + 2\pi/M_{\rm KK}$ $_{\rm (Kaluza-Klein)}$

- antisymmetric b.c. for adjoint fermions: masses $\sim M_{\rm KK}$
- adjoint scalars not protected by gauge symmetry: also masses $\sim M_{\rm KK}$

$\to$ dual to pure-glue YM theory
3+1-dimensional at scales $\ll M_{\rm KK}$

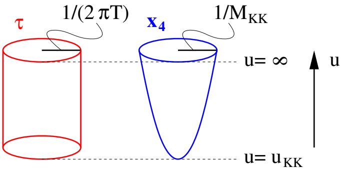
but supergravity approximation needs weak curvature,
cannot take limit $M_{\rm KK} \to \infty$

Deconfinement phase transition

Thermal circle in Euclidean time τ in addition to compactified x_4
Hawking-Page transition when $2\pi T = M_{\rm KK}$ (thus ~ 1 GeV ?)

Confined phase

$$ds^2 = \left(\frac{u}{R}\right)^{3/2}\left[d\tau^2 + d\mathbf{x}^2 + f(u)dx_4^2\right] + \left(\frac{R}{u}\right)^{3/2}\left[\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right]$$

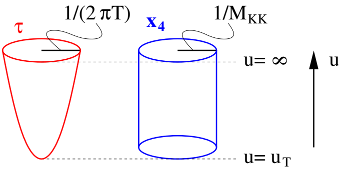


$$M_{\rm KK} = \frac{3}{2}\frac{u_{\rm KK}^{1/2}}{R^{3/2}} \qquad f(u) \equiv 1 - \frac{u_{\rm KK}^3}{u^3}$$

Cigar topology in x_4-u subspace $\longrightarrow$

Deconfined phase

$$ds^2 = \left(\frac{u}{R}\right)^{3/2}\left[\tilde f(u)d\tau^2 + \delta_{ij}d\mathbf{x}^2 + dx_4^2\right] + \left(\frac{R}{u}\right)^{3/2}\left[\frac{du^2}{\tilde f(u)} + u^2 d\Omega_4^2\right]$$



$$T = \frac{3}{4\pi}\frac{u_T^{1/2}}{R^{3/2}} \qquad \tilde f(u) \equiv 1 - \frac{u_T^3}{u^3}$$

Cigar in τ-u = **Euclidean black hole**

Glueballs in confined phase

$\exists$ scalar and tensor glueballs corresponding to 5D dilaton Φ and graviton G_{ij}
Csaki, Ooguri, Oz & Terning 1999

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Type-IIA supergravity compactified on x_4-circle many more modes:
Constable & Myers 1999; Brower, Mathur & Tan 2000

Mode Sugra fields J^{PC}	S_4 G_{44} 0^{++}	T_4 Φ, G_{ij} $0^{++}/2^{++}$	V_4 C_1 0^{-+}	N_4 B_{ij} 1^{+-}	M_4 C_{ij4} 1^{--}	L_4 G^α_α 0^{++}
n=0	7.30835	22.0966	31.9853	53.3758	83.0449	115.002
n=1	46.9855	55.5833	72.4793	109.446	143.581	189.632
n=2	94.4816	102.452	126.144	177.231	217.397	277.283
n=3	154.963	162.699	193.133	257.959	304.531	378.099
n=4	228.709	236.328	273.482	351.895	405.011	492.171

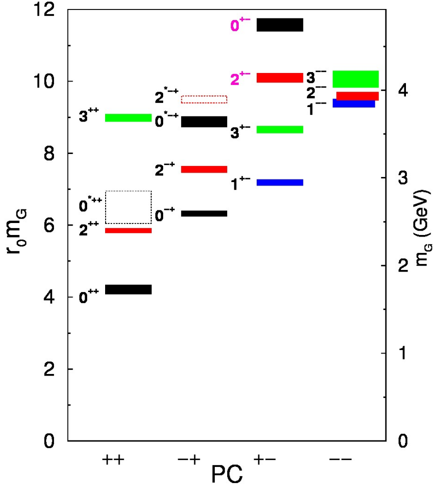
Lowest mode not from dilaton, but from "exotic polarization" – in 11D notation:

$$\underline{\delta g_{44}} = -\frac{r^2}{L^2} f\, H(r)G(x), \quad \delta g_{\mu\nu} = \frac{r^2}{L^2}\left[\frac{1}{4}H(r)\eta_{\mu\nu} - \left(\frac{1}{4} + \frac{3R^6}{5r^6 - 2R^6}\right)H(r)\frac{\partial_\mu\partial_\nu}{M^2}\right]G(x)$$

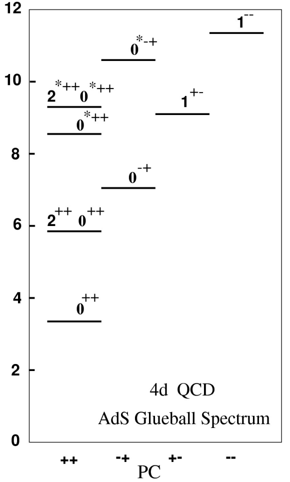
$$\delta g_{11,11} = \frac{r^2}{L^2}\frac{1}{4}H(r)G(x), \quad \delta g_{rr} = -\frac{L^2}{r^2} f^{-1}\frac{3R^6 H(r)G(r)}{5r^6 - 2R^6}, \quad \delta g_{r\mu} = \frac{90 r^7 R^6 H(r)\partial_\mu G(x)}{M^2 L^2 (5r^6 - 2R^6)^2}$$

Lattice glueballs vs. supergravity glueballs

Morningstar & Peardon hep-lat/9901004:



Brower, Mathur & Tan 2000:

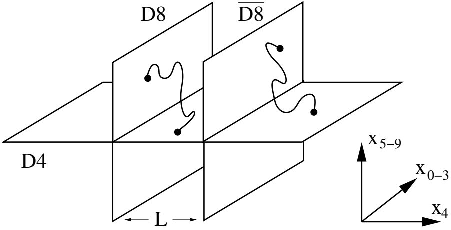


(mass scales matched on 2^{++}) $\rightarrow$ seemingly good qualitative agreement!

Sakai-Sugimoto model: Adding chiral quarks

T. Sakai, S. Sugimoto, Prog. Theor. Phys. 113, 843 (2005)
add N_f D8- and $\overline{\text{D8}}$-branes, separated in x_4, $N_f \ll N_c$ (probe branes)

	0	1	2	3	4	5	6	7	8	9
D4	x	x	x	x	x					
D8/$\overline{\text{D8}}$	x	x	x	x		x	x	x	x	x



4-8, 4-$\bar{8}$ strings
$\rightarrow$ fundamental, massless
chiral fermions

flavor symmetry
$U(N_f)_L \times U(N_f)_R$

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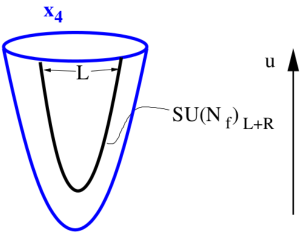

4-8, 4-$\bar{8}$ strings
→ fundamental, massless
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spontaneously broken because D8-$\overline{\text{D8}}$ have to join in cigar-shaped topology

for now: maximal separation in x_4 (antipodal on x_4 circle): $L = \pi/M_{\text{KK}}$

Quantitative predictions

Matching

1. $m_\rho \approx 776$ MeV fixes $\boxed{M_{\rm KK} = 949 \text{ MeV}}$ ($\Rightarrow T_{deconf} = 151$ MeV)
2. matching $f_\pi^2 = \frac{\lambda N_c}{54\pi^4} M_{\rm KK}^2$ gives $\boxed{\lambda = g_{\rm YM}^2 N_c \approx 16.63}$ [Sakai&Sugimoto 2005-7]
   (matching instead large-N_c lattice result [Bali et al. 2013] for $m_\rho/\sqrt{\sigma}$ gives $\lambda \approx 12.55$)

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   (matching instead large-N_c lattice result [Bali et al. 2013] for $m_\rho/\sqrt{\sigma}$ gives $\lambda \approx 12.55$)

yields (for $N_c = 3$ and $\lambda = 16.63 \ldots 12.55$):

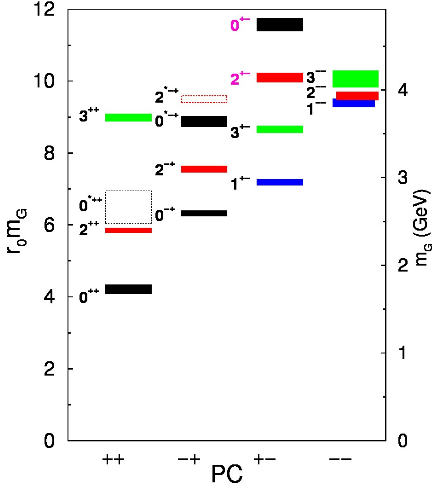
- $m_{a_1}^2/m_\rho^2 \approx 2.4$ (versus 2.5 from experiment!)
- nonzero η' mass from anomaly inflow: Witten-Veneziano formula with $m_{\eta'} = \frac{\sqrt{N_f/N_c}}{3\sqrt{3}\pi} \lambda M_{\rm KK} \approx 967 \ldots 730$ MeV for $N_f = 3$ (exp.: 958 MeV !)
- decay rate of ρ meson
  $\Gamma_{\rho\to 2\pi}/m_\rho = 0.1535 \ldots 0.2034$ (exp.: 0.191(1))
- decay rate for $\omega \to 3\pi$ (from Chern-Simons part of D8 action)
  $\Gamma_{\omega\to 3\pi}/m_\omega = 0.0033 \ldots 0.0102$ (exp.: 0.0097(1))
- gluon condensate [Kanitscheider, Skenderis & Taylor JHEP 0809]
  $C^4 \equiv \langle \frac{\alpha_s}{\pi} F_{\mu\nu}^2 \rangle = \frac{4}{3^7\pi^4} N_c \lambda^2 M_{\rm KK}^4 \simeq 0.0126 \ldots 0.0072\ {\rm GeV}^4$
  classical SVZ value: 0.012 GeV^4 (lattice higher but with large subtraction ambiguities)

Lattice vs. supergravity glueballs

seemingly good qualitative agreement by matchup up 2^{++}

(but AdS spectrum somewhat stretched. . .)

Morningstar & Peardon hep-lat/9901004:



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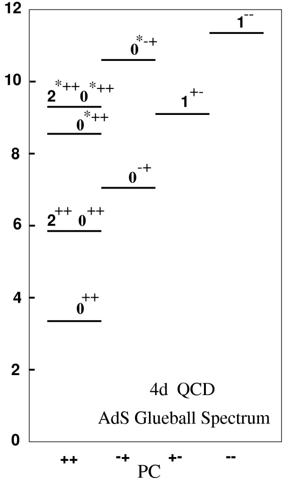

Lattice vs. supergravity glueballs in Sakai-Sugimoto model

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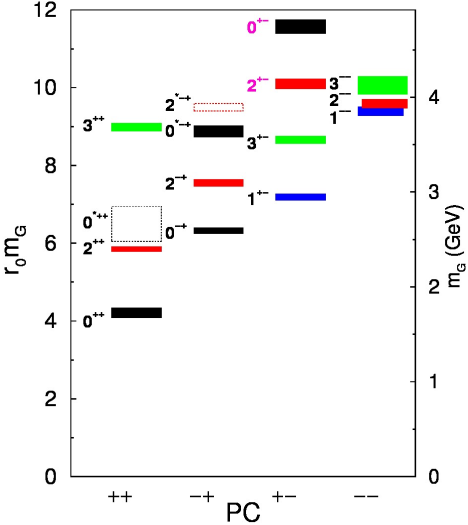

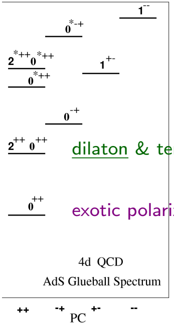

dilaton & tensor 1487 MeV

exotic polarization 855 MeV

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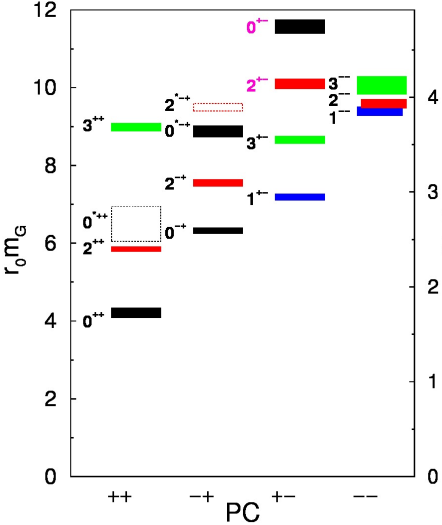
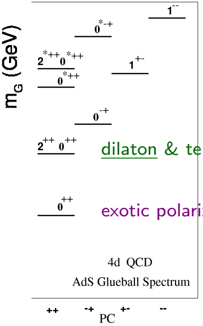


now good match lowest $0^{++} \leftrightarrow$ dilaton



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Lattice vs. supergravity glueballs in Sakai-Sugimoto model

Should exotic polarization (δG_{44} with x_4 the compactified direction of SYM_{4+1}) be excluded as lowest glueball mode?

- possibly not part of spectrum of holographic QCD in limit $M_{\mathrm{KK}} \to \infty, \lambda \to 0$ (already asked by Constable & Myers)
- simpler bottom-up AdS/QCD have dilaton mode as dual for lowest glueball

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- simpler bottom-up AdS/QCD have dilaton mode as dual for lowest glueball
- next lowest scalar mode ~ 1487 MeV is (predominantly) dilaton mode (induces metric perturbations other than δG_{44})

Glueball-$\bar{q}q$ couplings in Sakai-Sugimoto model

Gravitational modes stable in confined background, but
can calculate *effective action for glueball-$\bar{q}q$ interactions*

done for lowest (exotic) mode by
Hashimoto, Tan & Terashima, Phys.Rev. D77 (2008) 086001, arXiv:0709.2208

revisited, corrected, and extended to other modes by
Brünner, Parganlija & AR, arXiv:1501.07906

For example: Vertices of one glueball and two (massless) pions
for "exotic" mode:

$$S_{G_E\pi\pi} = \mathrm{Tr} \int d^4x \frac{1}{2} \partial_\mu \pi \, \partial_\nu \pi \left(\breve{c}_1 \eta^{\mu\nu} - c_1 \frac{\partial^\mu \partial^\nu}{M_E^2} \right) G_E$$

for "predominantly dilatonic" mode:

$$S_{G_D\pi\pi} = \mathrm{Tr} \int d^4x \frac{1}{2} \partial_\mu \pi \, \partial_\nu \pi \; \tilde{c}_1 \left(\eta^{\mu\nu} - \frac{\partial^\mu \partial^\nu}{M_D^2} \right) G_D$$

with $\{c_1, \breve{c}_1, \tilde{c}_1\} = \{62.66, 16.39, 17.23\} \times \lambda^{-1/2} N_c^{-1} M_{\mathrm{KK}}^{-1}$

and many more: $S_{G\rho\rho} \propto \lambda^{-1/2} N_c^{-1}$, $S_{G\rho\pi\pi} \propto \lambda^{-1} N_c^{-3/2}, \ldots$

Glueball decay rates in Sakai-Sugimoto model

Results for decay into two pions:

Exotic mode: $\Gamma_{G_E \to \pi\pi}/M_E \approx \dfrac{13.79}{\lambda N_c^2} \approx 0.092 \ldots 0.122 \quad (M_E \approx 855\text{MeV})$

Dilaton mode: $\Gamma_{D \to \pi\pi}/M_D \approx \dfrac{1.359}{\lambda N_c^2} \approx 0.009 \ldots 0.012 \quad (M_D \approx 1487\text{MeV})$

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Most likely *experimental candidates* for meson with dominant scalar glueball content: $f_0(1500)$ or $f_0(1710)$ (the latter favored by Janowski et al. arXiv:1408.4921)

$$\Gamma^{(\text{ex})}(f_0(1500) \to \pi\pi)/(1505\text{MeV}) = 0.025(3)$$

$$\Gamma^{(\text{ex})}(f_0(1710) \to \pi\pi)/(1722\text{MeV}) = \begin{cases} 0.017(4) & (\text{BES}) \\ 0.009(2) & (\text{WA102}) \end{cases}$$

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NB: relative width of lowest (exotic) scalar mode much larger than next ones!?

- another hint that G_E should be discarded?

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- another hint that G_E should be discarded?
- or could it perhaps correspond to broad glueball component of σ-meson à la Narison 1998: QCD sum rules need very broad glueball around 1 GeV plus narrow glueball around 1.5 GeV
  (cp.: Janowski et al. 1408.4921: eLSM fit of $f_0(1710)$ as predominantly glue, but only with extremely large gluon condensate)

Glueball decay rates in Sakai-Sugimoto model (cont'd)

Full decay pattern:

decay $G_D \to 4\pi$ suppressed (below 2ρ threshold): $\Gamma_{G\to 4\pi}/\Gamma_{G\to 2pi} \sim \lambda^{-1} N_c^{-1}$,
while $f_0(1500) \to 4\pi$ dominant:

decay	Γ/M (PDG)	$\Gamma/M[G_D]$
$f_0(1500)$ (total)	0.072(5)	0.027...0.037
$f_0(1500) \to 4\pi$	0.036(3)	0.003...0.005
$f_0(1500) \to 2\pi$	0.025(2)	0.009...0.012
$f_0(1500) \to 2K$	0.006(1)	0.012...0.016
$f_0(1500) \to 2\eta$	0.004(1)	0.003...0.004

$\Rightarrow$: $f_0(1500)$ seemingly disfavored

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$f_0(1710) \to \pi\pi$ OK,
but $f_0(1710)$ decays predominantly into $2K$!

— not reproduced by (chiral) WSS model,
but might be different in mass-deformed WSS (under investigation)

cf. mechanism of "chiral suppression of scalar glueball decay" (Chanowitz 2005)

Glueball decay rates in Sakai-Sugimoto model (cont'd)

Tensor glueball in WSS and extrapolated to higher mass:

decay	M	$\Gamma/M[T(M)]$
$T \to 2\pi$	1487	0.013...0.018
$T \to 2K$	1487	0.004...0.006
$T \to 2\eta$	1487	0.0005...0.0007
T (total)	1487	$\approx 0.02 \ldots 0.03$
$T \to 2\rho \to 4\pi$	2000	0.135...0.178
$T \to 2\omega \to 6\pi$	2000	0.045...0.059
$T \to 2\pi$	2000	0.014...0.018
$T \to 2K$	2000	0.010...0.013
$T \to 2\eta$	2000	0.0018...0.0024
T (total)	2000	$\approx 0.16 \ldots 0.21$

With a mass of 2 GeV, the relative width turns out to be comparable with that of the comparatively broad tensor meson $f_2(1950)$, which has $\Gamma/M = 0.24(1)$.

Very narrow (unconfirmed) candidate $f_J(2220)$ not compatible with WSS

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After fitting just m_ρ to fix $M_{\rm KK} = 949$ MeV

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- good prediction of deconfinement/chiral transition temperature,
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after fitting f_π or $m_\rho/\sqrt{\sigma}$ to also fix 't Hooft coupling at $\lambda = 16.63 \ldots 12.55$

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Warrants further studies!

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Plans:

- inclusion of nonzero mass for strange quark
- mixing with quarkonia (suppressed by $N_c^{-\frac{1}{2}}$)