Newton-Cartan geometry for Lifshitz holography

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Based on work with Roel Andringa, Eric Bergshoeff, Ergin Sezgin
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+ work with Eric Bergshoeff, Jelle Hartong (arXiv:1409.5555)

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  1. Show how Newton-Cartan geometry makes its appearance in Lifshitz holography
  2. Give bottom-up construction of this geometry, that elucidates certain aspects of Lifshitz holography

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  1. Temporal metric of rank 1 : $\tau_{\mu\nu} = \tau_\mu \tau_\nu$
  2. Spatial metric of rank 3 (d) : $h^{\mu\nu}$, with $h^{\mu\nu}\tau_\nu = 0$

Newton-Cartan Geometry

- A connection can be introduced via metric compatibility and $\Gamma^{\rho}_{[\mu\nu]} = 0$:

$$\nabla_\mu \tau_\nu = 0\,, \qquad \nabla_\mu h^{\nu\rho} = 0\,,$$
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- Vielbein formulation appears in modern applications. E.g. in Lifshitz holography.

Lifshitz holography

- Lifshitz holography attempts to describe strongly coupled field theories invariant under space-time translations, spatial rotations and non-relativistic dilatations:

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- Typical models consist of EH + Λ + massive vector.

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- Asymptotic behaviour of metric that is asymptotically Lifshitz (Ross;Chemissany, Geissbühler, Hartong, Rollier; Christensen, Hartong, Obers, Rollier):

$$ds^2 = e^{\Phi}\frac{dr^2}{r^2} + \left(-e_\mu{}^0 e_\nu{}^0 + e_\mu{}^a e_\nu{}^a\right) dx^\mu dx^\nu \,,$$

with

$$e^0 = r^{-z}\tau_{(0)\mu}(x)dx^\mu + \cdots \,, \quad e^a = r^{-1}e_{(0)\mu}{}^a(x)dx^\mu + \cdots \,.$$

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- These 'boundary data' play the role of sources for operators in dual field theory.

Newton-Cartan in holography

- $\tau_{(0)\mu\nu} = \tau_{(0)\mu}\tau_{(0)\nu}$, $h_{(0)\mu\nu} = e_{(0)\mu}{}^{a} e_{(0)\nu}{}^{b} \delta_{ab}$ hint at Newton-Cartan structure.

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- Vielbein formulation. Can this be understood/constructed in a more intrinsic way? What are the local symmetries?
- Note that $\tau_{(0)\mu}$, $e_{(0)\mu}{}^{a}$ are only determined up to anisotropic dilatations.

$$\delta\tau_{(0)\mu} = z\Lambda_D(x)\tau_{(0)\mu}\,, \qquad \delta e_{(0)\mu}{}^{a} = \Lambda_D(x) e_{(0)\mu}{}^{a}\,.$$

  Suggests looking for a conformal, non-relativistic space-time symmetry algebra.

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- Gauging = associating gauge fields, transformation rules and covariant curvatures to all generators.

symmetry	generators	gauge field	parameters	curvatures
time translations	H	τ_μ	$\zeta(x^\nu)$	$R_{\mu\nu}(H)$
space translations	P^a	$e_\mu{}^a$	$\zeta^a(x^\nu)$	$R_{\mu\nu}{}^a(P)$
boosts	G^a	$\omega_\mu{}^a$	$\lambda^a(x^\nu)$	$R_{\mu\nu}{}^a(G)$
spatial rotations	J^{ab}	$\omega_\mu{}^{ab}$	$\lambda^{ab}(x^\nu)$	$R_{\mu\nu}{}^{ab}(J)$
central charge transf.	Z	m_μ	$\sigma(x^\nu)$	$R_{\mu\nu}(Z)$
dilatations	D	b_μ	$\Lambda_D(x^\nu)$	$R_{\mu\nu}(D)$
spec. conf. transf.	K	f_μ	$\Lambda_K(x^\nu)$	$R_{\mu\nu}(K)$

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- τ_μ, ${e_\mu}^a$ play role of vielbeins of boundary of asymptotically Lifshitz space-time.

Torsional Newton-Cartan and the Schrödinger algebra

- Other fields can be made dependent via curvature constraints. E.g. for the spin connections:

$$R_{\mu\nu}{}^{a}(P) = 2\partial_{[\mu}e_{\nu]}{}^{a} - 2\omega_{[\mu}^{ab}e_{\nu]b} - 2\omega_{[\mu}{}^{a}\tau_{\nu]} - 2b_{[\mu}e_{\nu]}{}^{a} = 0\,,$$
$$R_{\mu\nu}(Z) = 2\partial_{[\mu}m_{\nu]} - 2\omega_{[\mu}{}^{a}e_{\nu]a} = 0\,.$$

Allow one to solve $\omega_{\mu}{}^{ab}$ and $\omega_{\mu}{}^{a}$ in terms of τ_{μ}, $e_{\mu}{}^{a}$, m_{μ}. A similar story holds for f_{μ}. Note necessity of m_{μ}.

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- What about b_μ? The time-like component is pure gauge:

$$\delta_K b_\mu = \Lambda_K \tau_\mu\,.$$

  The spatial components are found as solutions of the constraint

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- Note that now

$$\partial_{[\mu}\tau_{\nu]} \neq 0 \qquad \text{but} \qquad \tau_{[\mu}\partial_{\nu}\tau_{\rho]} = 0 \qquad \Rightarrow \qquad \text{torsion}\,.$$

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- Useful for calculations of e.g. boundary stress-tensor. Guideline in setting up holographic dictionaries.