Hadron spectroscopy from lattice QCD: progress and prospects

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Plan

- Introduction and background
  - A consumers guide to Lattice QCD
    - brief discussion of discretisation and calculating correlators
    - practicalities (aka compromises) and consequences
- Discussion and selected results
  - parallel tracks for progress
  - precision spectroscopy of single hadron states including excited and exotic states
  - spectroscopy of scattering states - progress and challenges
- Summary
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Many details and topics omitted for time constraints - APOLOGIES!
A LATTICE QCD PRIMER

Start from the QCD Lagrangian:

\[ \mathcal{L} = \bar{\Psi} (i \gamma^\mu D_\mu - m) \Psi - \frac{1}{4} G_\mu^a G^{\mu \nu}_a \]

Gluon fields on links of a hypercube;
Quark fields on sites: approaches to fermion discretisation -
Wilson, Staggered, Overlap.;
Derivatives \( \rightarrow \) finite differences.

Solve the QCD path integral on a finite lattice with spacing \( a \neq 0 \) estimated stochastically
by Monte Carlo. Can only be done effectively in a Euclidean space-time metric (no useful
importance sampling weight for the theory in Minkowski space).

Observables determined from (Euclidean) path integrals of the QCD action

\[ \langle O \rangle = \frac{1}{VZ} \int DUD\bar{\Psi}D\Psi O[U, \bar{\Psi}, \Psi] e^{-S[U, \bar{\Psi}, \Psi]} \]
A RECIPE FOR (MESON) SPECTROSCOPY

- Construct a basis of local and non-local operators $\bar{\Psi}(x)\Gamma D_i D_j \ldots \Psi(x)$ from distilled fields [PRD80 (2009) 054506].

- Build a correlation matrix of two-point functions

$$C_{ij} = \langle 0 | \mathcal{O}_i \mathcal{O}_j^\dagger | 0 \rangle = \sum_n \frac{Z^n_i Z^n_j}{2E_n} e^{-E_nt}$$

- Ground state mass from fits to $e^{-E_nt}$

- Beyond ground state: Solve generalised eigenvalue problem

$$C_{ij}(t) v_j^{(n)} = \lambda^{(n)}(t) C_{ij}(t_0) v_j^{(n)}$$

- Eigenvalues: $\lambda^{(n)}(t) \sim e^{-E_nt} \left[ 1 + O(e^{-\Delta Et}) \right]$ - principal correlator yields energies

- Eigenvectors: related to overlaps $Z_i^{(n)} = \sqrt{2E_n e^{E_nt_0}/2} v_j^{(n)\dagger} C_{ji}(t_0)$ - used in spin id.
Practicalities of a lattice calculation

for spectroscopy
1. Working in a finite box at finite grid spacing

- Identify region where physics doesn’t change with $a$ or $V$ or recover continuum QCD by extrapolation.

Costly but a regular feature in lattice calculations now.
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2. Simulating at physical quark masses

- Computational cost grows rapidly with decreasing quark mass: $m_q = m_{u,d}$ costly. Care needed vis location of decay thresholds and identification of resonances.

- Heavy quarks need special care: $c$-quark relativistically; $b$-quark with EFTs.

Physical light quark simulations possible. Heavy quark systematics understood.
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Use lattice simulations to study quark mass dependence!
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- Traditional point-to-all algorithms for quark propagation don’t allow for all Wick contractions.
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- Lorentz symmetry broken at $a \neq 0$ so $SO(4)$ rotation group broken to discrete rotation group of a hypercube. Identify states according to this symmetry.

Spin identification at finite lattice spacing: 0707.4162, 1204.5425
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5. Working in Euclidean time.

- Gives access to energies via \( C(t) \sim e^{-E_n t} \)
- Scattering matrix elements not directly accessible from Euclidean QFT [Maiani-Testa theorem].

Lüscher formalism and generalisations allow indirect access.
New (and not so new) Ideas for Old Problems

- **Anisotropic lattices**
  - $a_t \ll a_s$: improving resolution for better measurement.

- **Distillation**
  - breakthrough idea for quark propagation enabling precision spectroscopy including for isoscalars and exotics.

J. Bulava’s talk yesterday

- **Operator construction & spin id**
  - allows for robust spin assignment at finite lattice spacing and for high spins.

M. Hansen’s talk yesterday

- **Extension of scattering methodology to coupled channels**
  - Lüscher’s idea from ’90s extended to many scenarios enabling resonance/scattering parameters.
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Together these ideas have led to rapid recent progress
Lattice Hadron Spectroscopy
precision & pioneering results
SPECTROSCOPY: TWO STRATEGIES FOR PROGRESS

Below thresholds: stable “gold-plated” quantities characterised by

- Using well-tried and tested and validated methods.
- High statistics and improved actions for precise results
- Different actions in agreement.
- Most/All systematic errors accounted for e.g. continuum extrapolation, finite volume, physical light quark masses, Nf, EM effects.

Beautiful examples in the light sector - [C. Alexandrou talk]

Above thresholds: calculation of high spin & exotic states relatively new

- Physics of molecular/multi hadron states needs relevant operators.
- No continuum extrapolation
- Relatively heavy pions ← already changing
- Improvements underway now that methods are proven

Systematics already being addressed for “simpler” resonances - [J. Bulava talk]
Benchmark Spectroscopy:
states below thresholds - charmonium as an example
CHARMONIUM BELOW THRESHOLD - “GOLD-PLATED”

- High statistics, improved actions with different fermions.
- Simulation at/near $m_q^{\text{phys}}$ and multiple $a$.
- Discretisation errors $O(am_c)$ and $O(am_b)$ under control.

No disconnected diagrams in $c\bar{c}$ spectrum: OZI suppressed - assumed to be small
⇒ Mixing with lighter states not included.
Beyond benchmark spectroscopy: excited and exotic states
Spectroscopy of single hadrons: light isoscalar spectrum

Hadron Spectrum Collaboration PRD83 (2011) 111502

Includes all relevant disconnected diagrams, using distillation.
**Single Hadron States: Charmonium**

Precision calculation of high spin \((J \geq 2)\) and exotic states

**Caveat Emptor**

- Only single-hadron operators
- Physics of multi-hadron states appears to need relevant operators
- No continuum extrapolation
- \(m_\pi = 391\text{MeV}\)
- No disconnected contributions to \(\eta_c\)

Repeated With \(m_\pi = 230\text{MeV}\) with no change to pattern of states. JHEP 1612 (2016) 089
**CHARM(ONIUM) WITH $m_\pi \sim 230$ MeV**

No change to the pattern, structure of states. Also for $D(s)$. [JHEP 1612 (2016) 089]
Expect a large overlap with operators $\mathcal{O} \sim F_{\mu \nu}$

**Charmonium**

**Light Baryons**

Hadron Spectrum Collaboration

Lightest hybrid supermultiplet and excited hybrid supermultiplet same pattern and scale in meson and baryon, heavy and light [HadSpec:1106.5515] sectors.
Spectroscopy above decay thresholds
some examples with light and charm states
Scattering in a Euclidean Theory

Recall, lose direct access to scattering in (Euclidean) lattice calculations

- The large Euclidean time limit dominated by threshold or off-shell states. Maiani-Testa, PLB (1990).
- Analytic continuation of numerical (euclidean) correlators is an ill-posed problem.

Lüscher found a way to extract $\pi\pi \rightarrow \pi\pi$ scattering information in the elastic region from $\text{LQCD}$ . [NPB354, 531-578 (1991)]

- Use the finite volume as a tool
- Related lattice energy levels in a finite volume to a decomposition of the scattering amplitude in partial waves in infinite volume

$$\det \left[ \cot \delta(E_n^*) + \cot \phi(E_n, \vec{P}, L) \right] = 0$$

and $\cot \phi$ a known function (containing a generalised zeta function).

- Also known from perturbative NR-QM. Huang-Yang PRD105 (1957)
- To use this idea many technical improvements were needed. Eg need disconnected diagrams and energy levels at extraordinary precision. This is why it has taken a while ...
The method has been generalised for

- moving frames; non-identical particles; multiple two-particle channels, particles with spin


**Coupled channel analyses:**

- Relation to finite volume spectrum for coupled-channel scattering of hadron pairs given by \( \det [1 + i \rho \cdot t \cdot (1 + i \mathcal{M})] = 0 \)
- depends on several unknowns at any given energy
- extract information from *all* energy levels in a region via parameterisation of \( t \) with eg K-matrix.
- Lots of technology to optimise operators makes this work well. [e.g. Dudek et al PRD86 (2012) 034031].
Light mesons

- $\pi\pi$ in $I = 2, 1$ and isoscalar $\pi\pi \rightarrow \sigma$ PRD90 (2014) 099902, PRD92 (2015) 094592, PRL 118 (2017) 022002
- $\pi K - \eta K$ PRD91 (2015) 054008
- $\pi\eta - K\bar{K} - \pi\eta'$ $\rightarrow a_0$ PRD93 (2016) 094506
- Radiative transitions including resonant $\rho \rightarrow \pi\gamma^*$ PRL115 (2015) 242001, PRD93 (2016) 114508

Charm mesons

- $D\pi - D\eta - D_s\bar{K}$ JHEP1610 (2016) 011
- Preliminary studies of radiative transitions (PoS Lattice2016) and tetraquarks.
\( \pi\pi \) in P-wave, \( I=1 \)

coupled \( \pi\pi, K\bar{K} \) scattering in P-wave, PRD92 (2015) 094592

- Includes coupled channel \( \pi\pi, K\bar{K} \) at \( m_\pi = 236 \text{MeV} \).
- \( m_R = 790(2) \text{MeV}; g_R = 5.688(70)(26) \)
Isoscalar $\pi\pi$ scattering and the $\sigma$

includes all Wick contractions [enabled by distillation]
\[ D\pi - D\eta - D_s\bar{K} \text{ SCATTERING} \]

- Combined S & P wave analysis; 3 coupled channels in S-wave; 47 energy levels from 3 volumes, \( m_\pi = 391 \text{ MeV} \).
- Find a bound state just below threshold: \( m = (2275.9 \pm 0.9) \text{ MeV} \).
- \( D\pi \) threshold is \( (2276.4 \pm 0.9) \text{ MeV} \) and cf \( D^*_0 (2400) \).
- In P-wave \( (1^-) \) a deeply bound pole - cf \( D^*_0 (2007) \).
- In D-wave \( (2^+) \) a narrow resonance - cf \( D^*_2 (2460) \).
- Repeating at lighter pion mass.
Tackling the XYZs
X(3872) - A FIRST LOOK (NO COUPLED CHANNELS)

Prelovsek & Leskovec 1307.5172

Padmanath, Lang, Prelovsek 1503.03257

ground state: $\chi_{c1}(1P)$

$D\bar{D}^*$ scattering mix: pole just below threshold

Location of threshold; finite vol effects controlled?

Also results from Lee et al 1411.1389

Within 1MeV of $D^0\bar{D}^{0*}$ and 8MeV of $D^+\bar{D}^*$ thresholds: isospin breaking effects important?

X(3872) not found if $c\bar{c}$ not in basis.
**$Z_c^+$ - First look on the lattice (no coupled channels)**

Manifestly exotic hadron i.e. does not fit in the quark model picture.

**Prelovsek, Lang, Leskovec, Mohler: 1405.7615**

- 13 expected 2-meson e’states found (black)
- no additional state below 4.2GeV
- no $Z_c^+$ candidate below 4.2GeV
- an appropriate basis is crucial!

Similar conclusion from Lee et al [1411.1389] and Chen et al [1403.1318]

Why no eigenstate for $Z_c$? Is $Z_c^+$ a coupled channel effect? Work needed!
Recent (preliminary) work - charm tetraquarks

\[ I^G(J^{PC}) = 1^+ (1^+ -) \]: No candidate state for \( Z_c^+ \) found.

\[ \eta_{c\bar{c}} \] only tetraquark ops
\[ J/\psi \pi \] only meson-meson ops
\[ \eta_{c\bar{c}} \] full basis of ops

from G.Cheung, 
HadSpec

Charged \( X(3872) \) in \( I = 1, \bar{c}c\bar{d}u \)?
No candidate state seen. **BUT** Isospin breaking and unstable \( \rho \) not included.
**Other recent work: \( Z_c^+ \)**

A coupled-channel analysis by HAL QCD[1602.03465].

Challenges:
- The \( Z_c^+ \) (as other XYZ states) lies above several thresholds and so decays to several two-meson final states
- Requires a coupled-channel analysis for a rigorous treatment
- On lattice the number of relevant coupled-channels is large for high energies.

\[ \pi J/\psi - \rho \eta_c - \bar{D}D^* \]

- Potential method, not robustly tested.
- Suggest \( Z_c^+ \) a threshold cusp.

Other lattice results are needed to test this result.
Summary & Outlook

- Lattice QCD provides a robust formalism for calculating non-perturbative phenomena.
- New ideas are allowing rapid progress for the spectroscopy of resonances.
- Precision lattice calculations of excited and exotic hadron states available
  - includes hybrids and other exotics treated as “single-hadron” states.
- Studies of resonances, including multiple two-particle channels underway.
- Expect significant progress in next few years in e.g. X,Y,Z but there are particular challenges in quarkonia
  - heavy quark discretisation
  - a proliferation of thresholds and coupled-channels
  - including disconnected contributions in $c\bar{c}$ still technically difficult.
- Numerically precision achievable, control of other “traditional” systematics will be a challenge.
- Many theoretical challenges remain e.g. three-particle scattering.
- The $B$ and $\Upsilon$ systems are relatively unexplored.
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Thanks for listening!
A disconnected aside...

- for precision spectroscopy of low-lying states: verify size of disconnected diagrams (OZI suppressed in charm)
- what is the contribution for higher-lying (exotic) states?

Number of techniques on the market. Distillation works very well: tractable and statistically precise.

The correlator is

\[
\begin{pmatrix}
C_{ll} - 2D_{ll} & -\sqrt{2}D_{ls} & -\sqrt{2}D_{lc} \\
-\sqrt{2}D_{sl} & C_{ss} - D_{ss} & -\sqrt{2}D_{sc} \\
-\sqrt{2}D_{cl} & -D_{cs} & C_{cc} - D_{cc}
\end{pmatrix}
\]
\[ \Delta(1^{--}) = -17(16)\text{MeV} \]