Gluon confinement

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Berlin, Germany

Overview

- Why can gluons not be observed?

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- Possible mechanisms

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- Predictions based on the mechanisms

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- Predictions based on the mechanisms
- Testing the predictions
- Summary

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  - Much harder measurement than for quarks
  - Less stringent limits

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  - Much harder measurement than for quarks
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- Other confinement mechanism?

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  - Sufficient to show in one gauge

Positivity violation for the gluon propagator

- Landau gauge

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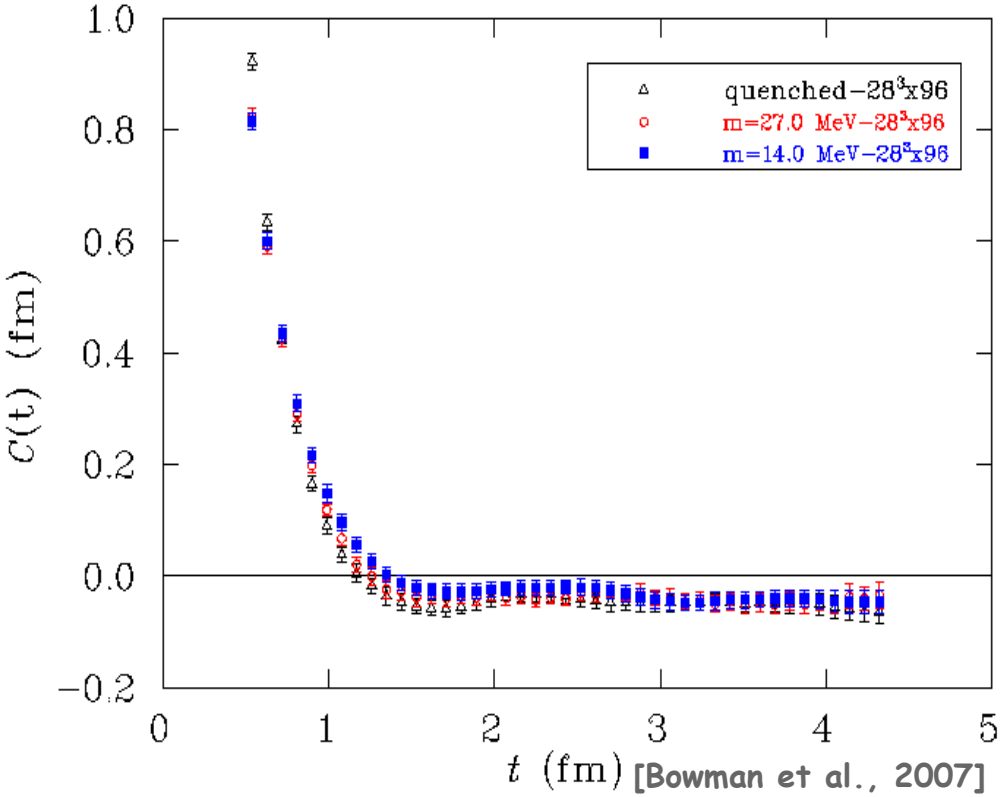

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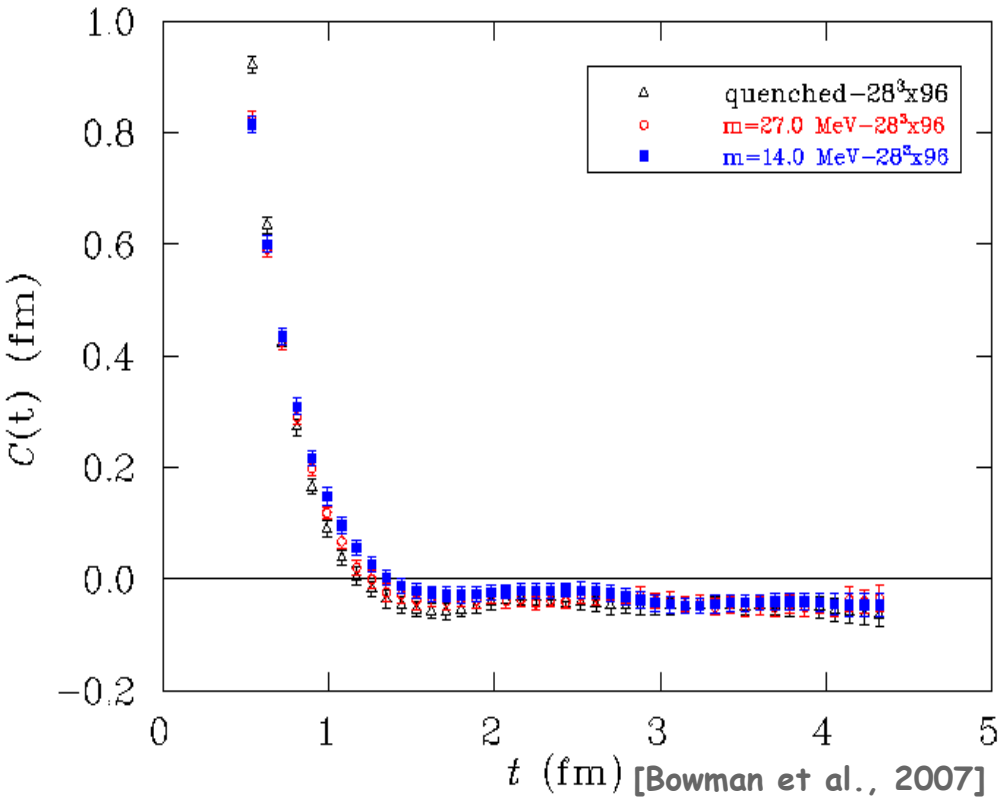

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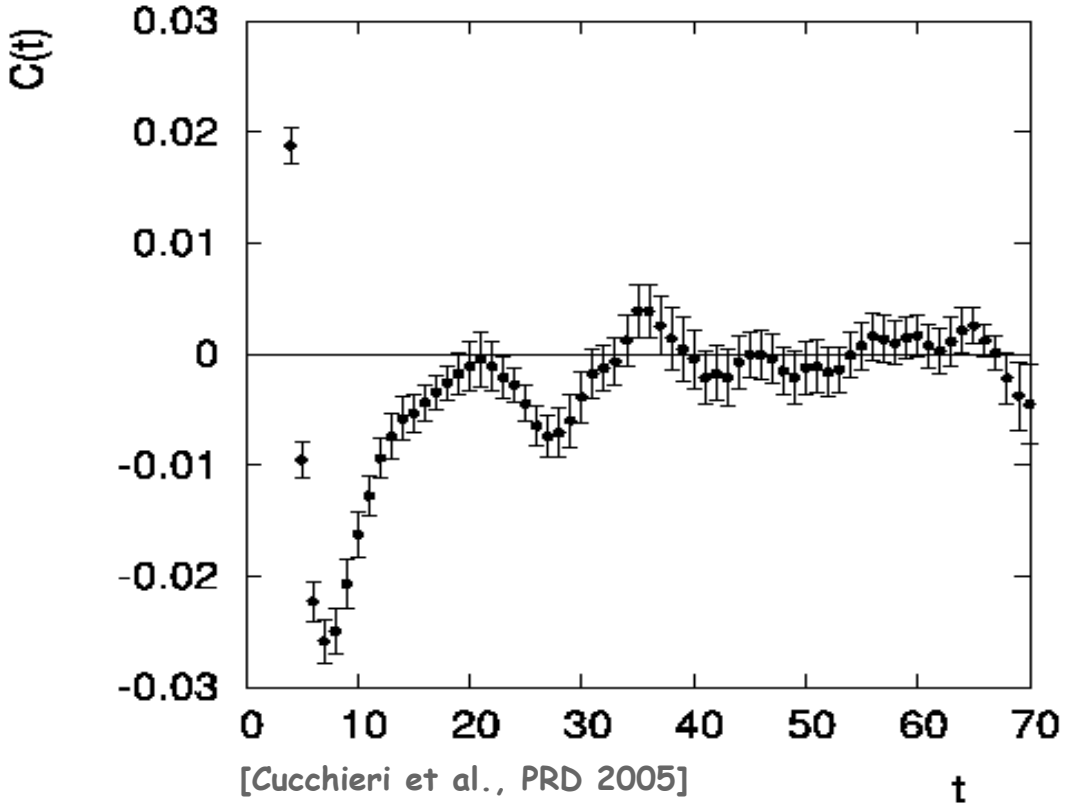

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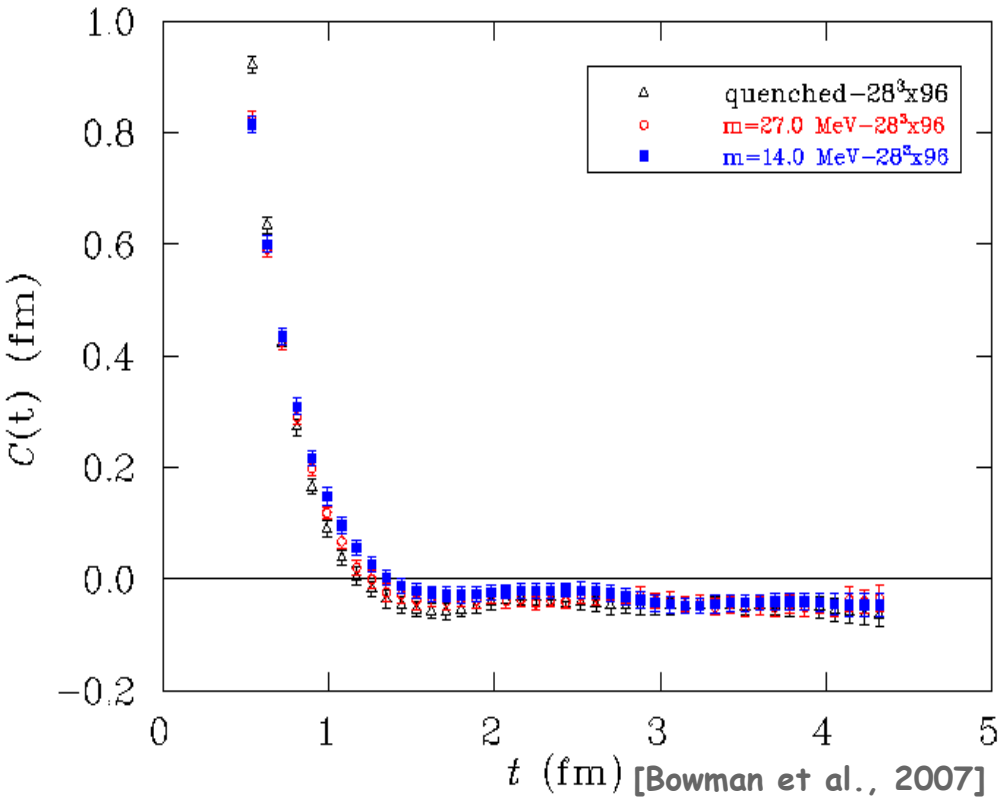

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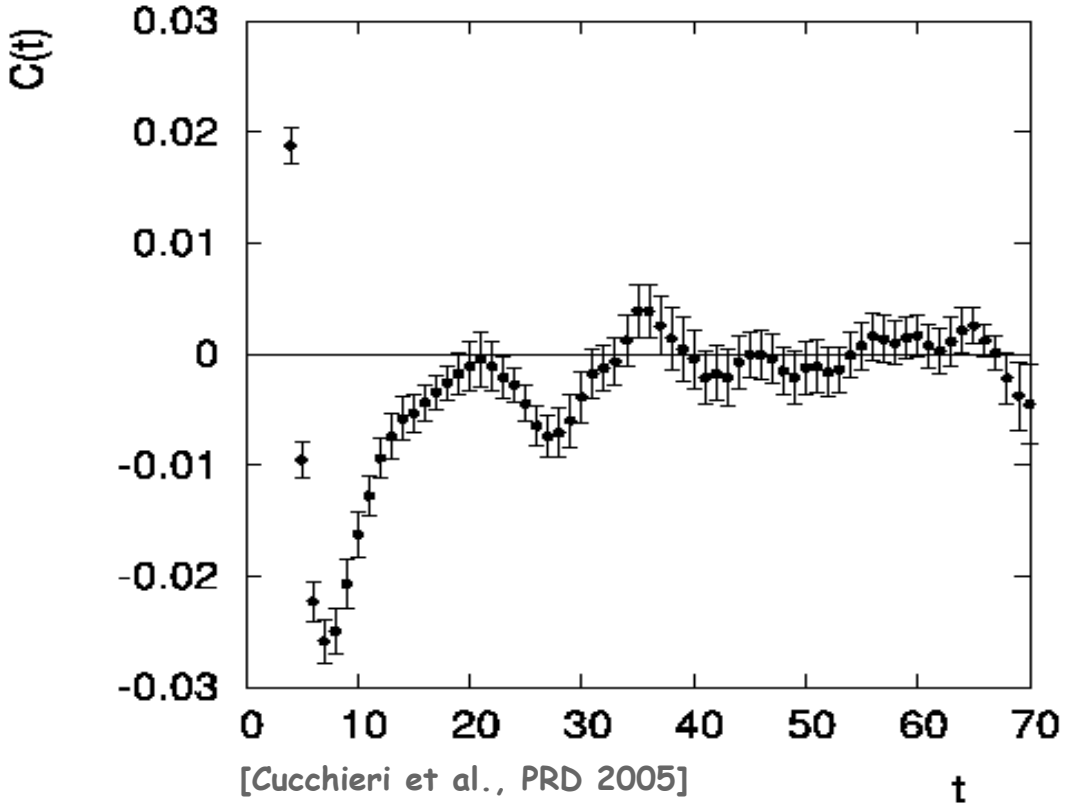

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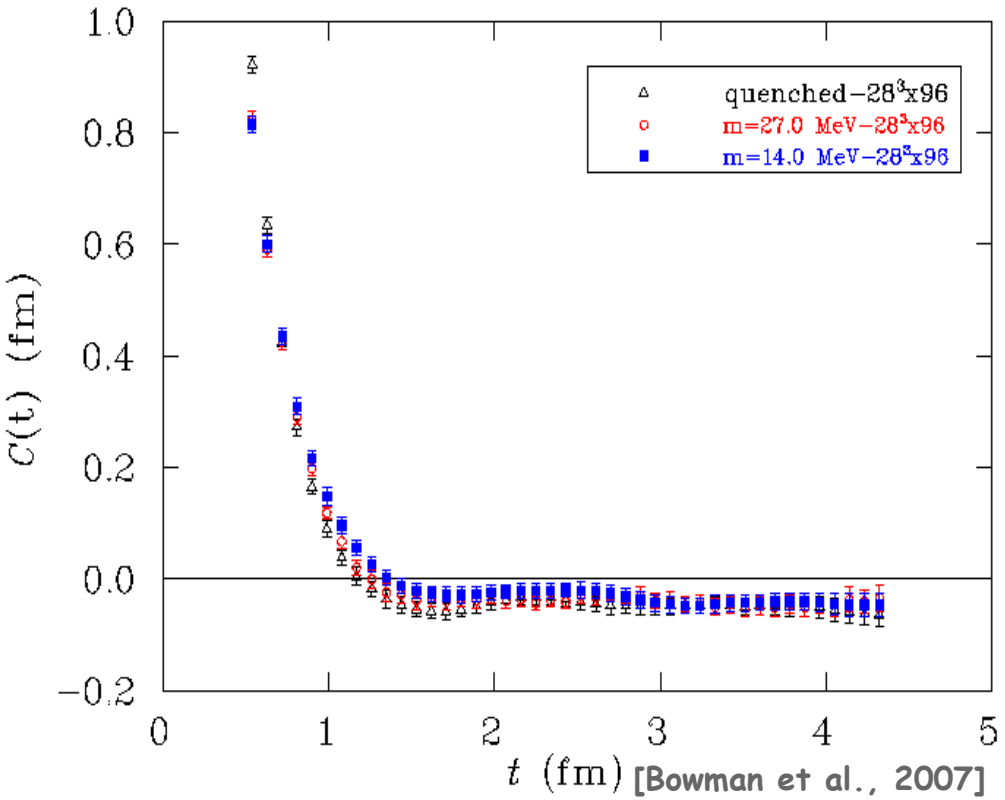

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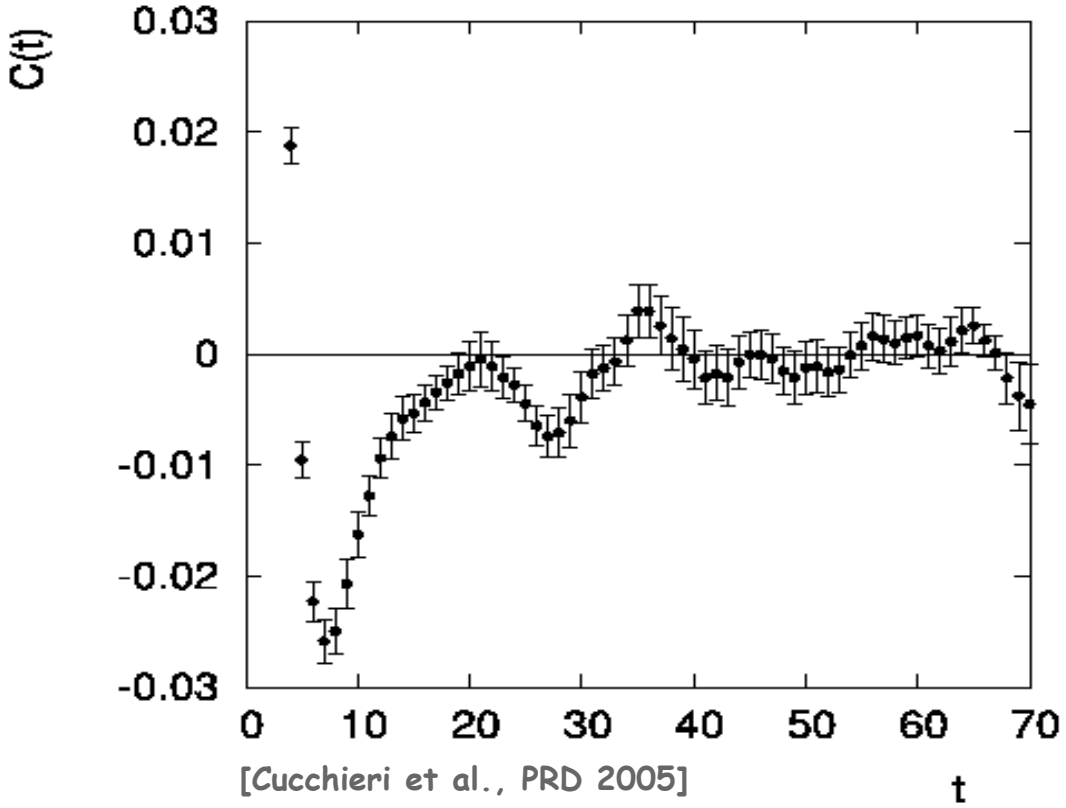

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- Landau gauge
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- Unquenching does not change anything
- Clear positivity violation – gluons are not physical particles

Spectral functions

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$$Propagator = One\ particle\ part + \int dq^2 \frac{spectral\ function(q^2)}{p^2+q^2}$$

  - Sum rule for gluons
    - Oehme-Zimmermann superconvergence relation

$$Overlap\ with\ one\ particle + \int dq^2\ spectral\ function(q^2) = \frac{1}{Z_3} = 0$$

    - Z_3 (divergent) renormalization constant – violation already in perturbation theory

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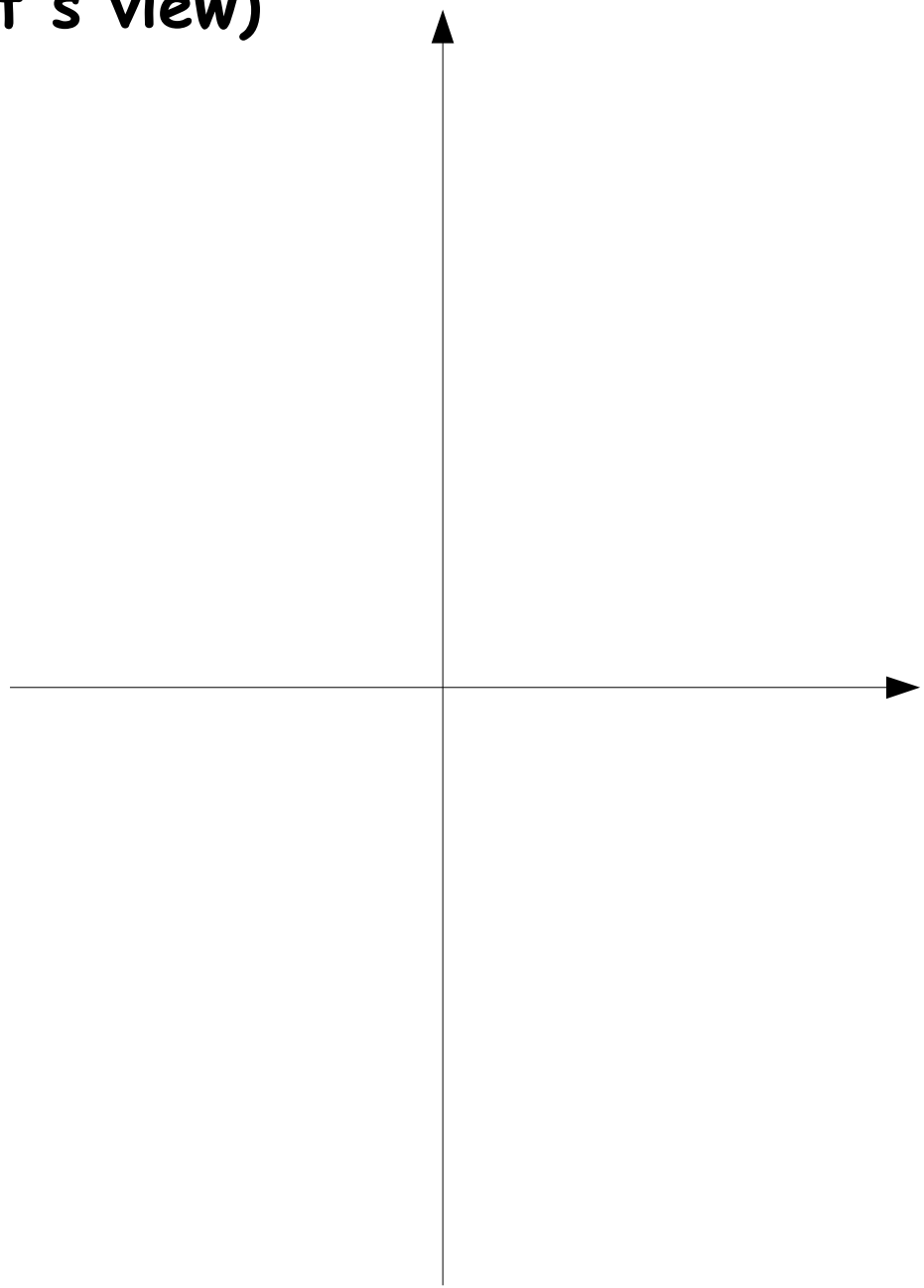
- Mechanism to connect field configurations and their consequences: Gribov-Zwanziger mechanism
  - Similar, but different basis: Kugo-Ojima scenario
    - Less investigated

Configuration space (artist's view)



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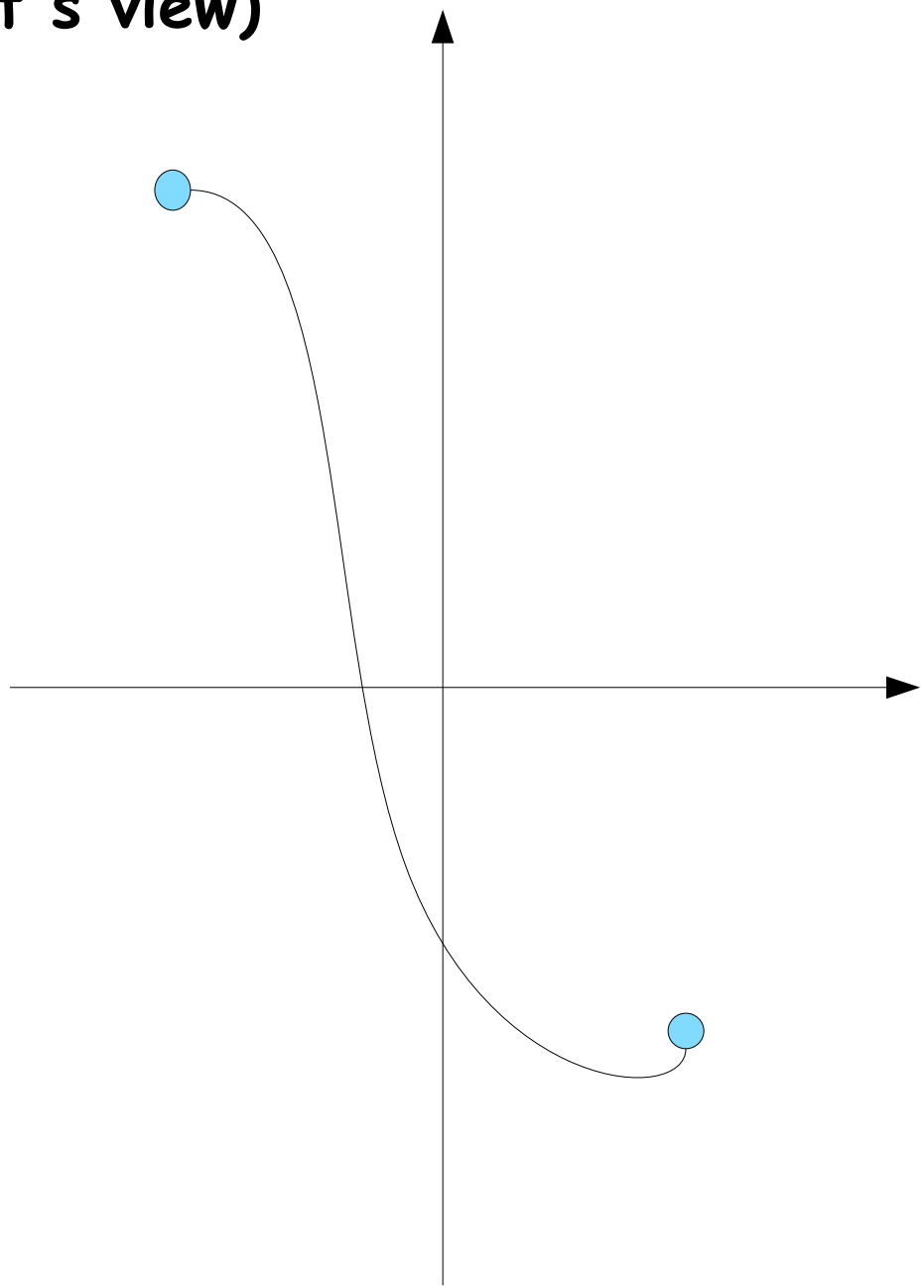

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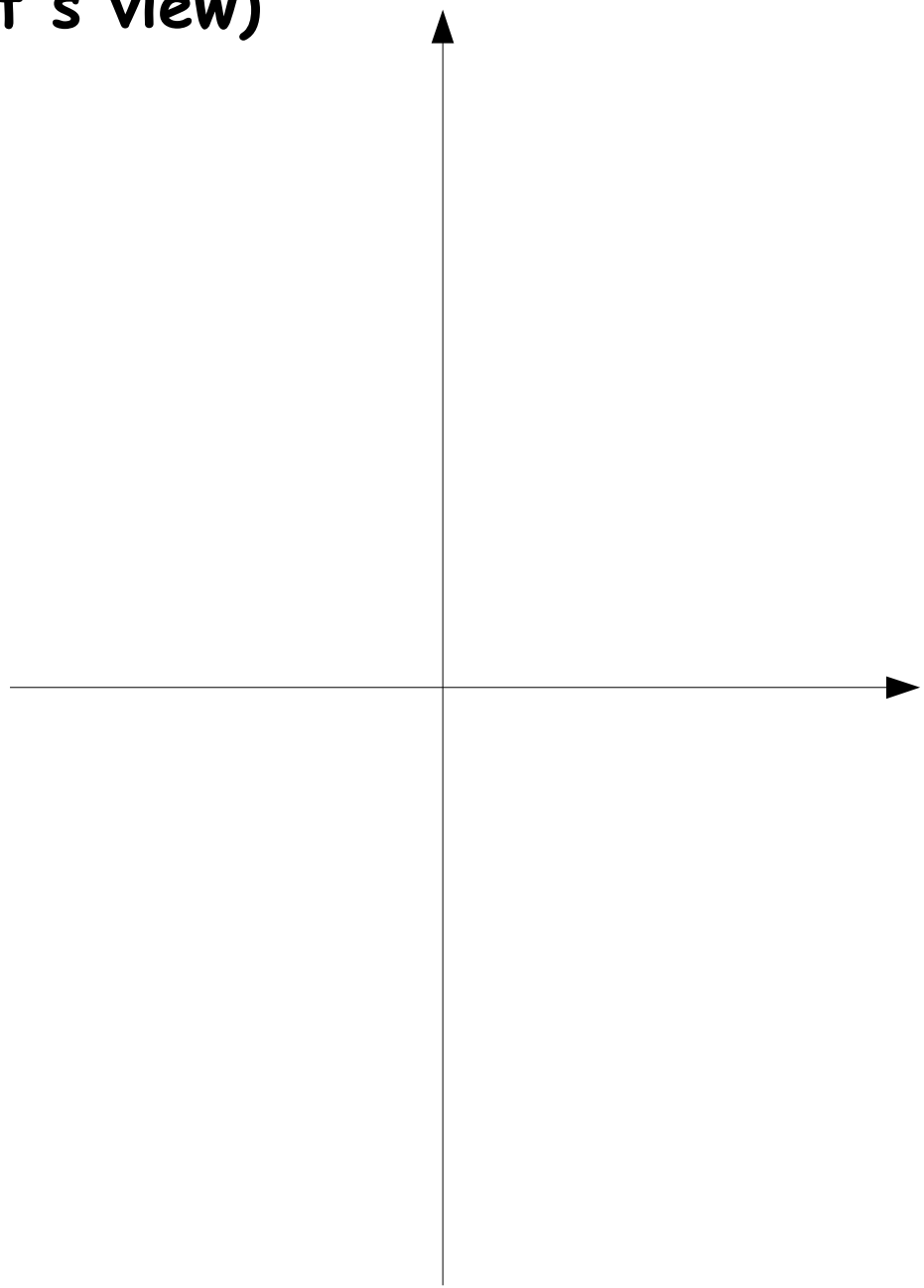

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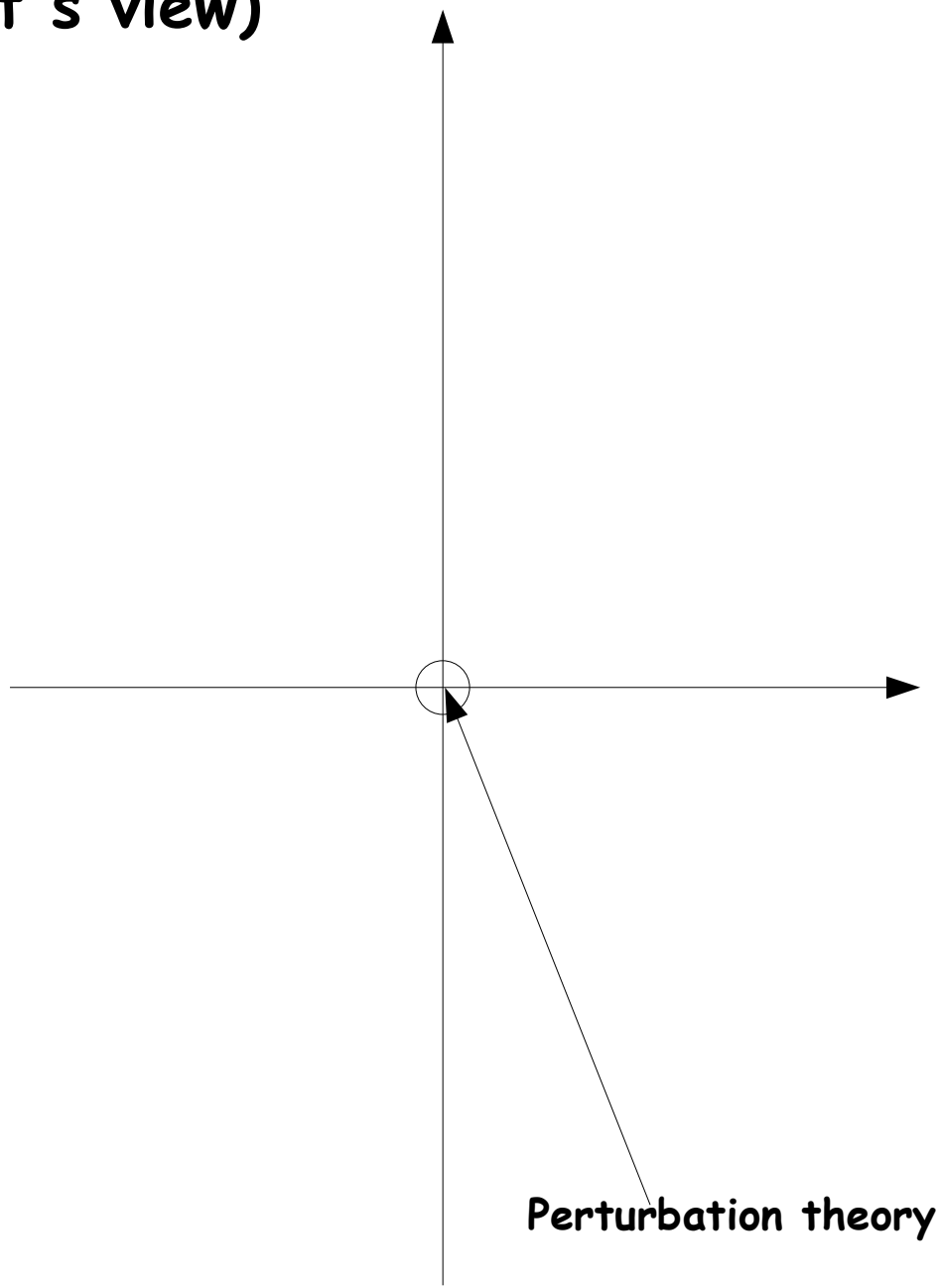

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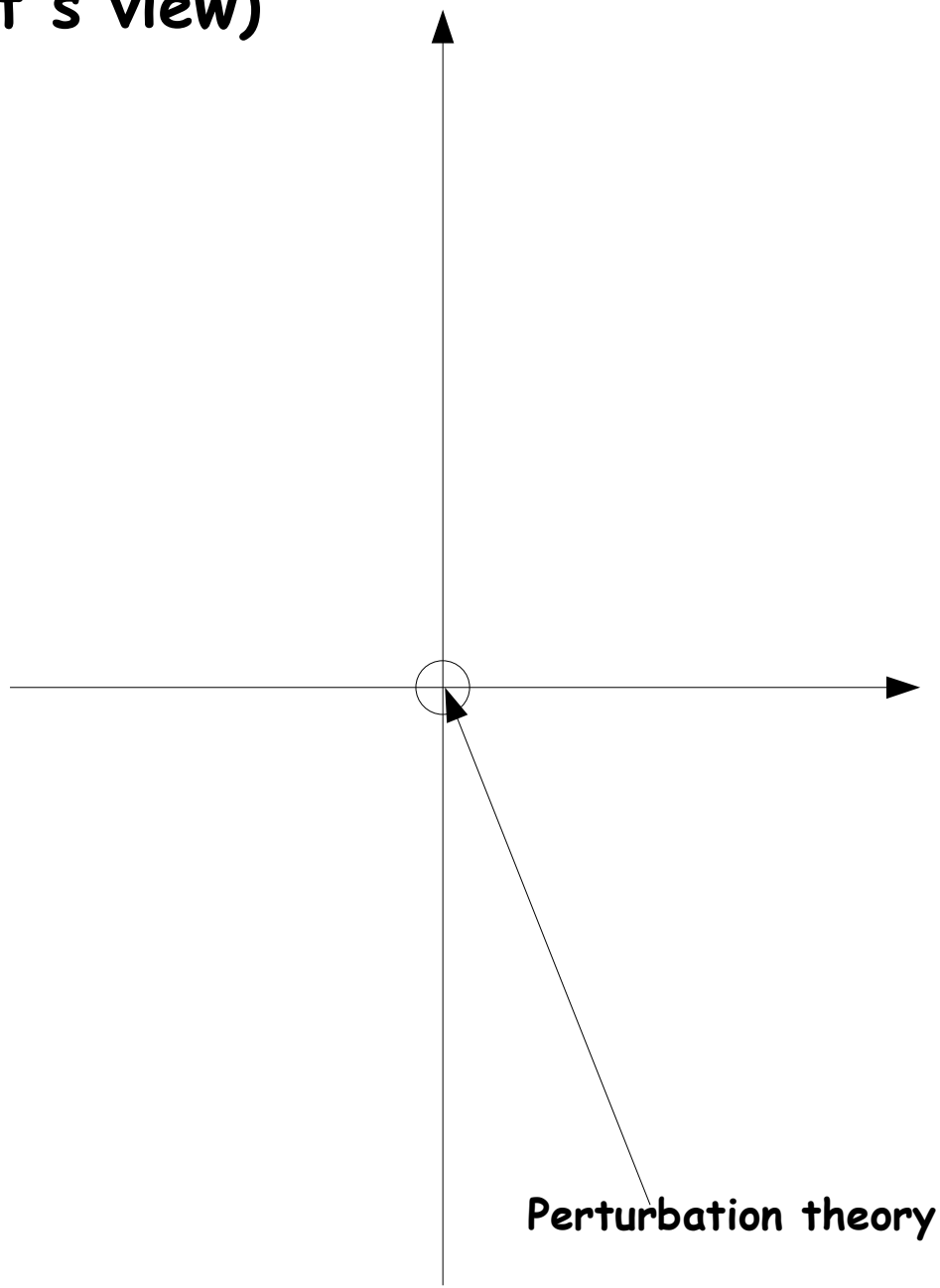

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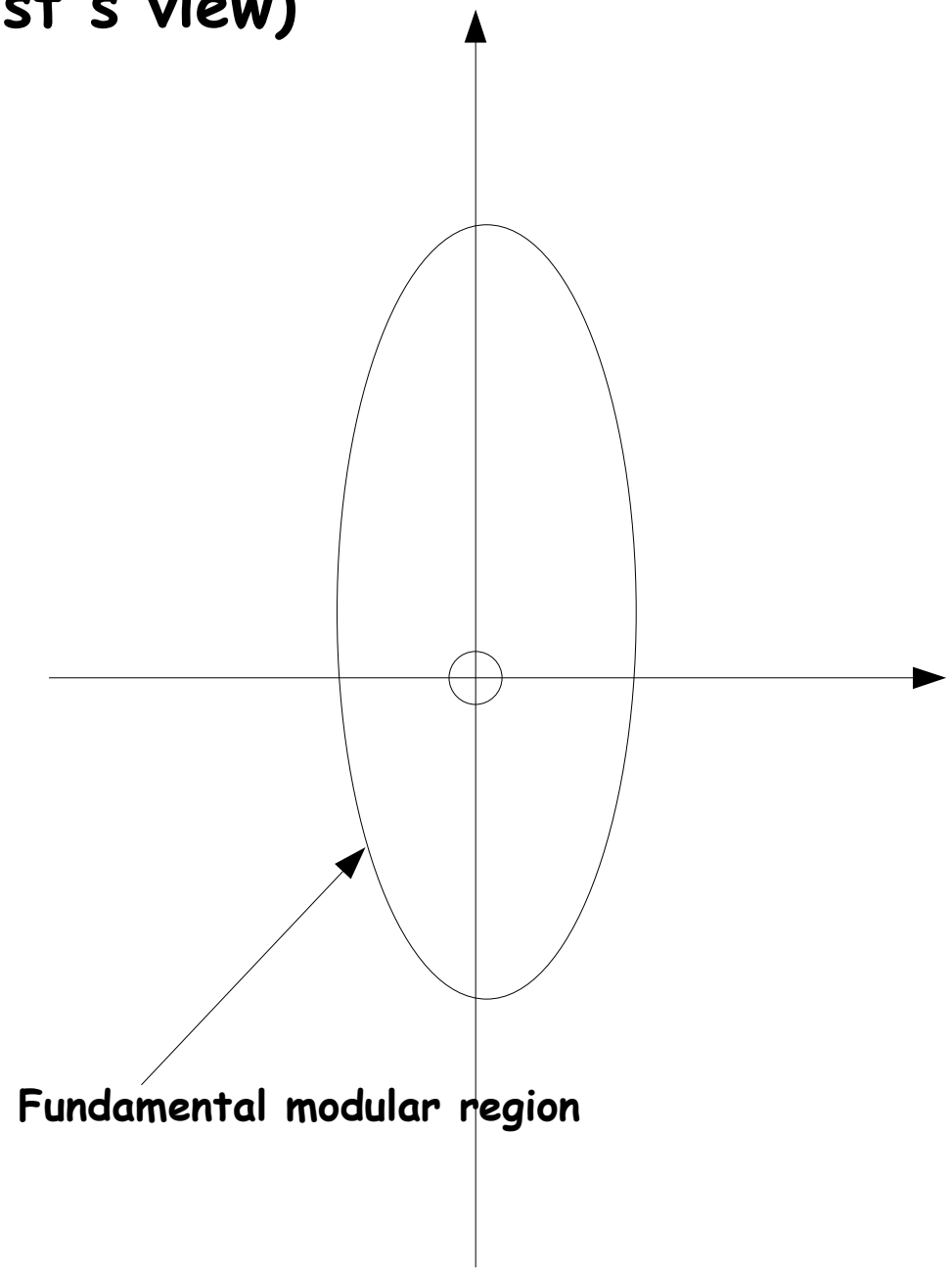

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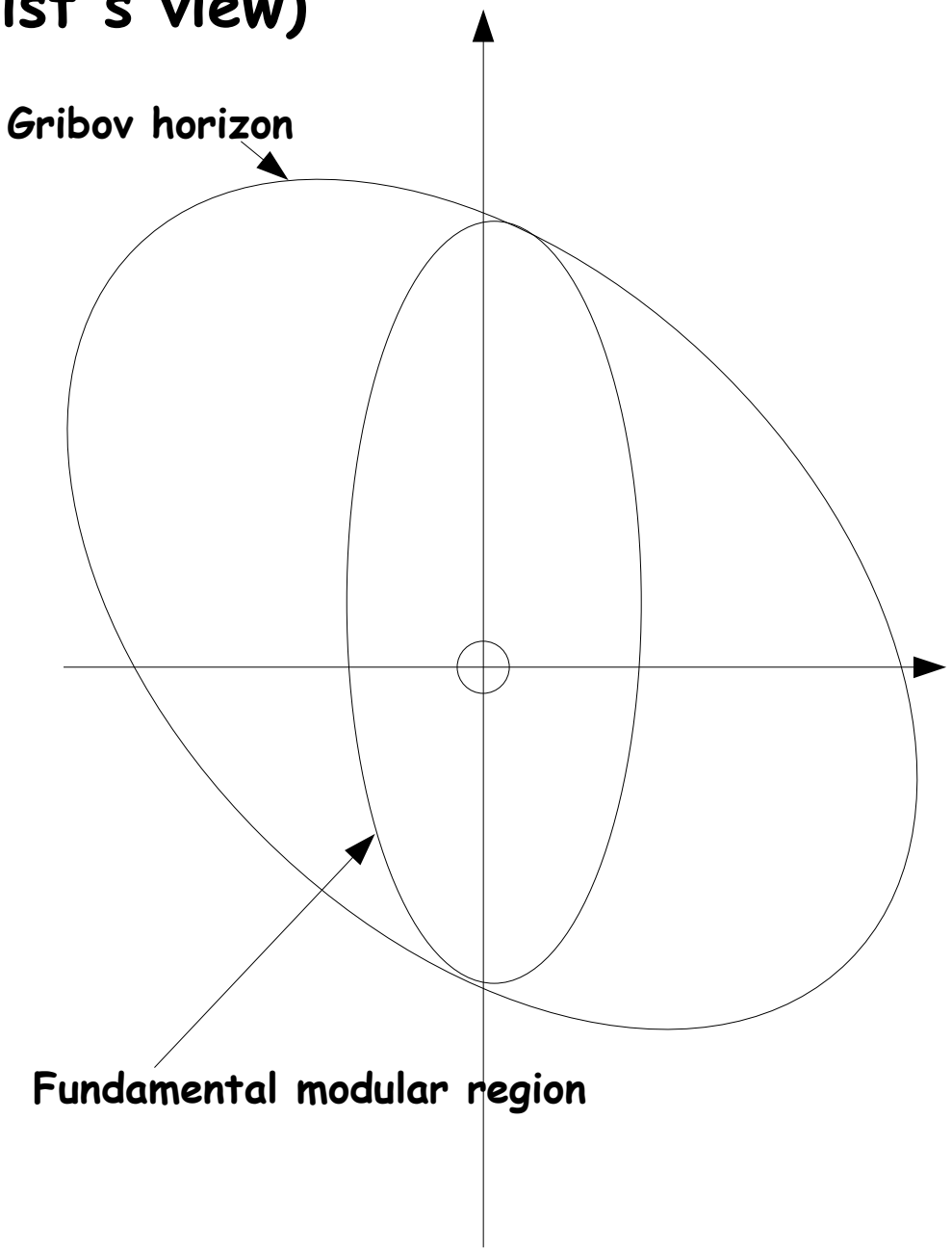

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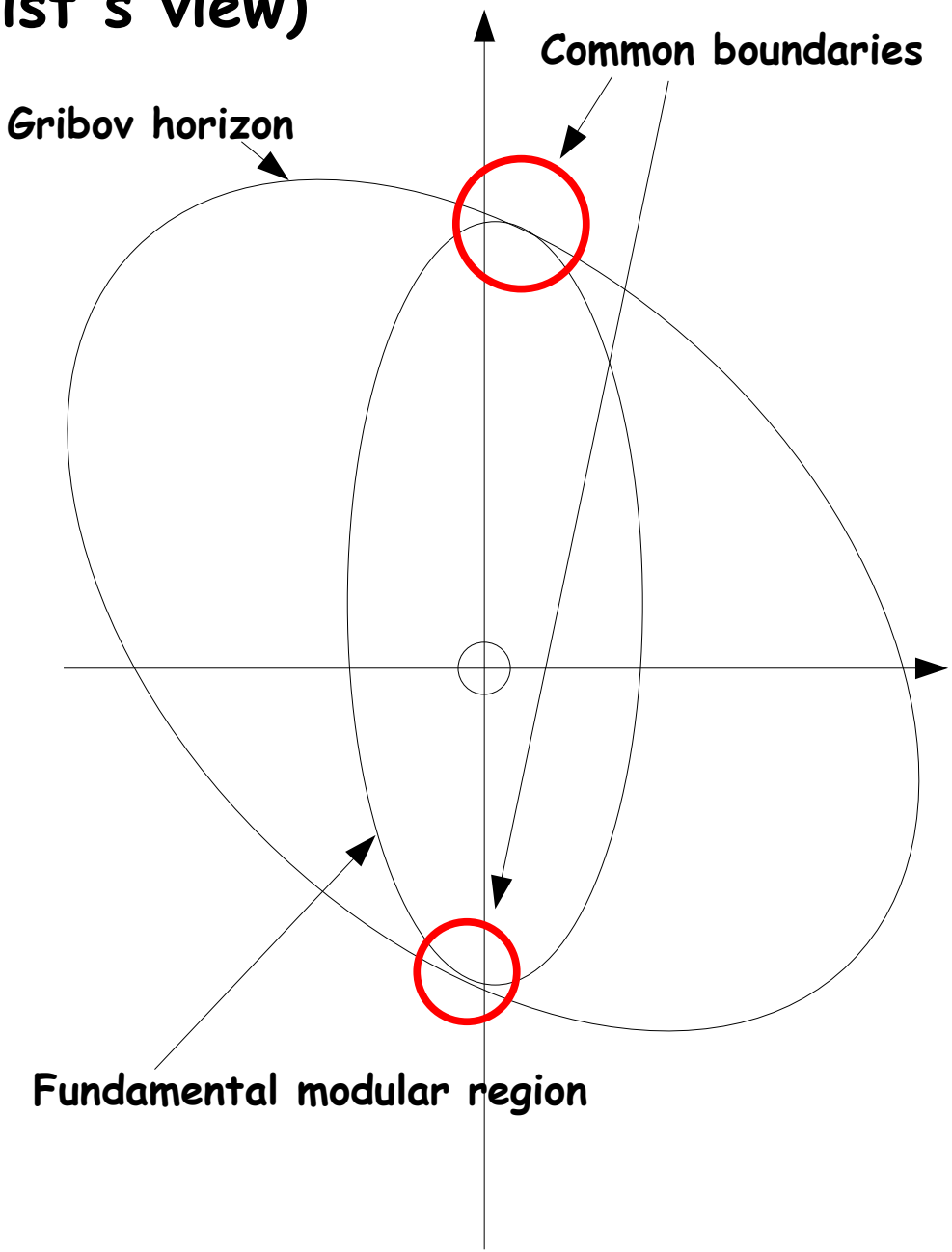

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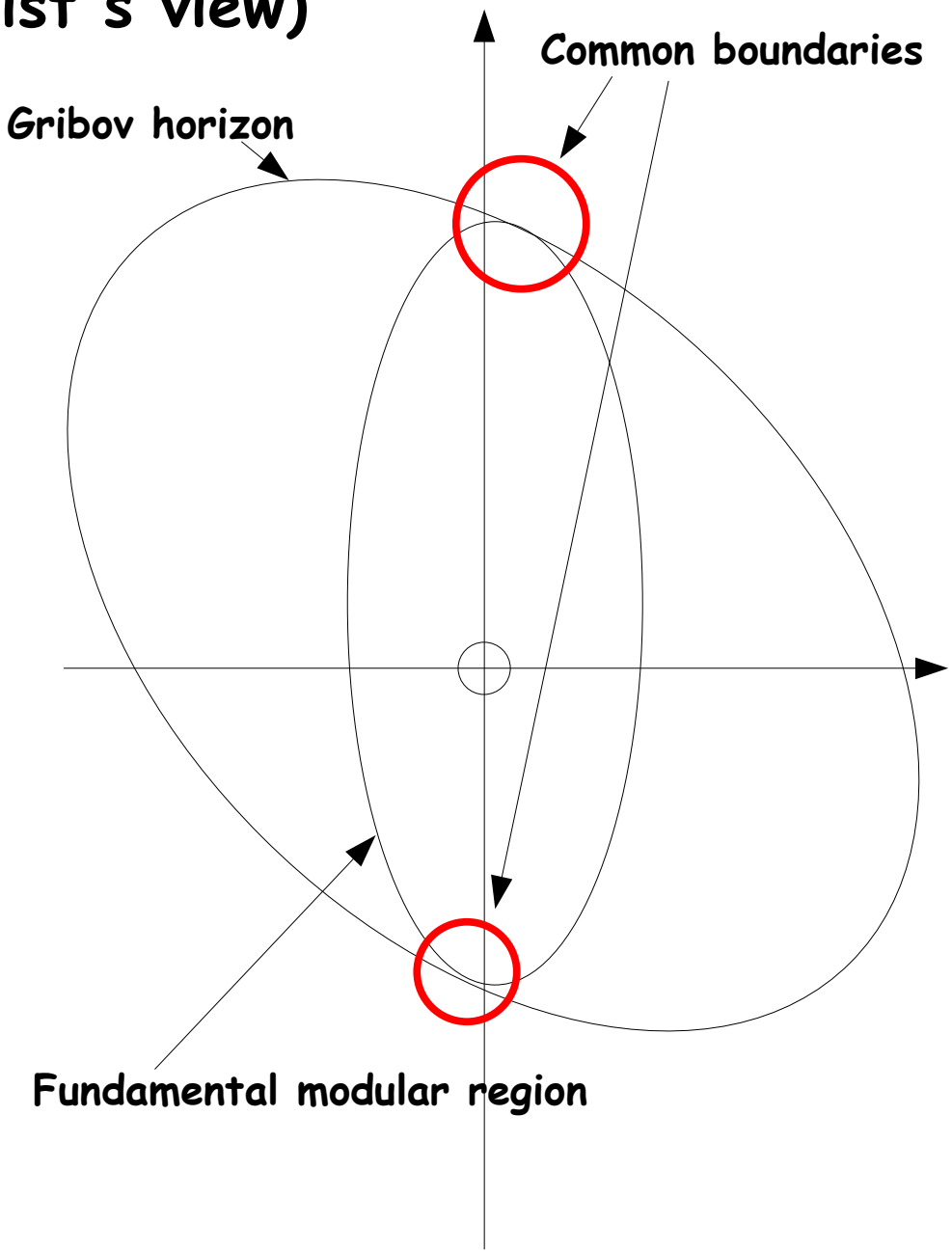

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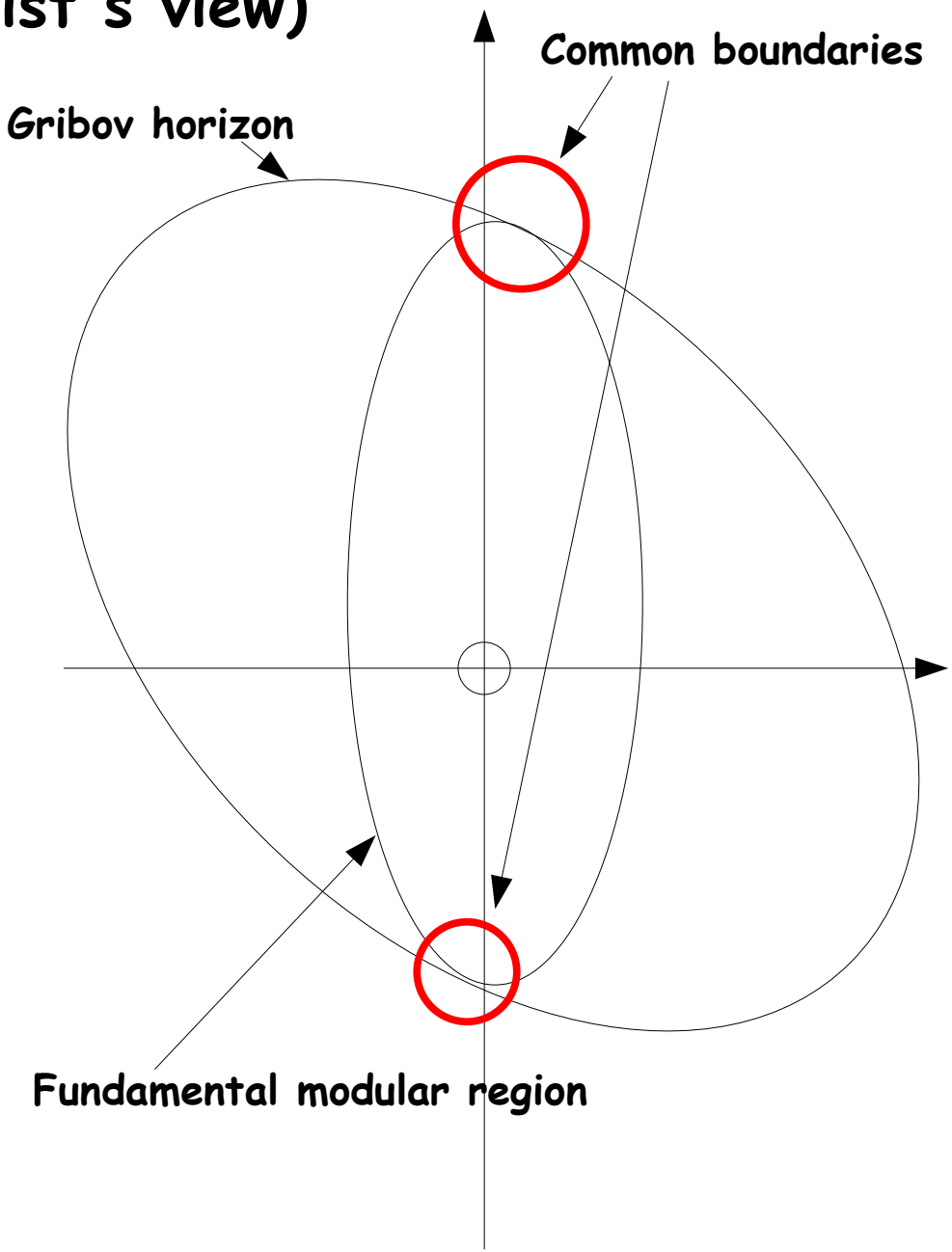

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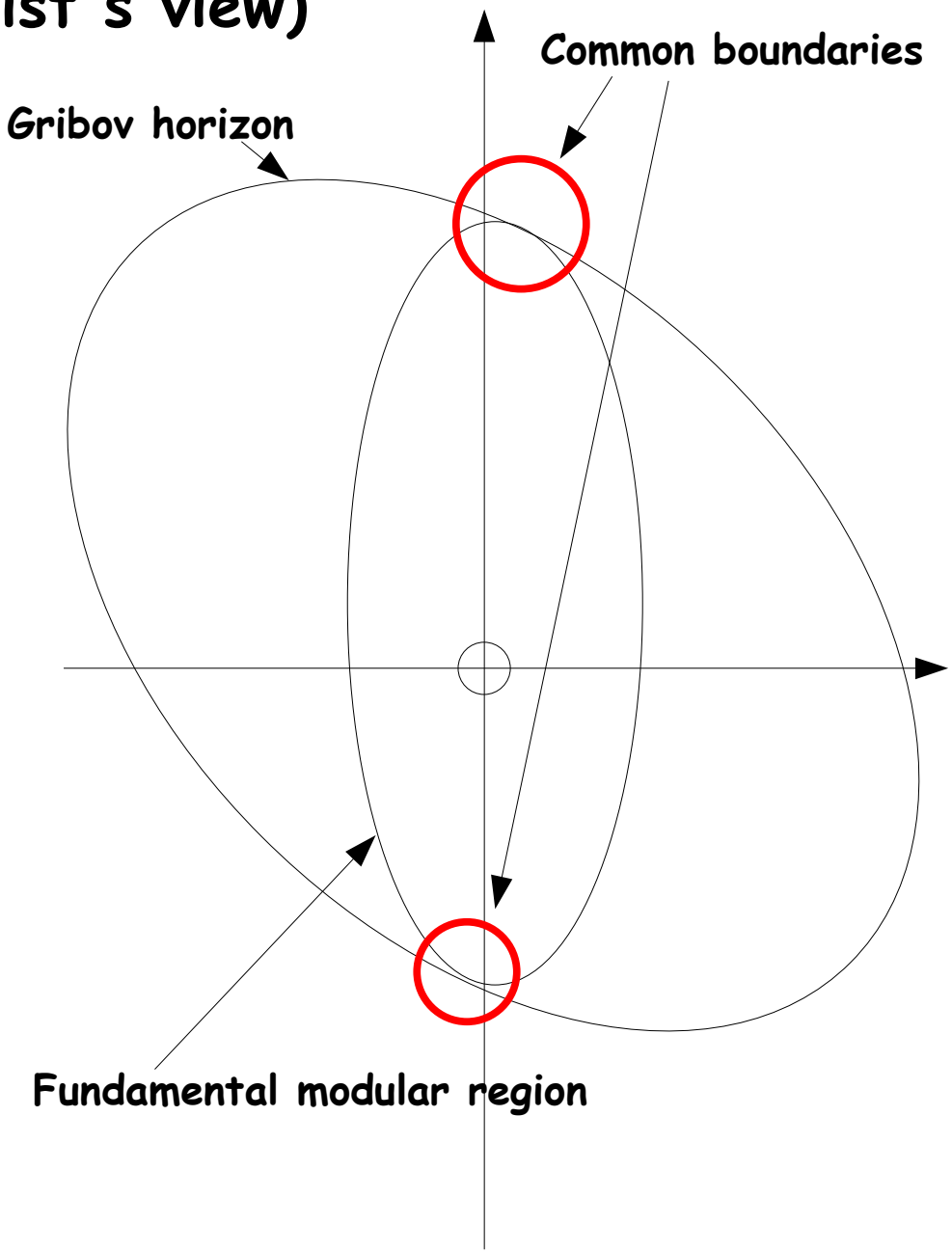

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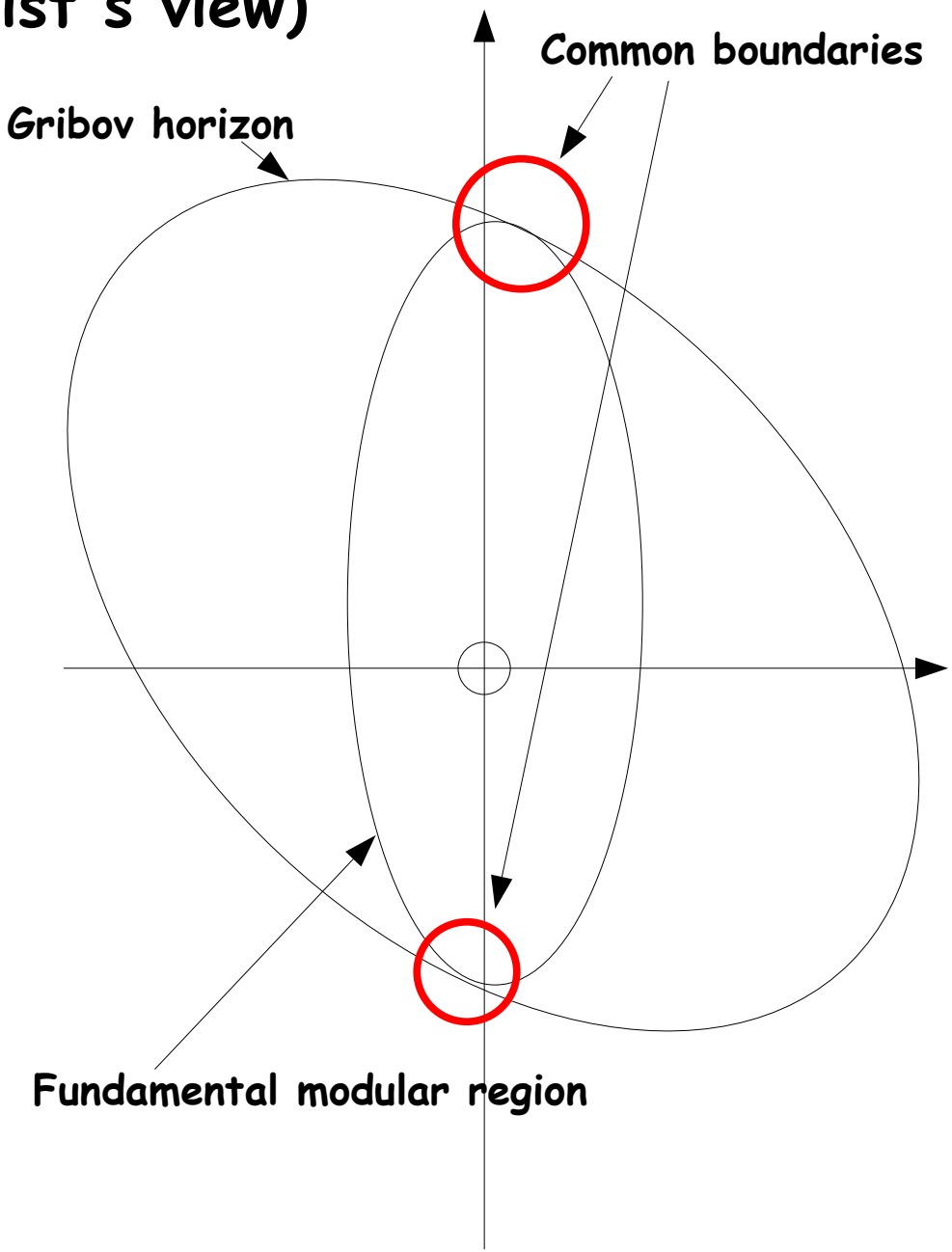
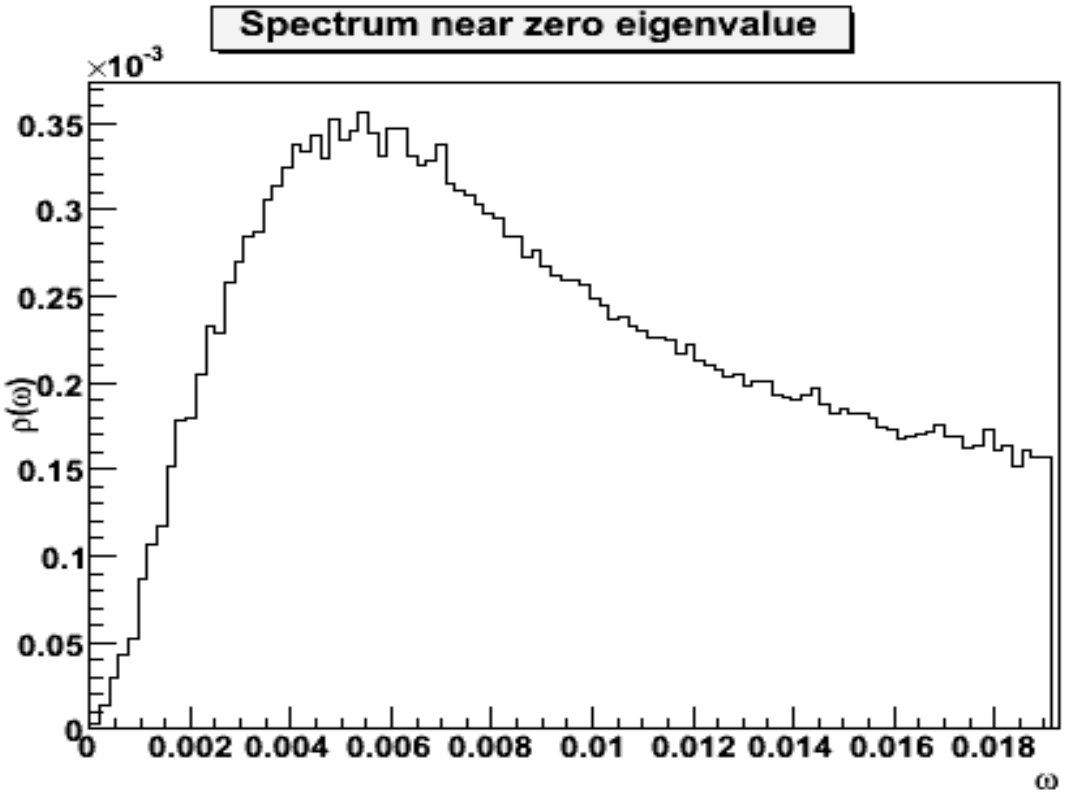
- Leads to positivity violating spectral functions



Landau-gauge Faddeev-Popov operator eigenspectrum



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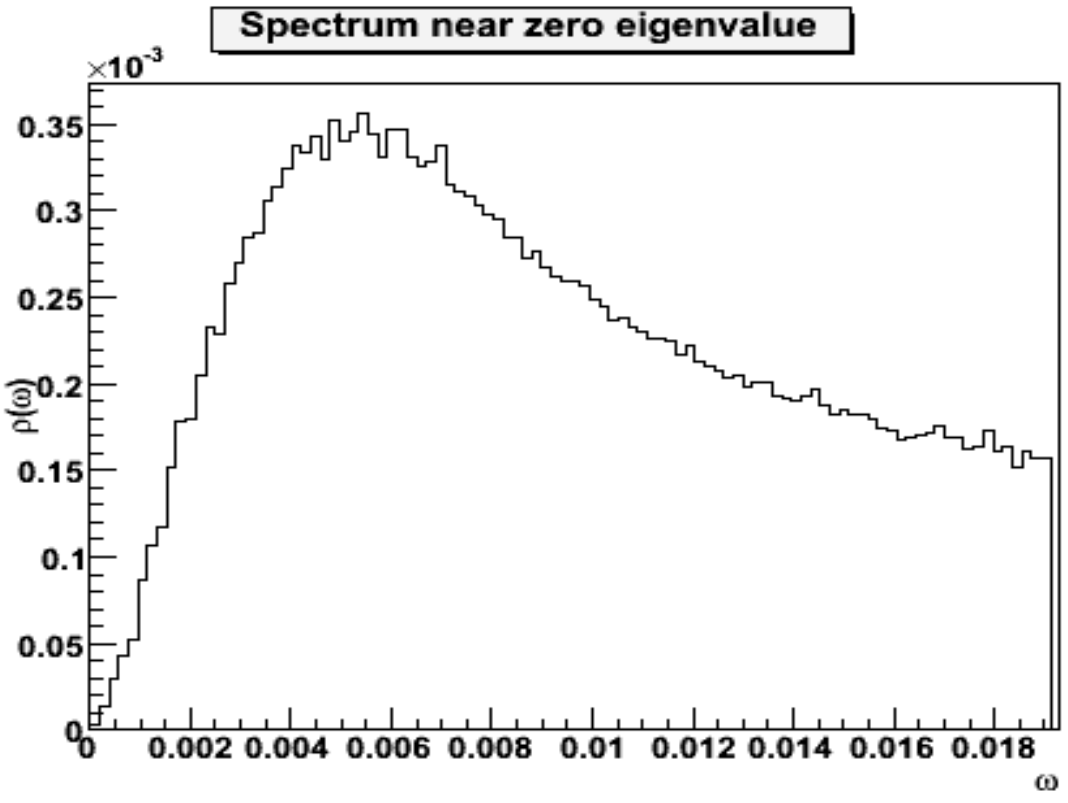

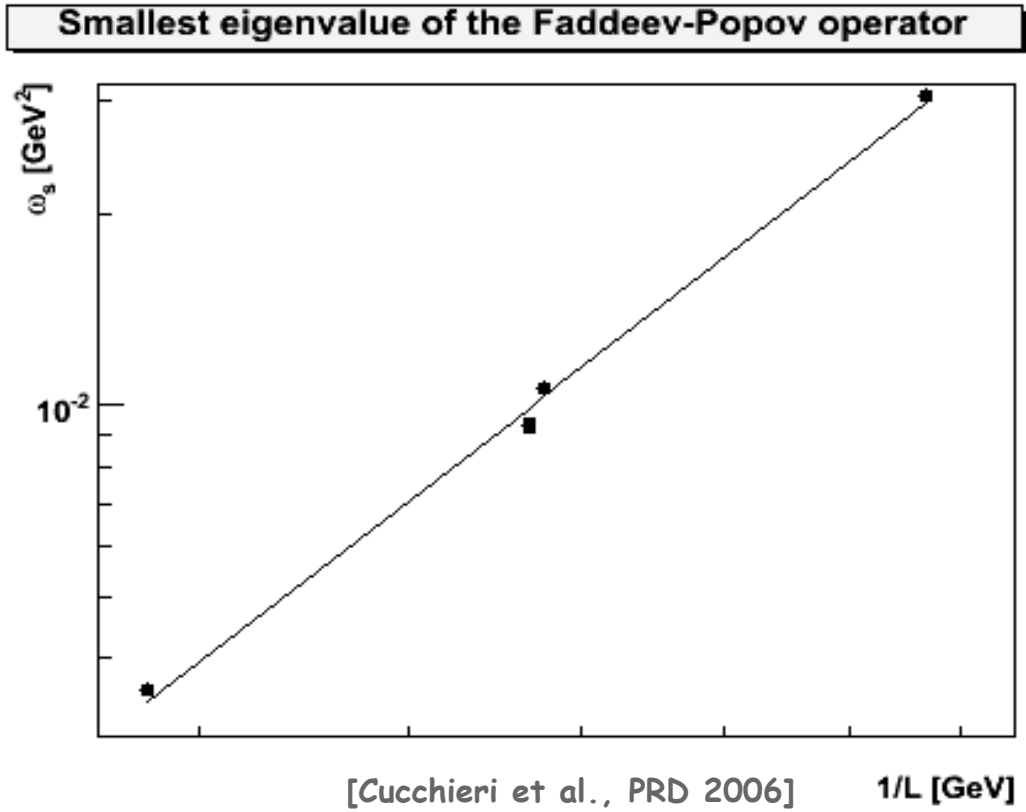

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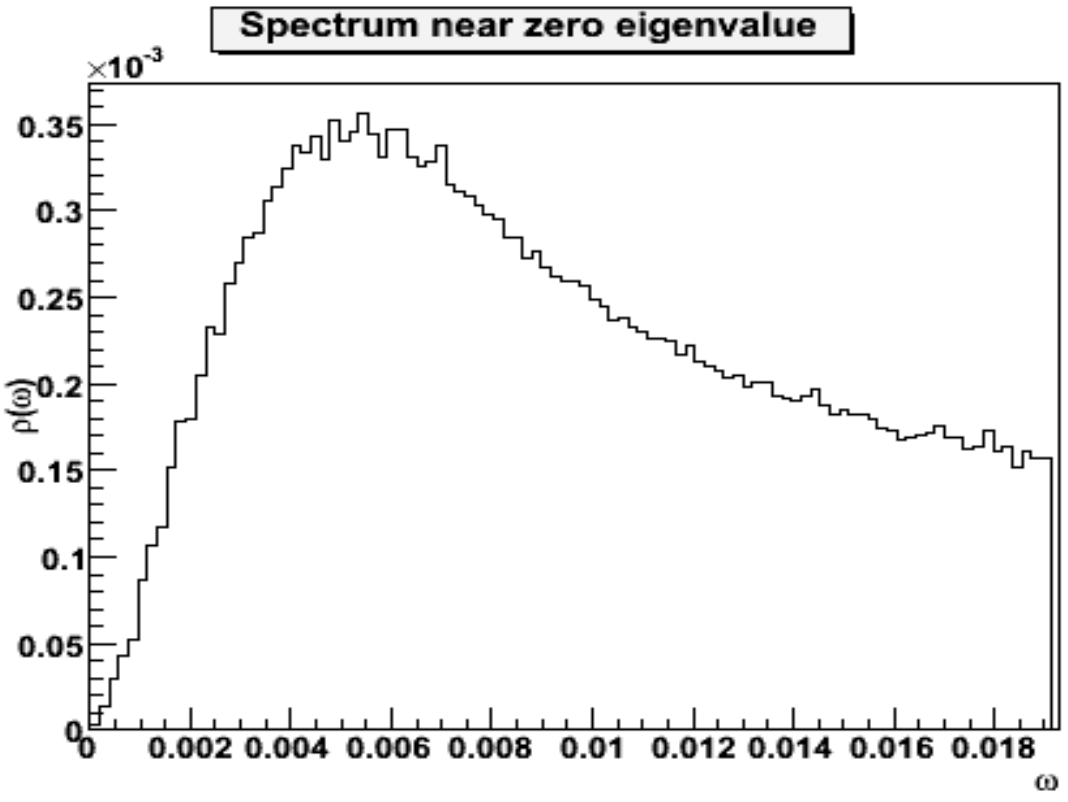

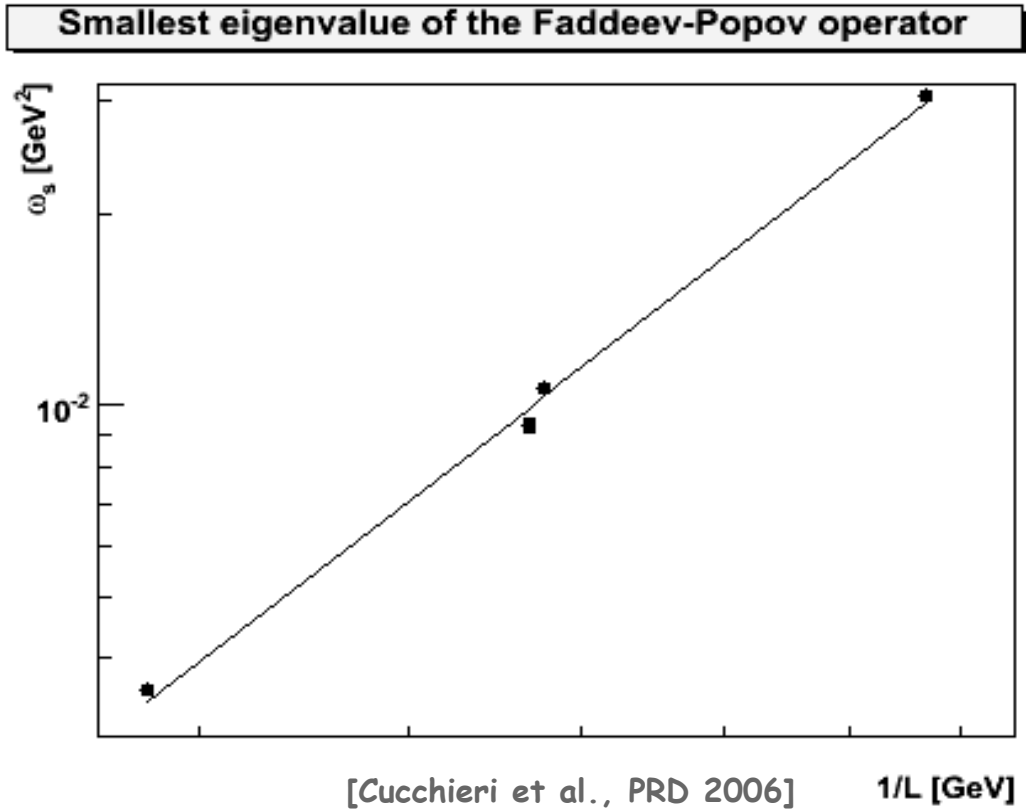

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- Average configuration in the continuum limit on the Gribov horizon
- Agrees with Gribov-Zwanziger scenario

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Landau gauge

- Lagrangian (gauge group SU(2) from here):

$$L = -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} - \bar{c}^a \partial_\mu D^{ab}_\mu c^b$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu$$

$$D^{ab}_\mu = \delta^{ab} \partial_\mu - g f^{abc} A^c_\mu$$

- Degrees of freedom:

  Gluons: A^a_μ

  Ghosts: $\bar{c}^a, c^a$

  (Intermediate states - not observable)

Propagators [Introduction: Alkofer & von Smekal, 2001]

- 2-point Green's functions are the propagators
- Gluon:

$$D^{ab}_{\mu\nu}(x-y) = \langle A^a_\mu(x) A^b_\nu(y) \rangle$$

$$D_{\mu\nu}(p) = (\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}) \frac{Z(p)}{p^2}$$

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- Ghost linked to the Faddeev-Popov operator

$$D^{ab}_G(x-y) \sim <(\partial_\mu D^{ab}_\mu)^{-1}> = <(\partial_\mu(\delta^{ab}\partial_\mu - g\, f^{abc} A^c_\mu))^{-1}>$$

3-point vertices in Landau gauge

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$$G^{A^3} = \Gamma^{tl} < AAA > / \left(\Gamma^{tl} DDD \Gamma^{tl} \right)$$

  - Mixes in general tensor structures
    - Here only in case of the three-gluon vertex

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  - Very general result [Watson et al. PRL 2001]

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Ghost-gluon vertex in 4d

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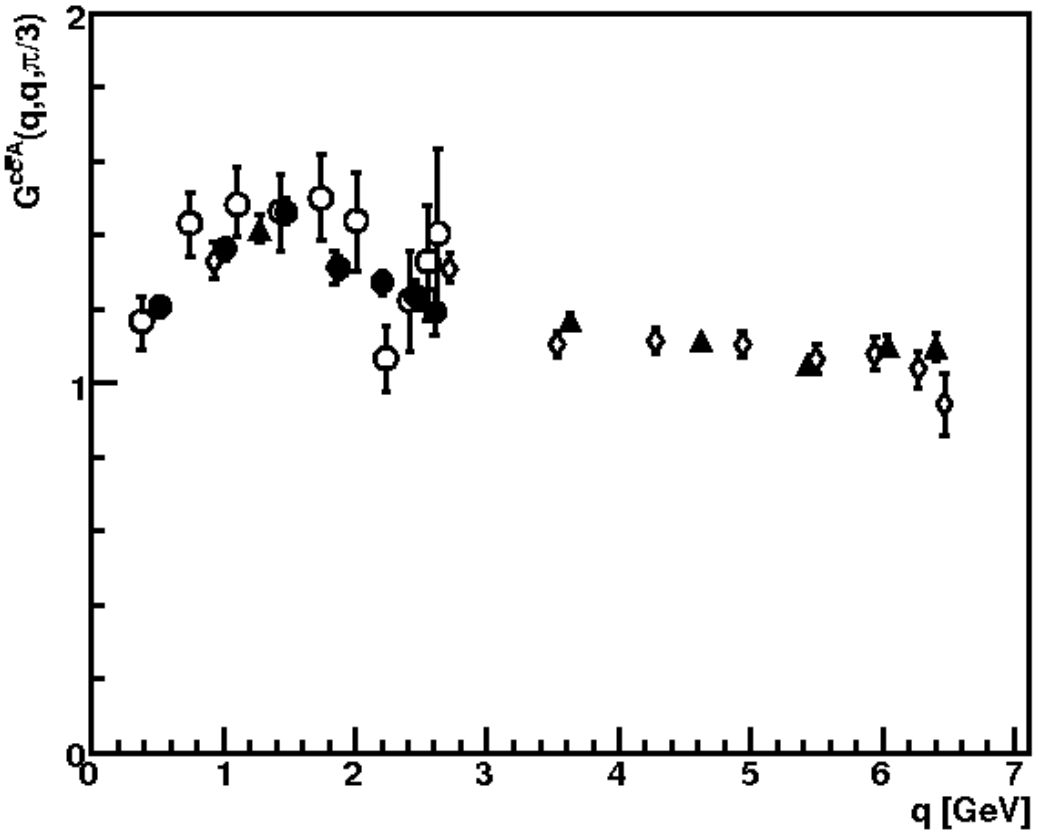

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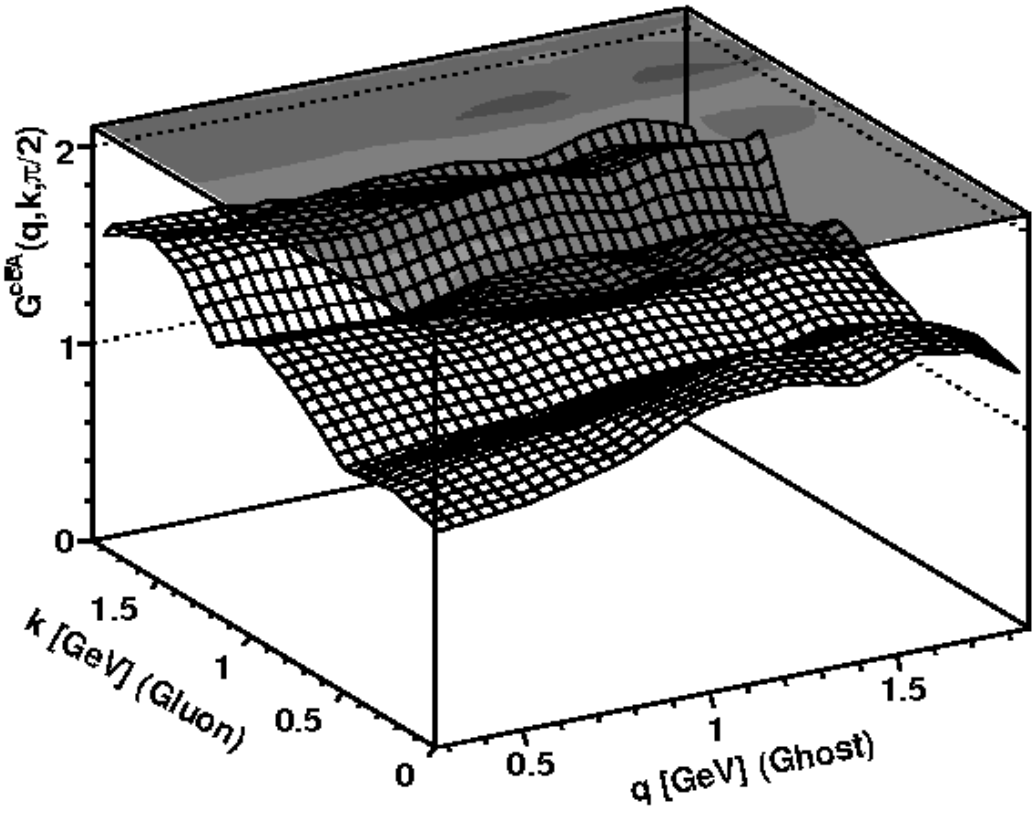

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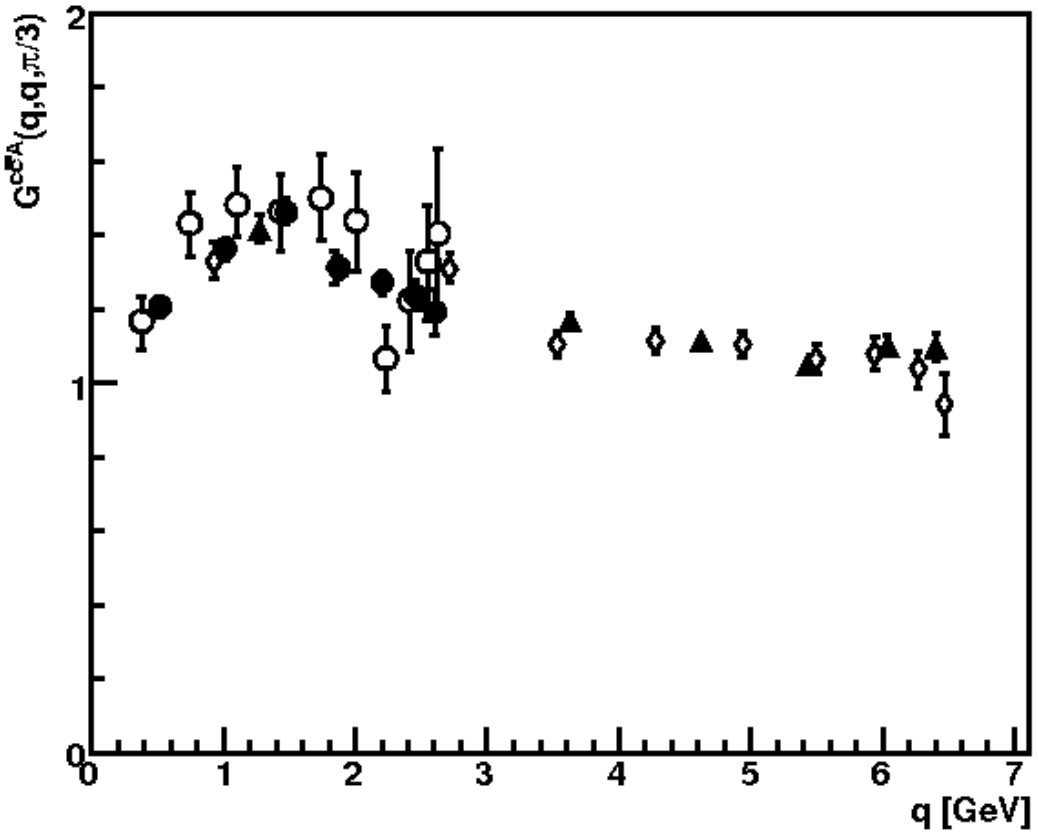

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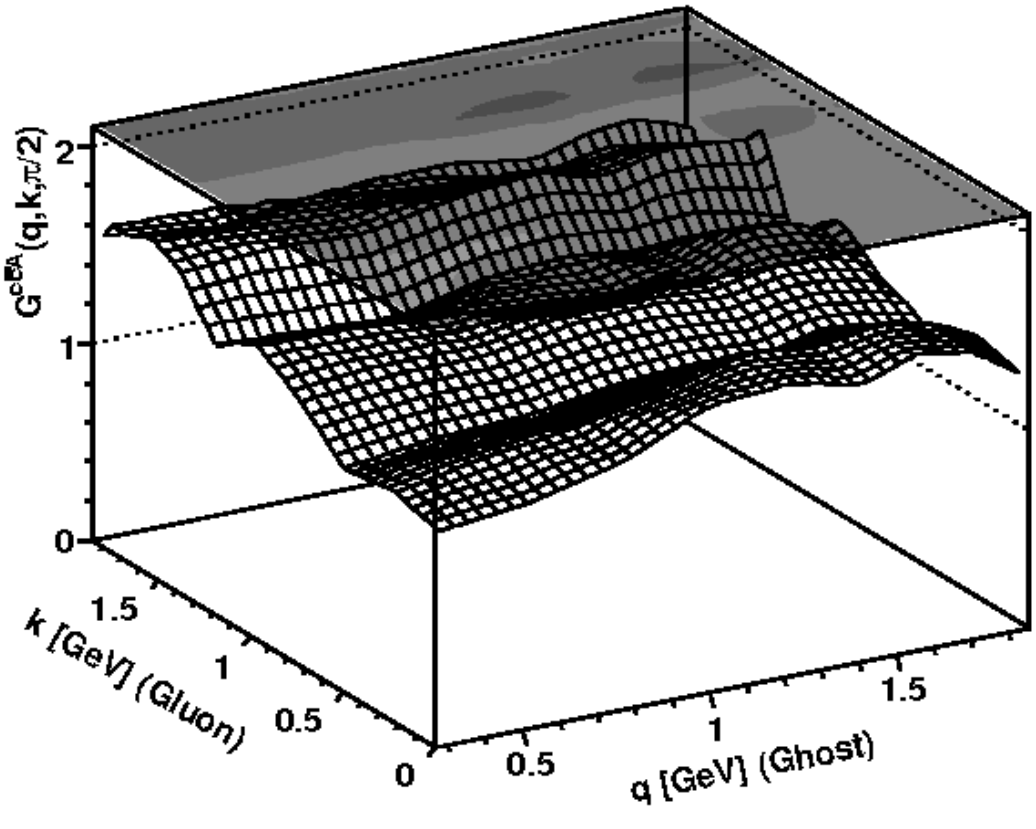

- Vertex in various momentum configurations is infrared finite and essentially tree-level-like
- Some structure at intermediate momenta

Ghost-gluon vertex in 3d

[Cucchieri et al., preliminary
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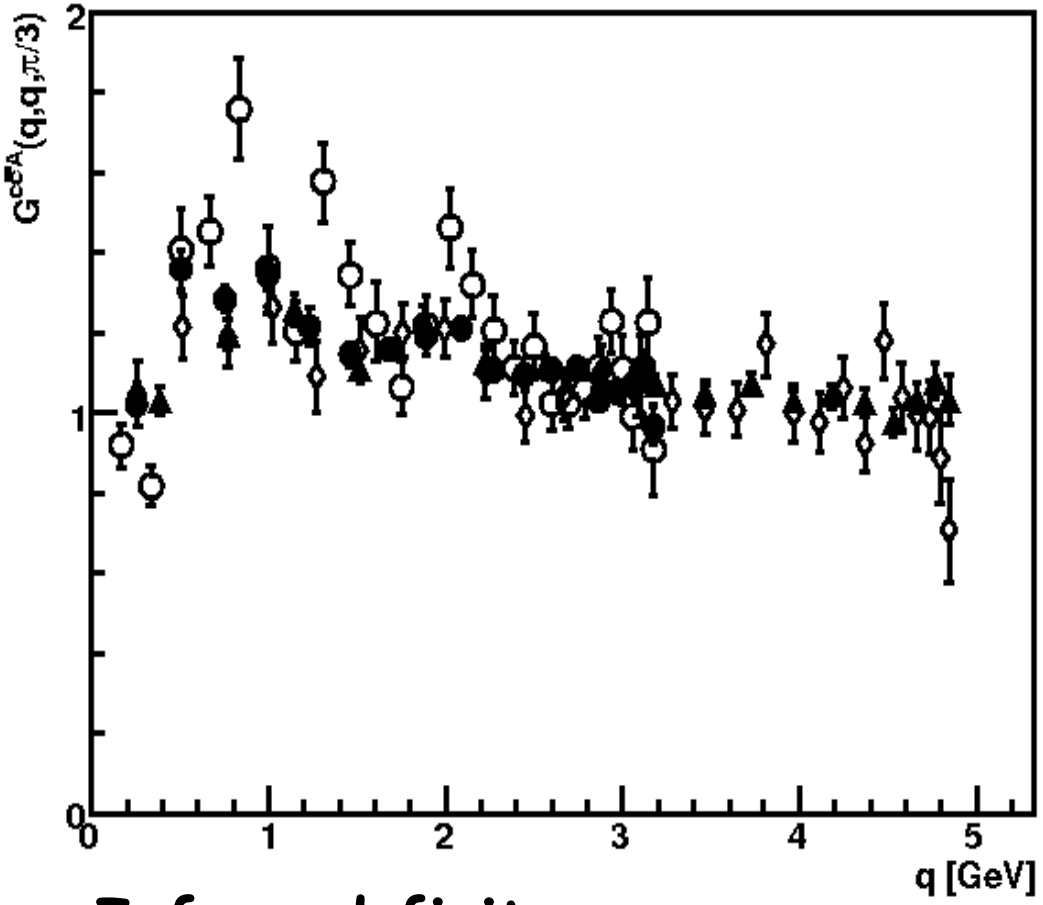

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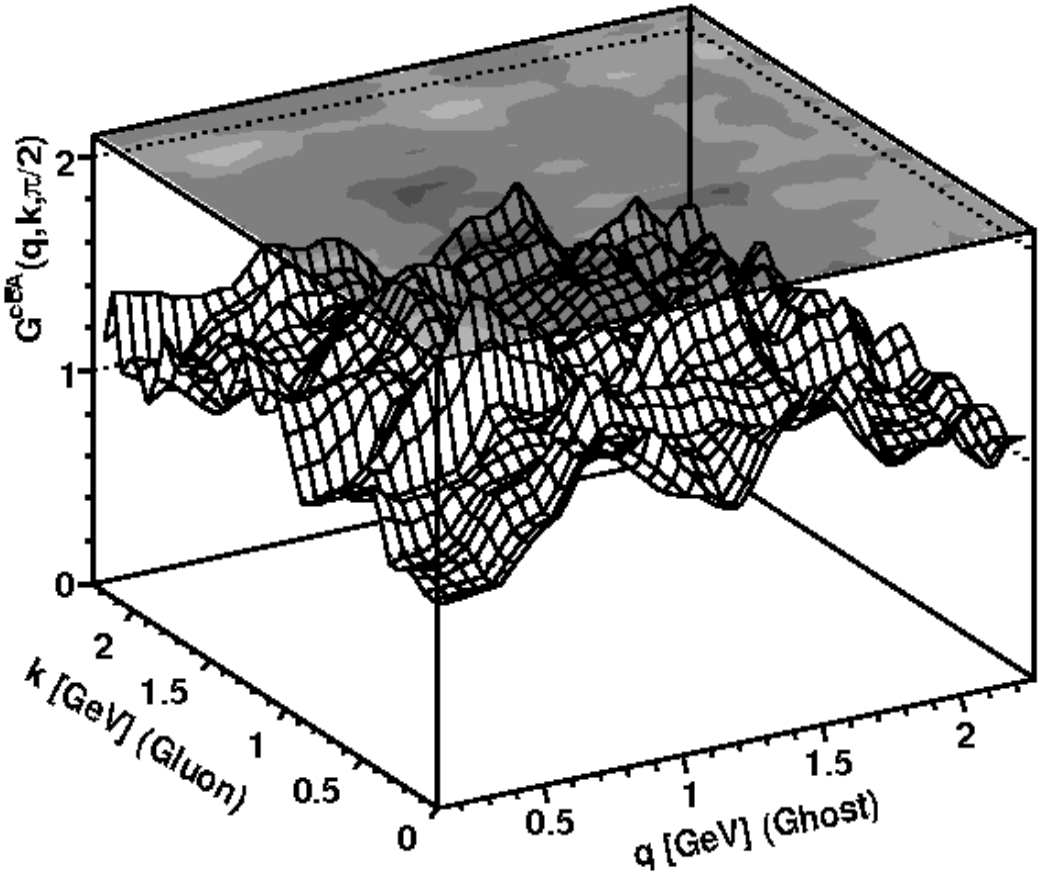
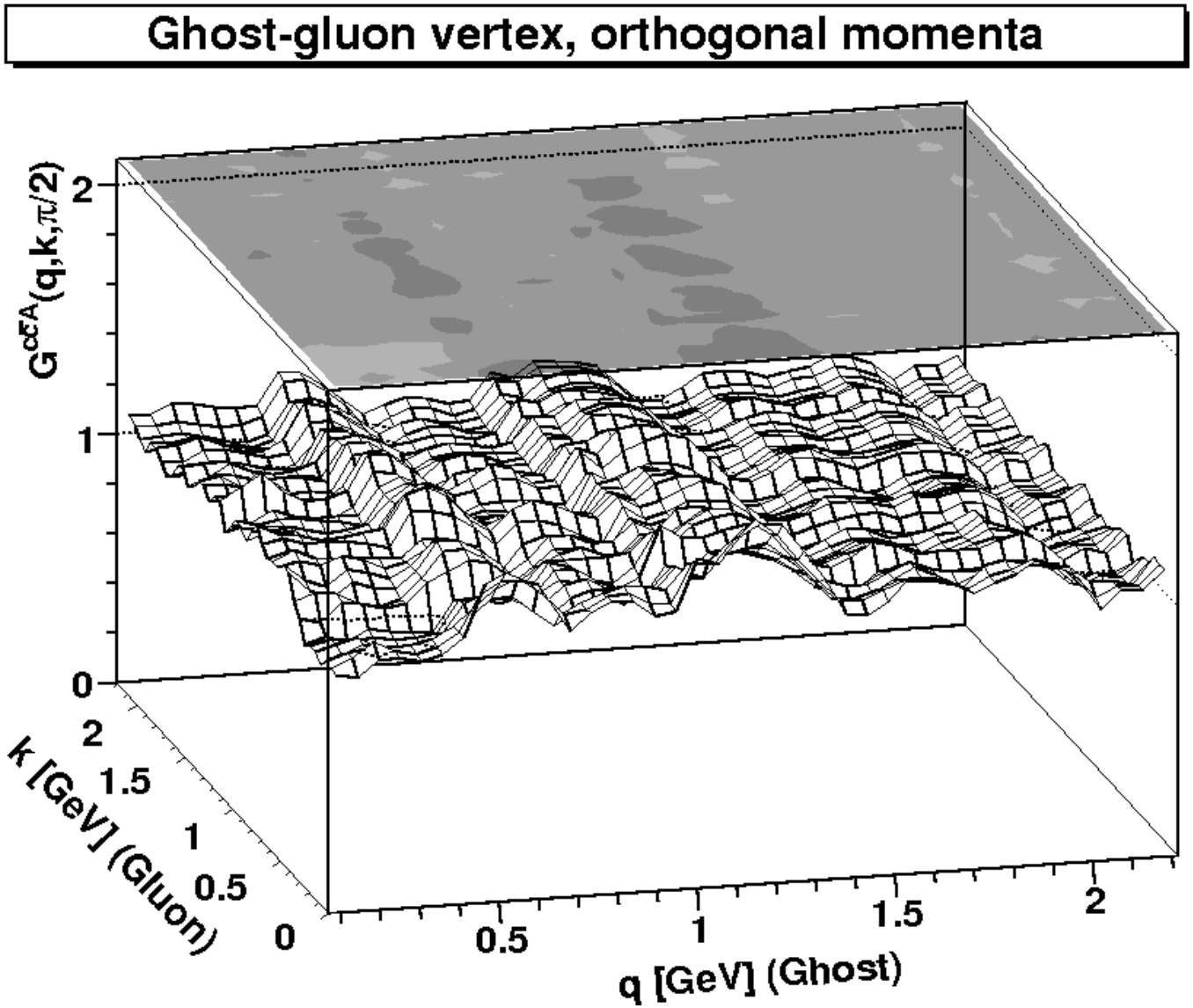


- Infrared finite
- Same as in 4d, but larger volumes
- More statistics needed

Ghost-gluon vertex in 2d

[Maas PRD 2007
120^2, beta=10]



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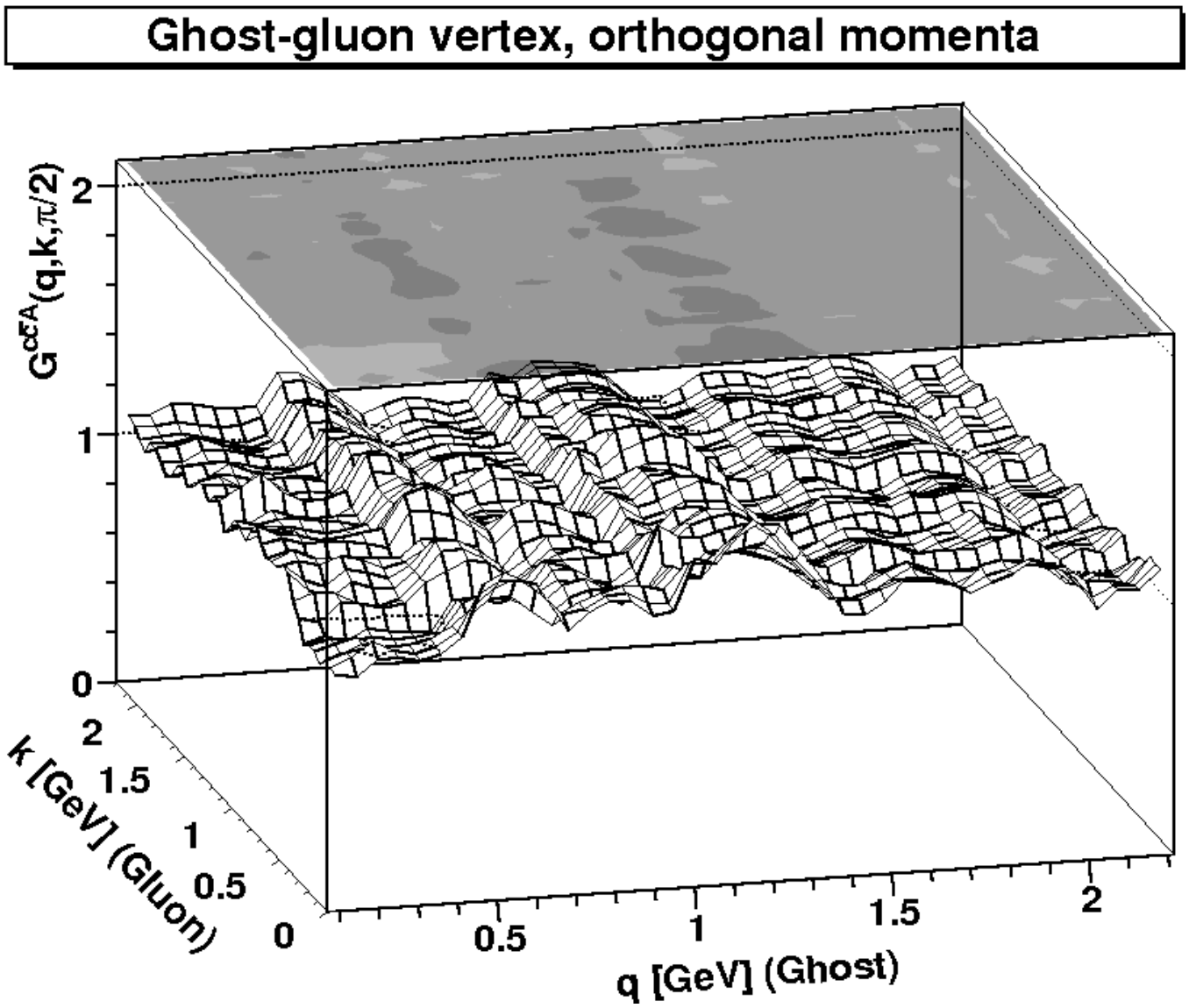

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- Assumption of infrared finite ghost-gluon vertex supported in all d

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- Infrared finite ghost-gluon vertex seems reasonable
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- Yields the numerical value of the exponents of all Green's functions under additional assumptions
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- Domination of the BRST-exact part of the action
  - Like in a topological field theory

General IR behavior

- A dressing function with n ghost legs and m gluon legs in d dimensions with all external momenta p of the same magnitude behaves as $\left(p^2\right)^{-(n/2-m)\kappa-(1-n/2)(d/2-2)}$ [Huber at al. 2007]

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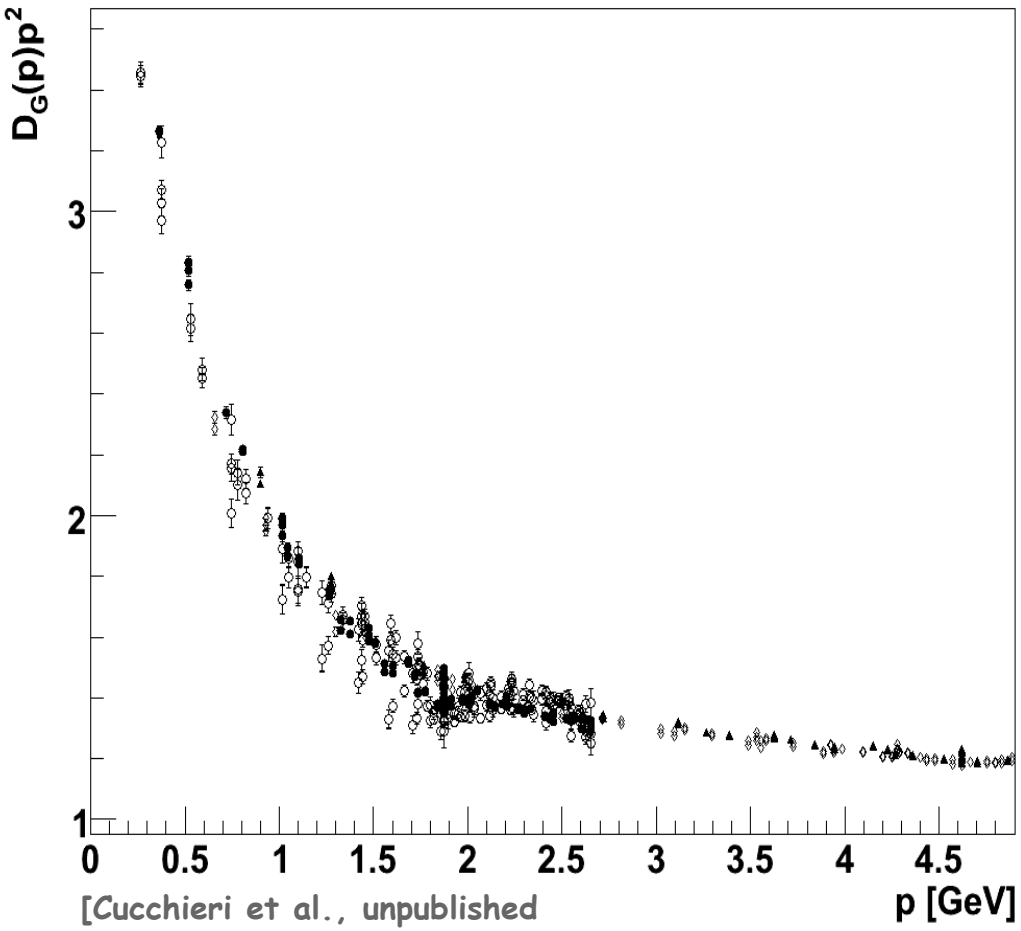
Ghost propagator in 4d

Ghost dressing function



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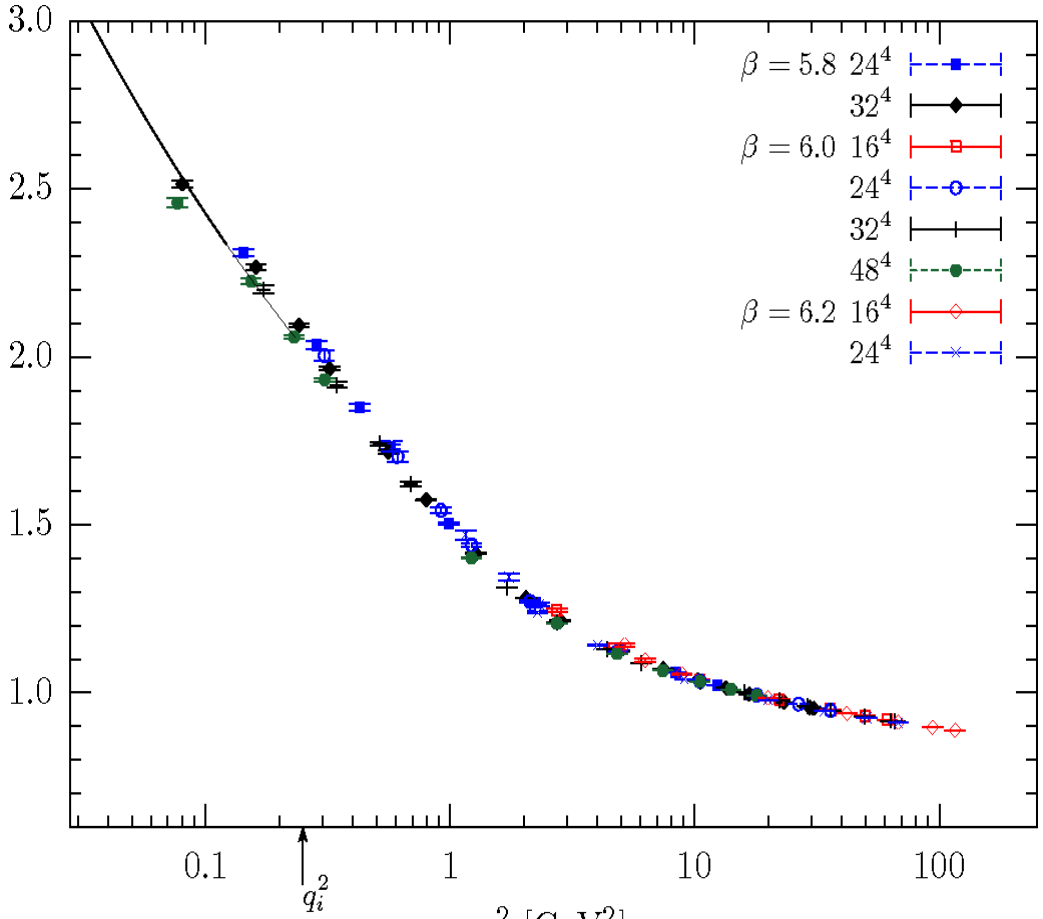

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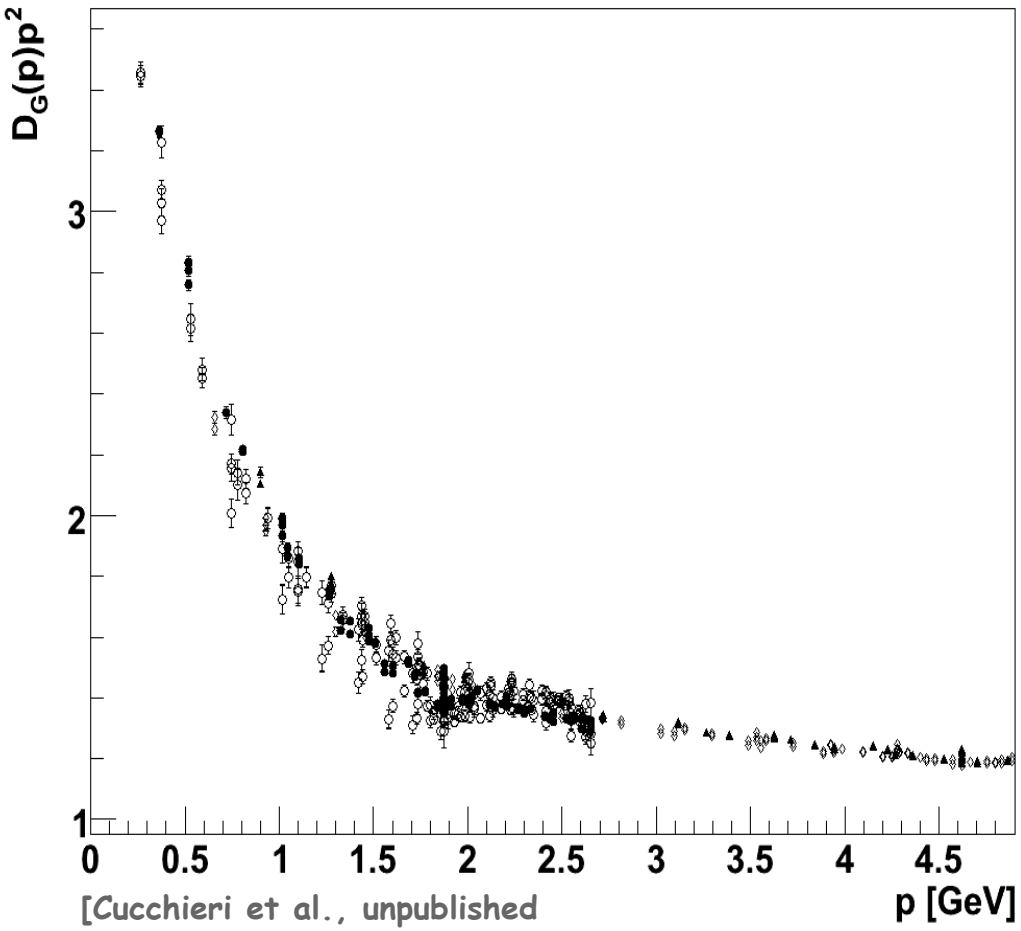

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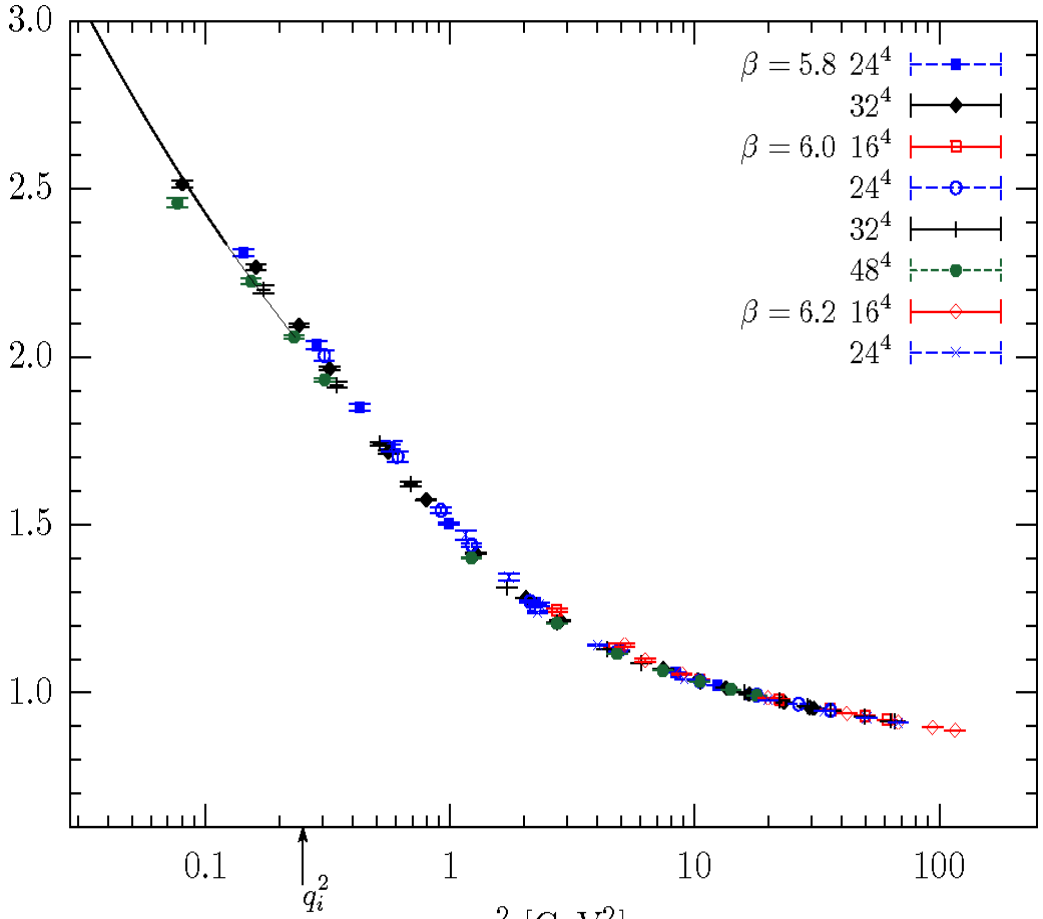

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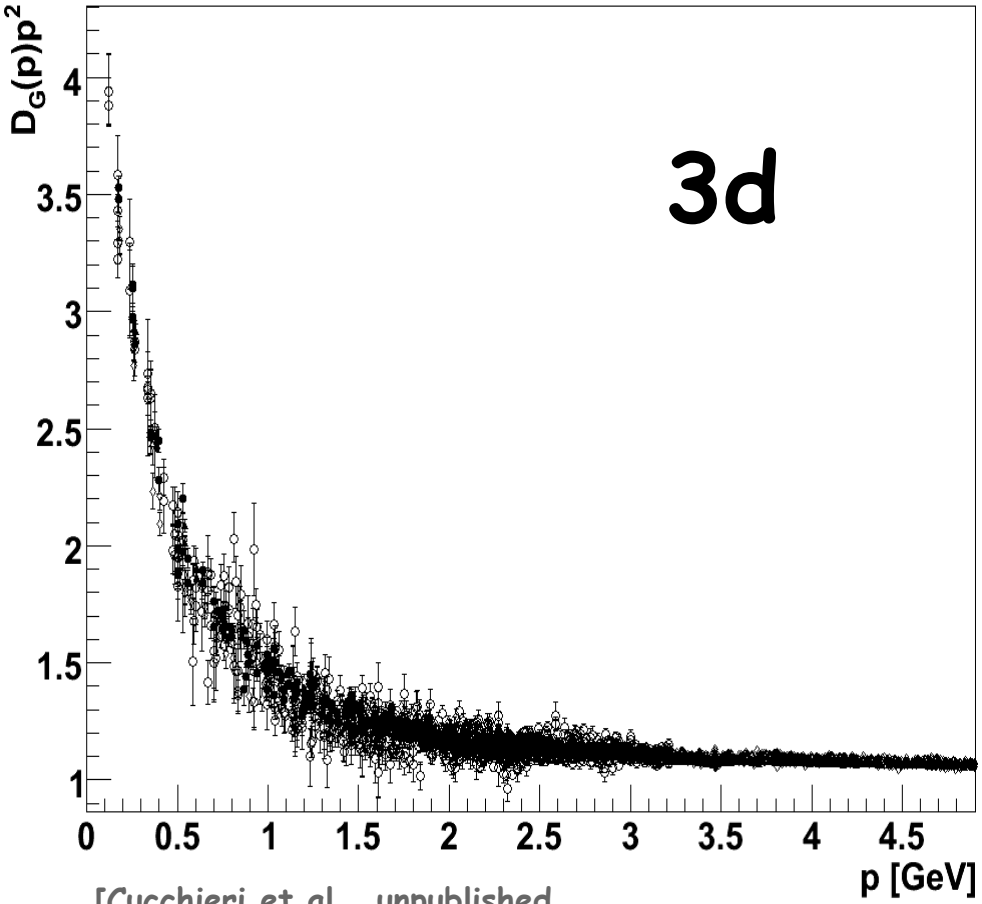
- Infrared divergent – as predicted
- Exponent too small comp. to prediction. Finite volume effect?

Ghost propagator in lower dimensions

Ghost dressing function



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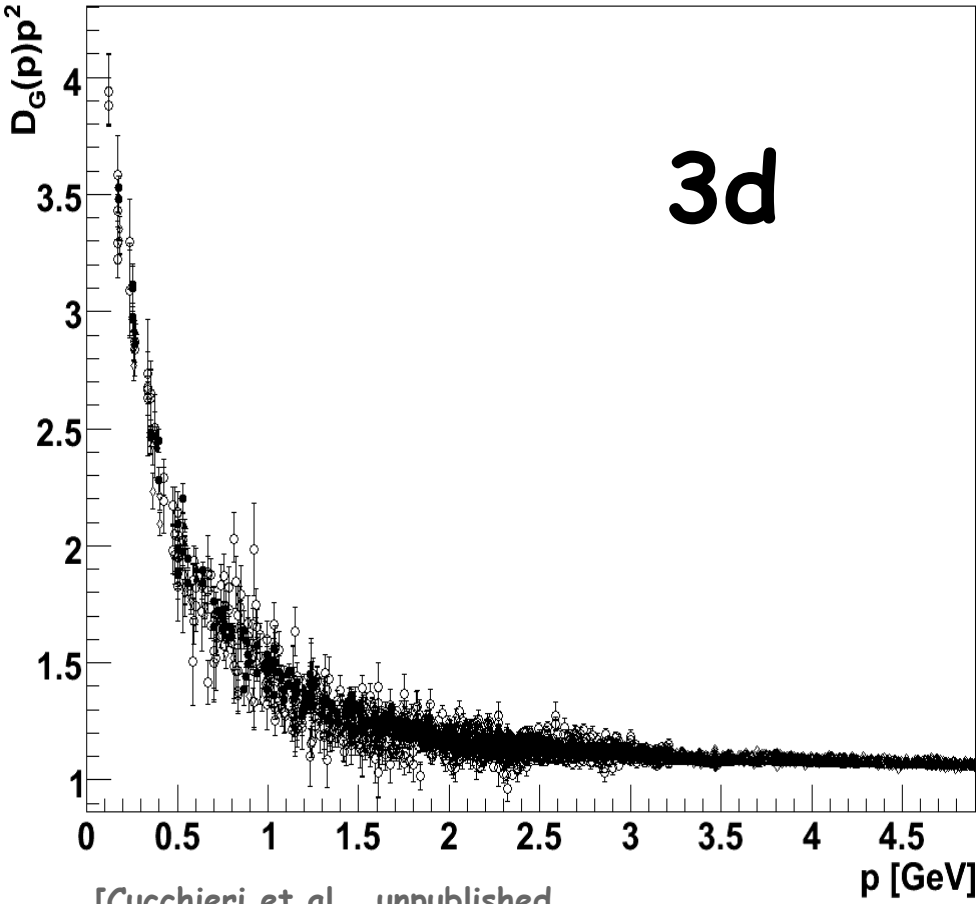

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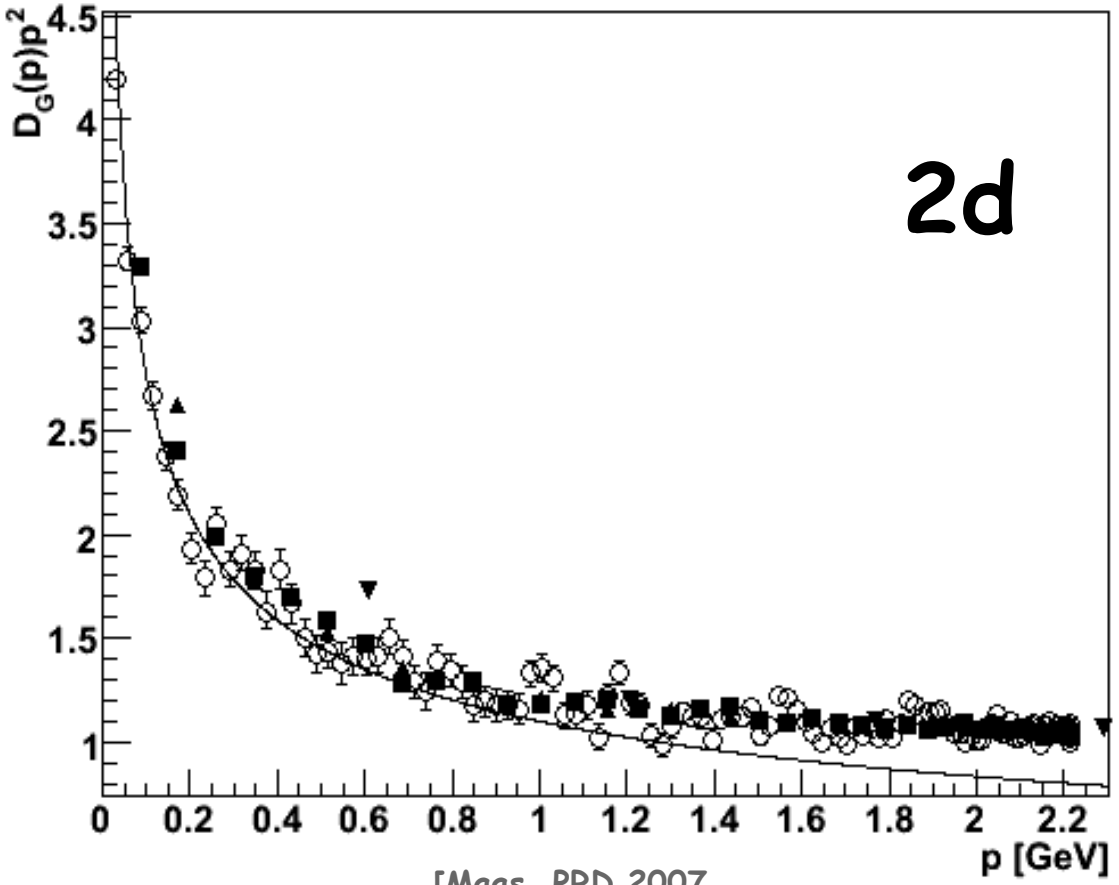

2d

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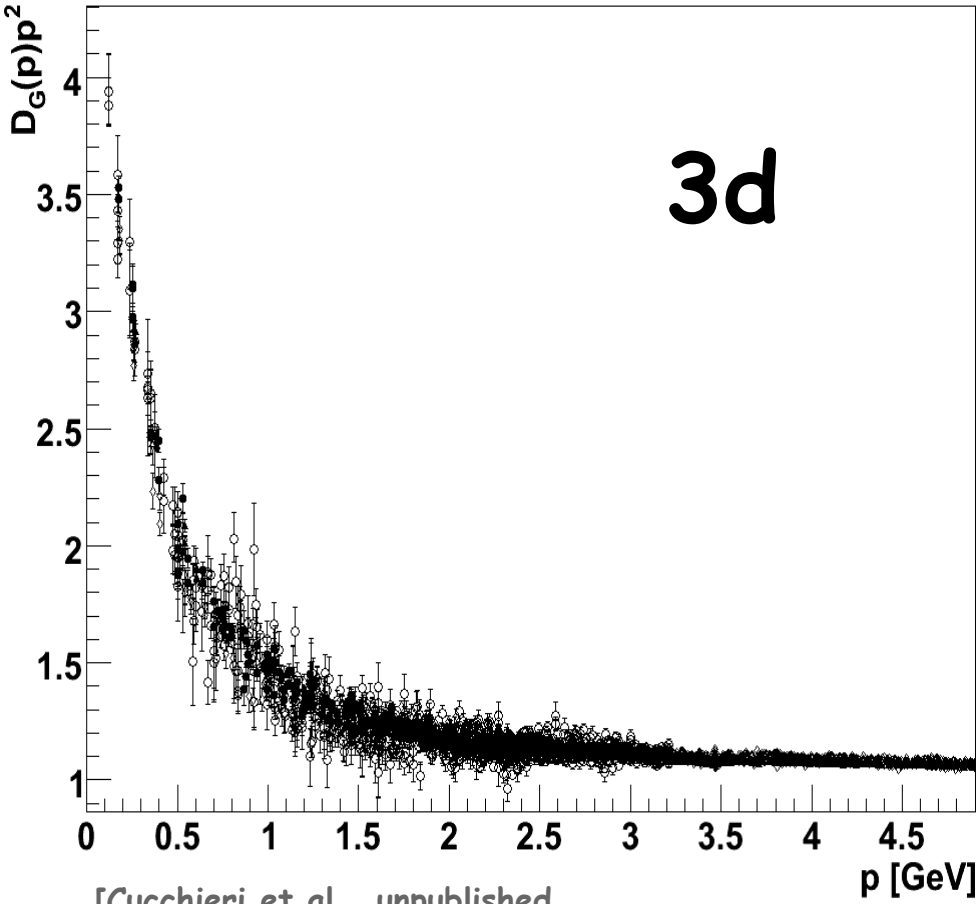

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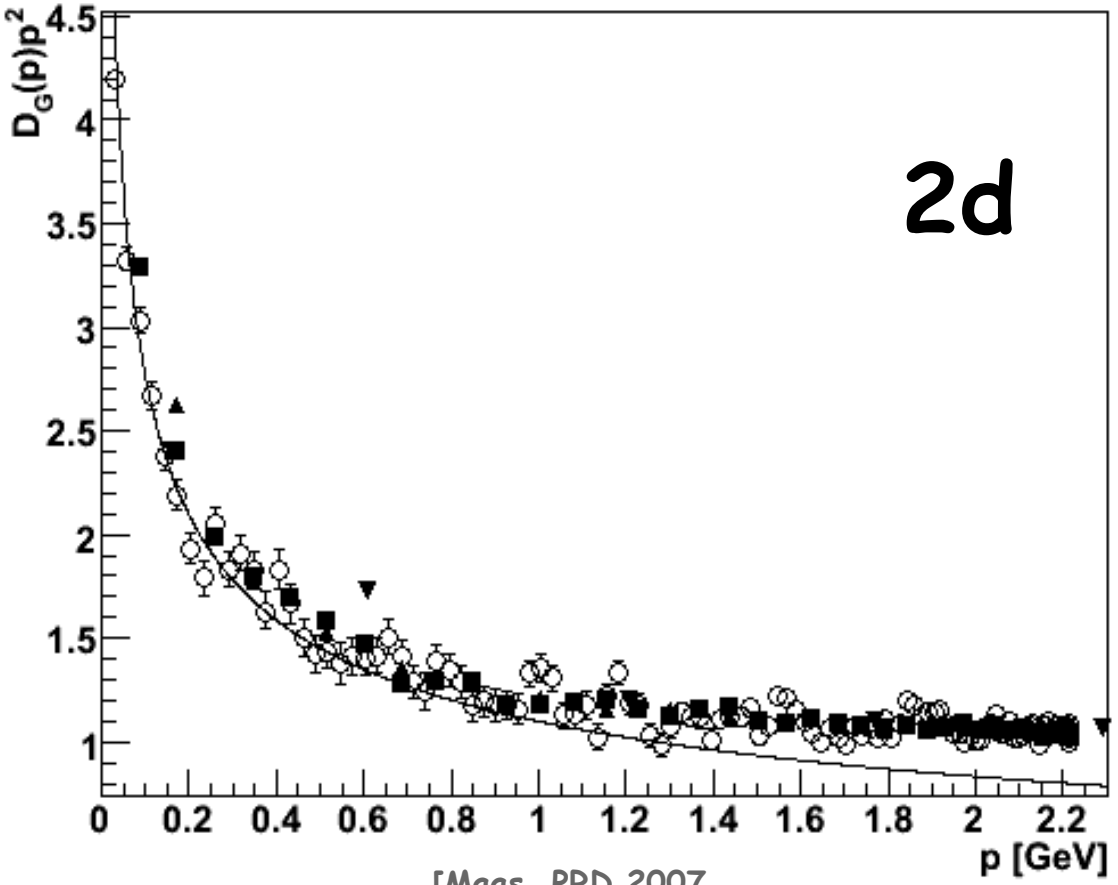

2d

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- Infrared enhanced
- Exponent in 2d agrees with predictions

Ghost propagator and finite volumes

[Fischer et al., Ann. Phys. in press]

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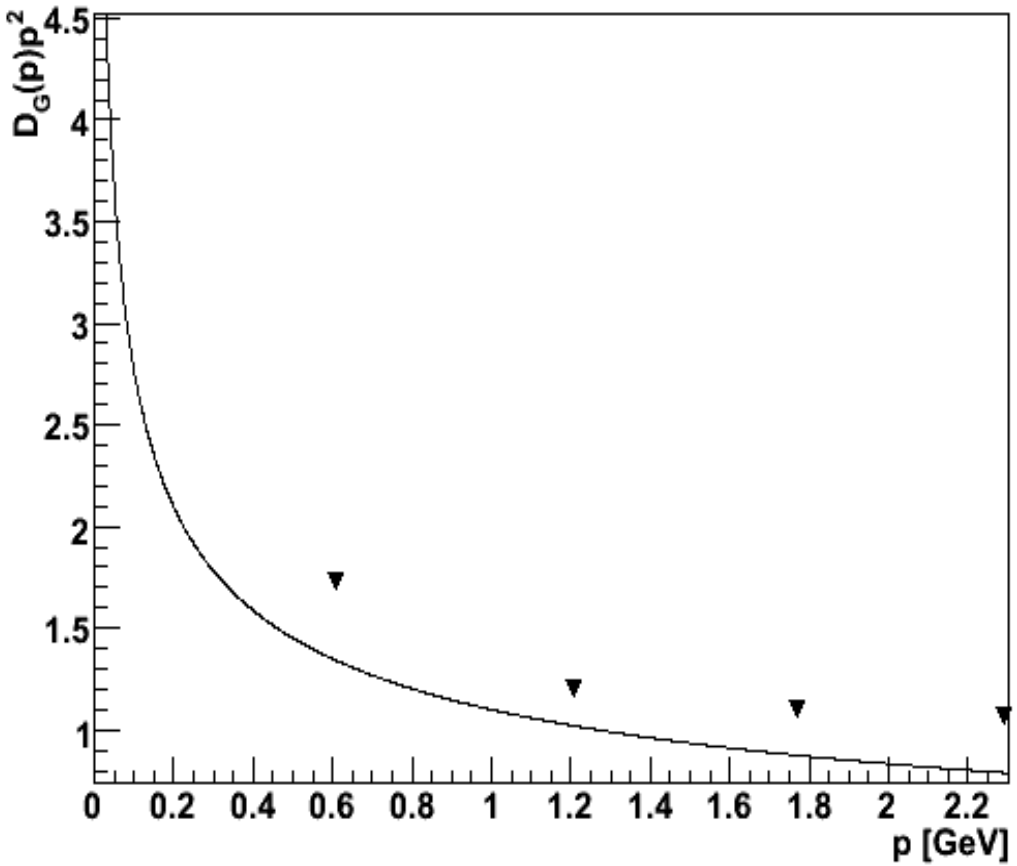

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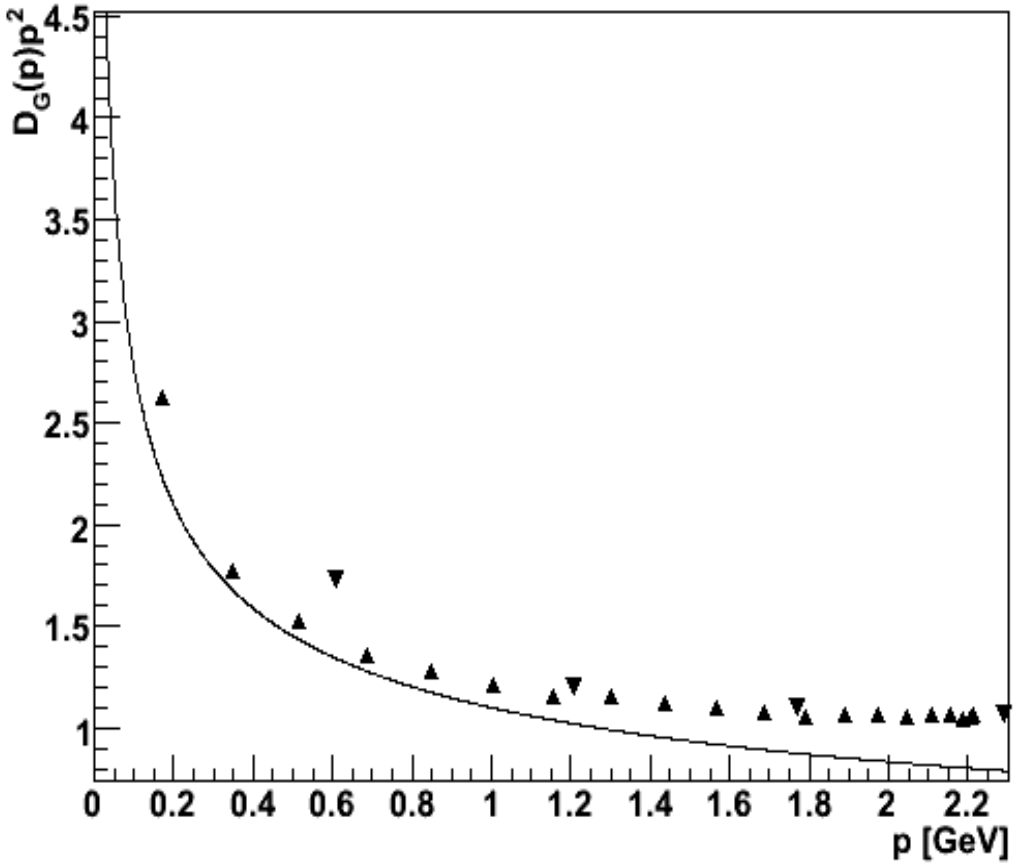

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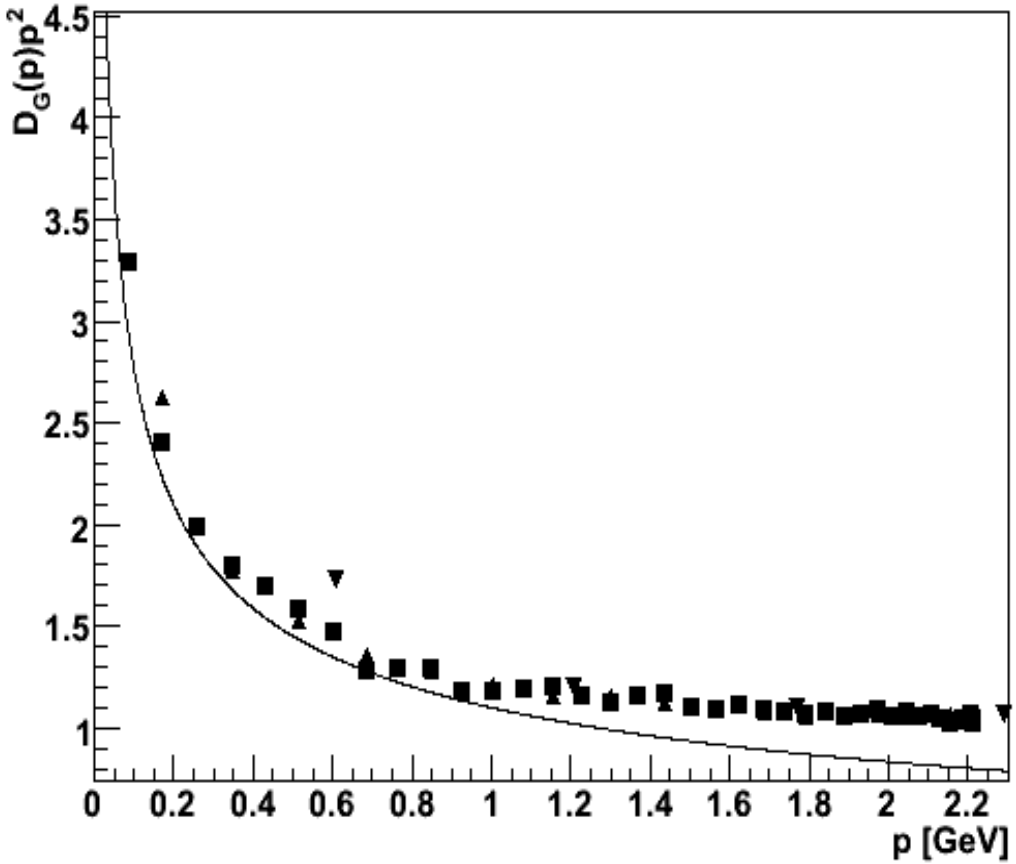

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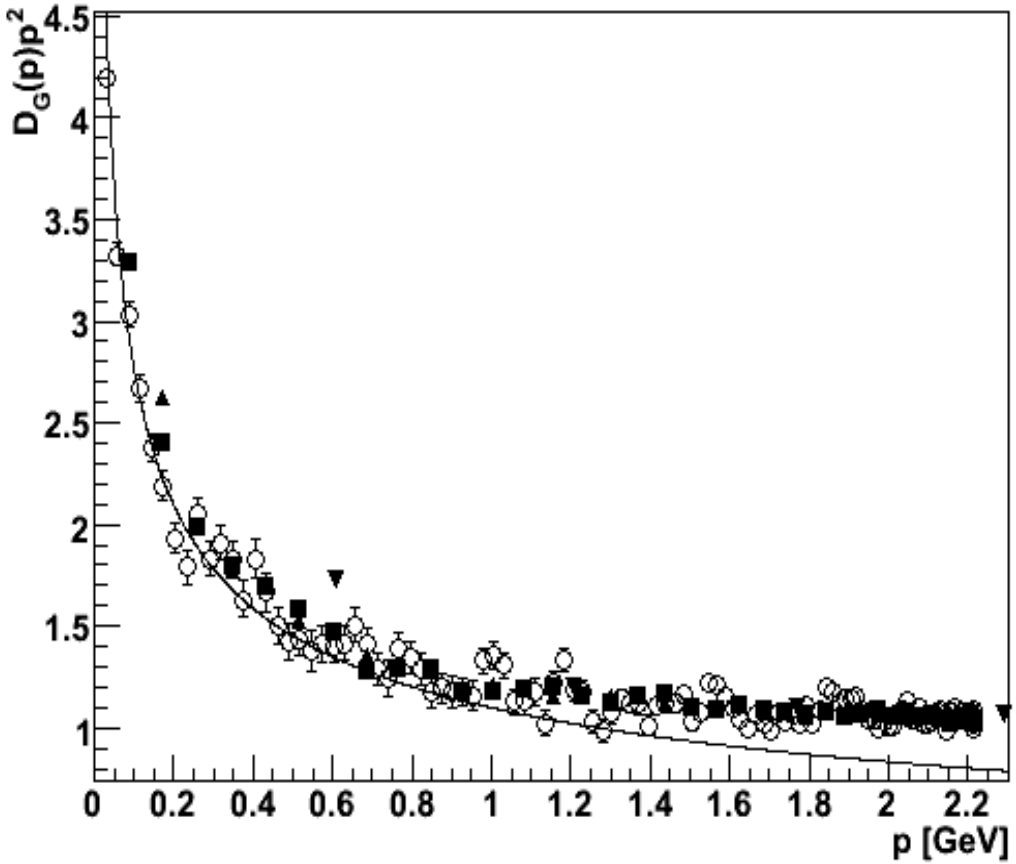

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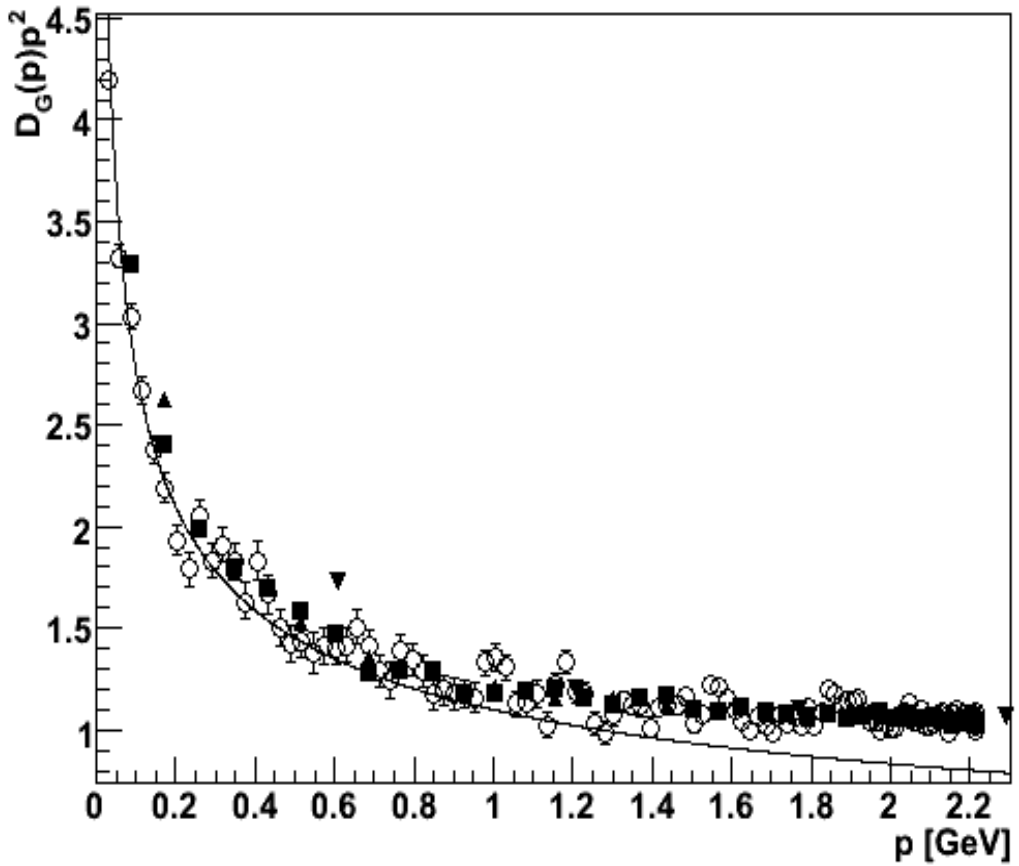

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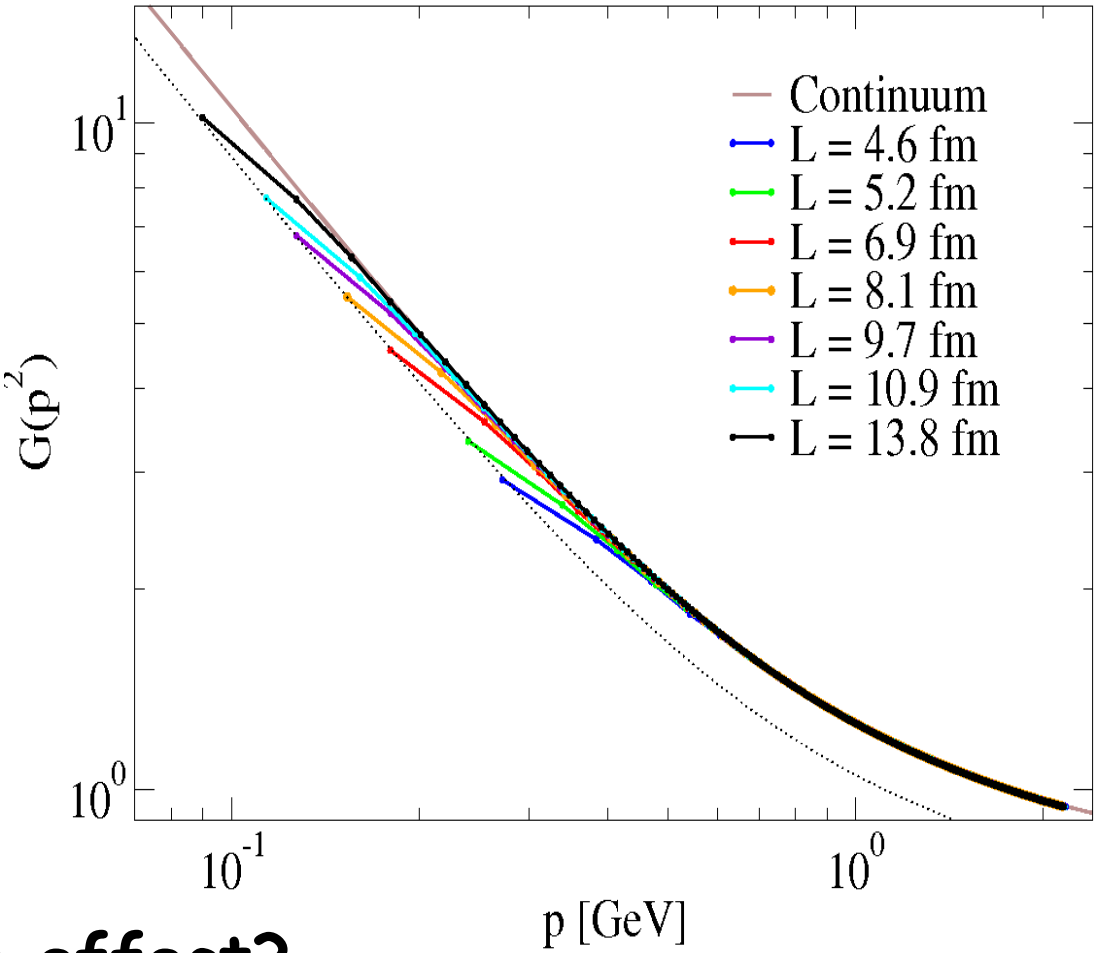

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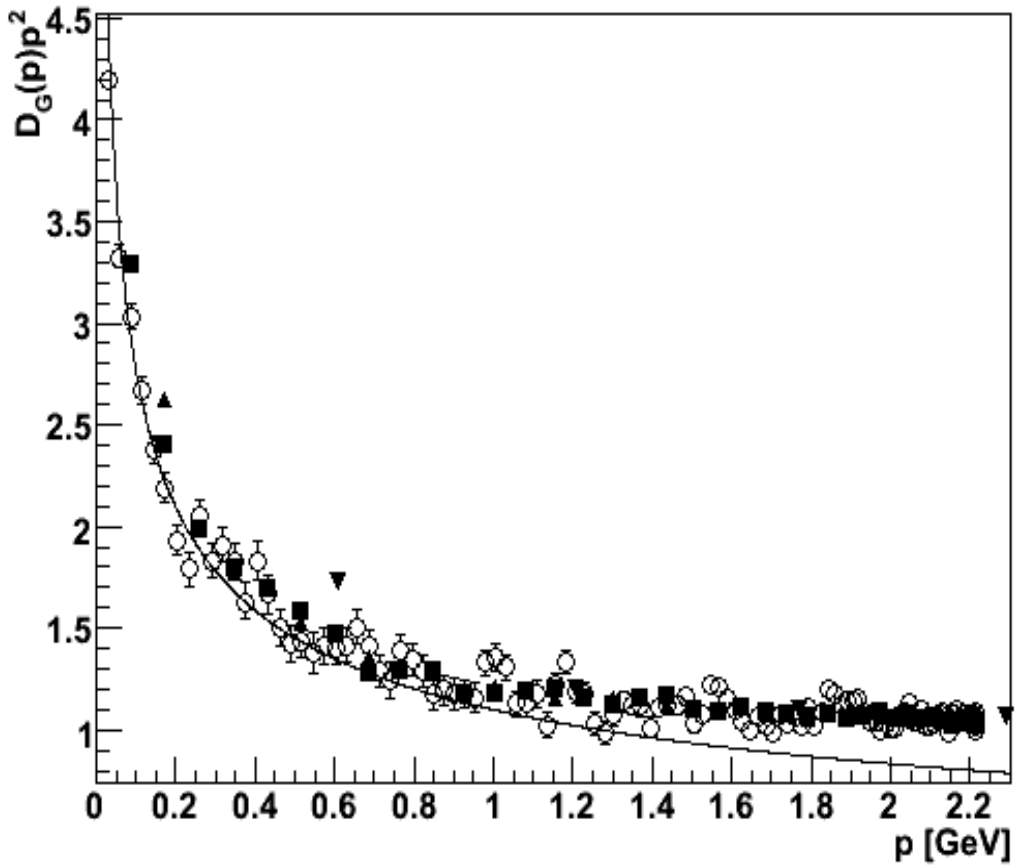

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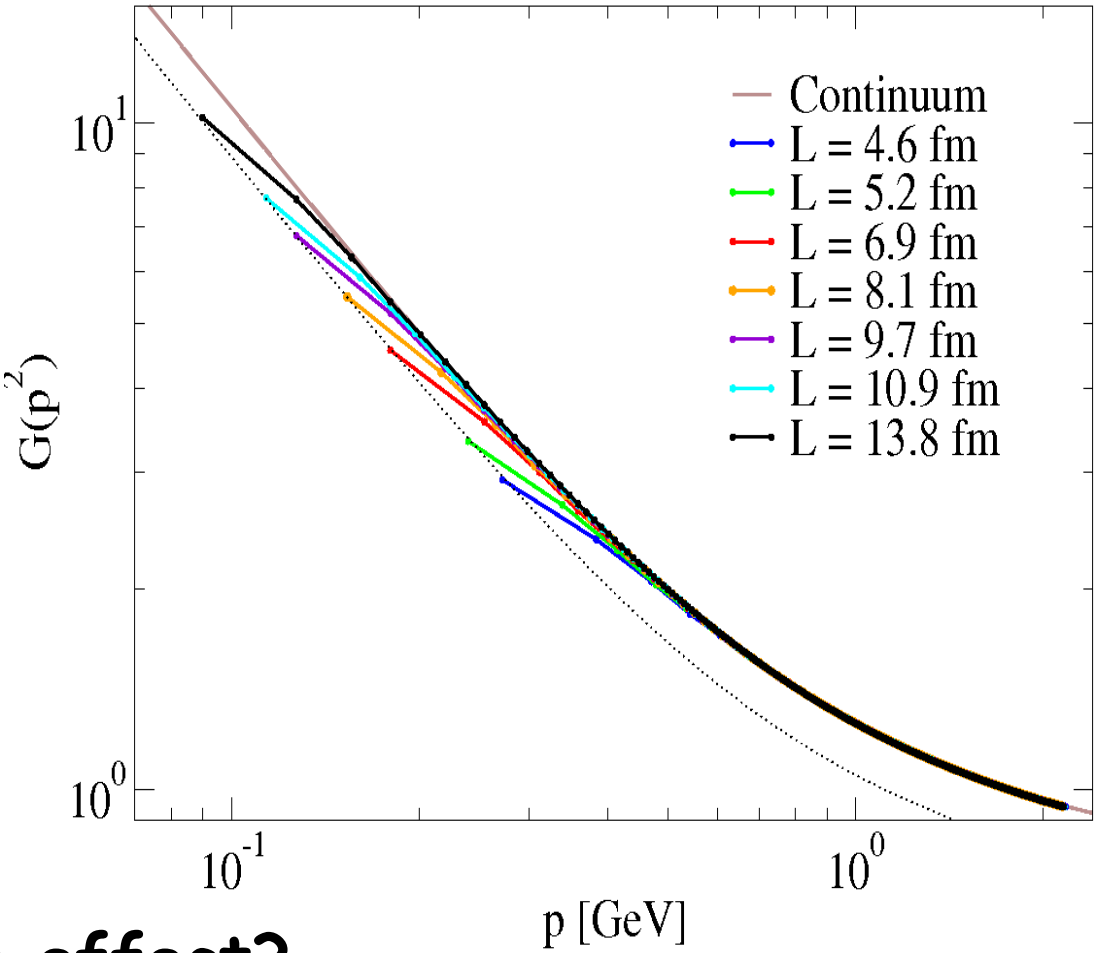

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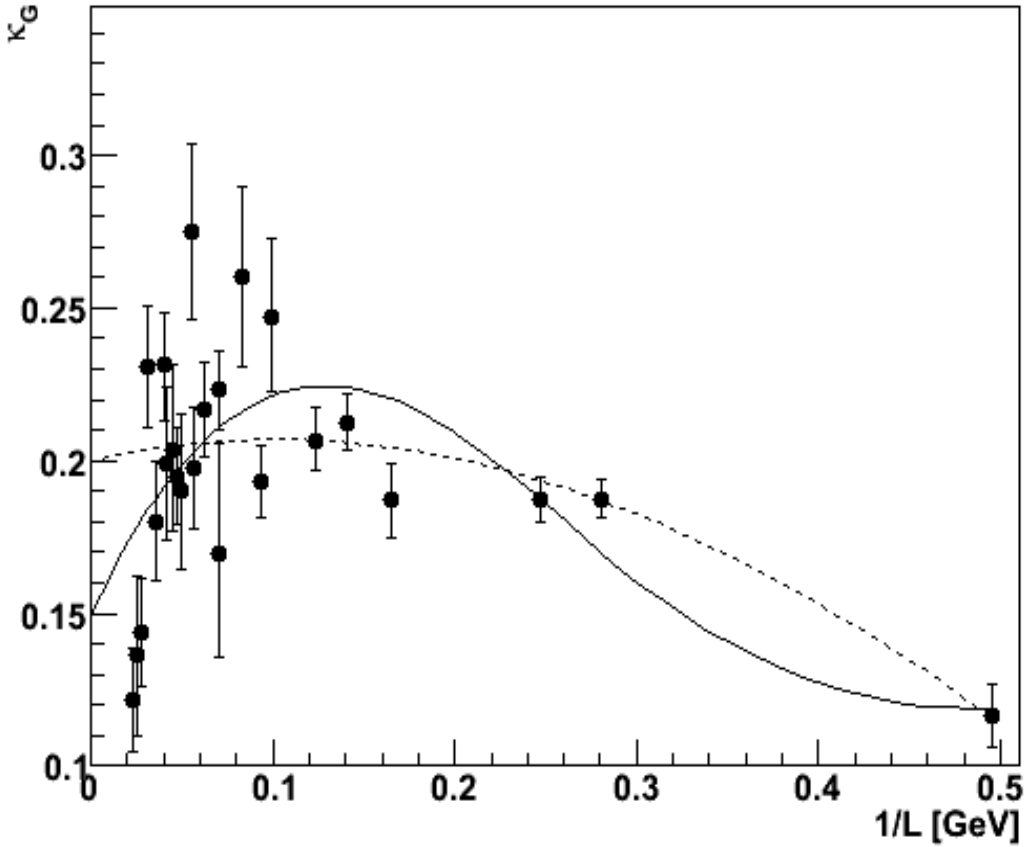
Ghost propagator and finite volumes

Ghost infrared exponent



[Maas, PRD 2007]

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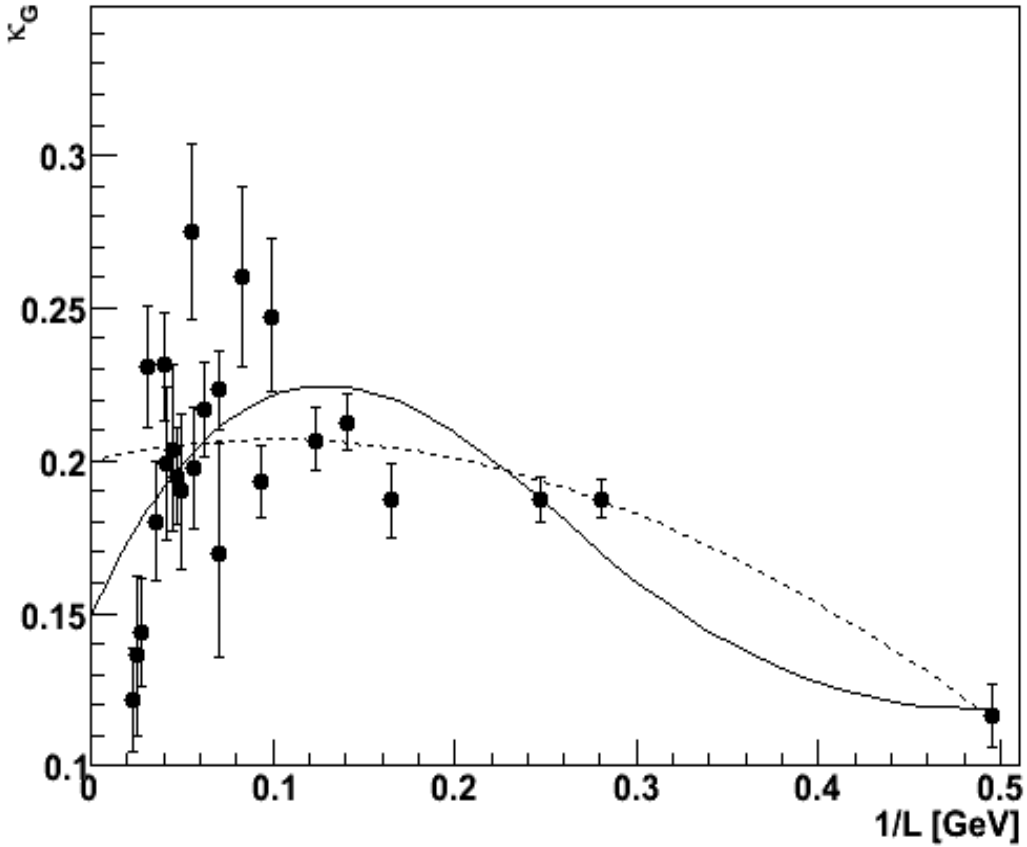

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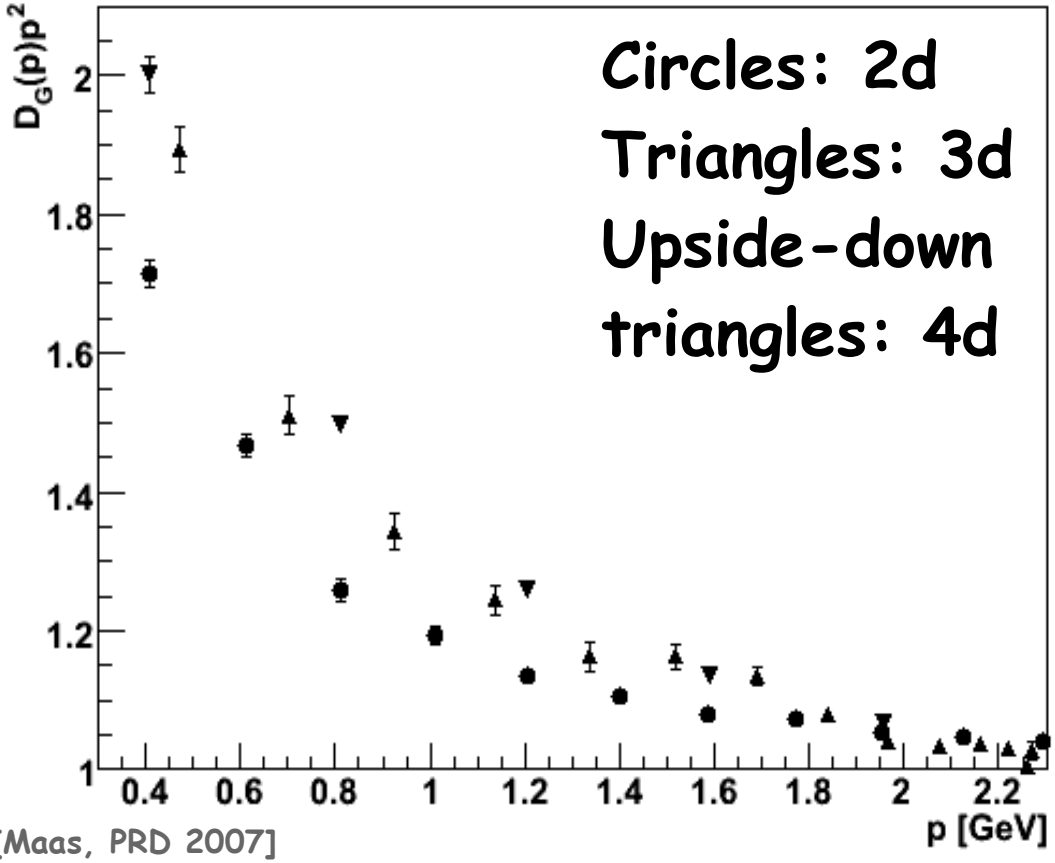

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Gluon propagator

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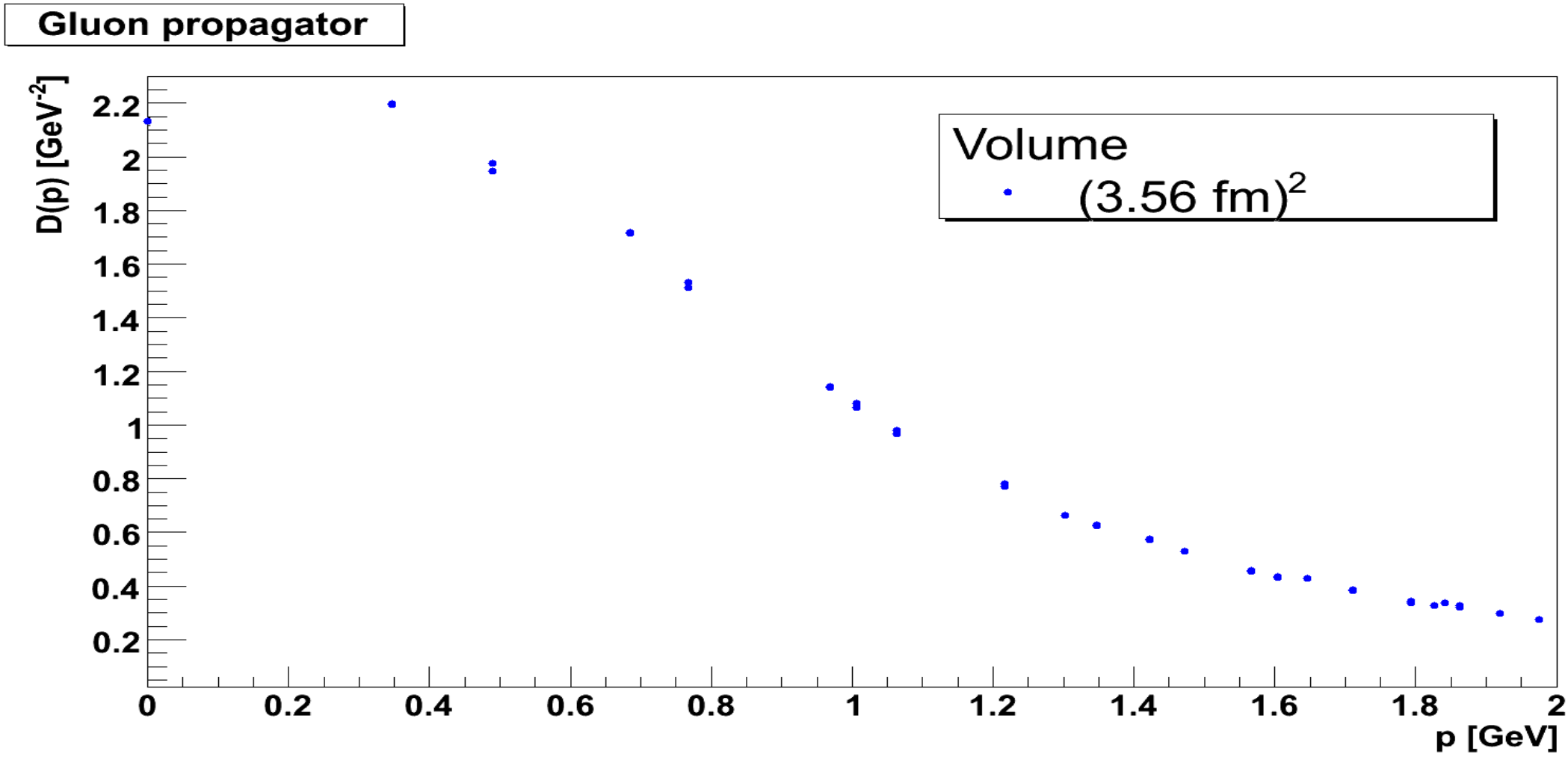
- Situation complicated by finite volume effects

Very large lattices needed

[Maas, PRD 2007]



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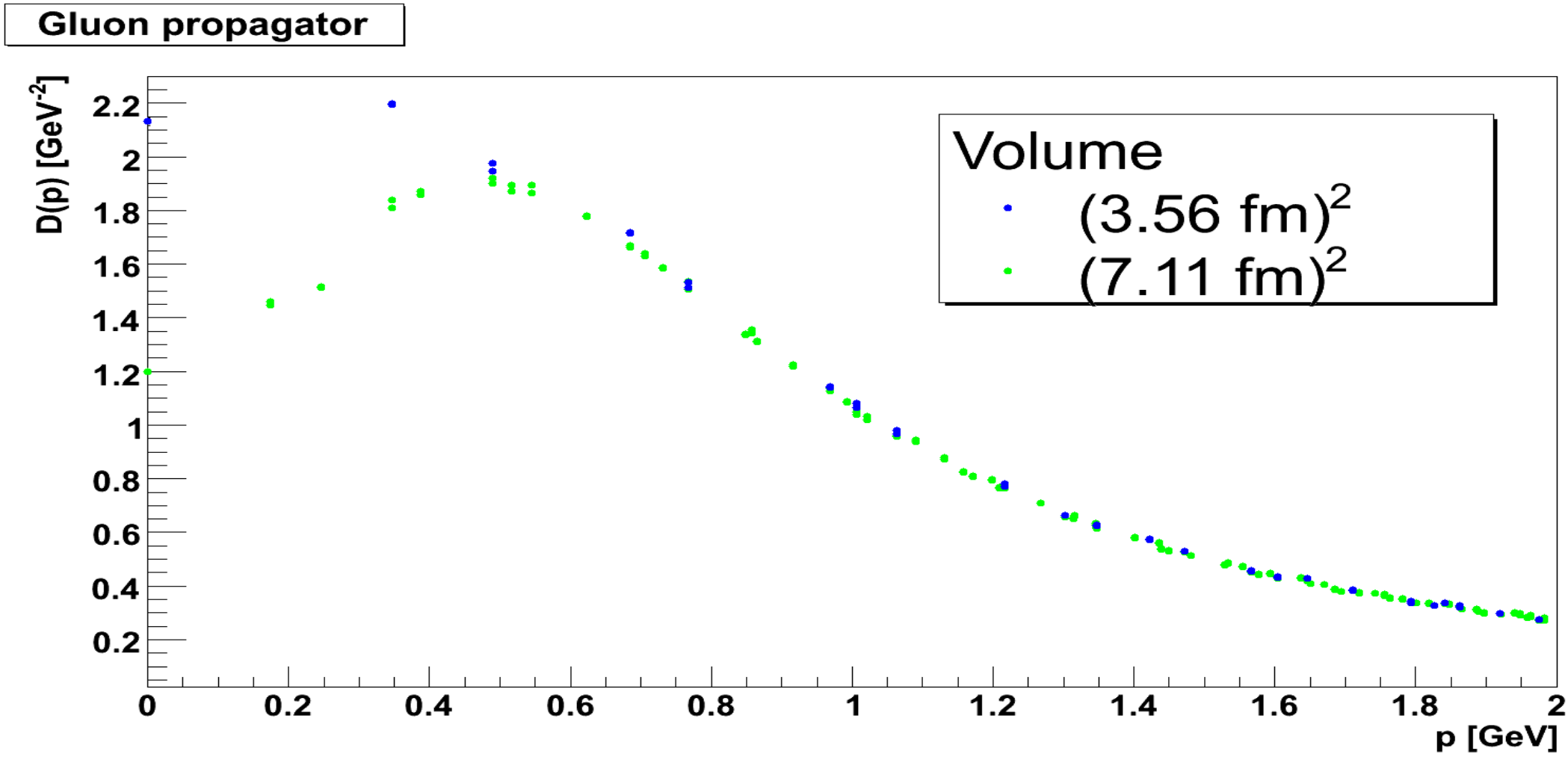

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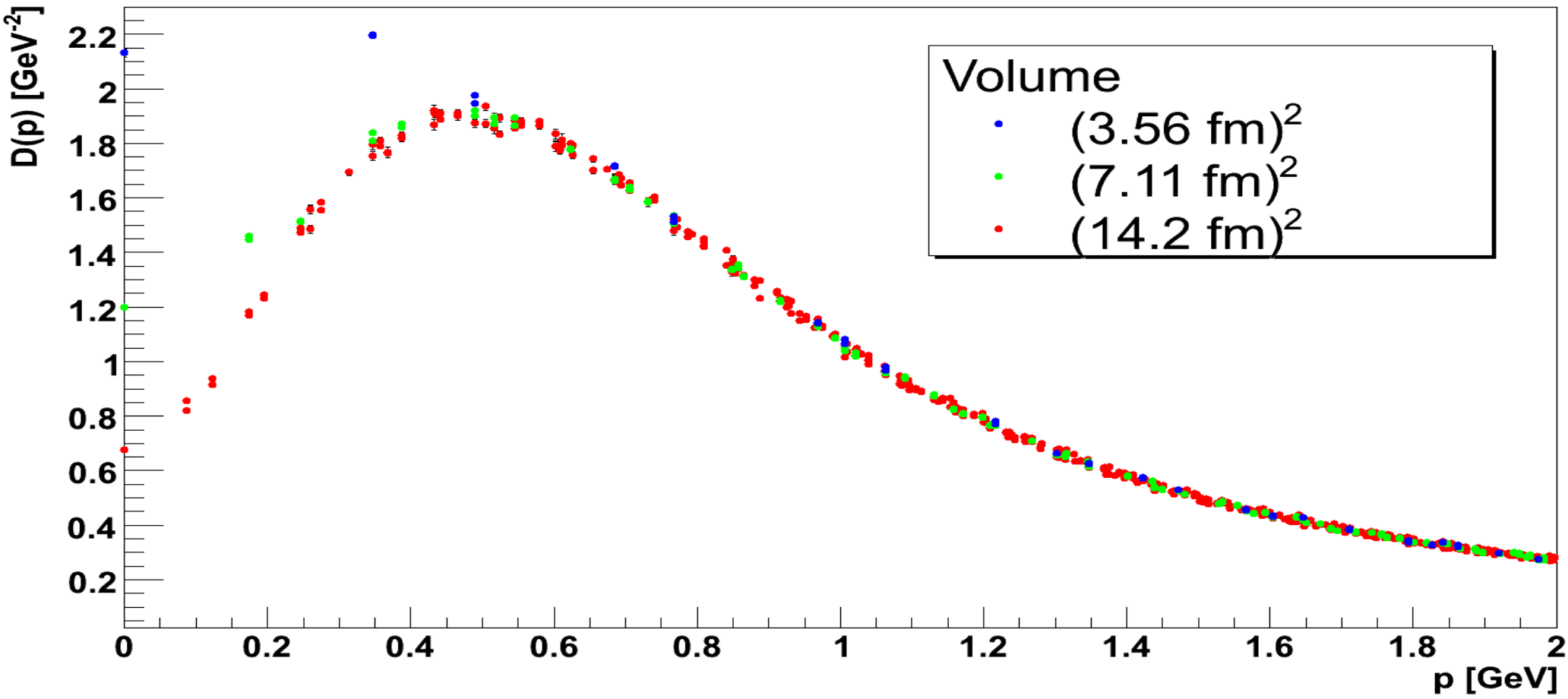

Very large lattices needed: 2d

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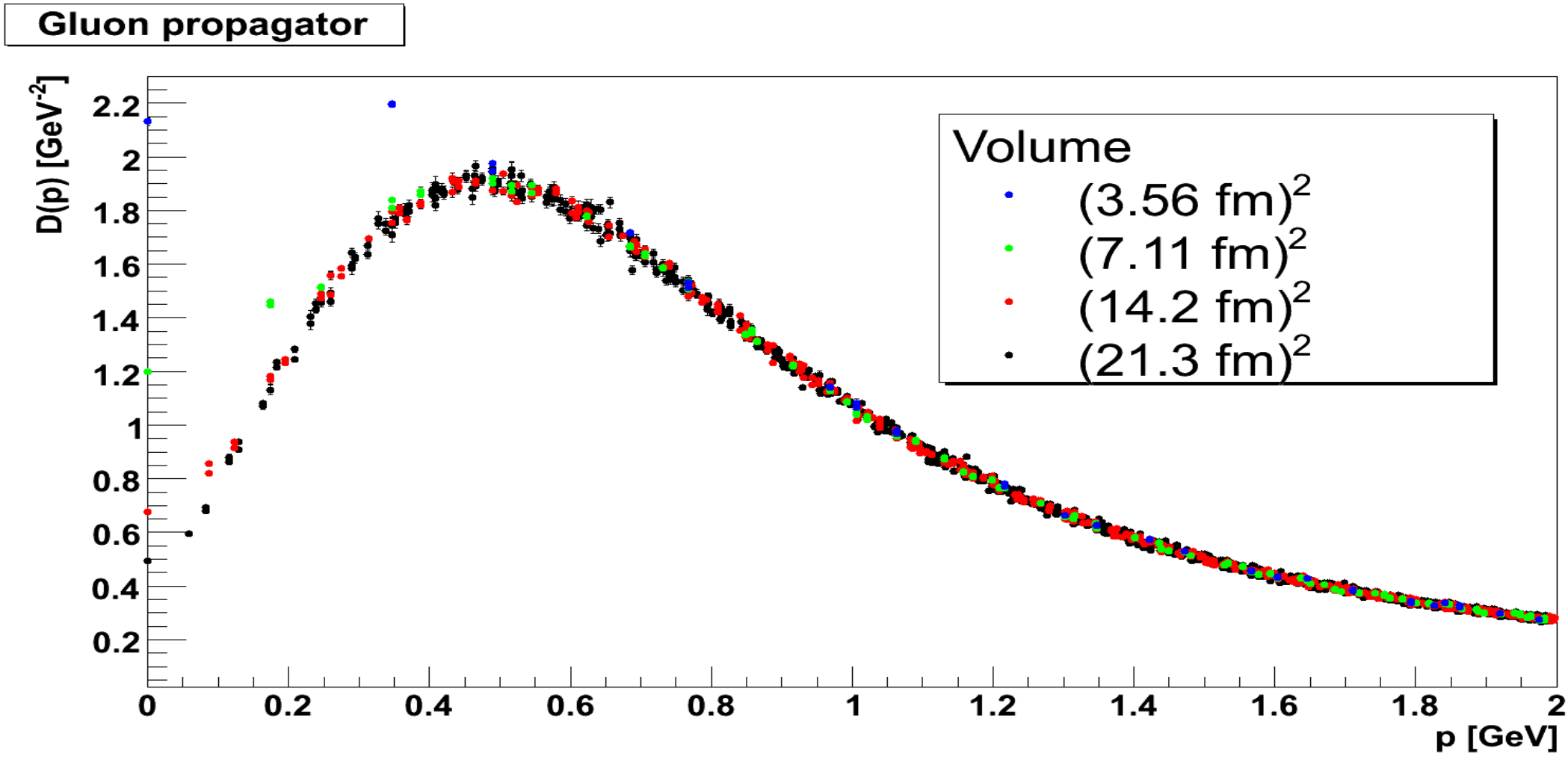

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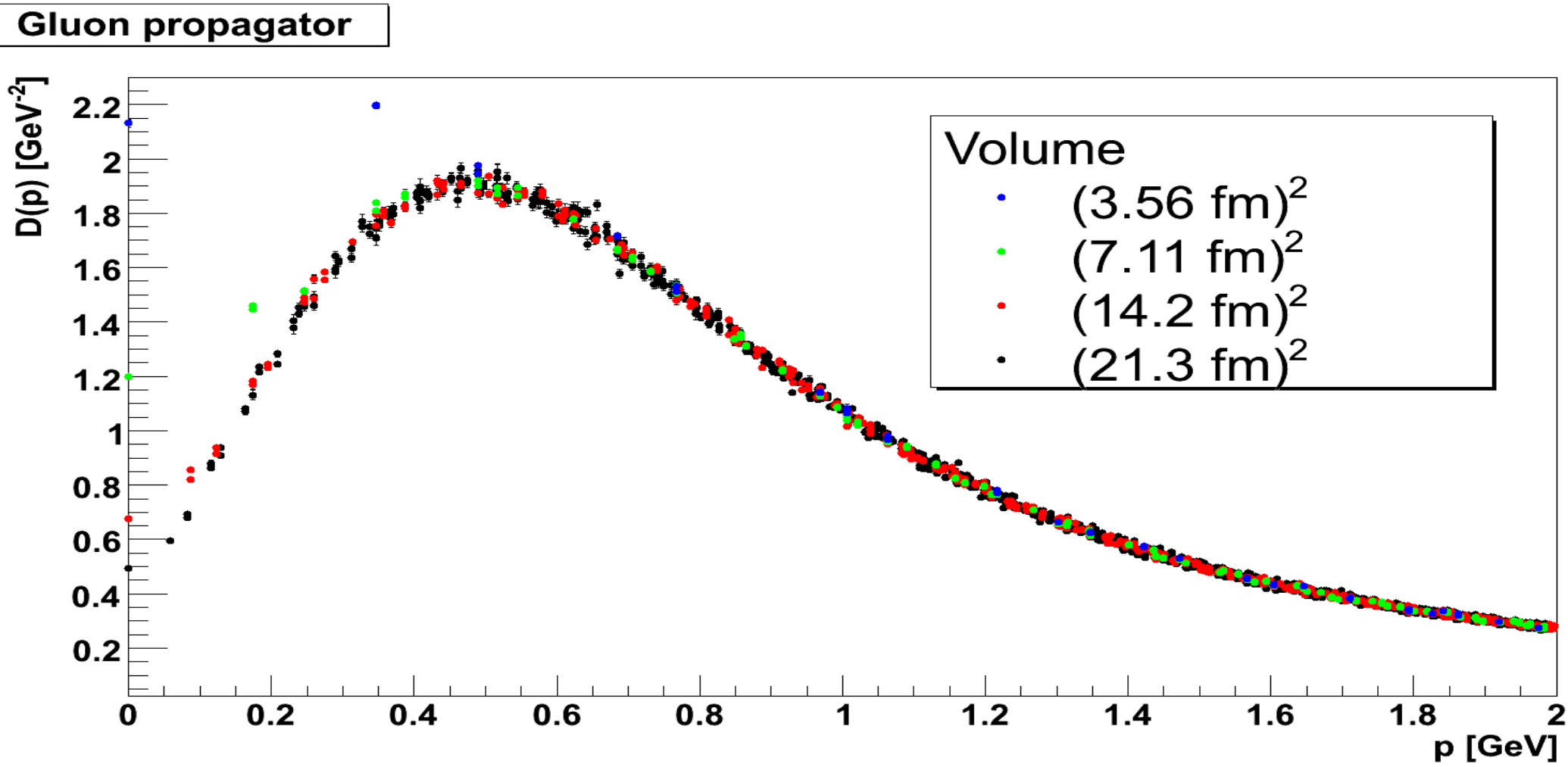
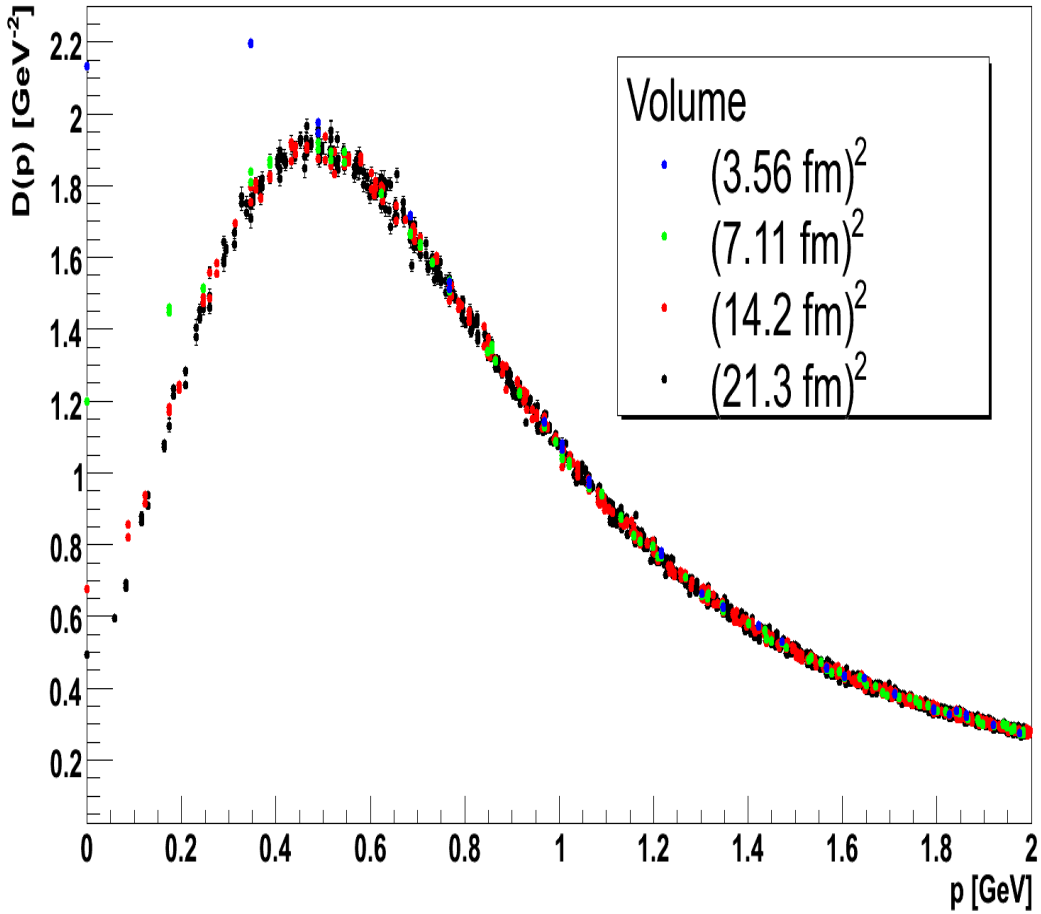
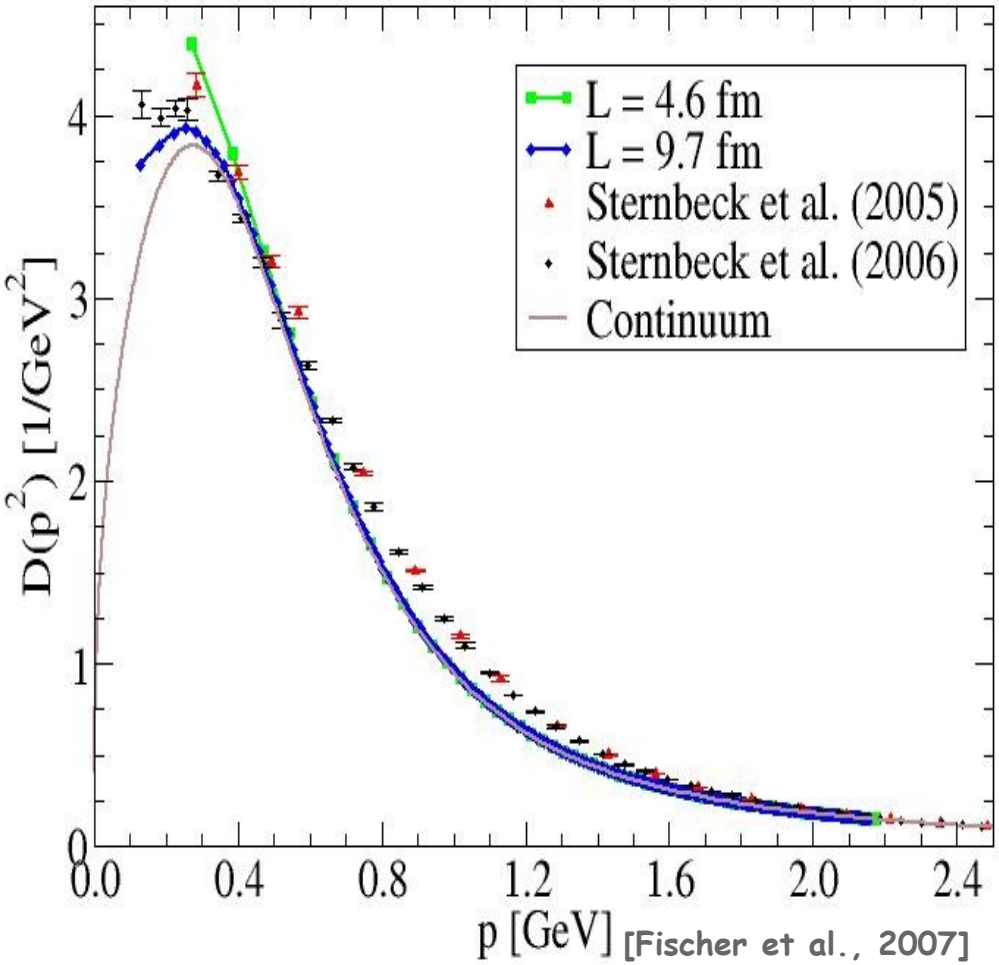


- Propagator goes from diverging to massive to infrared suppressed
- In accordance with functional methods

Lattice vs. DSE in a finite volume

Gluon propagator





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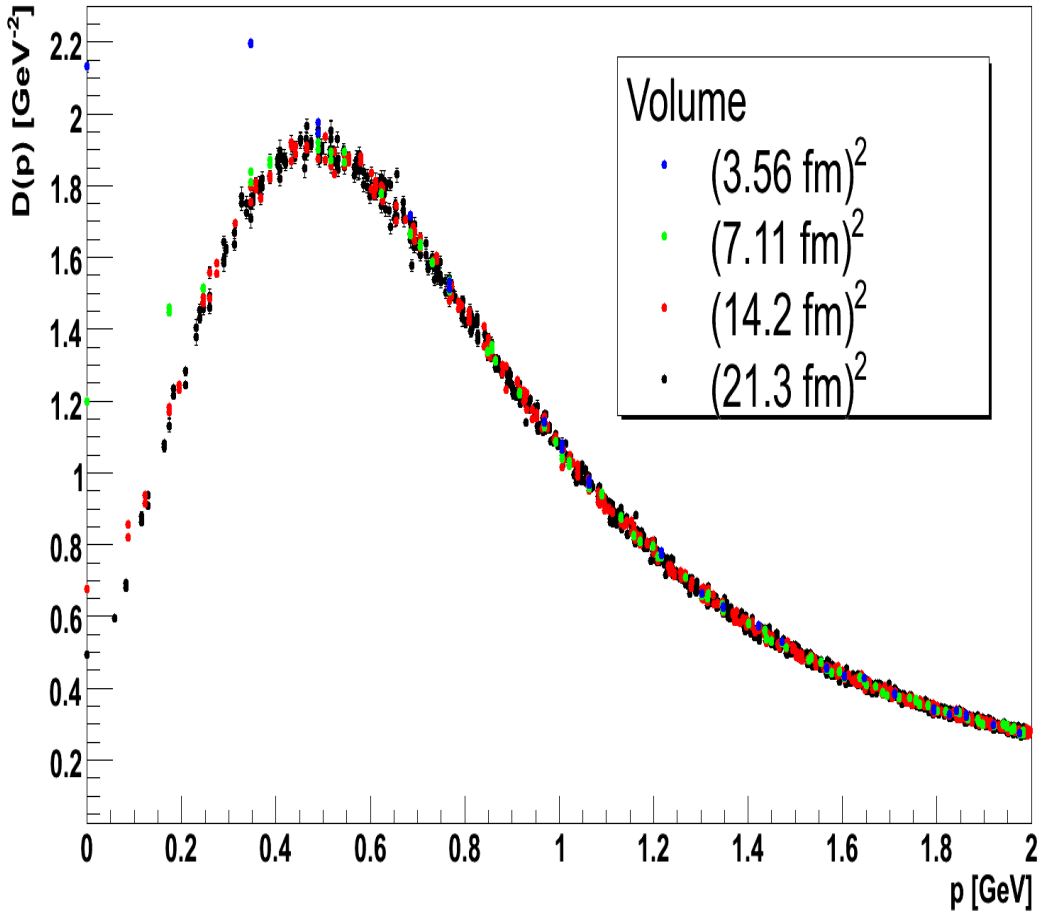

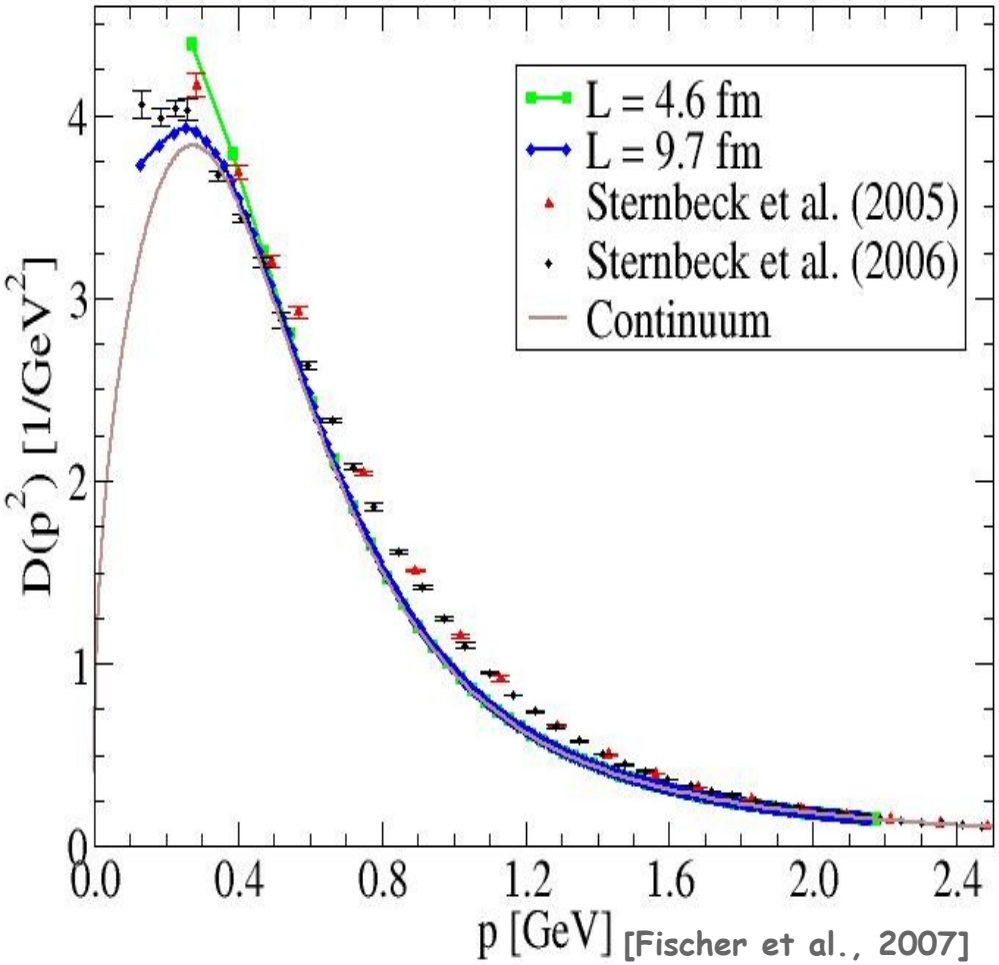

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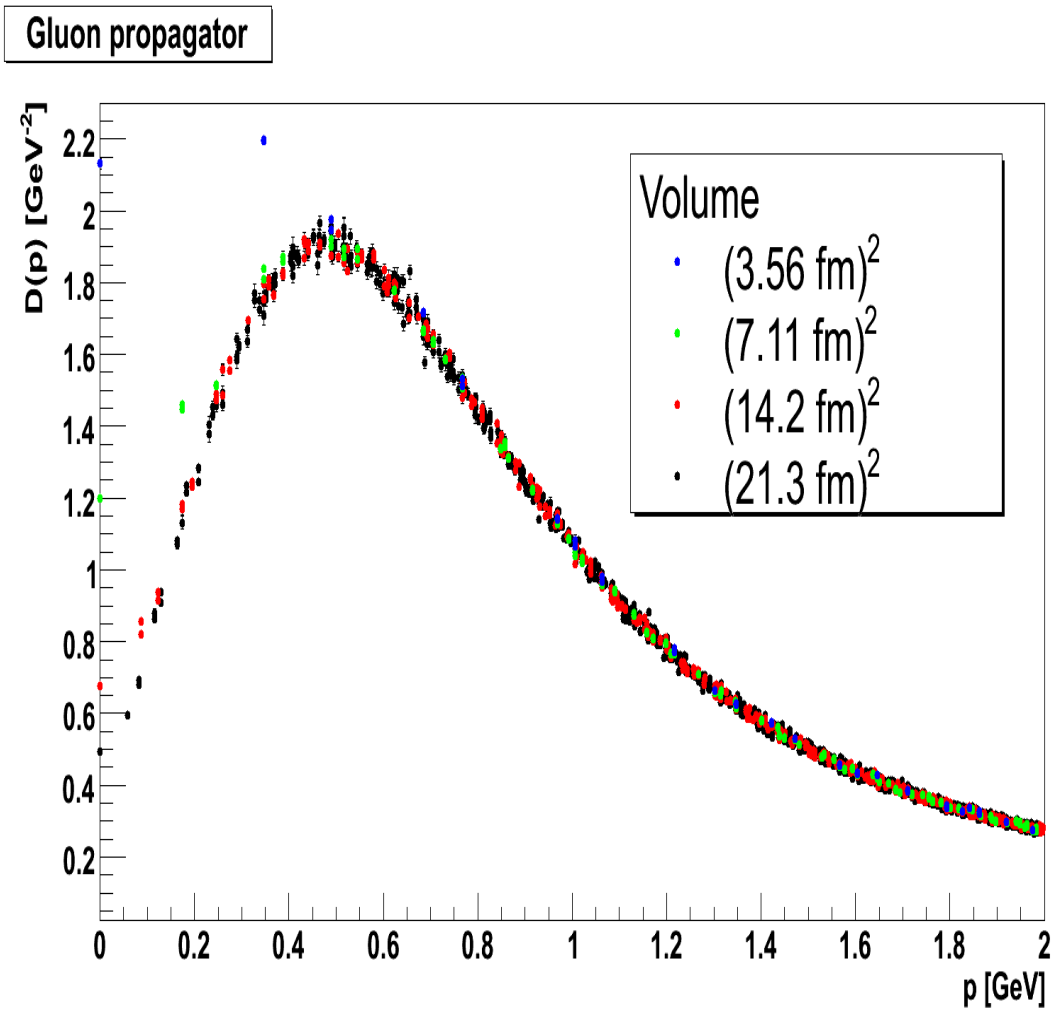

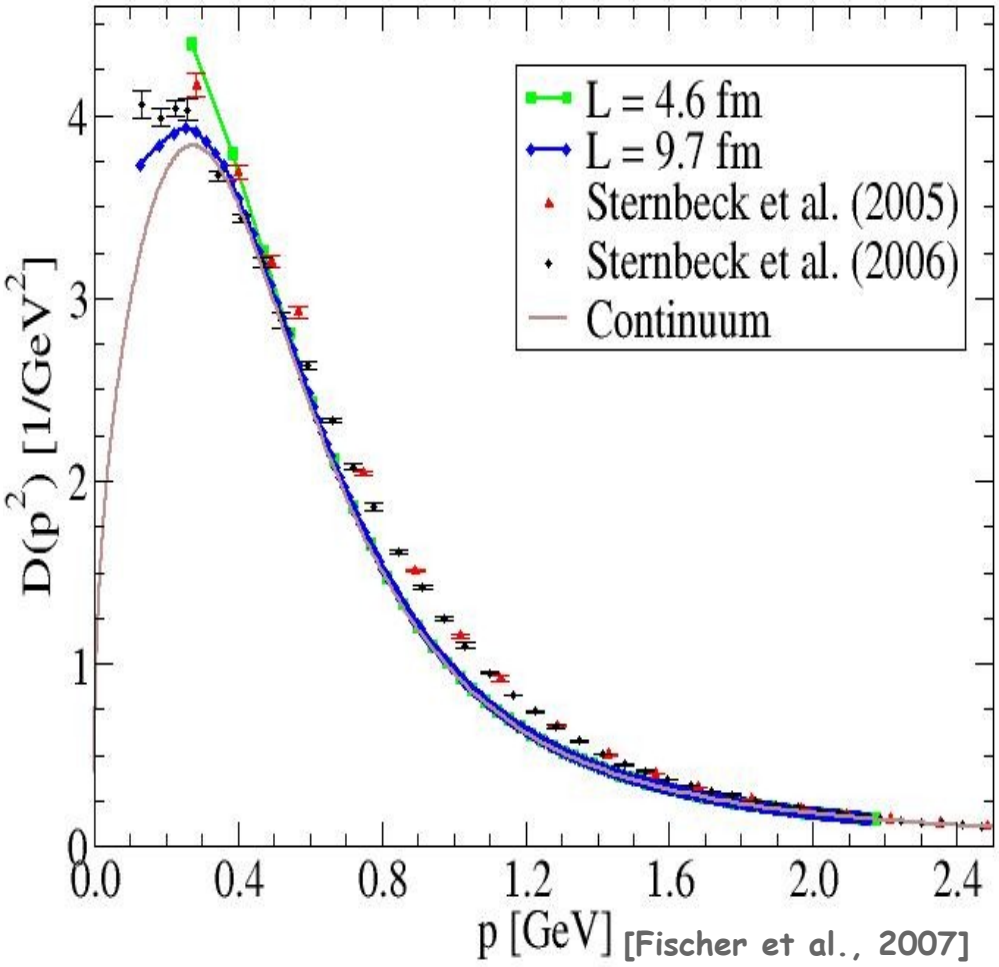

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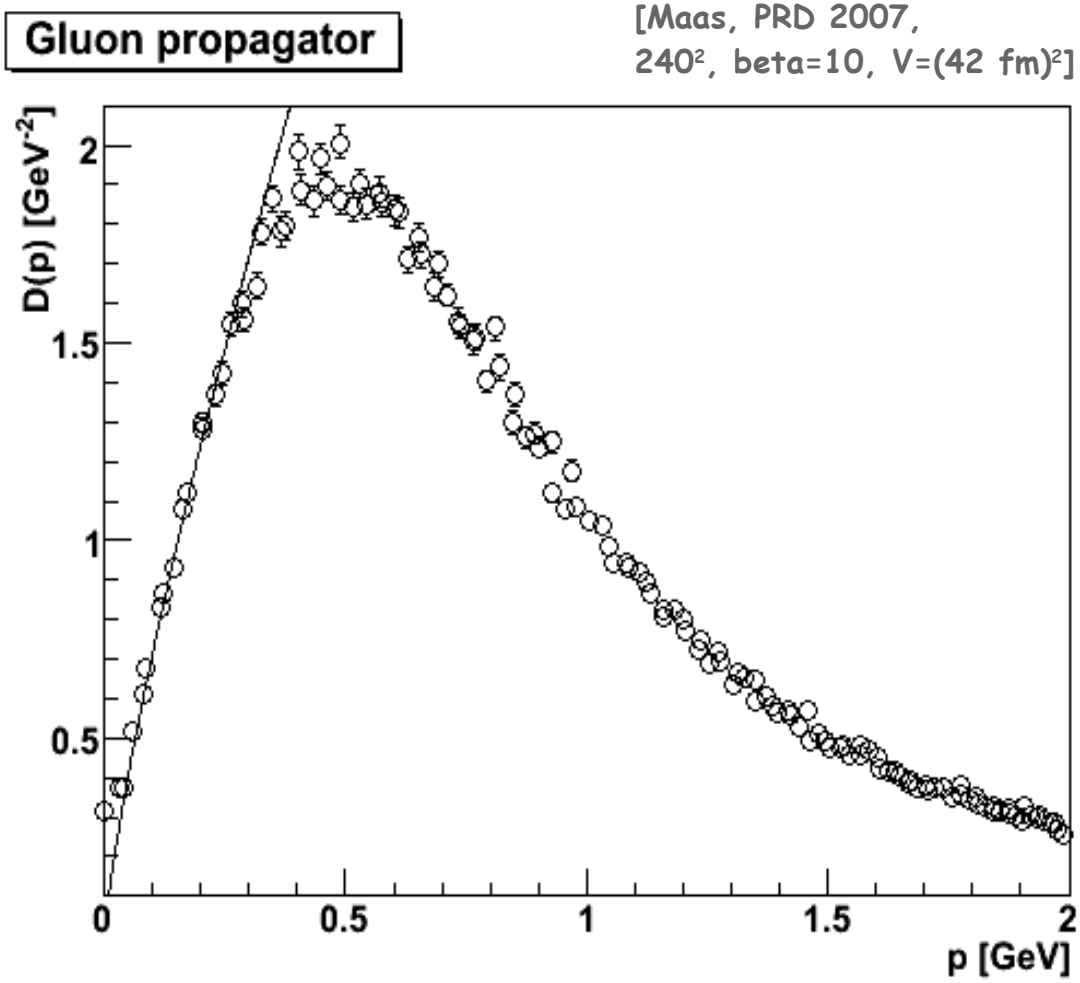
- Qualitative similar behavior
- Relevant length scale about 10-15 fm

Gluon propagator in 2d



- Strong infrared suppression

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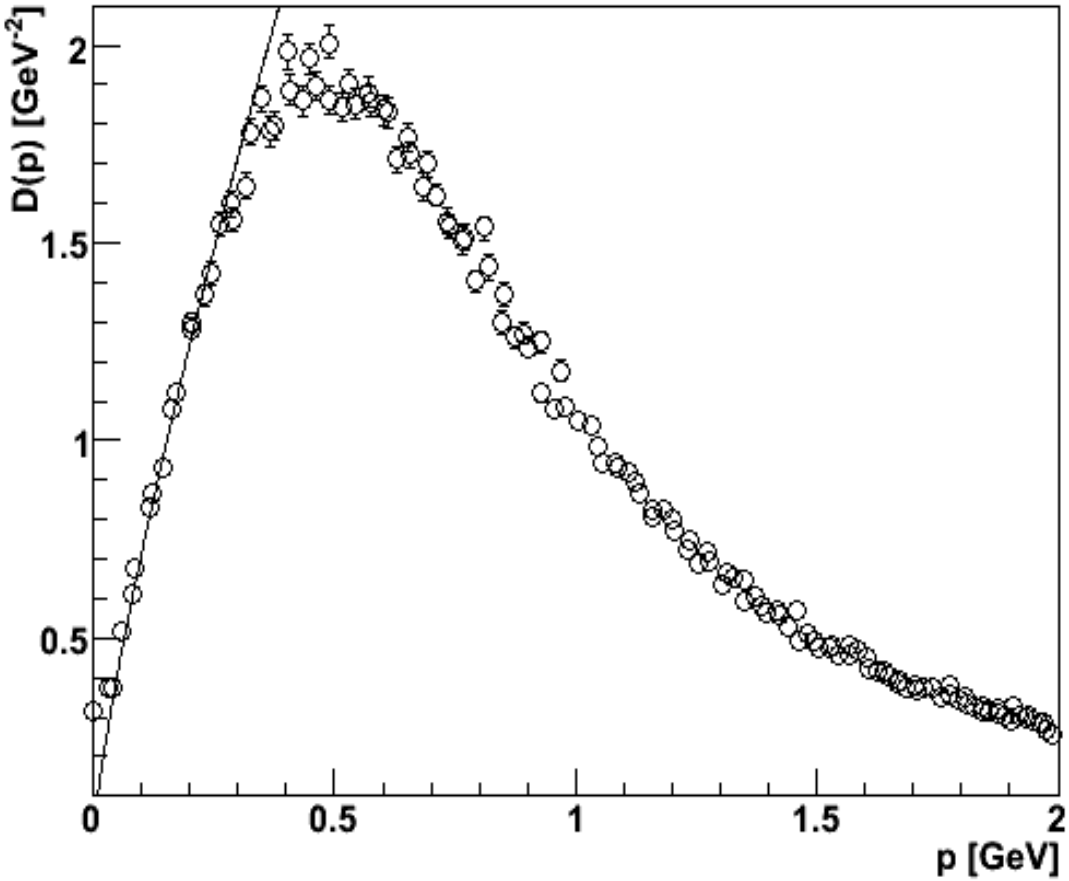
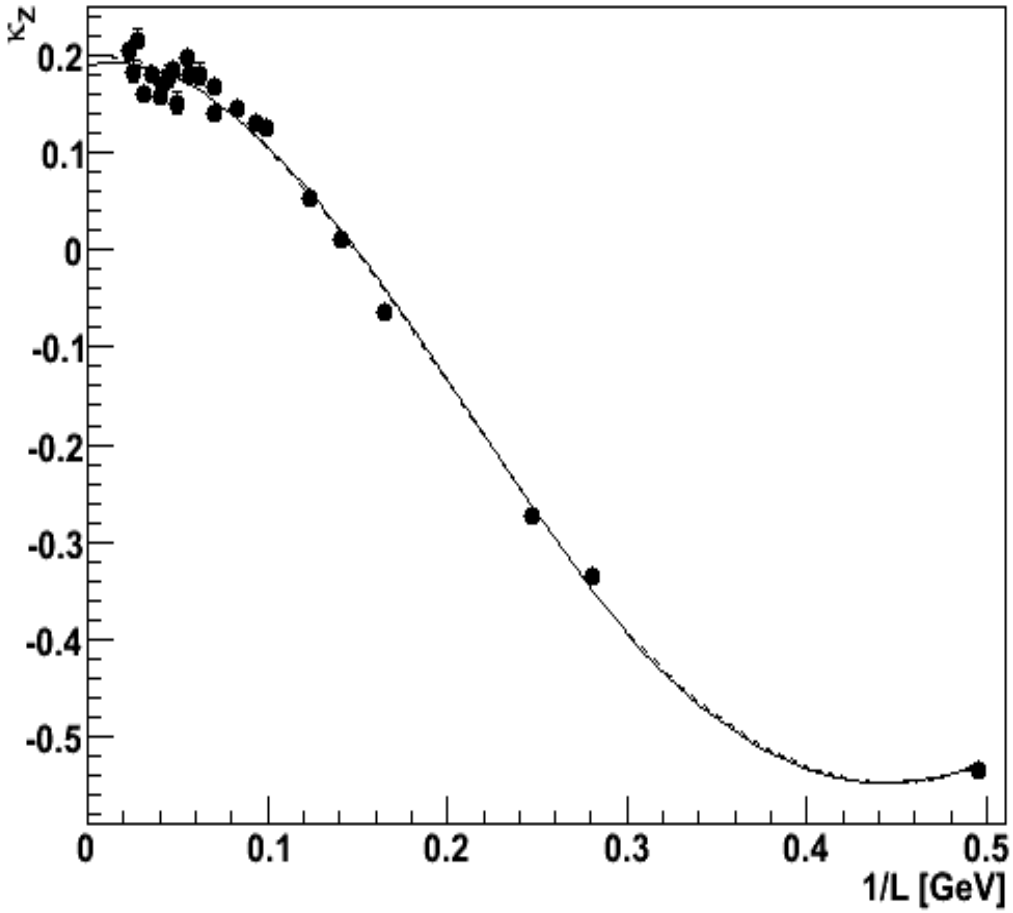
[Maas, PRD 2007,
240^2, beta=10, V=$(42\ fm)^2$]



Gluon infrared exponent

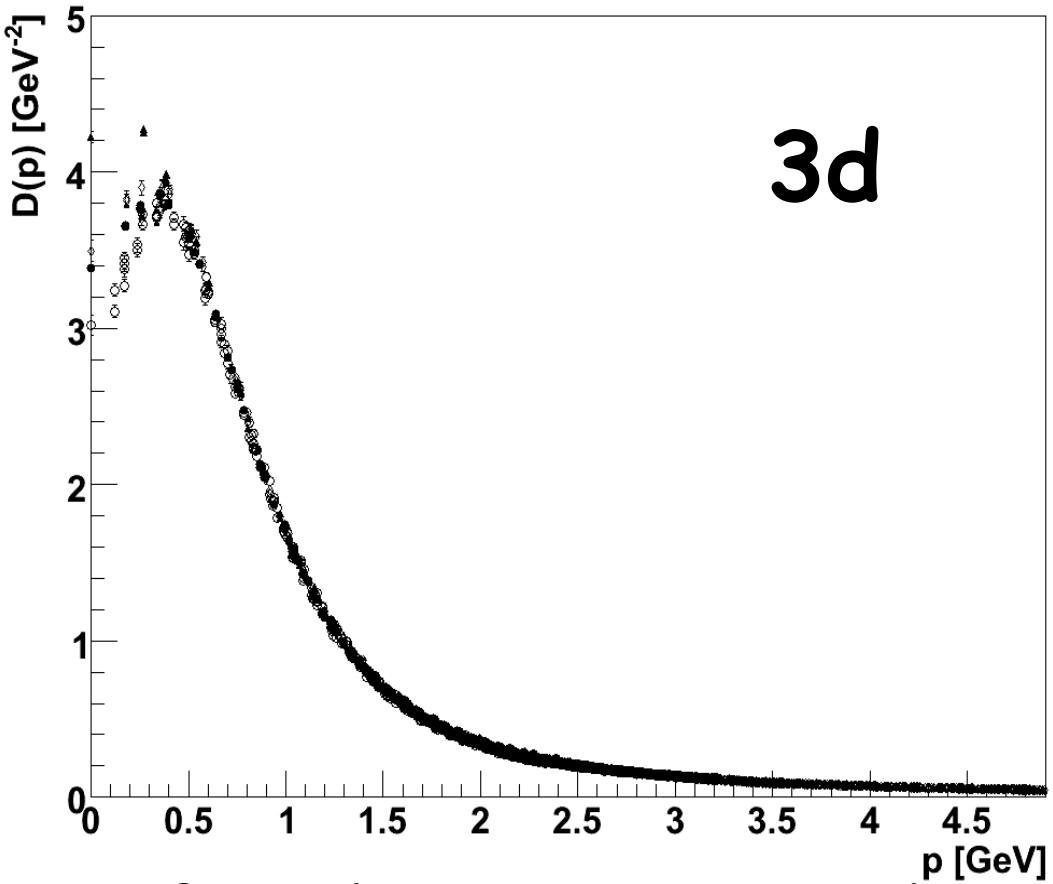


- Strong infrared suppression
- Exponent reaches the predicted numerical value

Gluon propagator in higher dimensions

Gluon propagator

[Cucchieri et al., unpublished
40^3, 60^3, beta=4.2, 6.0]



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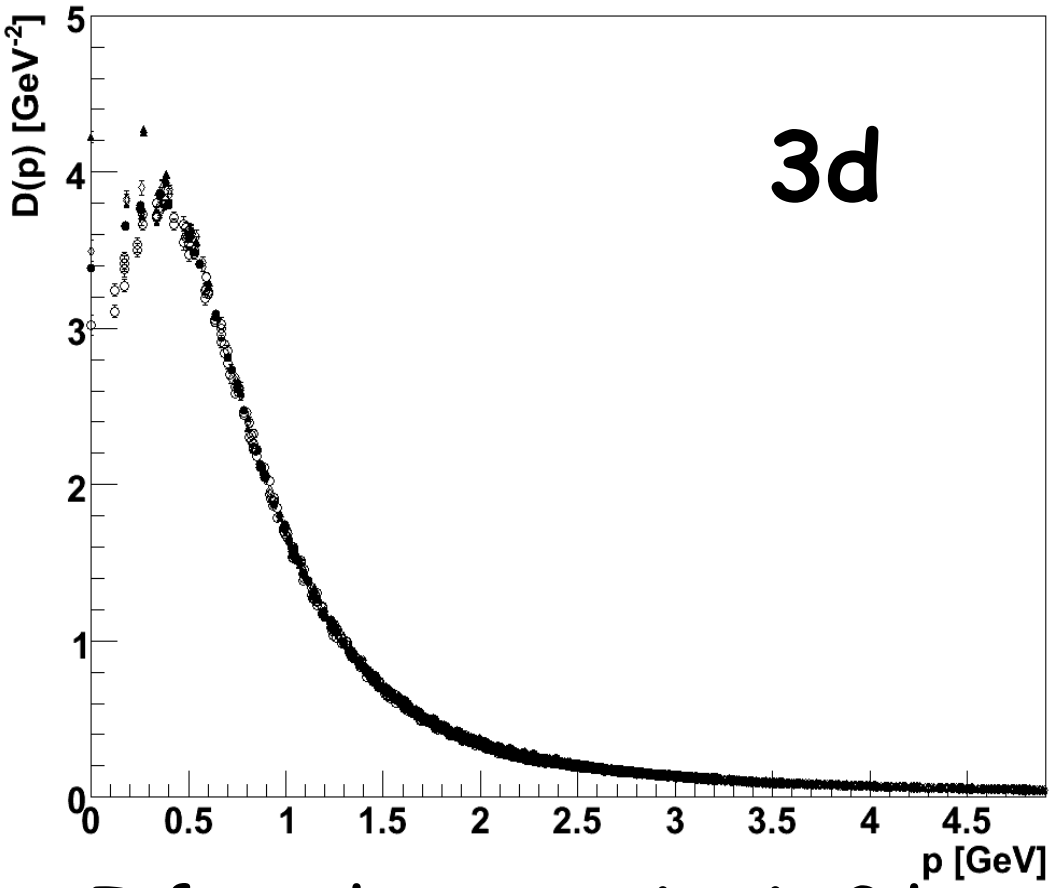

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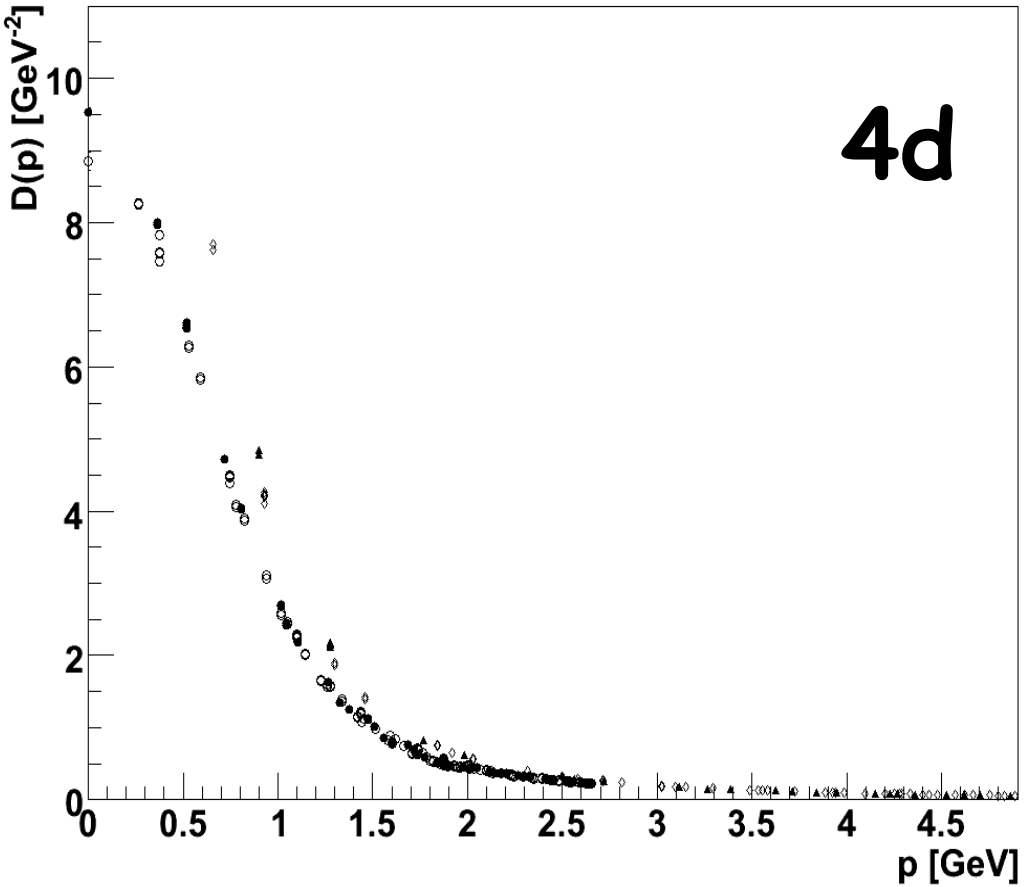

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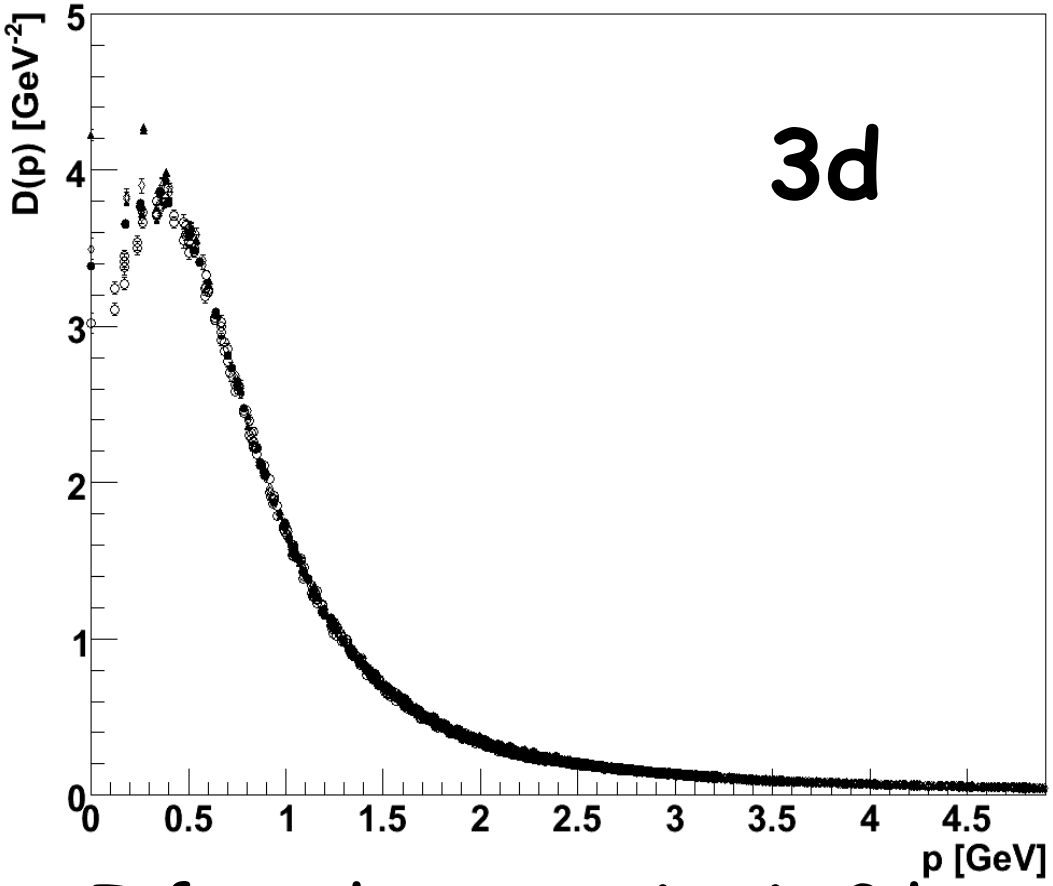

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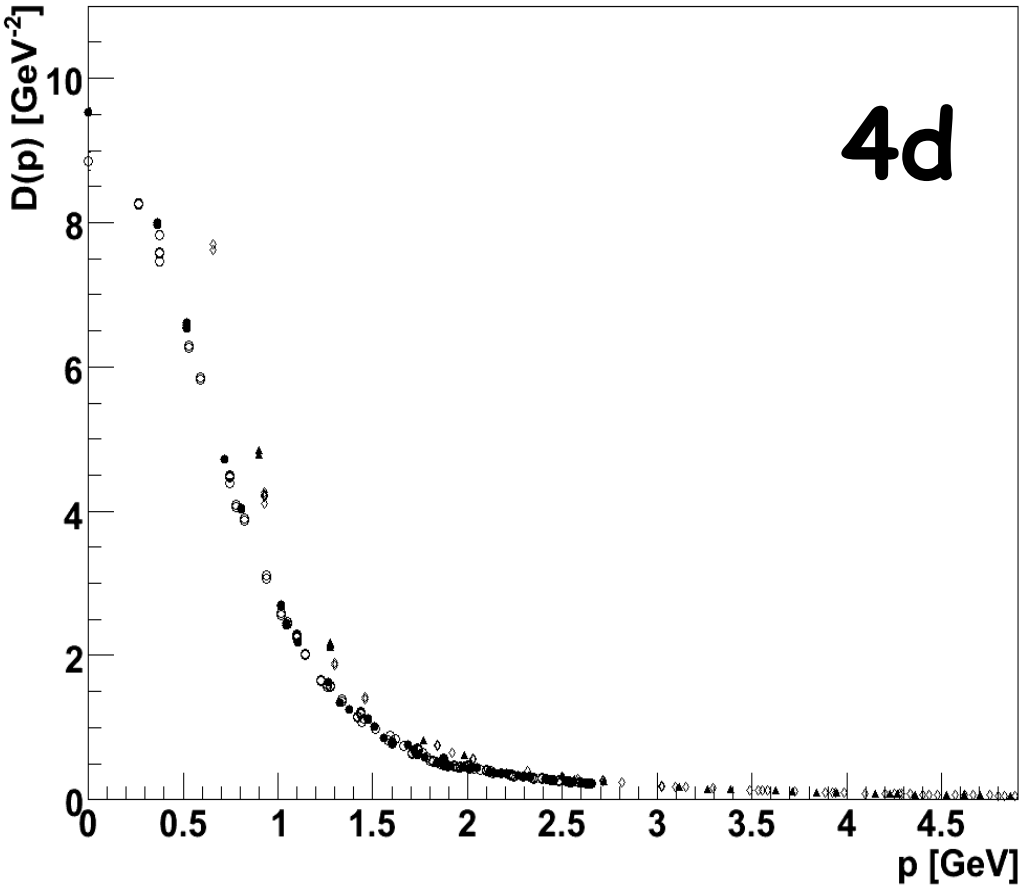

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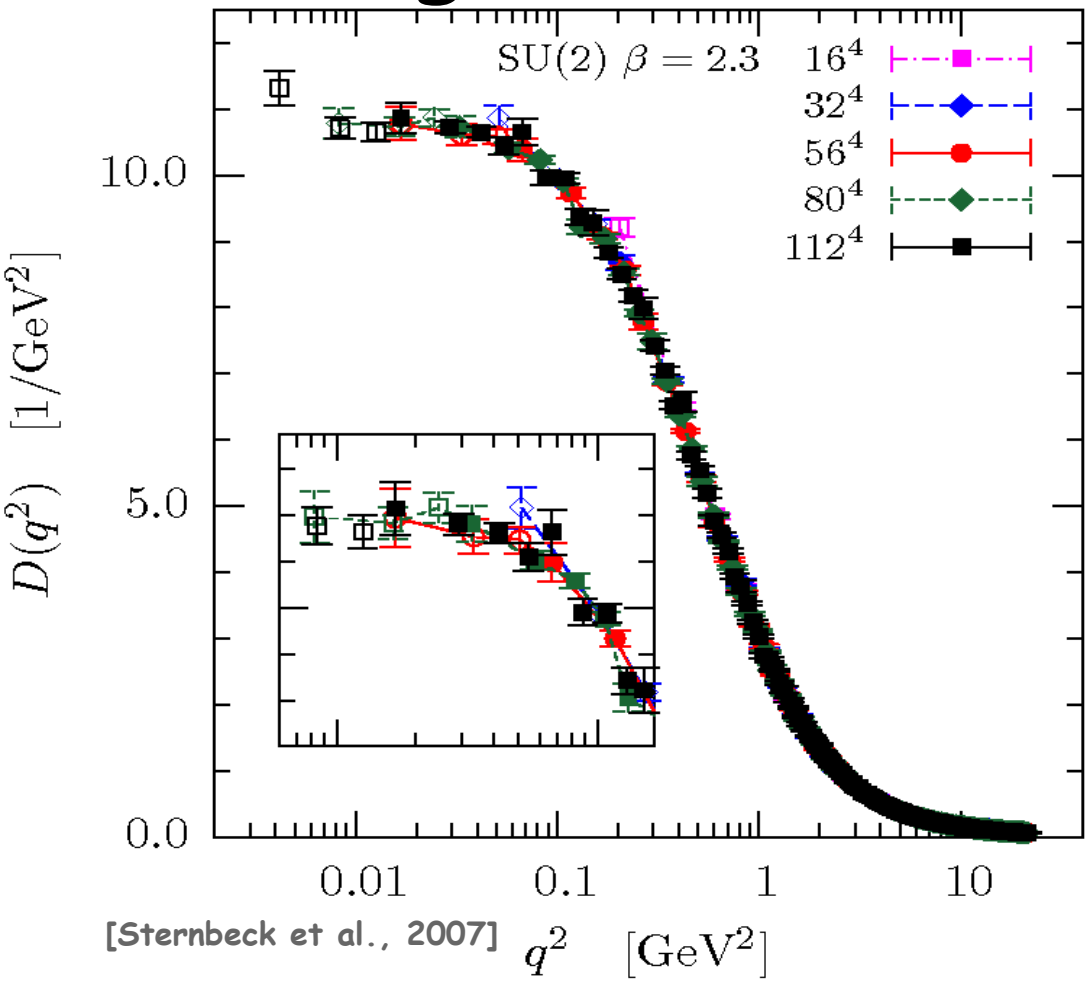
Gluon propagator in 4d and large volumes



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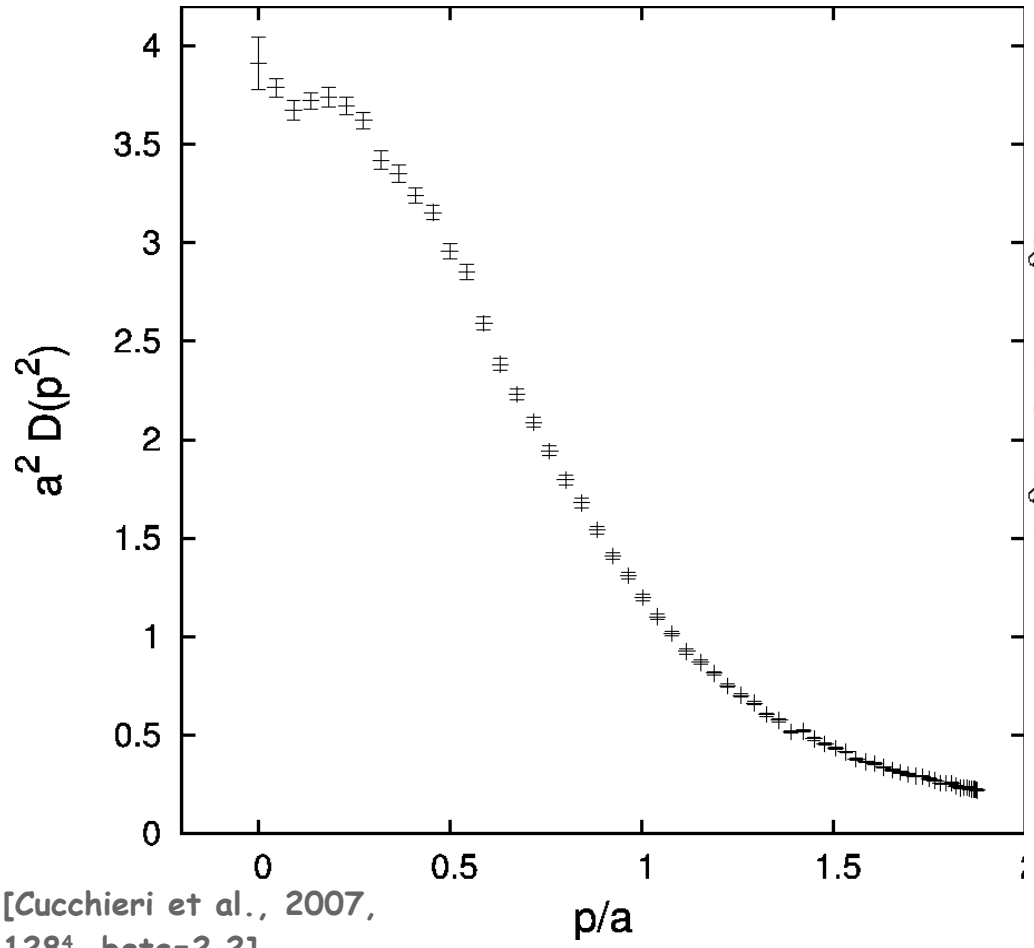

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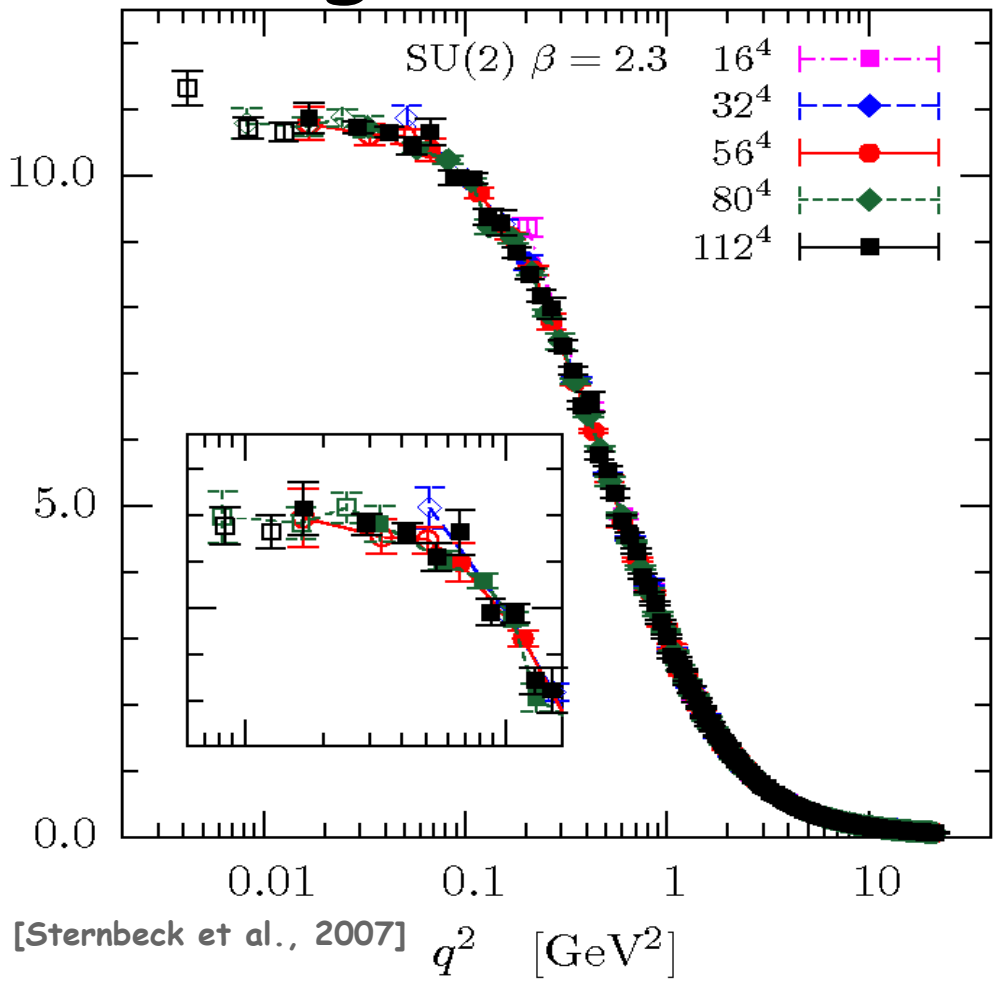

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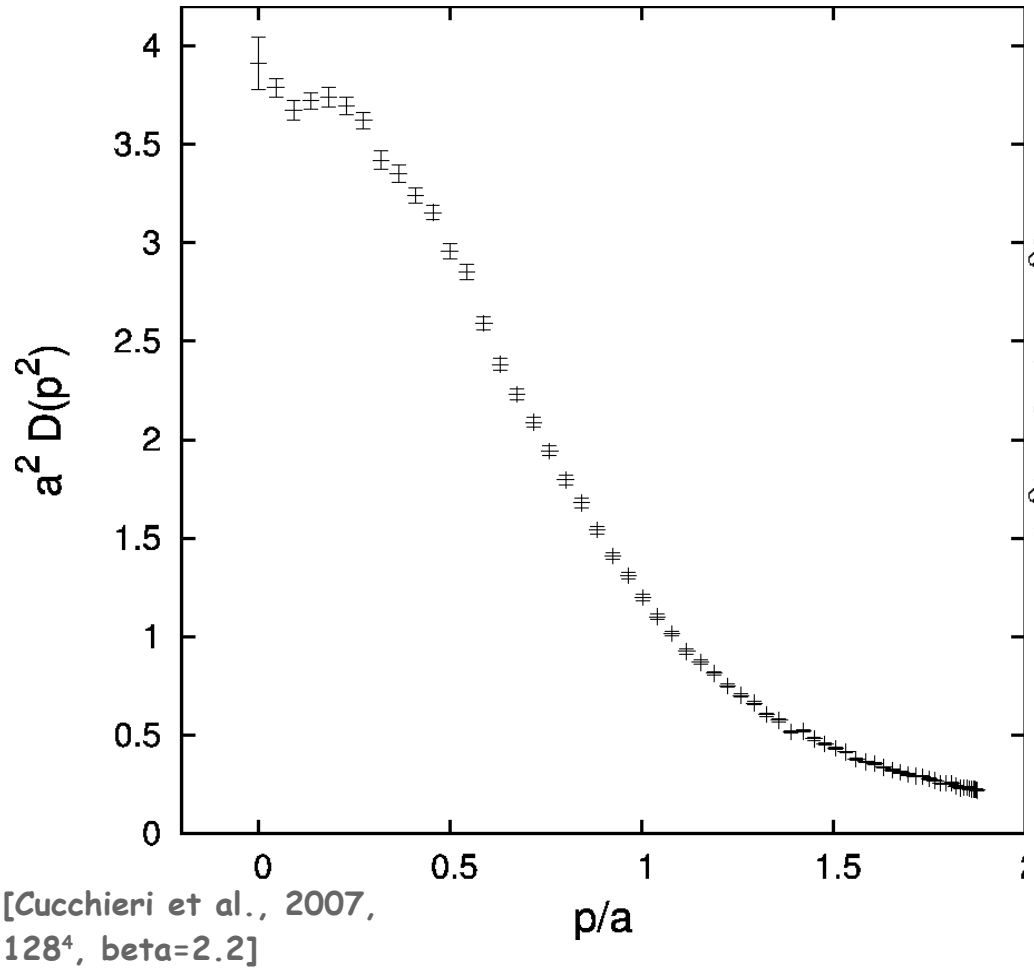

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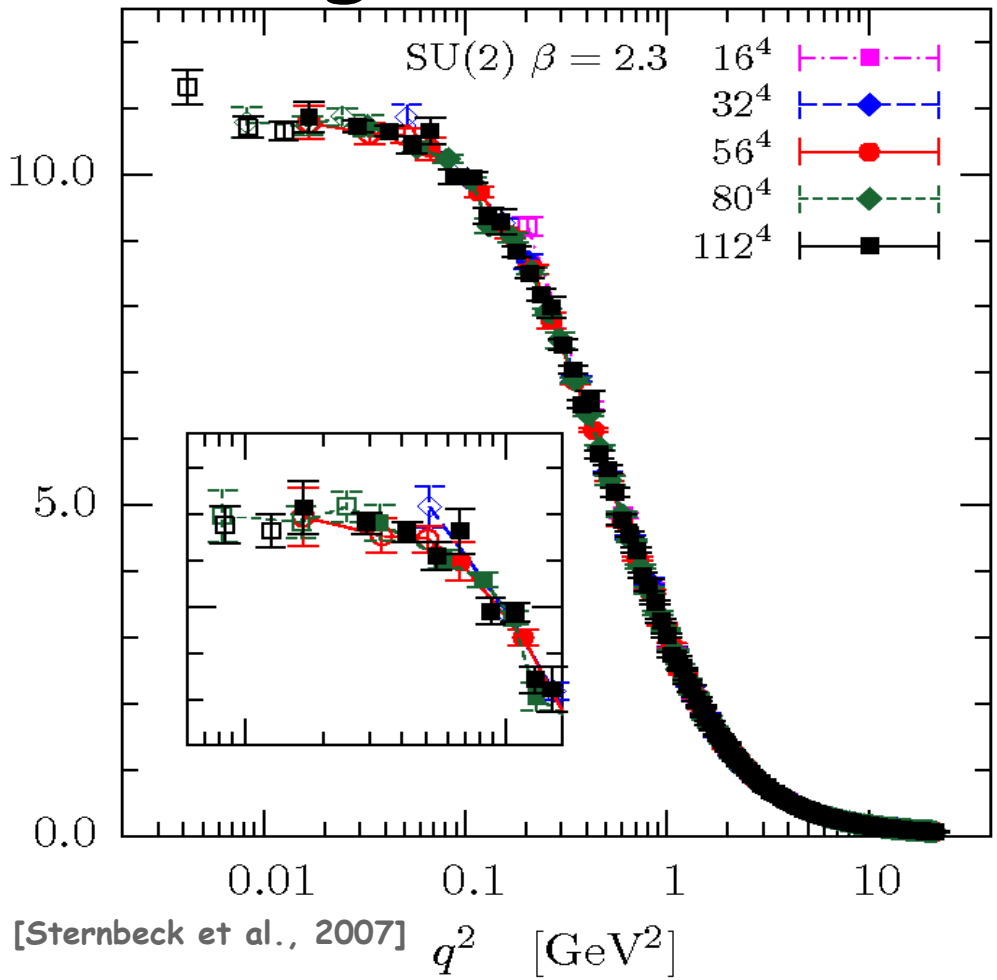

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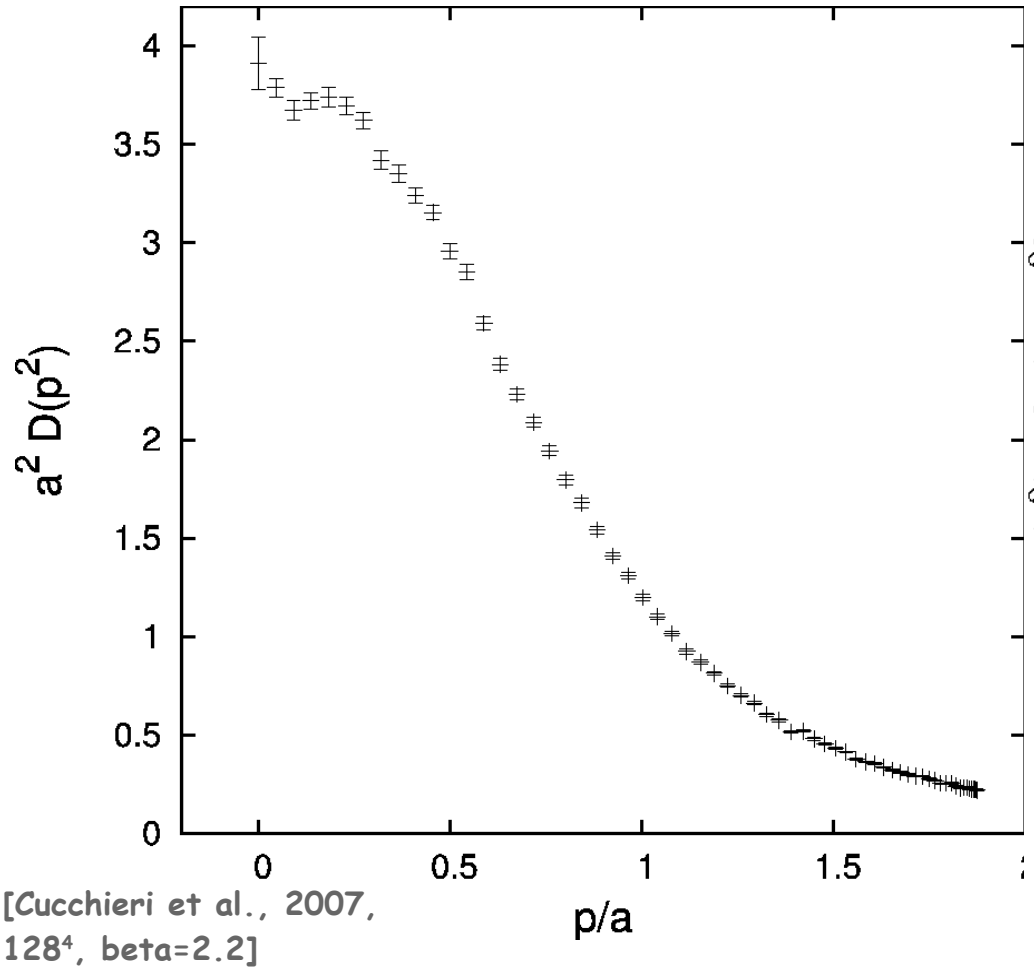

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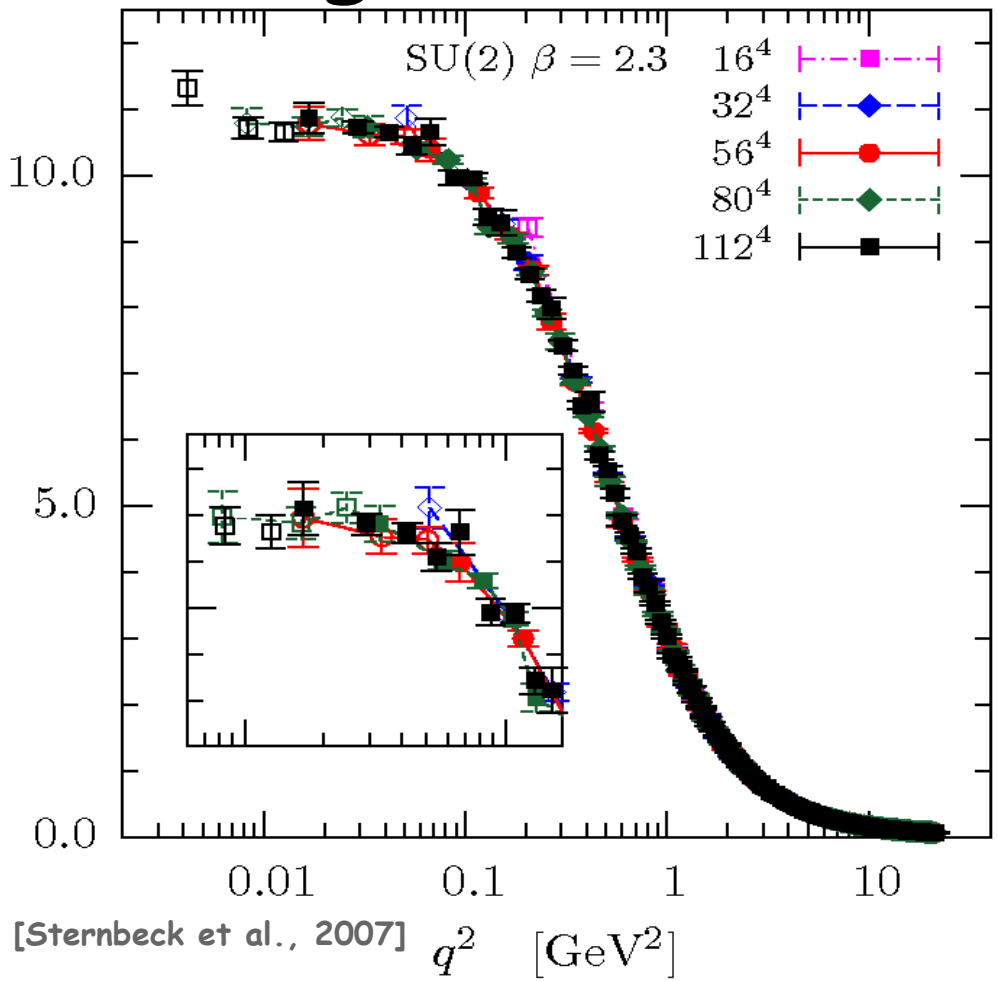

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- Genuine or other artifacts (Gribov-copies?) relevant?

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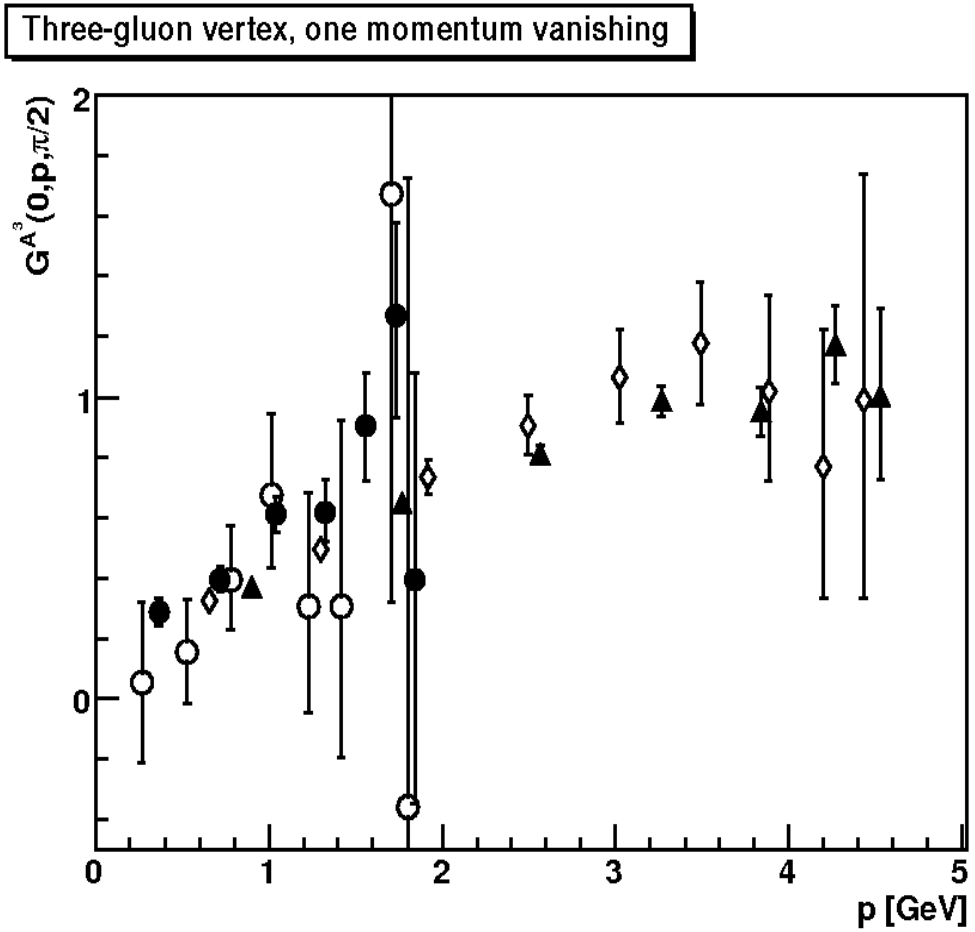
Three-gluon vertex in 4d



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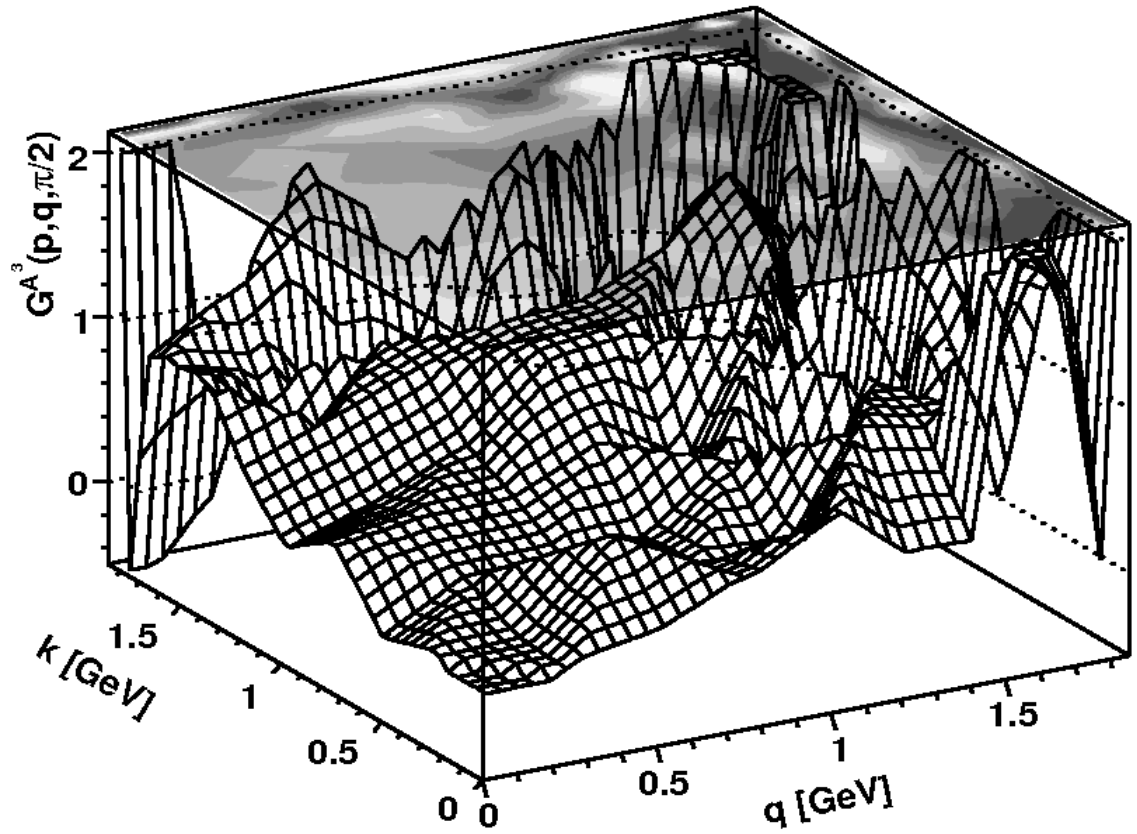
Three-gluon vertex in 4d

Three-gluon vertex, orthogonal momenta



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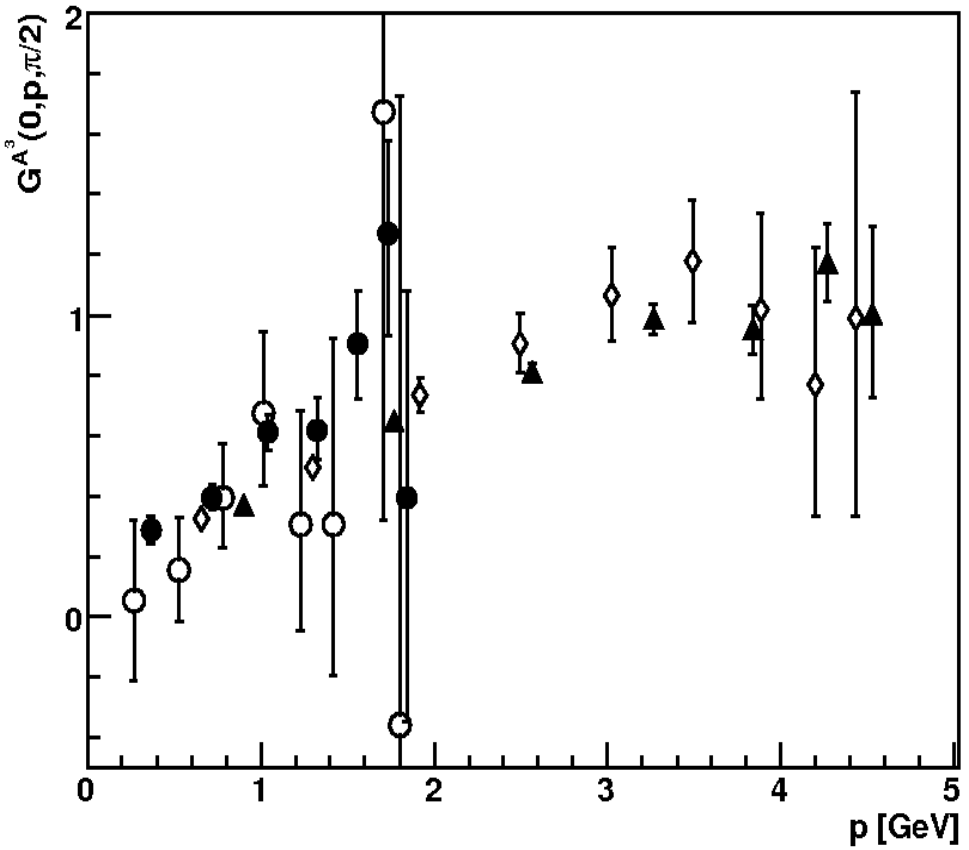

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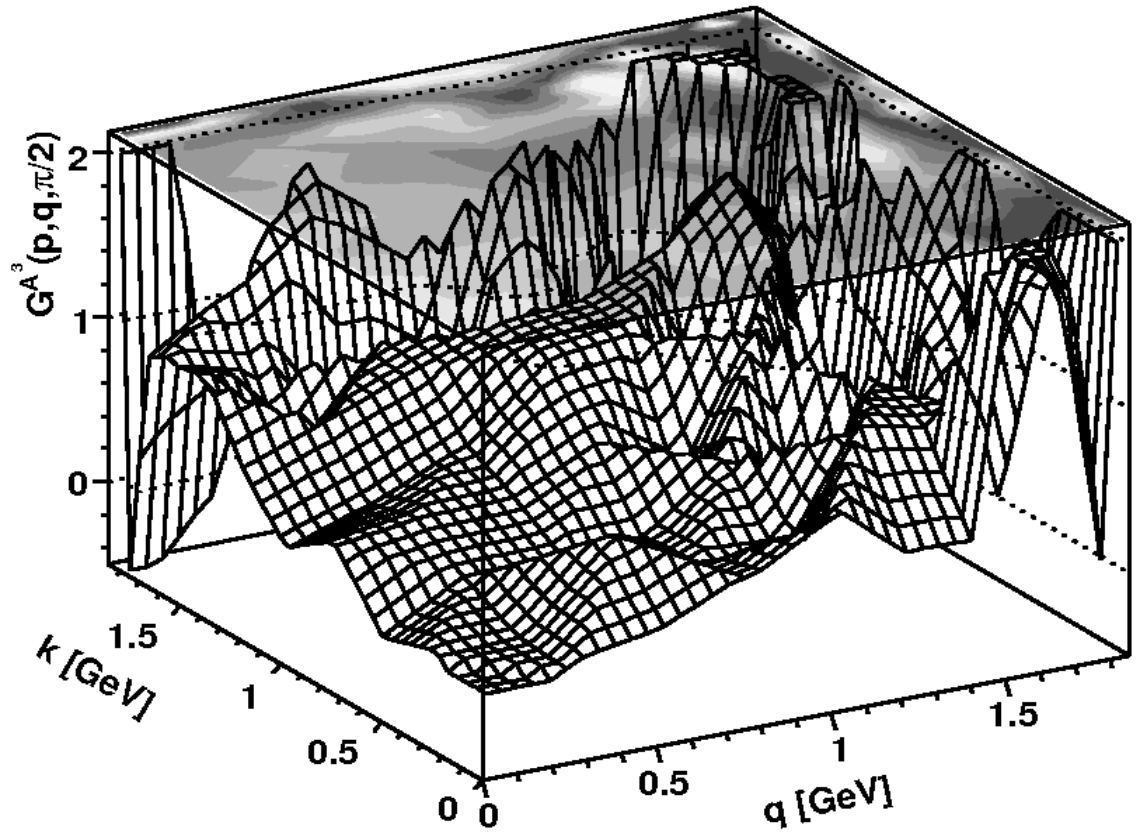

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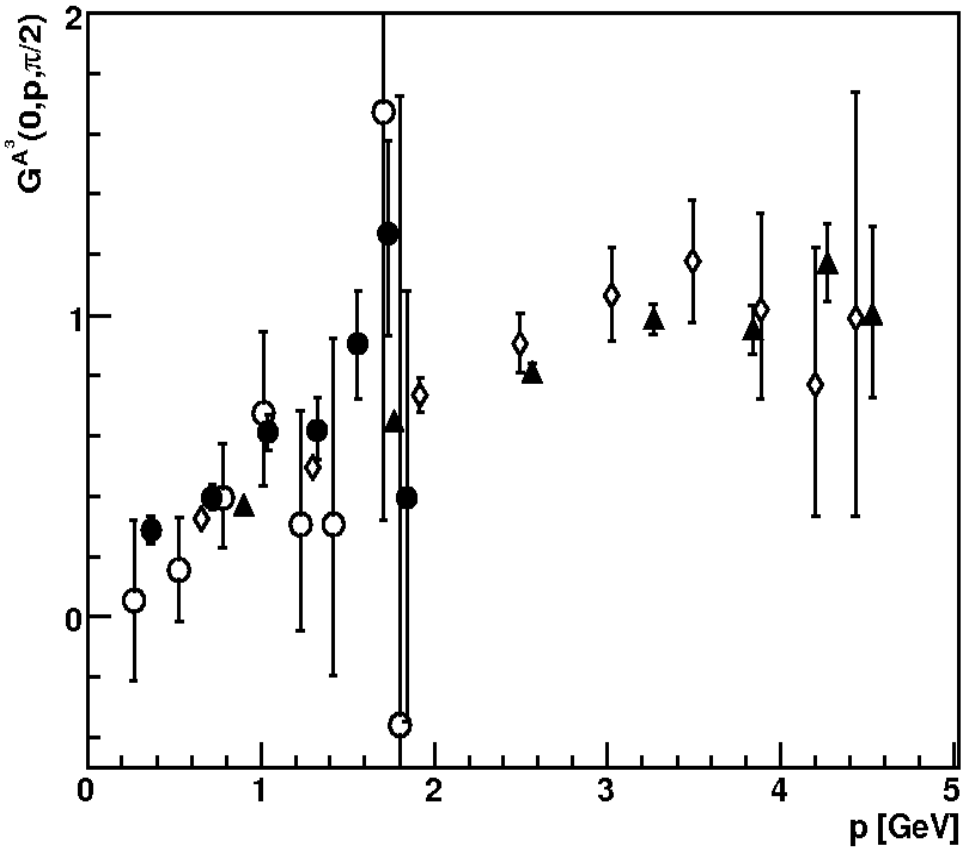

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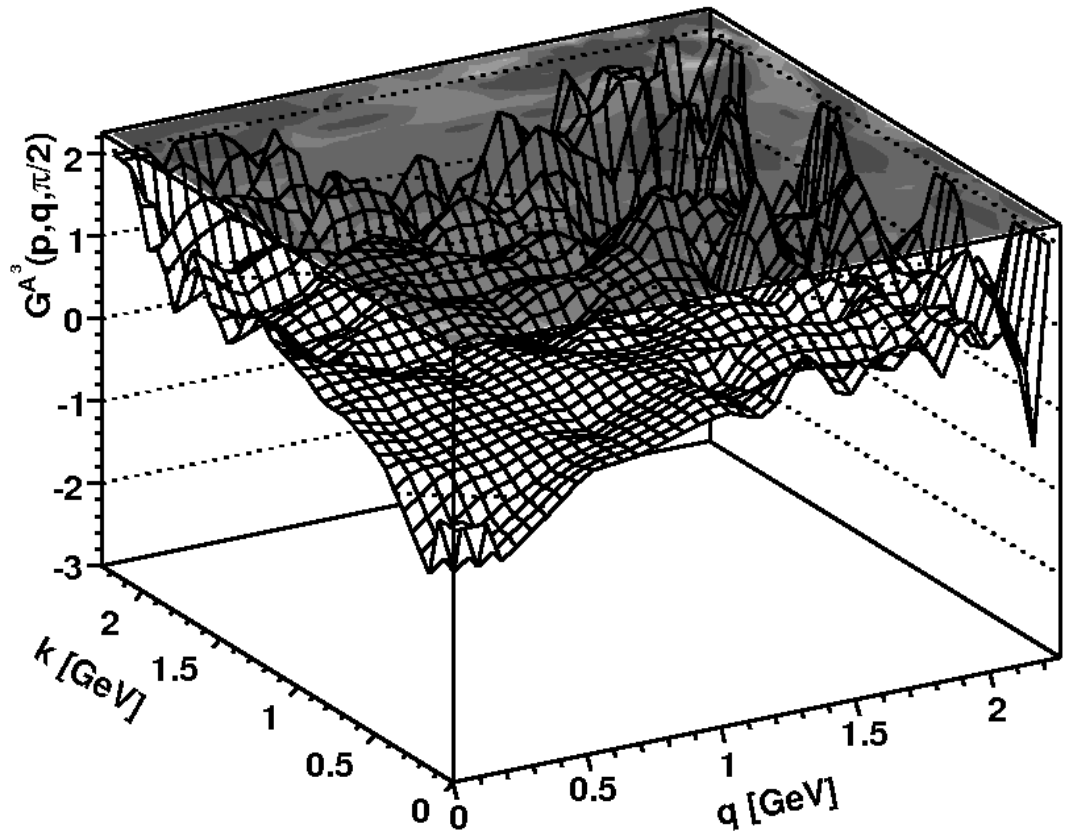
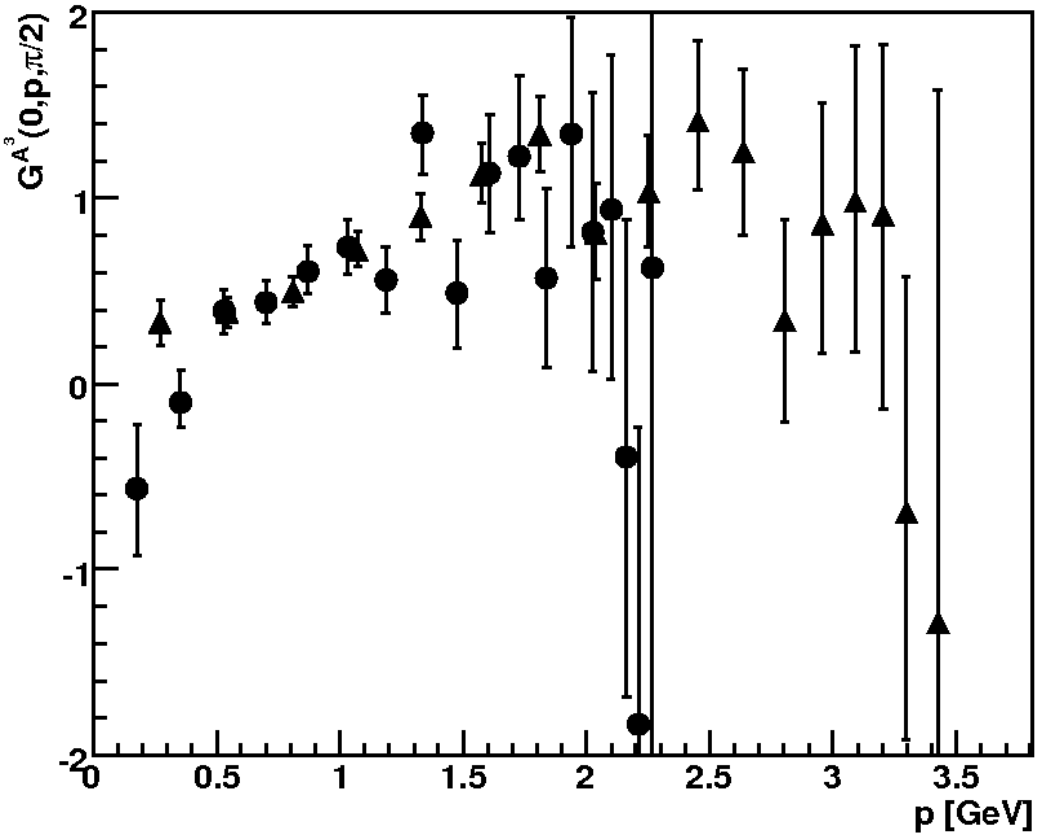


Three-gluon vertex, one momentum vanishing



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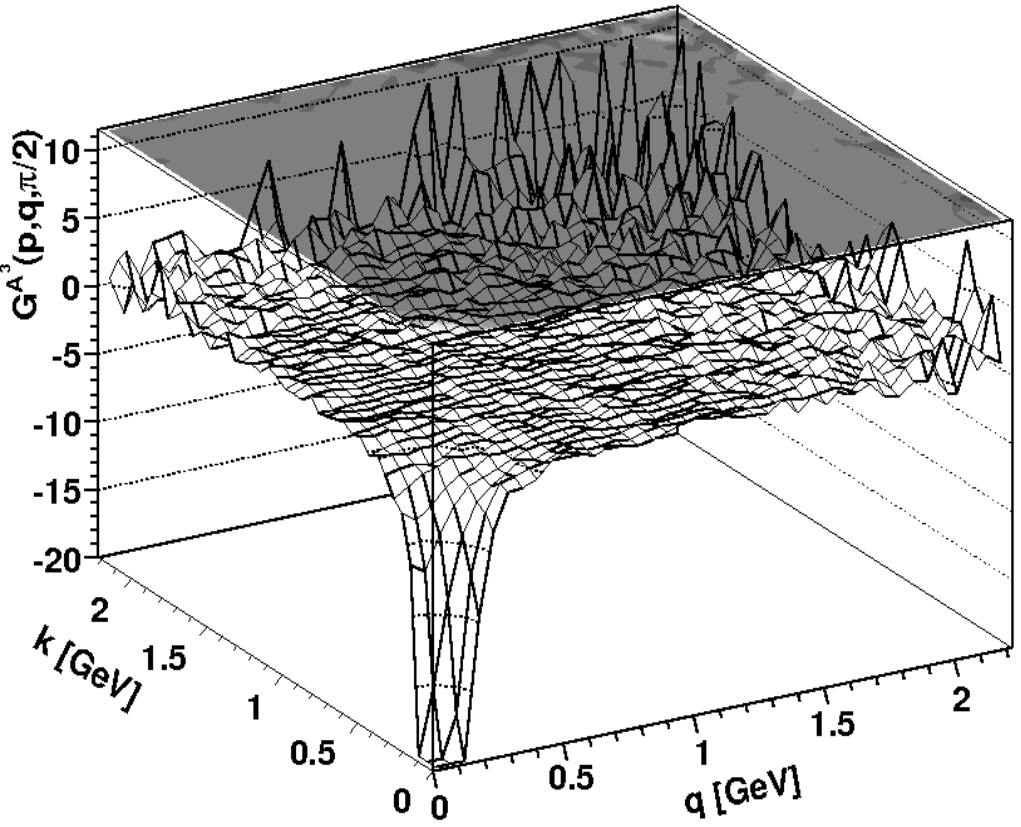
- Possible zero-crossing at small momenta
- Behavior at very small momenta open
- Infrared divergent after sign-change?

Three-gluon vertex in 2d

[Maas, PRD 2007]

Three-gluon vertex, orthogonal momenta



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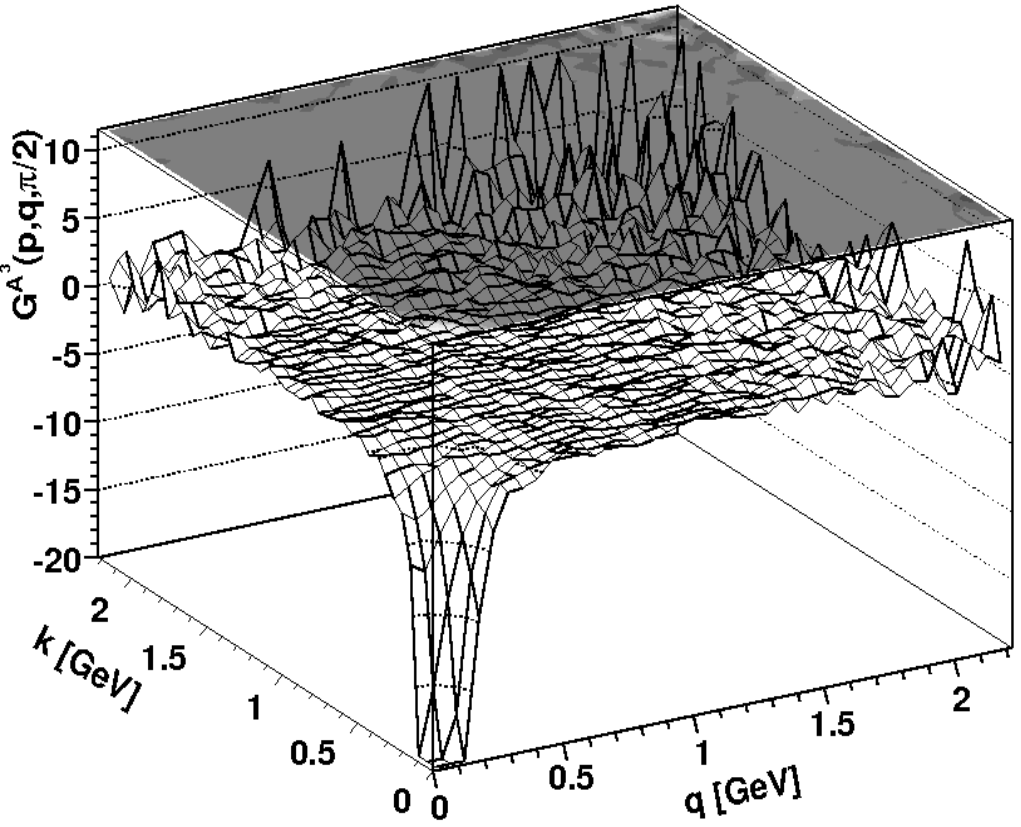

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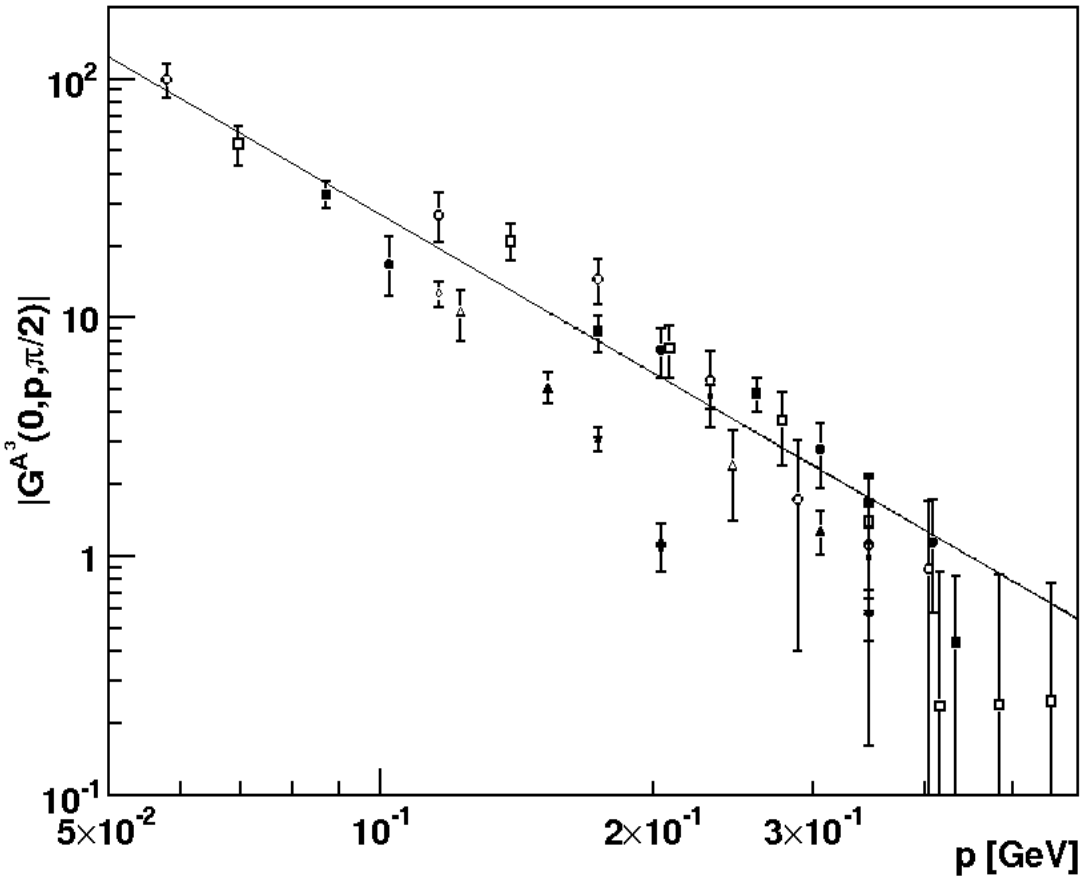

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- Test other predictions!

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Propagators as a function of gauge group (2d)

Gluon propagator

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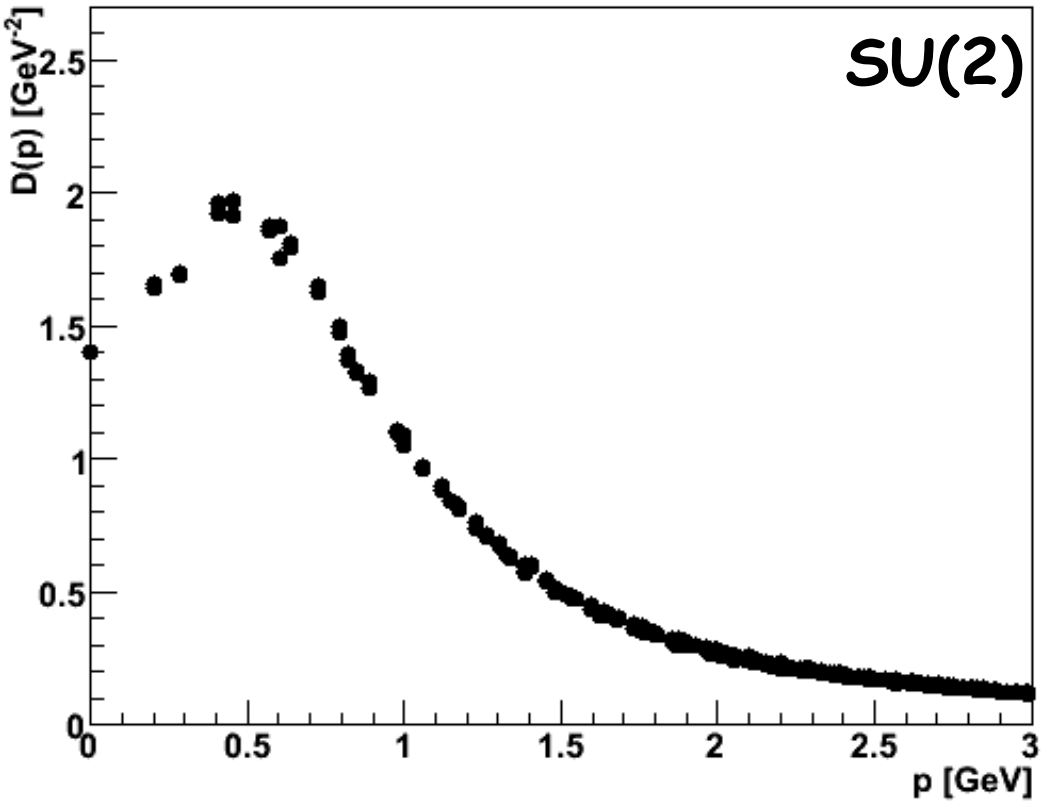
34^2, beta=10]



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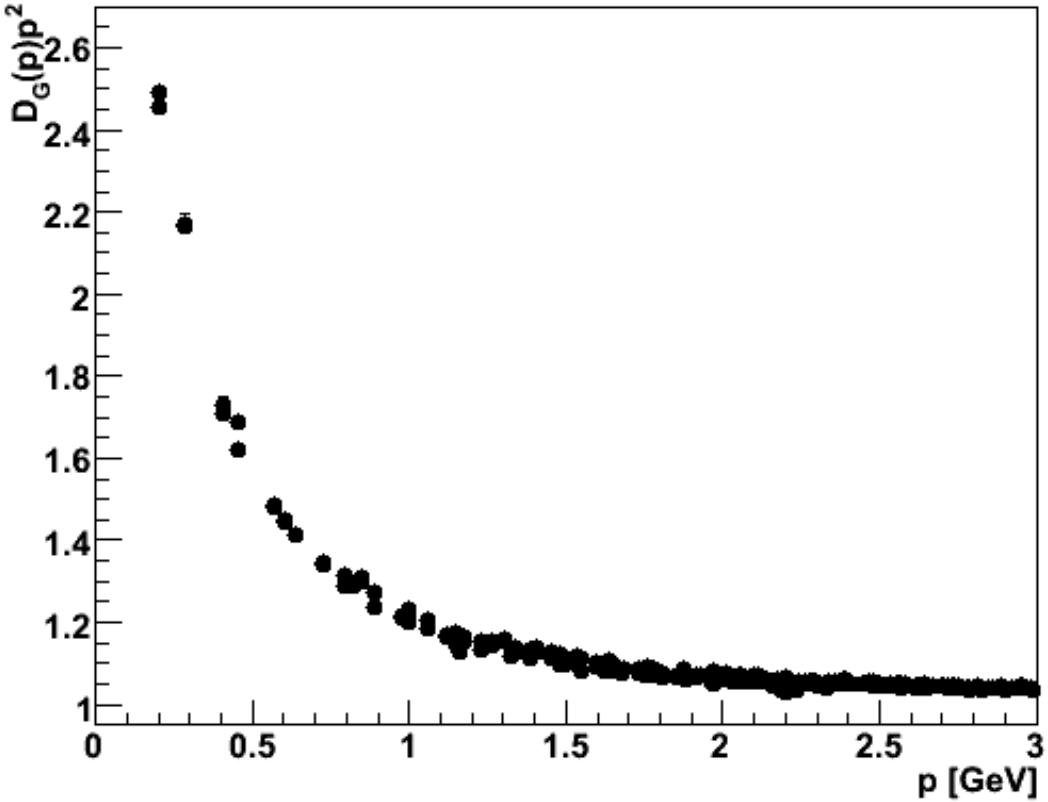

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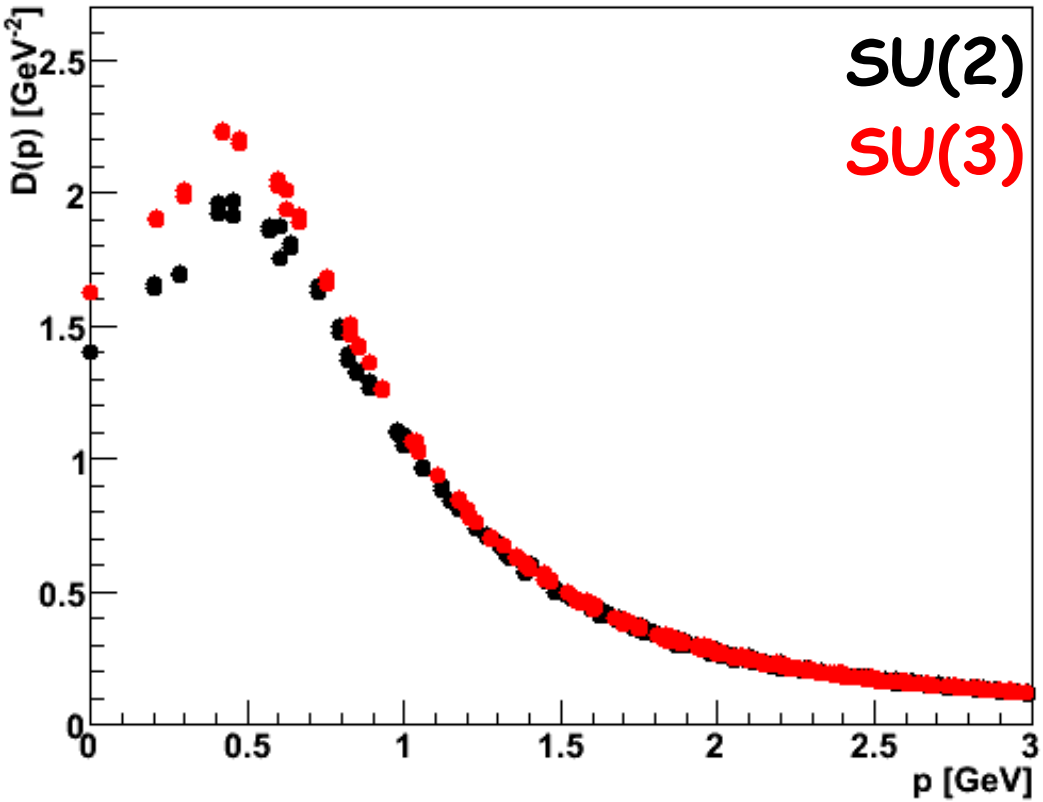

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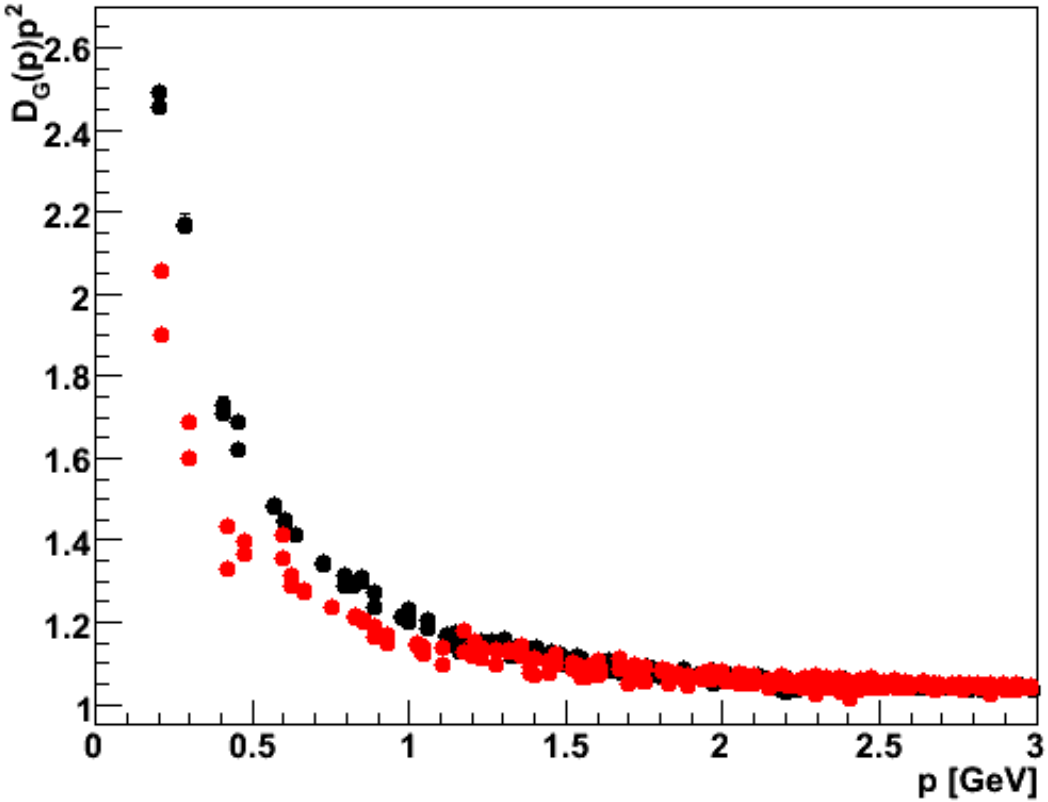

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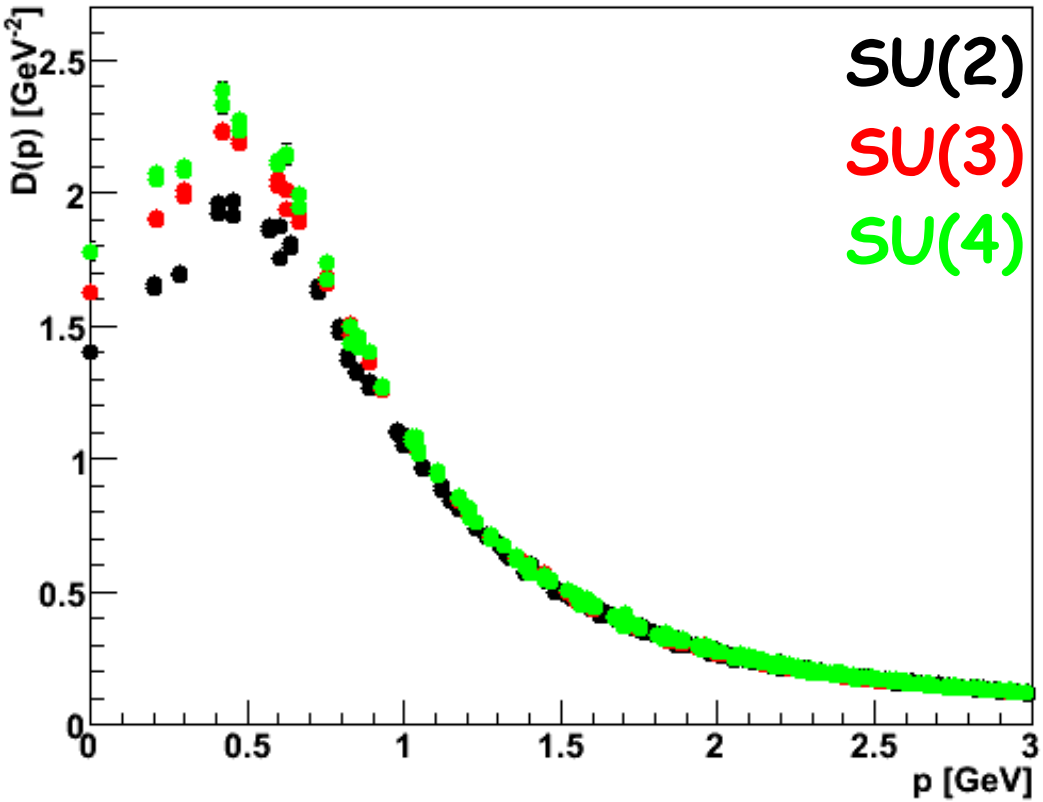

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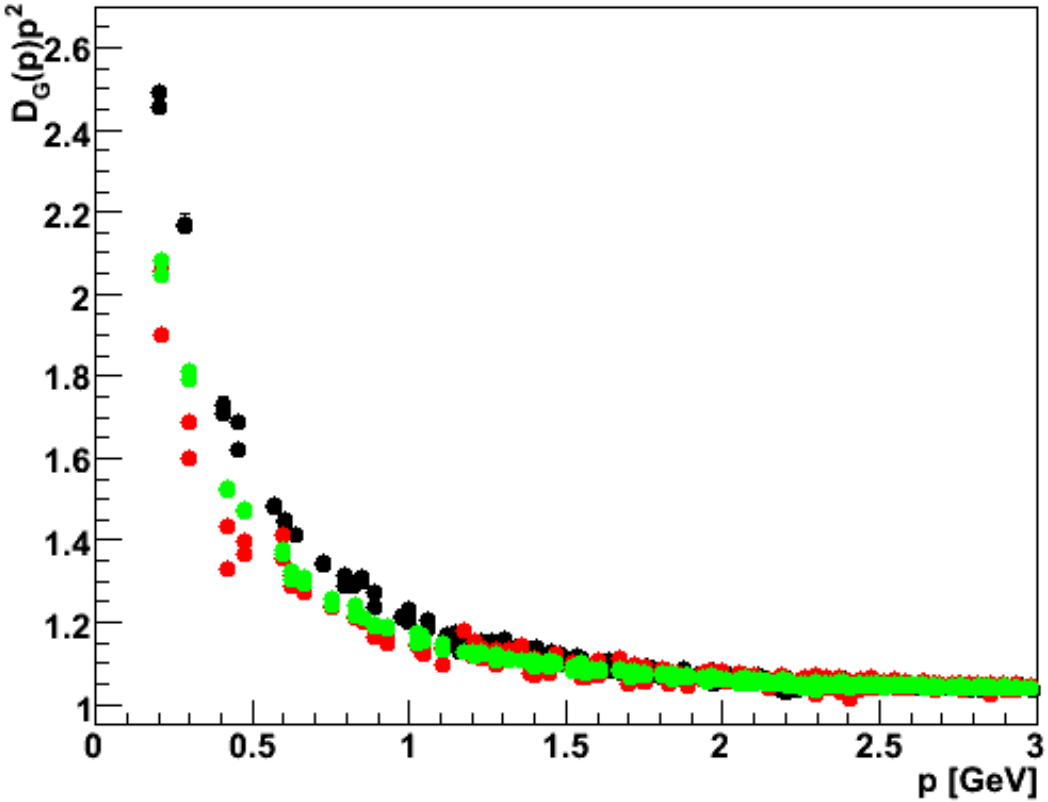

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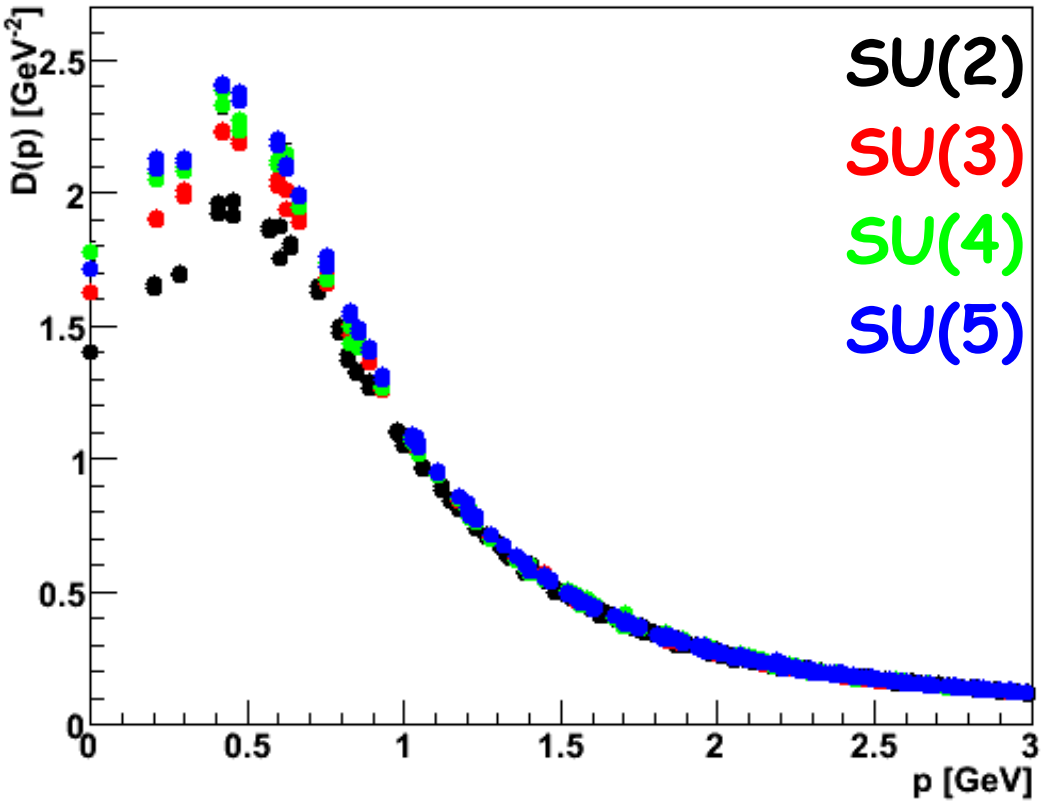

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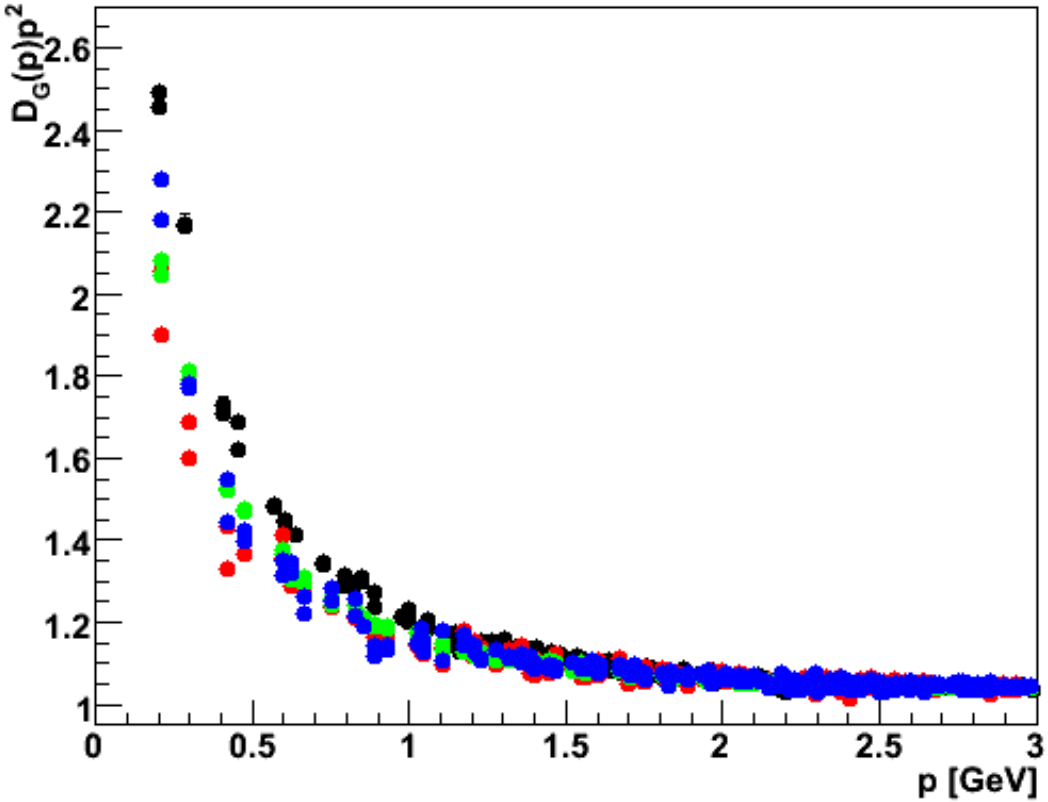

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- Same in 3d and 4d (only SU(2/3)) [Cucchieri et al. 2007, Sternbeck et al. 2007]

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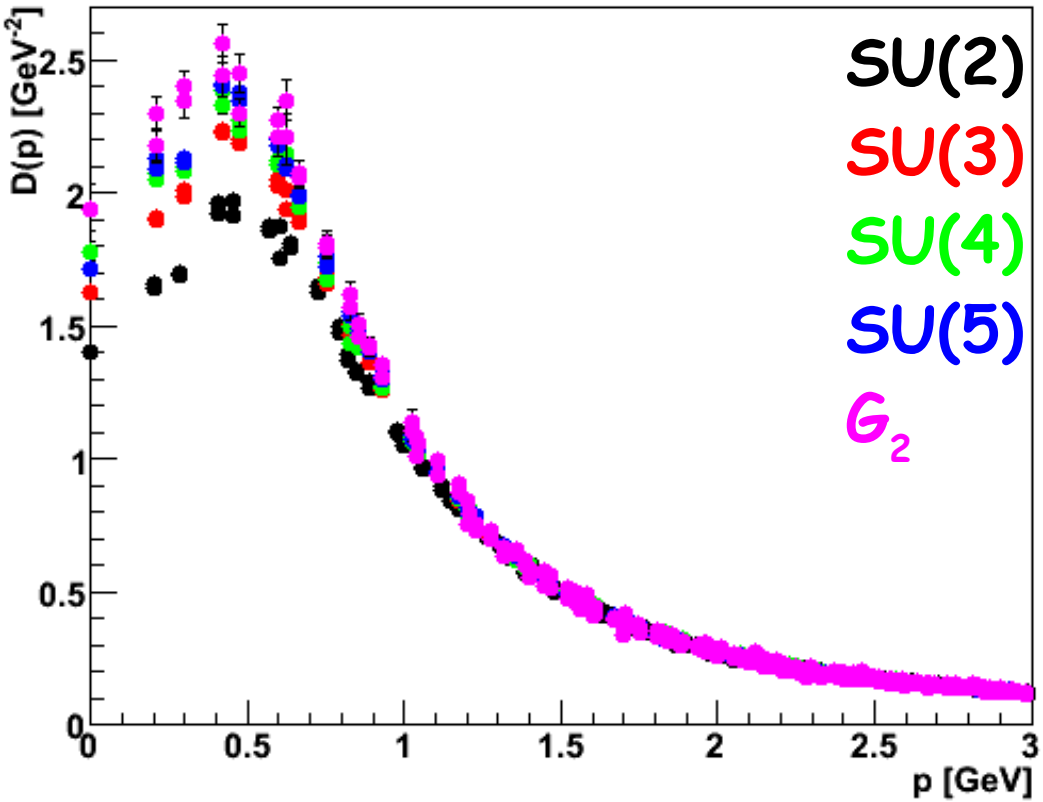

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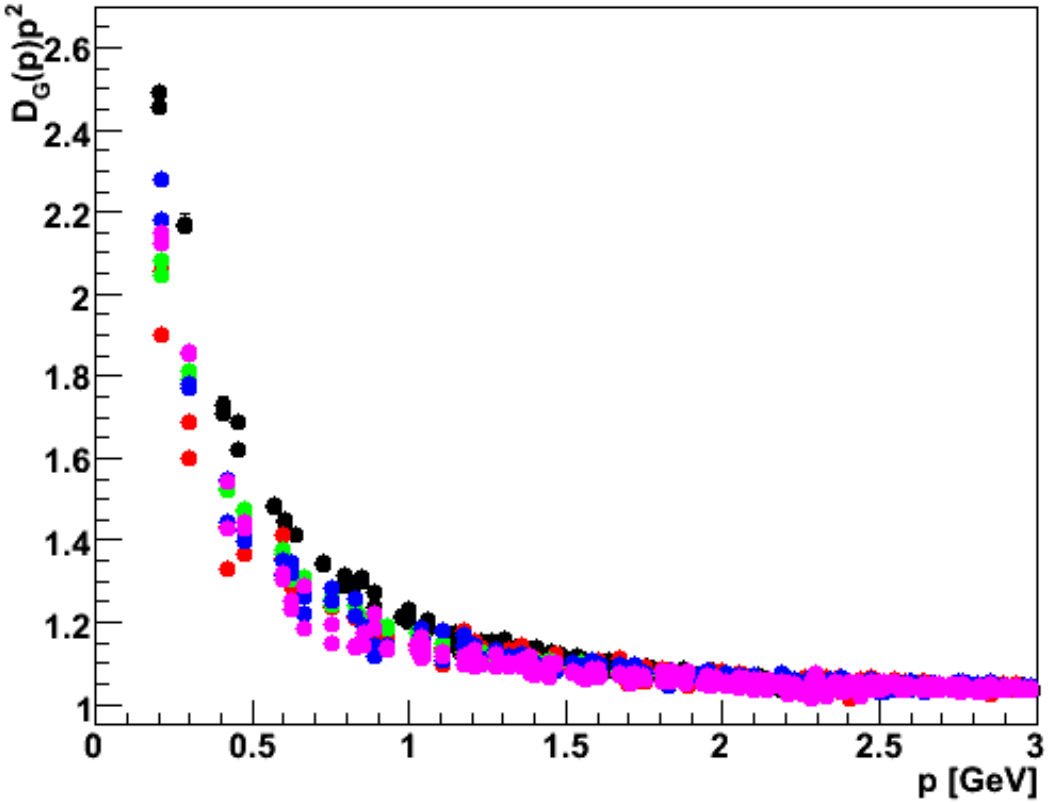

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- Same in 3d and 4d (only SU(2/3)) [Cucchieri et al. 2007, Sternbeck et al. 2007]
- Agrees with prediction of functional methods

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- Finite density: No lattice results yet

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  - Surely in 2d, likely in 3d, unsure in 4d
  - Many effects prevent definite conclusions
- Many results compatible between lattice and functional methods
- Many open questions, though

Collaboration

- A. Cucchieri, T. Mendes (Uni. of São Paulo, Brazil)
- Š. Olejník (Slovak Academy of Sciences, Slovakia)
- C. S. Fischer (Darmstadt), J. M. Pawlowski (Heidelberg), L. v. Smekal (Adelaide, Australia)
- Supported by the DFG
  - Some e-prints on the topic: 0704.0772[hep-lat], hep-lat/0701011, hep-lat/0610006, hep-th/0610011, hep-th/0511307, hep-ph/0506066, hep-ph/0408074

Topical workshop

415th Wilhelm & Else Heraeus Seminar

"Quarks and Hadrons in Strong QCD"

17th-20th March 2008 – St. Goar – Germany

Invited speakers include:

R. Alkofer, A. Cucchieri, A. di Giacomo, H. Gies, J. Greensite, M. Pennington, S. Sorella, A. Thomas, A. Wipf, S. Yuri, D. Zwanziger

http://crunch.ikp.physik.tu-darmstadt.de/qhqcd

Organizers: C. S. Fischer & A. Maas