Confinement of Gluons

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UNI GRAZ

Overview

- QCD in the non-perturbative domain

Supported by the FWF

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- Confinement of gluons

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- Propagators and vertices

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- Propagators and vertices
- Connection to topology and quark confinement
- Summary

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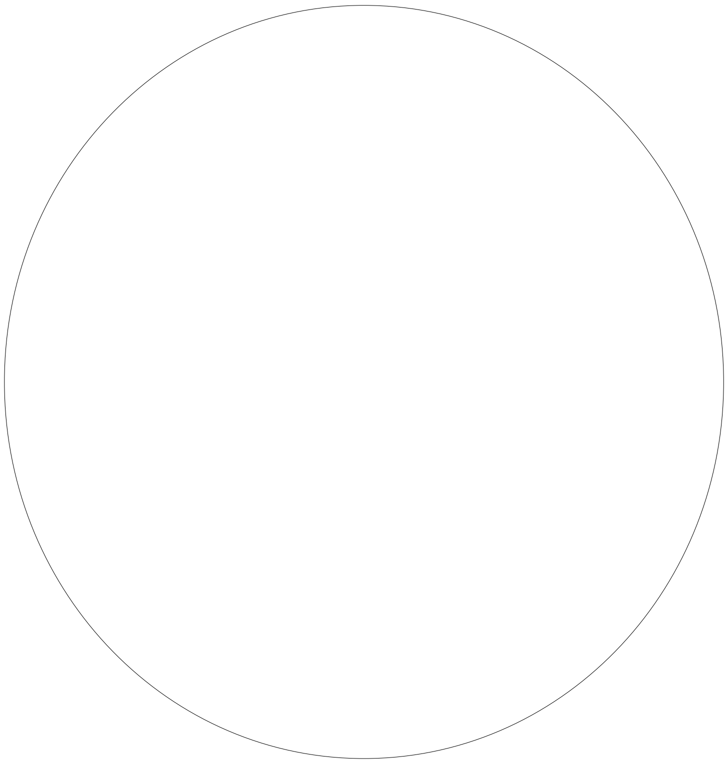

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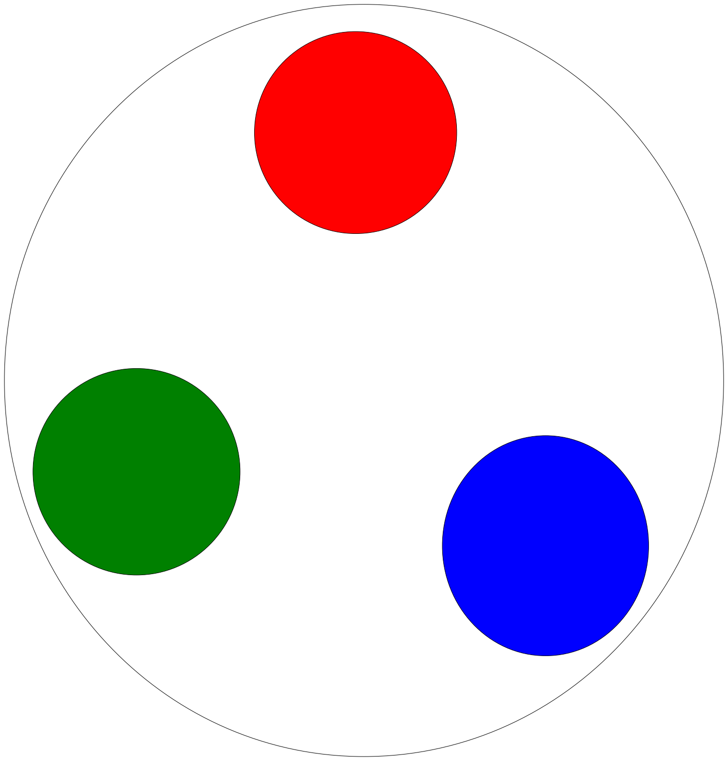

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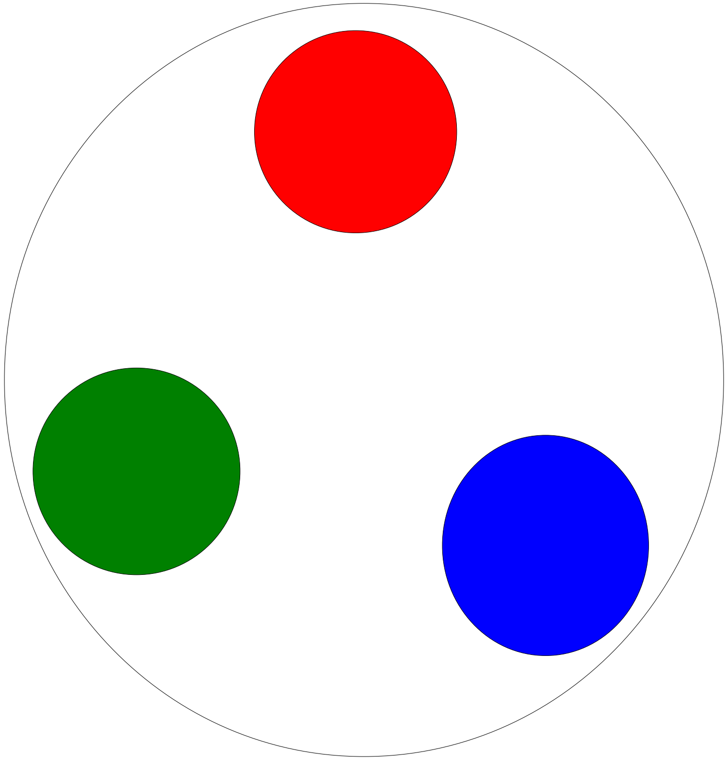

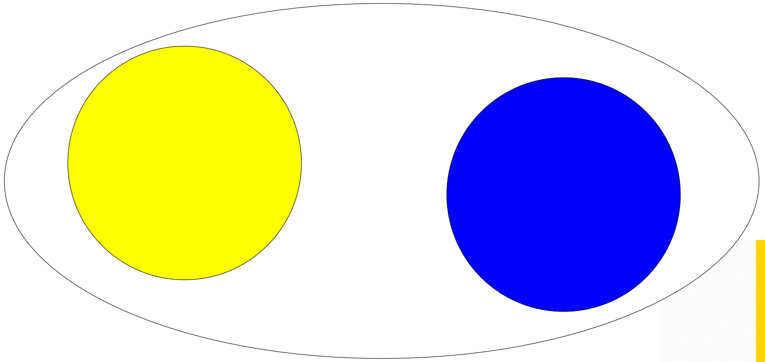

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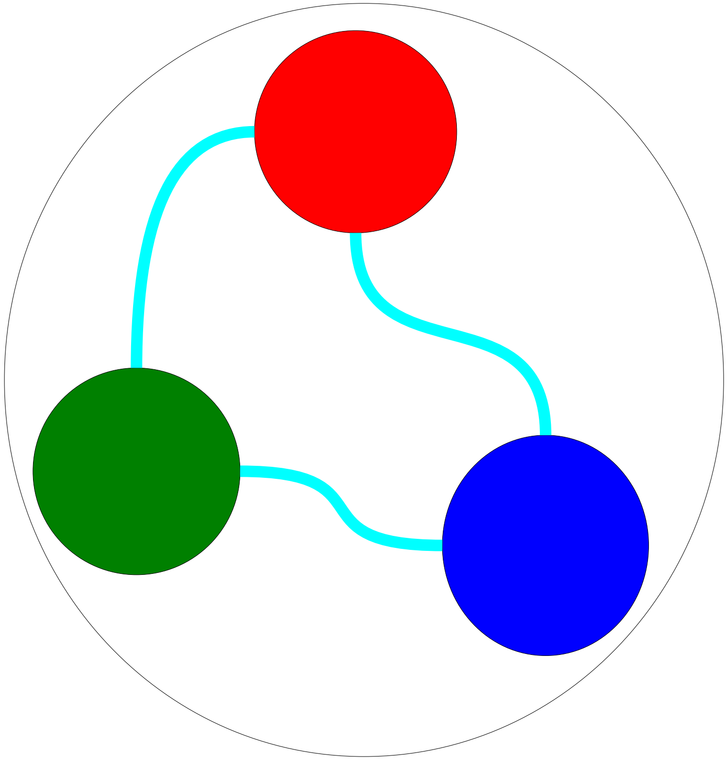

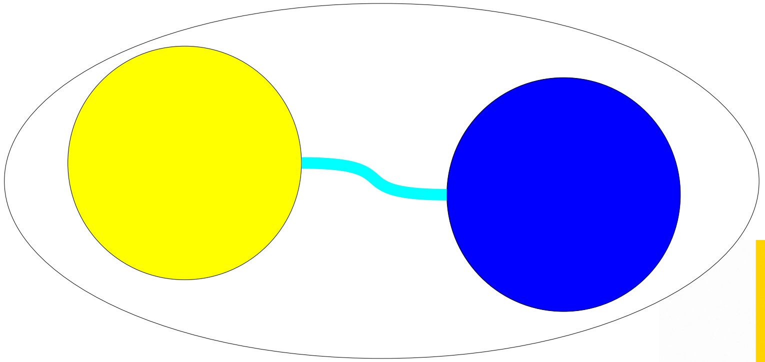

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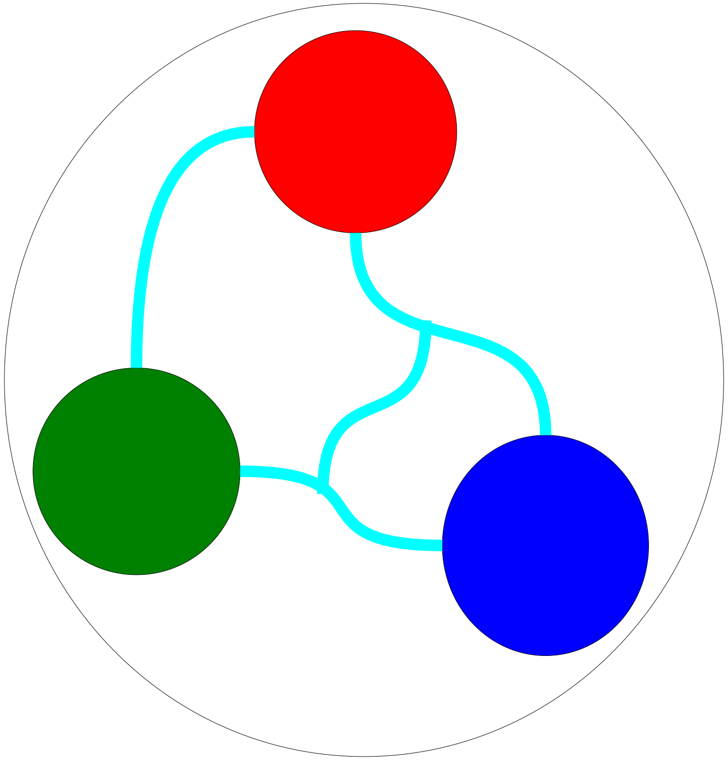

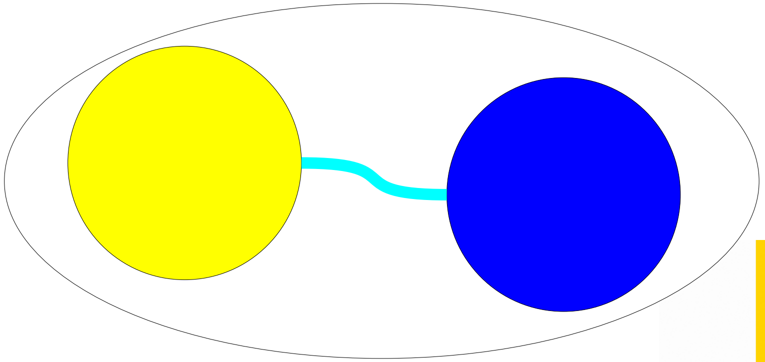

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- Quarks and gluons cannot be observed as individual particles: Confinement
  - Measured with very high precision

What is confinement - Experiment

- Experiment tells...
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  - No confinement in perturbation theory for d>2
  - Non-perturbative effect

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- Restrict to Yang-Mills theory: Only gluons

Yang-Mills Theory

- Lagrangian:

$$L = -\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f^{abc} A_\mu^b A_\nu^c$$

- Degrees of freedom:

Gluons: A_μ^a

Gauge-fixing

- Yang-Mills theory is a gauge theory
  - Gauge transformations $A_\mu^a \rightarrow A_\mu^a + (\delta^{ab}\partial_\mu - g f^{abc} A_\mu^c)\phi^b(x)$ with arbitrary $\phi^a(x)$ change the gauge fields, but leave physics invariant: Gauge orbits

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- E.g. Landau gauge condition $\partial_\mu A_\mu^a = 0$

Unique gauge-fixing [For an introduction: Sobreiro & Sorella, 2005]

- In perturbation theory: Local gauge condition
  - Landau gauge: $\partial_\mu A_\mu^a = 0$

(Perturbative) Landau gauge

- Lagrangian:

$$L=-\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu}-\bar{c}^a\partial_\mu D^{ab}_\mu c^b$$

$$F^a_{\mu\nu}=\partial_\mu A^a_\nu-\partial_\nu A^a_\mu-gf^{abc}A^b_\mu A^c_\nu$$

$$D^{ab}_\mu=\delta^{ab}\partial_\mu-gf^{abc}A^c_\mu$$

- Degrees of freedom:

Gluons: A^a_μ

Ghosts: $\bar{c}^a, c^a$

(Auxiliary fields - not observable)

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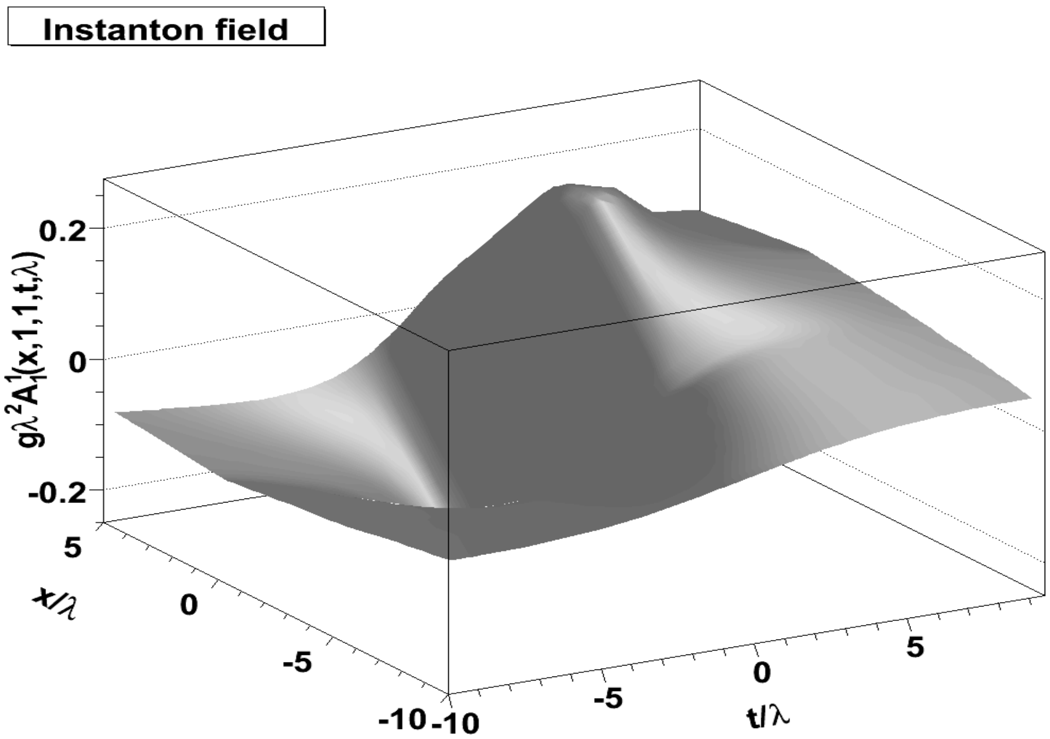
- There are no local gauge conditions known, which select a unique gauge field configuration [Singer CMP 1978]
  - Non-local conditions possible

Example: Instanton [Maas, EPJC 2006]



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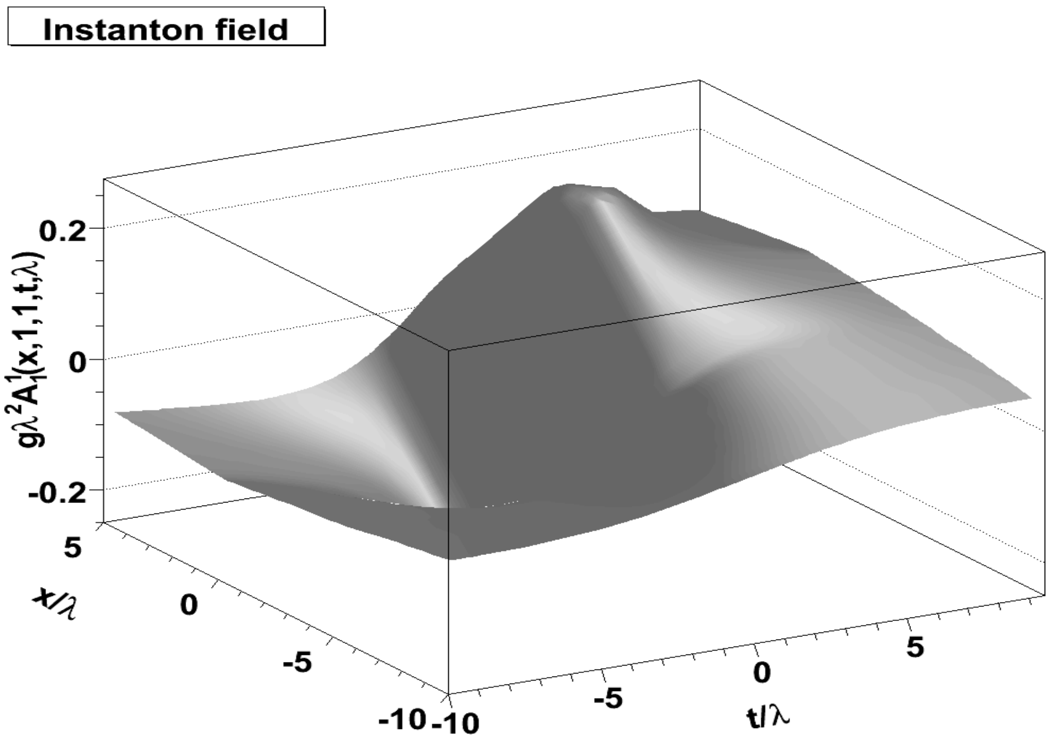

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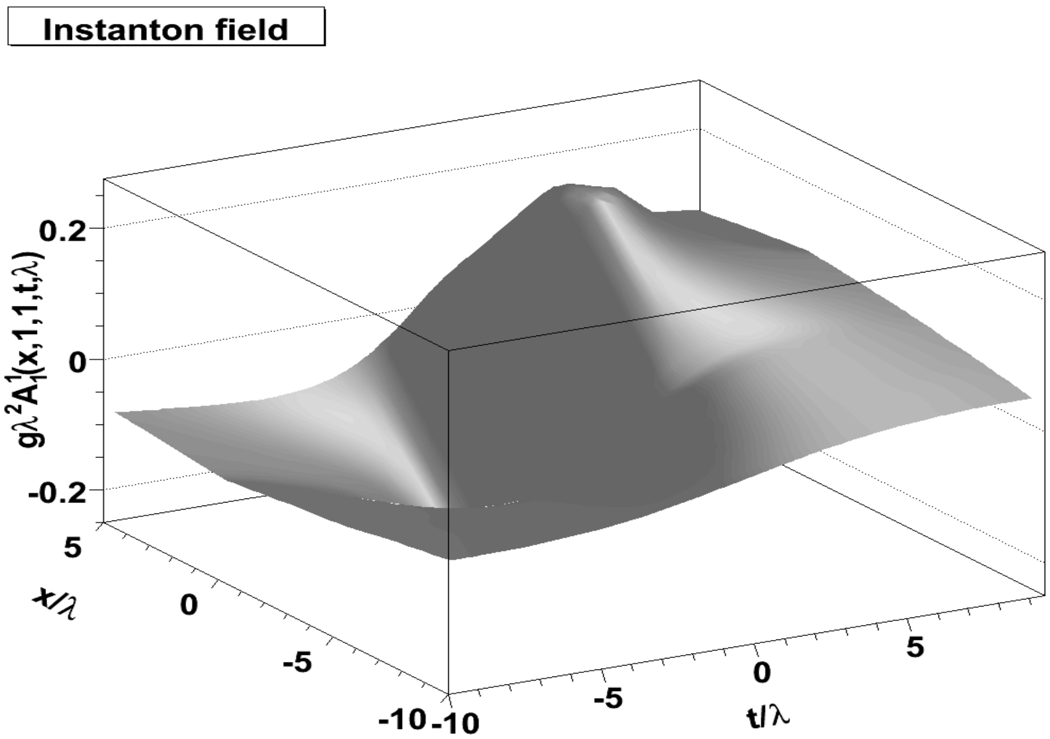

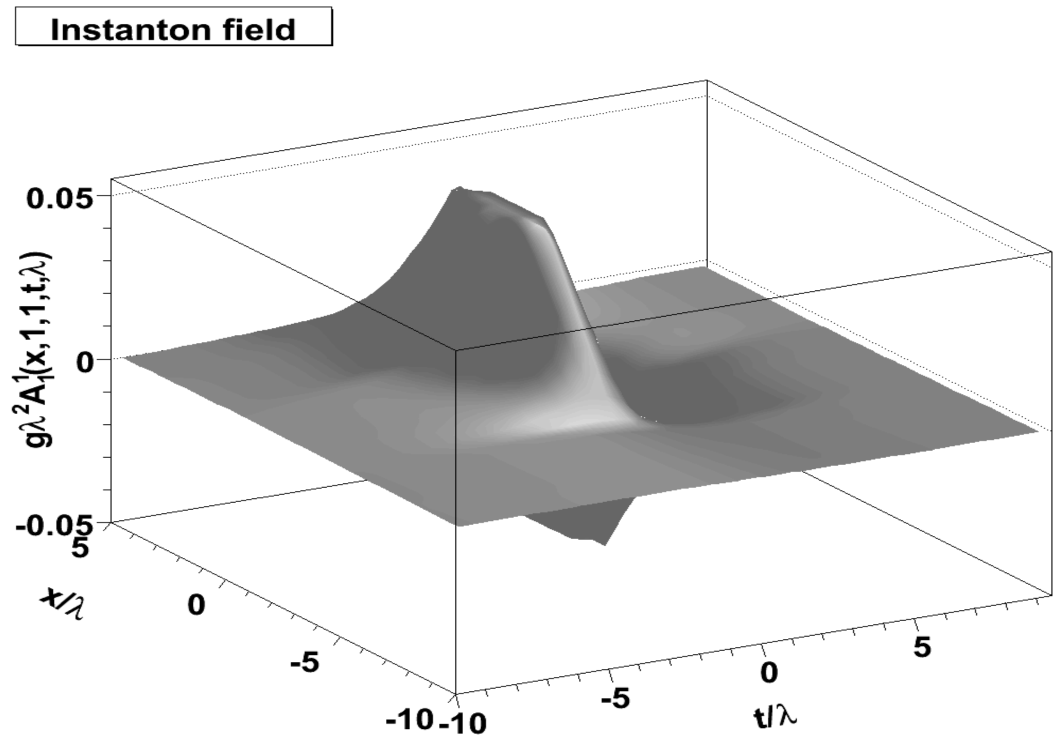

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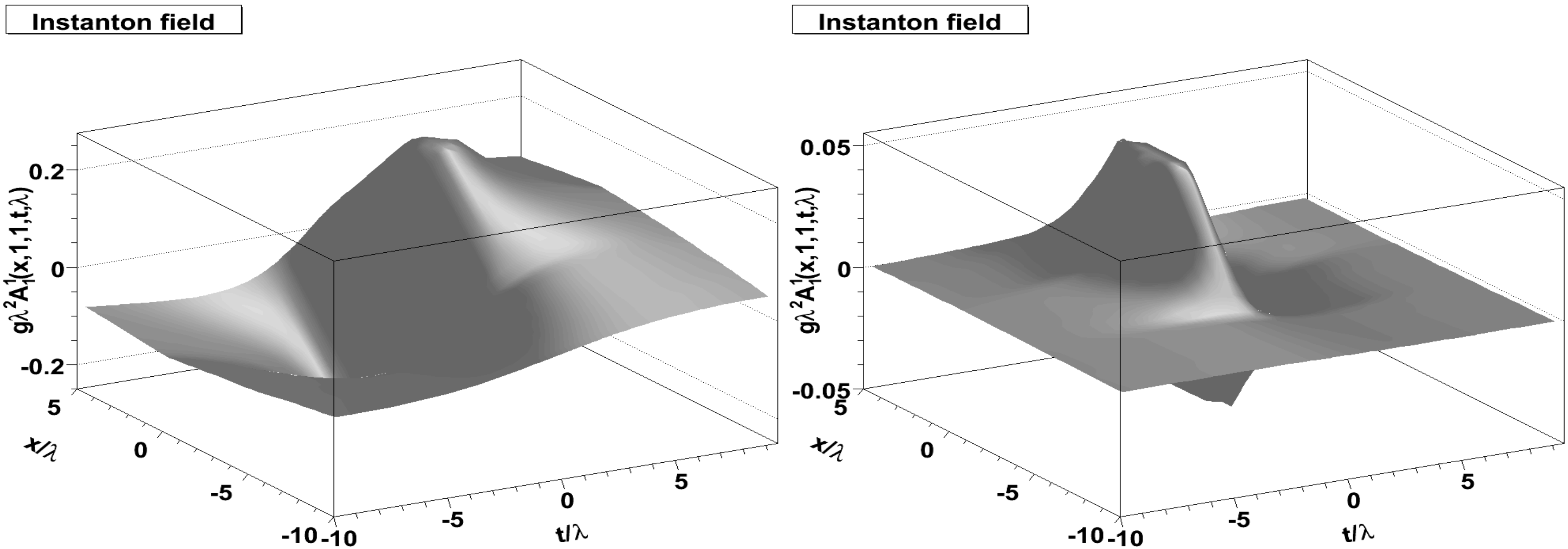

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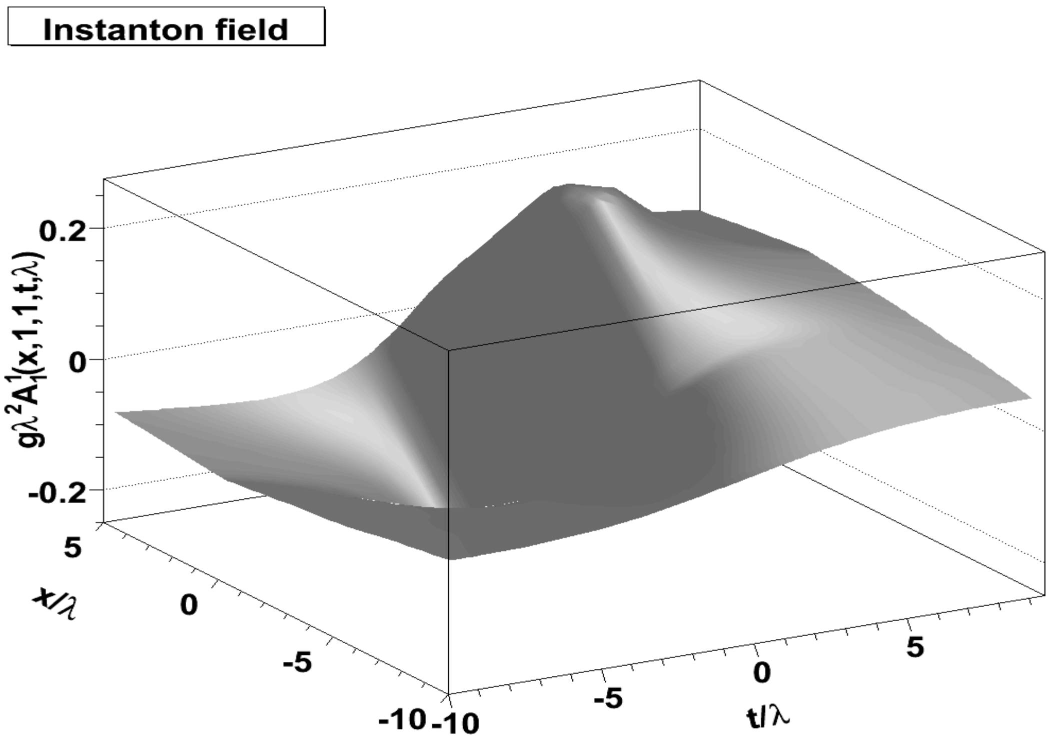

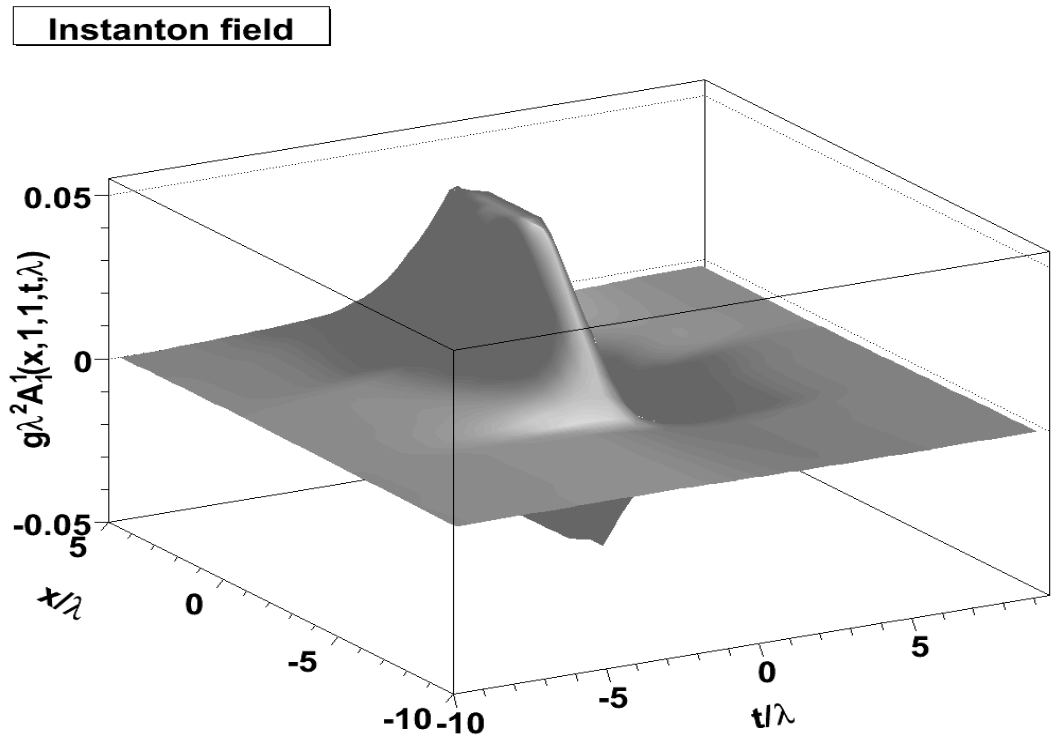

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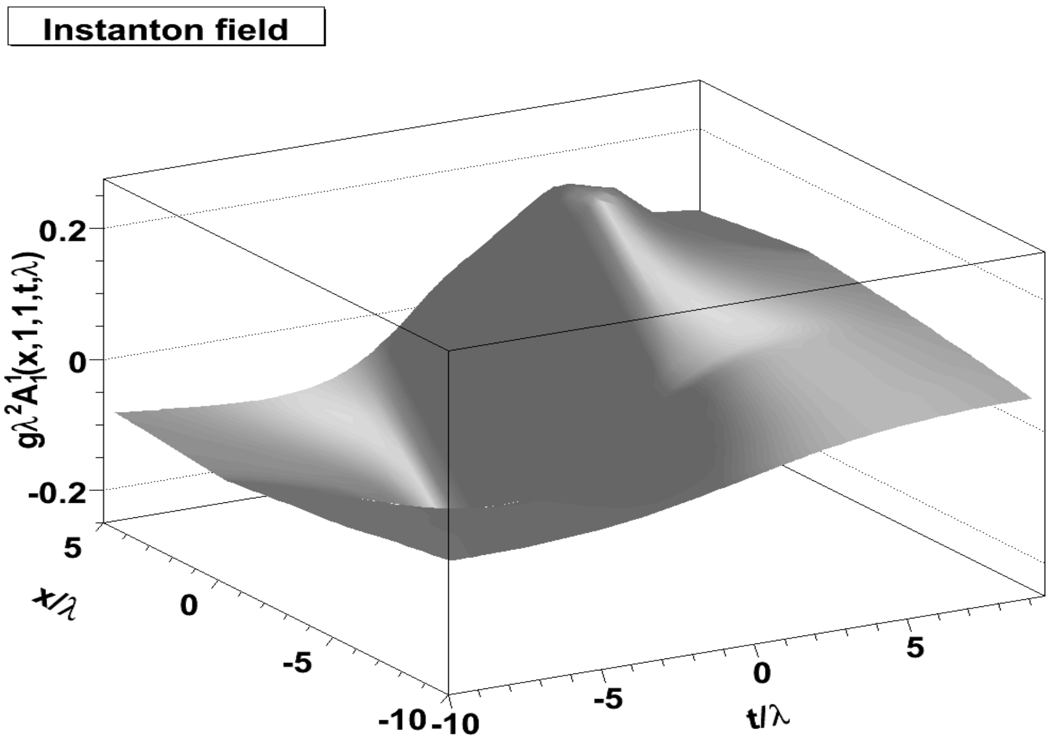

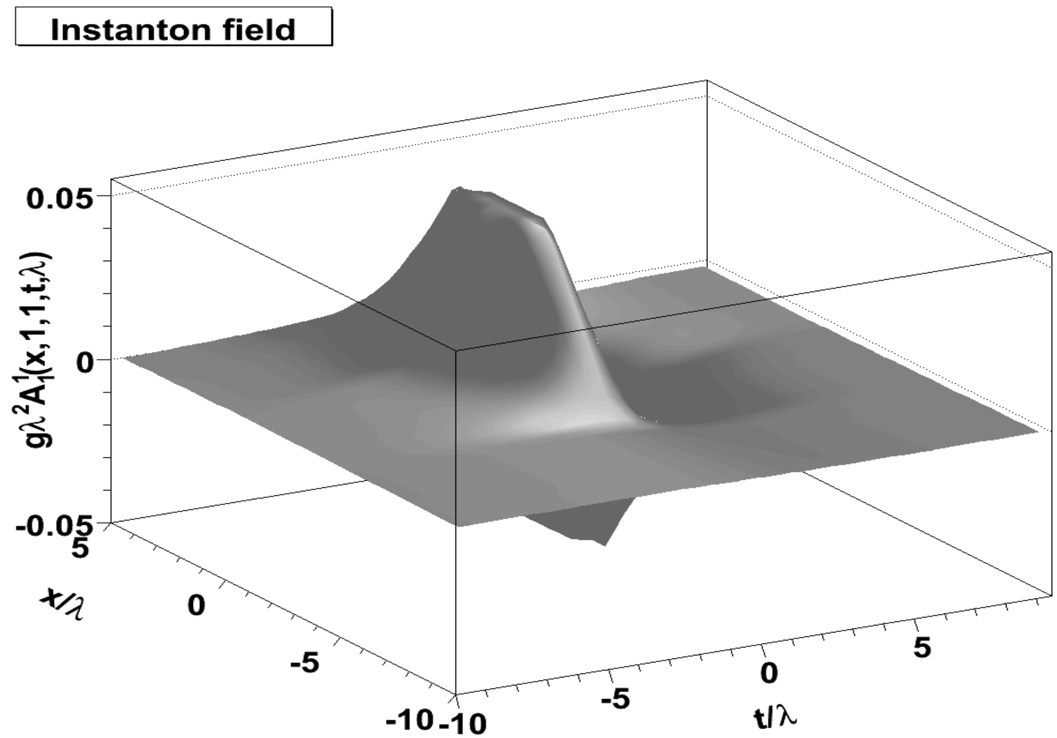

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  - Non-perturbative: Depends on $1/g$

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- Choice: Leave the global color symmetry unfixed

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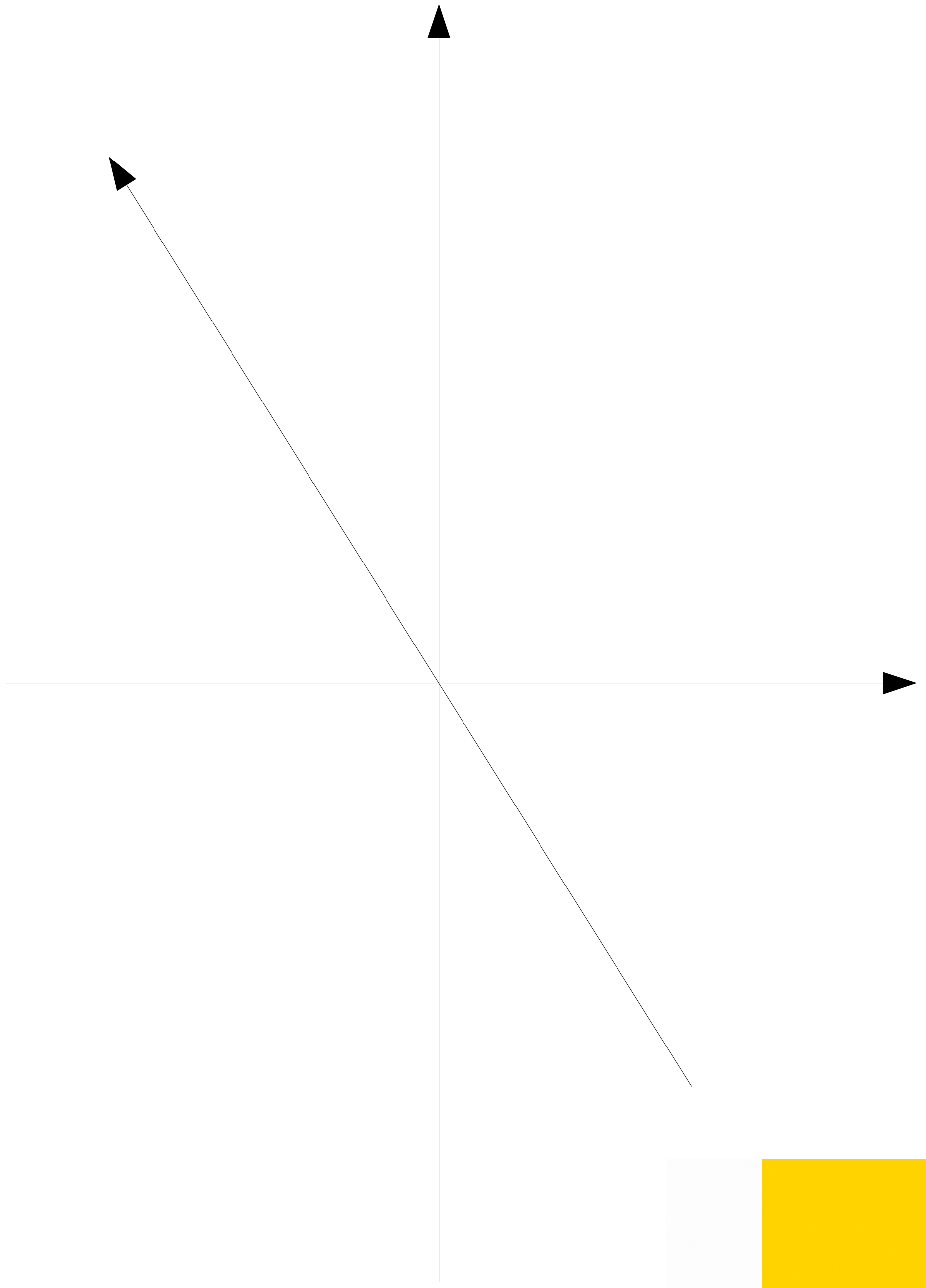
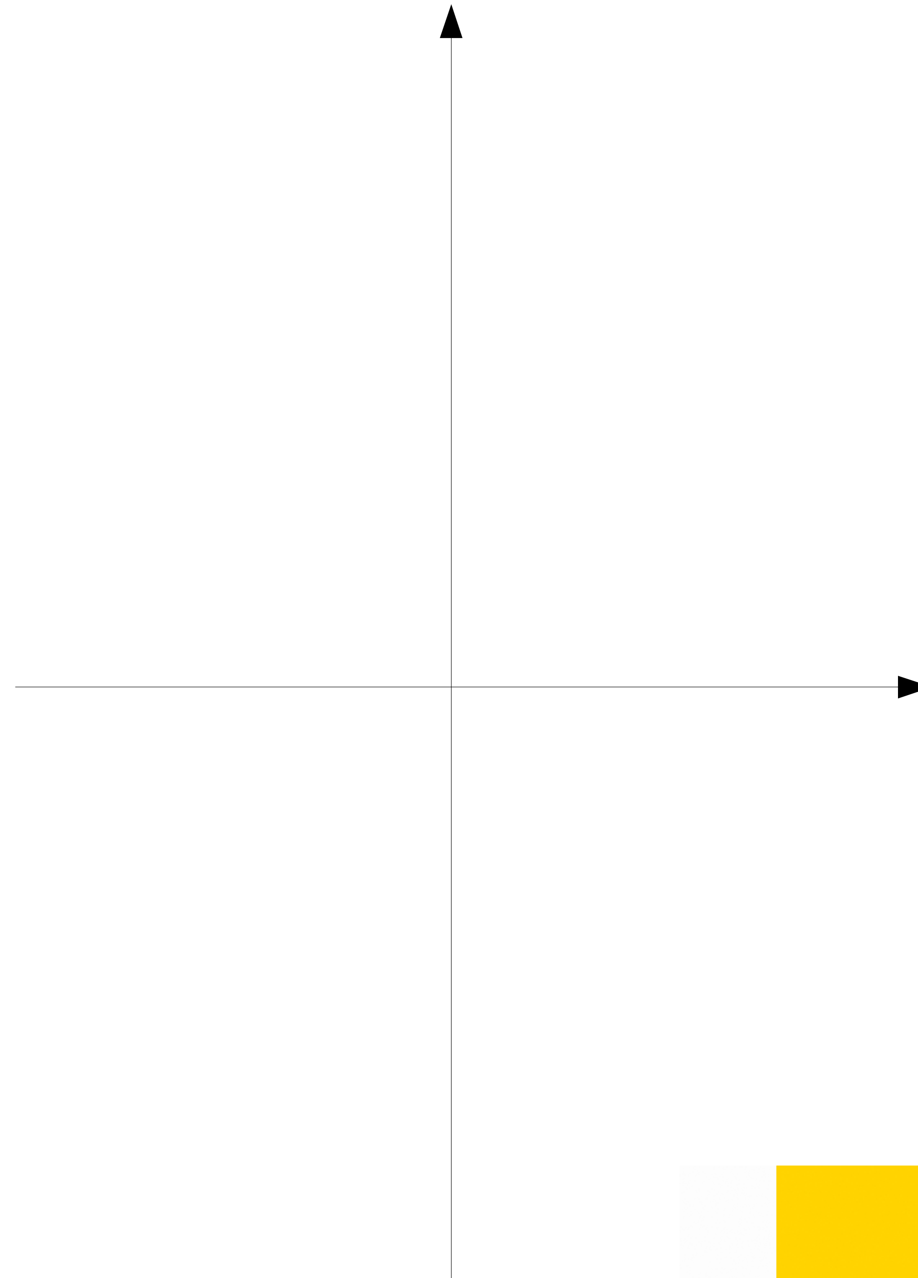
  - Average over the complete residual orbit: Hirschfeld-like
  - Average over some specificed subset: Minimal Landau gauge

Configuration space (artist's view)



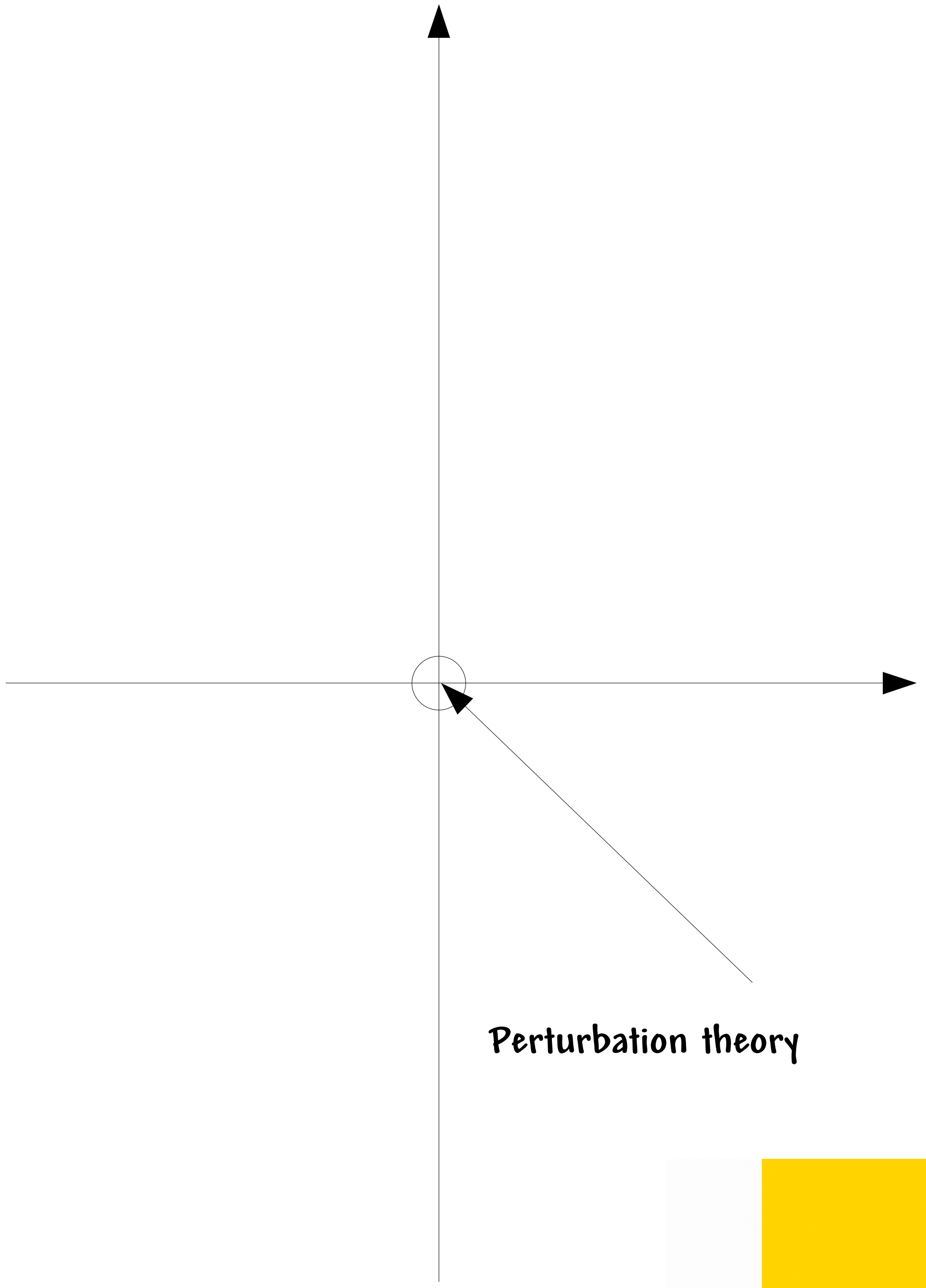
Configuration space (artist's view)

- Impose Landau gauge condition
  - Reduces configuration space to a hypersurface
  - Only residual gauge orbits left



Configuration space (artist's view)

- Perturbation theory is applicable close to the origin
- Non-perturbative physics probes the complete hypersurface



Configuration space (artist's view) [Gribov NPA 1978, Zwanziger 1993...2003]

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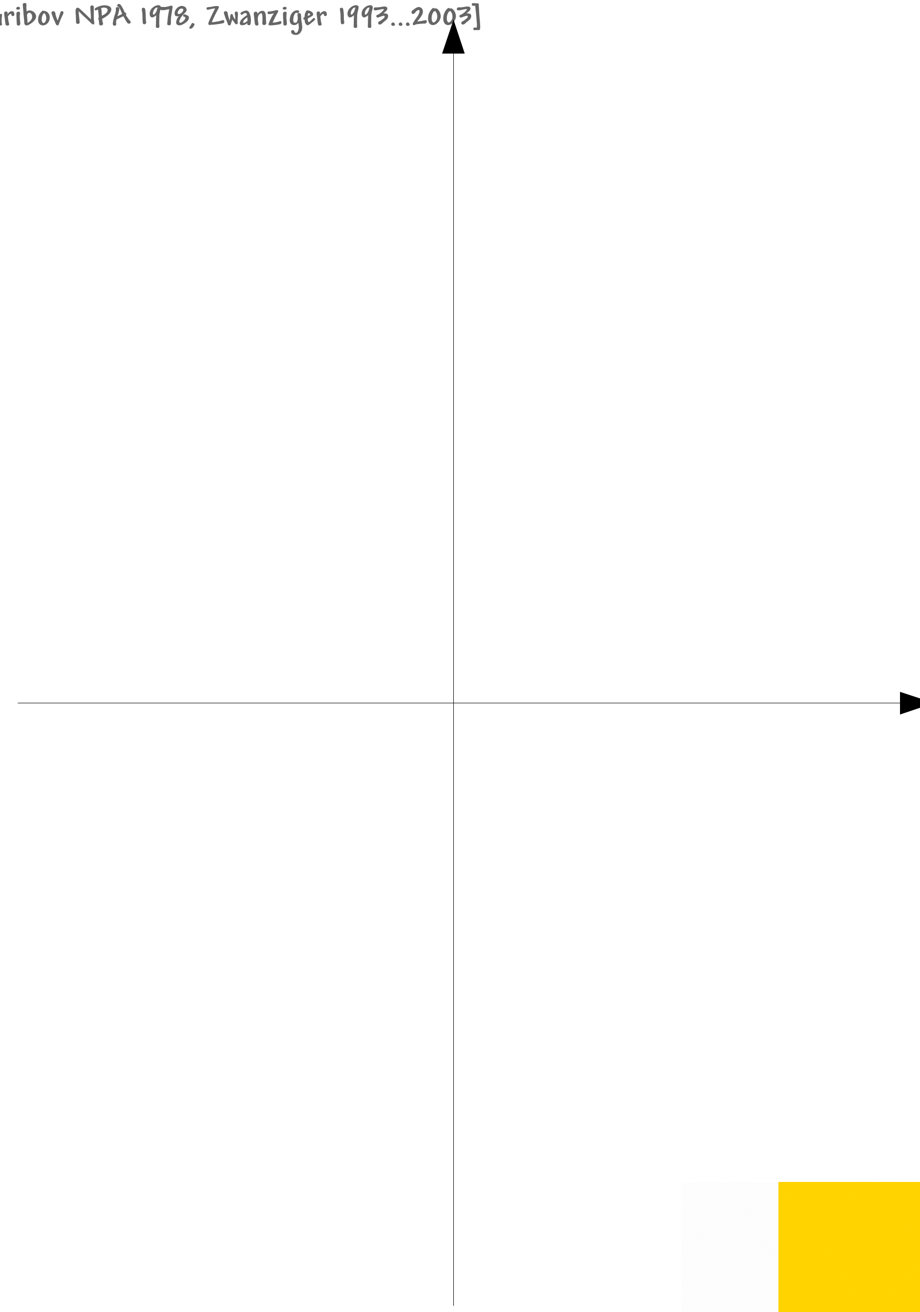

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- A local minium of

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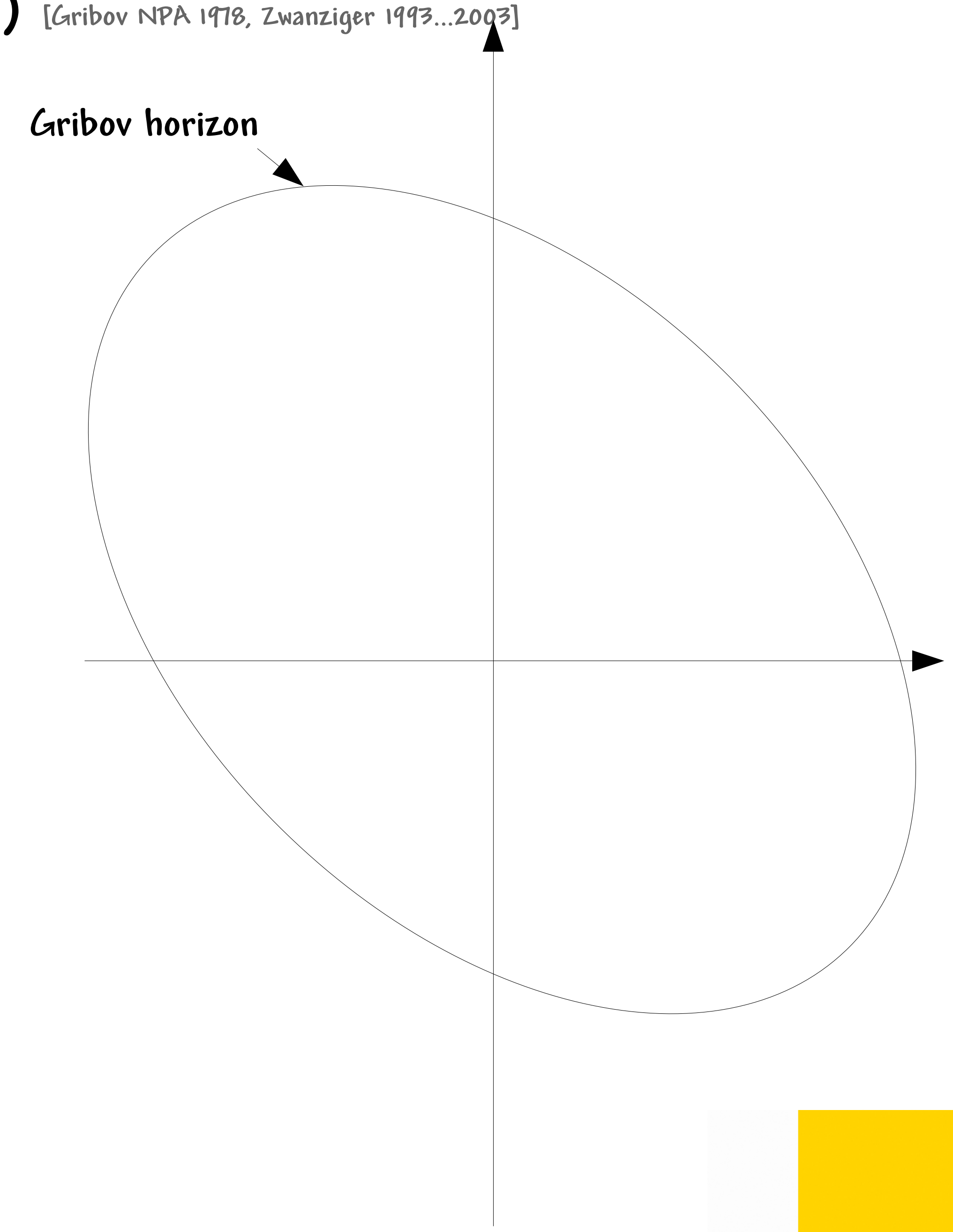
defines the bounded first Gribov region



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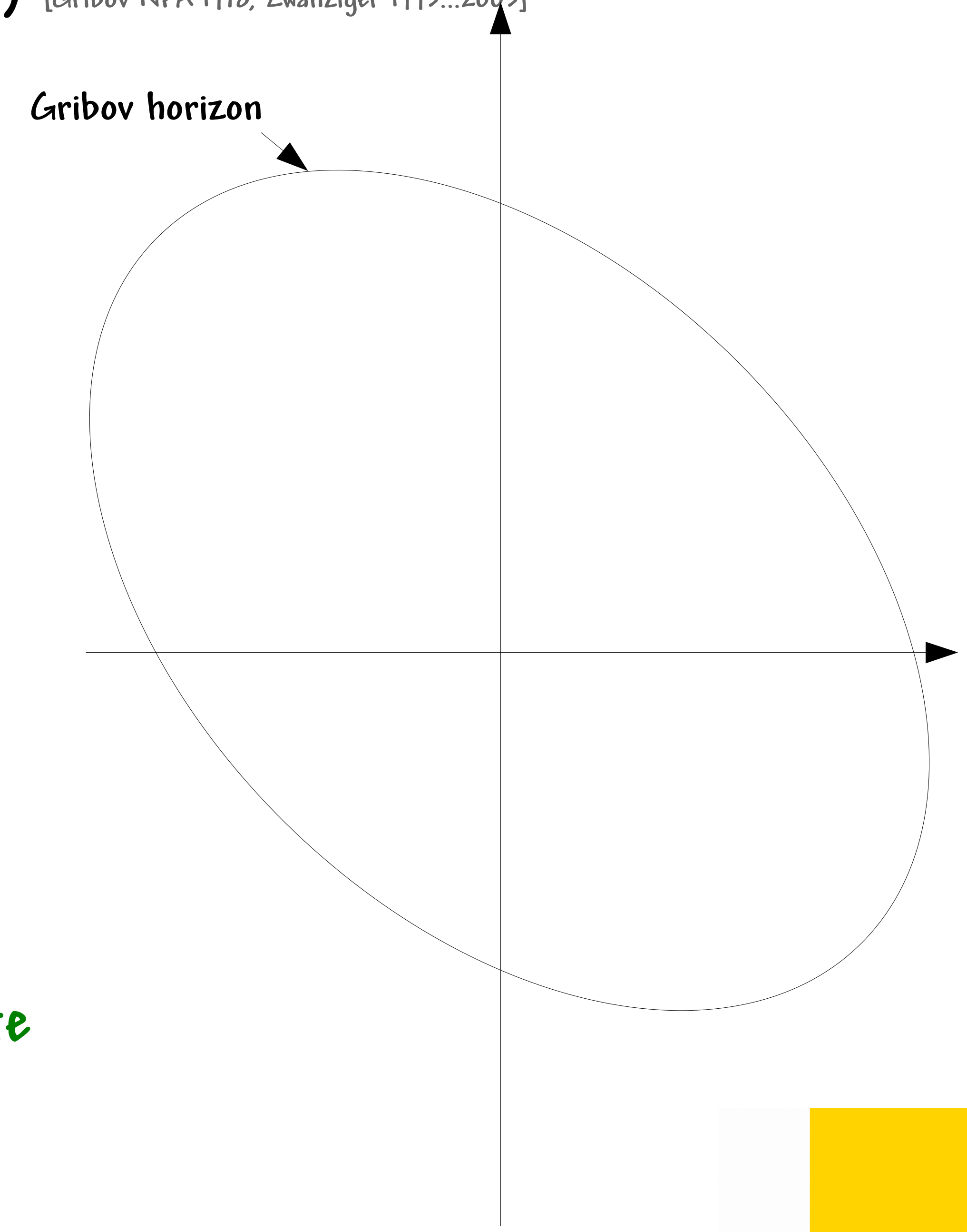

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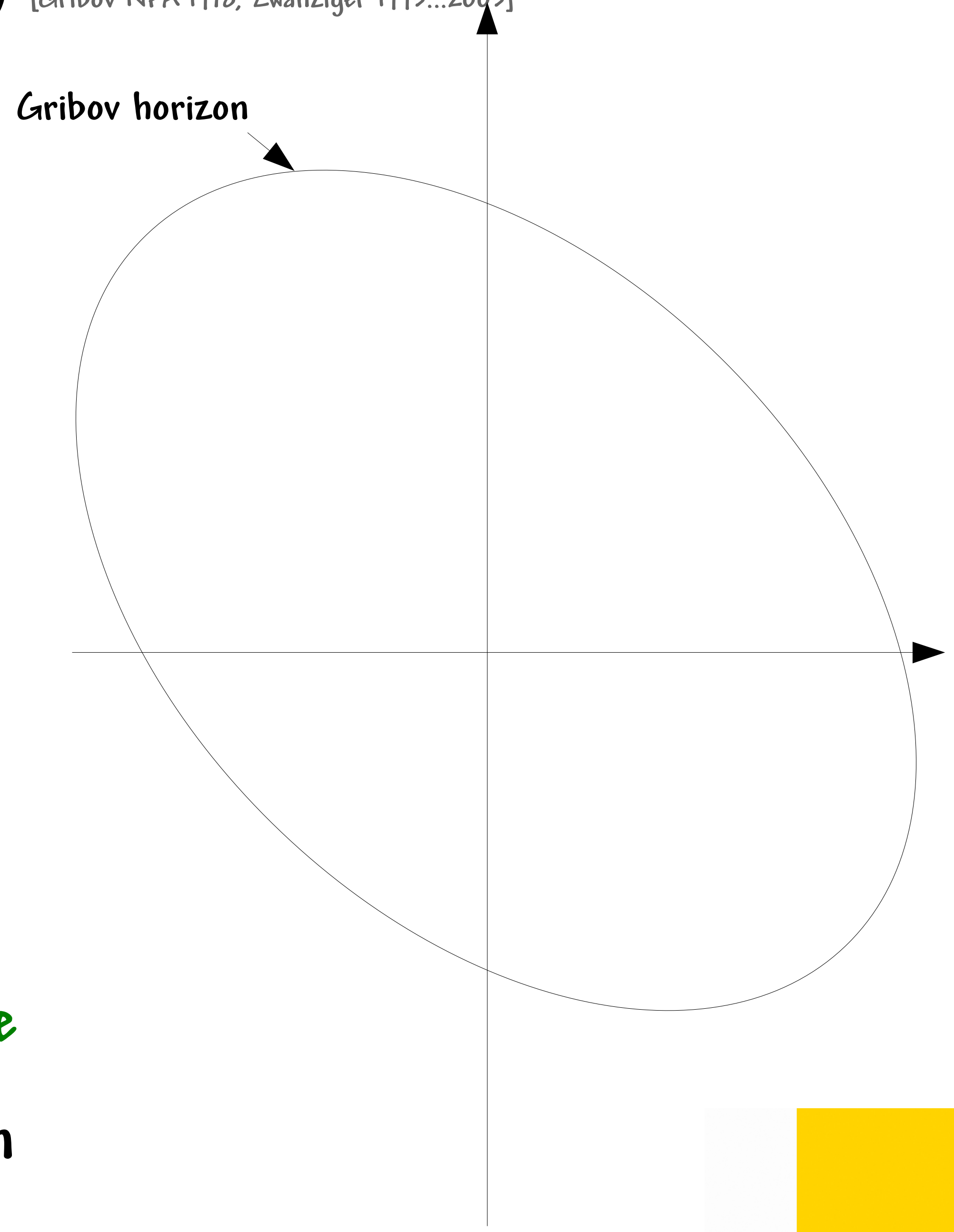
- Selecting an element from this region is minimal Landau gauge



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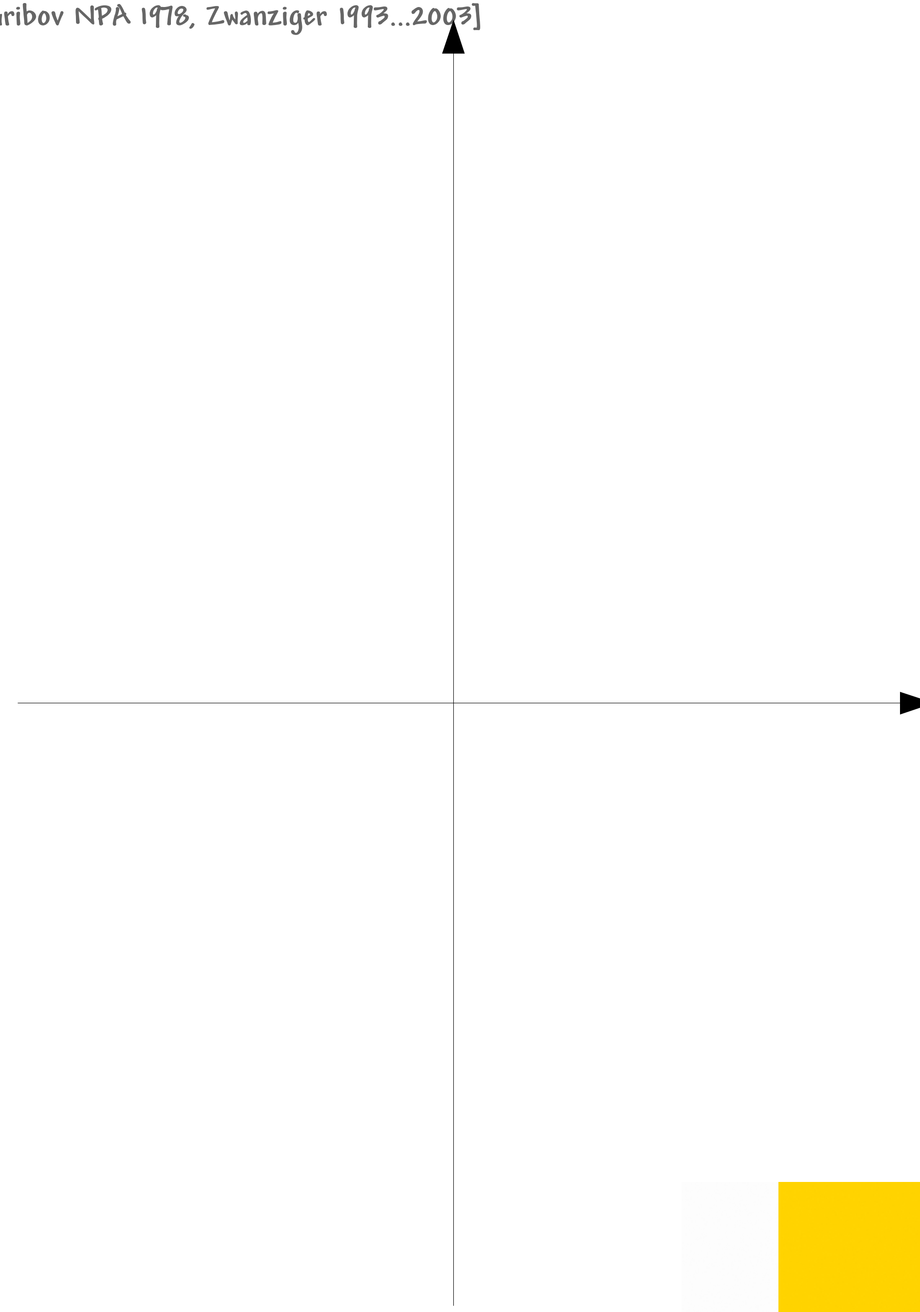

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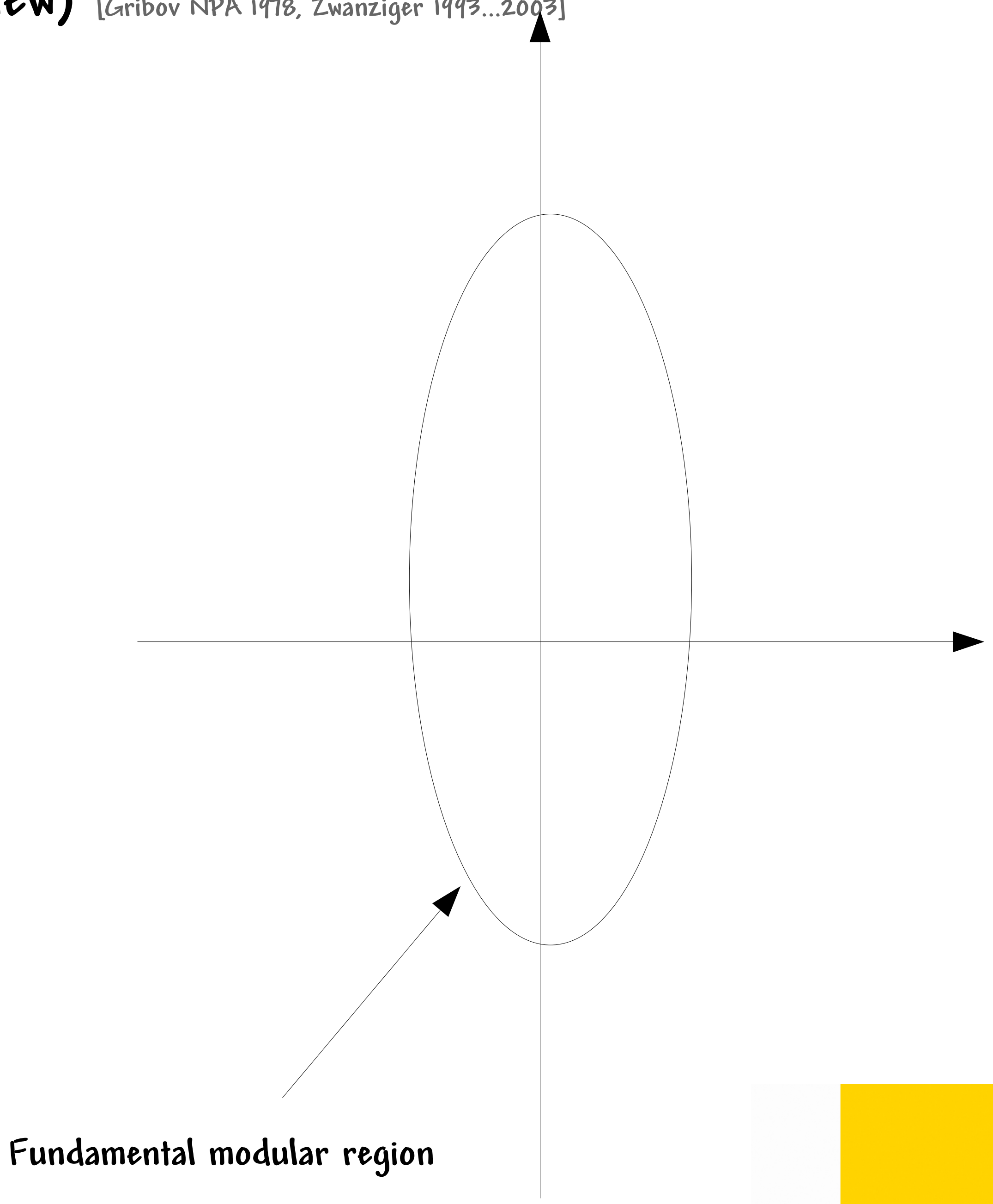

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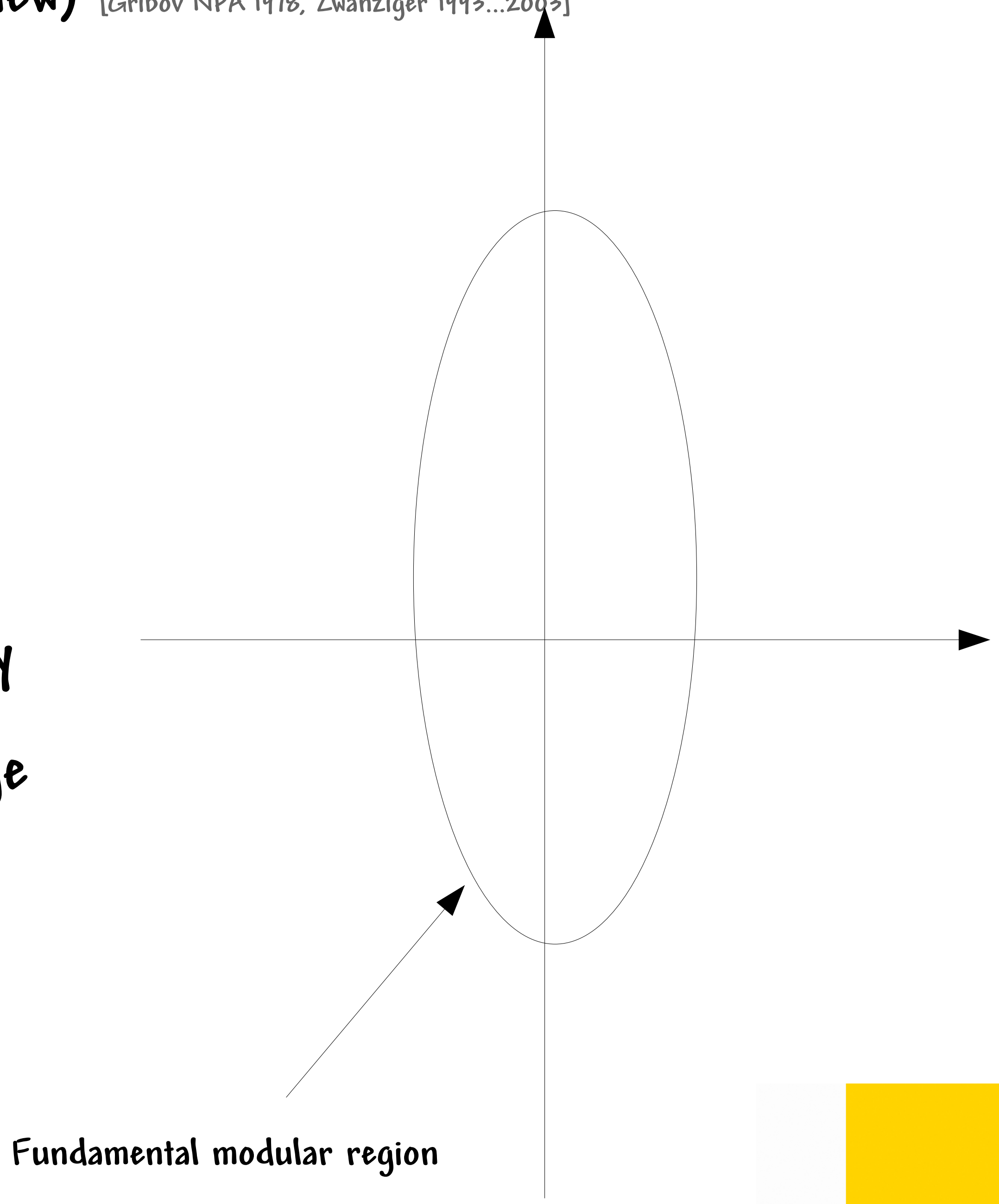
  - Unique: Absolute Landau gauge



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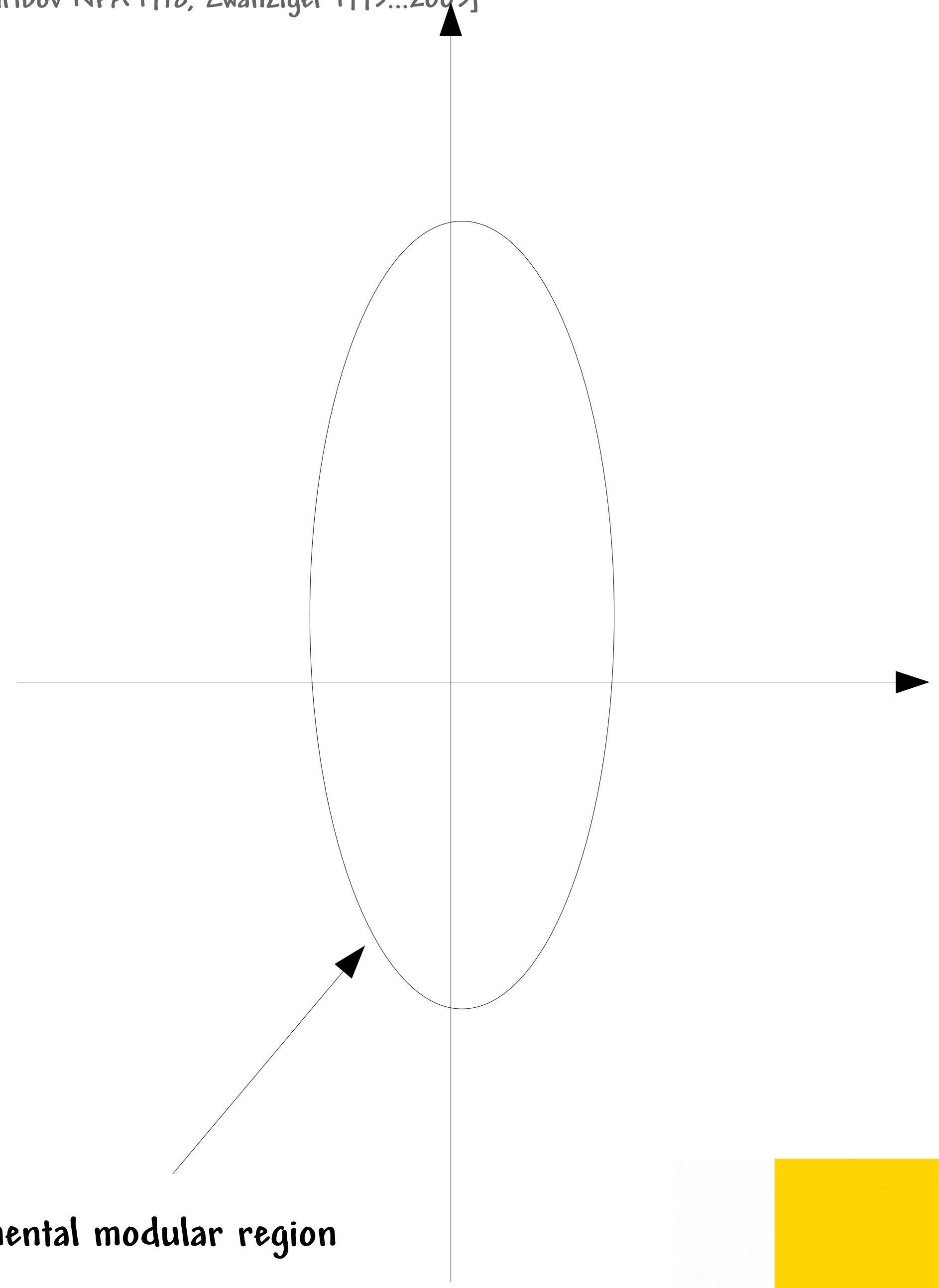
- Equivalent: Take the representative of the gauge orbit, which minimizes the trace of the gluon propagator

  $$\int dp\, D_{\mu\mu}^{aa}(p)$$



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- Here: Absolute Landau gauge

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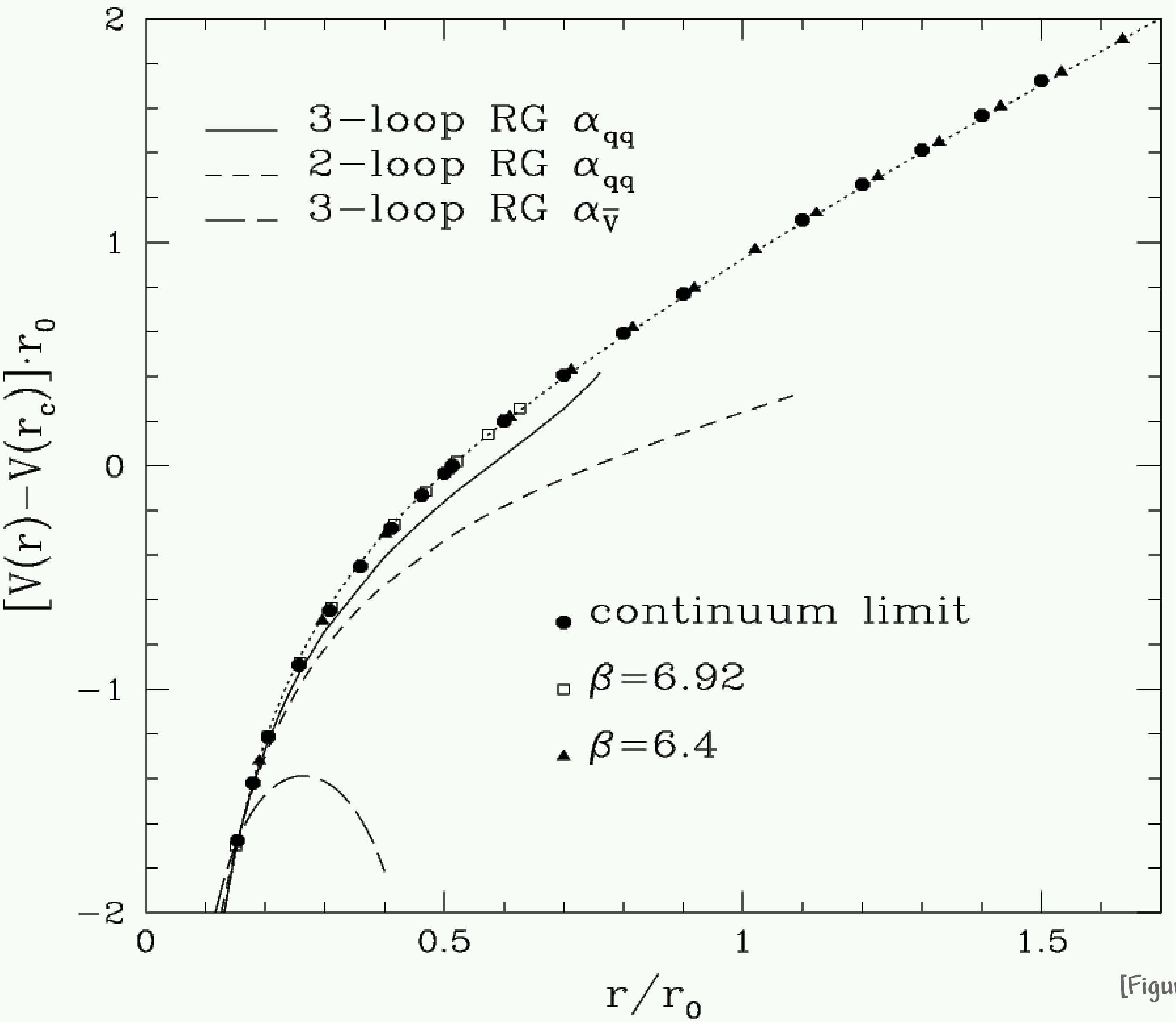
- In general: Vanishing of all colored expectation values
  - Necessary and sufficient
- Confining potentials, e.g. linear rising
  - Works only for non-dynamical objects – useless for gluons
  - Leads to (perfect) screening for dynamical objects

Confinement of quarks

- Inter-quark potential confining



[Figures from Sommer et al., 2001)]

Confinement of quarks

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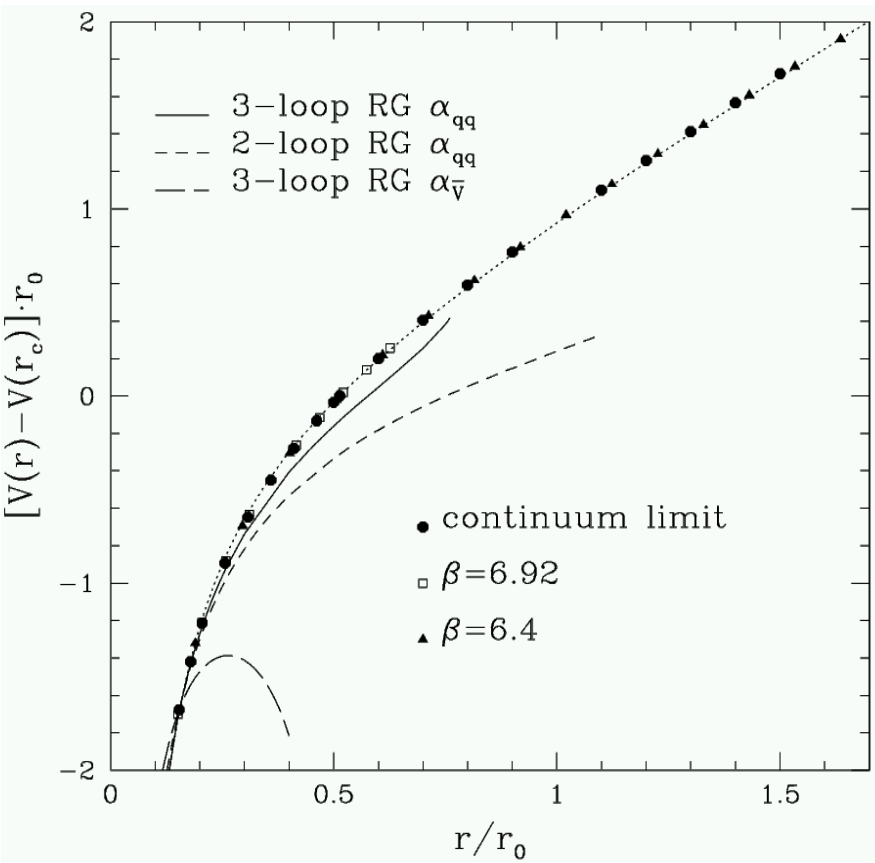
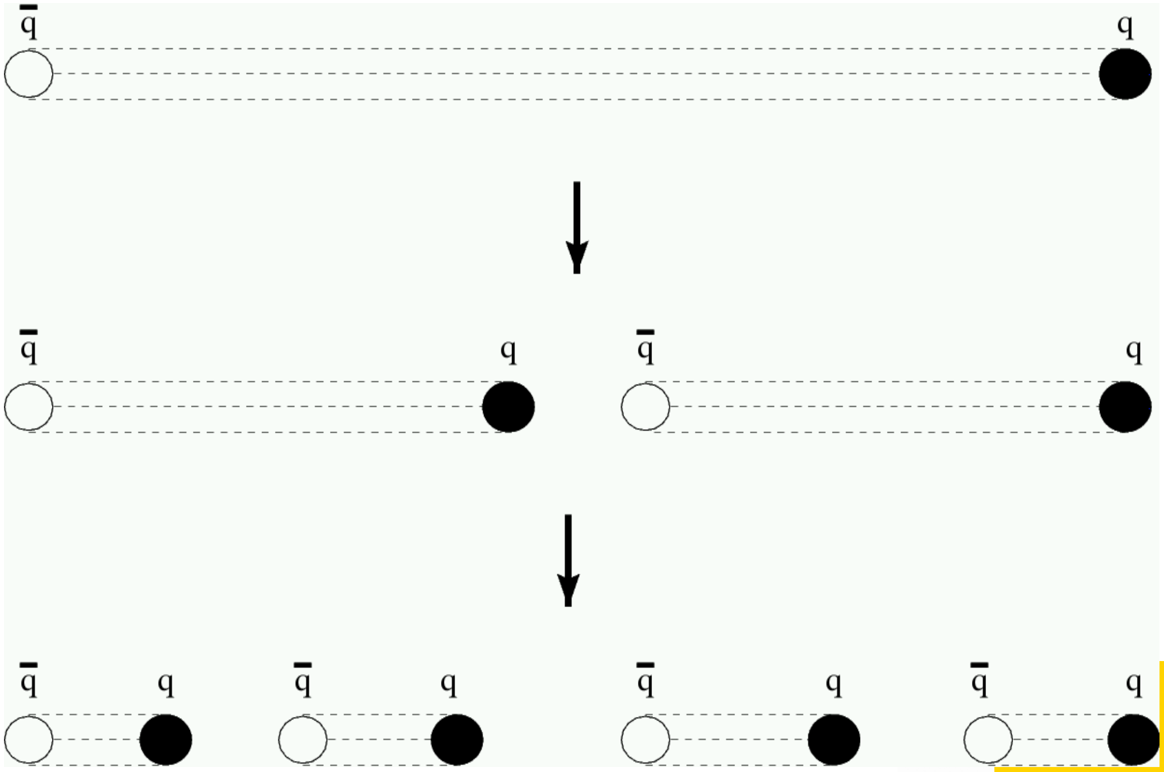


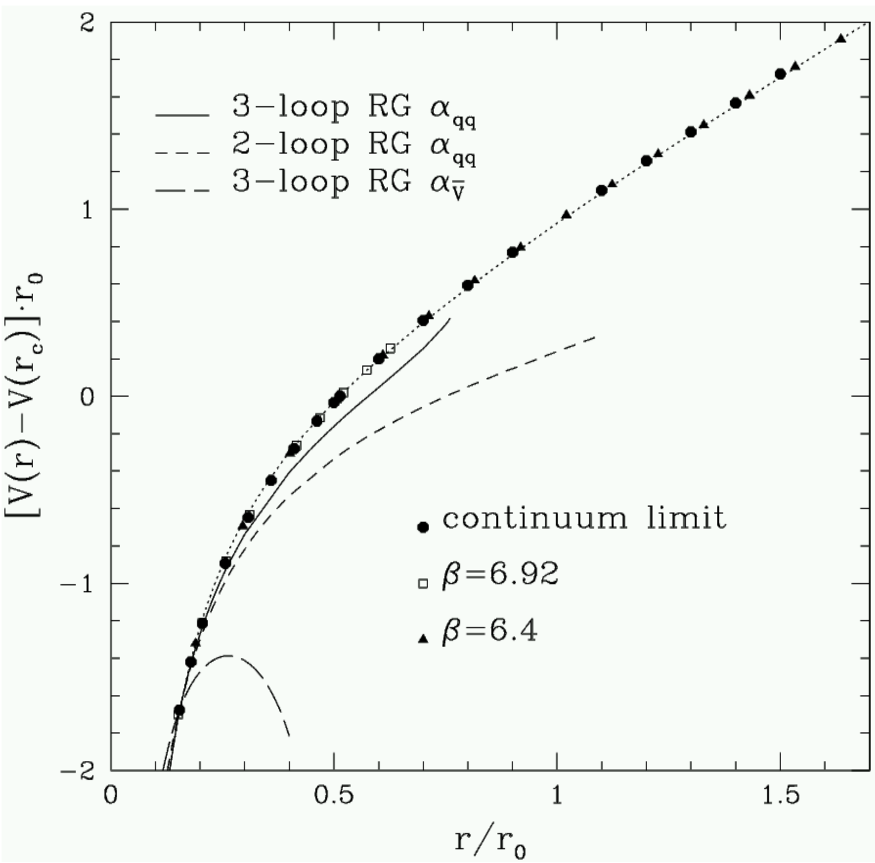


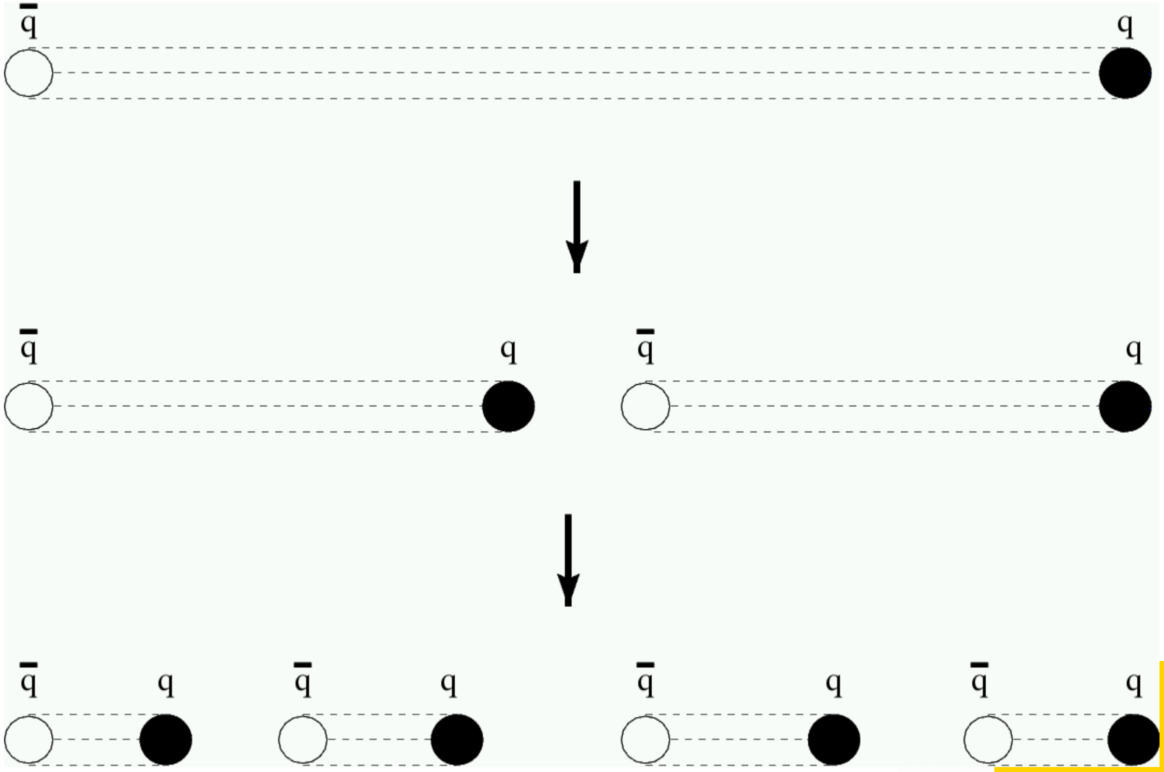
[Figures from Sommer et al., 2001 (left) and from Greensite, 2003 (right)]

UNI GRAZ

Confinement of quarks

- Inter-quark potential confining
  - Flattened by string breaking
- Origin of linear potential?





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  - Sum rule for gluons

$$Overlap\ with\ one\ particle + \int dq^2\, spectral\ function(q^2) = \frac{1}{Z_3} = 0$$

    - Z_3 (divergent) renormalization constant

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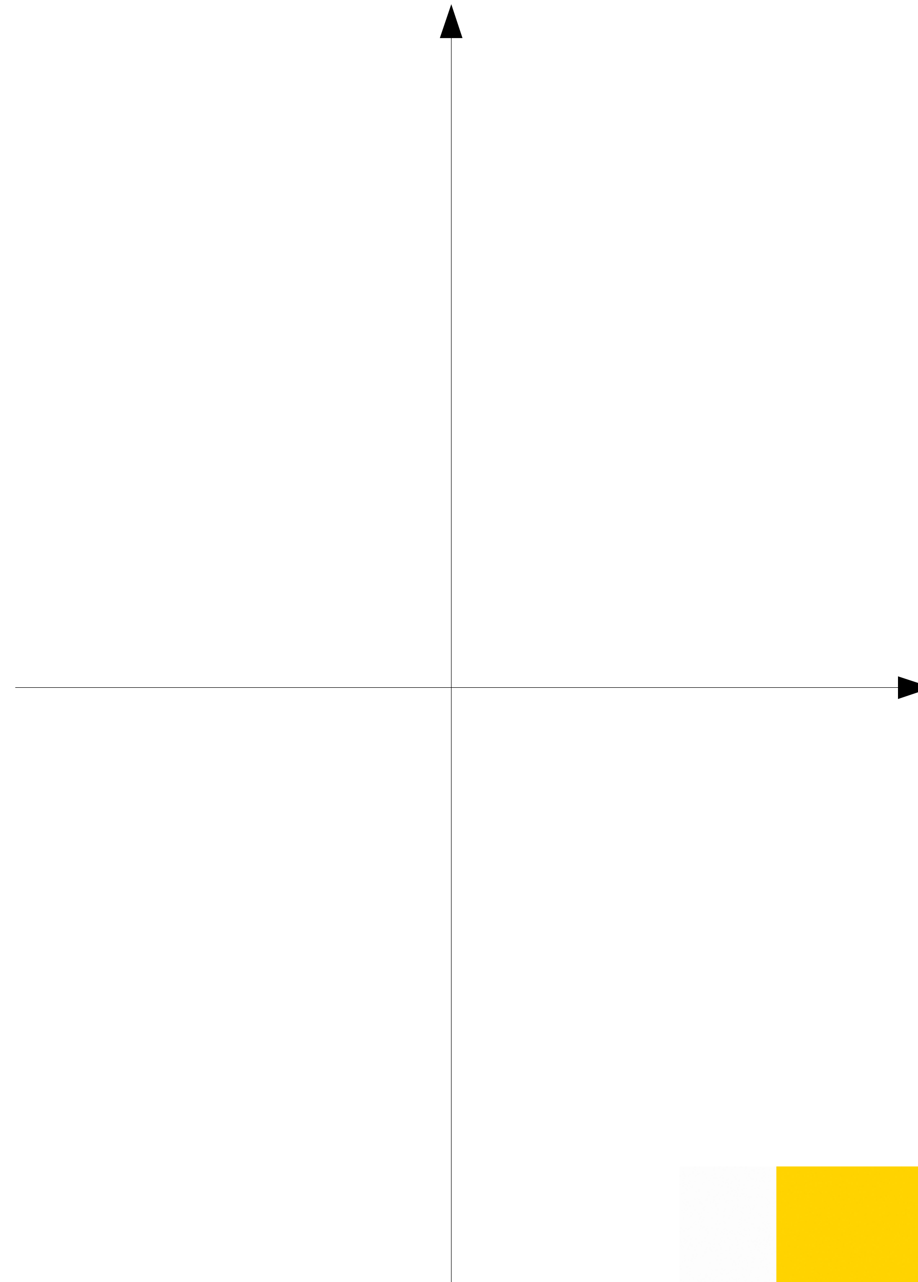

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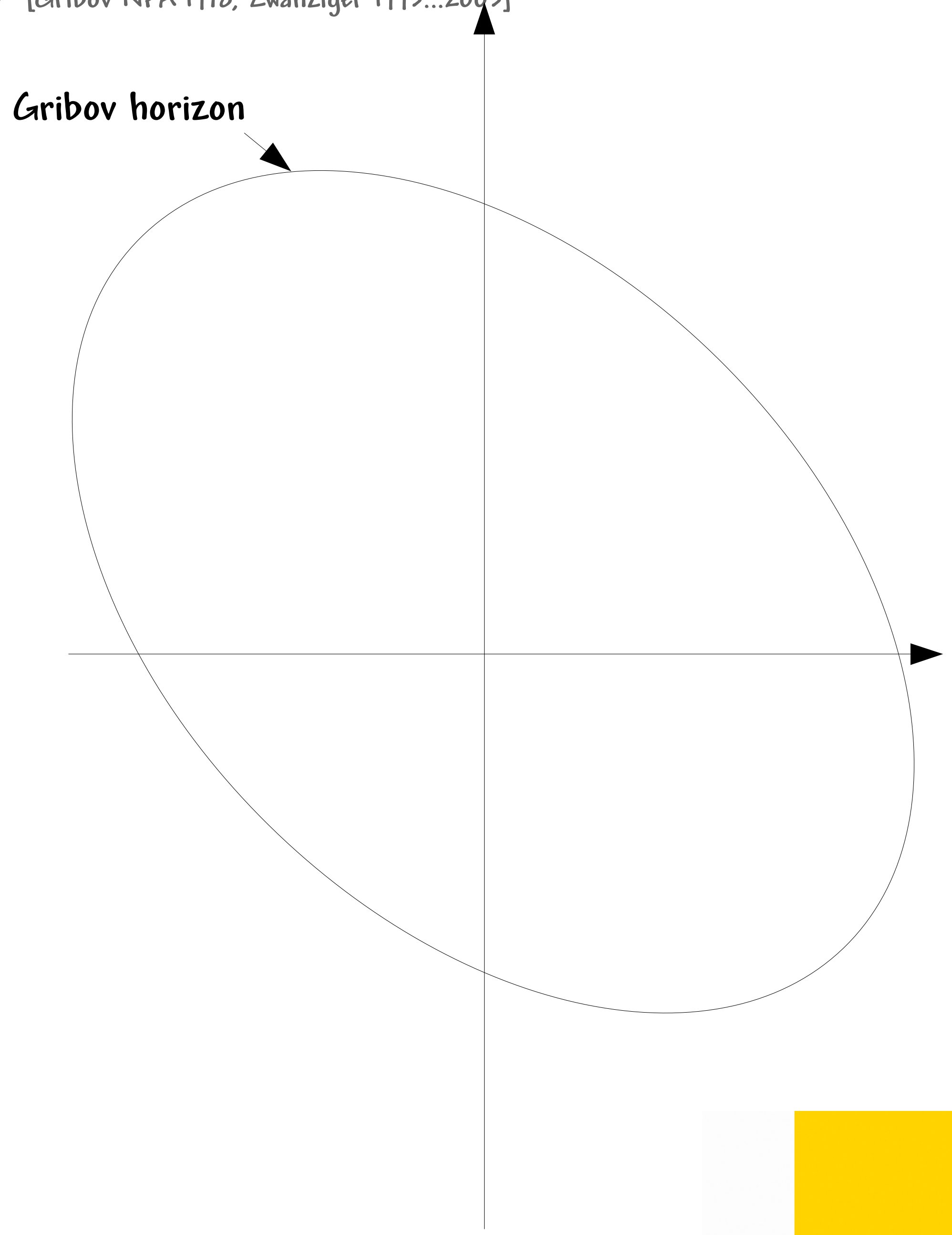

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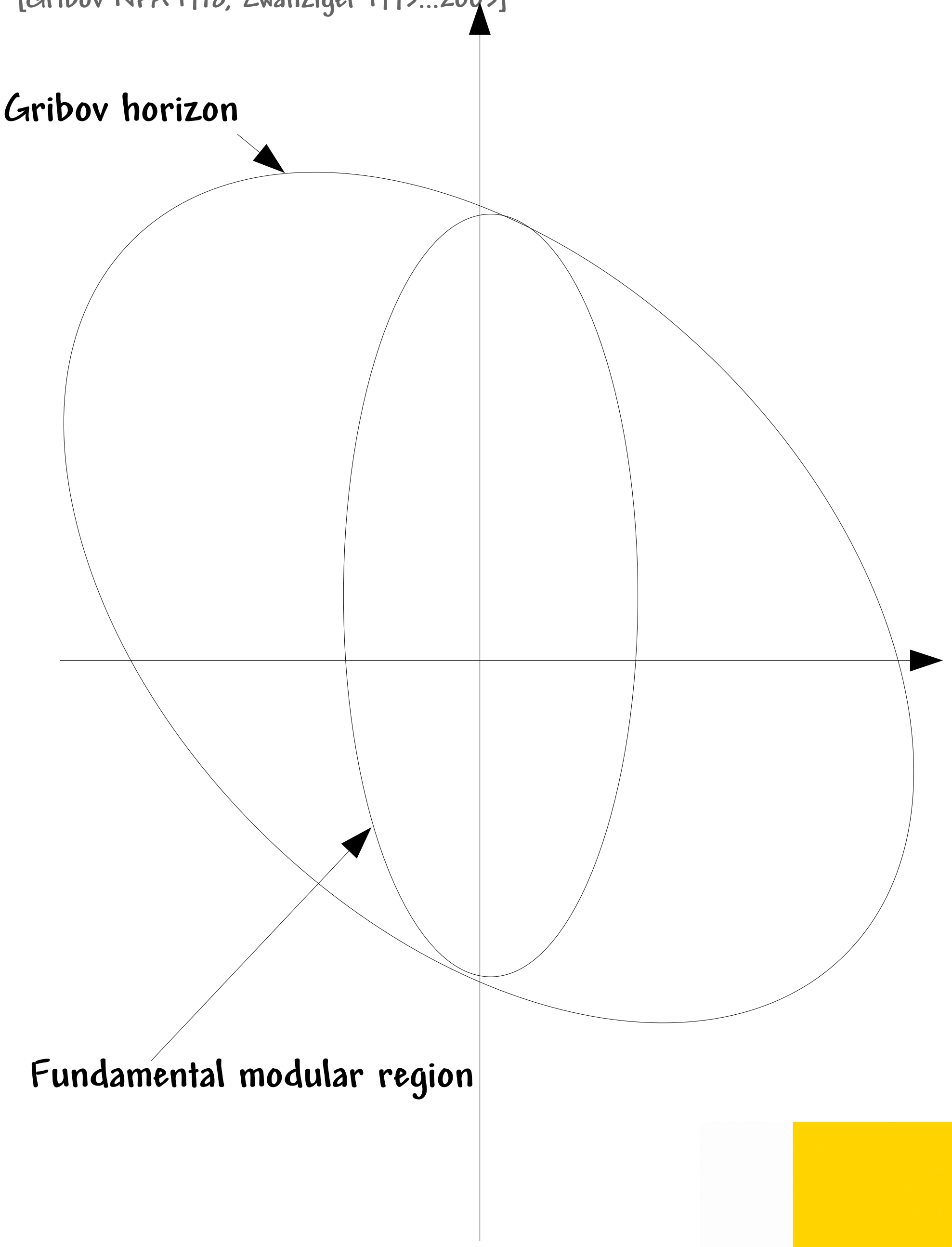

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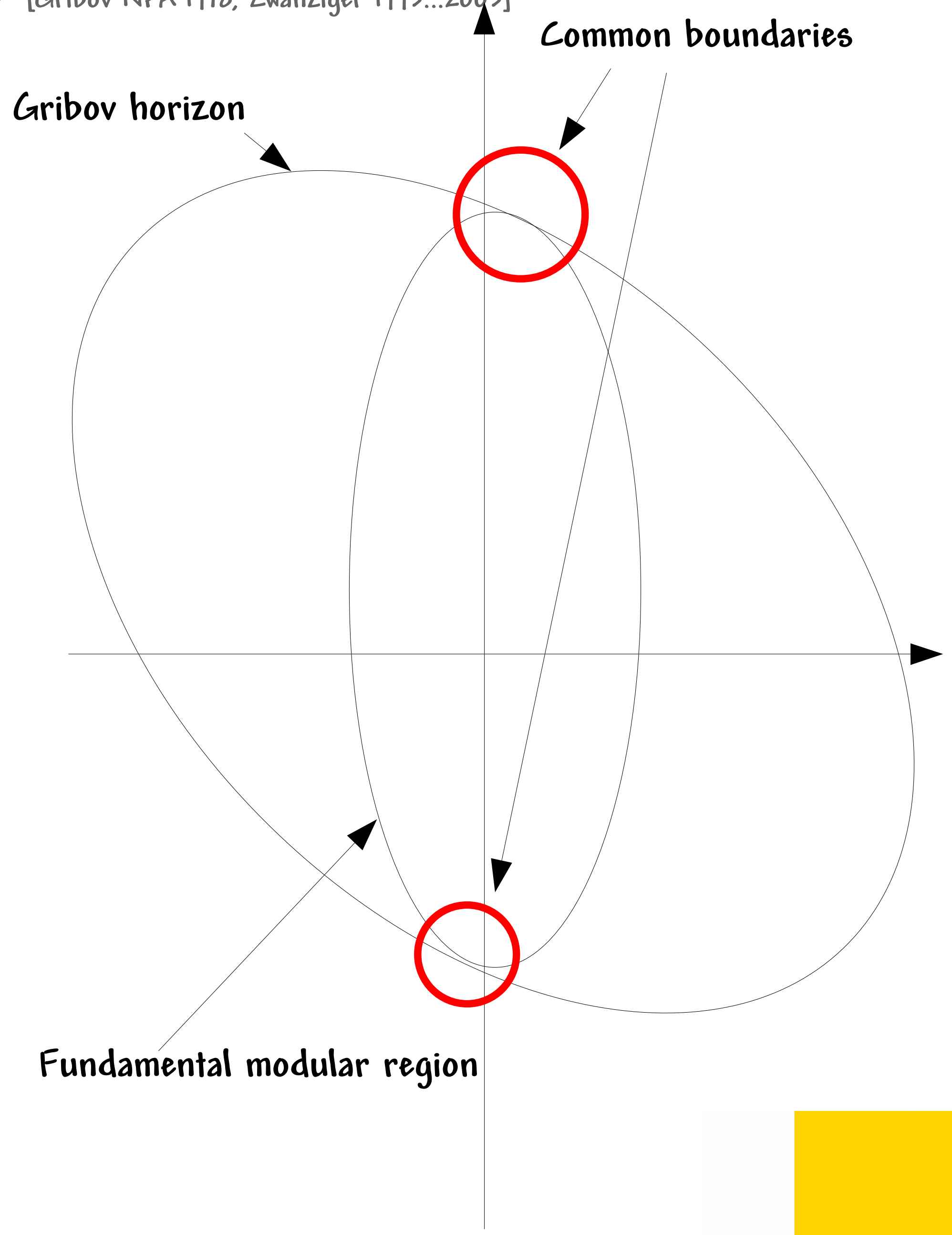

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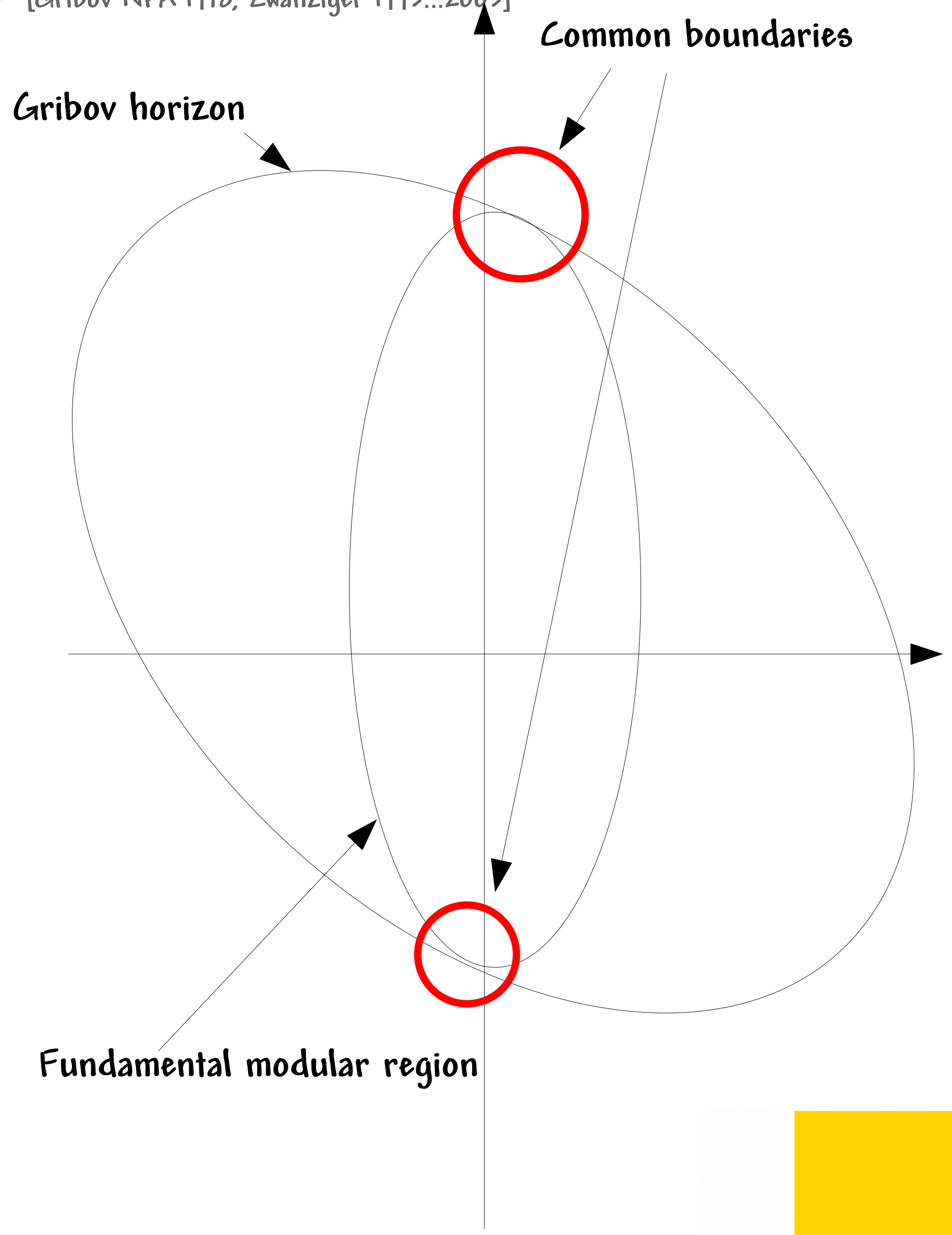

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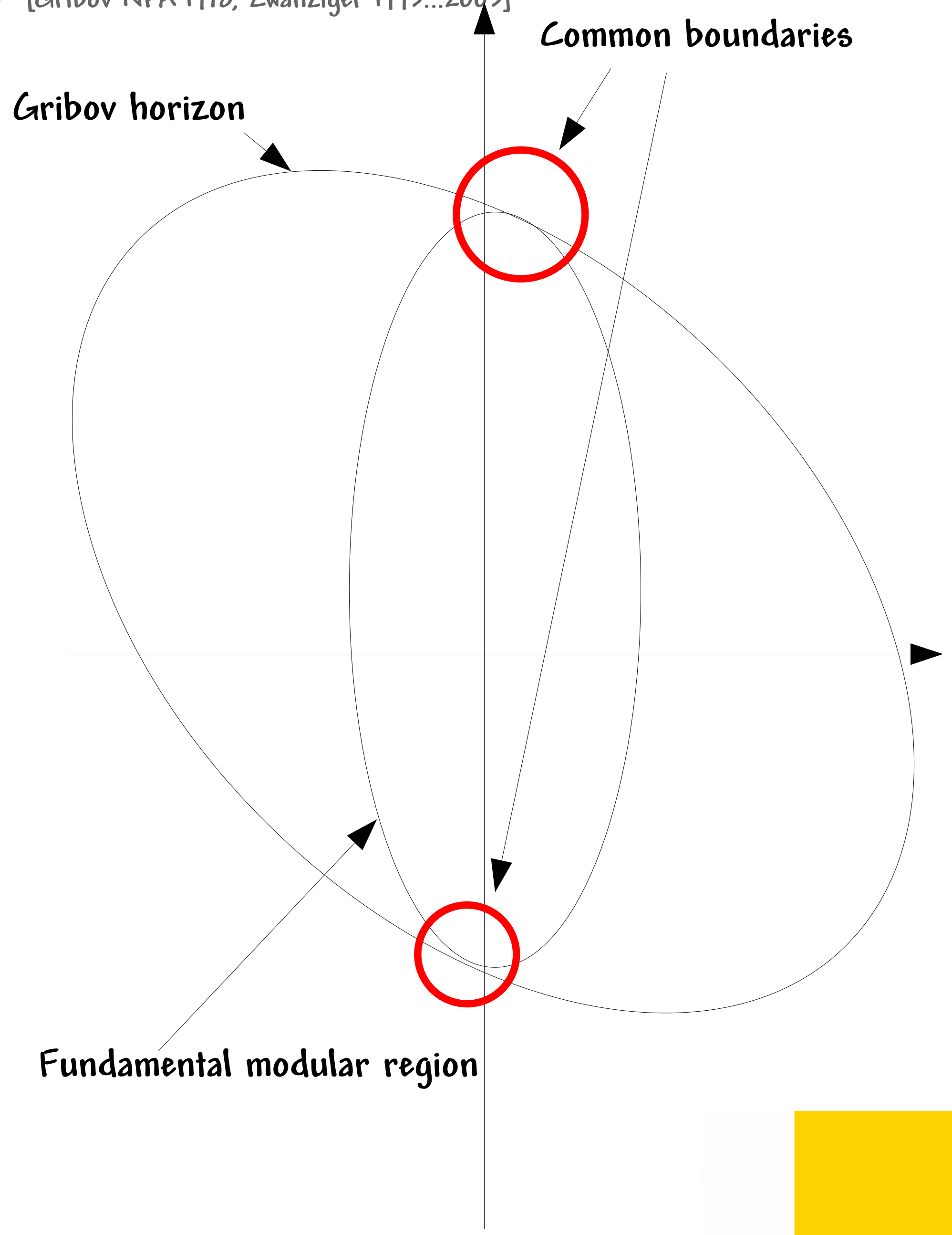

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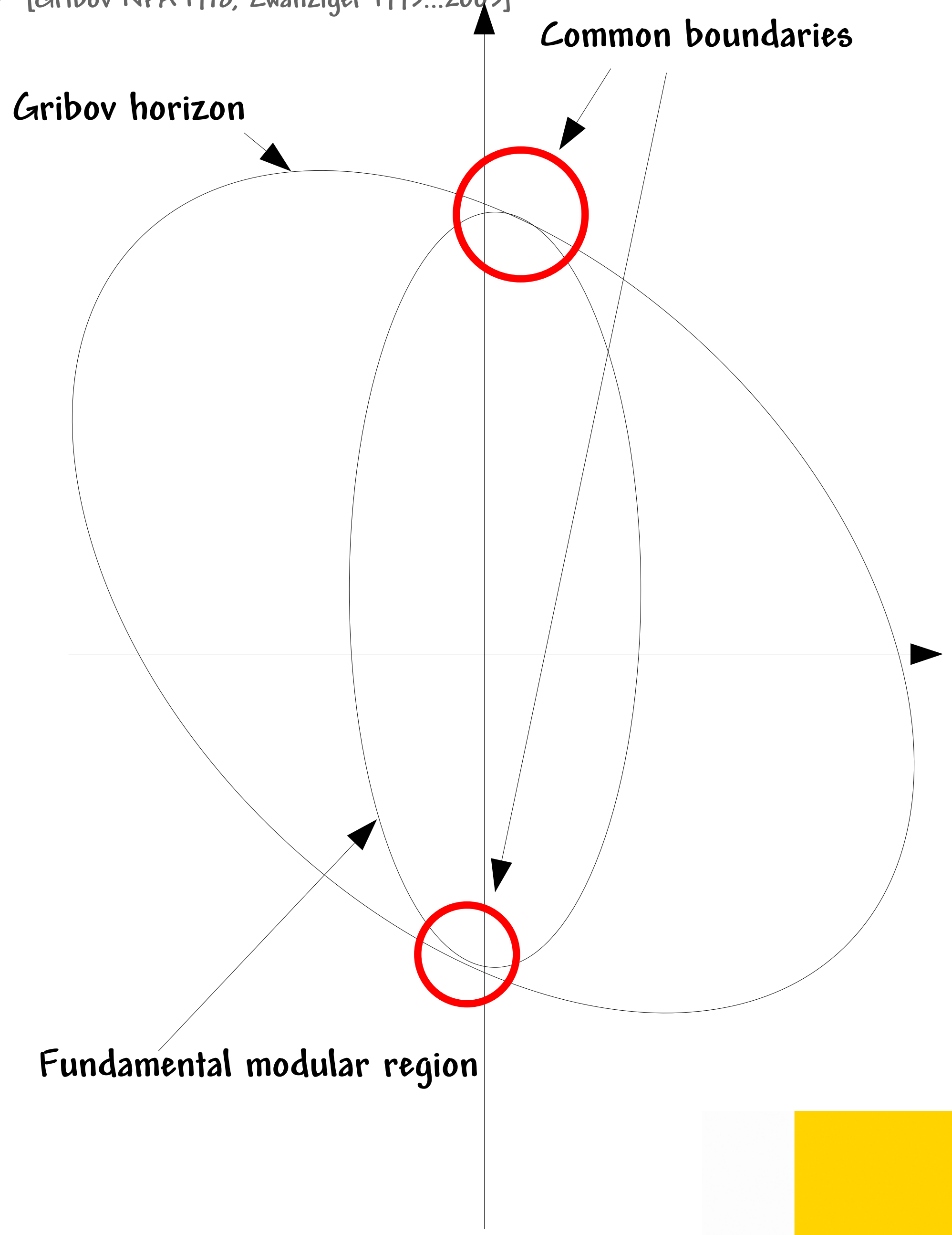

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- Simplest non-vanishing Green's functions in Yang-Mills theory: 2-point functions
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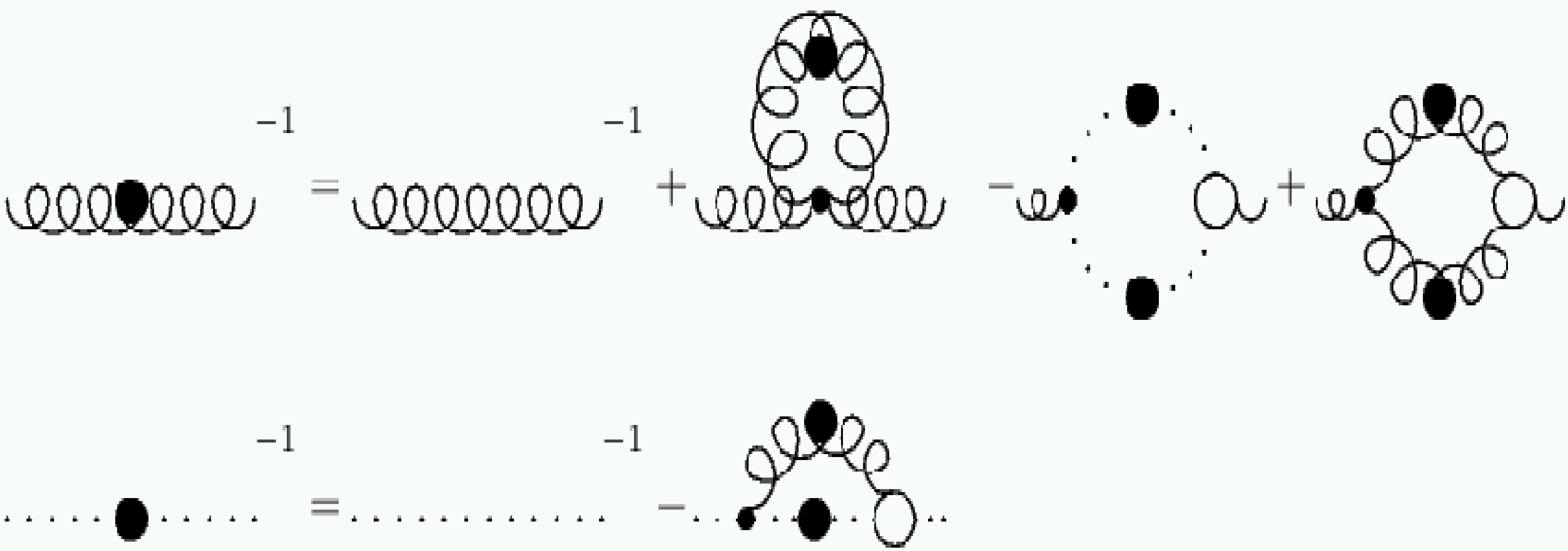

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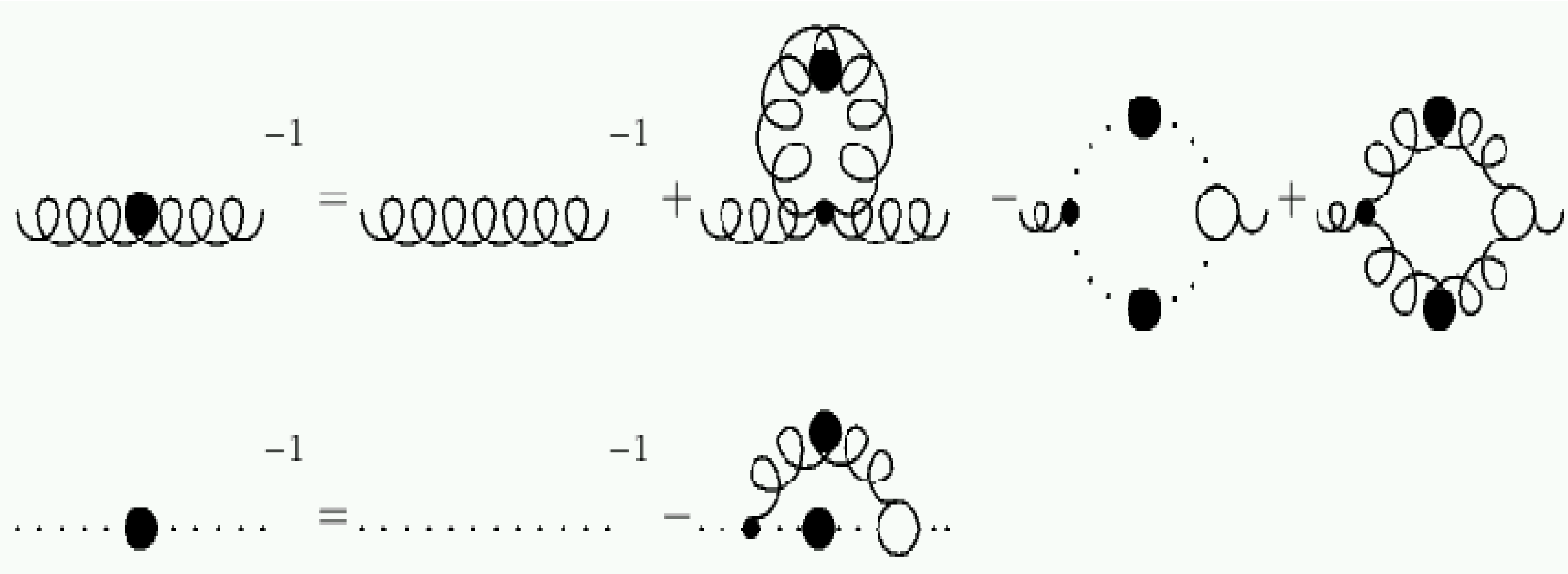

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  - Number of Gribov copies increases exponentially with volume
    - NP-hard
    - Minimal and absolute Landau gauge conincide in small volumes

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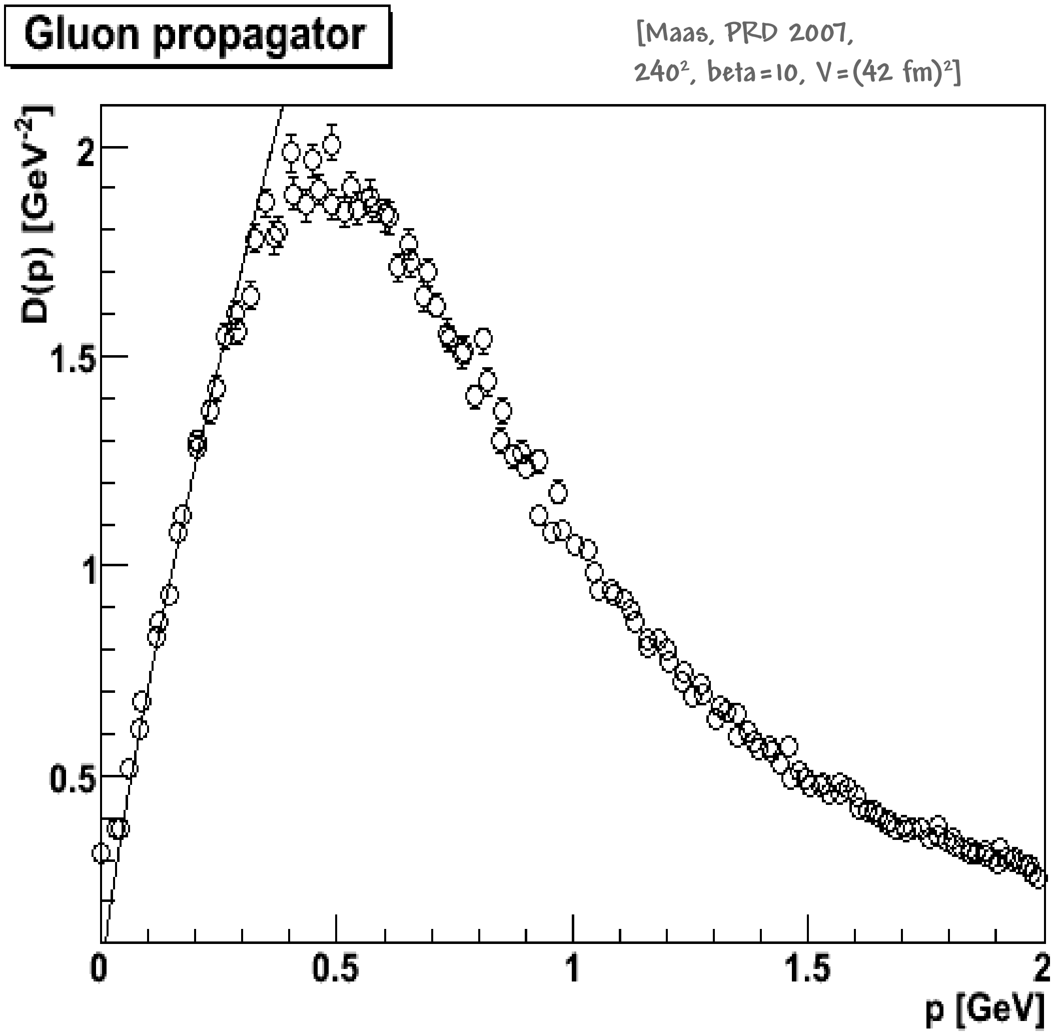

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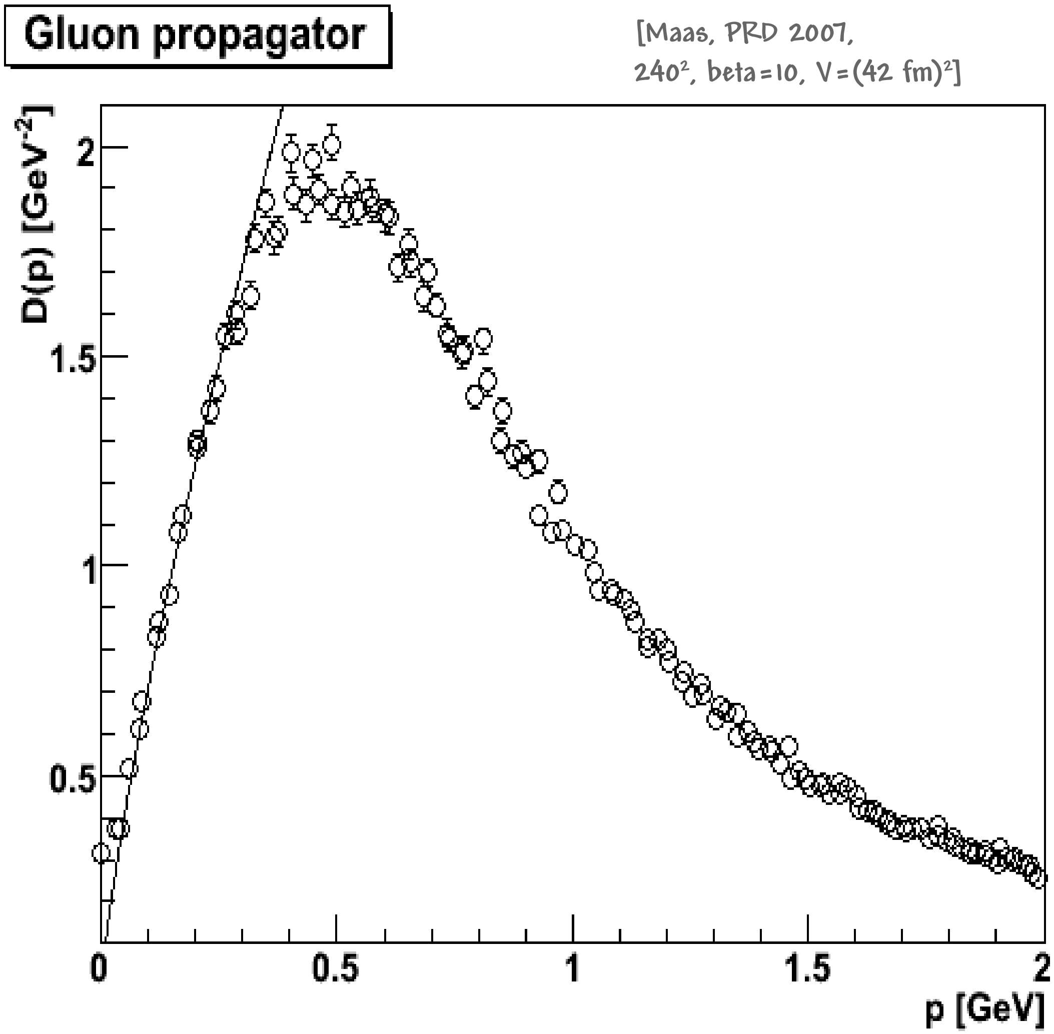
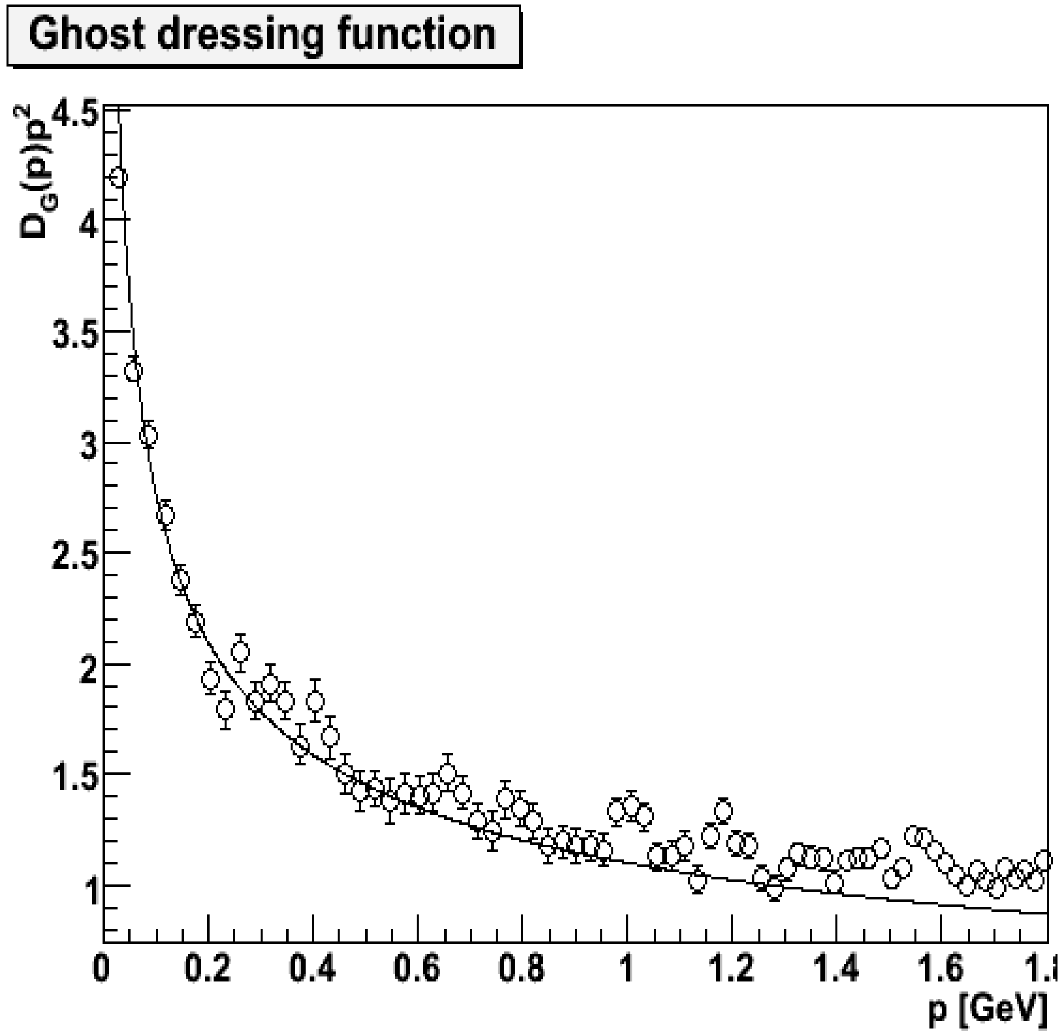
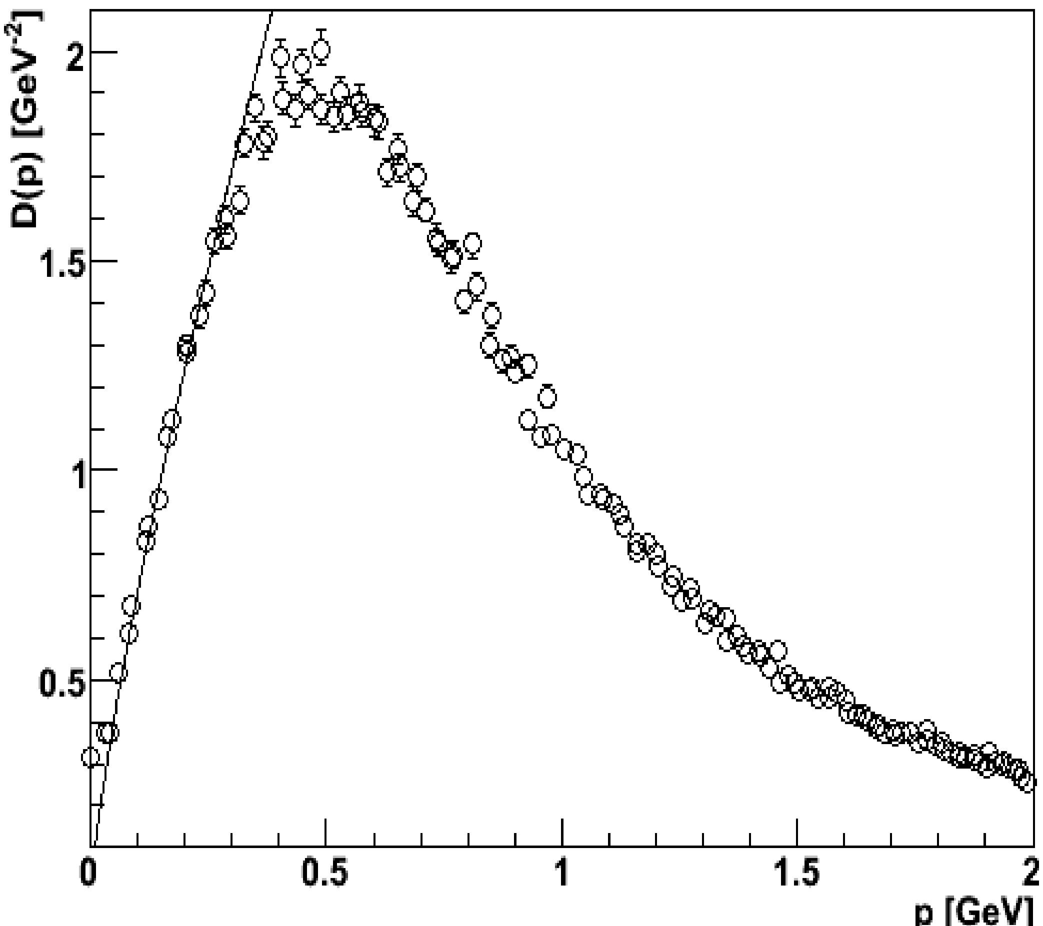




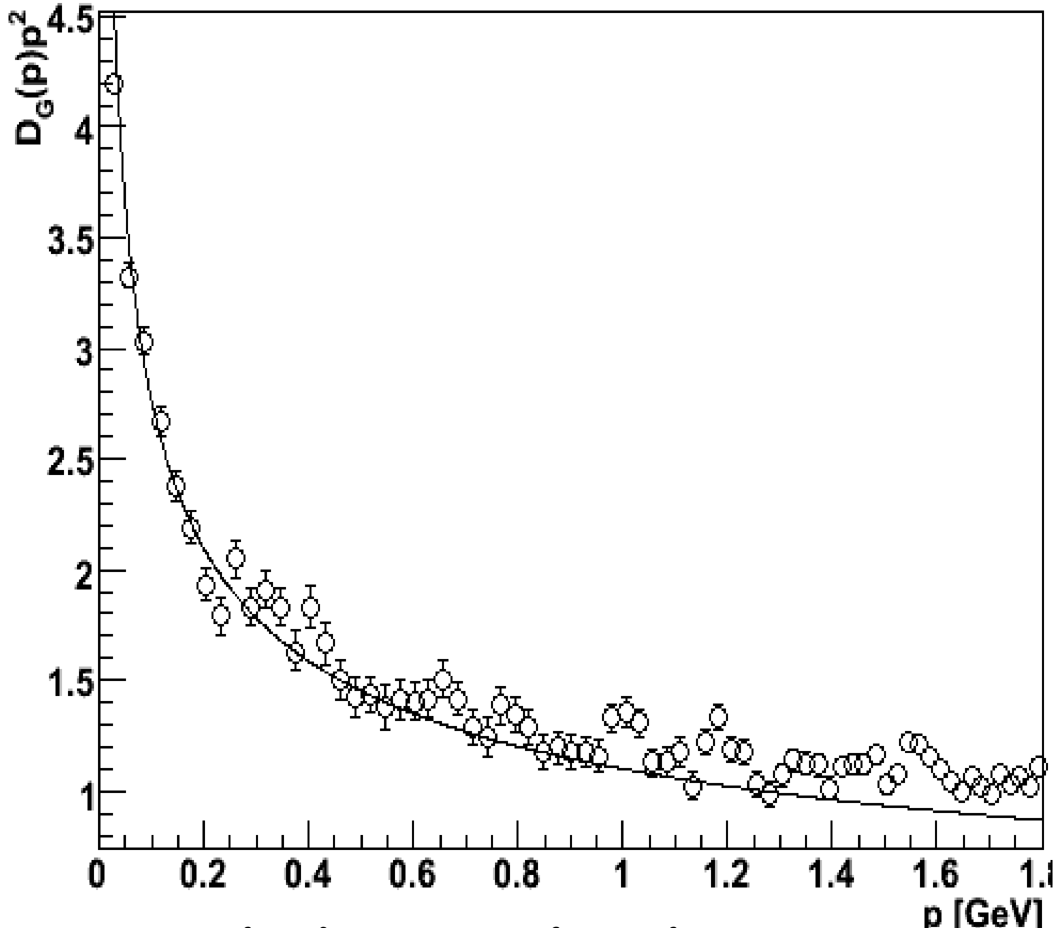
Propagators in two dimensions

Gluon propagator

[Maas, PRD 2007, 240^2, beta=10, V=(42 fm)^2]

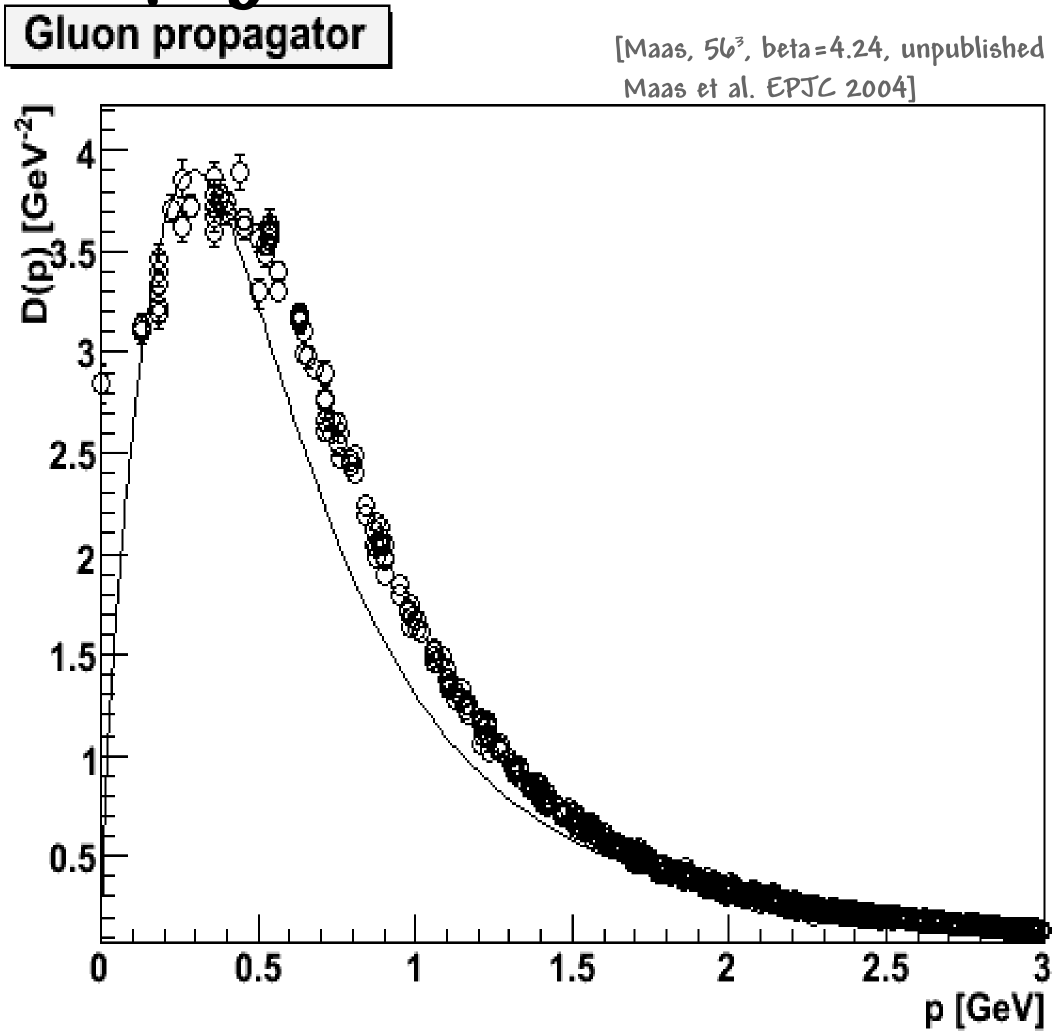


Ghost dressing function



- Quantitative in agreement – same exponent κ on the lattice and in the continuum
- Only asymptotic behavior known in the continuum
- Minimal and absolute Landau gauge results agree almost

Propagators in three dimensions



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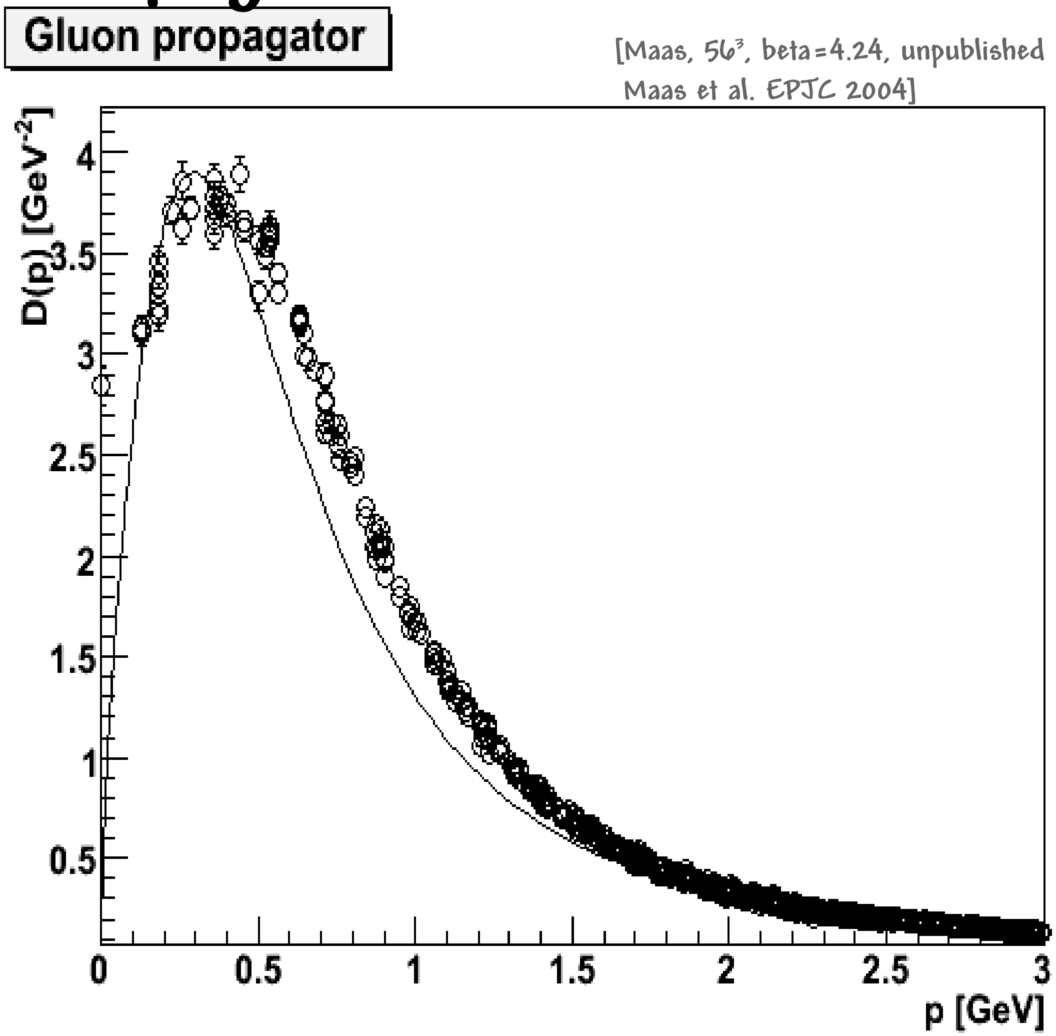

[Maas, 56^3, beta=4.24, unpublished
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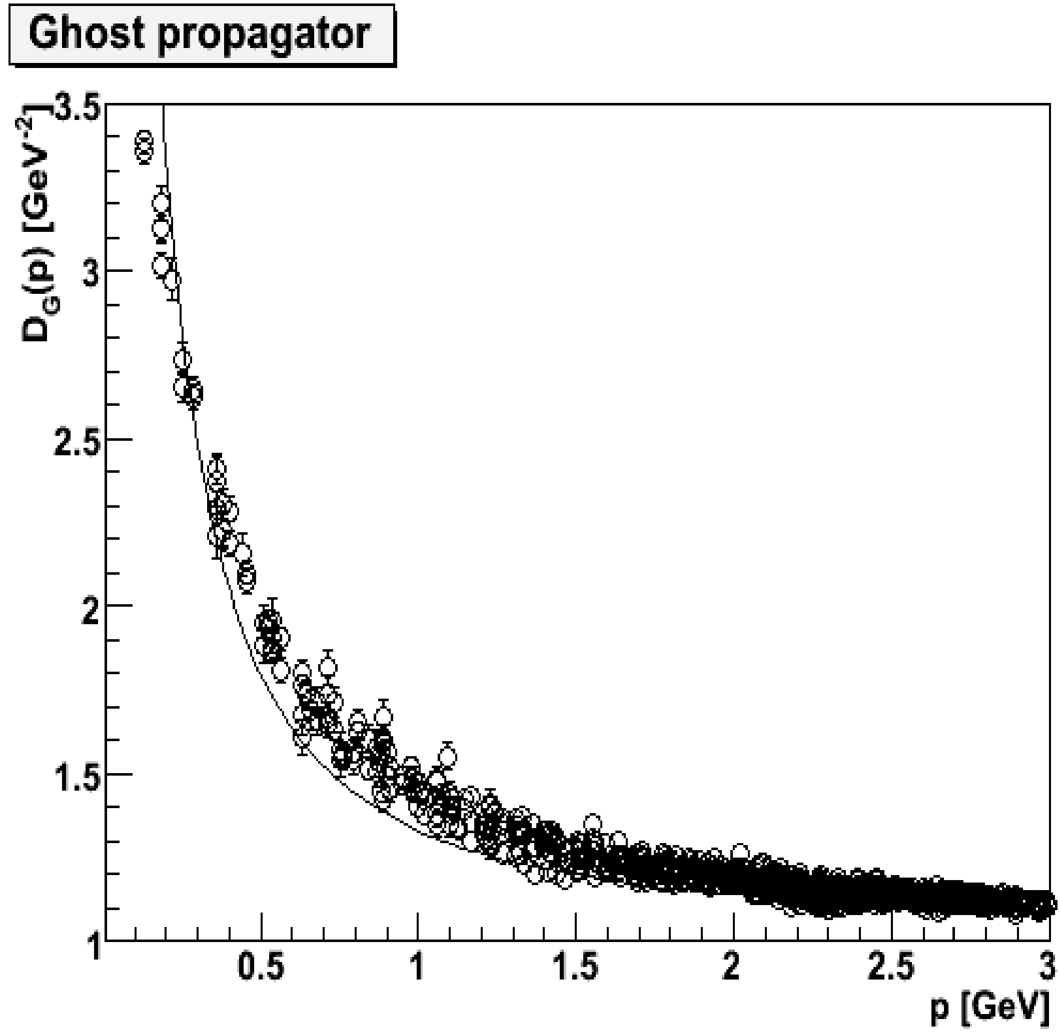

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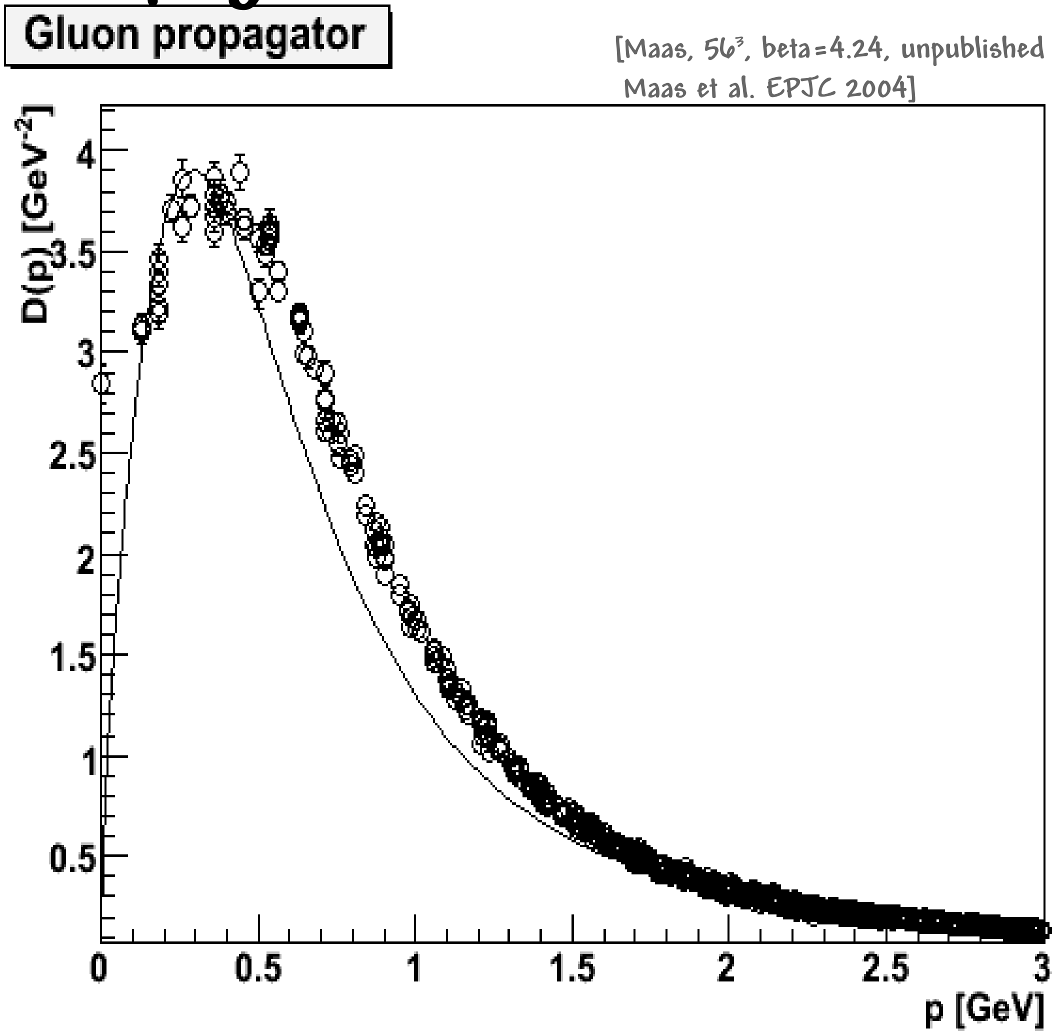
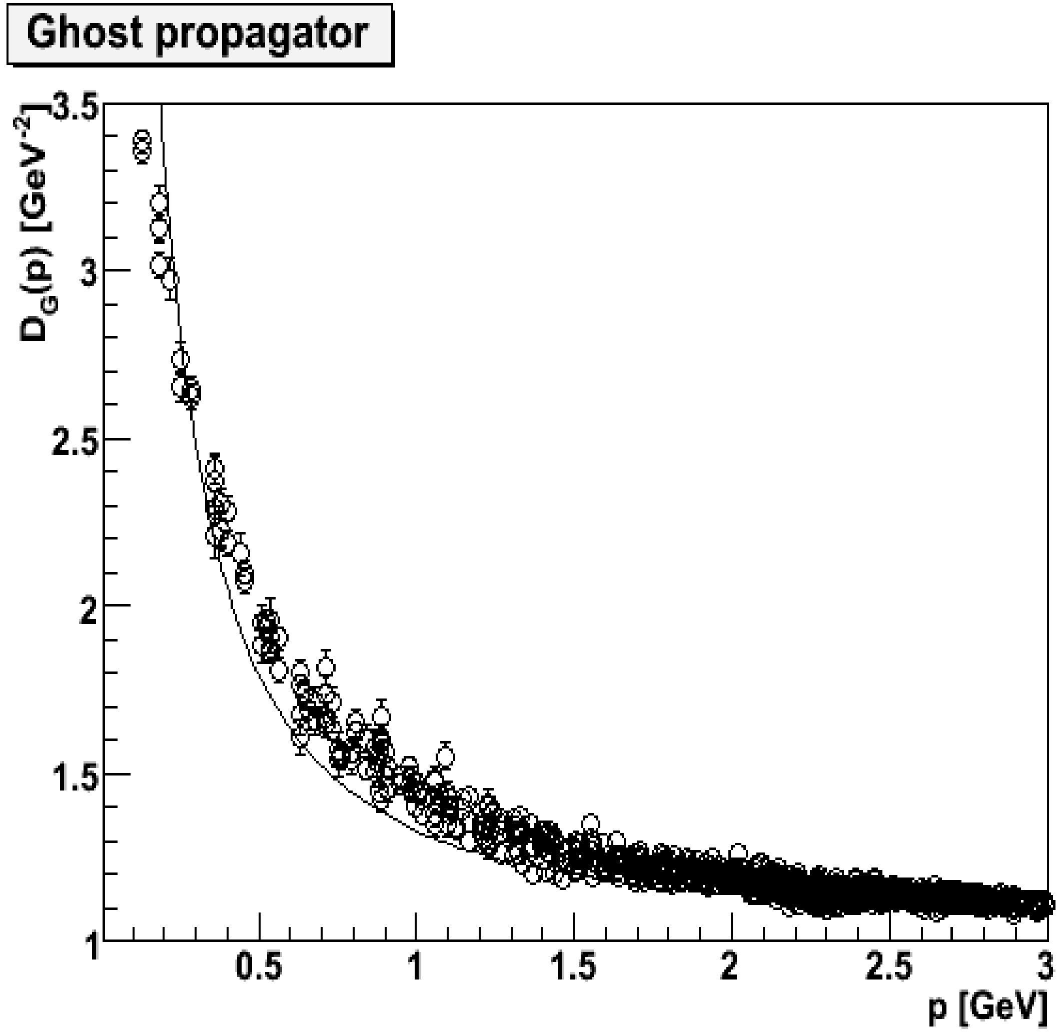
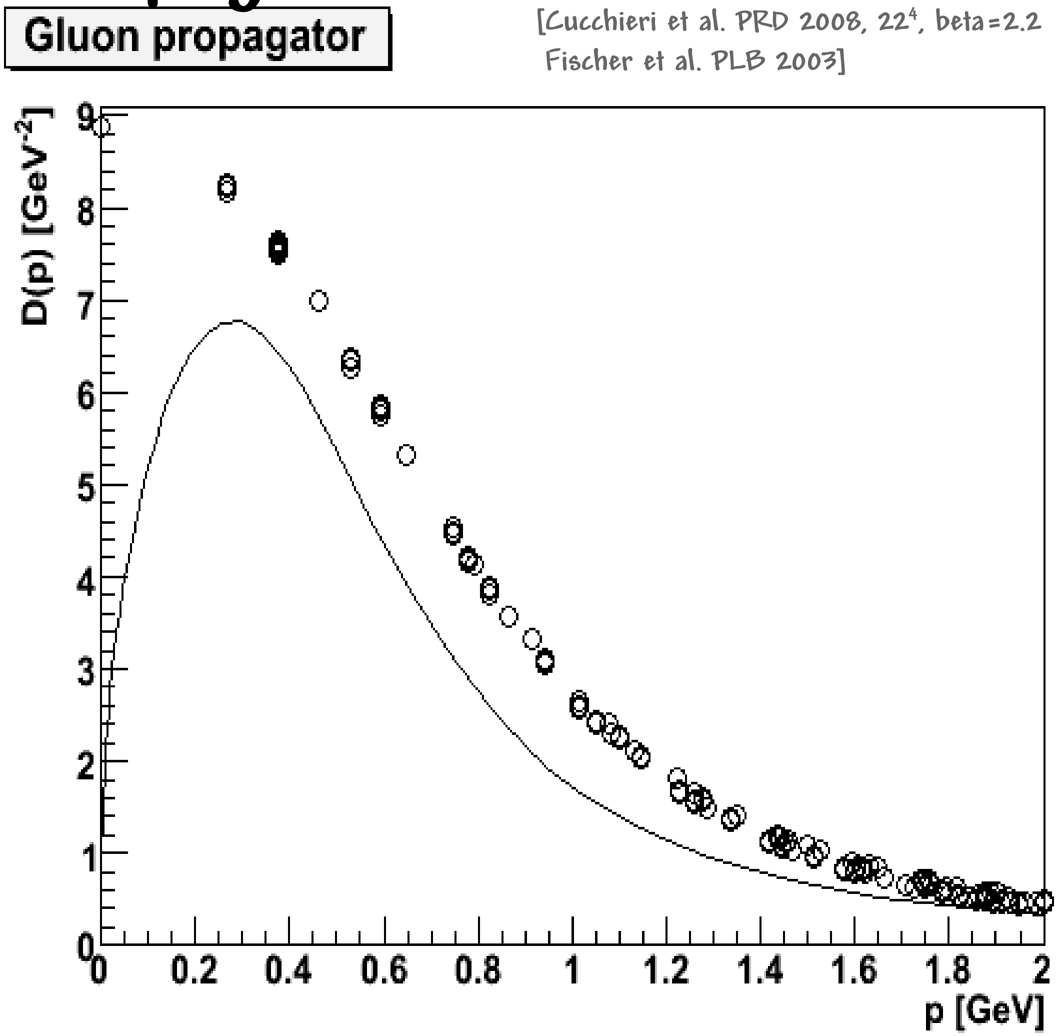




- Differences between minimal and absolute Landau gauge up to 1 GeV
  - Minimal Landau gauge leads to a massive behavior
- Qualitative agreement between lattice and continuum results

Propagators in four dimensions

[Cucchieri et al. PRD 2008, 22^4, beta=2.2
Fischer et al. PLB 2003]



Propagators in four dimensions

Gluon propagator

[Cucchieri et al. PRD 2008, 22^4, beta=2.2
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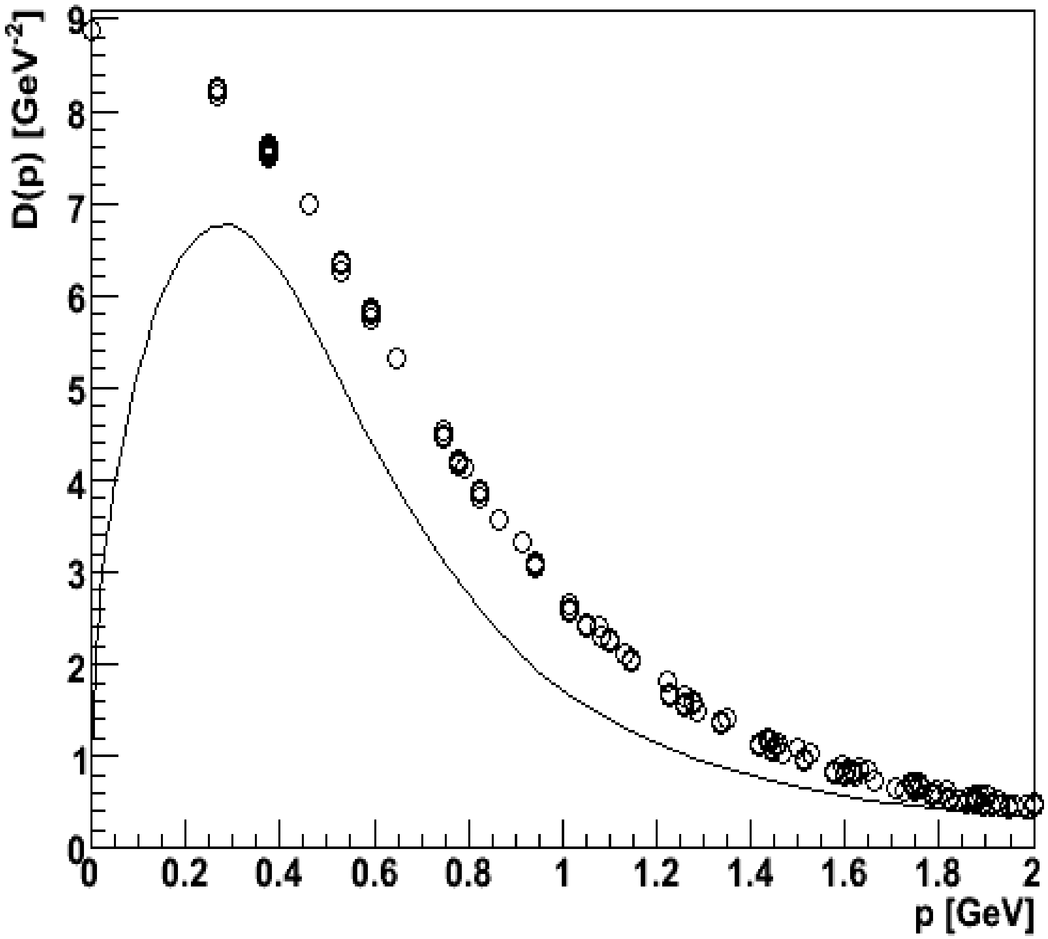
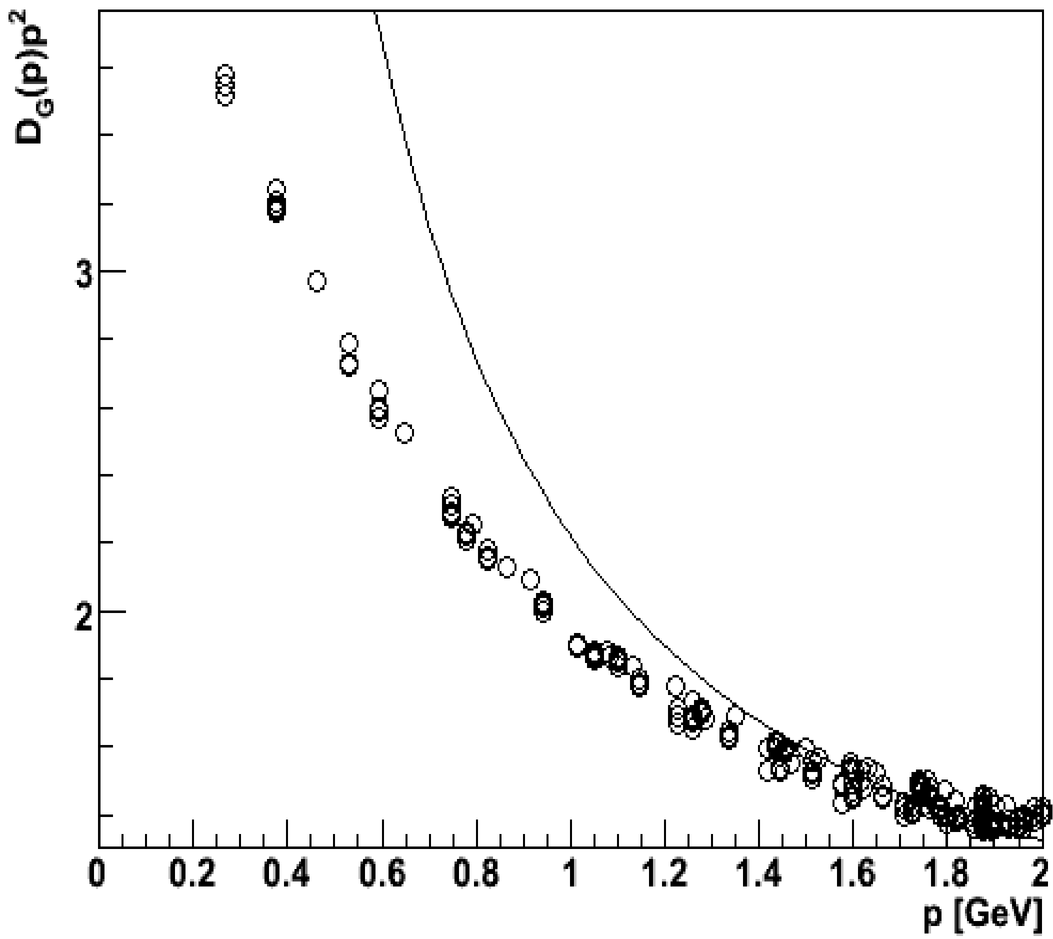
Ghost dressing function





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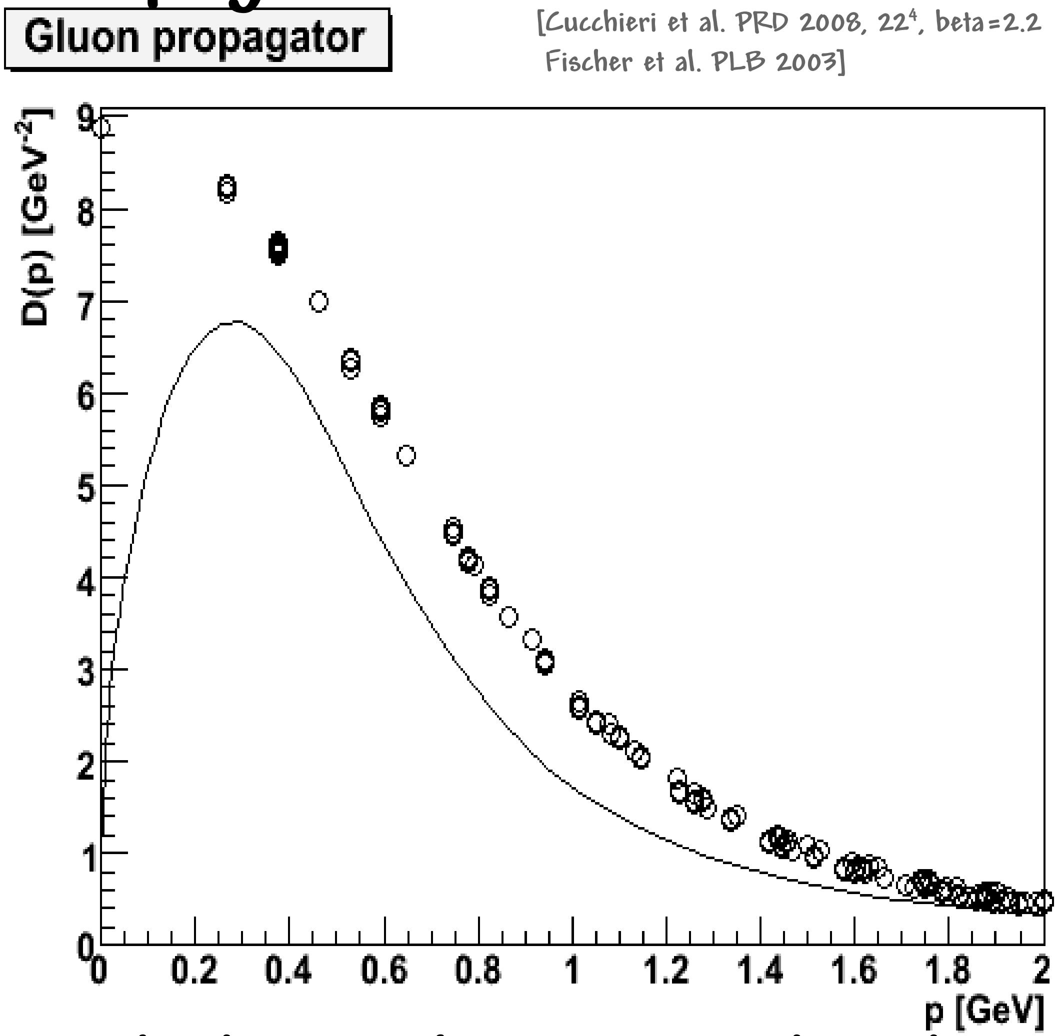

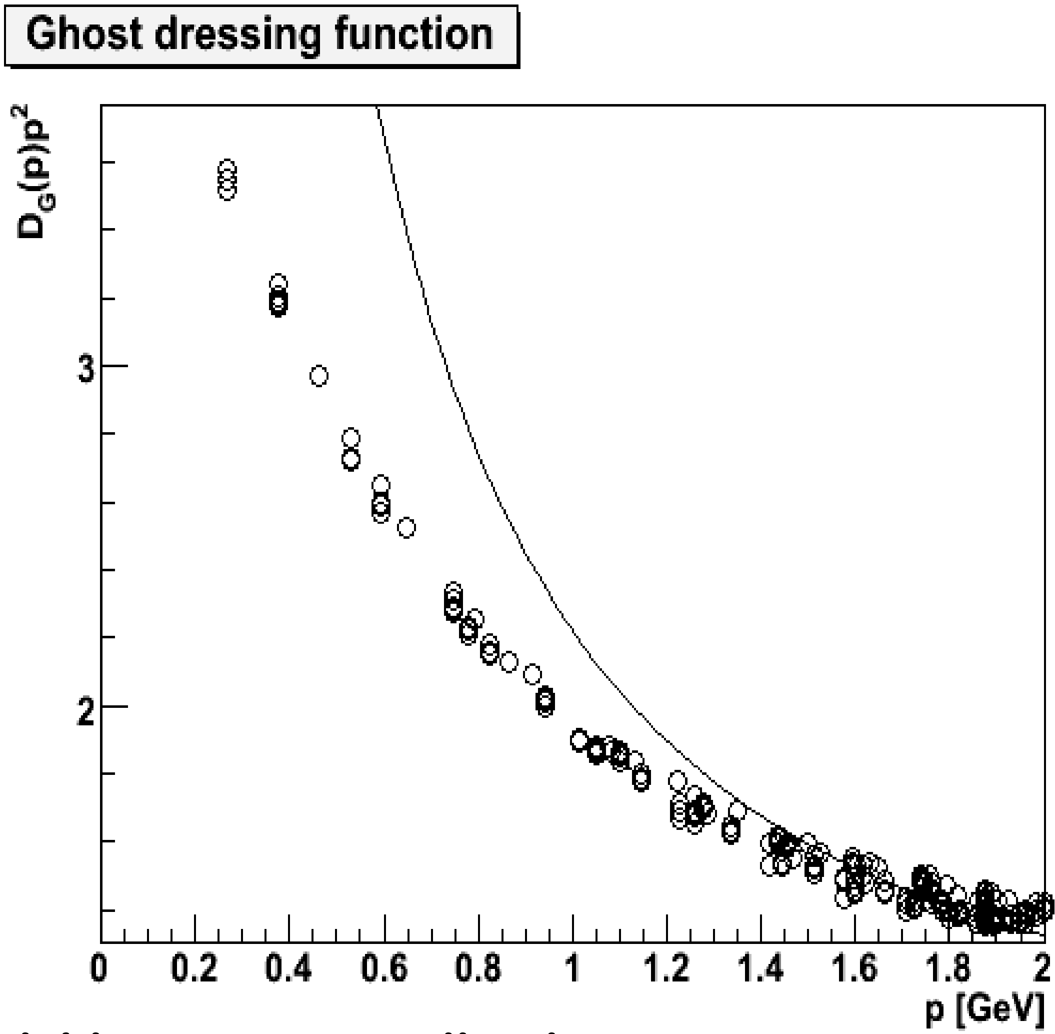

- Absolute Landau gauge results only available on very small volumes
- Infrared dominated by finite volume artifacts
- Functional methods give predictions for infinite volume approach

Vertices in Landau gauge

- Two 3-point vertices in Landau gauge

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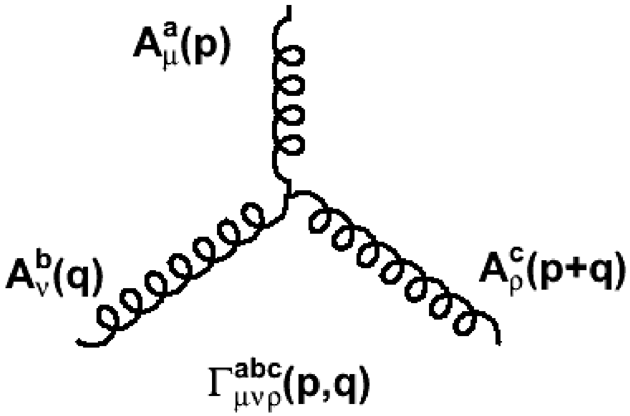
  - Two at tree-level and beyond
  - Describe gluon self-interaction



Vertices in Landau gauge

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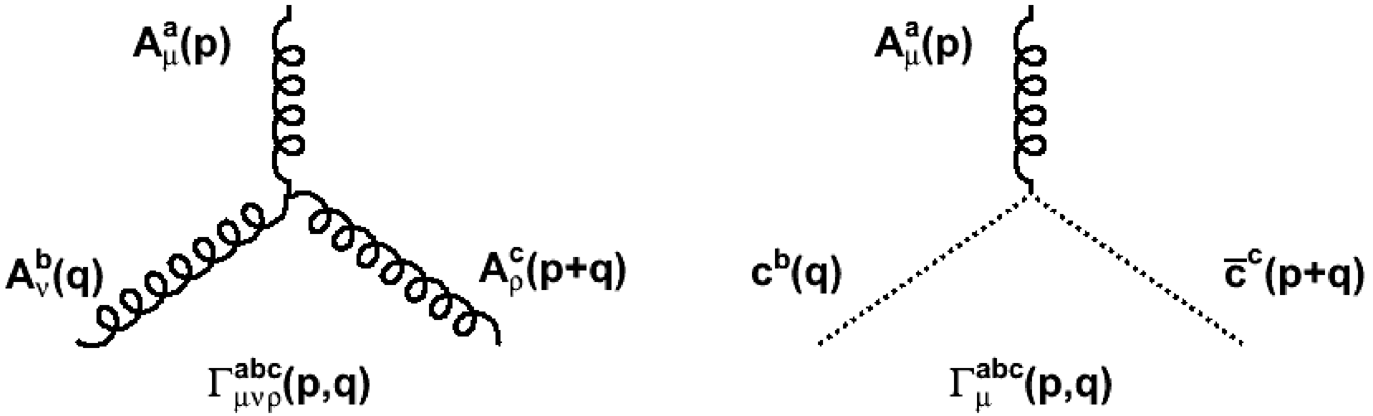
  - Two at tree-level and beyond
  - Describe gluon self-interaction and ghost-gluon self-interaction



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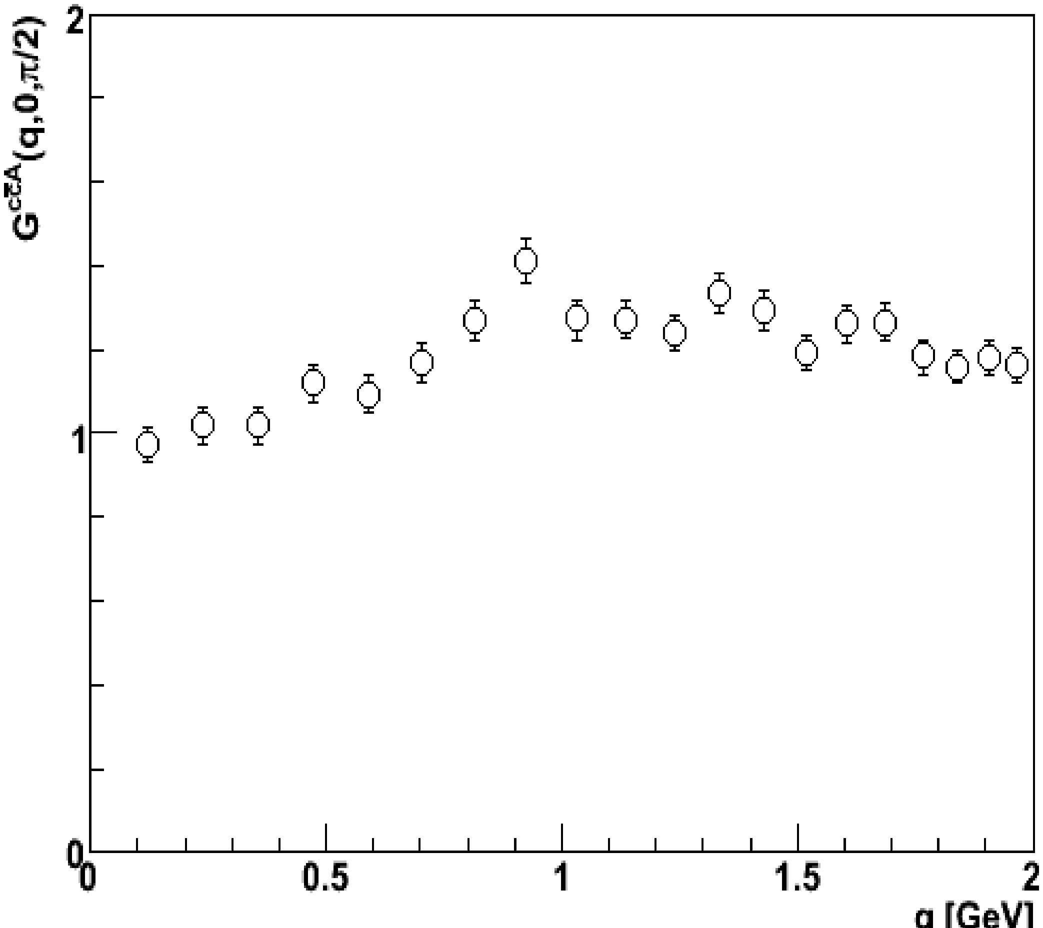
- Two independent momenta: Take to be at 90° here

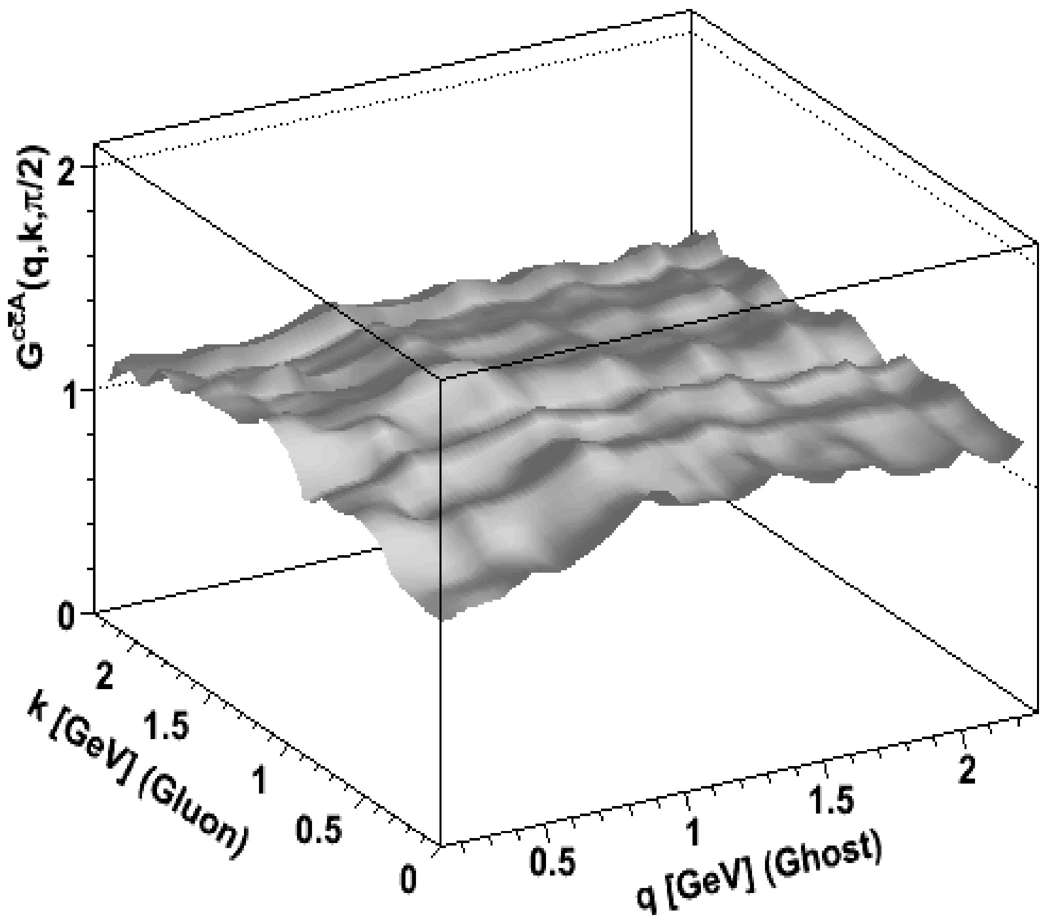
Ghost-gluon vertex in 3d

[60³@beta=4.2, Cucchieri et al., PRD 2008]

Ghost-gluon vertex, one momentum vanishing



Ghost-gluon vertex, orthogonal momenta

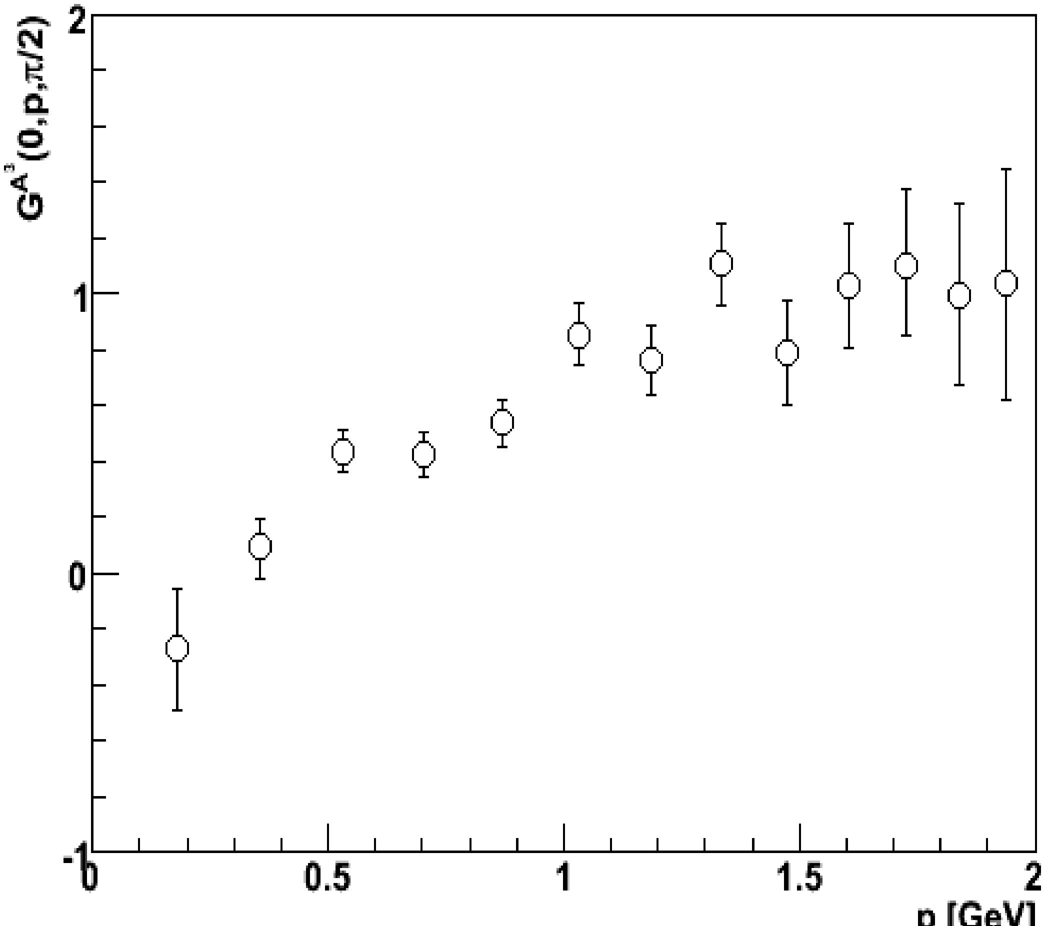


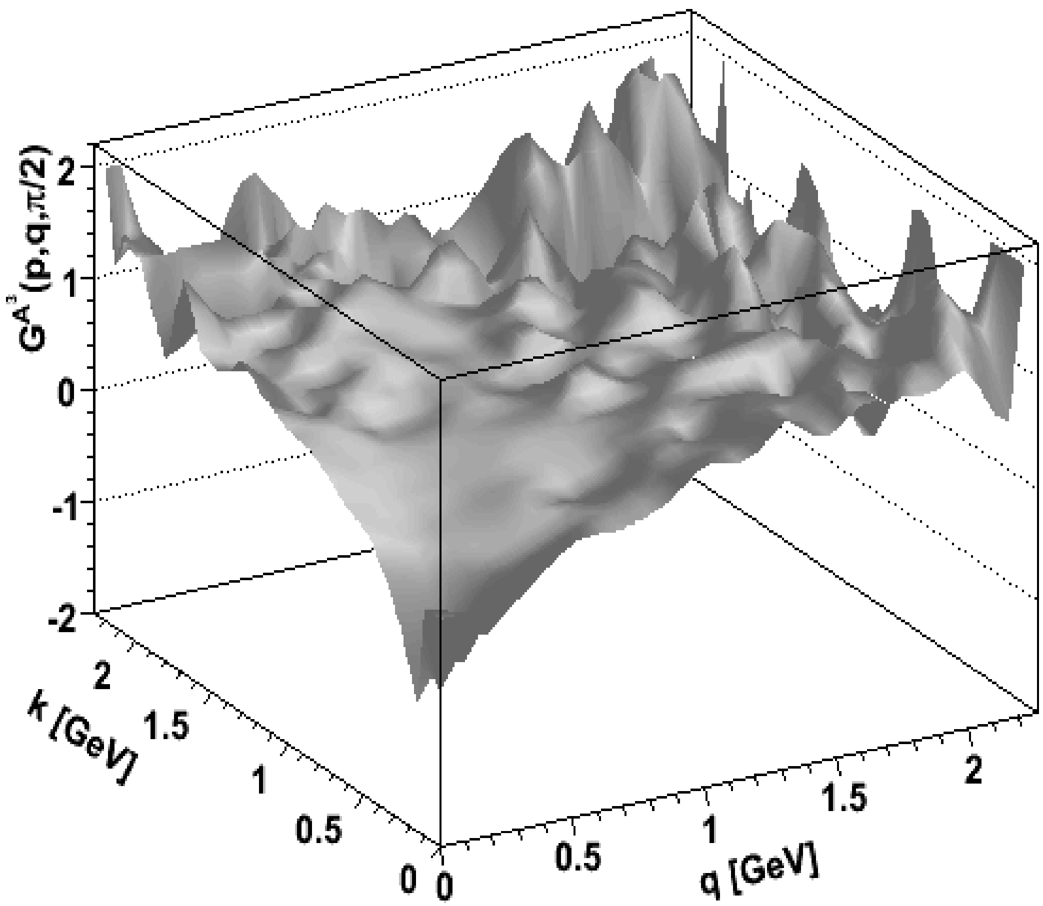
- Essentially constant, only some structure at 1 GeV
- Same in 2d and 4d
- In agreement with DSE predictions [Schleifenbaum et al., PRD 2005]

Three-gluon vertex in 3d

[40^3@beta=4.2, Cucchieri et al., PRD 2008]

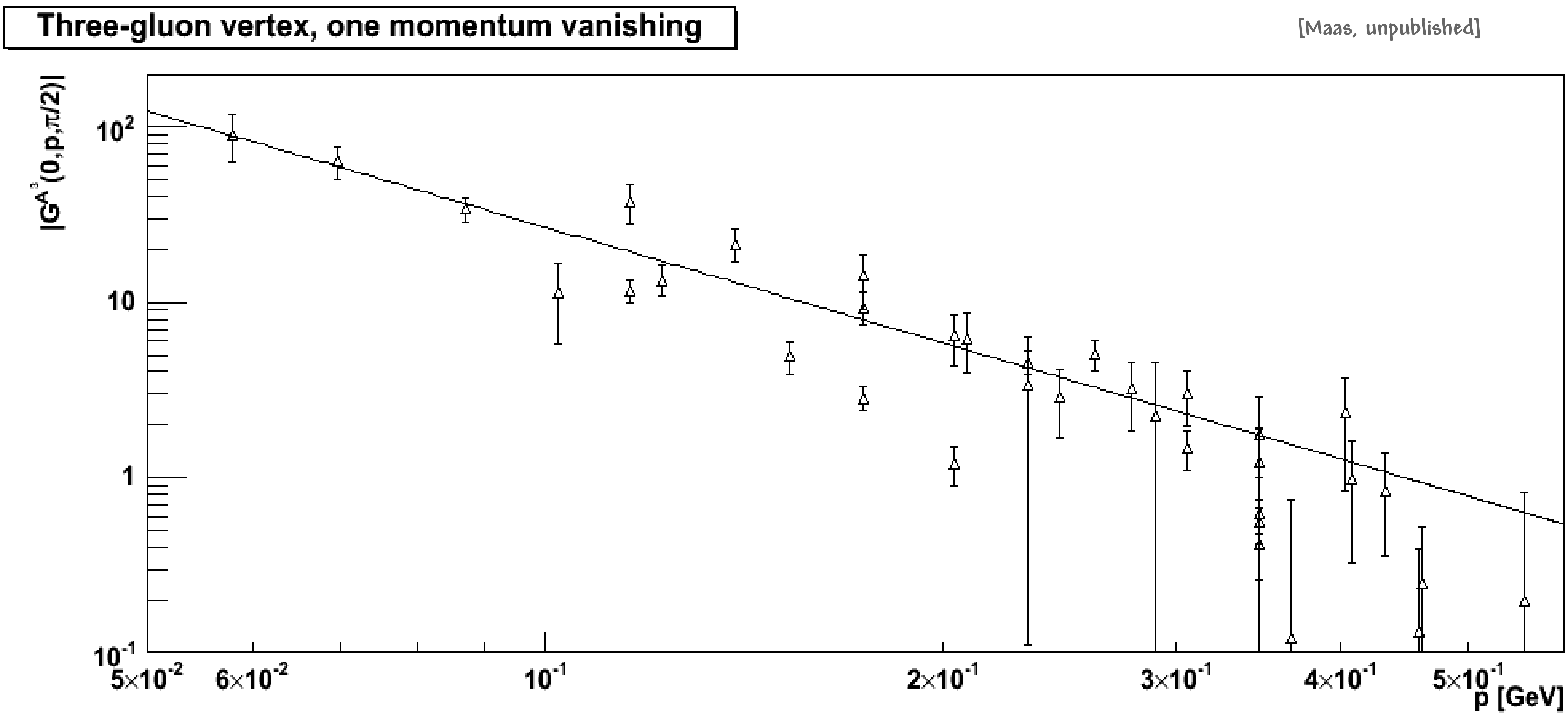
Three-gluon vertex, one momentum vanishing	Three-gluon vertex, orthogonal momenta





- Has a zero at low momenta, and negative infrared divergence
- Suppression at low momenta observed in 4d
- Divergence expected from functional studies, pre-factor unknown

Far infrared of three-gluon vertex: Two dimensions



- Results agree quantitatively with predicted infrared exponent
- But different kinematics: Scale dependence and angular dependence separable?

Summary on Green's functions

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- Lattice calculations and functional methods agree for two- and three point functions in absolute Landau gauge
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- Consistent with the confinement scenarios of Gribov and Zwanziger and of Kugo and Ojima

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    - Construct potential from the correlation functions
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      - Requires knowledge of vertices
    - Determine connection to the source of the potential
      - The origin is assumed to be topological field configurations

Topological configurations

- Vortices

Topological configurations

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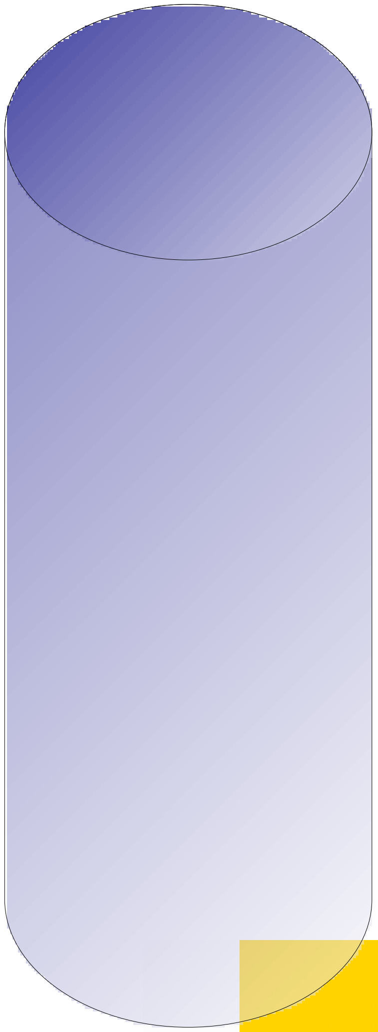
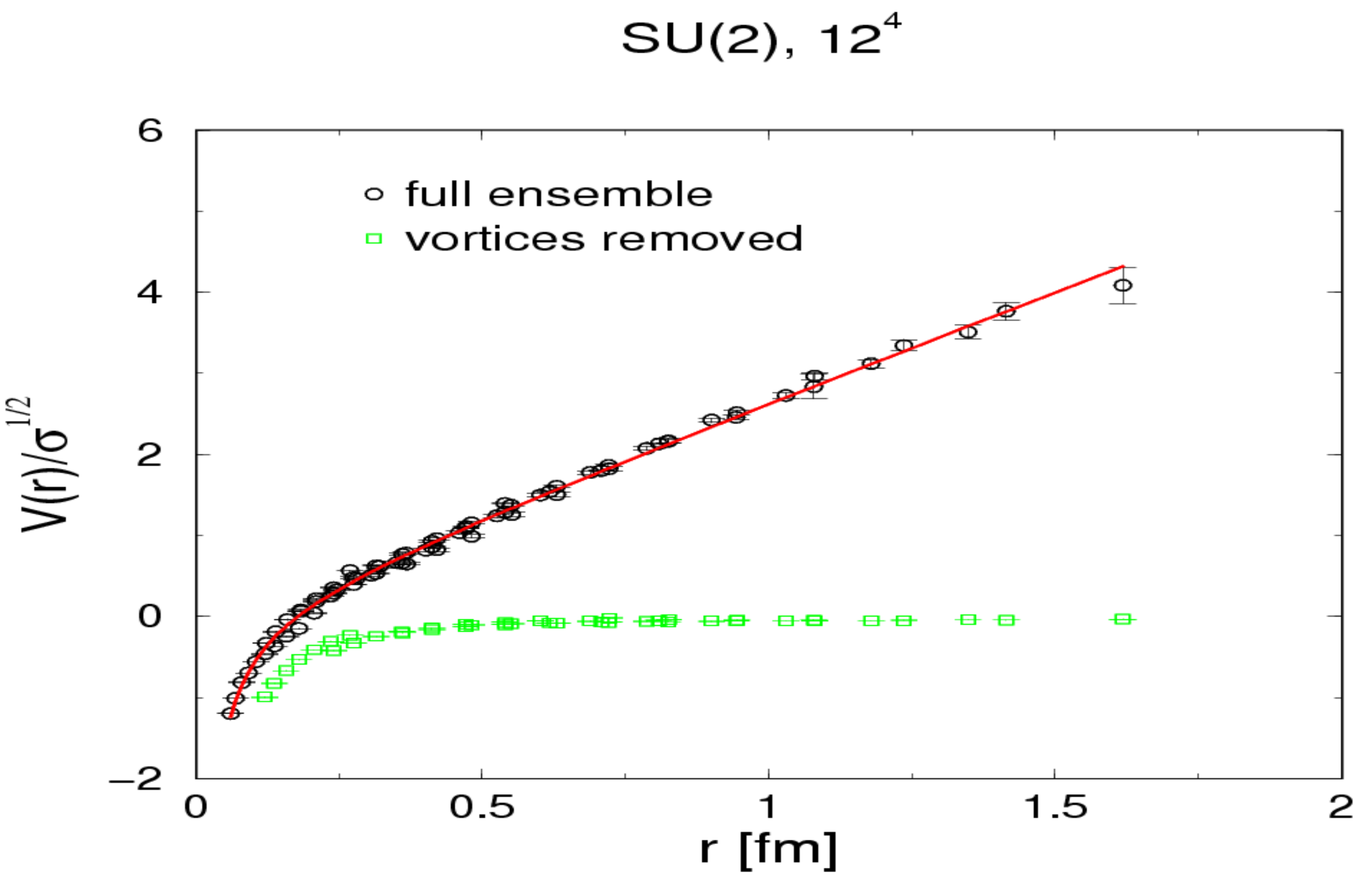
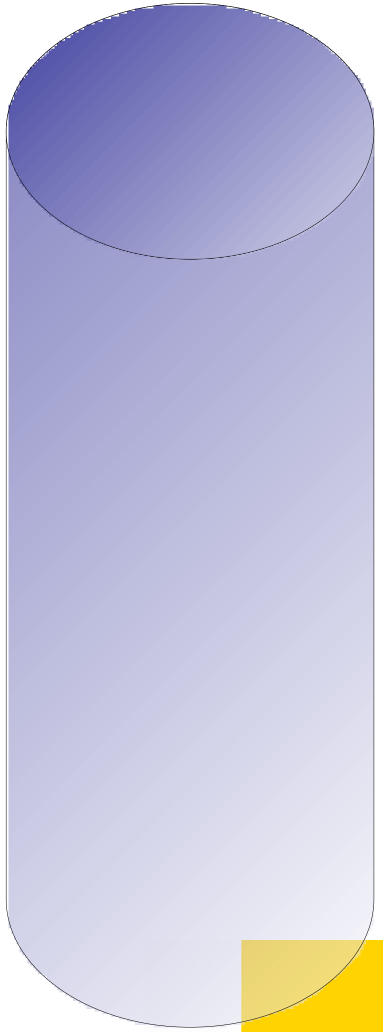
  - String-like objects - world surface



Topological configurations

- Vortices
  - String-like objects - world surface
  - Influence the confinement potential





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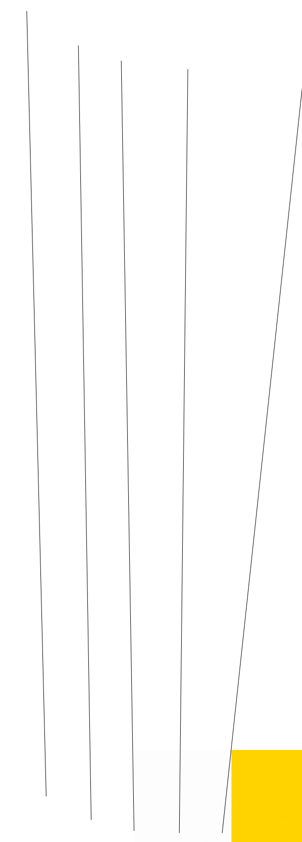

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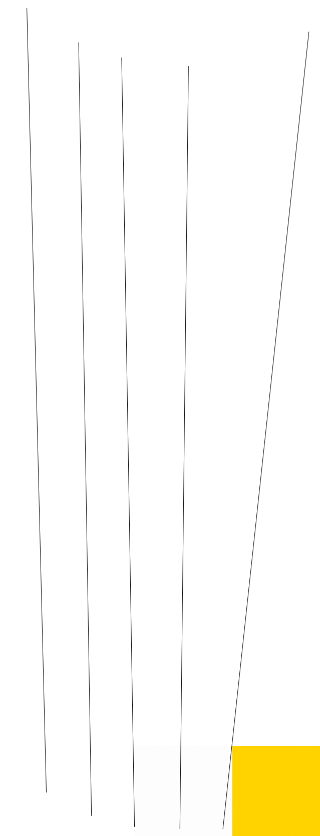

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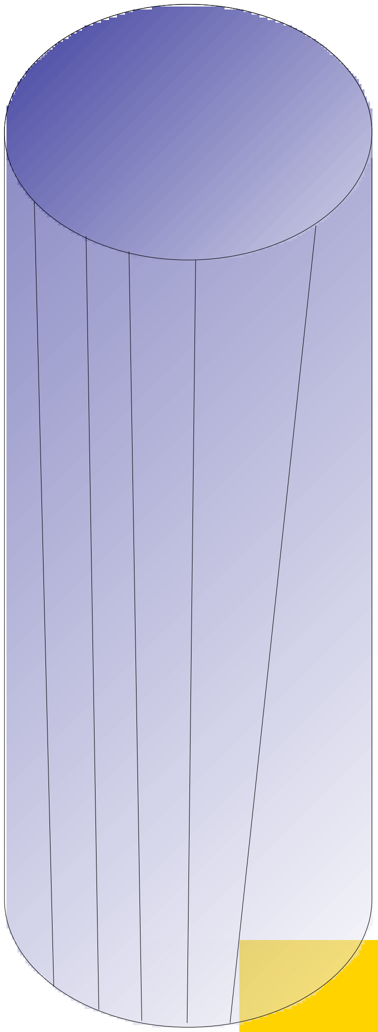
    - Monopoles move on the surface of vortices

[Greensite, 2003, Reinhardt, NPB 2002, Boyko et al., NPB 2006]



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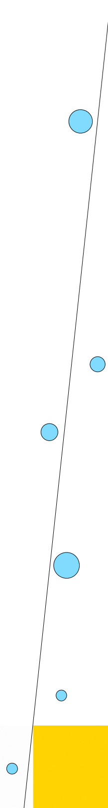

Topological configurations

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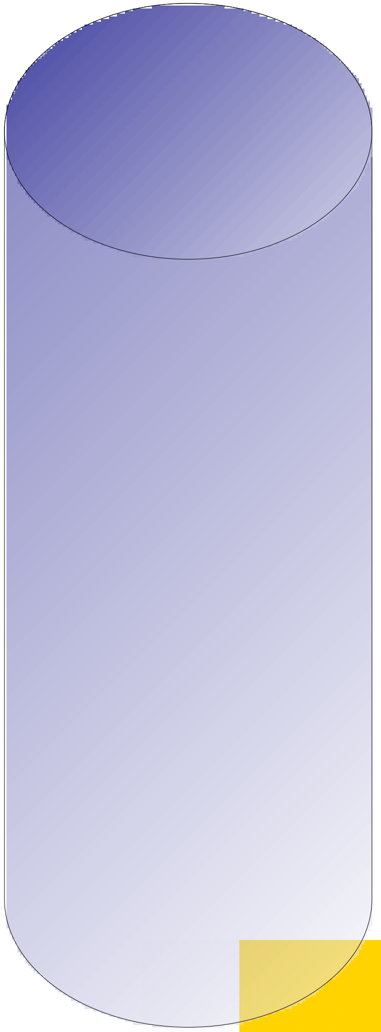

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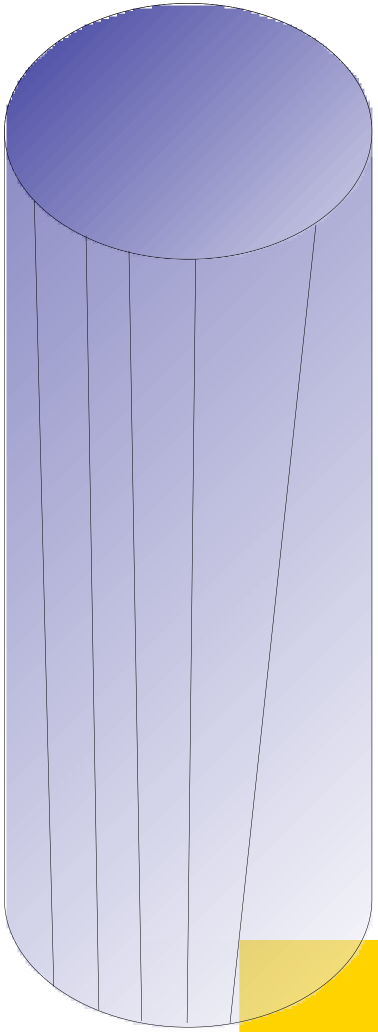

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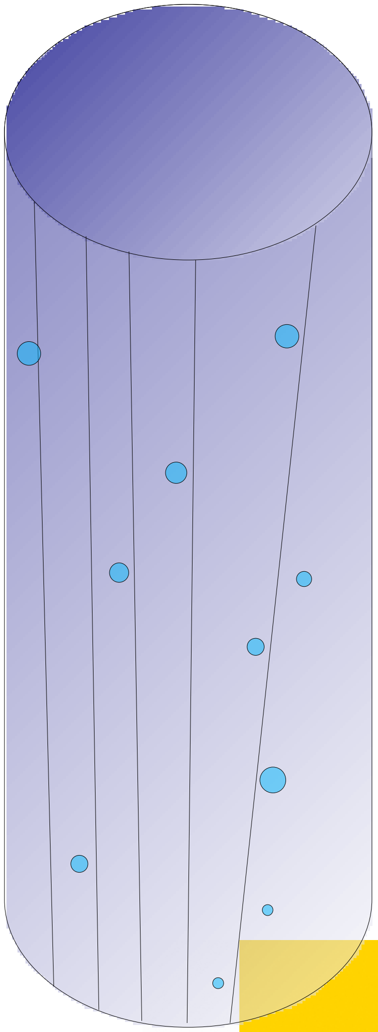

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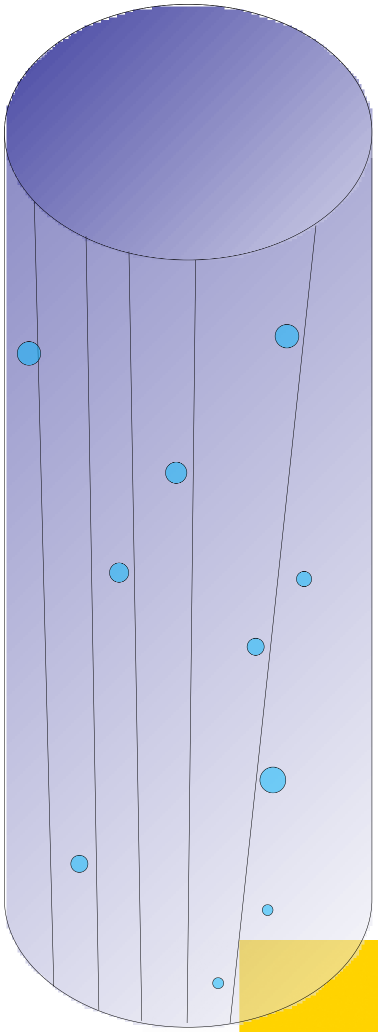

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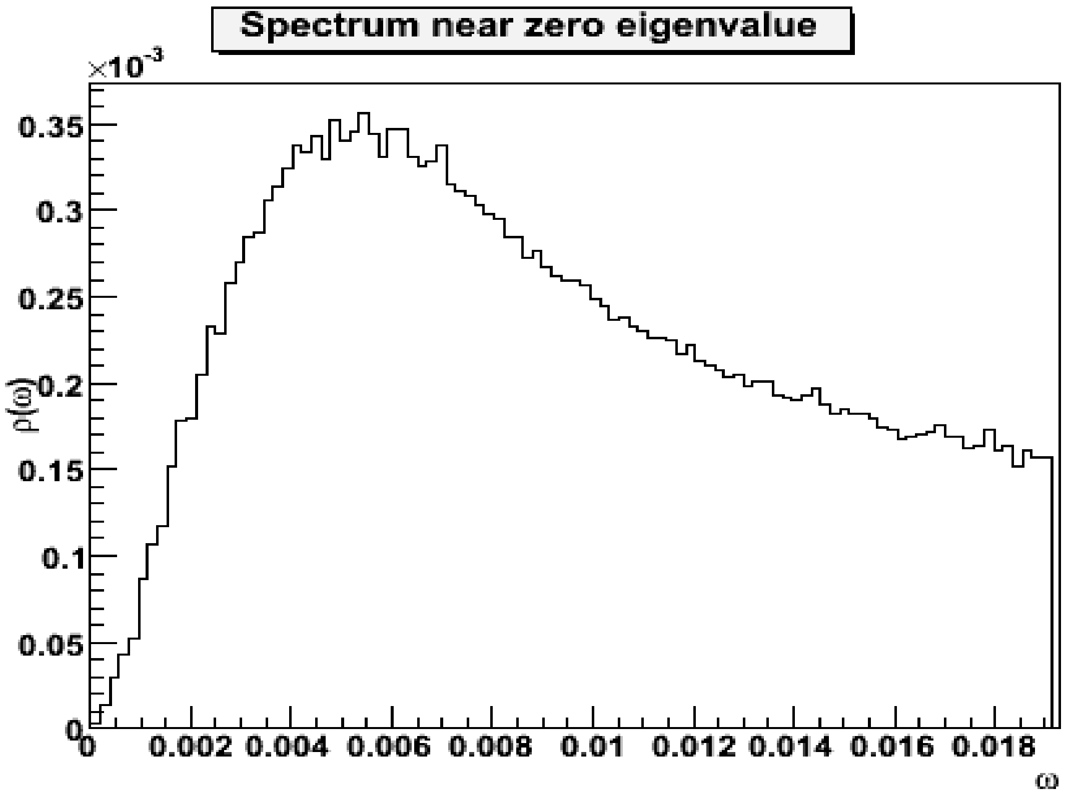
- How relates this to the gluons?

To answer: Return to the Faddeev-Popov operator

Landau-gauge Faddeev-Popov operator eigenspectrum



[Cucchieri et al., PRD 2006]

- Near zero enhanced compared to vacuum

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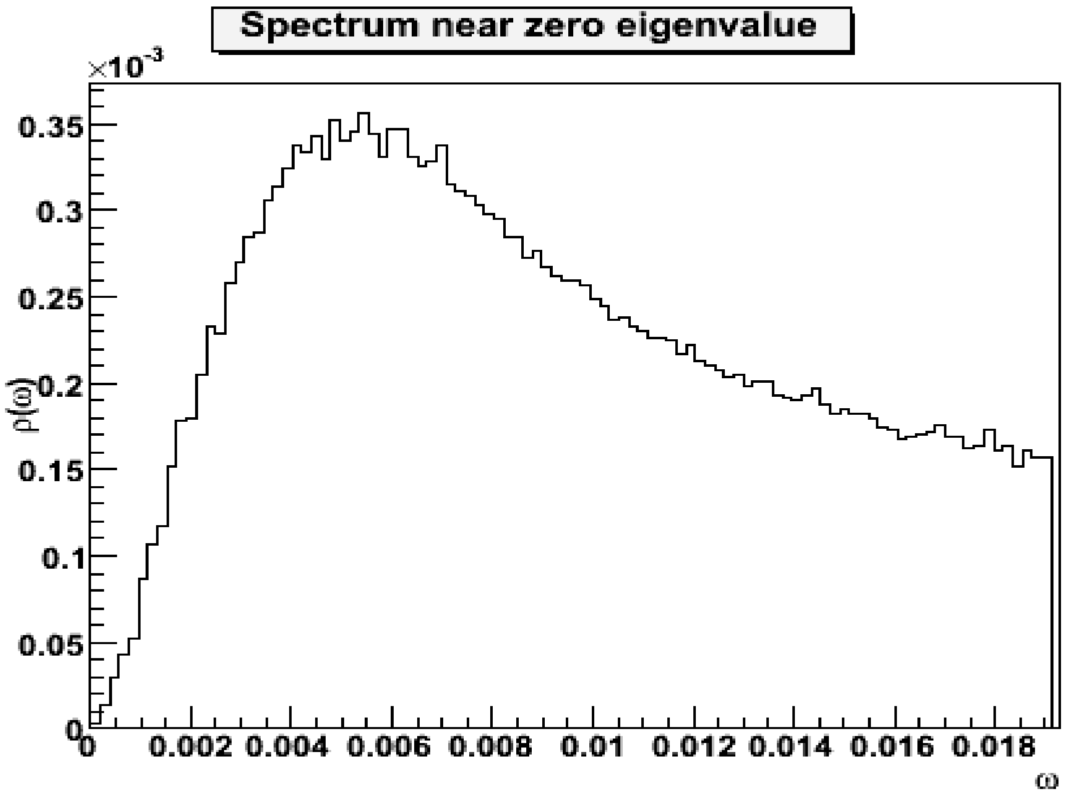

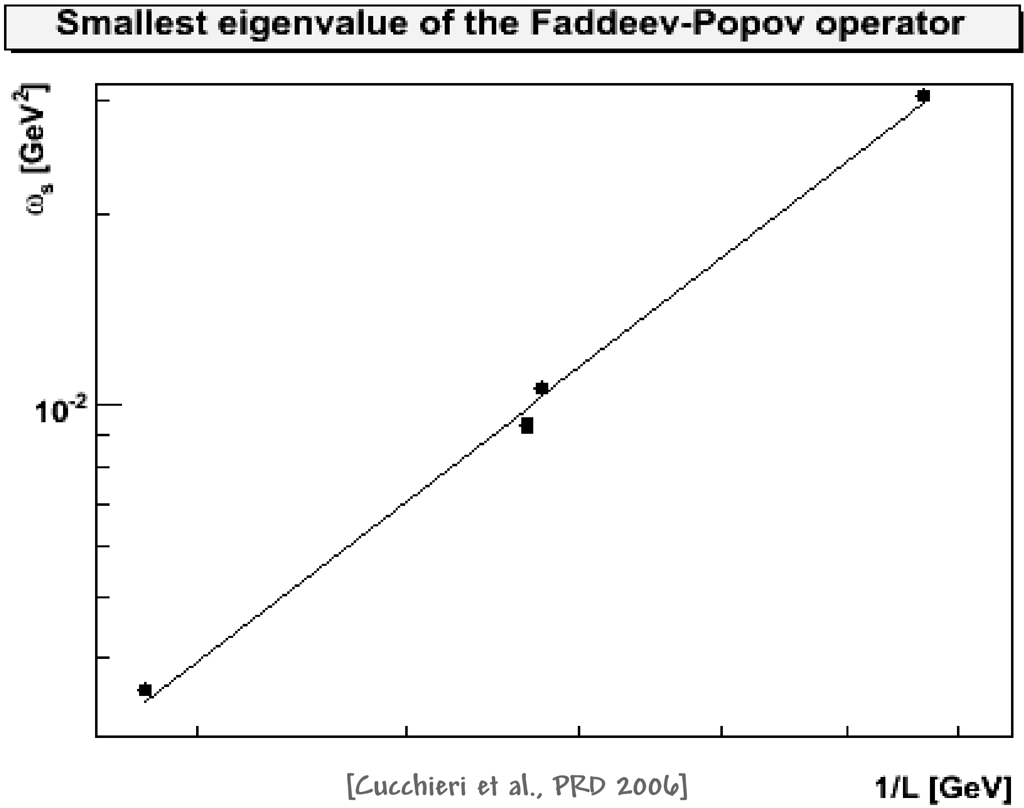

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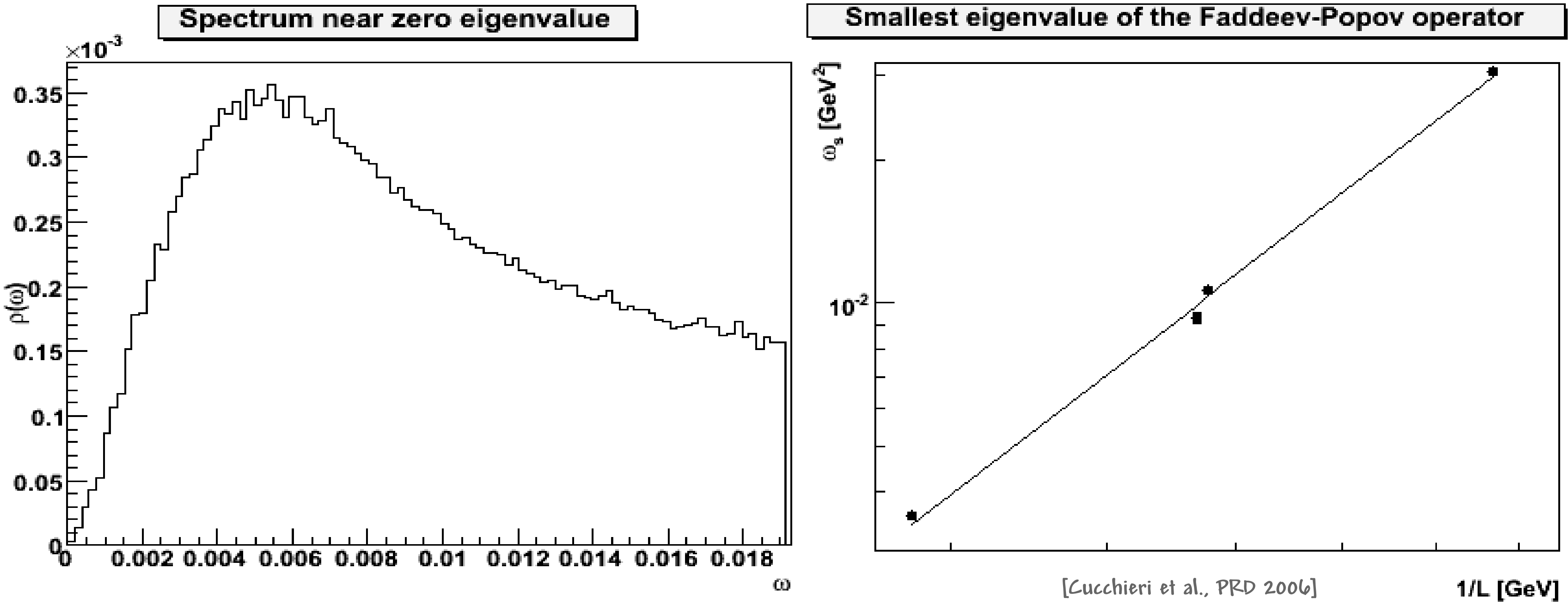

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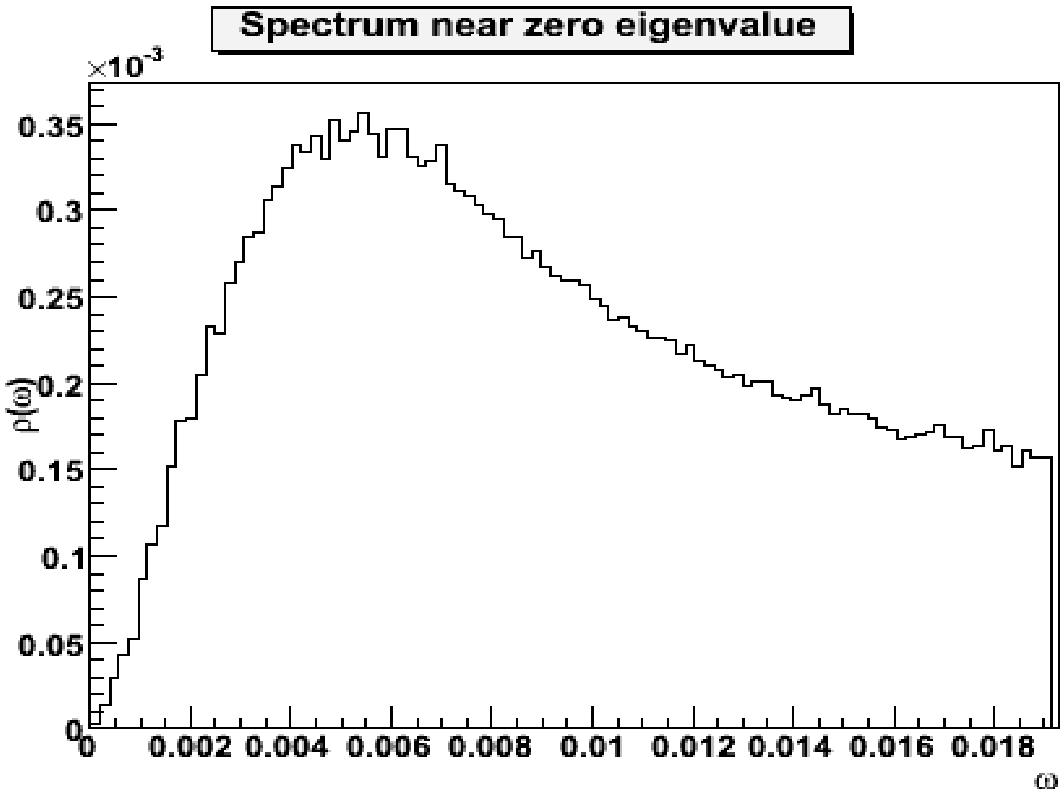

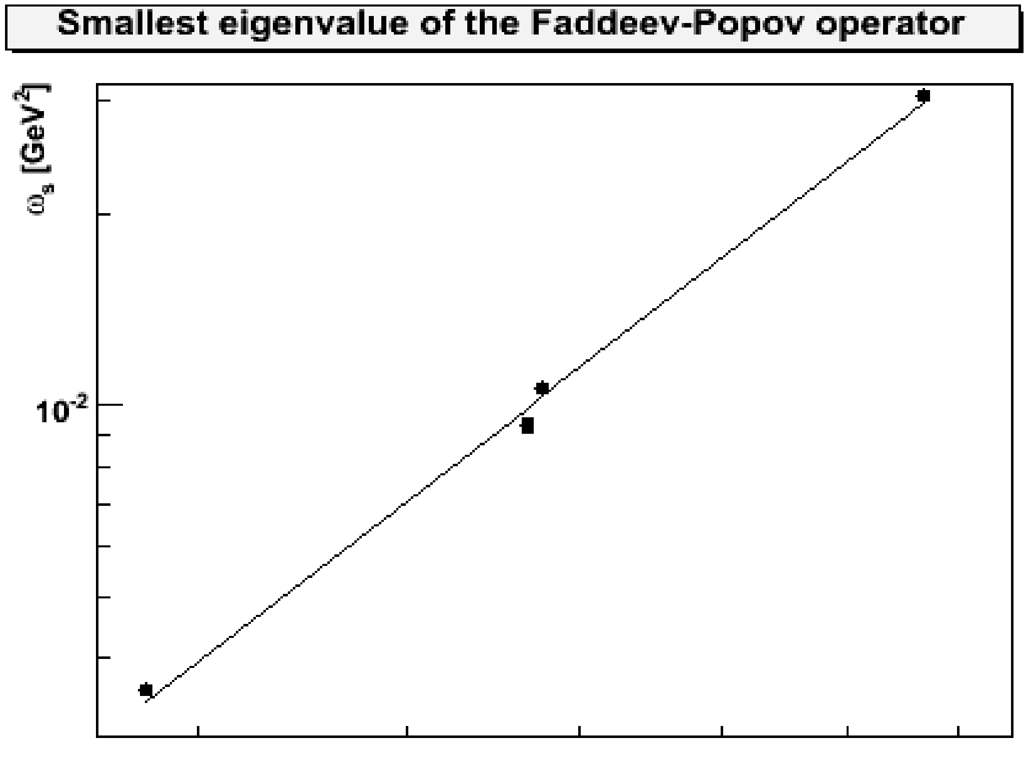

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- Leads to the infrared divergence of the ghost propagator

Ghost propagators and vortices

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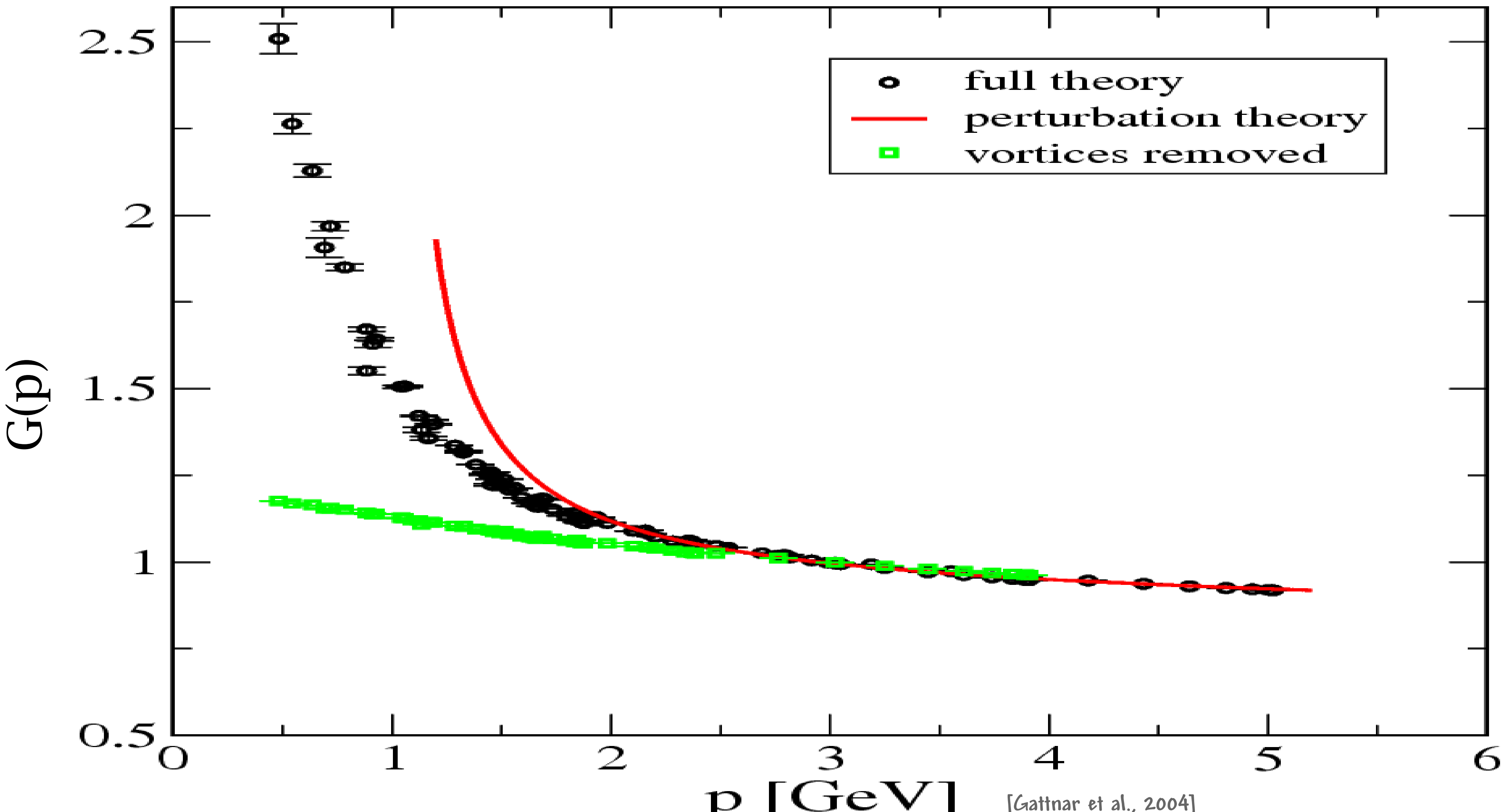

[Gattnar et al., 2004]

Ghost propagators and vortices

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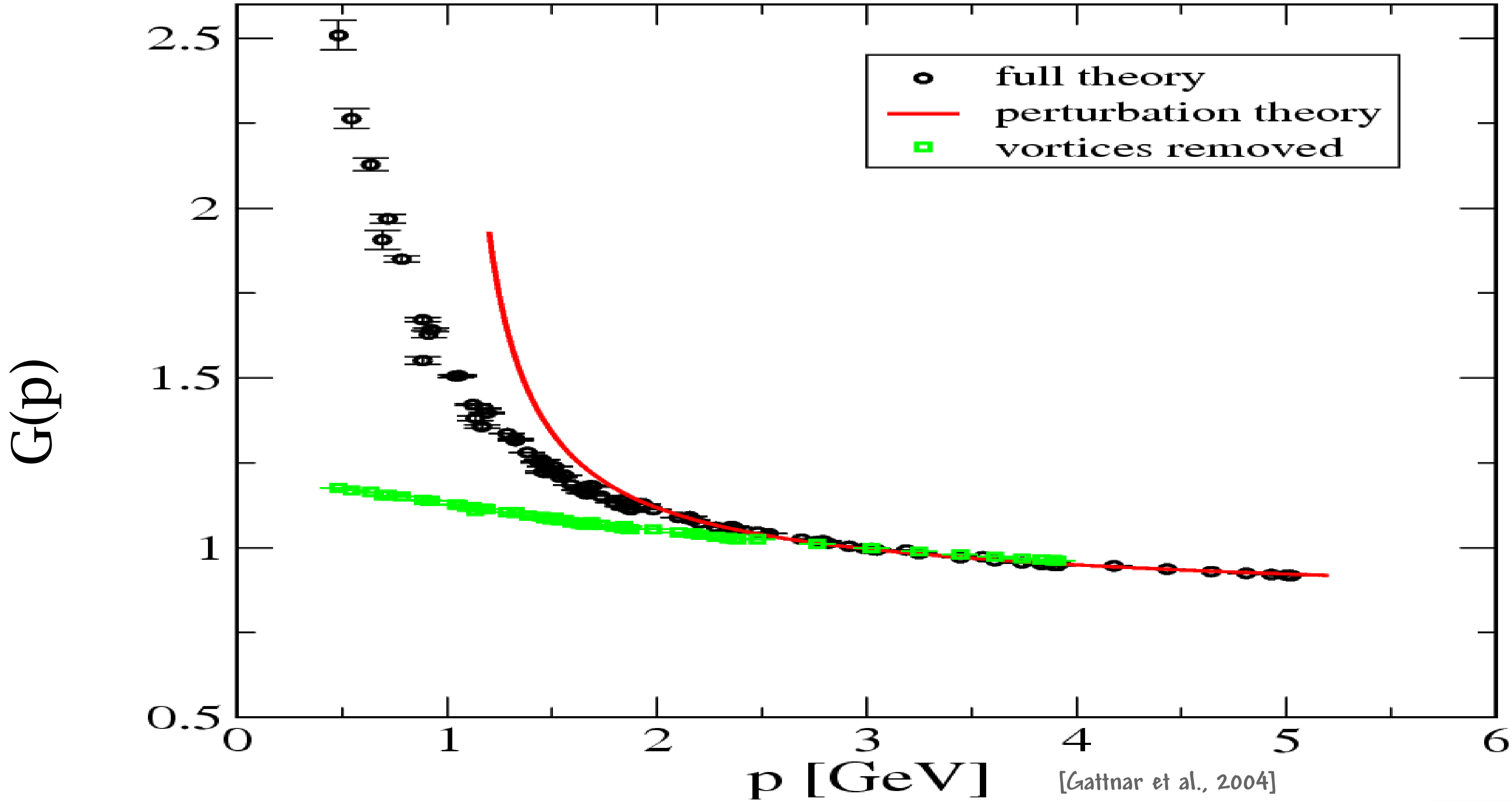
- Hints to a change of the FP-eigenspectrum



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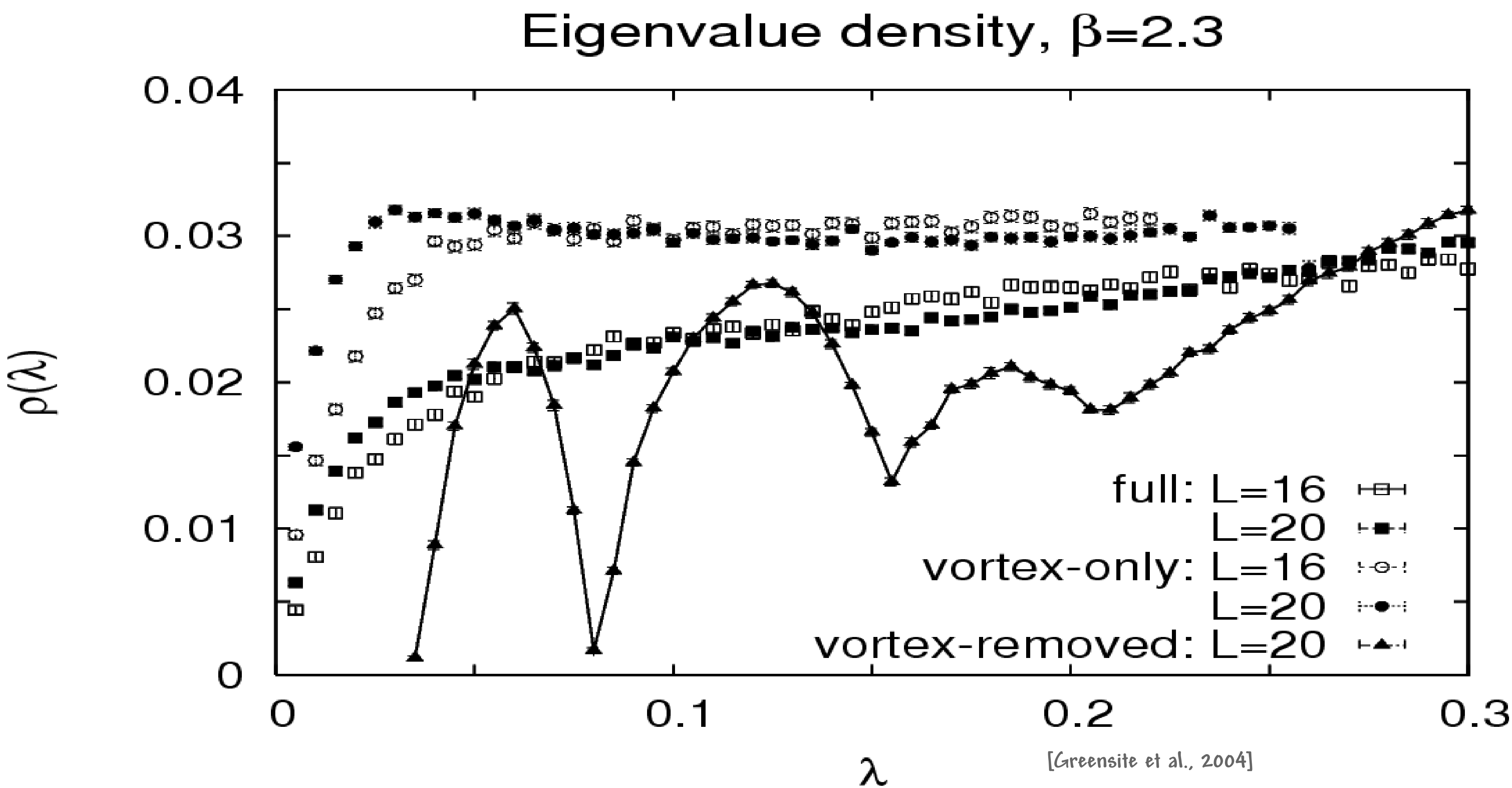
Vortices and the eigenspectrum of the FPO

- Removal of vortices in Coulomb gauge reduces enhancement of near-zero modes



[Greensite et al., 2004]

Analytical approach

- Study analytically the eigenspectrum of the FP-operator in topological background fields
- Use as background fields the field of
  - Instantons [Maas, EPJC 2006]
  - Monopoles [Maas, NPA 2007]
  - (Center) Vortices [Maas, EPJC 2006]

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- General form (for transverse fields)

$$-(\delta^{ab}\partial^2 + g f^{abc} A_\mu^c \partial_\mu)\phi^b = \omega^2 \phi^a$$

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- At least the eigenspectrum is necessary
- Eigenvectors do also play a role in the Gribov-Zwanziger scenario

Solutions

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- Results for a more complicated system?

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- Still more open questions than fully answered ones...