

Confinement of gluons

-

Operators, Green's functions and topological configurations

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9th of February 2007



Overview

- **Confinement**
 - What is it, how do we see it & why is it there?
 - Gribov-Zwanziger scenario & the **Faddeev-Popov operator**

Reviews: Alkofer et al. PR 2000, Greensite PNPP 2003, Fischer JPG 2005

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 - Gauge dependence

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 - **Green's functions**
 - Gauge dependence
- The role of **topological configurations**
- Summary & outlook

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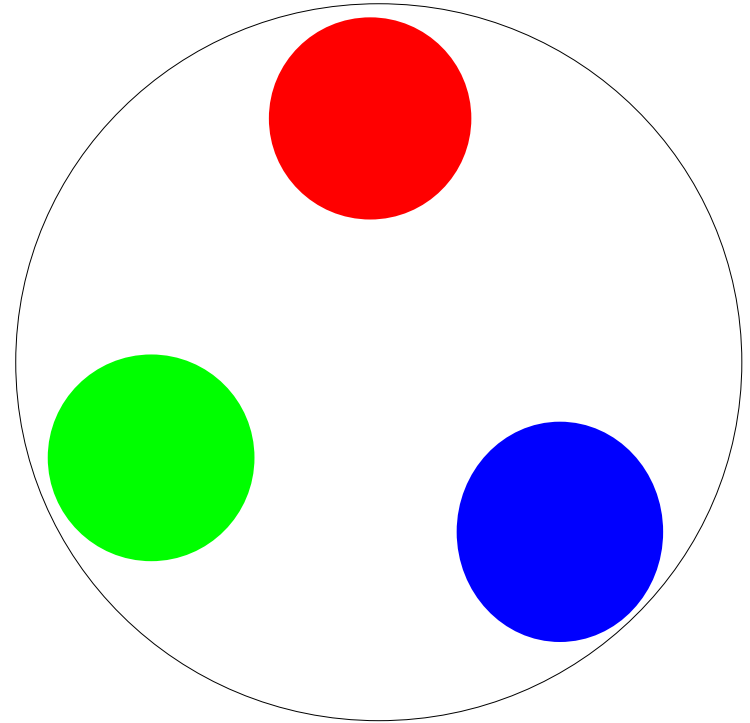
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 - Rutherford-like behavior at high energies
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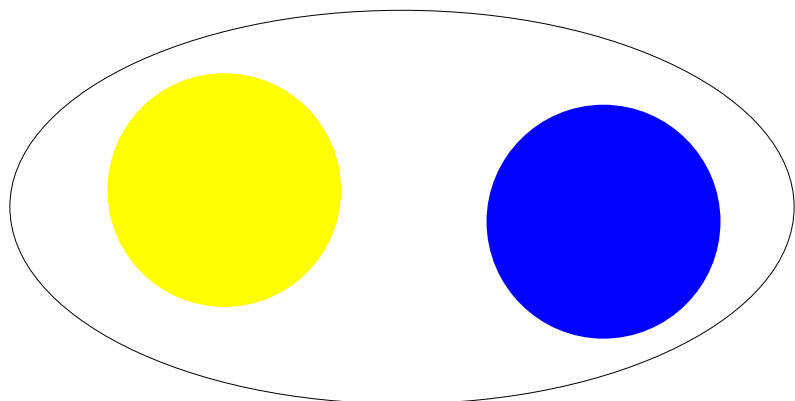
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- Substructure cannot be isolated:
Confinement

Substructure: Quarks and gluons

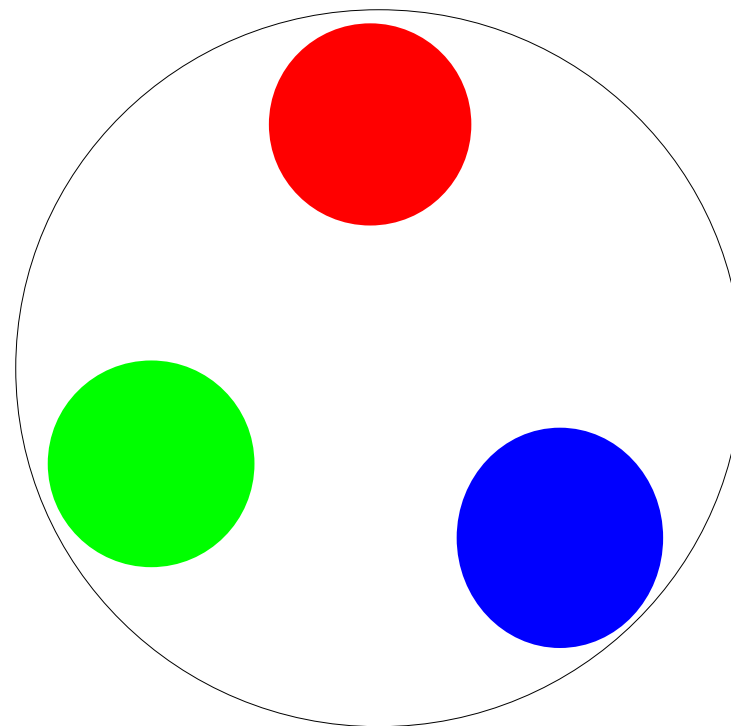
- Baryon: 3 (valence-)quarks



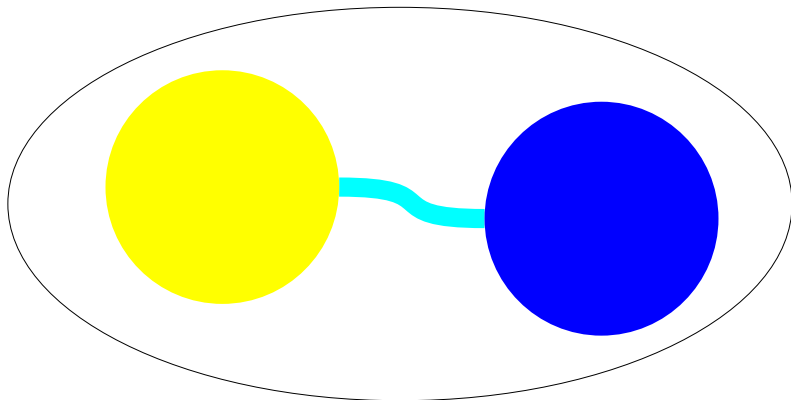
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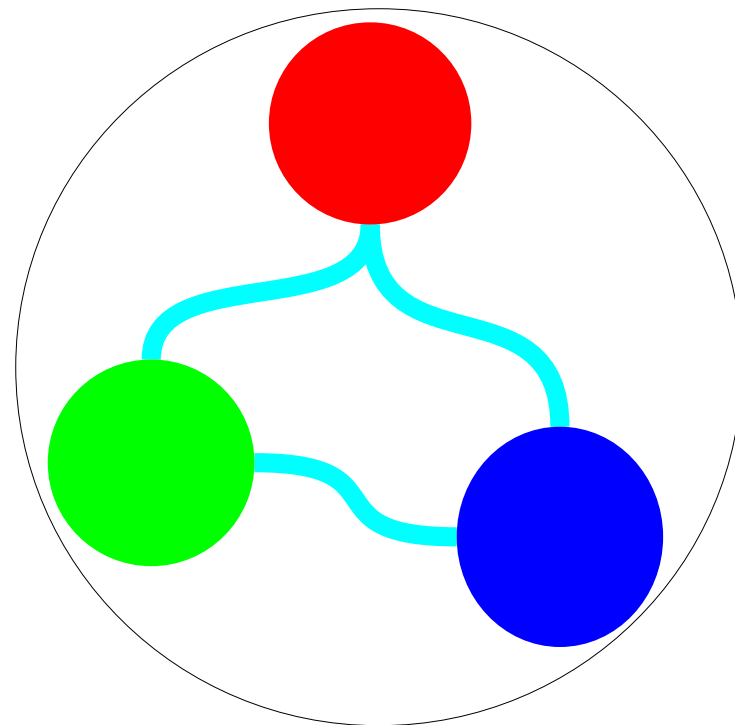
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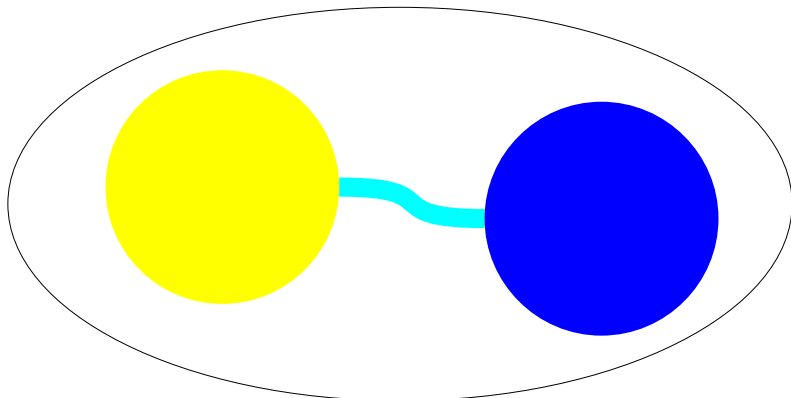
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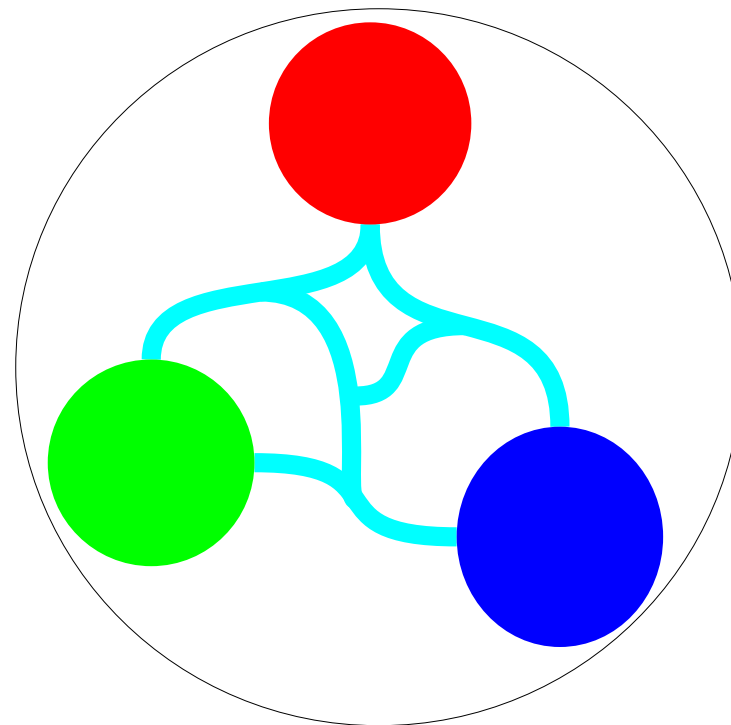
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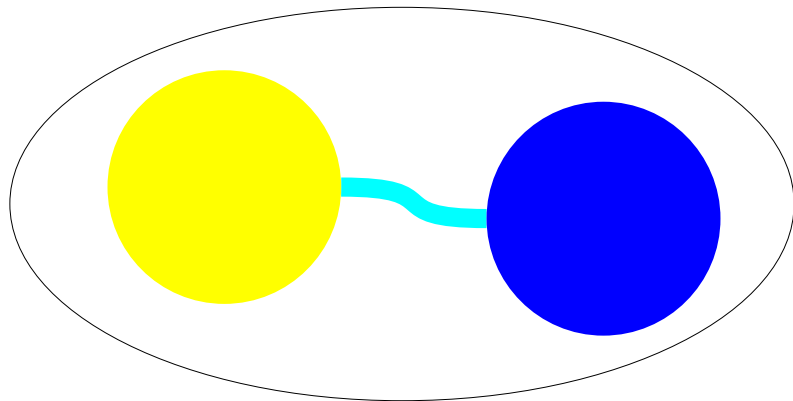
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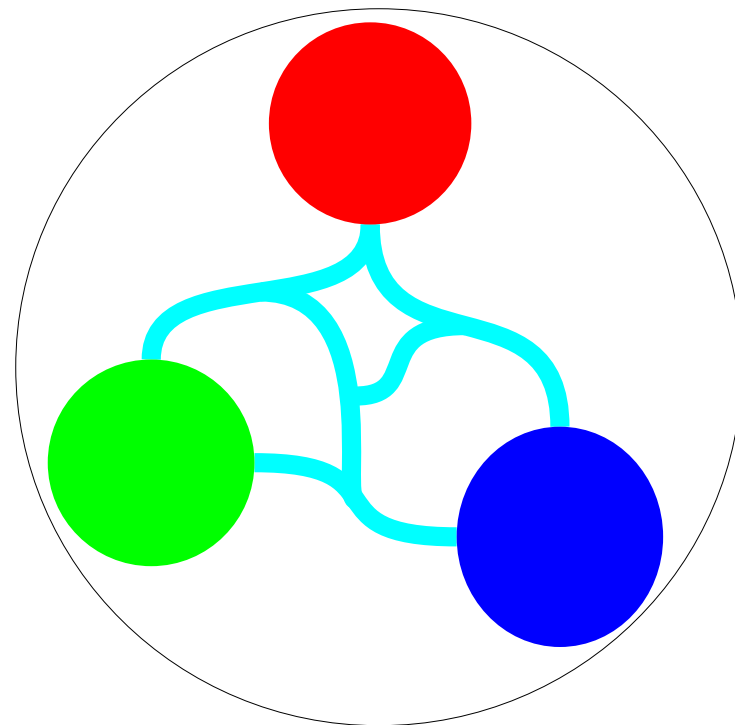
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Substructure: Quarks and gluons



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- Interactions mediated by gluons
- Gluons are also charged
- No free quarks and gluons: Confinement
 - Measured to very high precision for quarks



QCD - The theory of quarks and gluons

- **Quantum Chromodynamics** is a gauge theory

$$L = -\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \bar{\Psi} (\gamma_\mu D_\mu + m) \Psi$$

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- **Quarks (likely) subject to, but not source of confinement - Gluons alone: Yang-Mills theory**

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 - **How to establish confinement?**

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- **Sum rule for gluons**

$$\text{Overlap with one particle} + \int dq^2 \text{spectral function}(q^2) = \frac{1}{Z_3} = 0$$

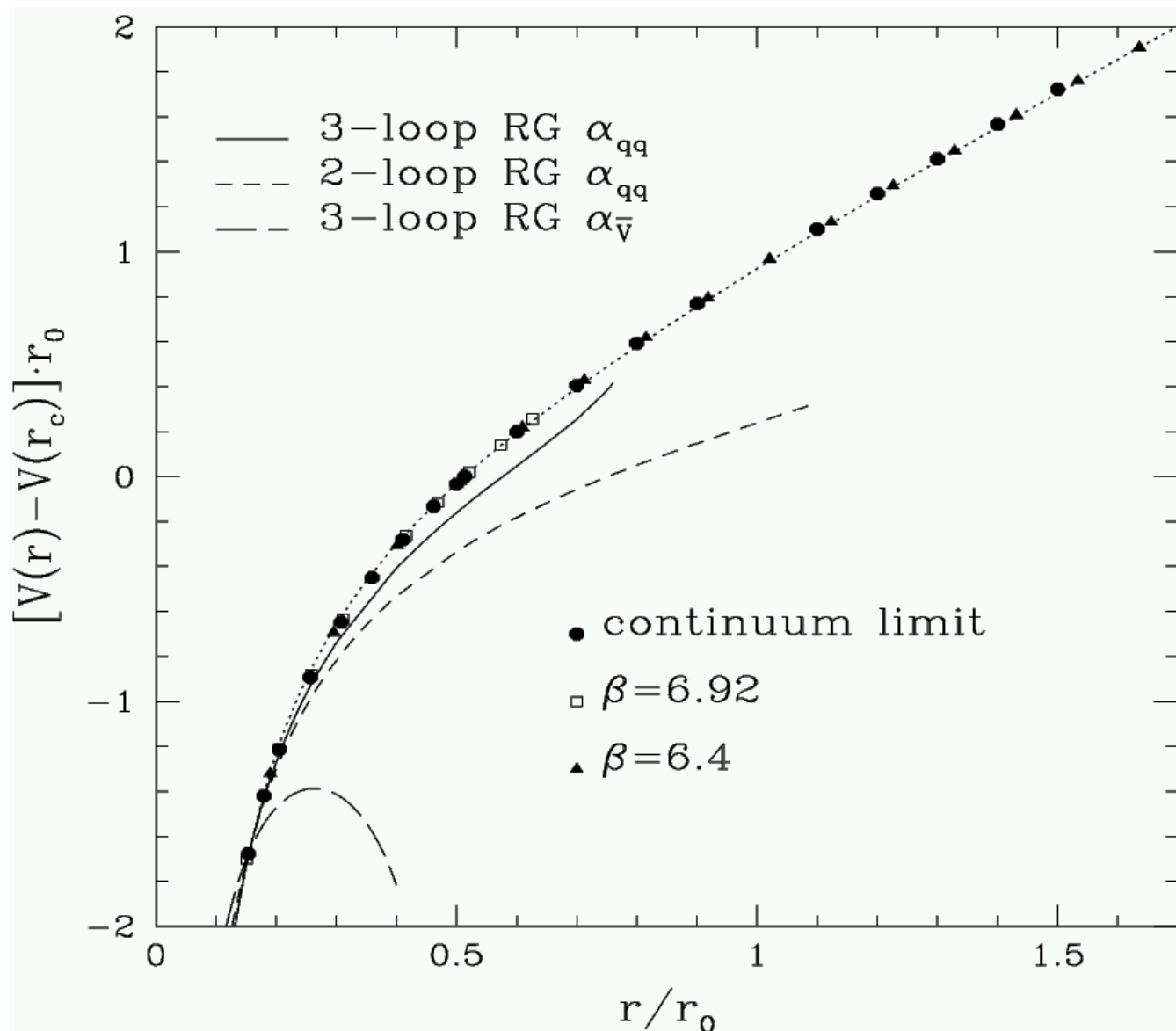
- Z_3 (divergent) renormalization constant
- **Likely also valid at finite temperature**

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Confinement of quarks

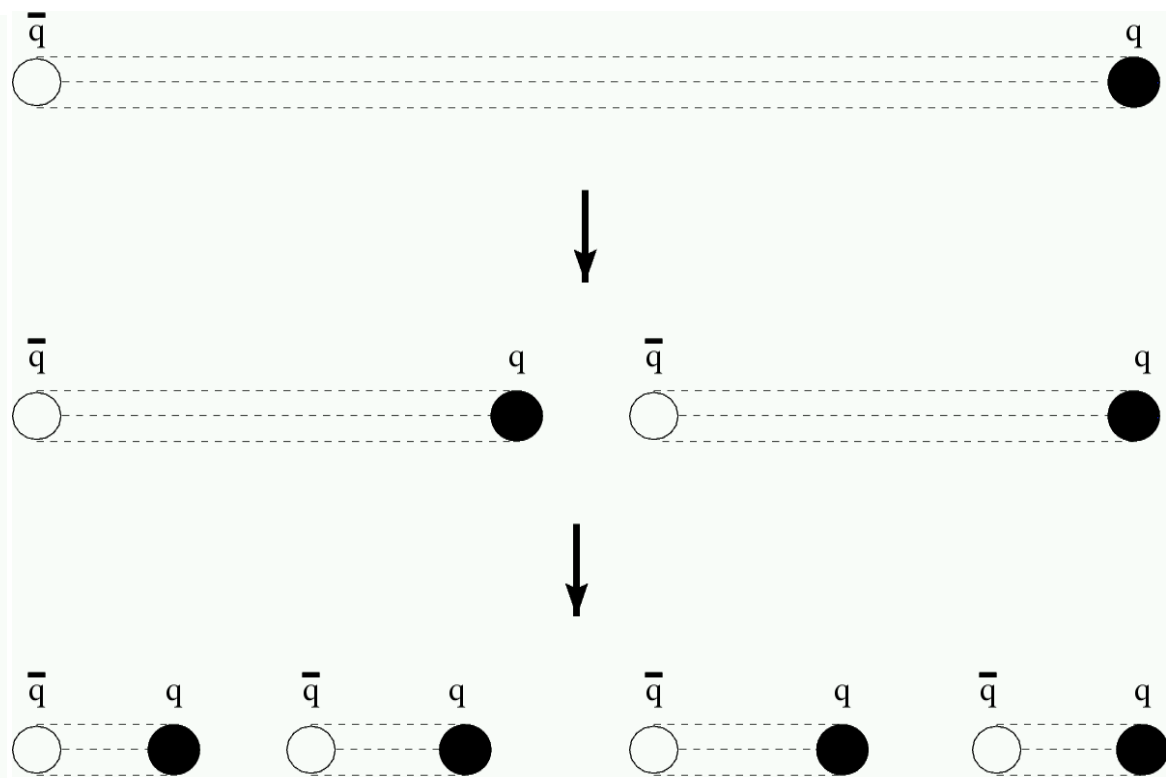
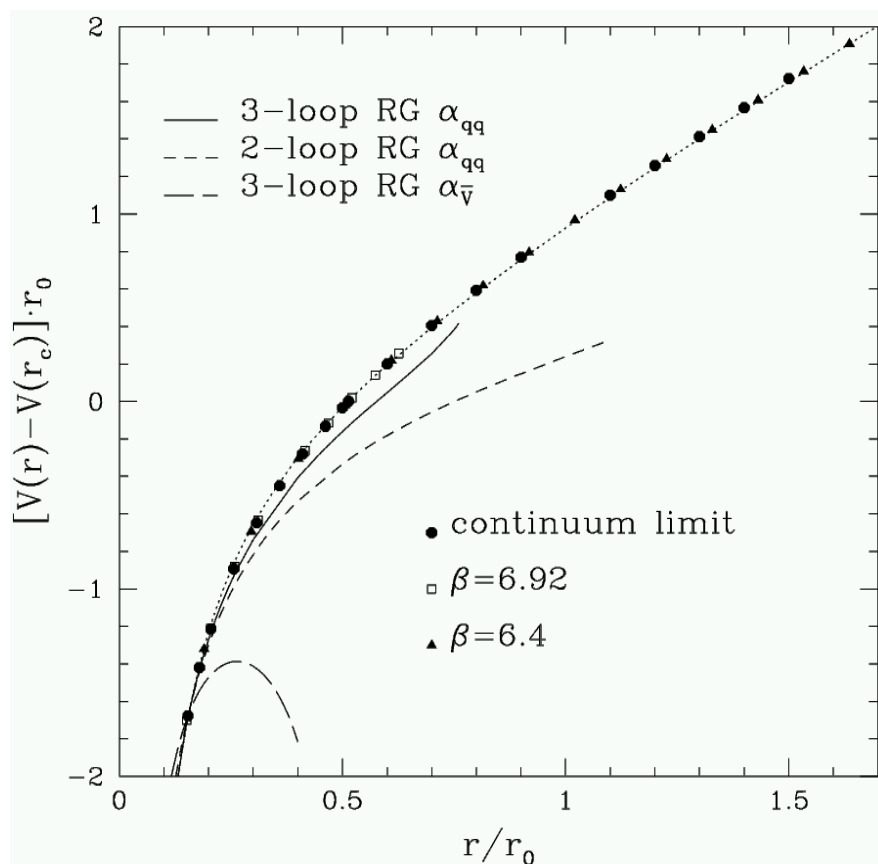
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[Figures from Sommer et al., 2001]

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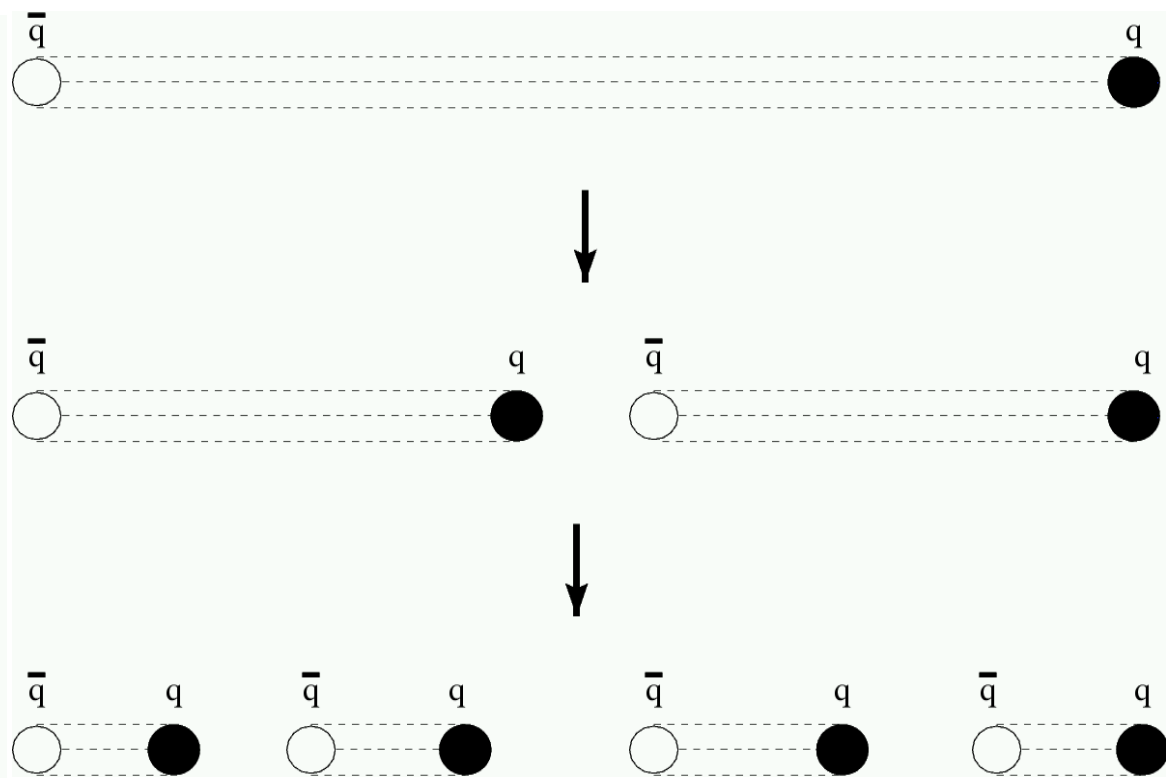
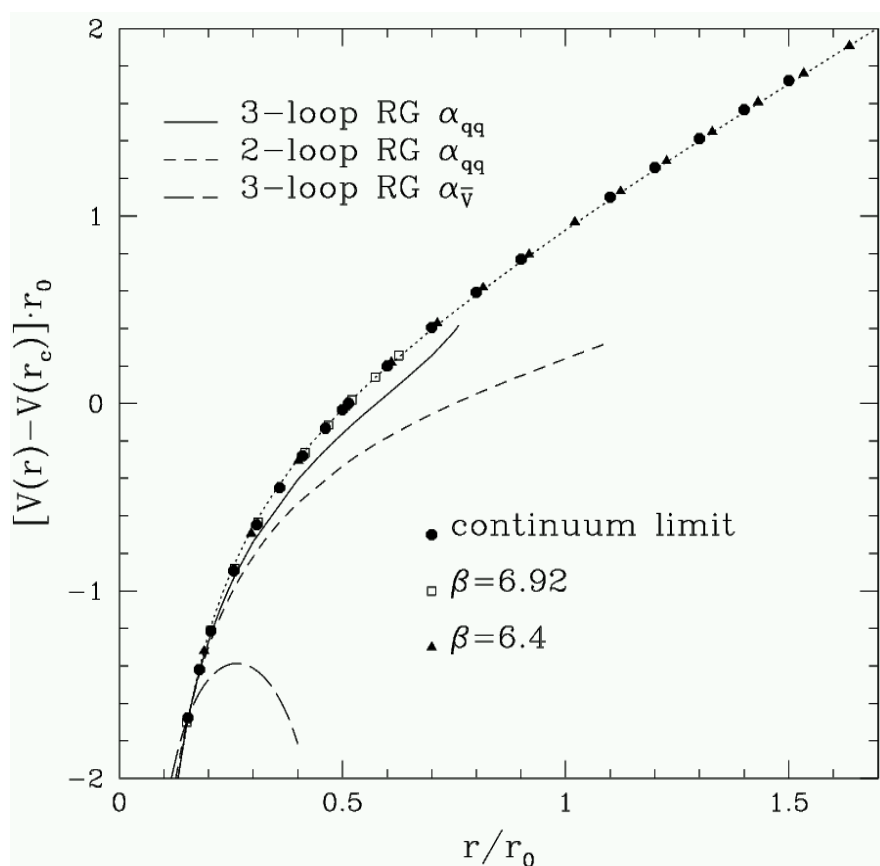
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[Figures from Sommer et al., 2001 (left) and from Greensite, 2003 (right)]

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- Origin of linear potential?



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 - Works only for non-dynamic objects

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 - Geometry of field configuration space: Gribov-Zwanziger
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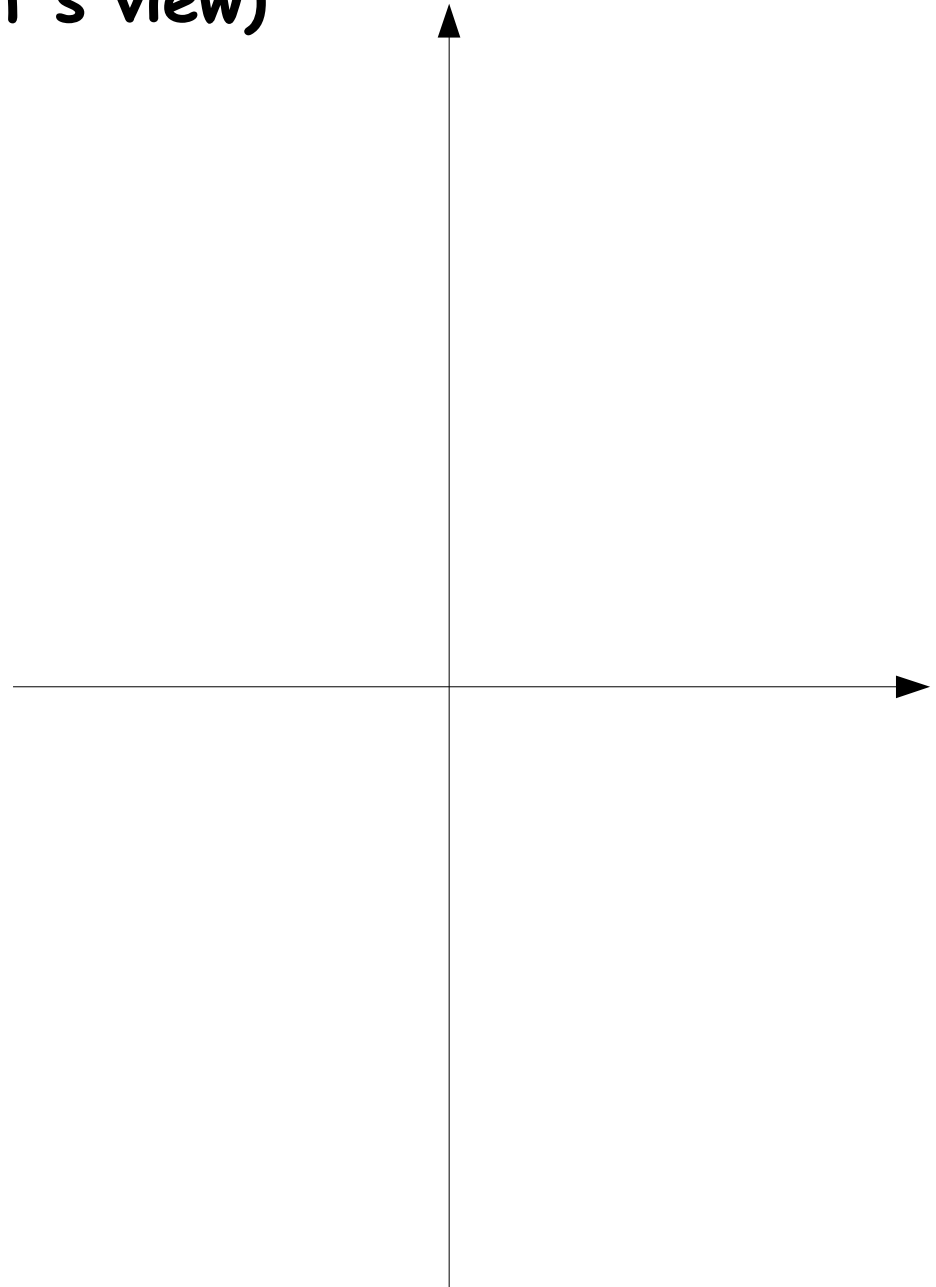
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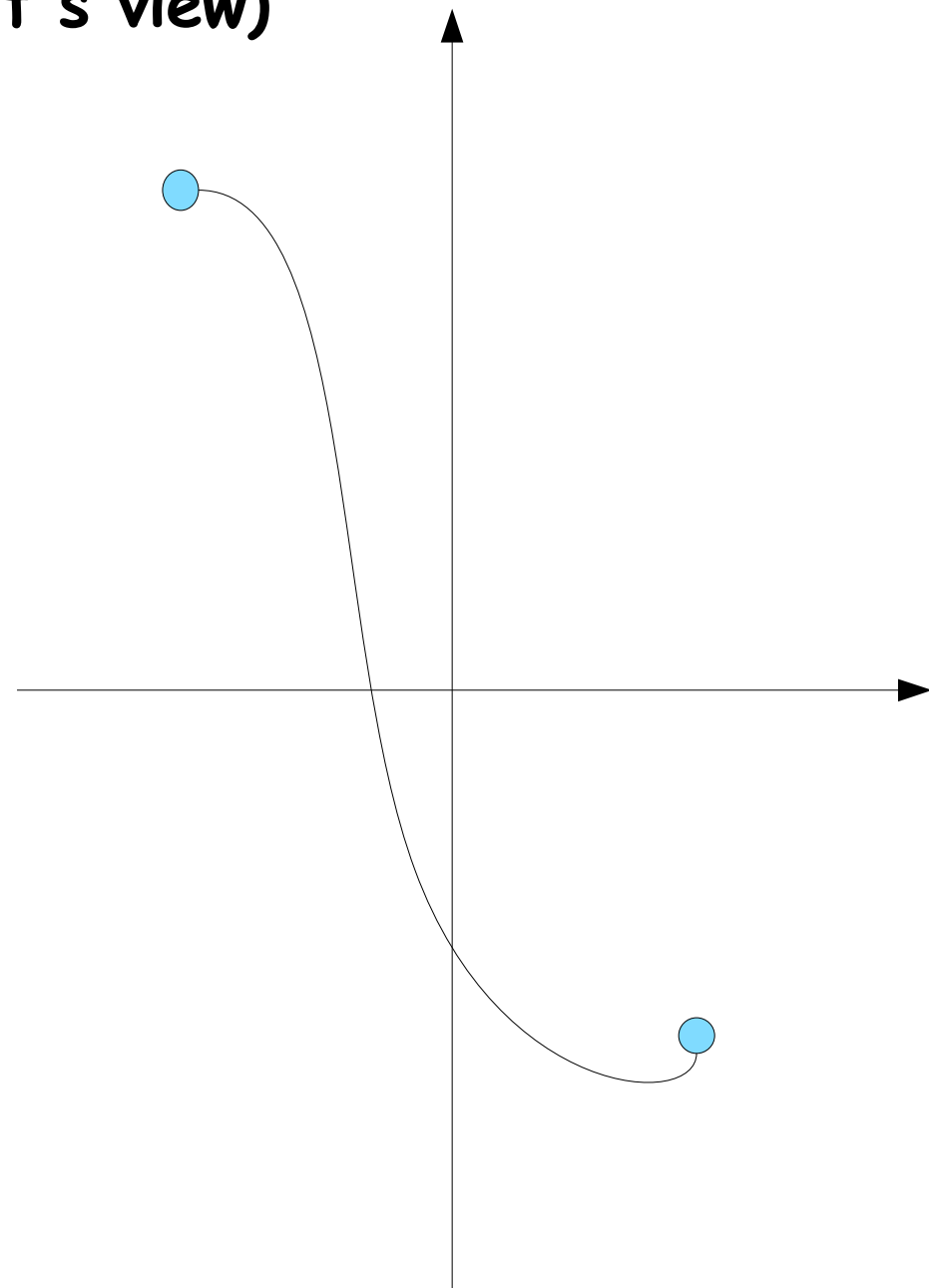
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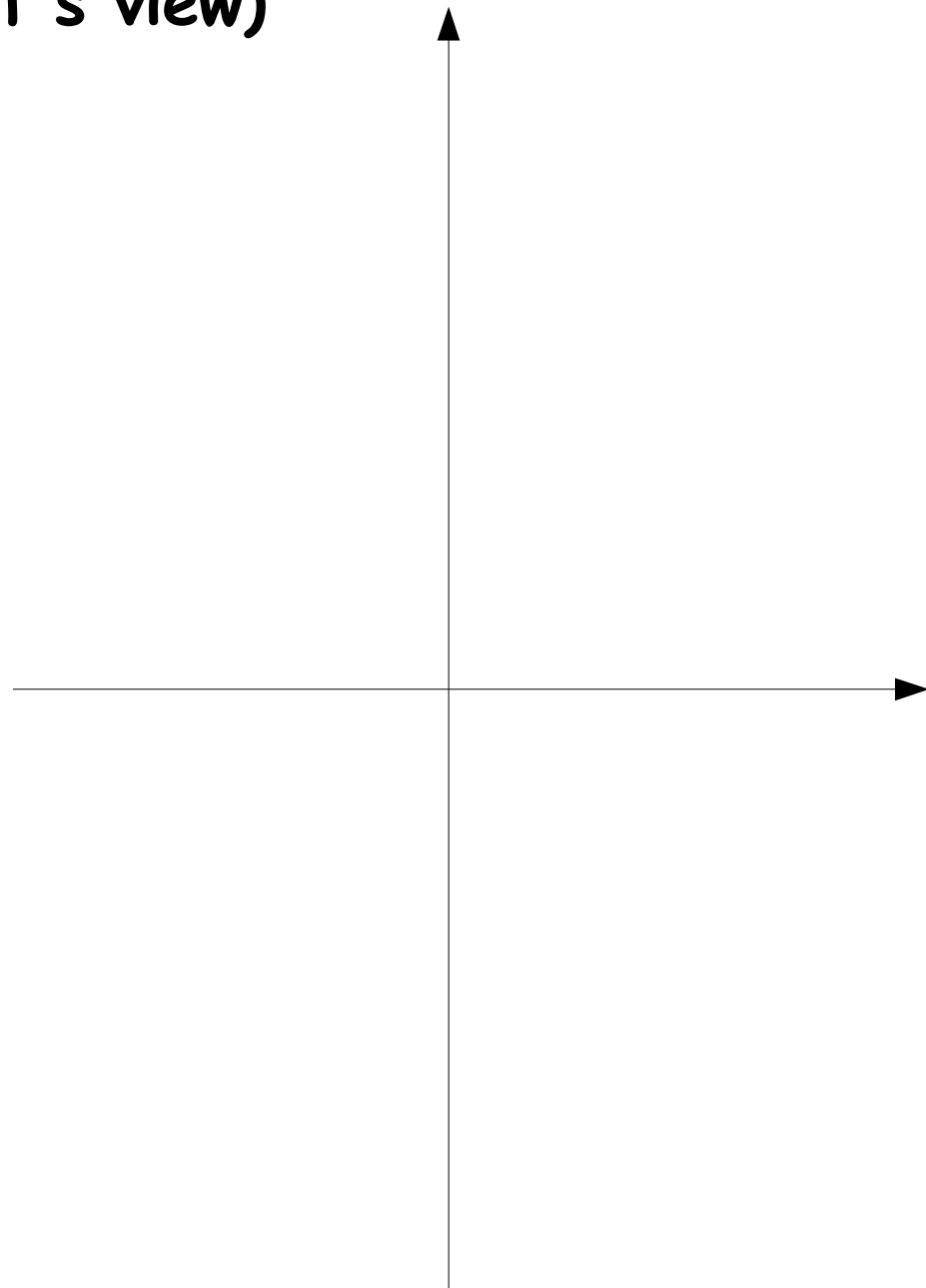
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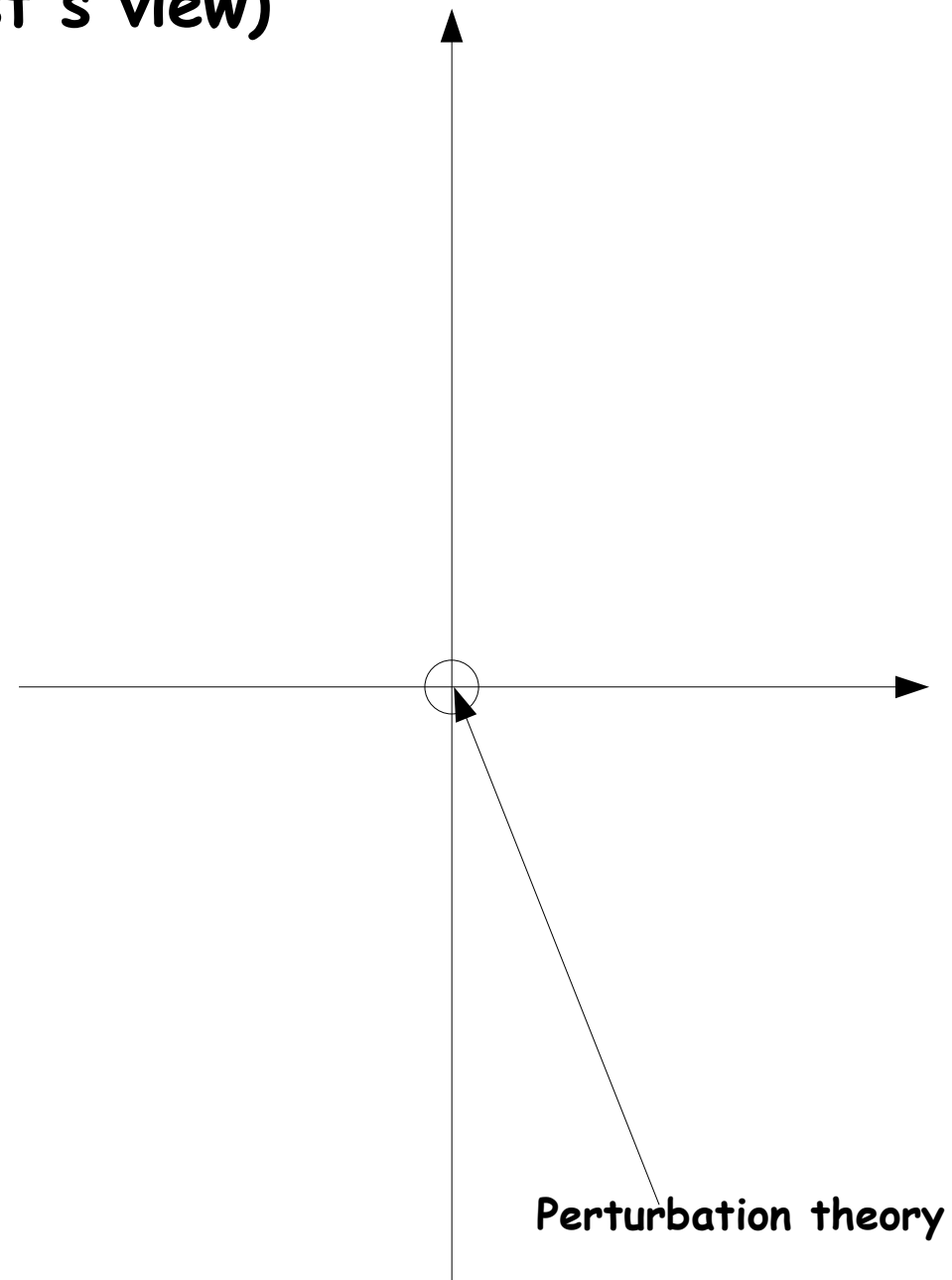
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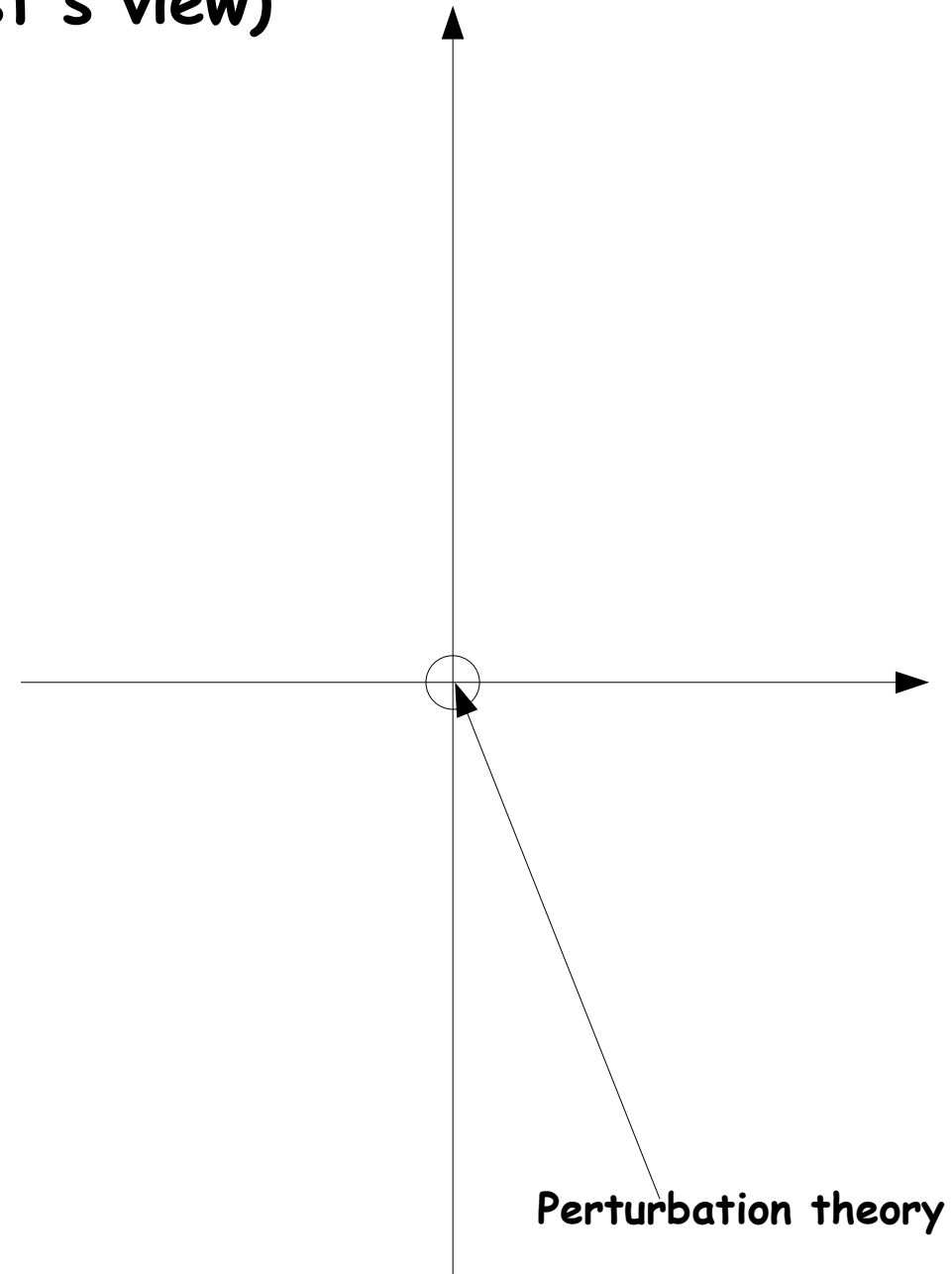
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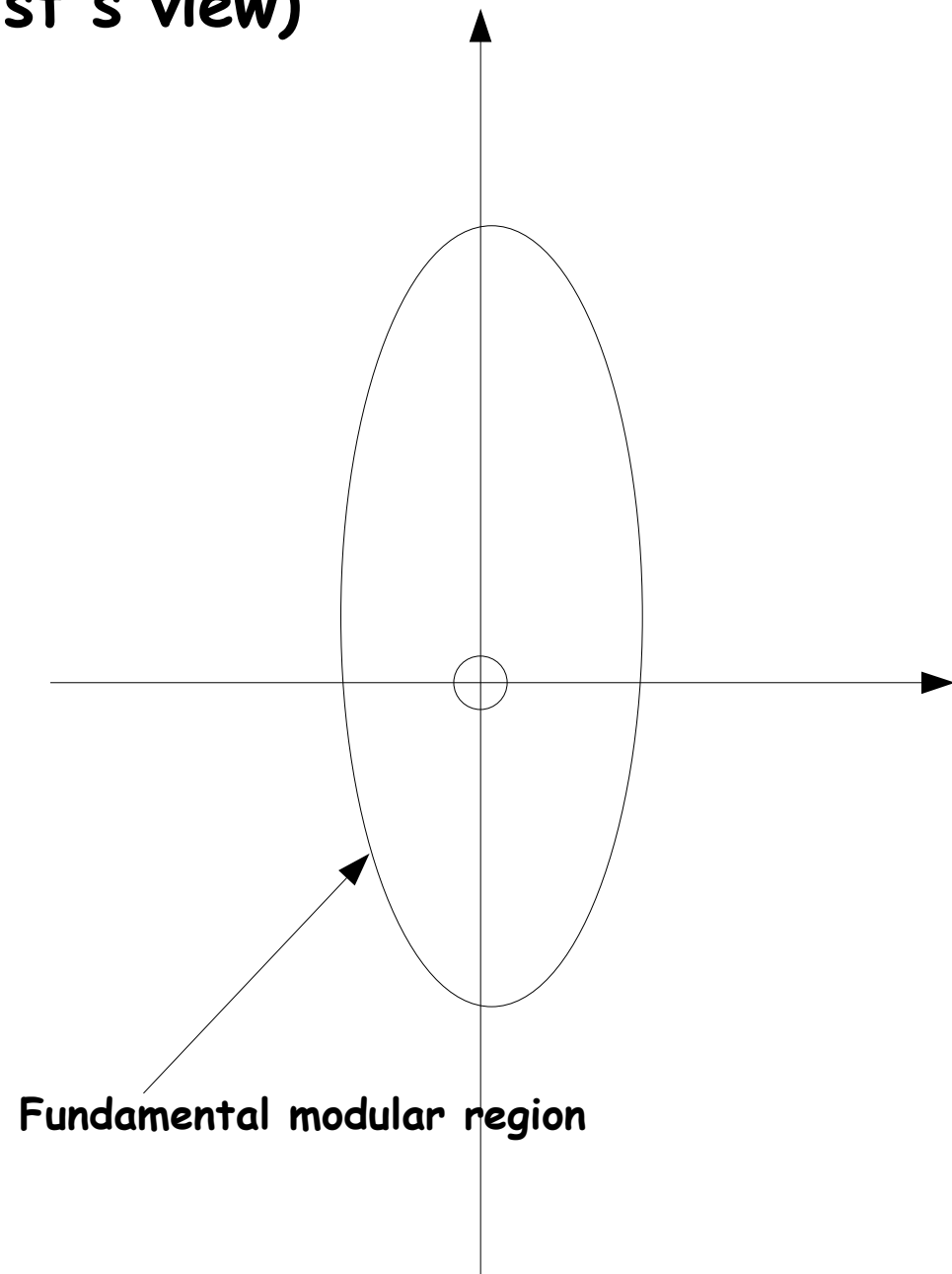
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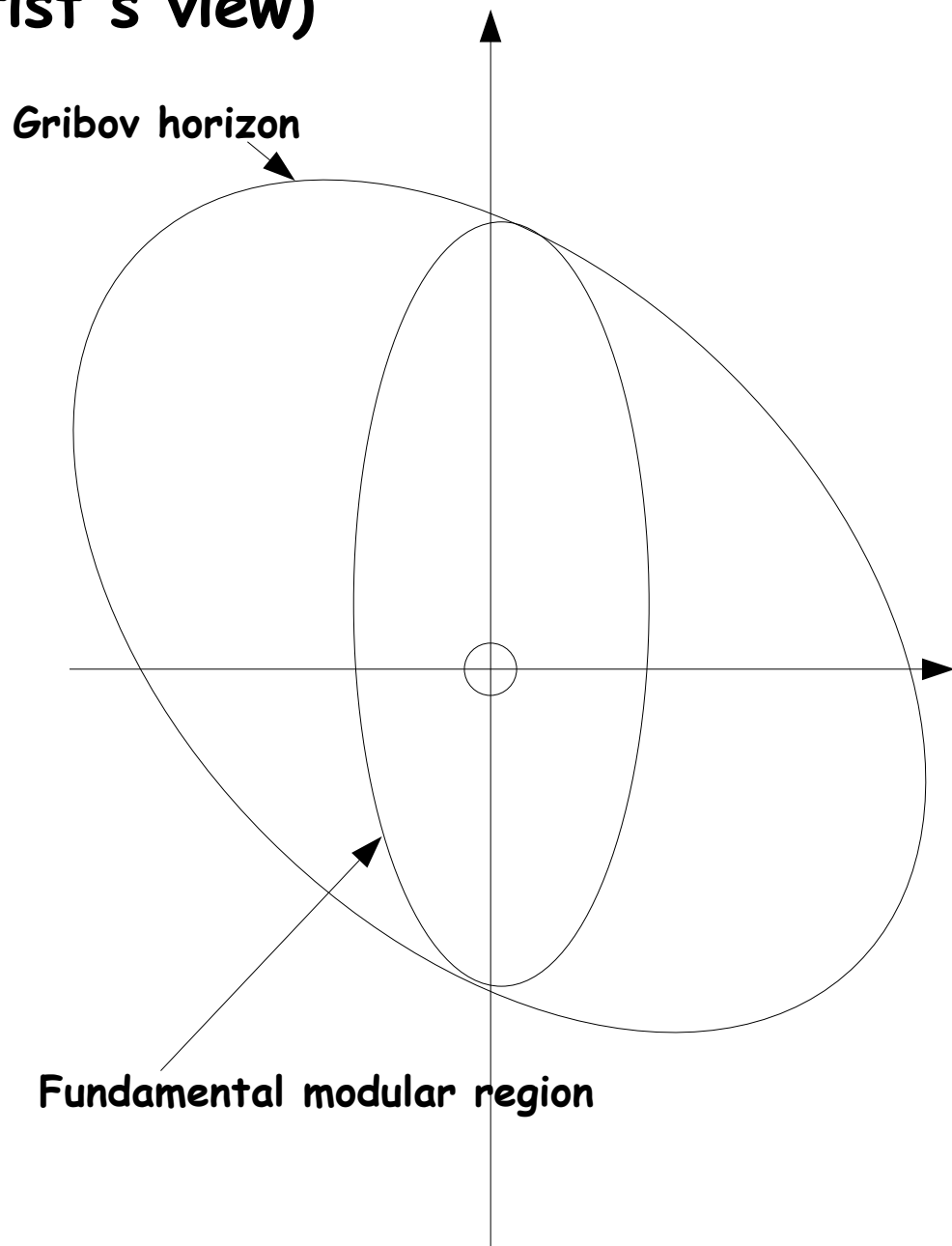
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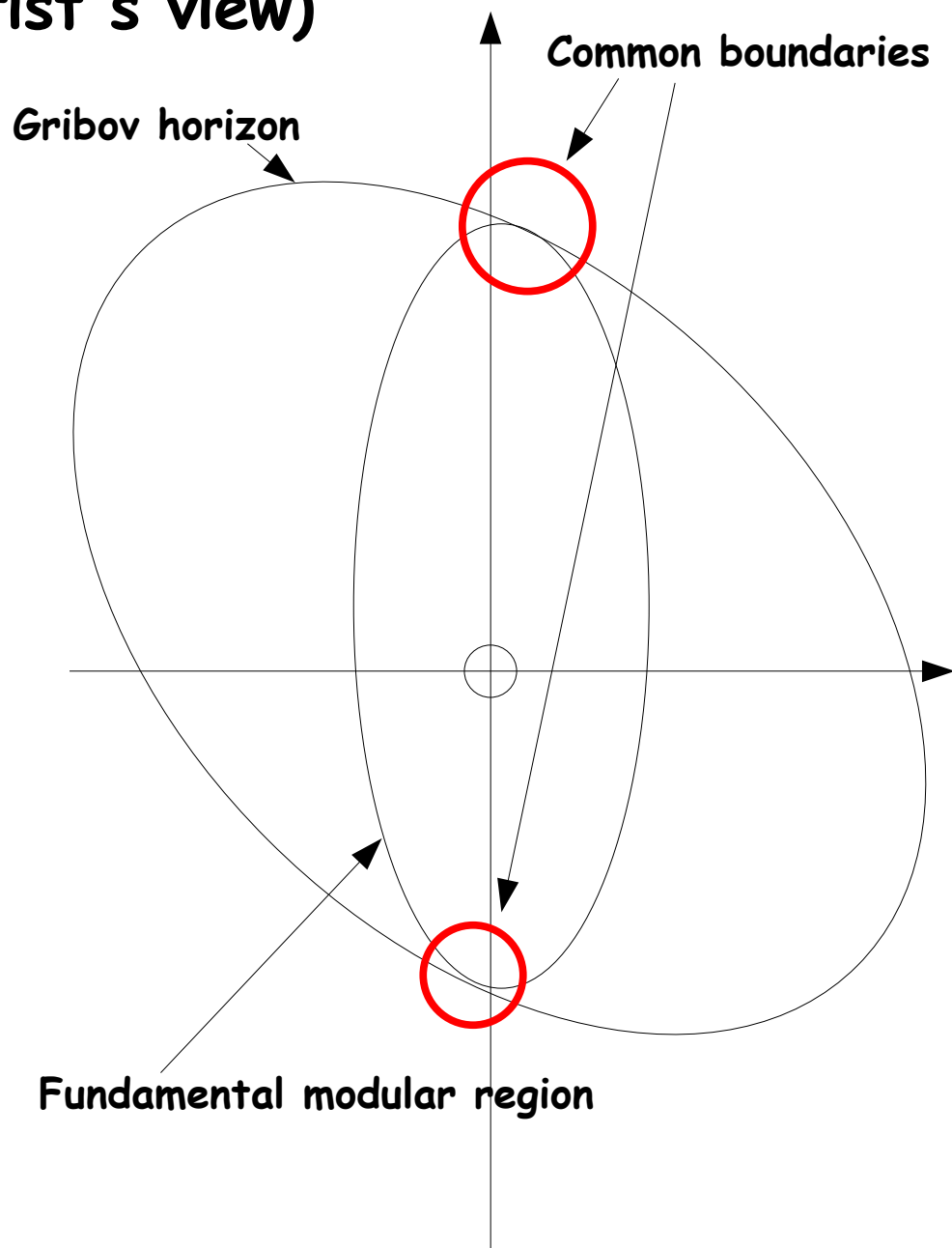
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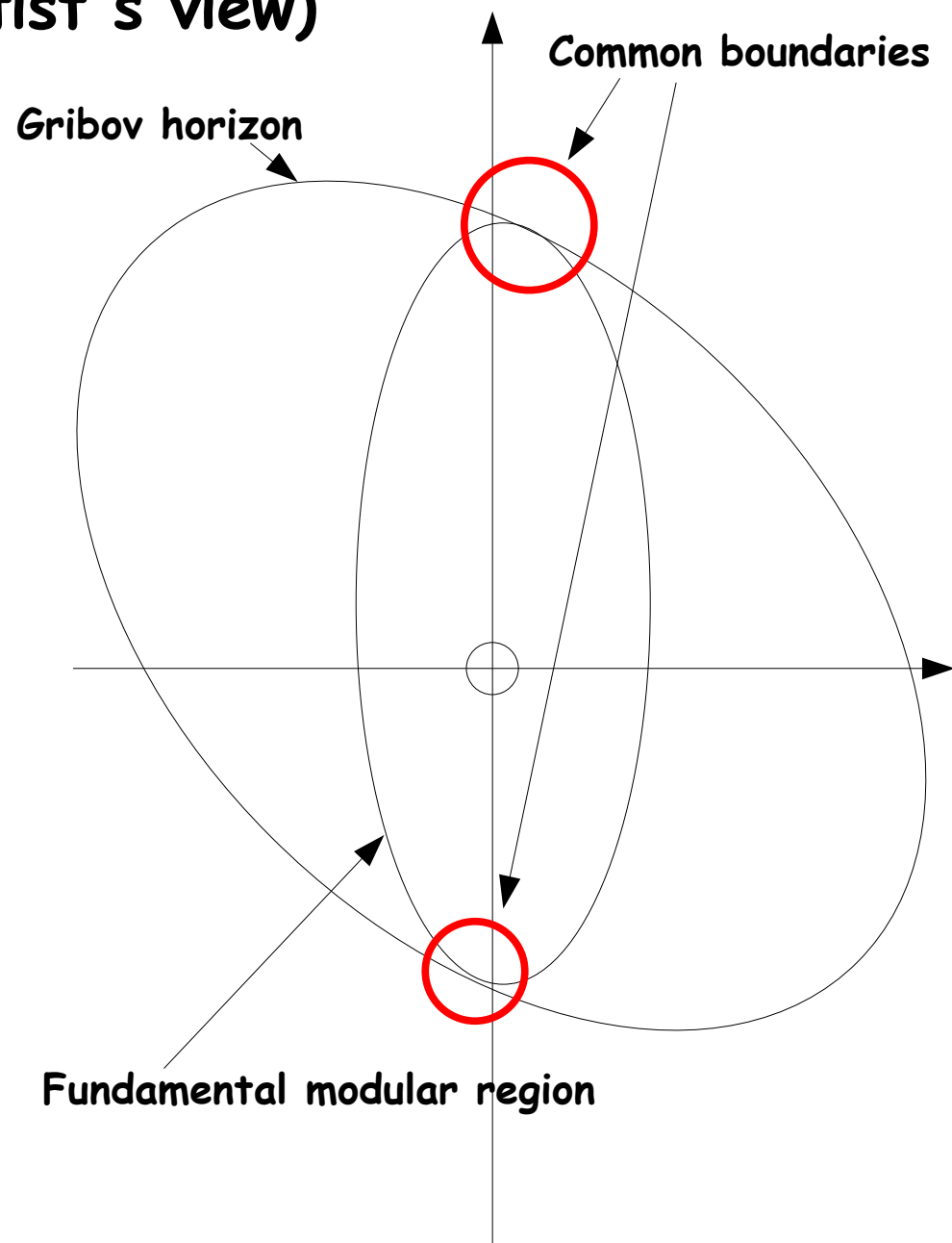
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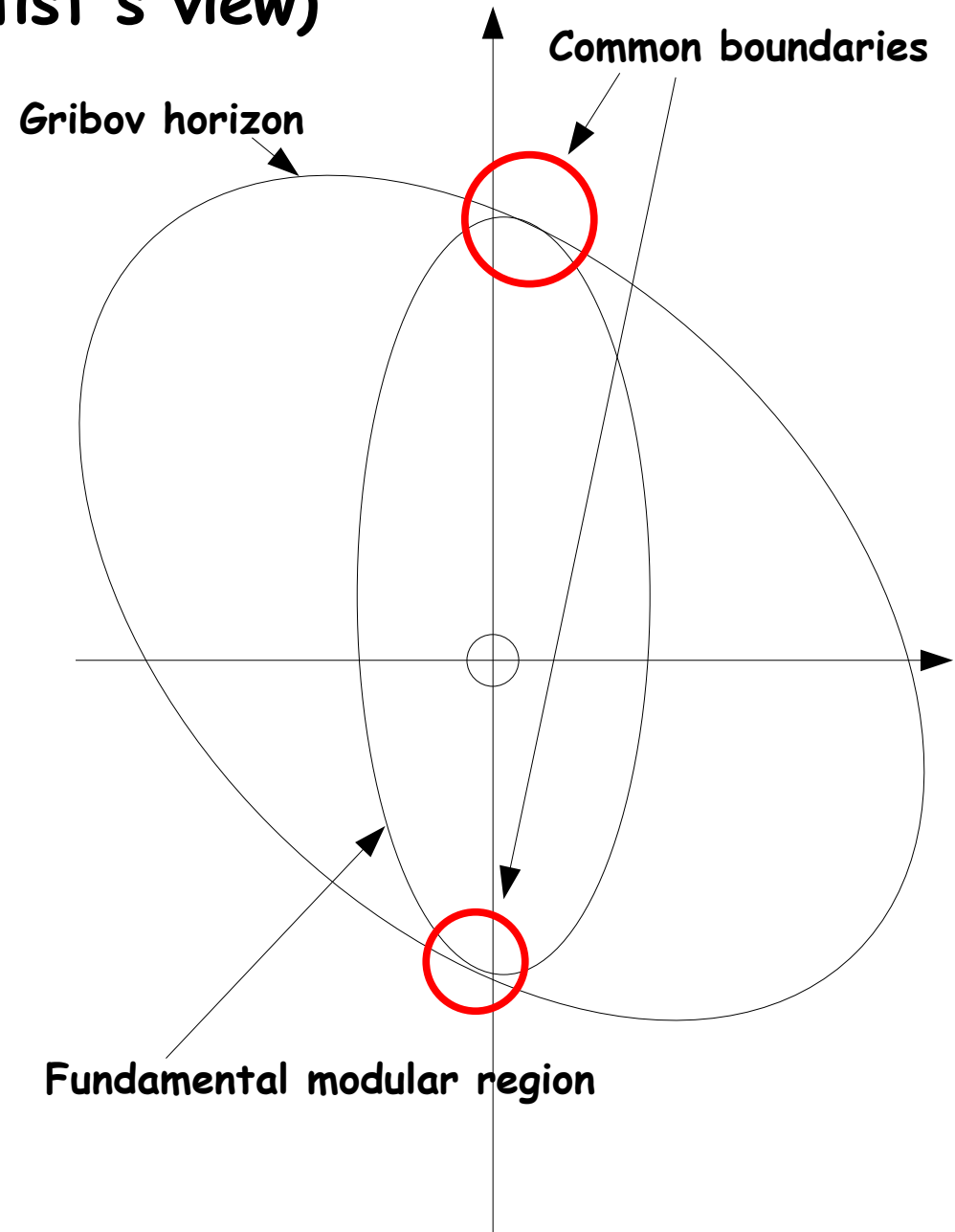
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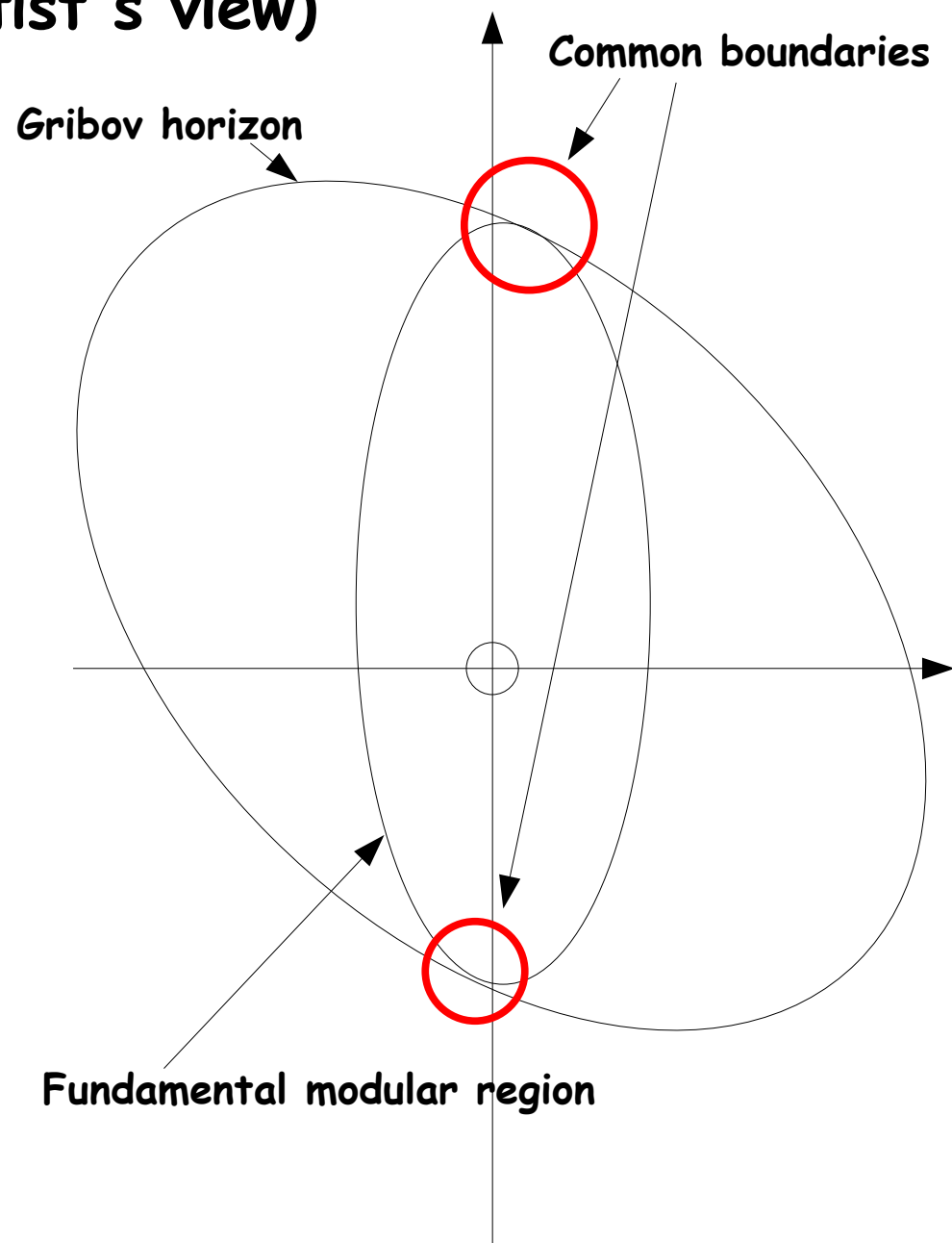
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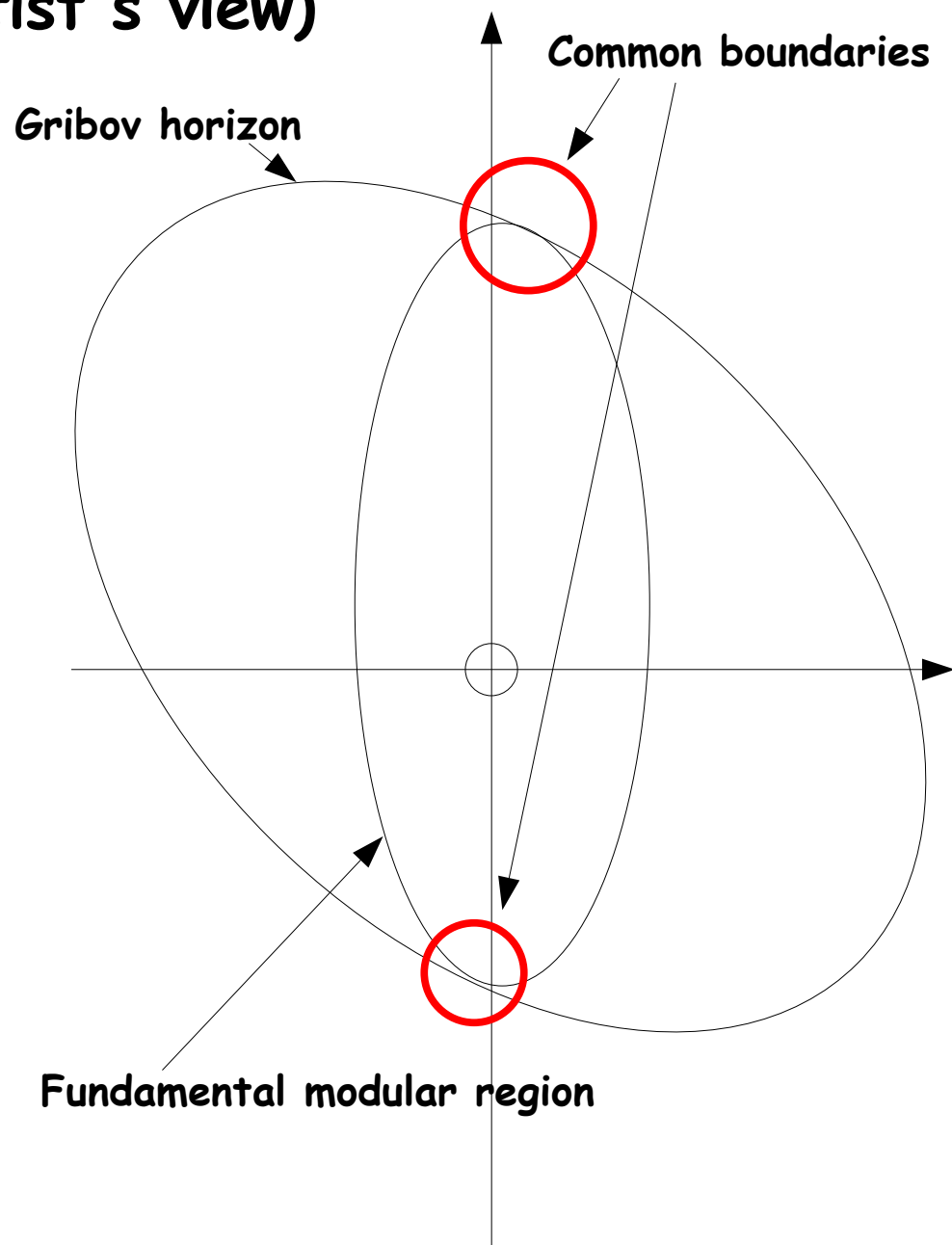
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- **Leads to positivity violating spectral functions**



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Lambda gauges

- (Symbolic) Lagrangian for Gaussian average:

$$L = -\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a - \bar{c}^a \partial'_\mu D_\mu^{ab} c^b$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - gf^{abc} A_\mu^b A_\nu^c$$

$$D_\mu^{ab} = \delta^{ab} \partial_\mu - gf^{abc} A_\mu^c$$

- Degrees of freedom:

Gluons: A_μ^a

Ghosts: \bar{c}^a, c^a

(Intermediate states - not observable)

Faddeev-Popov operator

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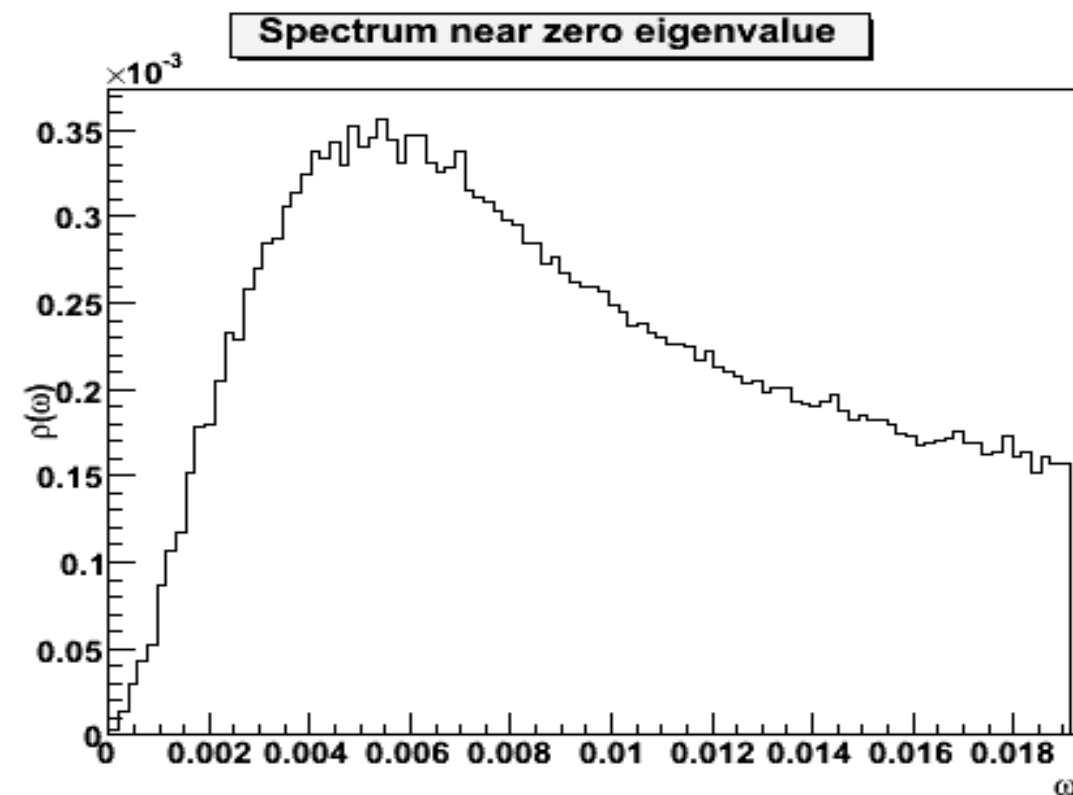
- **Second-order differential operator**
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- **Simpler in Landau gauge**

$$M^{ab} = -\delta^{ab} \partial_\mu \partial_\mu + g f^{abc} A_\mu^c \partial_\mu$$

- **No (non-trivial) zero-modes in the vacuum**

Landau-gauge Faddeev-Popov operator eigenspectrum

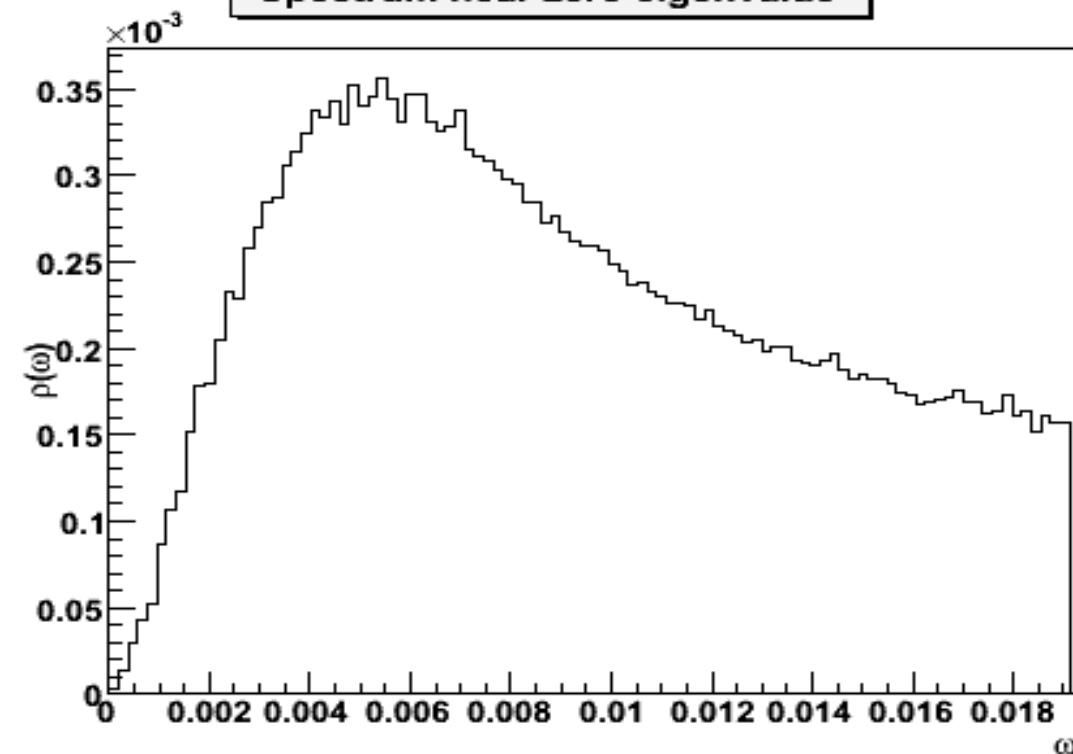


[Cucchieri et al., PRD 2006]

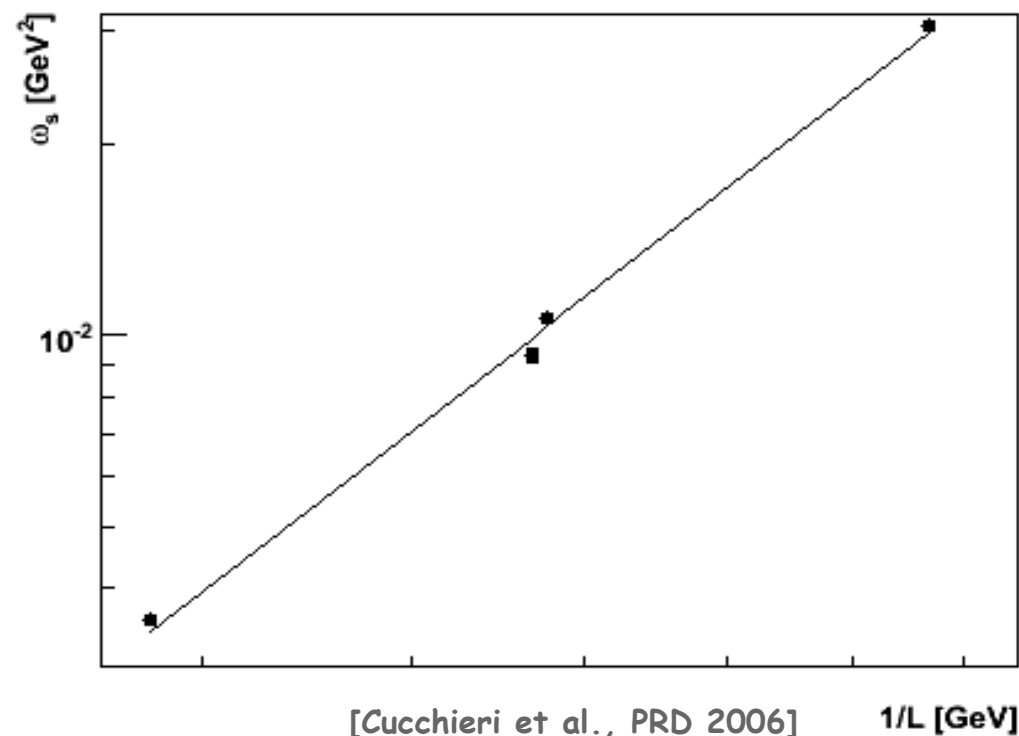
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Landau-gauge Faddeev-Popov operator eigenspectrum

Spectrum near zero eigenvalue



Smallest eigenvalue of the Faddeev-Popov operator



[Cucchieri et al., PRD 2006] 1/L [GeV]

- Near zero **enhanced** compared to vacuum
- Average configuration in the continuum limit on the Gribov horizon
- **Agrees with Gribov-Zwanziger** scenario

Propagators

[Introduction: Alkofer et al., PR 2001]

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- 2-point Green's functions are the **propagators**
- **Gluon** (only one dimension anisotrope):

$$D_{\mu\nu}^{ab}(x-y) = \langle A_{\mu}^a(x) A_{\nu}^b(y) \rangle$$

$$D_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_{\mu} p_{\nu}}{p^2} \right) \frac{Z(p)}{p^2} + \frac{p_{\mu} p_{\nu}}{p^2} \frac{L(p)}{p^2}$$

Propagators

[Introduction: Alkofer et al., PR 2001]

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Propagators

[Introduction: Alkofer et al., PR 2001]

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- **Ghost linked to the Faddeev-Popov operator**

$$D_G^{ab}(x-y) \sim \langle (\partial'_\mu D_\mu^{ab})^{-1} \rangle = \langle (\partial'_\mu (\delta^{ab} \partial_\mu - g f^{abc} A_\mu^c))^{-1} \rangle$$

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 - Infrared enhancement - **mediates long-range forces**

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 - No further approximations
 - Assumption: Wick-rotation works
 - All non-perturbative effects taken correct

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 - Exact results

Methods

- **Lattice calculations** - operators, Green's functions and topological objects
- **Functional methods** - Green's functions
- **Analytical methods** - operators and topological objects
- Other methods
- Combination of methods necessary
 - Successful cooperation

Landau gauge

- Gauge condition $\partial_\mu A_\mu^a = 0$

Landau gauge

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 - Simple propagator structure

$$D_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \frac{Z(p)}{p^2}$$

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- **Most intensively studied gauge so far**

Predictions

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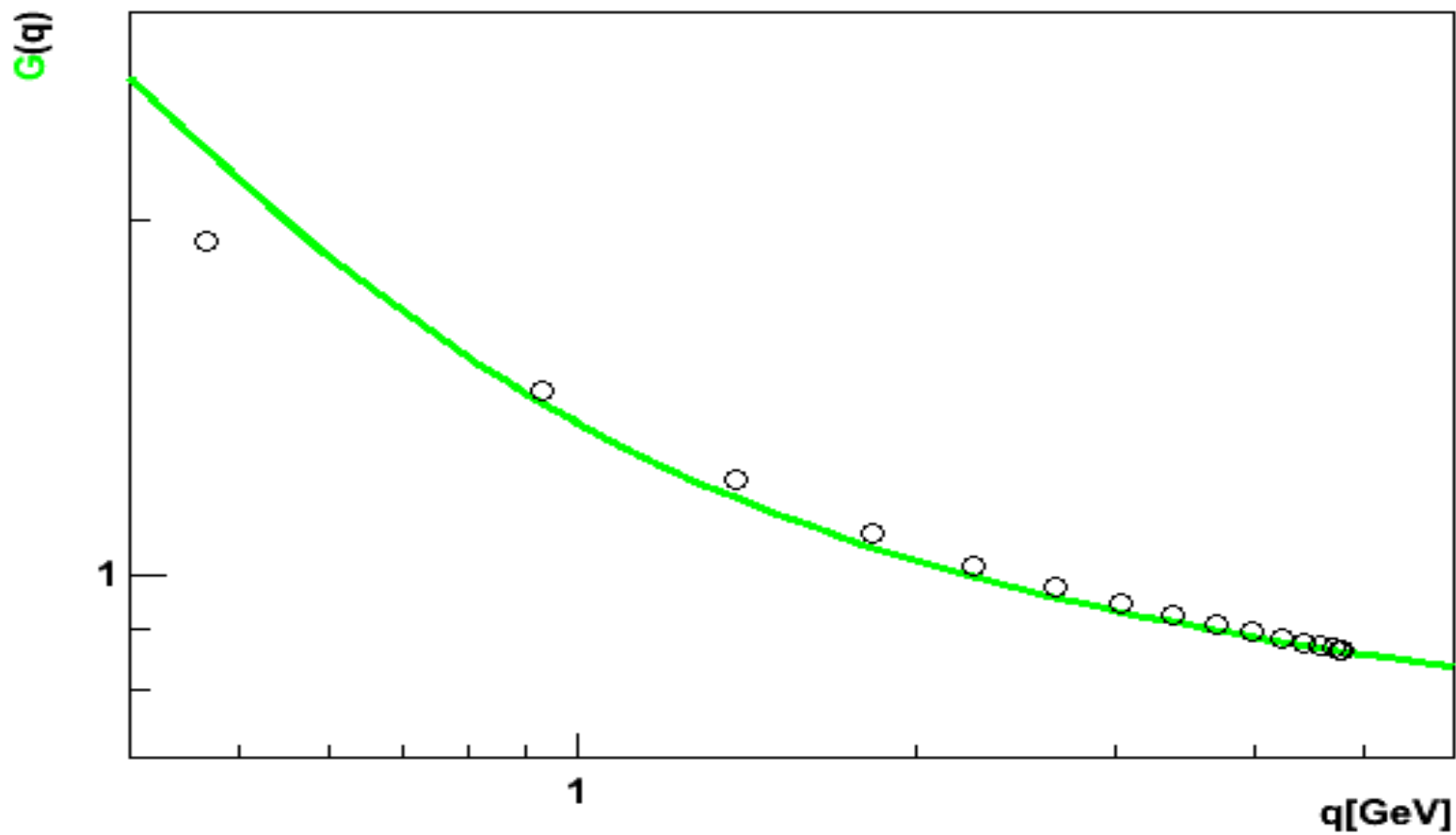
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- Numerical values of exponents under rather well-supported assumptions [Lerche et al., PRD 2002, Zwanziger PRD 2002]
 - Depend only on dimensionality

Ghost

[DSE: Fischer et al., PLB 2002,
Lattice 32⁴: Cucchieri et al., unpublished]

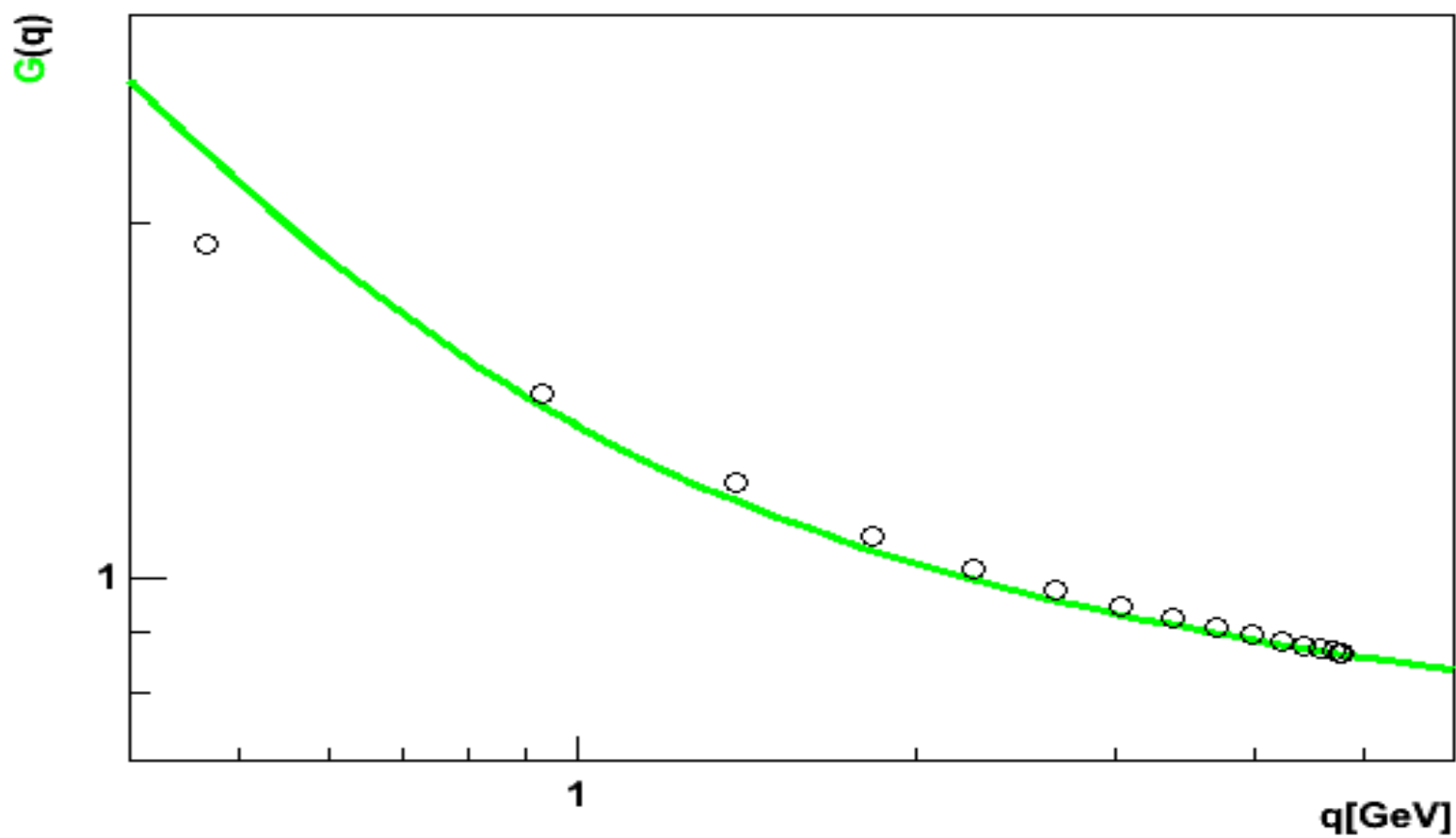
Ghost Dressing Function



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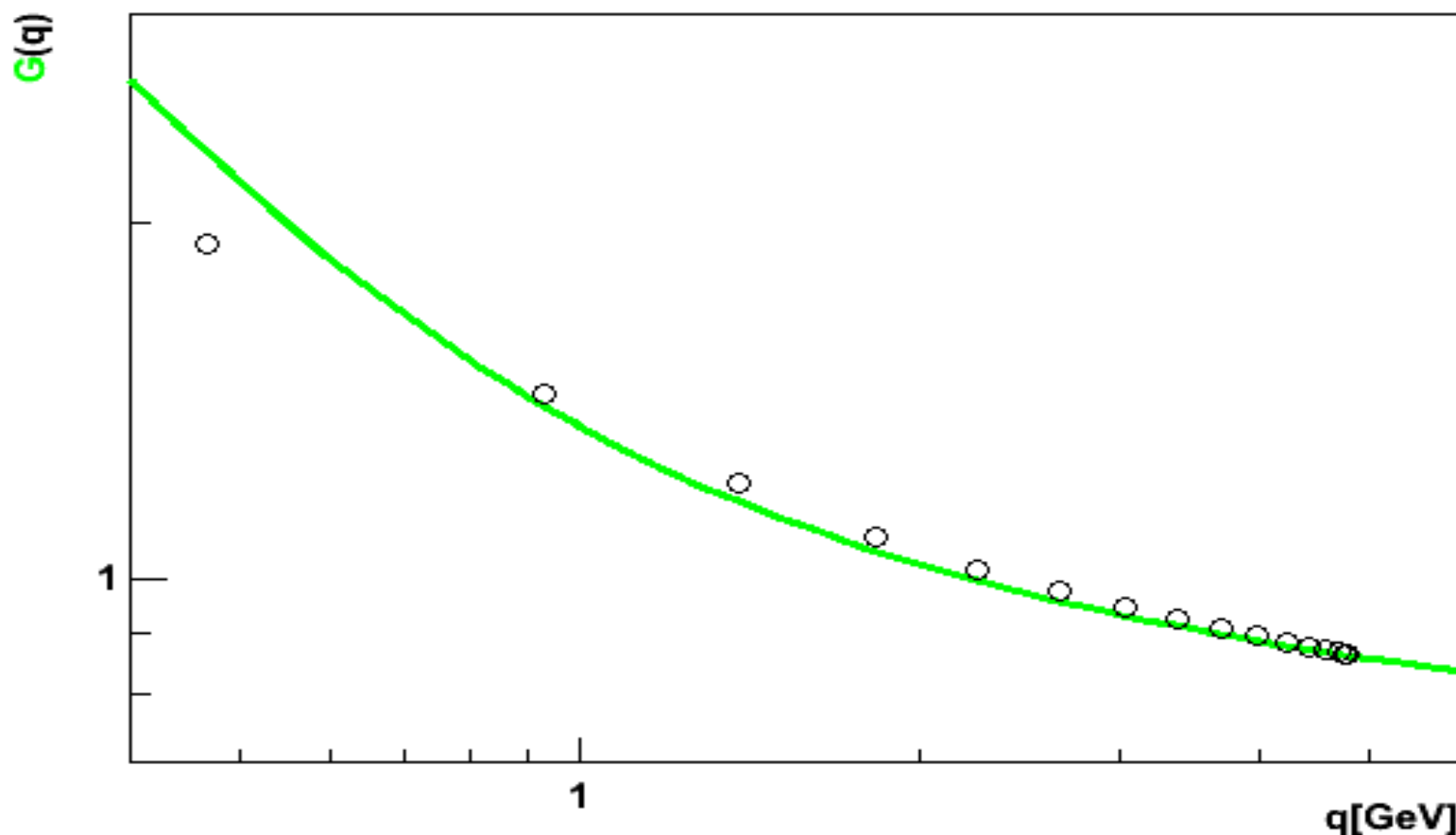


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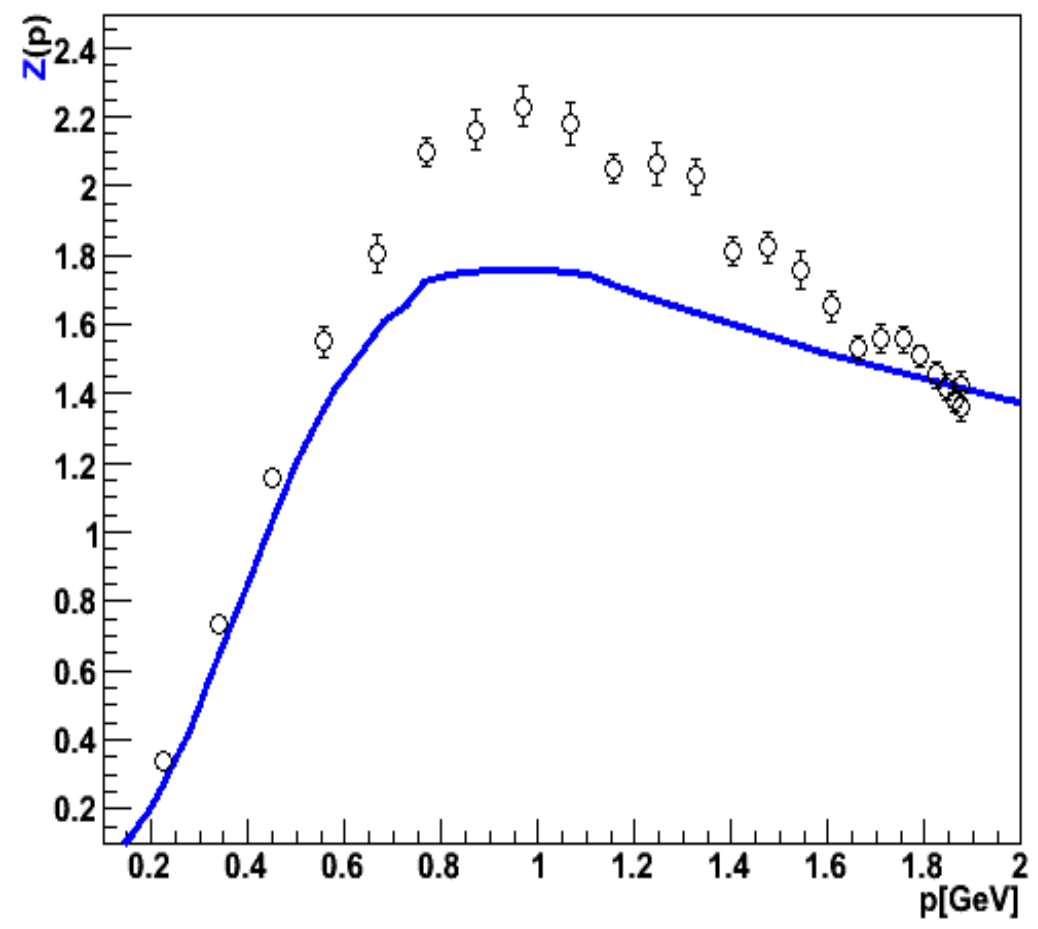
- IR divergent: Mediates long-range forces

- DSEs gives power-law: $G(p) \sim (p^2)^{-0.60}$ [Zwanziger PRD 2002, Lerche et al. PRD 2002]

- Confirmed by RG calculations [Pawlowski et al., PRL 2004]

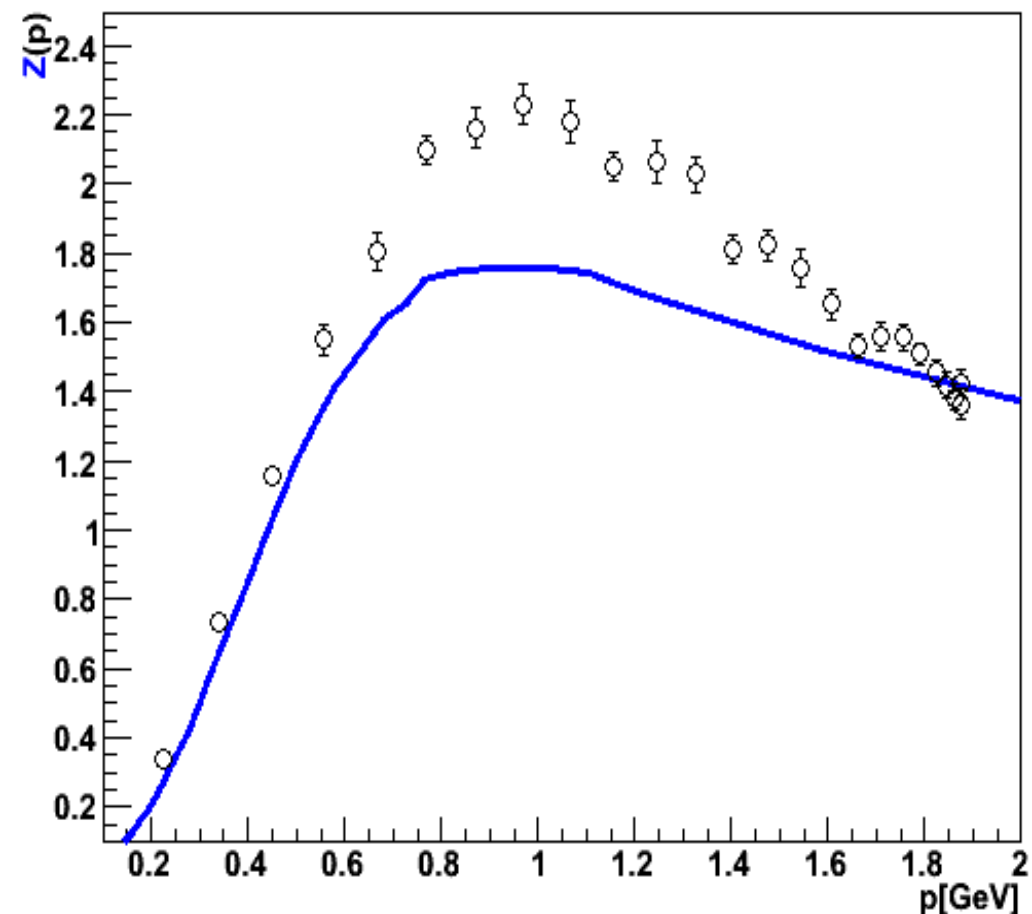
Gluon

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- Infrared vanishing - confined

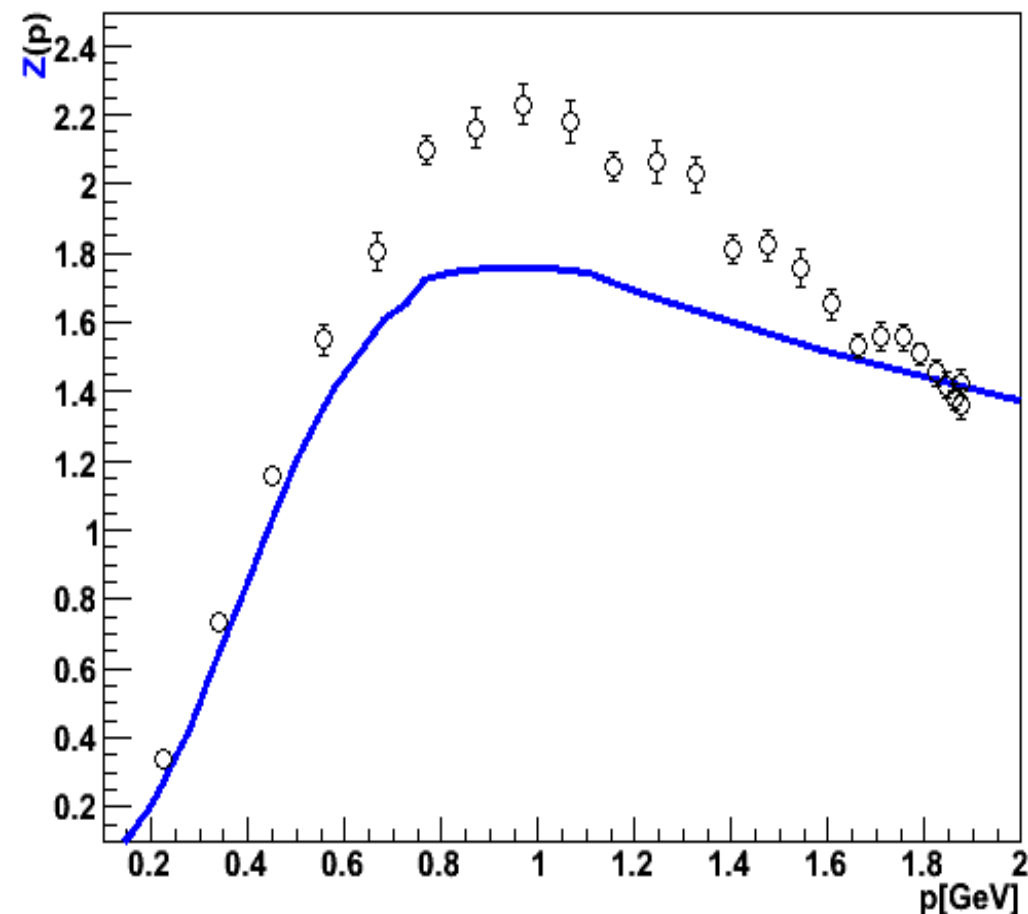
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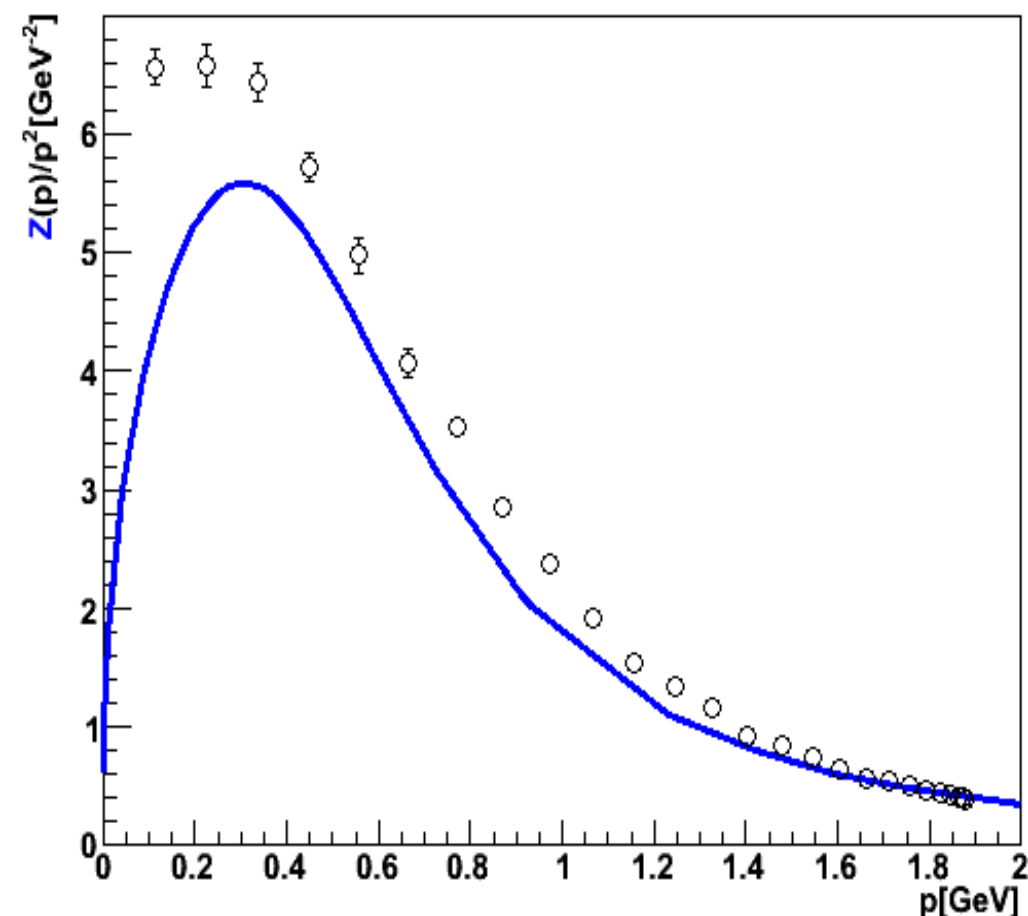
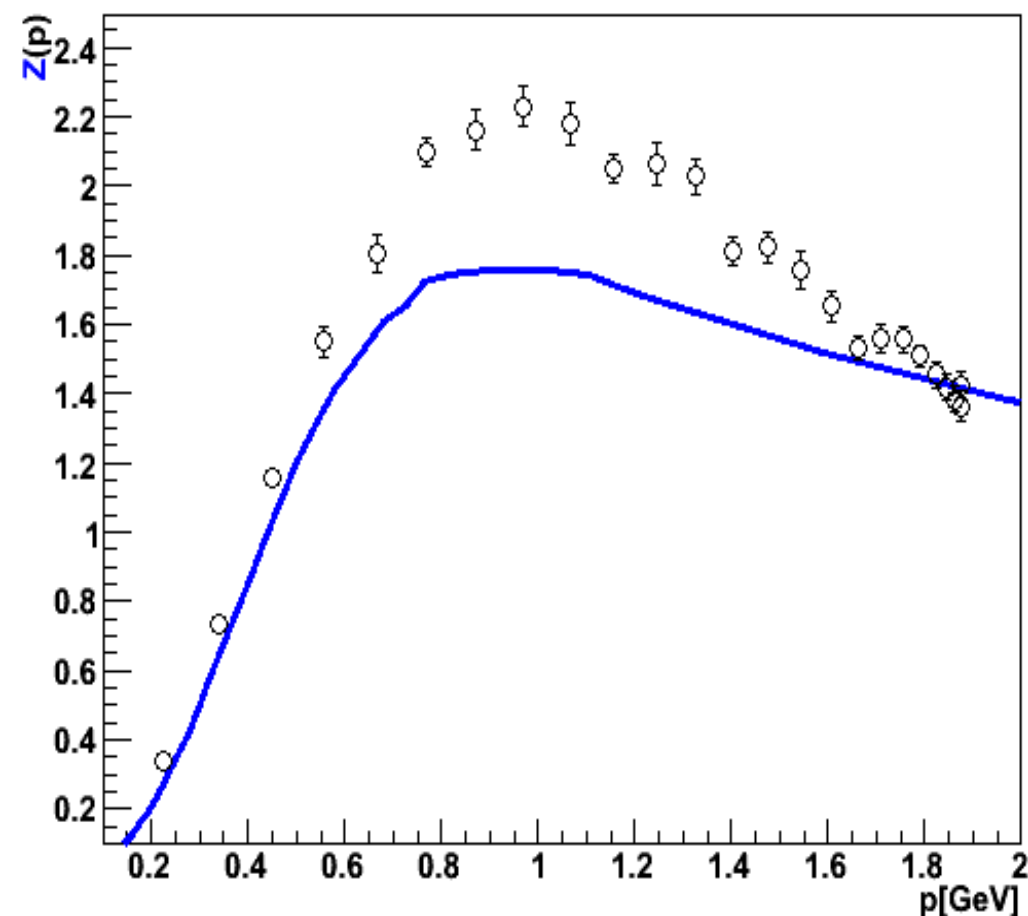
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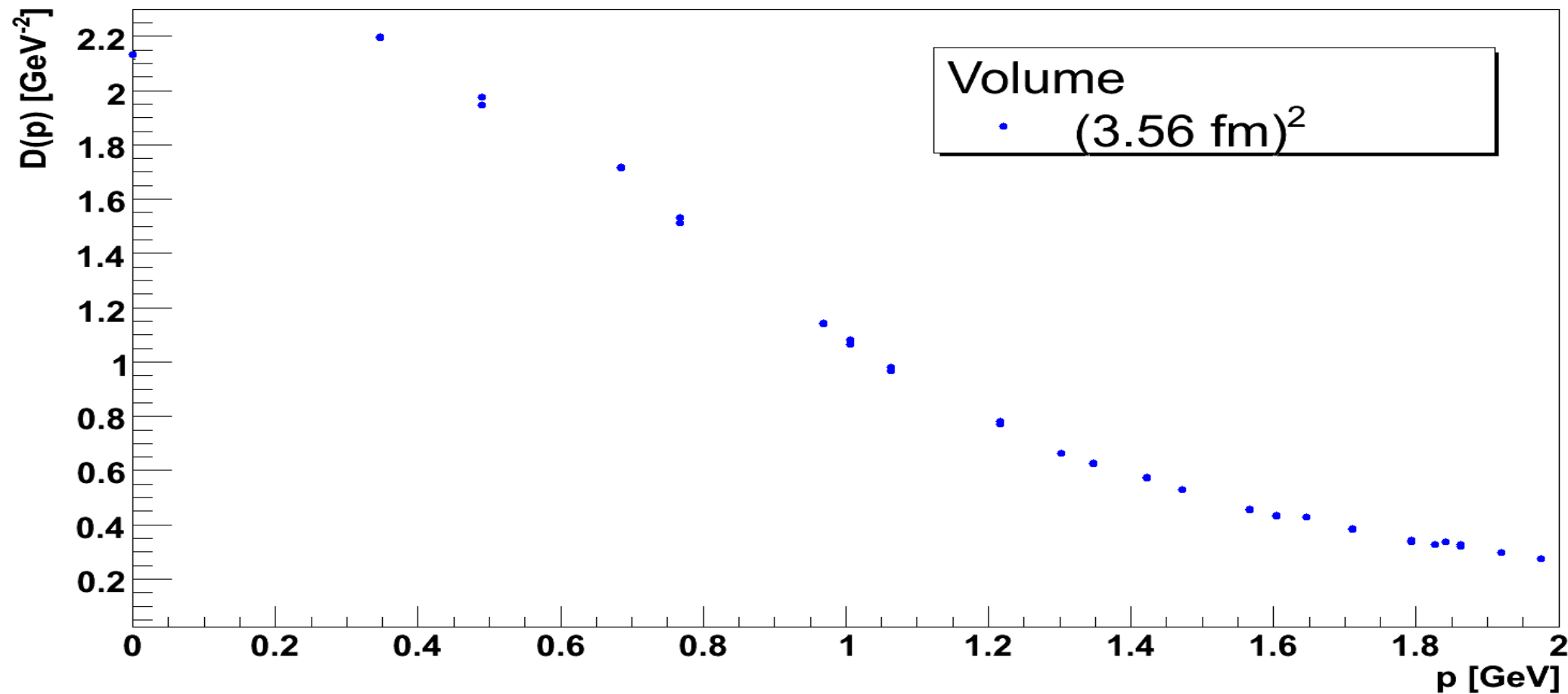
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Very large lattices needed: 2d

[Maas, unpublished]

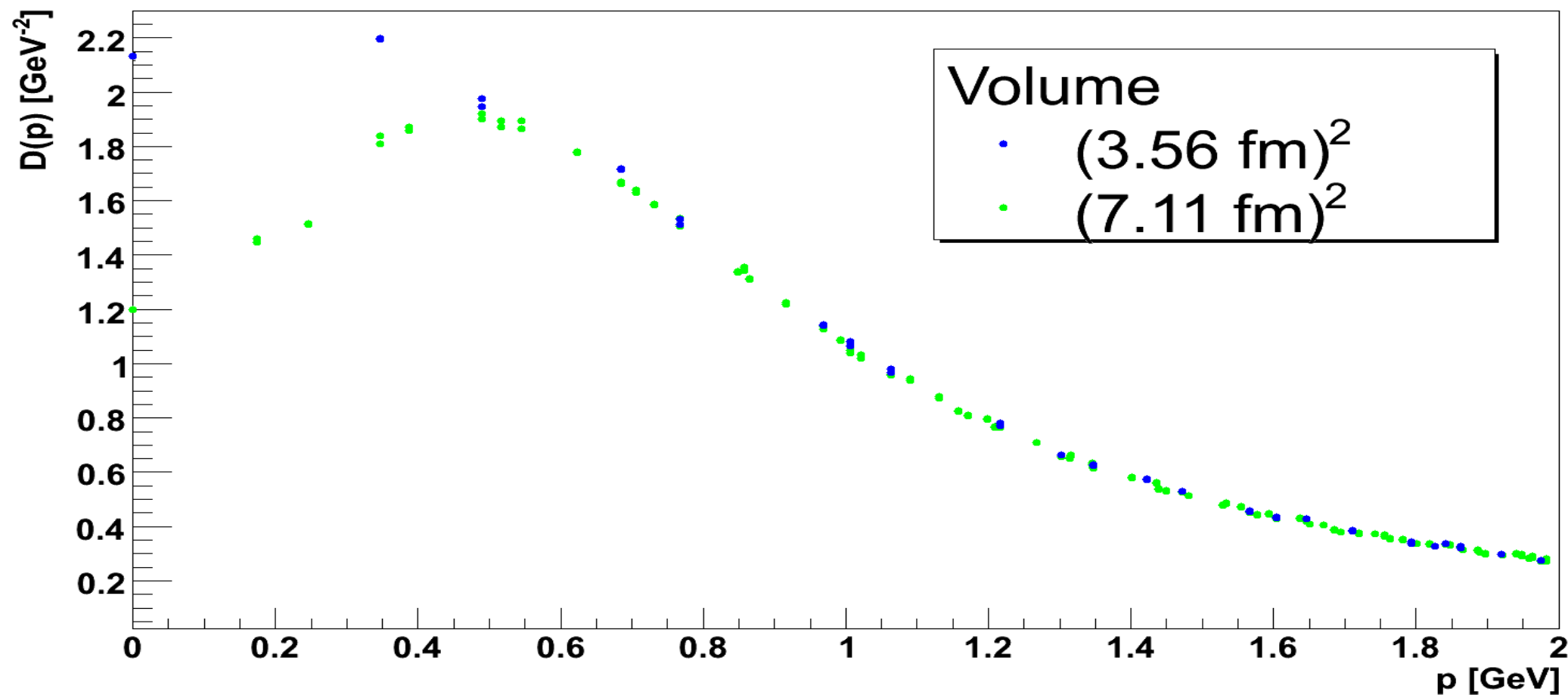
Gluon propagator



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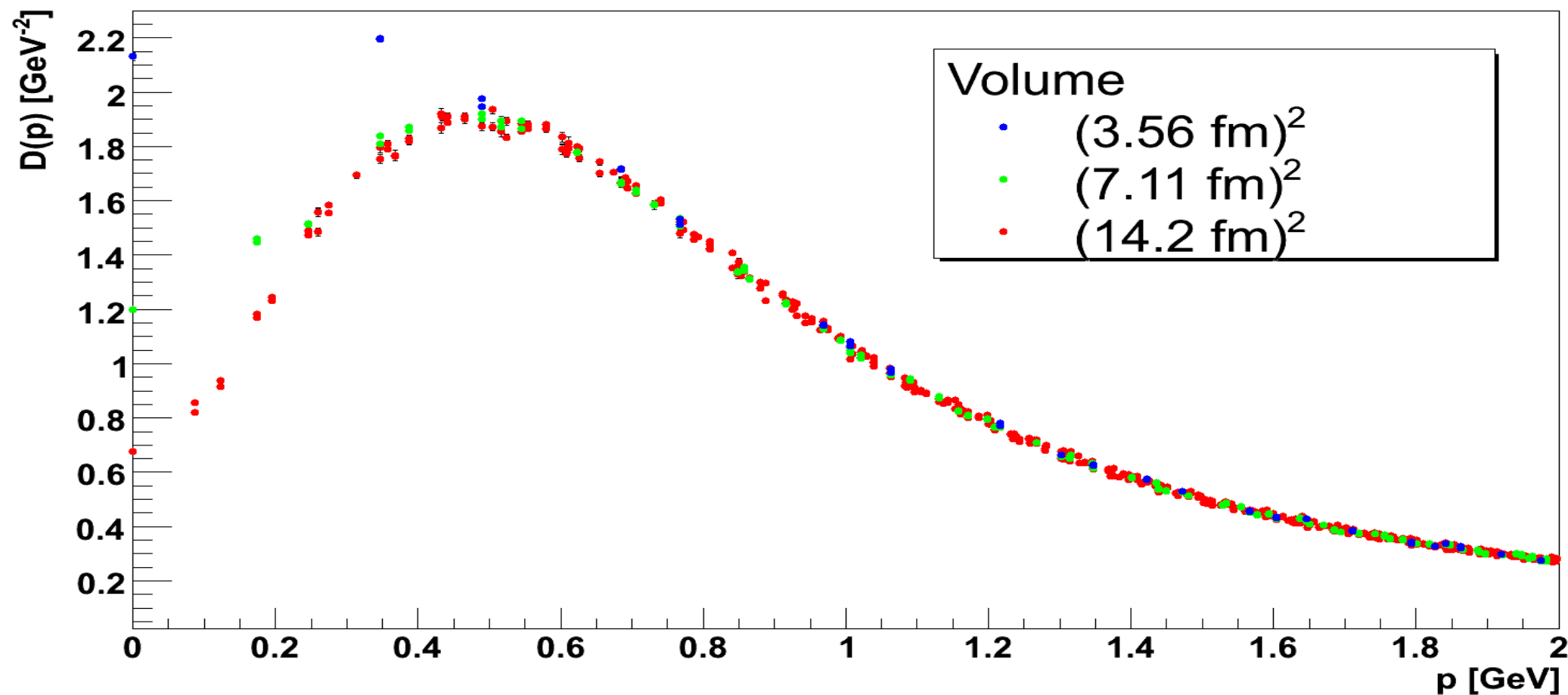
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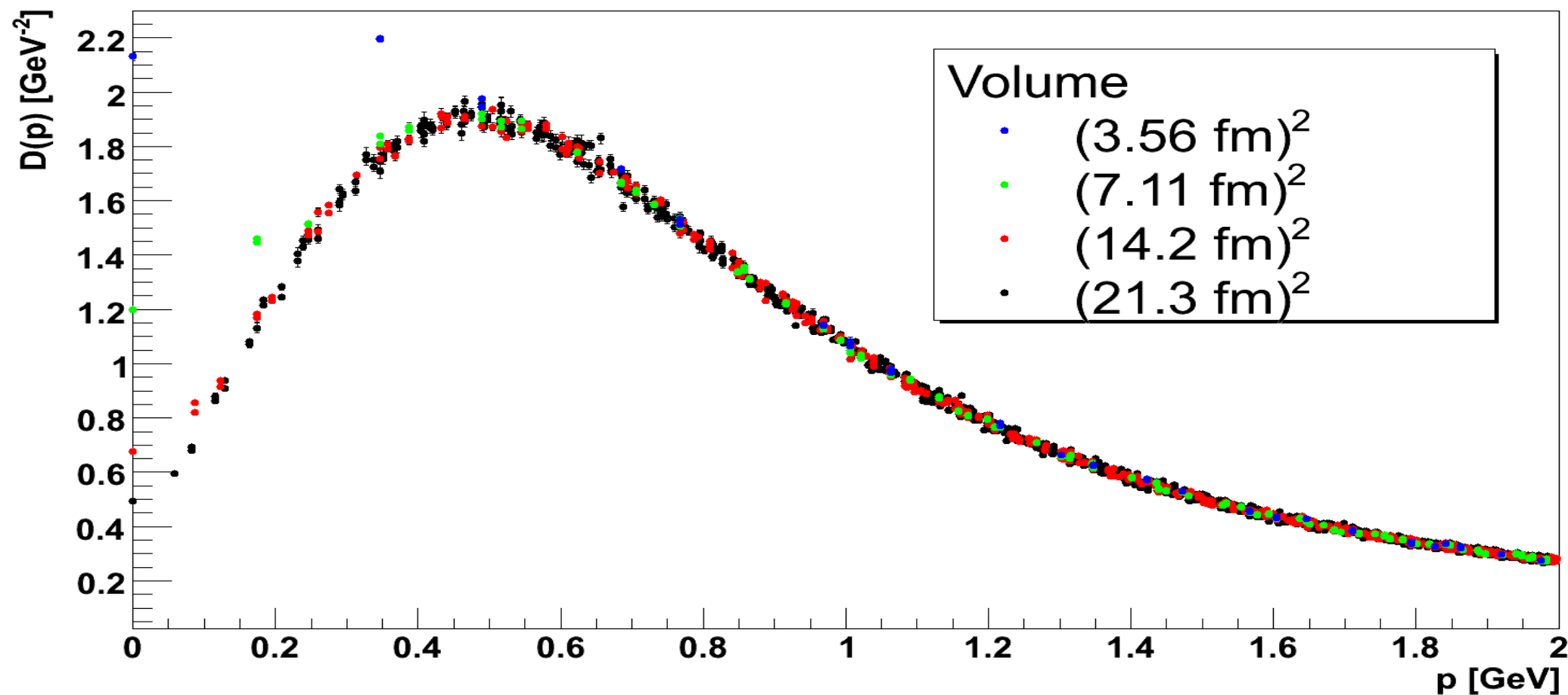
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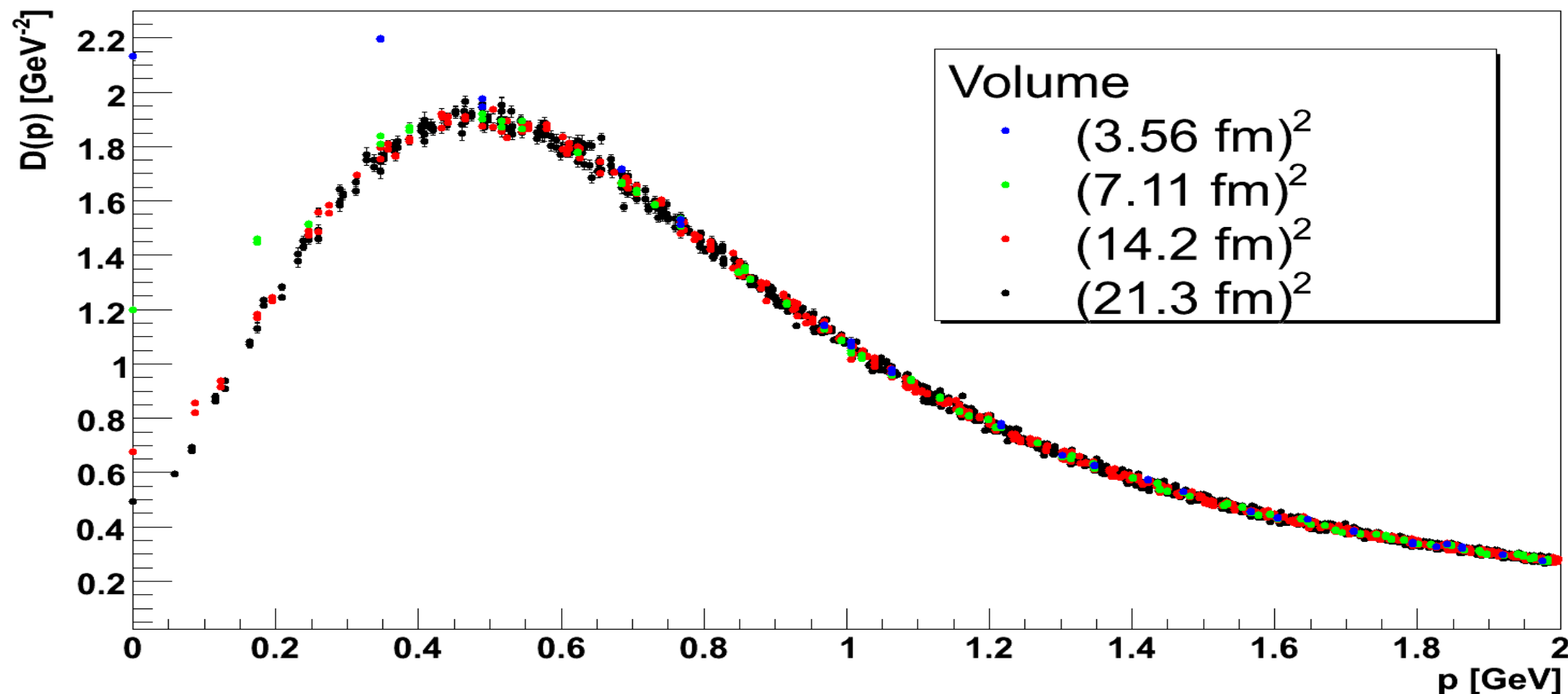
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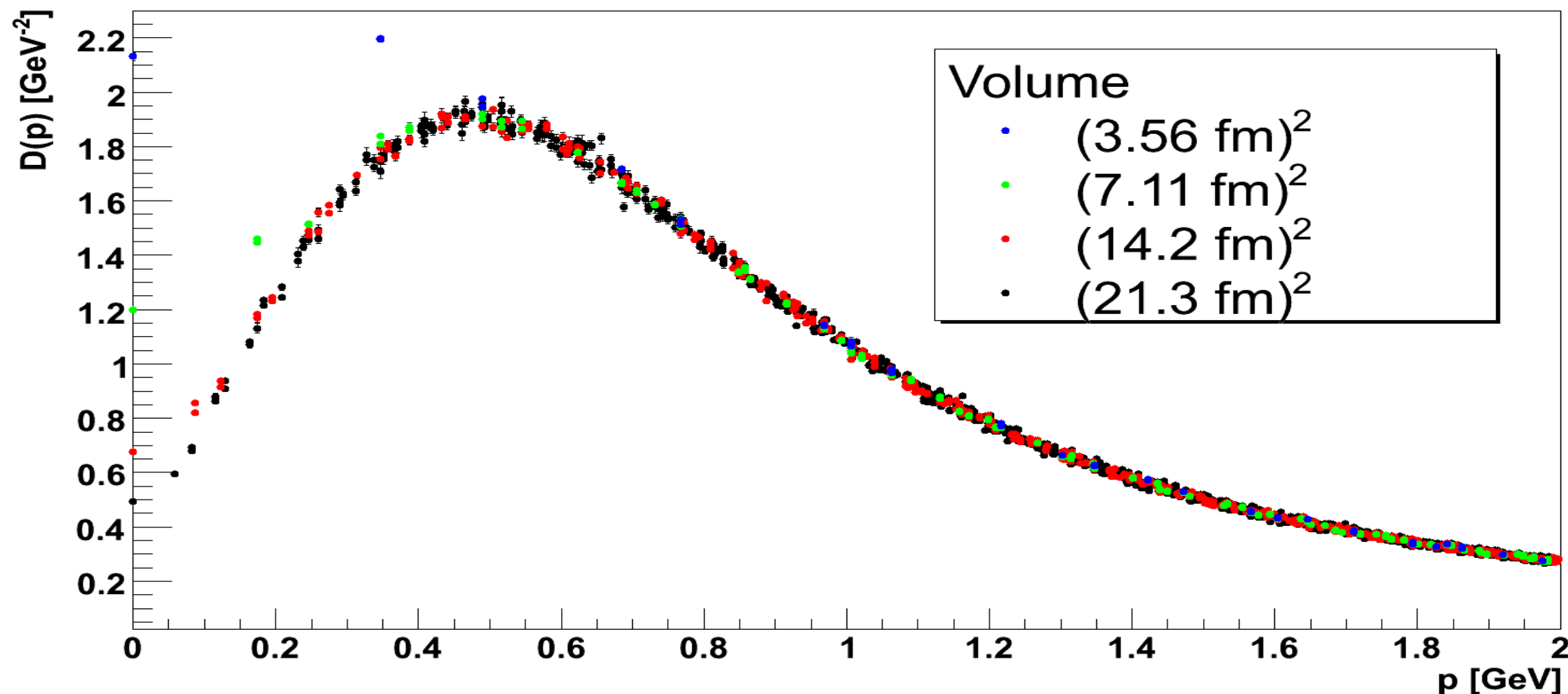


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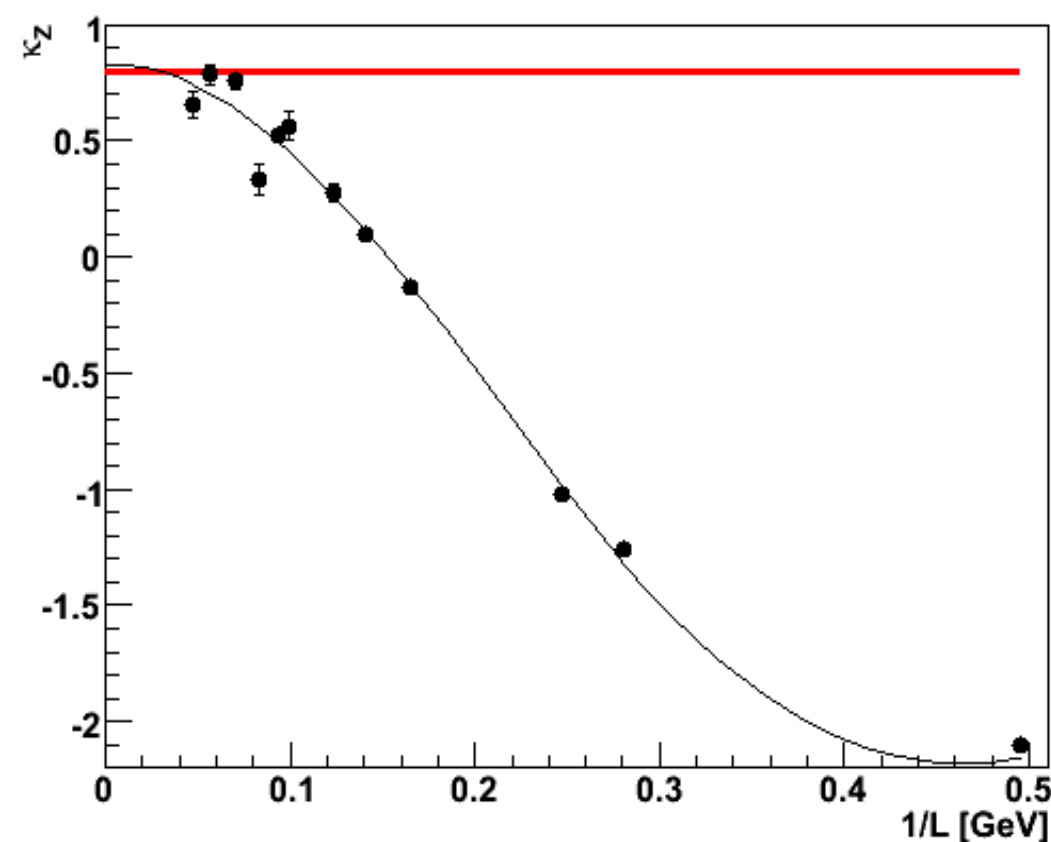
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- Effects less pronounced for the ghost

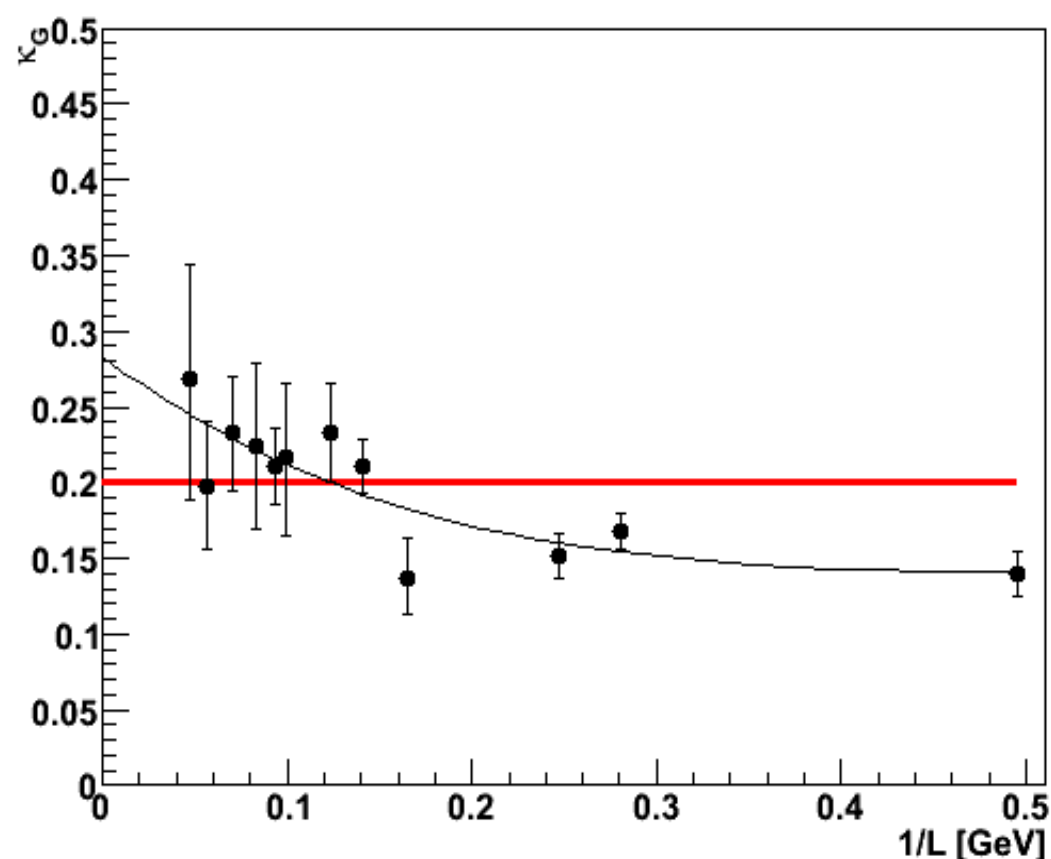
Possibility to extract exponents quantitatively

Gluon infrared exponent



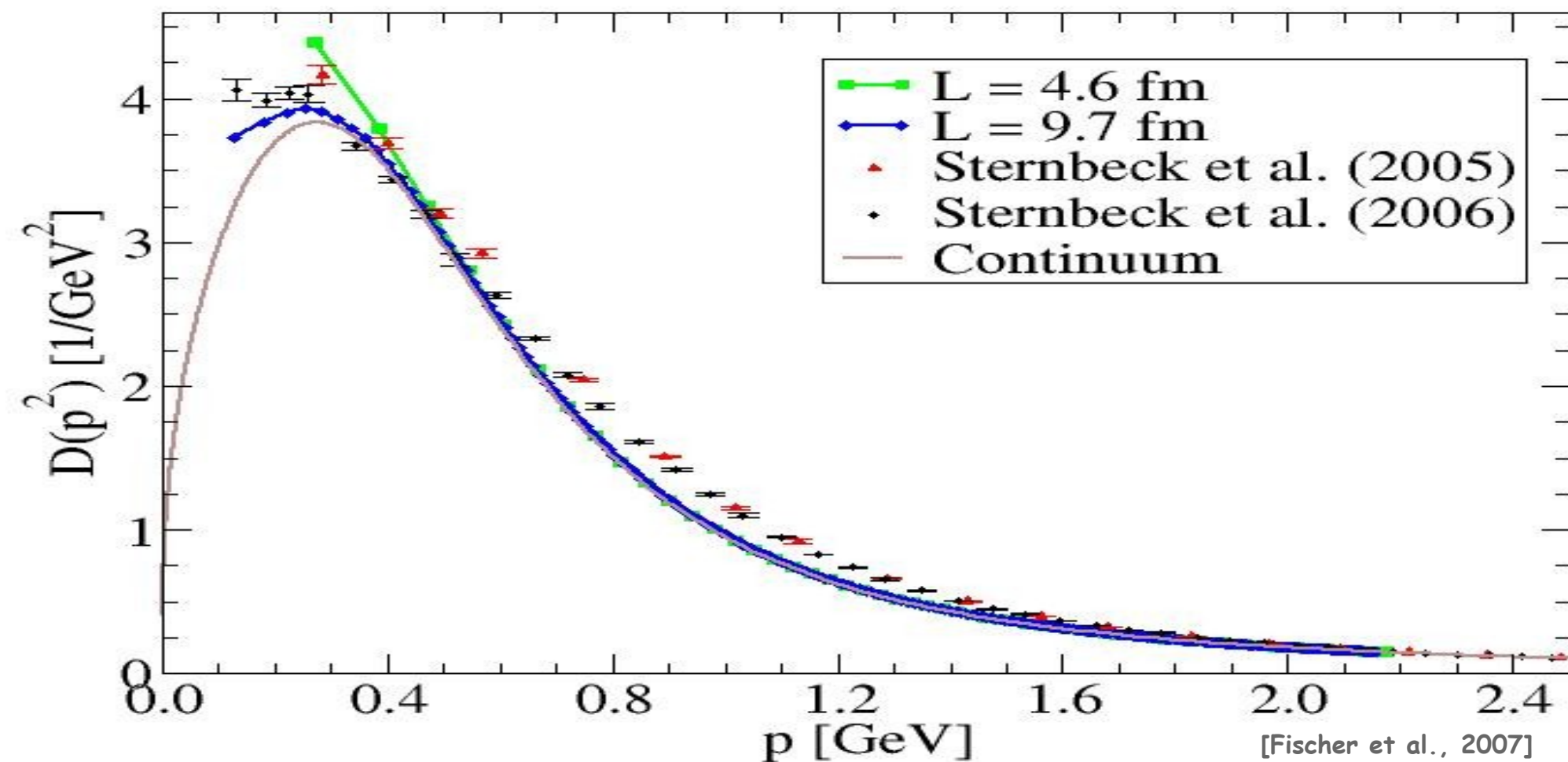
Ghost infrared exponent

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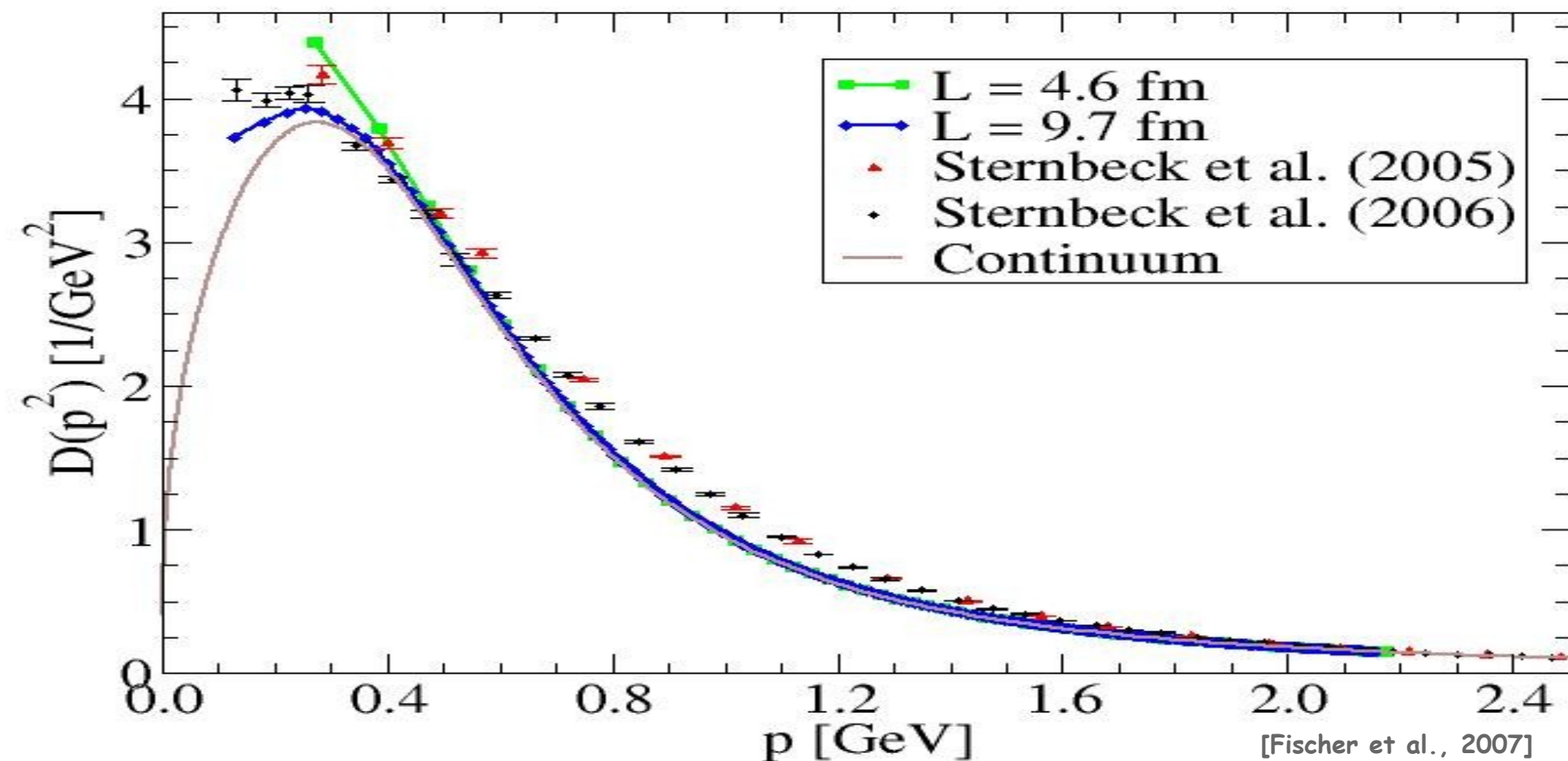
- Results compatible with predictions (red)
- Requires better statistics and larger volumes

Lattice vs. DSE in a finite volume



- Qualitative similar behavior

Lattice vs. DSE in a finite volume



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- Relevant length scale about 10-15 fm

Lambda gauges

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- **Lorentz invariance not manifest**
 - **Two (continuous) variables:** Energy, 3-momentum
 - **Glueon propagator: two tensor structures**
 - **Temporal:** $D_{00}(k_0, |\vec{k}|)$
 - **Spatial:** $D^{tr}(k_0, |\vec{k}|) = P_{ij} D_{ij}(k_0, |\vec{k}|)$

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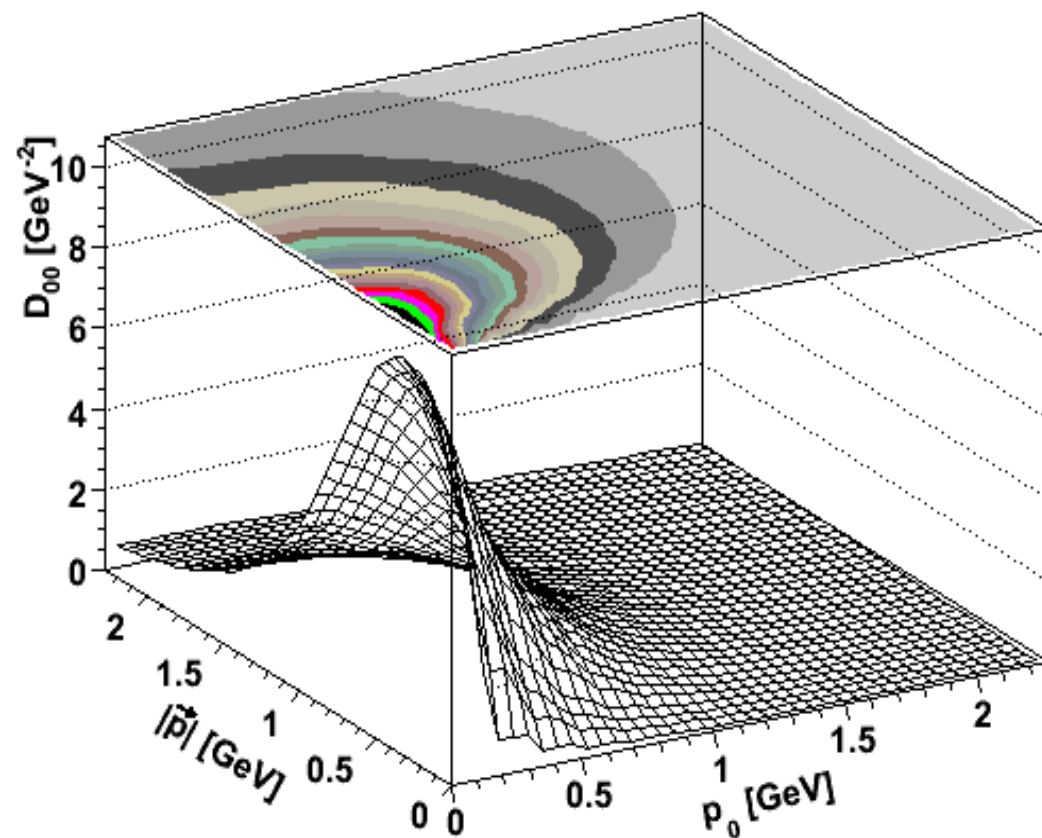
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 - **Transverse gluon propagator vanishing**

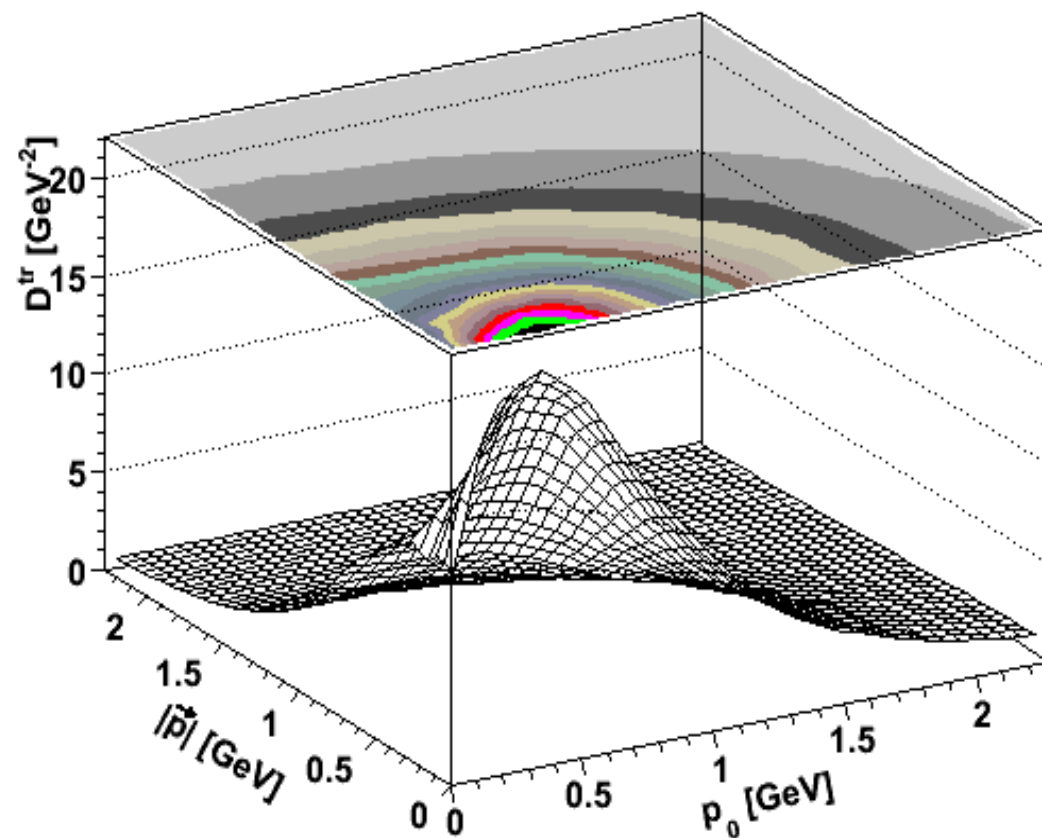
Gluon propagator - Landau

[Lattice 40^3 , $\beta=4.2$, $\lambda=1$:
Cucchieri et al., unpublished]

Temporal gluon propagator D_{00}



Spatial gluon propagator D^{tr}

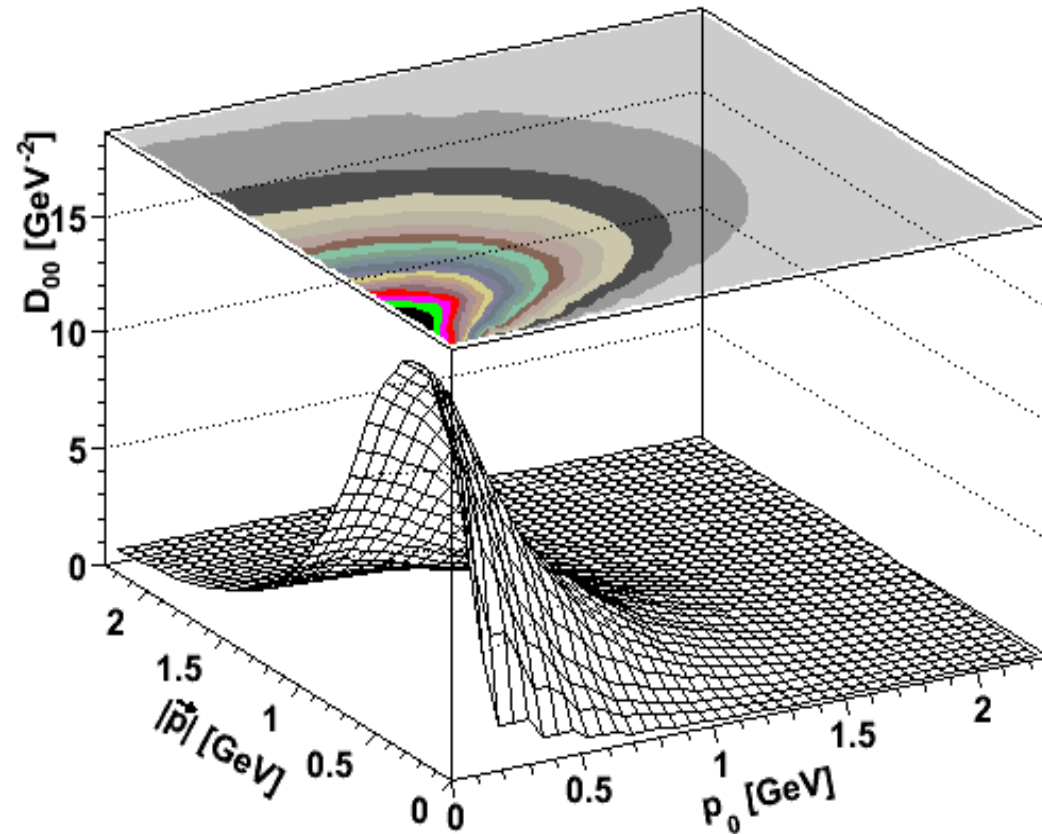


- Different in Landau gauge because the gluon is a vector particle
- Distinct maximum and infrared suppressed

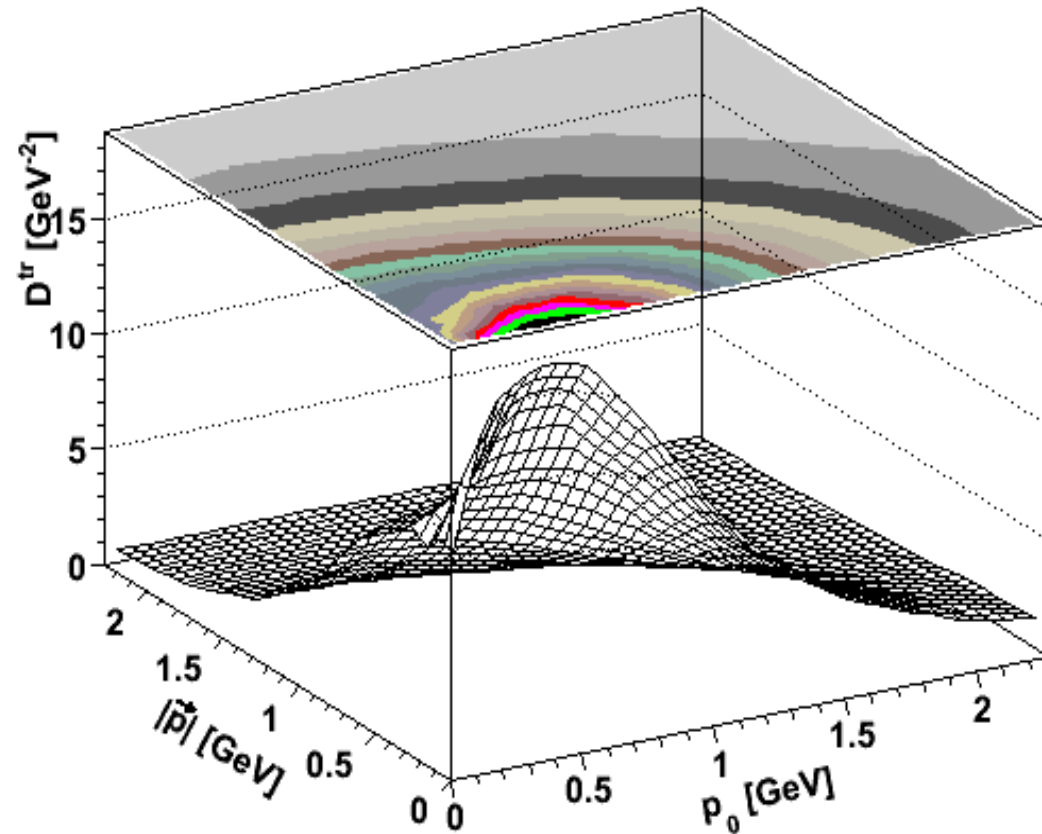
Gluon propagator

[Lattice 40^3 , $\beta=4.2$, $\lambda=1/2$:
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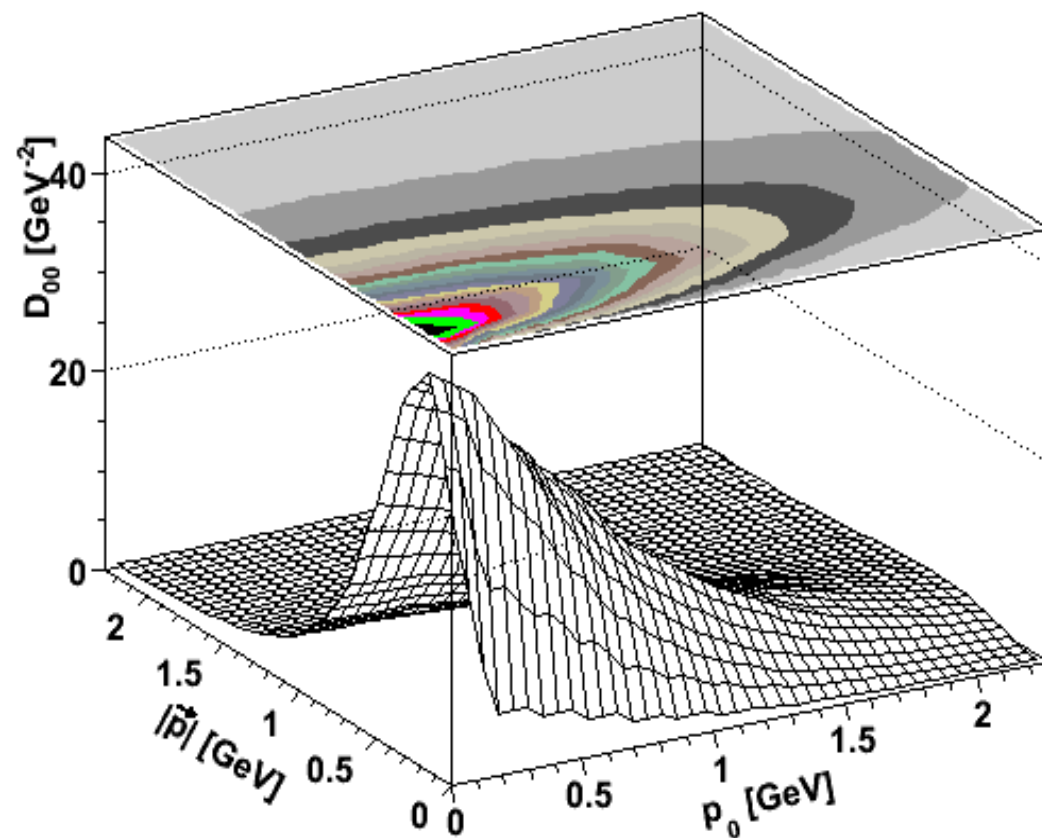


• No visible changes

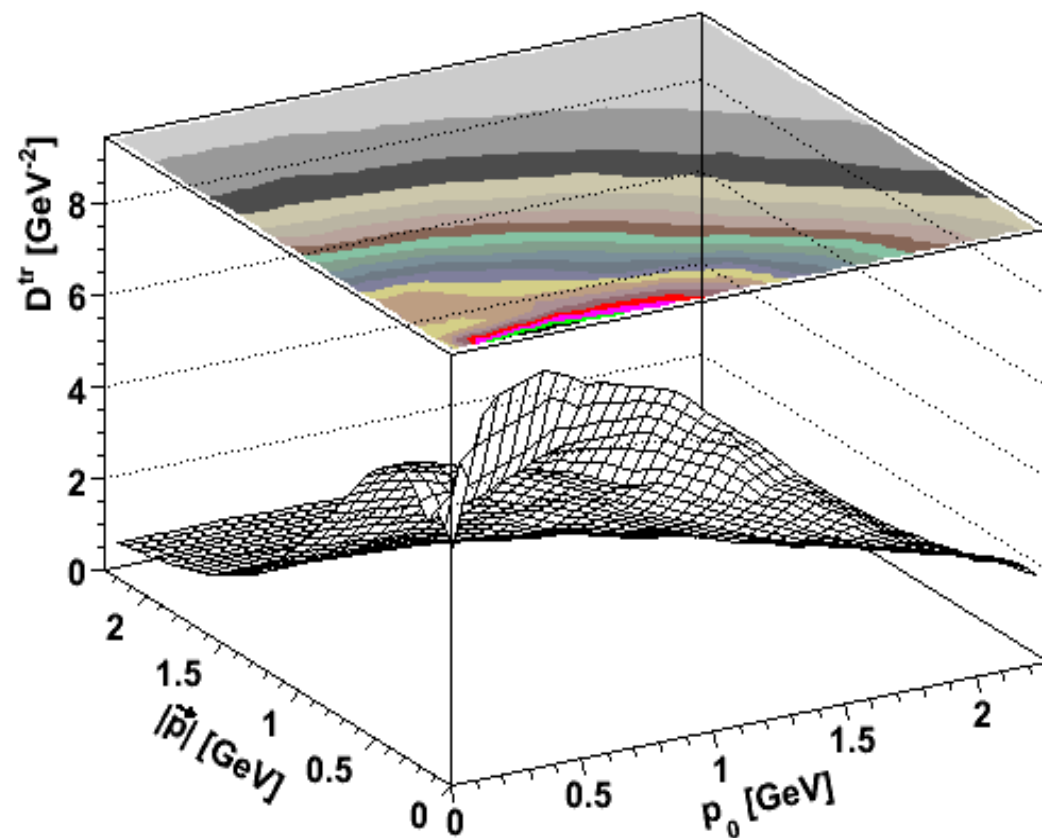
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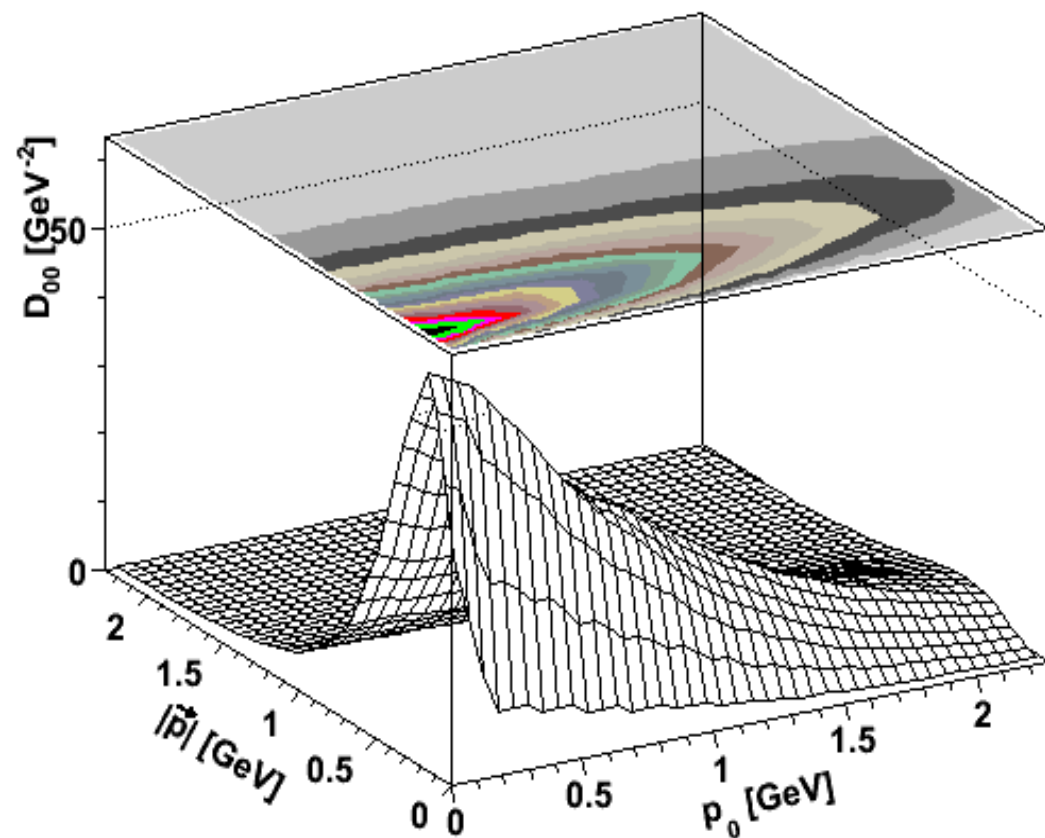


- Temporal: Maximum becomes more pronounced, more infrared
- Spatial: Spatial maximum at larger momenta

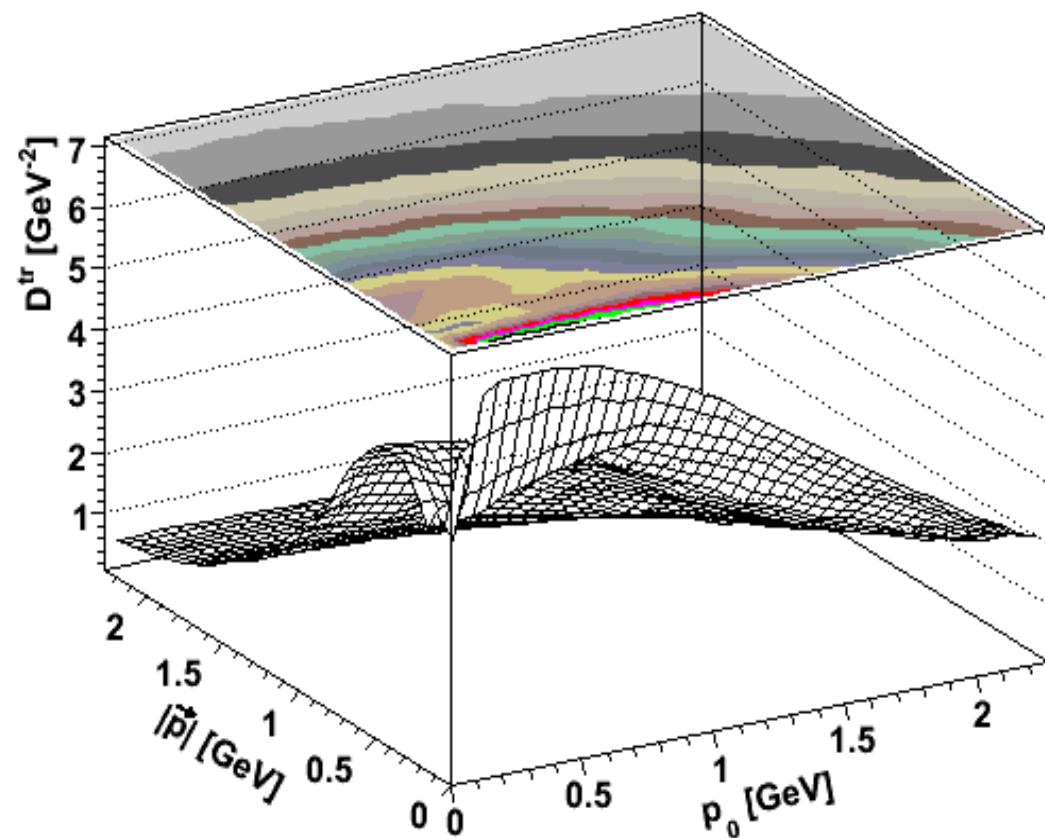
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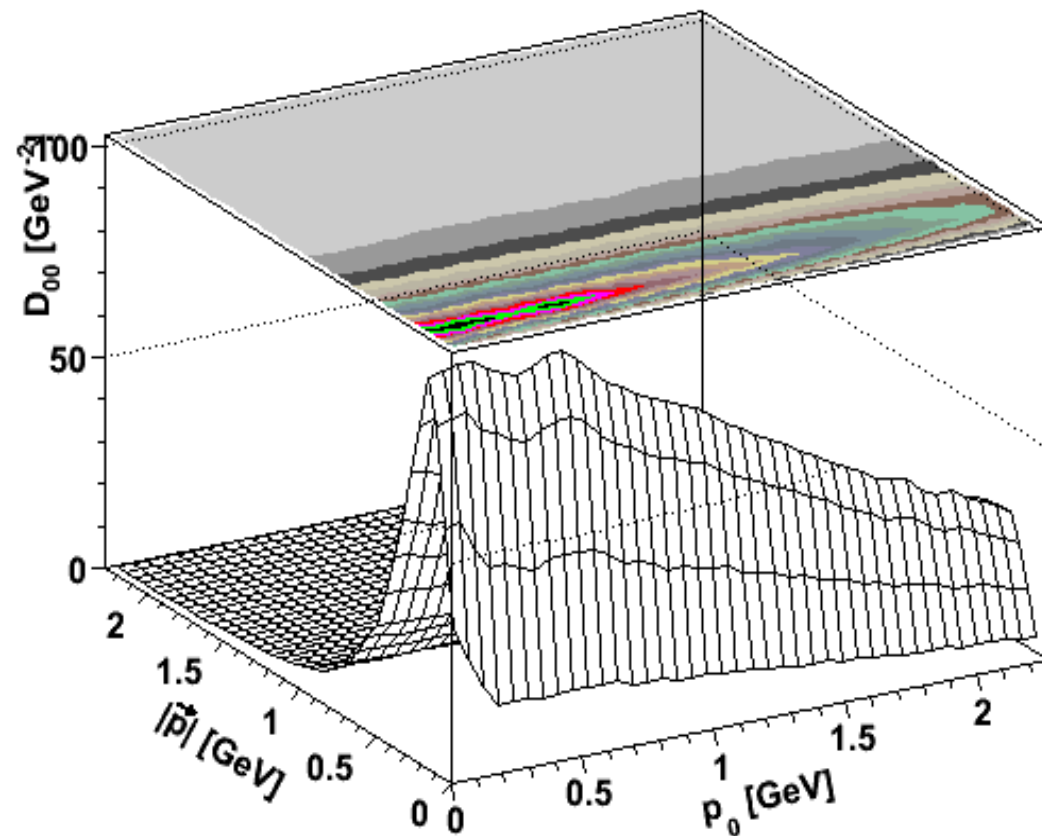


- Resolution of maxima more complicated
- Larger volumes required

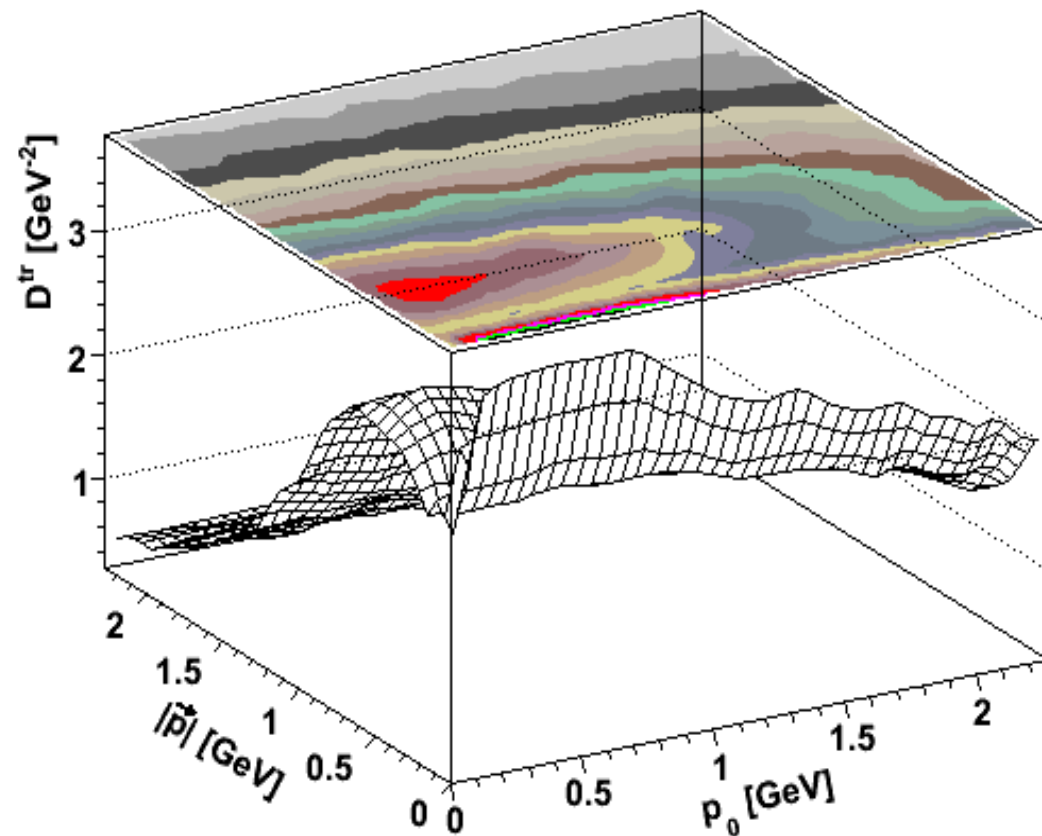
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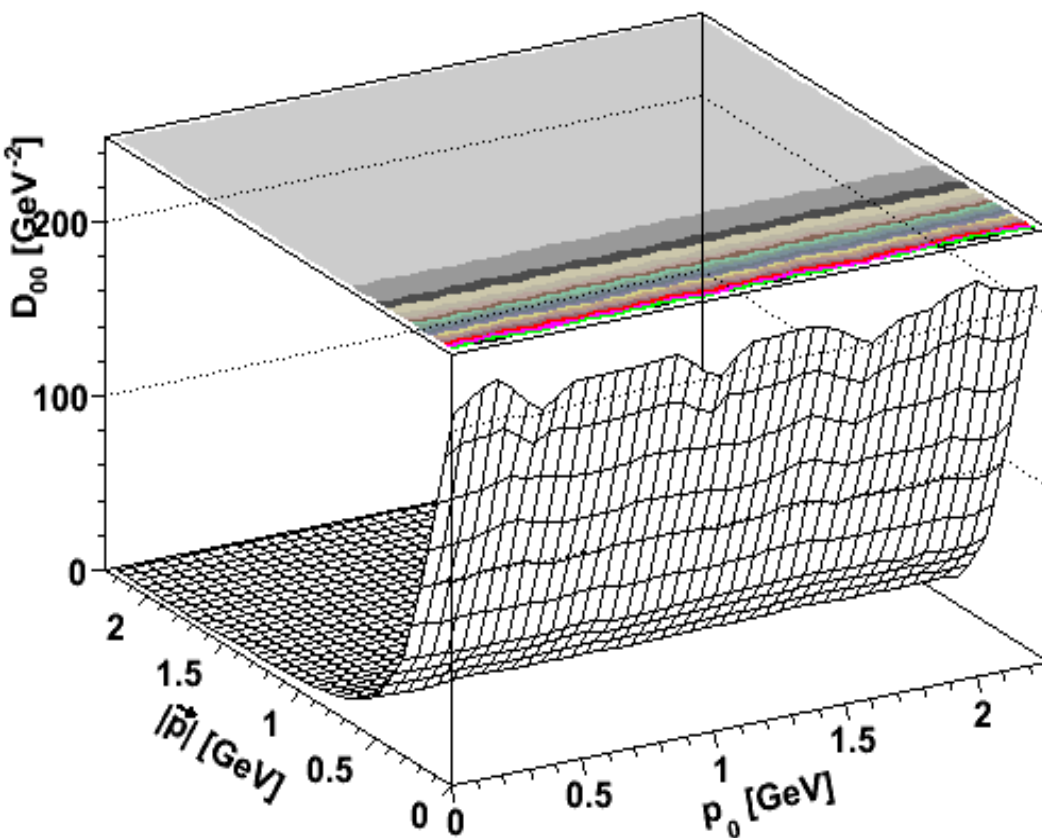


- Coulomb-like at large momenta - Landau-like at small momenta
- Separation momentum, decreasing with λ

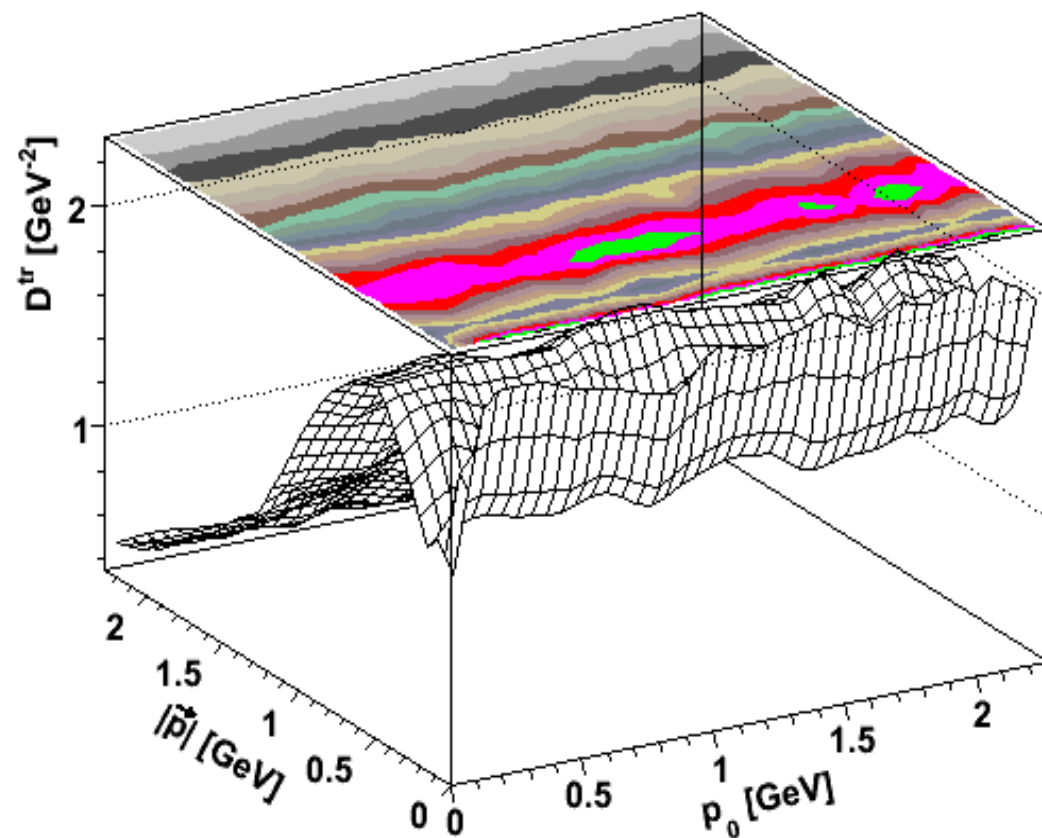
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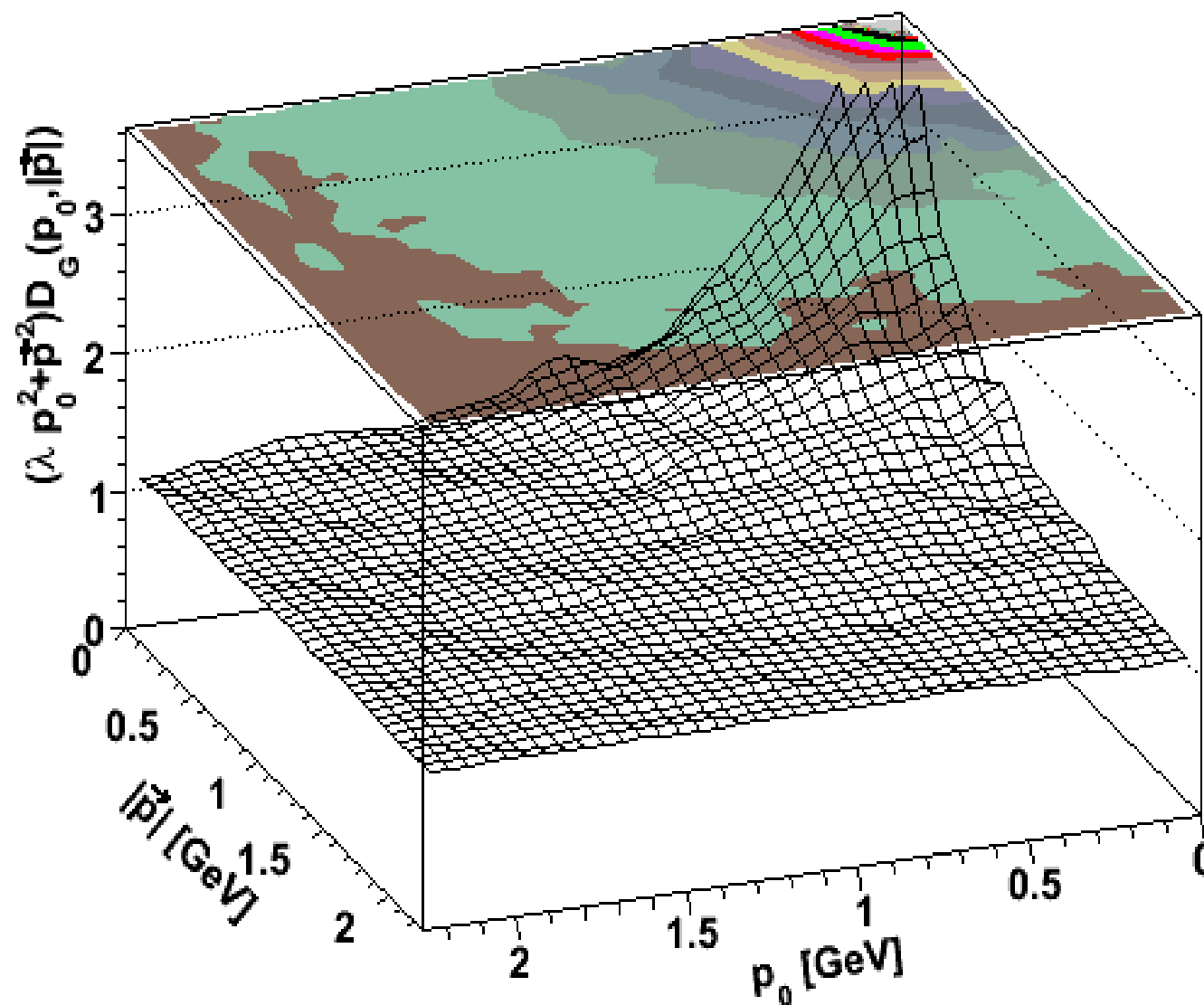
- Temporal propagator infrared divergent - discontinuous change
- Spatial propagator only vanishing at zero energy

Ghost dressing function

[Lattice 40^3 , $\beta=4.2$, $\lambda=1$:
Cucchieri et al., unpublished]

- Scalar particle
 - Isotrope
- Infrared strongly enhanced

Ghost dressing function

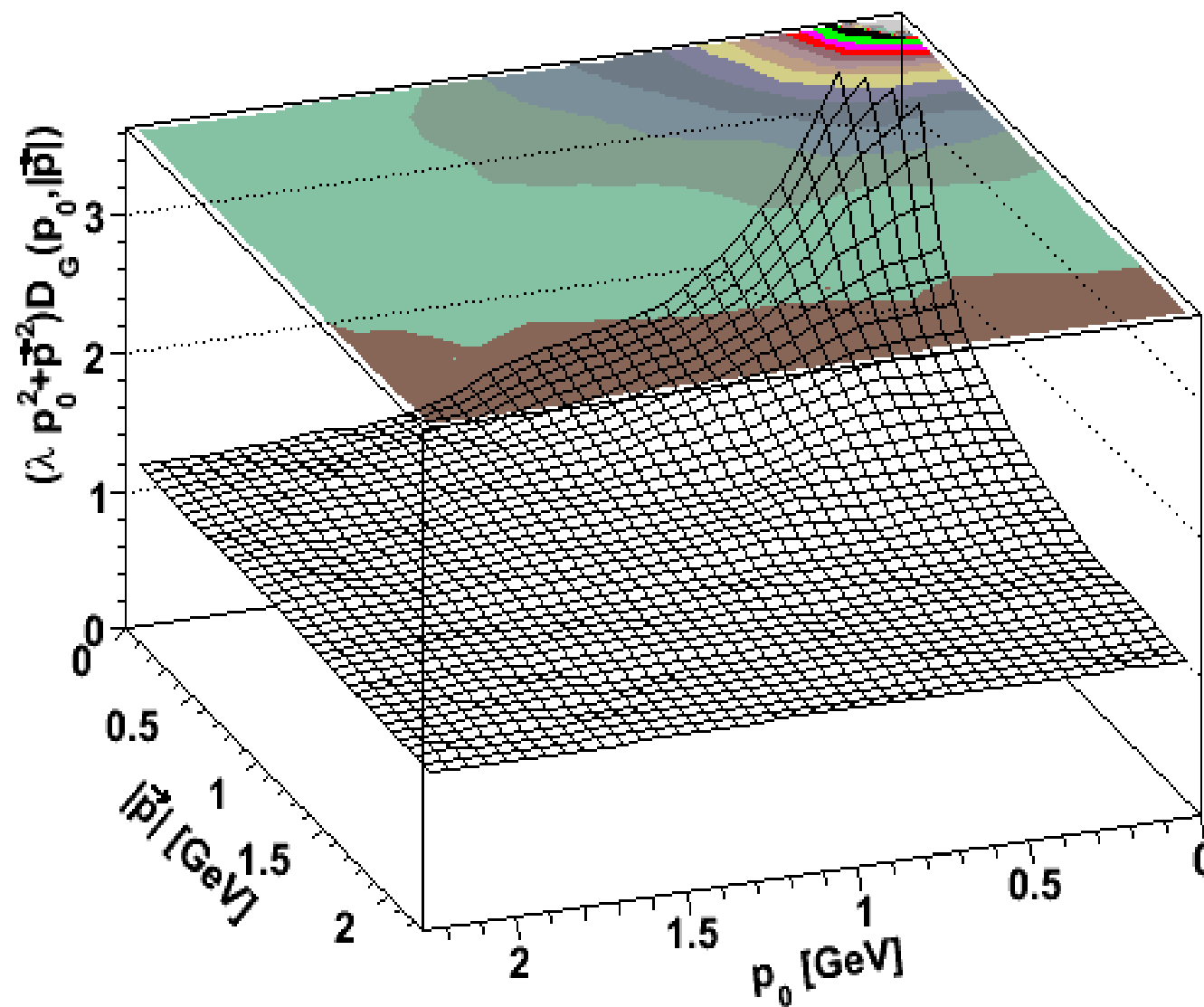


Ghost dressing function

[Lattice 40^3 , $\beta=4.2$, $\lambda=1/2$:
Cucchieri et al., unpublished]

- No visible change

Ghost dressing function

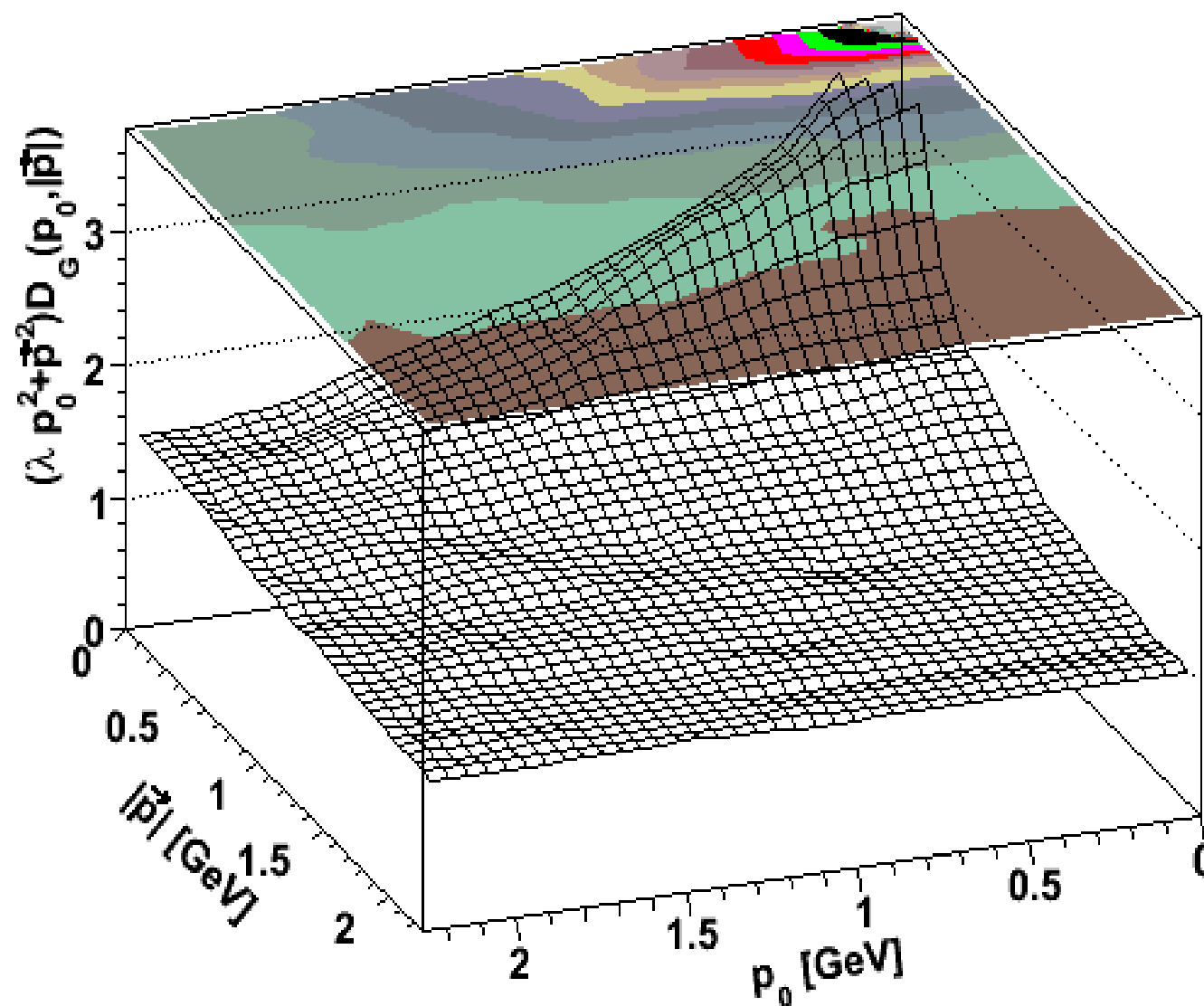


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- Increase at all temporal momenta for infrared spatial momenta

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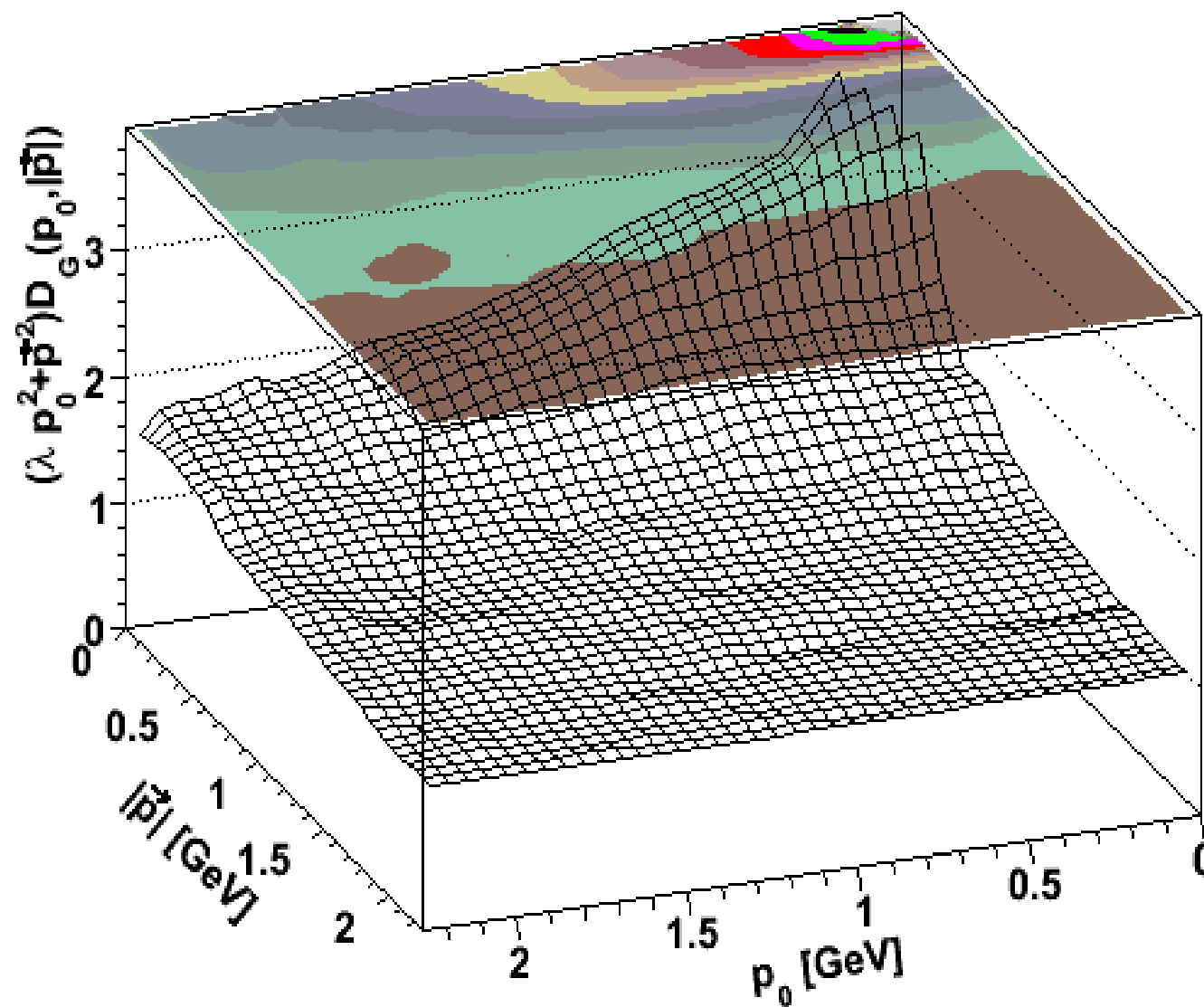


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[Lattice 40^3 , $\beta=4.2$, $\lambda=1/20$:
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Ghost dressing function

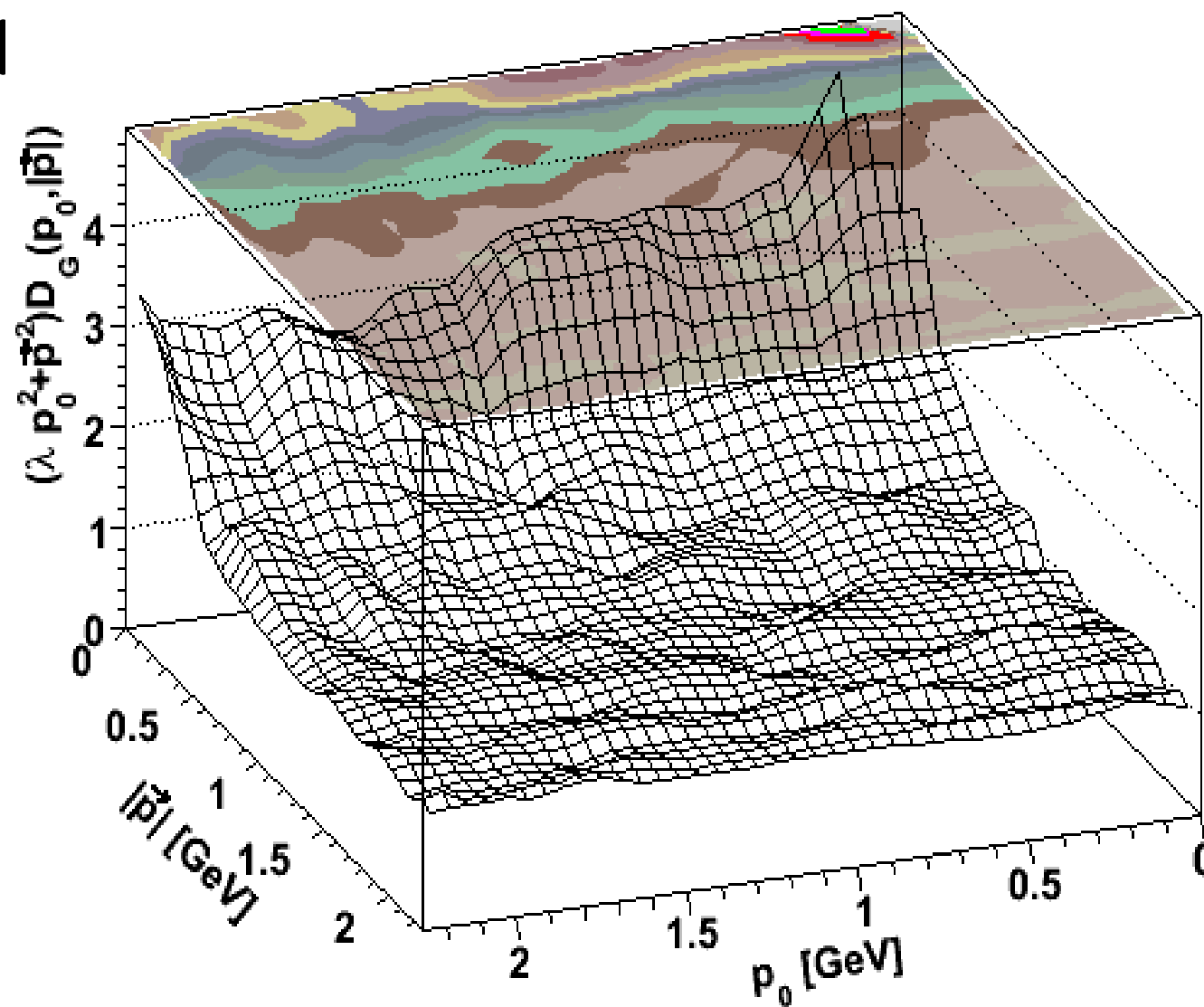


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[Lattice 40^3 , $\beta=4.2$, $\lambda=1/100$:
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Ghost dressing function



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- **Results in 4d similar** [Cucchieri et al., 2007]
 - Smaller volumes - stronger finite volume effects

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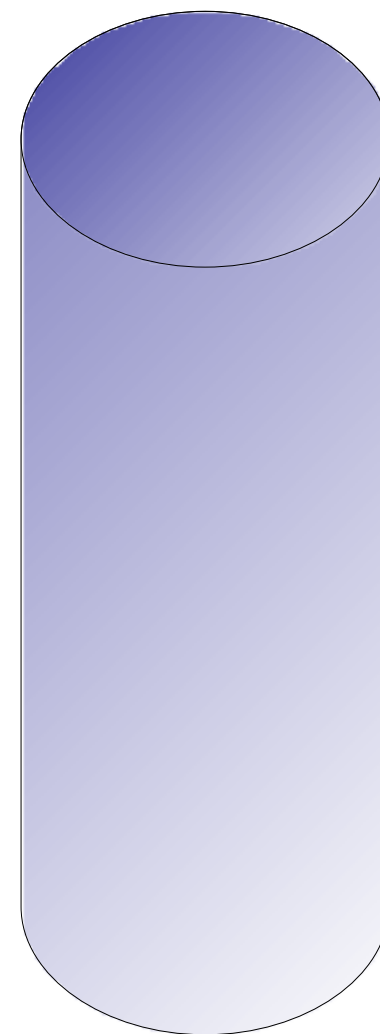
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 - **Determine connection to the source** of the potential

Topological configurations

- **Vortices**

Topological configurations

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 - String-like objects - world surface

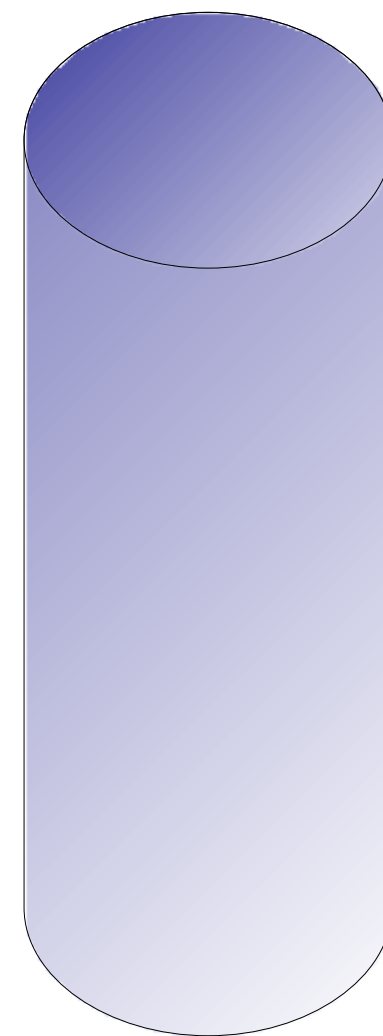
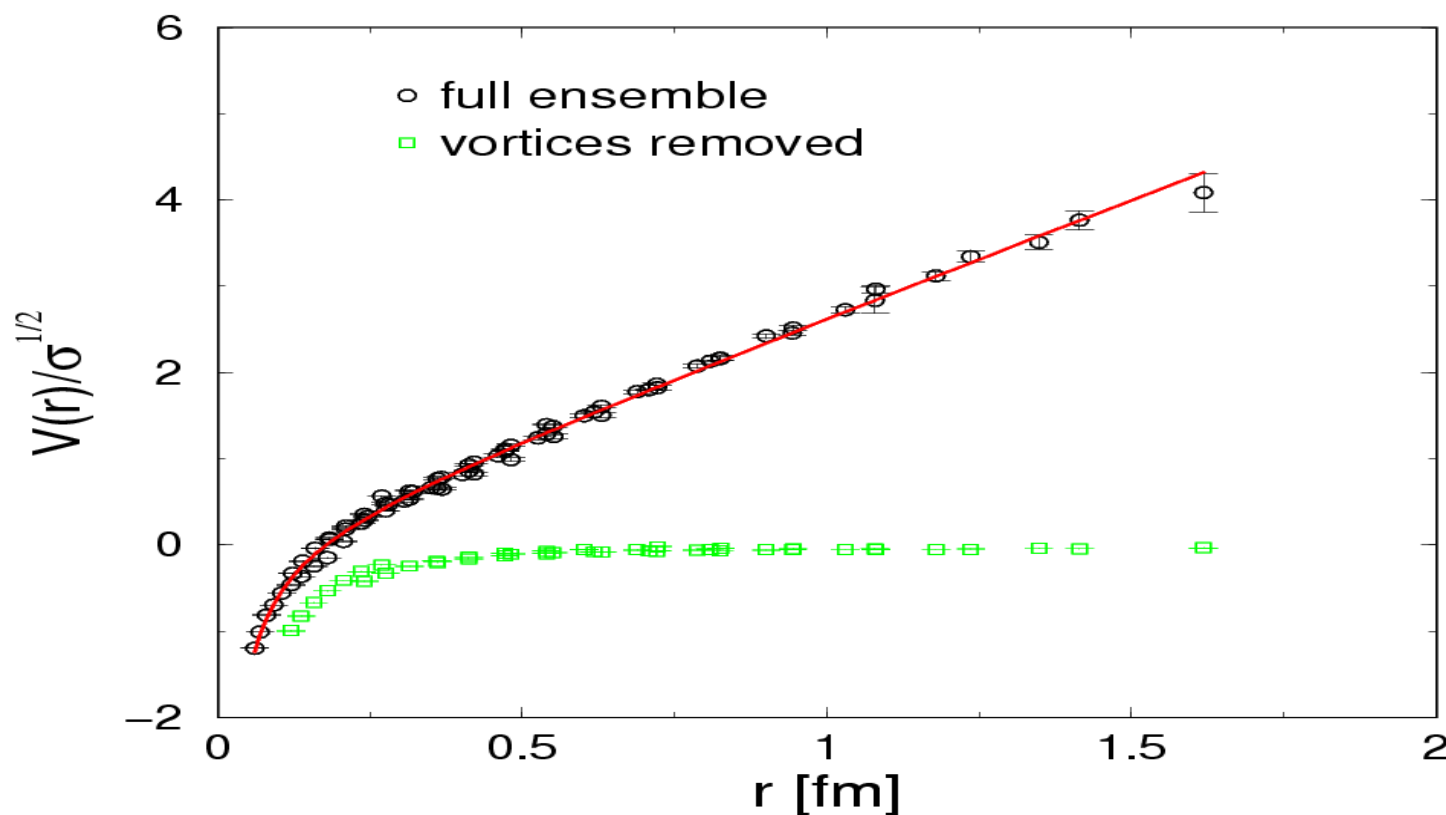


Topological configurations

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- Influence the confinement potential

$SU(2), 12^4$



Topological configurations

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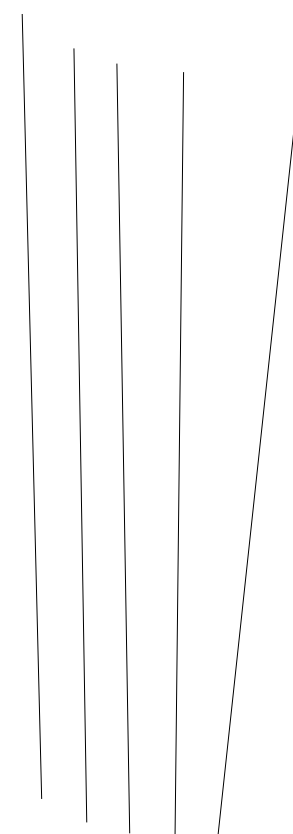
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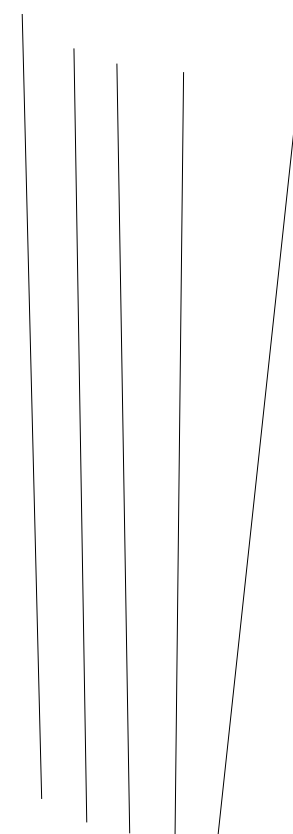
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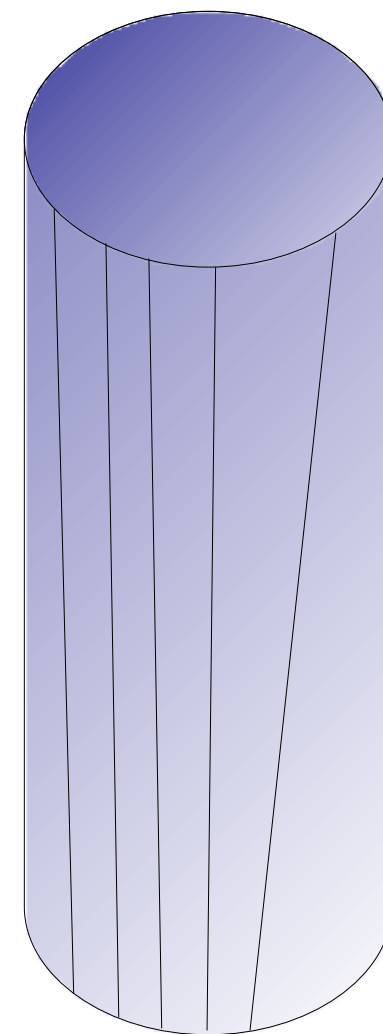
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[Greensite, 2003, Reinhardt, NPB 2002, Boyko et al., NPB 2006]



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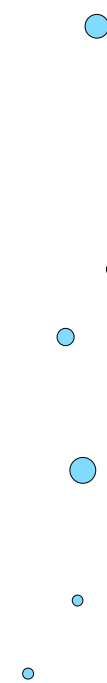
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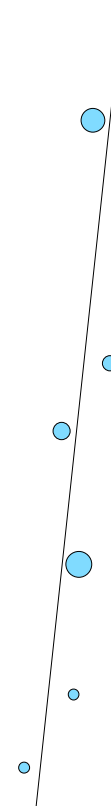
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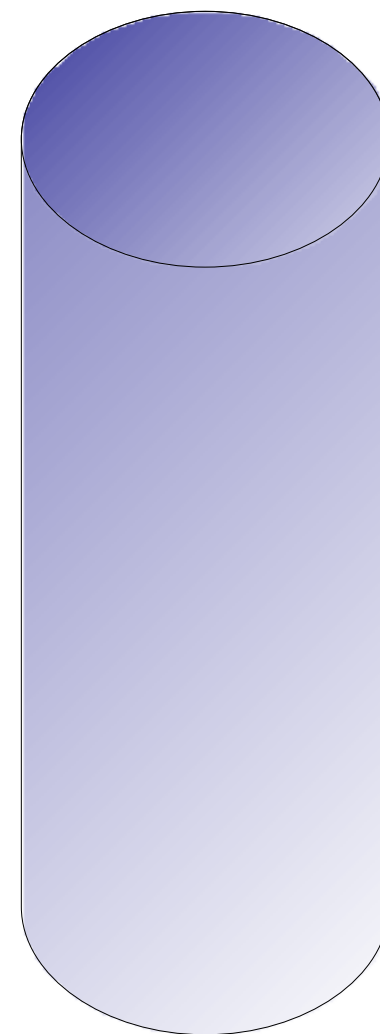
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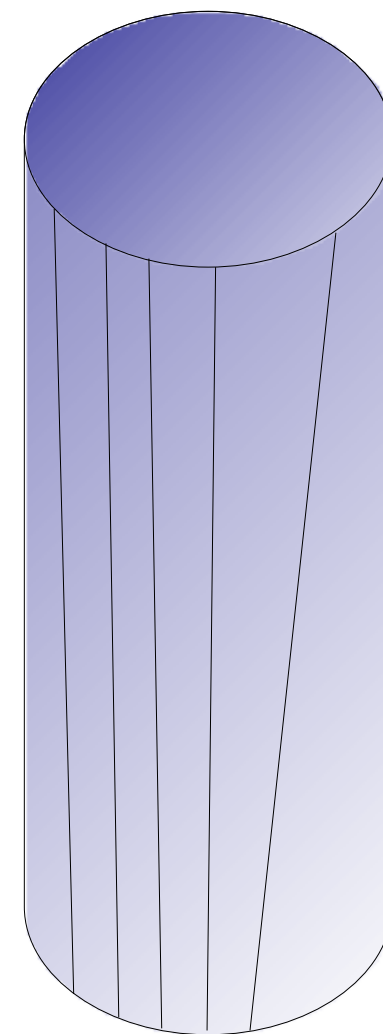
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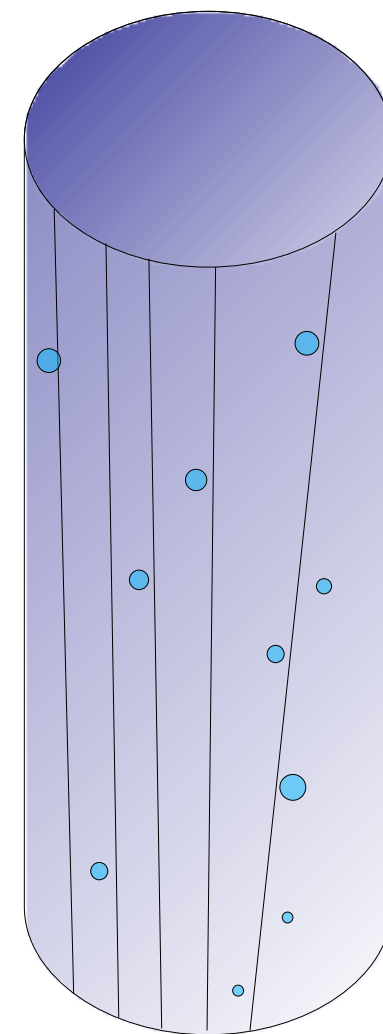
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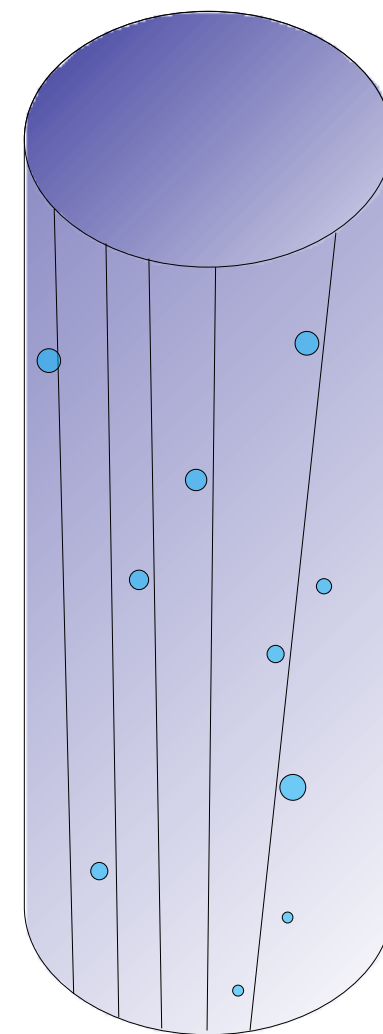
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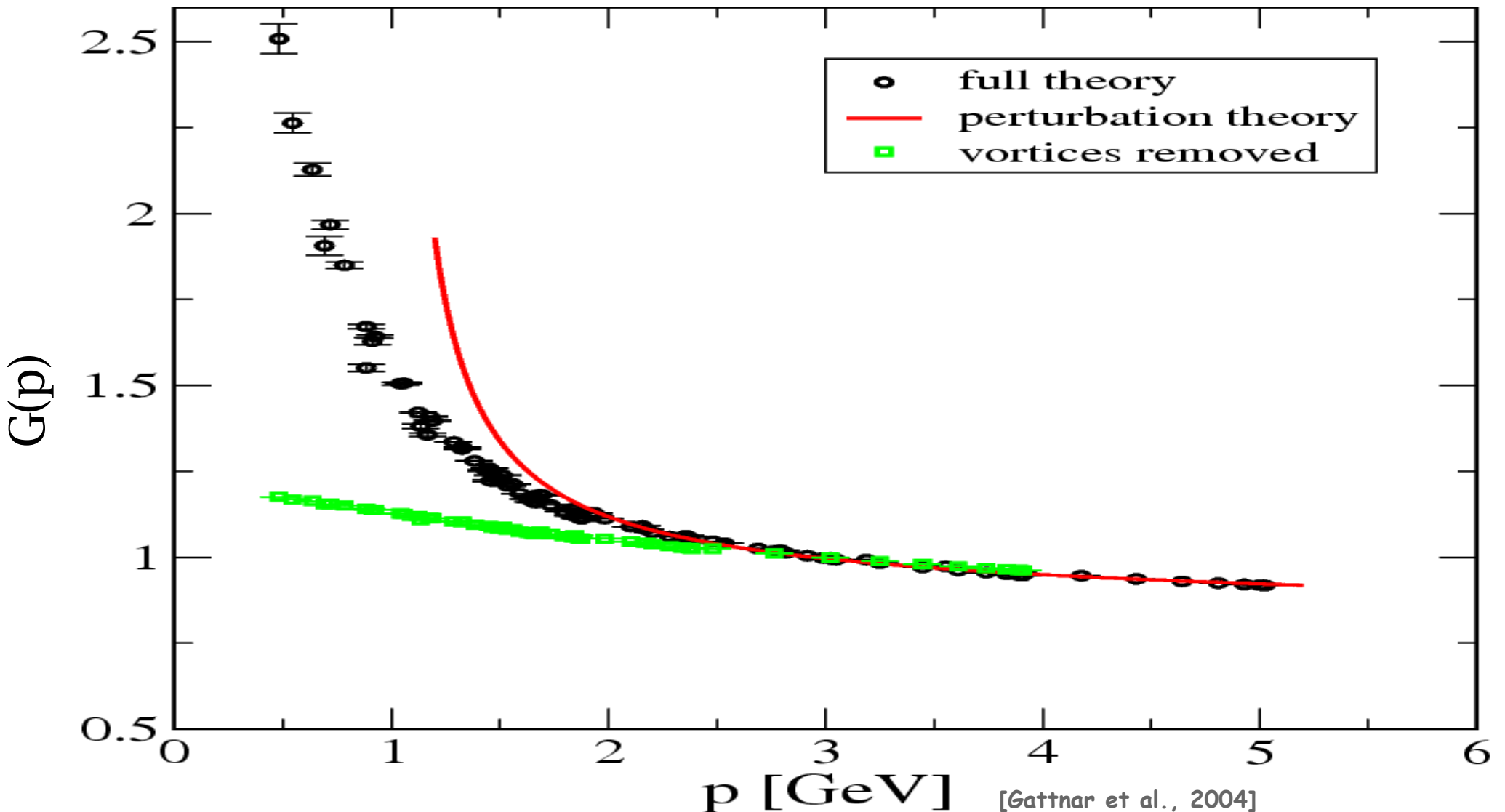
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To answer: Return to the Faddeev-Popov operator

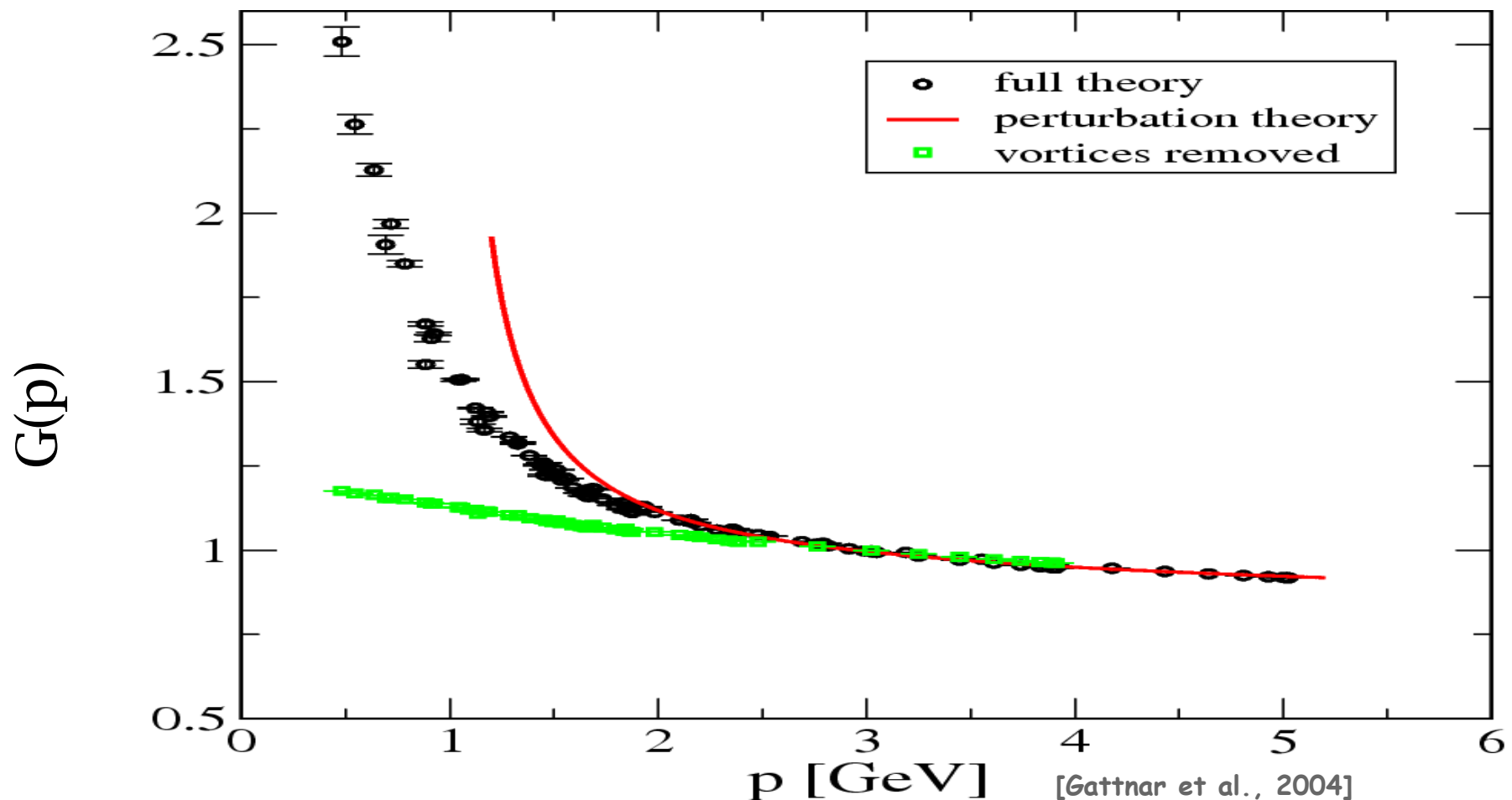
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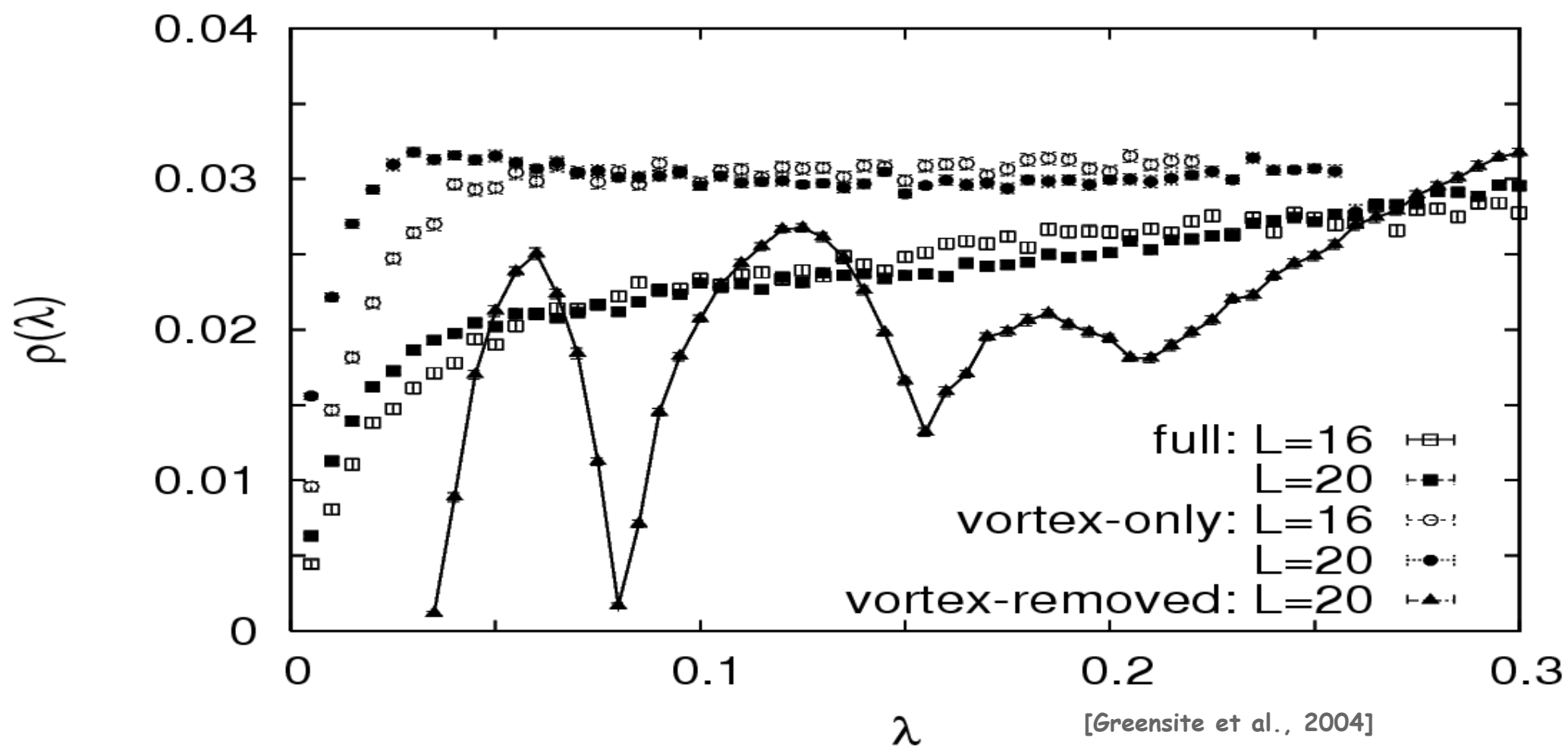
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- Hints to a change of the FP-eigenspectrum



Vortices and the eigenspectrum of the FPO

- Removal of vortices in Coulomb gauge reduces enhancement of near-zero modes

Eigenvalue density, $\beta=2.3$



Analytical approach

- Study analytically the **eigenspectrum** of the **FP-operator** in **topological background fields**
- Use as background fields the field of
- **Instantons** [Maas, EPJC 2006]
- **Monopoles** [Maas, 2006]
- **(Center) Vortices** [Maas, EPJC 2006]

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- General form (for transverse fields)

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- **Results for a more complicated system?**

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- Different gauge groups and more gauges

Collaborations

- A. Cucchieri, T. Mendes (Uni. of São Paulo, Brazil)
 - Lattice results on interpolating gauges and Landau gauge Faddeev-Popov operator
 - hep-lat/0701011, hep-lat/0610123, hep-lat/0610006, hep-lat/0605011
- C. S. Fischer (Darmstadt Uni. of Technology), J. M. Pawłowski (Uni. of Heidelberg), L. von Smekal (Uni. of Adelaide, Australia)
 - Finite volume DSEs
 - hep-ph/0701050
- Supported by the DFG and VEGA