Fundamental and Adjoint scalar fields coupled to Yang-Mills theory

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11.3.2010

Confinement

experimental facts:

- absence of free quarks in Nature $\Rightarrow$ quark confinement
- no free gluons $\Rightarrow$ colour confinement

no isolated particles in Nature with non-vanishing colour charge!

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theory:

- What could be responsible for this behaviour?

$\Rightarrow$ interaction between quarks

Wilson loop

Wegner-Wilson loop $W(\vec{r}, t)$:

- defined as the trace product of gauge variables along a closed oriented contour
- complex quantity, but real expectation value
- possible access to quark interaction
- connection between Wilson loop and the static potential between colour sources:

$$W(\vec{r}, t) = e^{Vt}$$

Static Potential

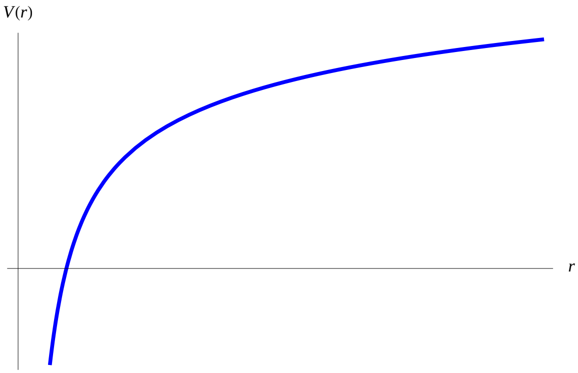
$$W(\vec{r}, t) = e^{Vt}$$

- ground state contribution $E_0(\vec{r})$ can be identified as static potential $V(\vec{r})$
- dominates in limit of large t

properties:

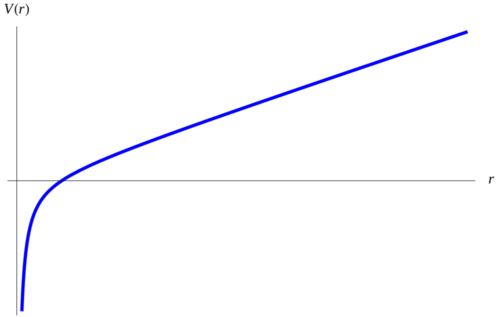
- potential cannot rise faster than linearly
- $V''(\vec{r}) \geq 0 \quad \Rightarrow$ convexity
- $V'(\vec{r}) \geq 0 \quad \Rightarrow$ monotonically rising

Static Potential



Confinement in SU(N)

fundamental:

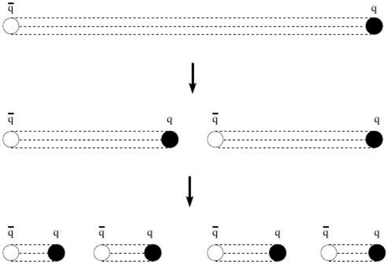


- linear rise of potential

String picture

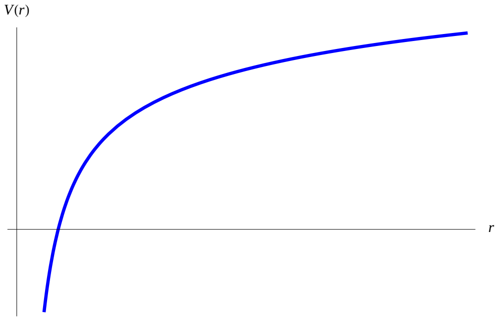
- electric flux between two colour sources is squeezed into thin, effectively 1-dim flux tube
- string breaking at large distances via particle-anti-particle pair creation

[J. Greensite '08]



Confinement in SU(N)

adjoint:



- string breaking

G(2)

Why G(2)?

- strcutural difference compared to SU(N) because of trivial center

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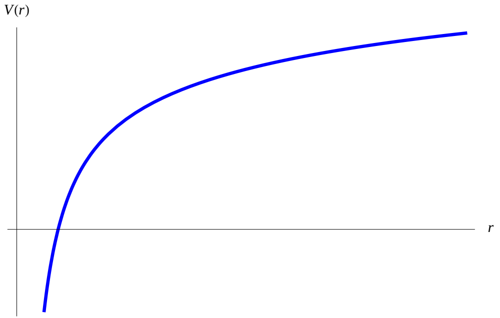
properties of G(2):

- simplest among exceptional Lie groups
- it's own universal covering group
- trivial center
- adjoint representation: 14-dim
- fundamental represenatation: 7-dim

Confinement in G(2)

fundamental and adjoint:



- string breaking

Question

What is responsible for confinement?

possible answer: interaction between quarks $\Rightarrow$ static potential

- potential shows representation dependence
- also gauge group dependent

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Can we see these differences also on the level of correlation functions?

Landau gauge Yang-Mills theory including scalar fields

Lagrangian:

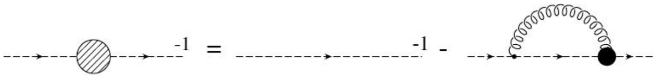
$$\mathcal{L} = \frac{1}{2} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{1}{2\zeta} \left(\partial_\mu A^a_\mu\right)^2 + \bar{c}^a \partial_\mu D^{ab}_\mu c^b + \left(D_{\mu,ij} \Phi^*_j\right) \left(D_{\mu,ik} \Phi_k\right)$$

$$-m^2 \Phi^*_i \Phi_i - \frac{h}{4!} \left(\Phi^*_i \Phi_i\right)^2$$

- $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu$
- $D^{ab}_\mu = \delta^{ab} \partial_\mu + g f^{abc} A^c_\mu$
- $D_{\mu,ij} = \delta_{ij} \partial_\mu - ig \left(\frac{t^a}{2}\right)_{ij} A^a_\mu$

Dyson-Schwinger equations

- nonperturbative method
- equations of motions for Green's functions
- derived with Mathematica package DoDSE [R. Alkofer, M.Q. Huber, K. Schwenzer '09]
- example: ghost propagator DSE



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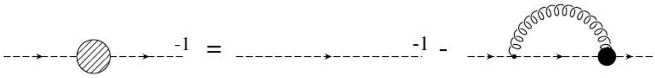

$\Rightarrow$ no difference in DSEs!

Infrared Powercounting Technique

- propagators parametrized by: $\Delta_{ij}(p) = P_{ij} \frac{Z(p^2)}{p^2}$
- power law ansatz for dressing function:

$$Z^{IR}(p^2) = a \cdot (p^2)^{\alpha}$$

- formula for infrared exponent of arbitrary diagram
- work directly on the level of the IREs

Infrared Powercounting Technique

- yields constraints for the infrared exponents of the diagrams
- important tool for identifying leading contributions in the DSEs

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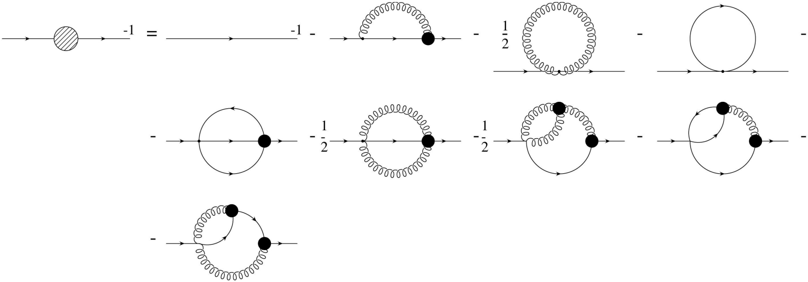
$\Rightarrow$ no difference between fundamental and adjoint scalar fields on this level!!

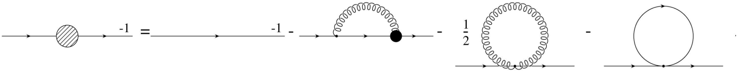
One-loop truncation

full scalar propagator DSE:



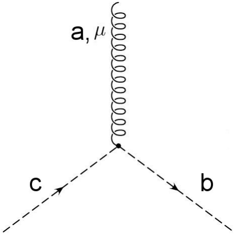
One-loop truncation

one-loop truncated scalar propagator DSE:



Colour structure

- colour structure of tree-level vertices is known from DSEs
- example: ghost-gluon vertex



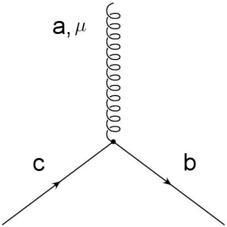
$$\sim igf^{abc}p^{\mu}$$

for calculation only colour structure relevant:

$$\sim f^{abc}$$

Colour structure

- colour structure of tree-level vertices can be calculated from DSEs
- different according to representation:



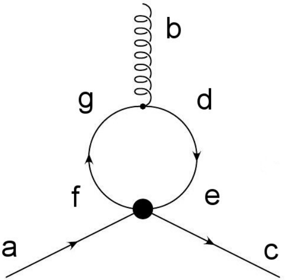
$\sim ig(t^a)_{ij}(q-k)_\mu$ fundamental
$\sim igf^{abc}(q-k)_\mu$ adjoint

for calculation:
$\sim (t^a)_{ij}$ fundamental
$\sim f^{abc}$ adjoint

- use them to calculate the coefficient for each appearing diagram

Example: swordfish diagram

scalars in fundamental representation:

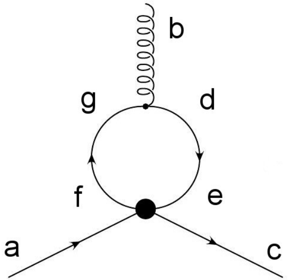


$$\sim \ (\delta_{ec}\delta_{af} + \delta_{fe}\delta_{ac}) \times \left(t^b\right)_{dg} \times (\delta_{de}\delta_{fg}) = ... =$$

$$= \left(t^b\right)_{ac}$$

Example: swordfish diagram

scalars in adjoint representation:

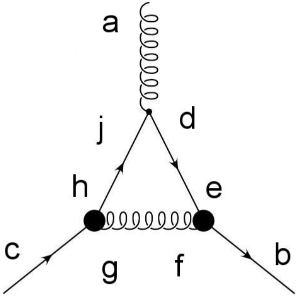


$$\sim \quad (\delta_{af}\delta_{ec} + \delta_{ae}\delta_{fc} + \delta_{ac}\delta_{fe}) \quad \times$$
$$\left(f^{bdg}\right) \times (\delta_{de}\delta_{fg}) = ... =$$

$$= \left(f^{bca} - f^{bca}\right) = 0$$

Example: difference between SU(N) and G(2)

scalars in fundamental representation:



$$\sim \left(C_F - \frac{1}{2}C_A\right)(t^a)_{cb}$$

- SU(2): $-\frac{1}{4}$
- SU(3): $-\frac{1}{6}$
- G(2): 0

4-point functions

- 3-point functions calculated by using identities for group invariants
- more difficult for 4-point functions:
  - construct an orthogonal basis so that results have a simple form
  - project colour structure on basis

Summary

Question: What is responsible for confinement?

possible answer: interaction between quarks $\Rightarrow$ static potential

- potential shows representation dependence
- also gauge group dependent

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Results and Outlook

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- difference in colour structure

work to be done:

- finishing calculations

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outlook:

- Do we have to look deeper?