On the Faddeev-Popov operator eigenspectrum in topological background fields

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Overview

- Confinement

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  - Quarks and topological excitations

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  - Gluons and the Faddeev-Popov operator

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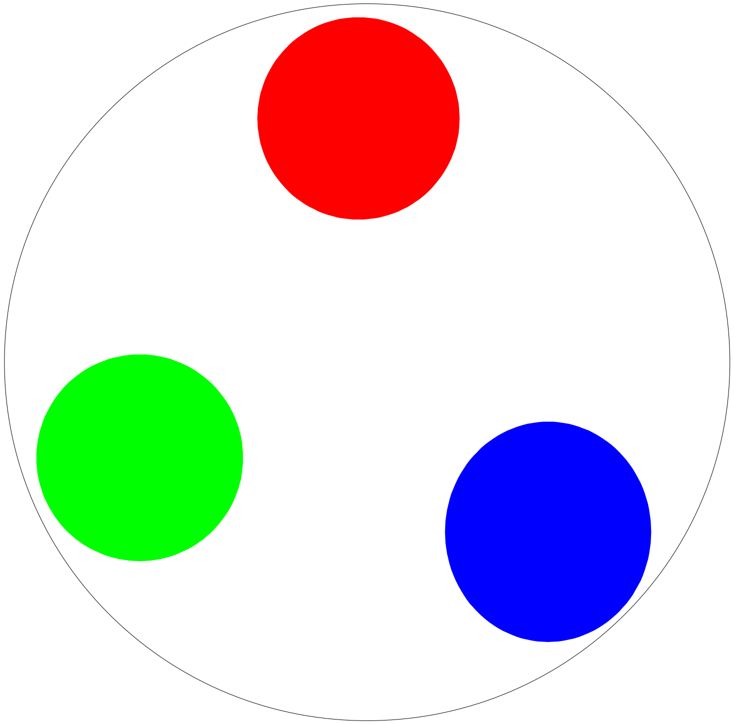
- Eigenspectrum of the Faddeev-Popov operator in topological fields
- Summary

[hep-th/0511307]

QCD degrees of freedom

- Baryon: 3 (valence-)quarks



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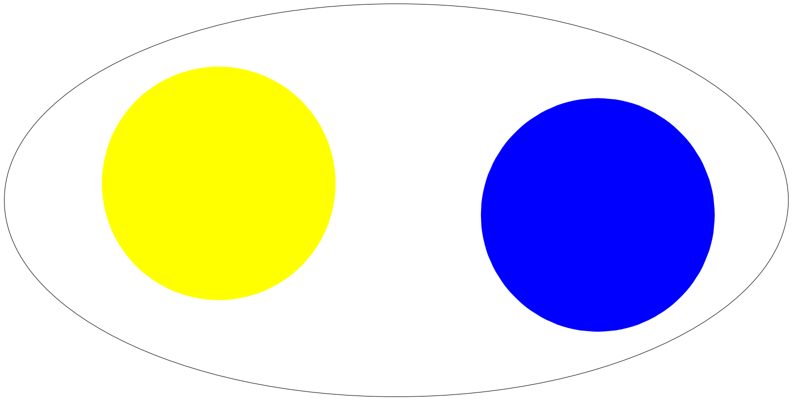

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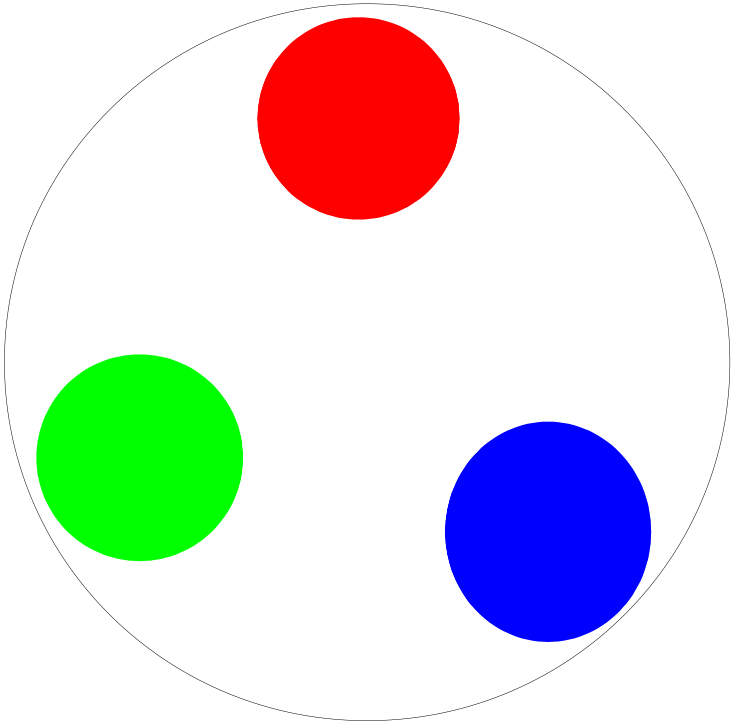
- Meson: 1 quark and 1 antiquark



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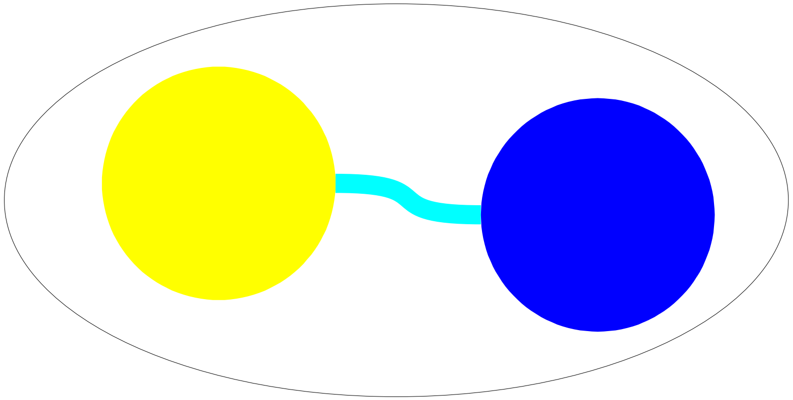

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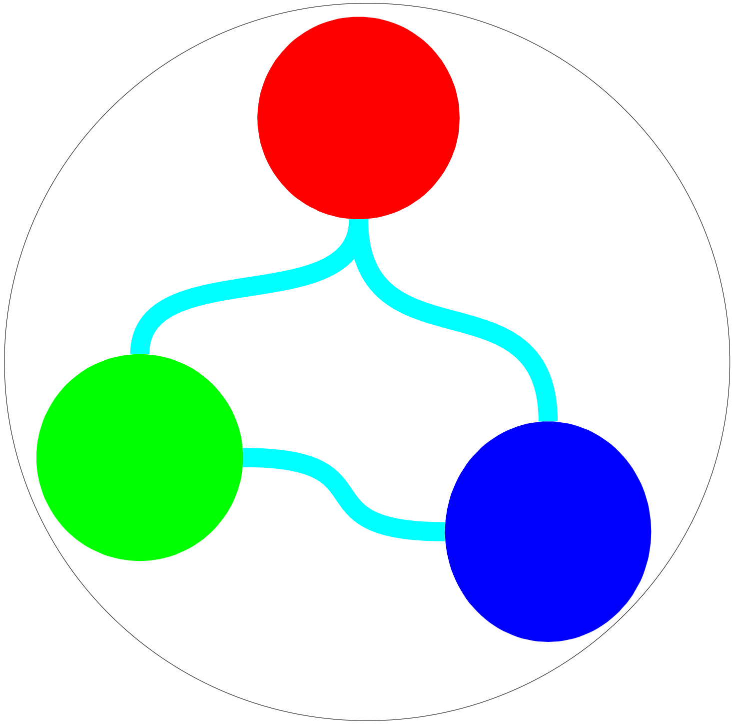

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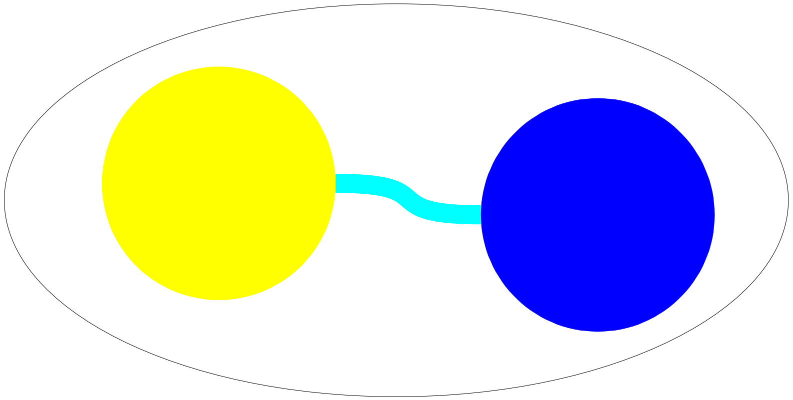

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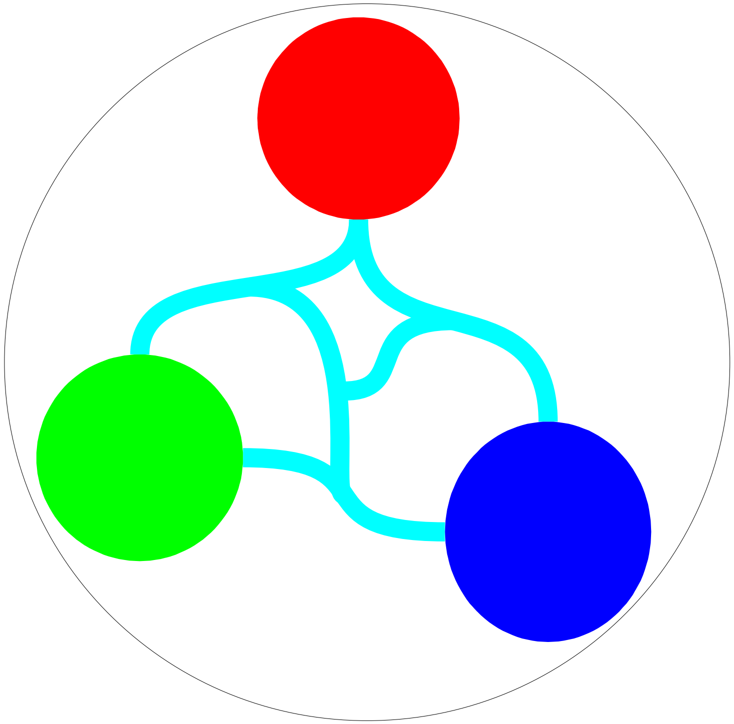

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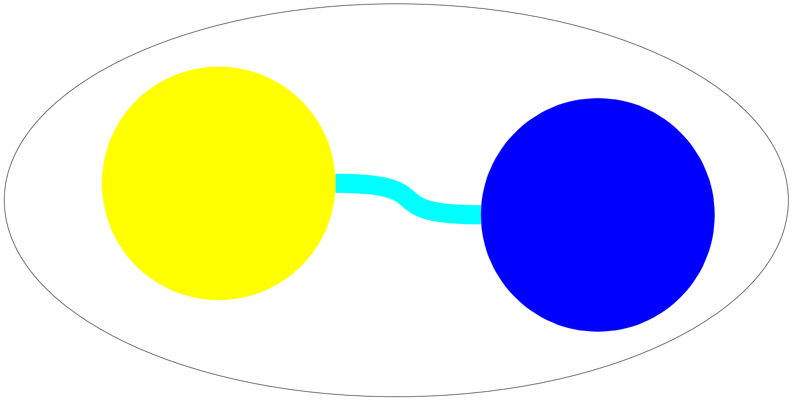

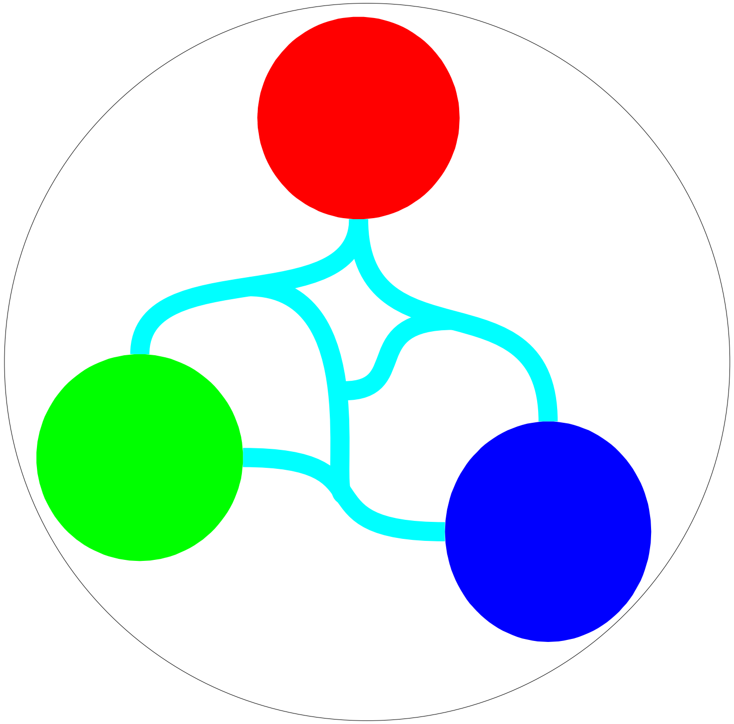

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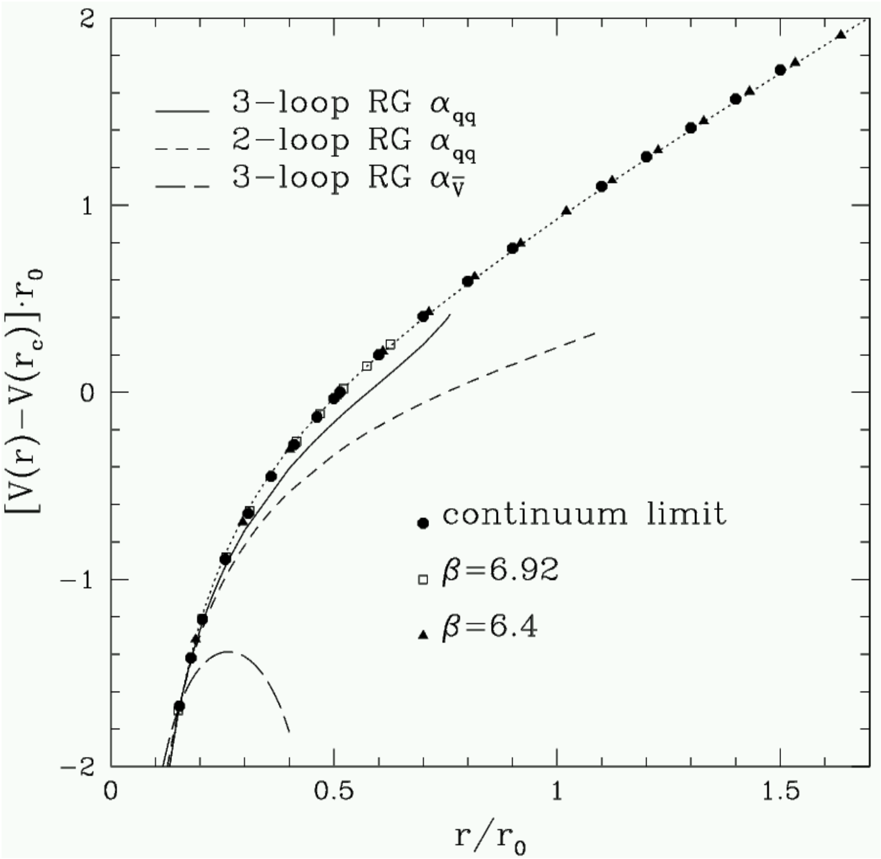
- Meson: 1 quark and 1 antiquark
- Interactions mediated by gluons
- Gluons are also charged
- No free quarks and gluons: Confinement
  - Measured to very high precision for quarks

Confinement of quarks

- Inter-quark potential confining



[Figure from Sommer et al., 2001]

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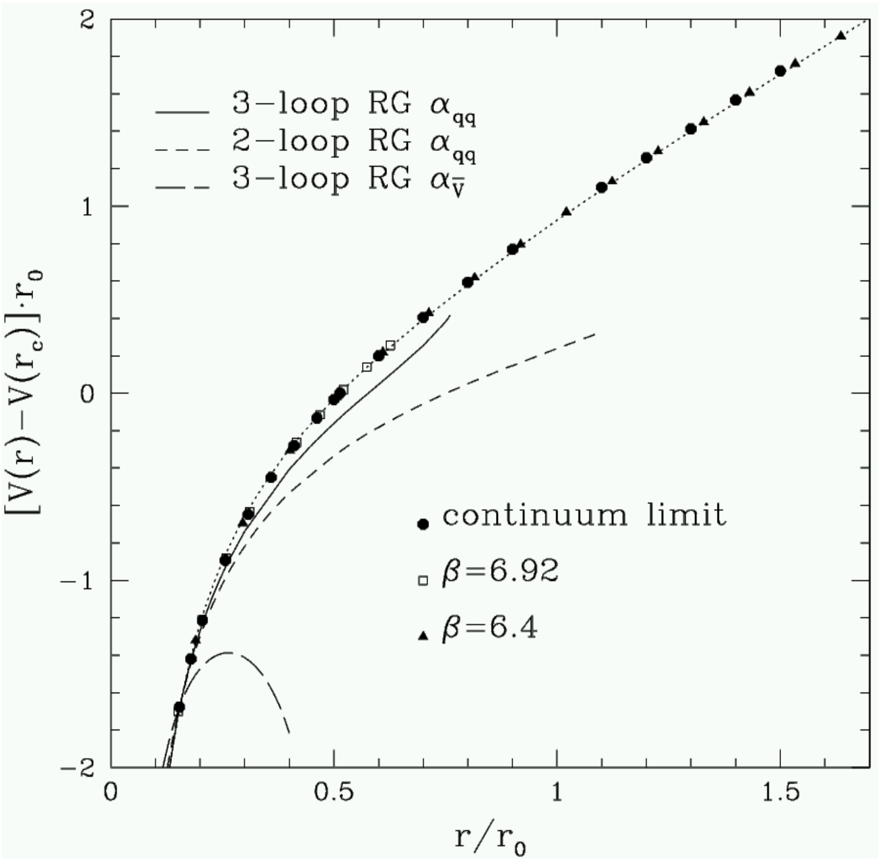

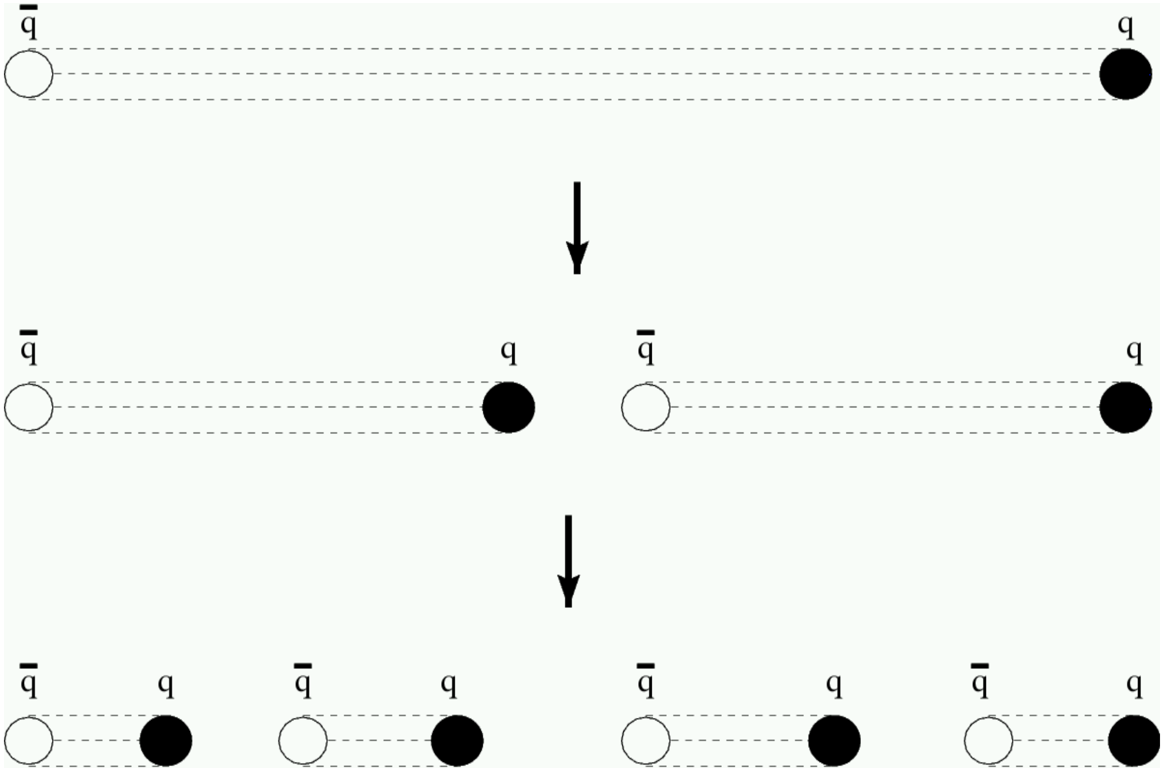

[Figures from Sommer et al., 2001 (left) and from Greensite, 2003 (right)]

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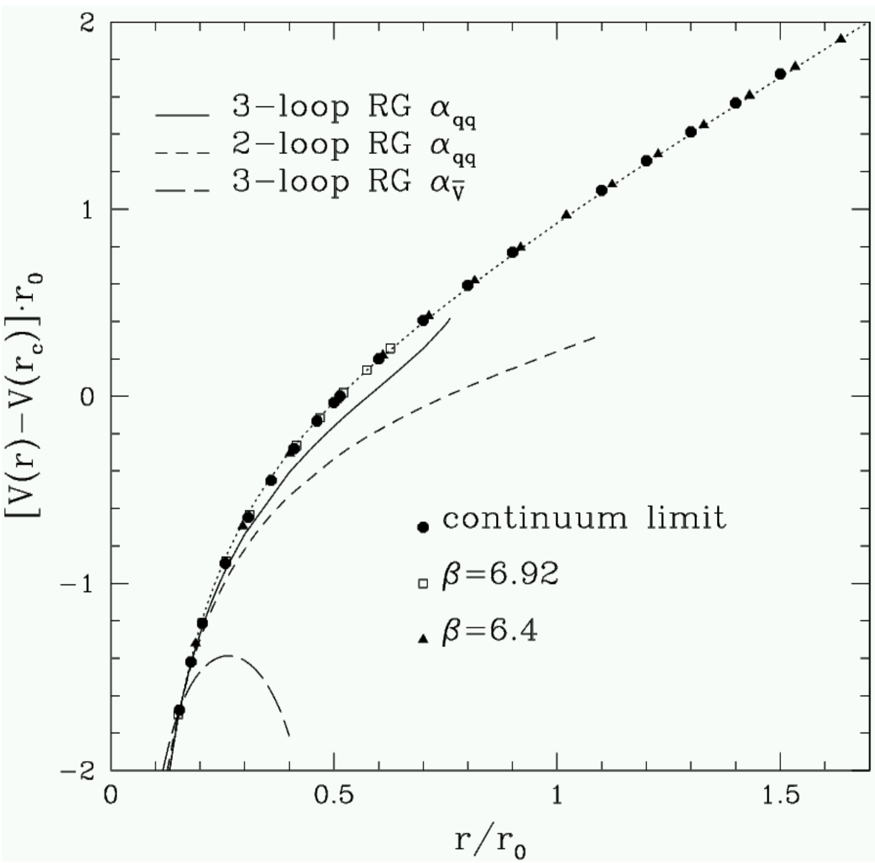
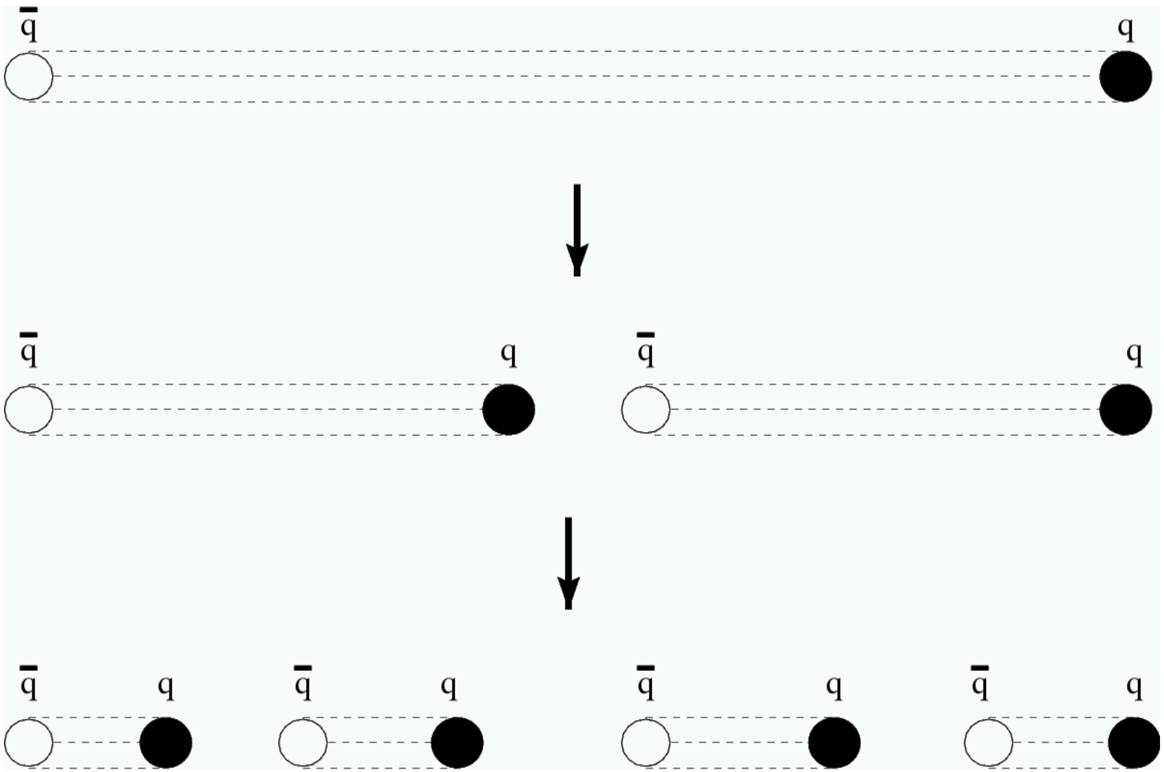
- Origin of linear potential?





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  - Monopoles
  - Merons
  - Vortices
  - ...
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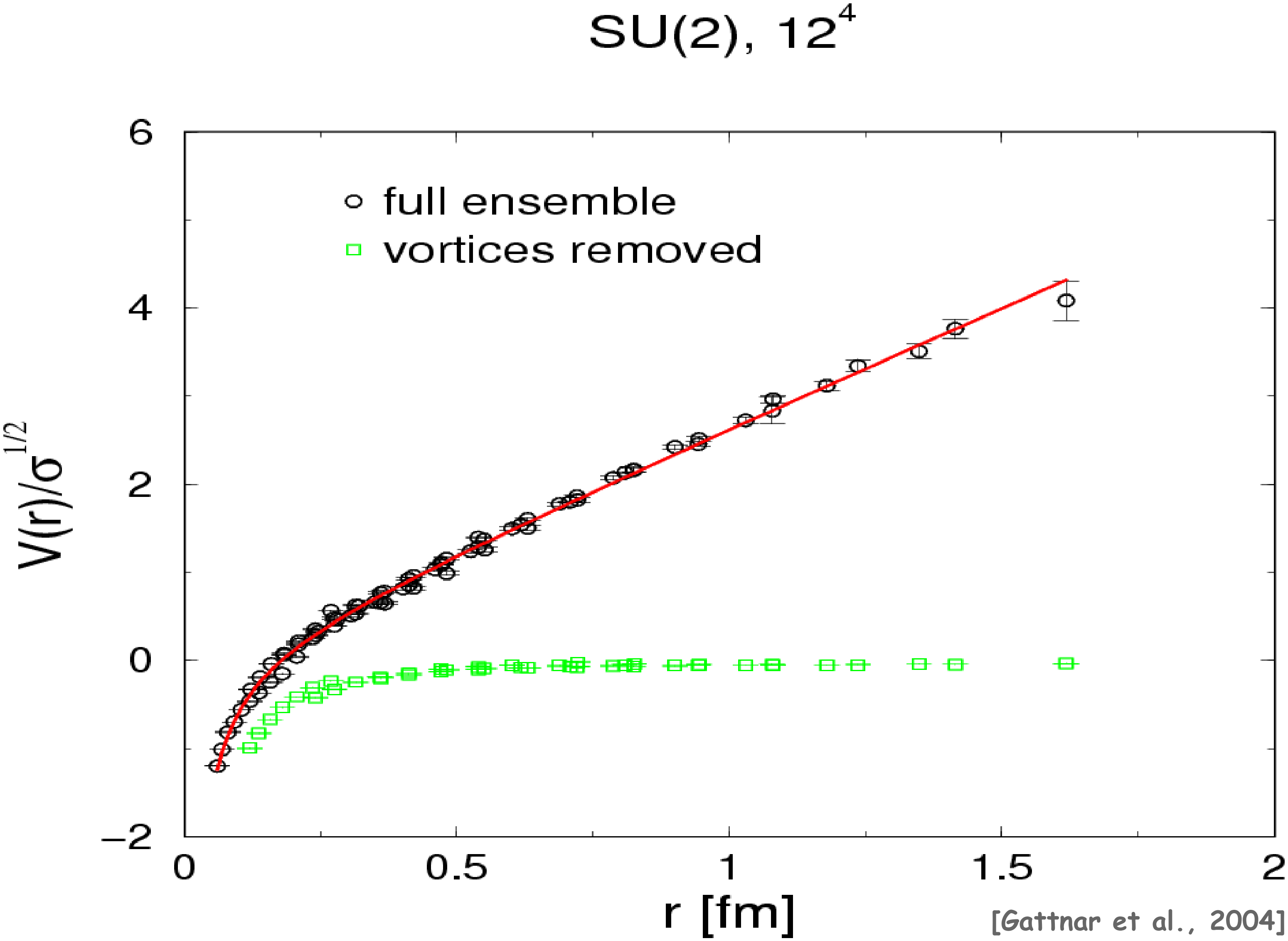

[Gattnar et al., 2004]

Vortices and quark confinement

- (Center-) Vortices very promising
- Removal of vortices removes quark confinement
  - Also chiral symmetry breaking
  - Many other attractive features
    - Casimir scaling,...
- Candidate for a confining excitation

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    - Common boundary of the first Gribov region and the fundamental modular region
- Both give simple criterion in Landau gauge

Landau-gauge and ghosts

- Gauge sector: Choose Landau gauge

$$L=-\frac{1}{4}F_{\mu\nu}^{a}F^{\mu\nu,a}-\bar{c}^{a}\partial^{\mu}D_{\mu}^{ab}c^{b}$$

$$F_{\mu\nu}^{a}=\partial_{\mu}A_{\nu}^{a}-\partial_{\nu}A_{\mu}^{a}-gf^{abc}A_{\mu}^{b}A_{\nu}^{c}$$

$$D_{\mu}^{ab}=\delta^{ab}\partial_{\mu}-gf^{abc}A_{\mu}^{c}$$

- Degrees of freedom:

Gluons: A_{μ}^{a}

Ghosts: $c^{a},\bar{c}^{a}$

(Intermediate states - not observable)

Propagators

[Introduction: Alkofer & von Smekal, 2001]

- **Ghost**:

$$D_G^{ab}(x-y) = <\bar{c}^a(x)c^b(y)>$$

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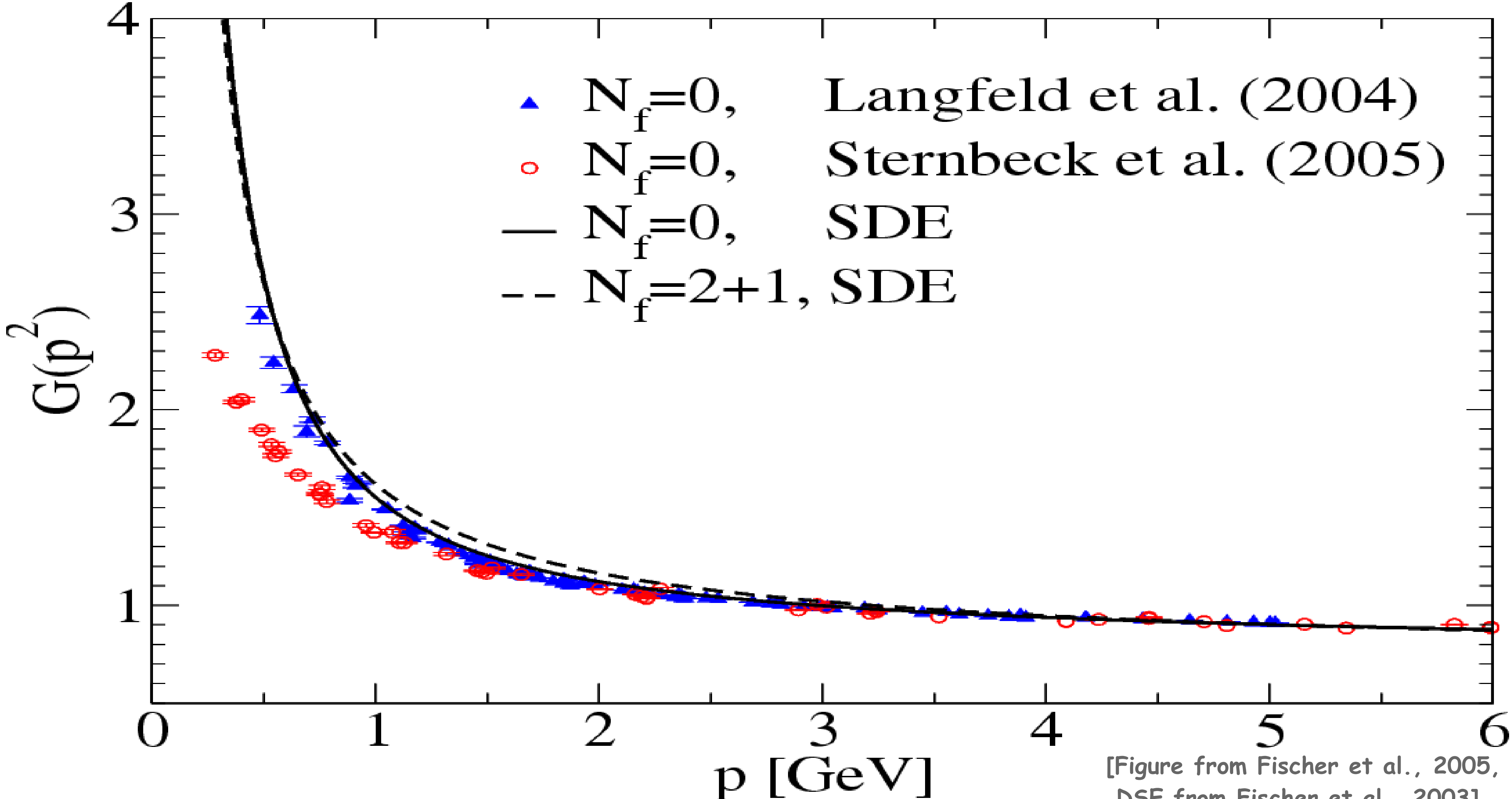
- Implies enhancement of zero or near-zero eigenspectrum of the Faddeev-Popov operator

Results on the ghost

- IR divergence in accordance with scenarios



[Figure from Fischer et al., 2005, DSE from Fischer et al., 2003]

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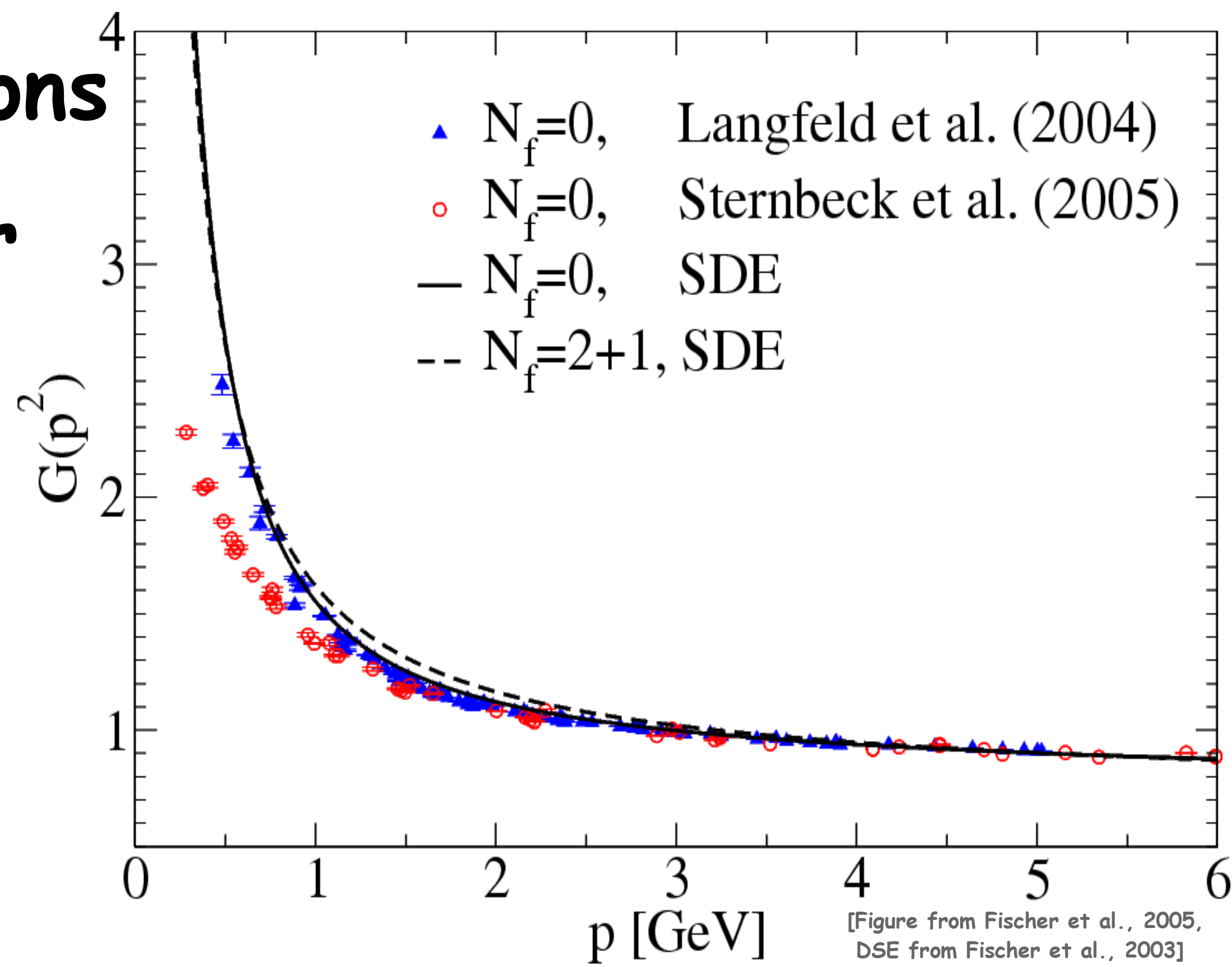

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Results on the ghost

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  - Renormalization group [Pawlowski et al., 2004]
- Corresponding results in Coulomb gauge [Feuchter et al., 2004]
  - Other gauges still unclear – under investigations

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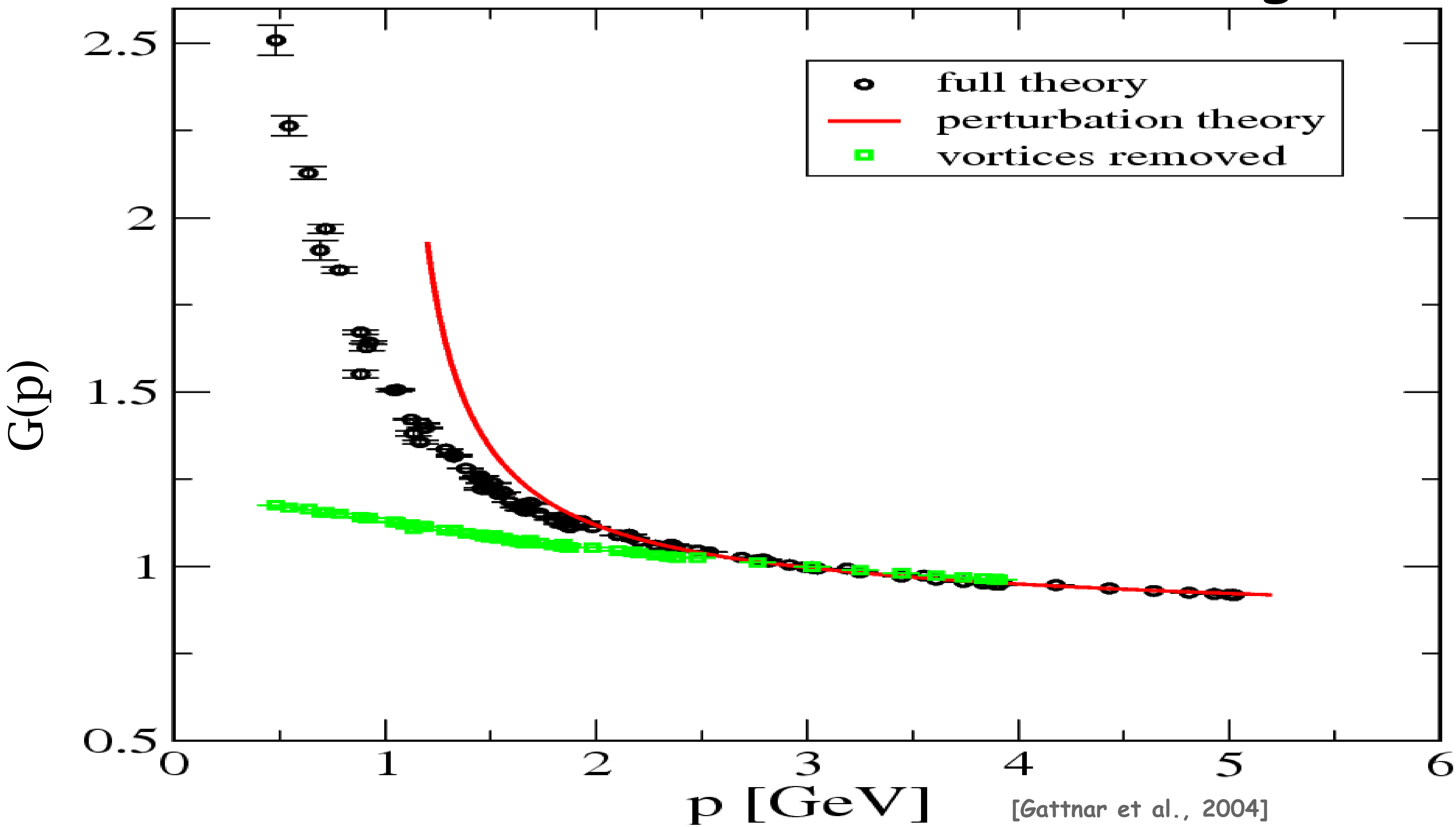
- Some results from lattice calculations

Ghost propagators and vortices

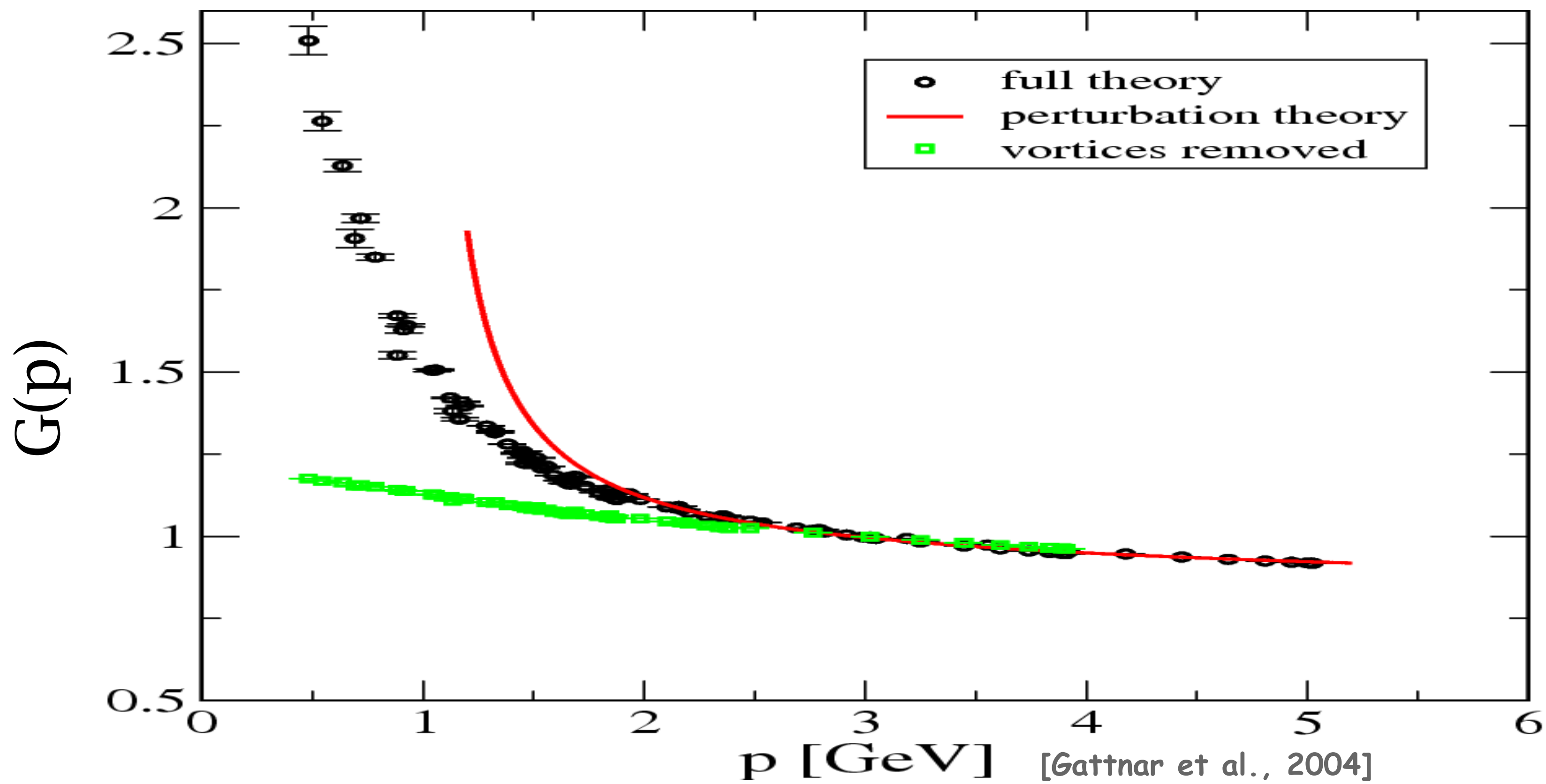
- Removal of vortices tames infrared divergence



[Gattnar et al., 2004]

Ghost propagators and vortices

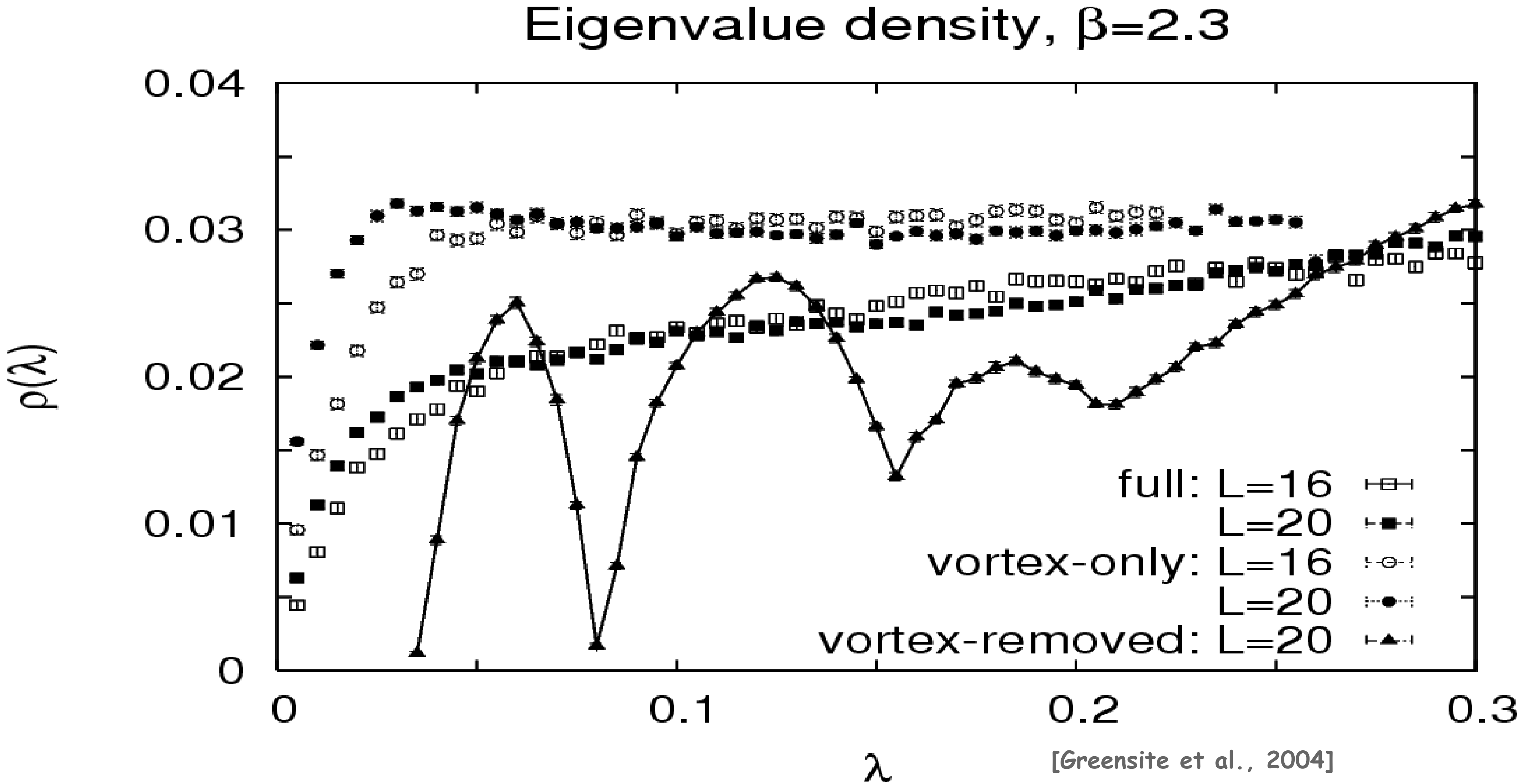
- Removal of vortices tames infrared divergence of ghosts in Landau gauge
- Hints to a change of the FP-eigenspectrum



[Gattnar et al., 2004]

Vortices and the eigenspectrum of the FPO

- Removal of vortices in Coulomb gauge reduces enhancement of near-zero modes



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Analytical approach

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- (Center) Vortices
  - Thick, string-like configurations
  - Should give an enhancement

Field configurations

- **Instanton**

$$A_\mu^a = \frac{1}{g} \frac{2}{r^2 + \lambda^2} r_\nu b_{\nu\mu}^a$$

[Bohm et al., 2001]

  - λ size, b constant matrices

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- Thick center vortex (cylindrical coordinates)

$$A_\eta^a = \frac{1}{g} \delta^{a3} \frac{\mu(\rho)}{\rho}$$

[Diakonov 1999, Diakonov et al., 2001]

  - Other components vanish, μ 'profile'-function
  - $\mu(0)=0$, $\mu(\infty)=2n+1$, n integer, $2n+1$ vortex flux

Eigenvalue equation

- **General form (for transverse fields)**

$$-(\delta^{ab}\partial^2 + g f^{abc} A_\mu^c \partial_\mu)\phi^b = \omega^2 \phi^a$$

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- Radial equations: (System of) ordinary differential equations

Vacuum

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- Non-localized: Correspond to scattering states in quantum mechanics

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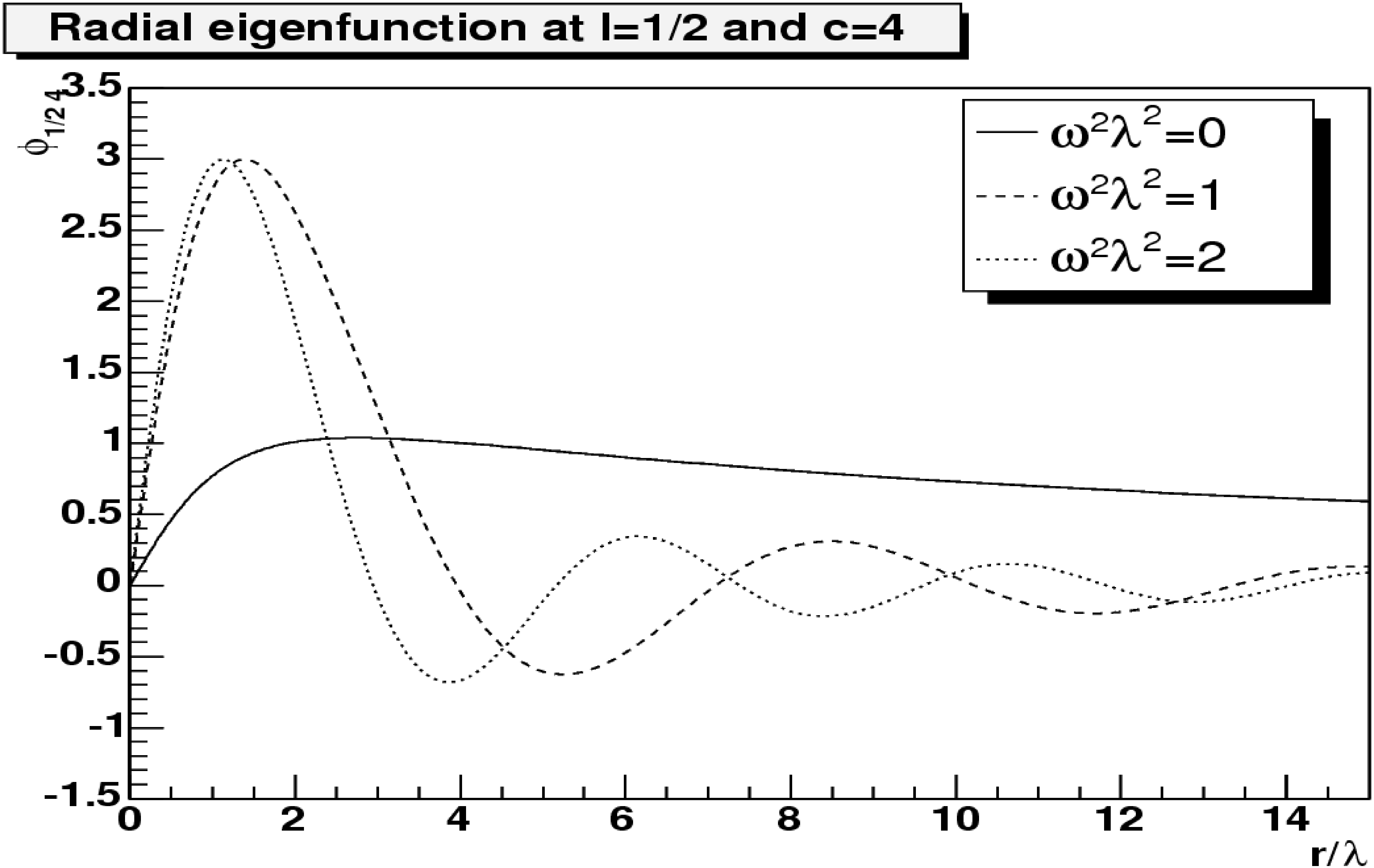
  - Belong to the region of field configuration space relevant to the Gribov-Zwanziger scenario

Radial part of zero-mode



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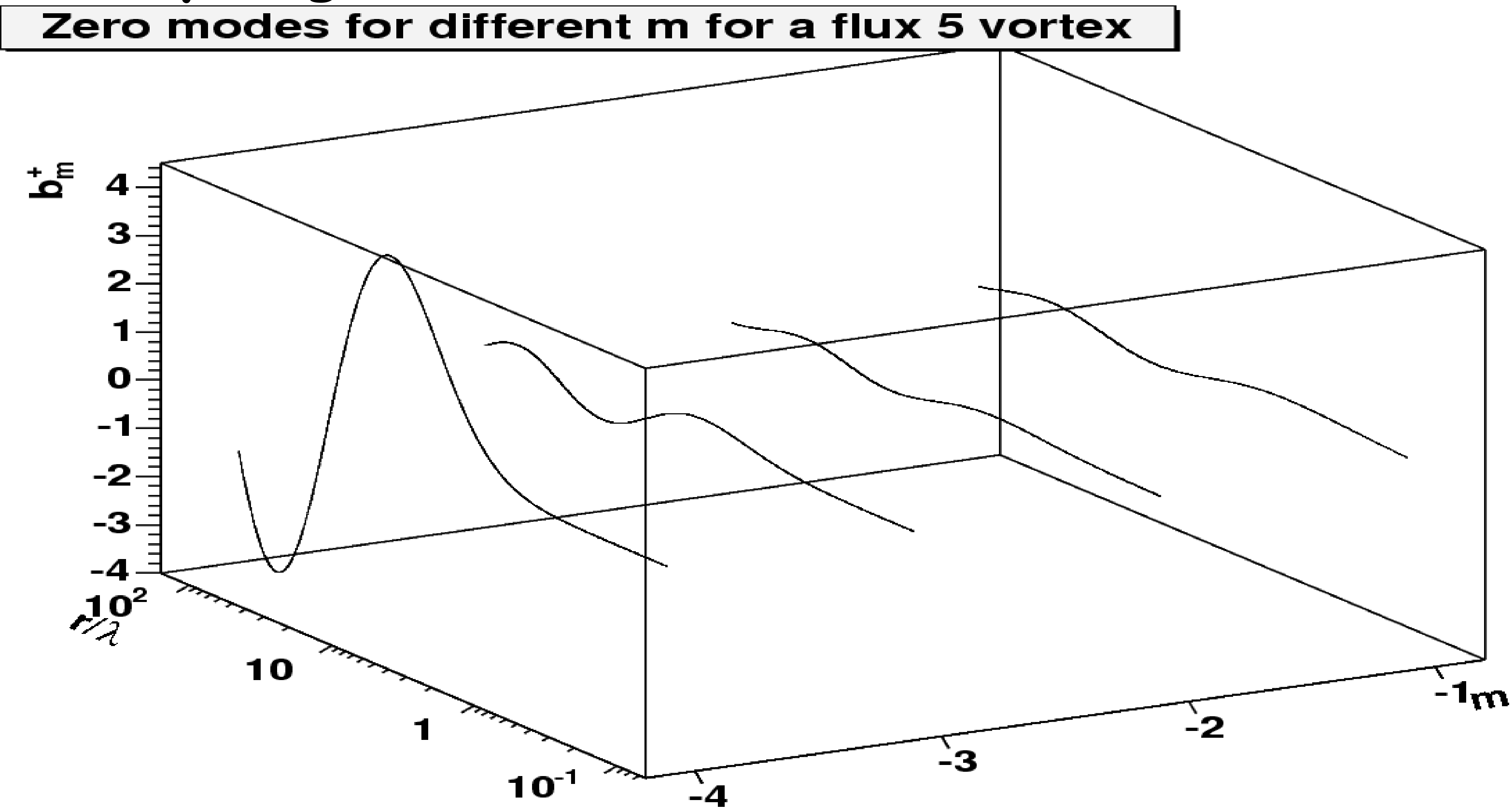
  - Belongs probably to the Gribov-Zwanziger region, if flux greater 1

Radial part of zero modes for flux 5

- Only "right-handed"- similar for left-handed



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- Fulfilled for $n>0$ – also on the lattice [Greensite et al., 2004]

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  - What does the instanton results signify?
- Origin or symptom?