Describing Gluons

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Pathways to Confinement
Rio de Janeiro
Brazil

Overview

- Gauge freedom in Yang-Mills theory

Supported by the FWF

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- Non-perturbative gauges

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- Comparing lattice and continuum

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- Gauge freedom in Yang-Mills theory
- Non-perturbative gauges
- Comparing lattice and continuum
- Correlation functions and physics

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Strong interactions

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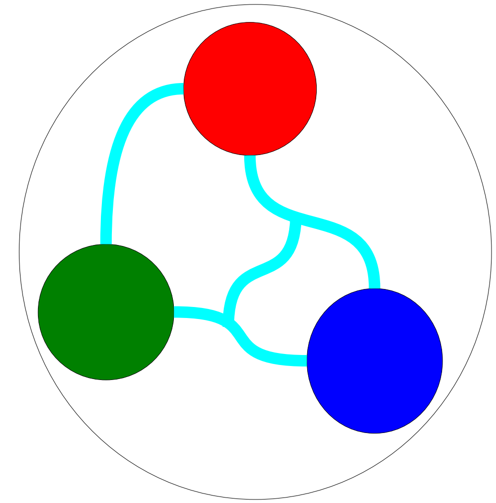
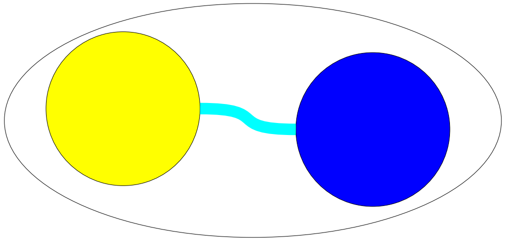
- Substructure can be described by QCD
  - Degrees of freedom are quarks and gluons





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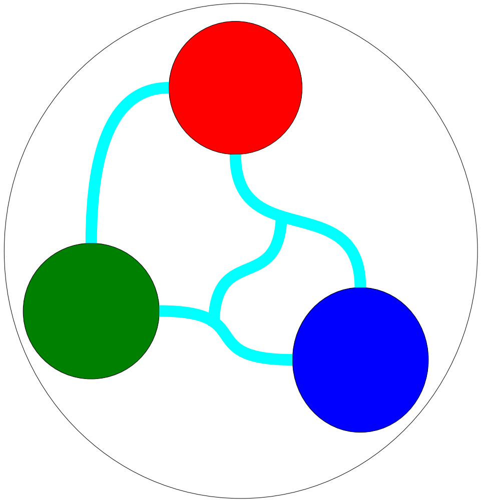
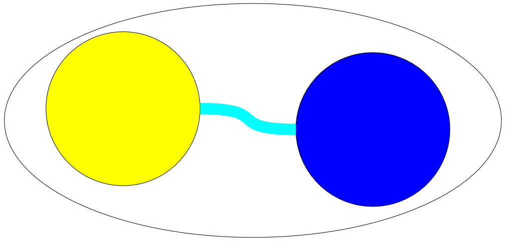
  - Confinement: Colored objects unobservable





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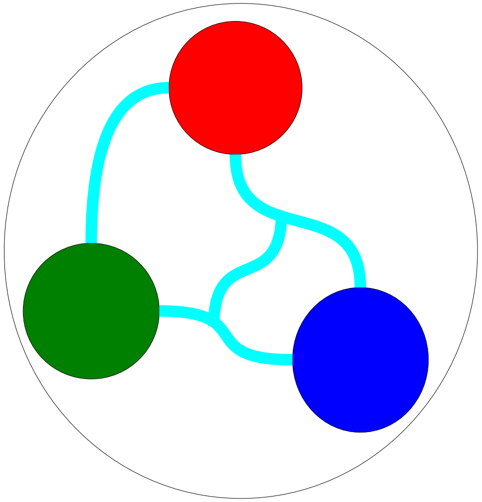

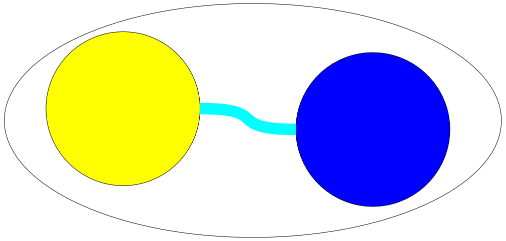

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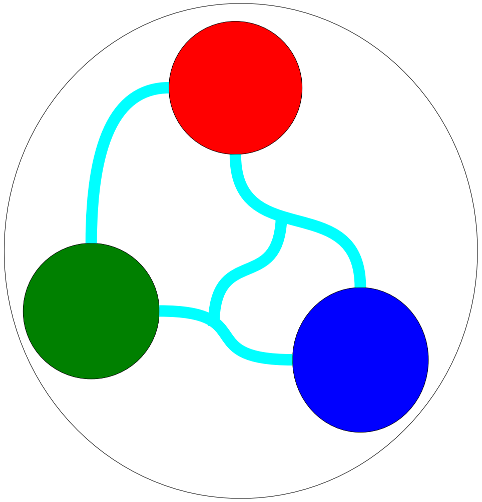

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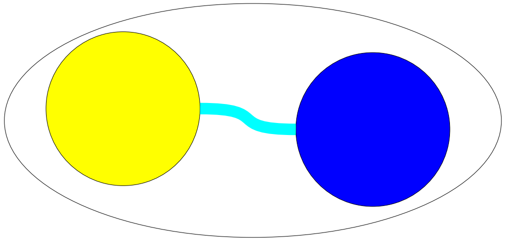

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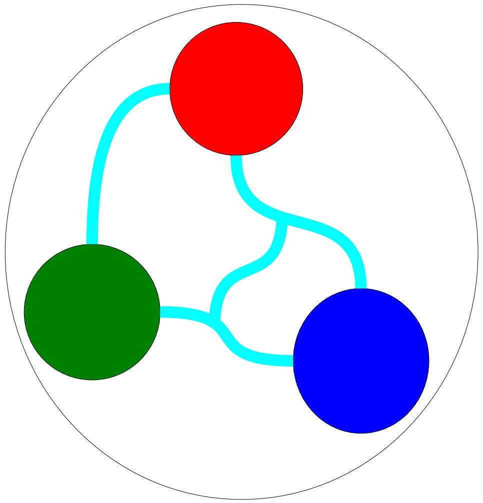

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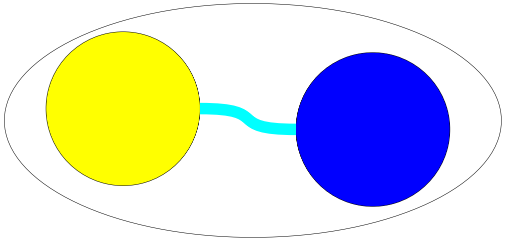

- Reduce complexity and ignore Quarks: Yang-Mills theory
  - Affects gluon properties only quantitatively

Yang-Mills Theory

- Lagrangian (after Wick rotation to Euclidean space-time):

$$L=-\frac{1}{4}F_{\mu\nu}^{a}F_{\mu\nu}^{a}$$

$$F_{\mu\nu}^{a}=\partial_{\mu}A_{\nu}^{a}-\partial_{\nu}A_{\mu}^{a}-gf^{abc}A_{\mu}^{b}A_{\nu}^{c}$$

- Degrees of freedom:

Gluons: A_{μ}^{a}

- g is the coupling constant, giving the strength of coupling
- f^{abc} are numbers, depending on the gauge group, SU(3) for QCD: quarks & gluon are organized in multiplets, just as with spin

Gauge-fixing

- Yang-Mills theory is a gauge theory
  - Gauge transformations $A_\mu^a \rightarrow A_\mu^a + (\delta^{ab}\partial_\mu - g f^{abc} A_\mu^c)\phi^b(x)$ with arbitrary $\phi^a(x)$ change the gauge fields, but leave physics invariant

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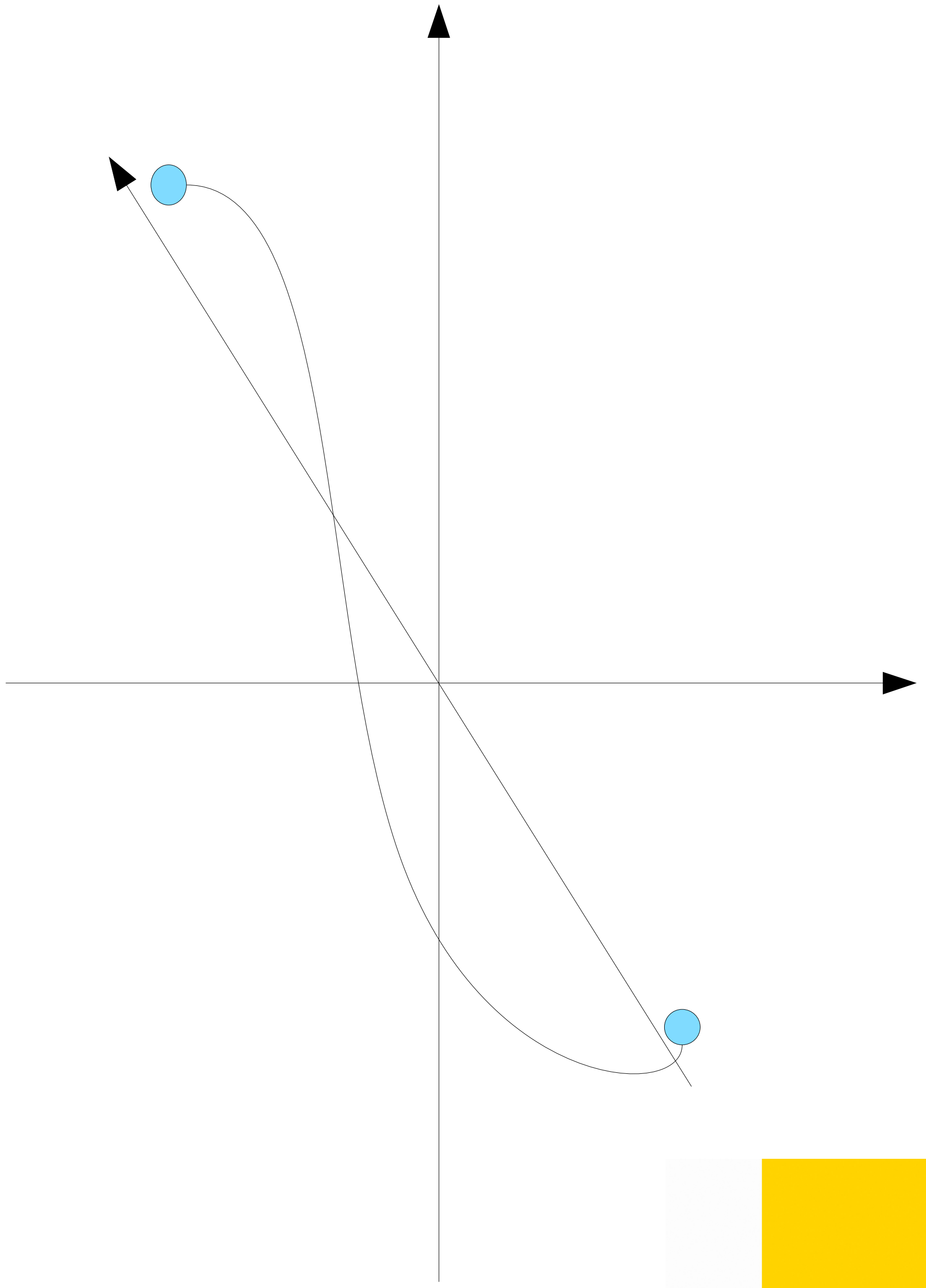
- Correlation functions are in general gauge-dependent
  - Gauge-fixing is required
- Example: Landau gauge condition $\partial_\mu A_\mu^a = 0$
  - Here only Landau gauge results will be considered
  - Many other gauges have been studied

Configuration space (artist's view)

- Gauge fields not unique
  - Gauge transformation does not change physics



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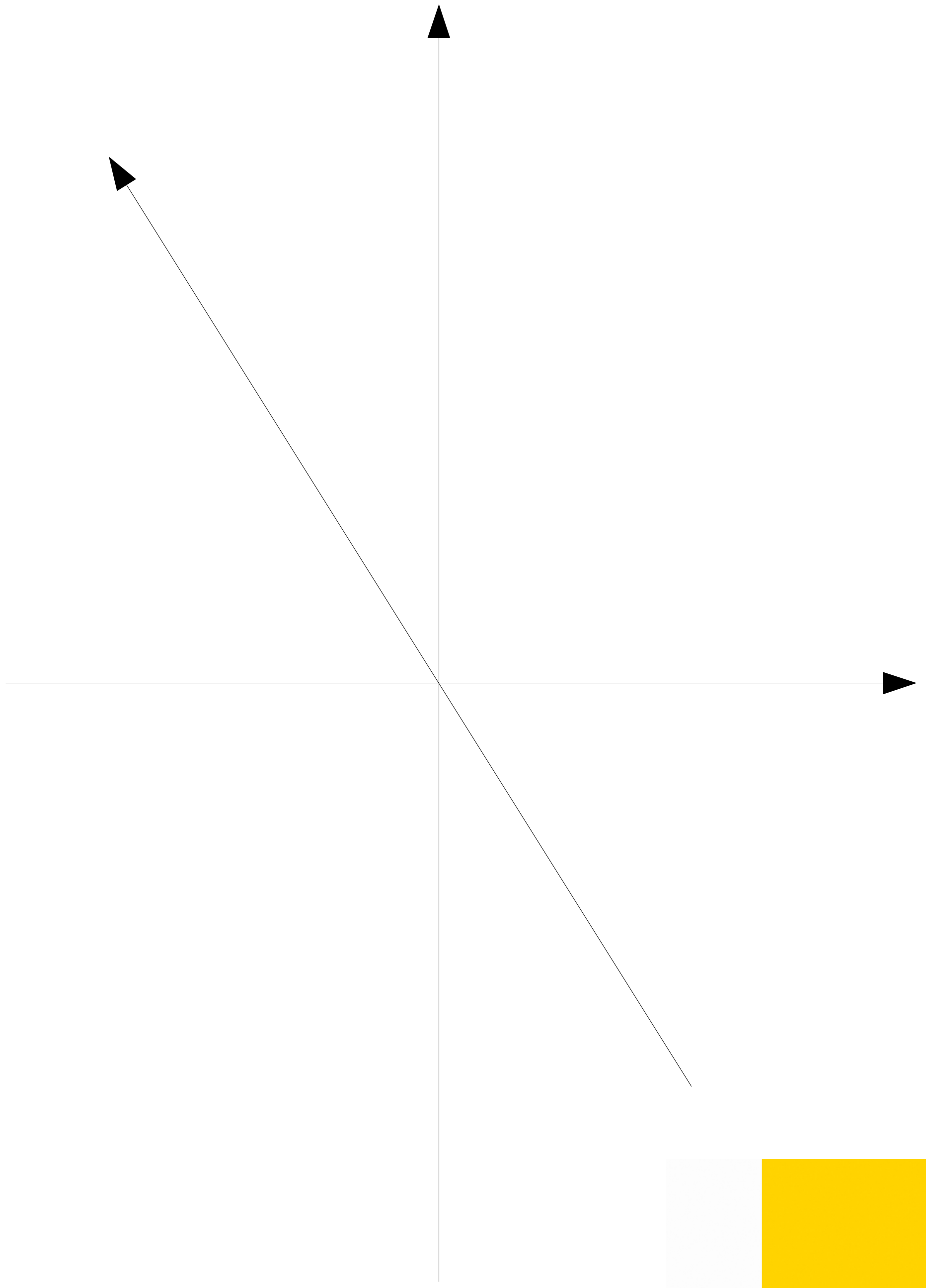

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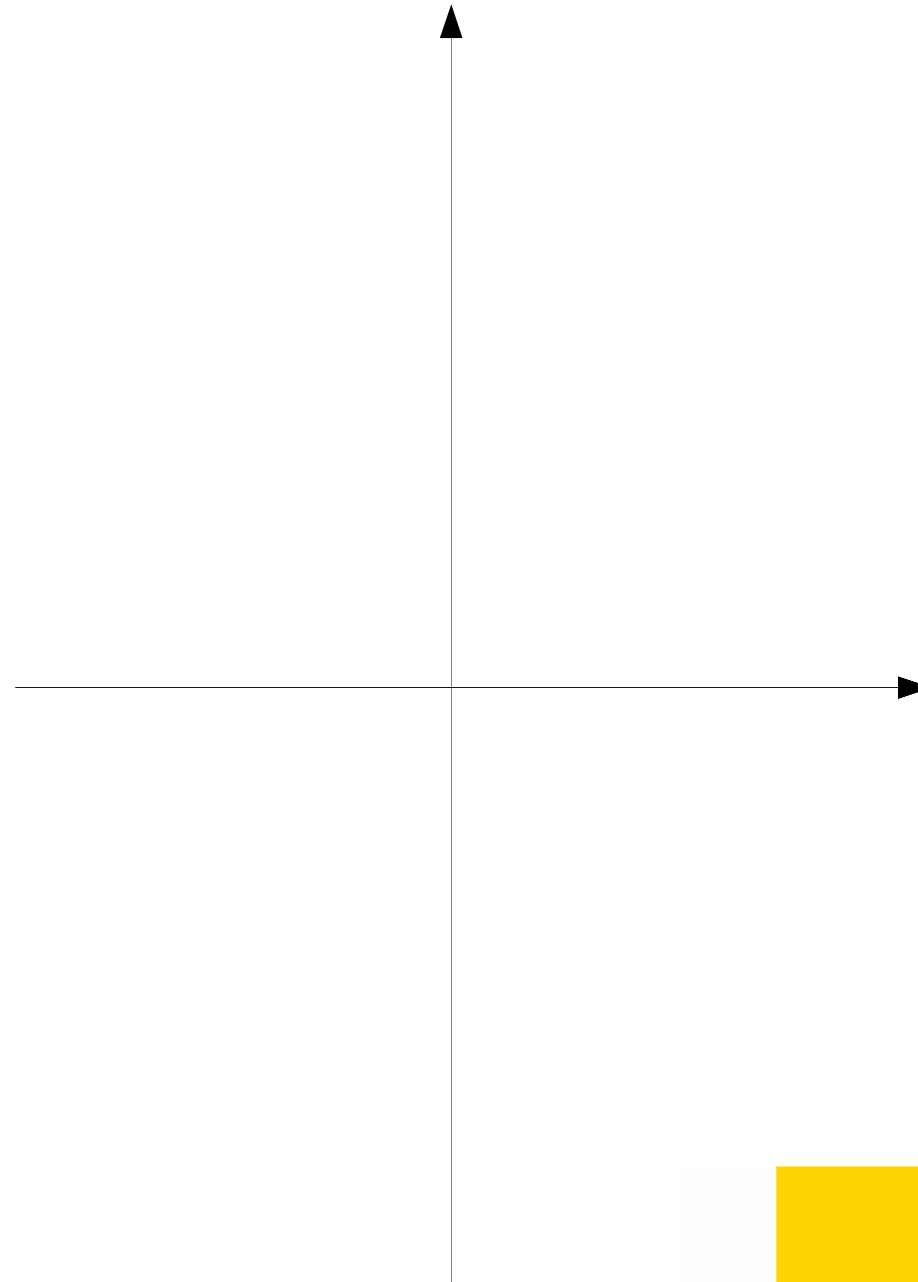
- Impose Landau gauge condition
  - Reduces configuration space to a hypersurface



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- Local gauge condition
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- This condition can be implemented using auxiliary fields, the so-called ghost fields
  - No physical objects: Pure mathematical convenience

(Perturbative) Landau gauge

- Lagrangian:

$$L=-\frac{1}{4}F_{\mu\nu}^a F_{\mu\nu}^a-\bar{c}^a\partial_\mu D_\mu^{ab}c^b$$

$$F_{\mu\nu}^a=\partial_\mu A_\nu^a-\partial_\nu A_\mu^a-gf^{abc}A_\mu^b A_\nu^c$$

$$D_\mu^{ab}=\delta^{ab}\partial_\mu-gf^{abc}A_\mu^c$$

- Degrees of freedom:

Gluons: A_μ^a

Ghosts: $\bar{c}^a, c^a$

- Ghosts interact with gluons: They have to be included

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- Sufficient for perturbation theory

Proceeding

- Once the gauge is fixed, all kind of (perturbative) calculations can be done
- Use Green's or correlation functions as basic entities

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- Full Green's functions contain all information
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- If having a non-vanishing color charge they change under gauge transformation
- Simplest non-zero Green's functions: 2-point functions or propagators
  - Expectation values of products of two field operators
  - 1-point functions vanish

Propagators

- In Landau gauge: Gluon and one auxiliary field: Ghost
- Gluon:

$$D_{\mu\nu}^{ab}(x-y) = \langle A_\mu^a(x) A_\nu^b(y) \rangle$$

$$D_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \frac{Z(p)}{p^2}$$

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- Ghost propagator can be expressed as a gluon operator, the inverse Faddeev-Popov operator

$$D_G^{ab}(x-y) \sim <(\partial_\mu D_\mu^{ab})^{-1}> = <(\partial_\mu(\delta^{ab}\partial_\mu - g f^{abc} A_\mu^c))^{-1}>$$

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- Combination of gauge-variant Green's functions yield gauge-invariant results
  - E.g. scattering cross-sections

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- Almost all perturbative calculations proceed via gauge-variant correlation functions
- Also for the non-perturbative physics?

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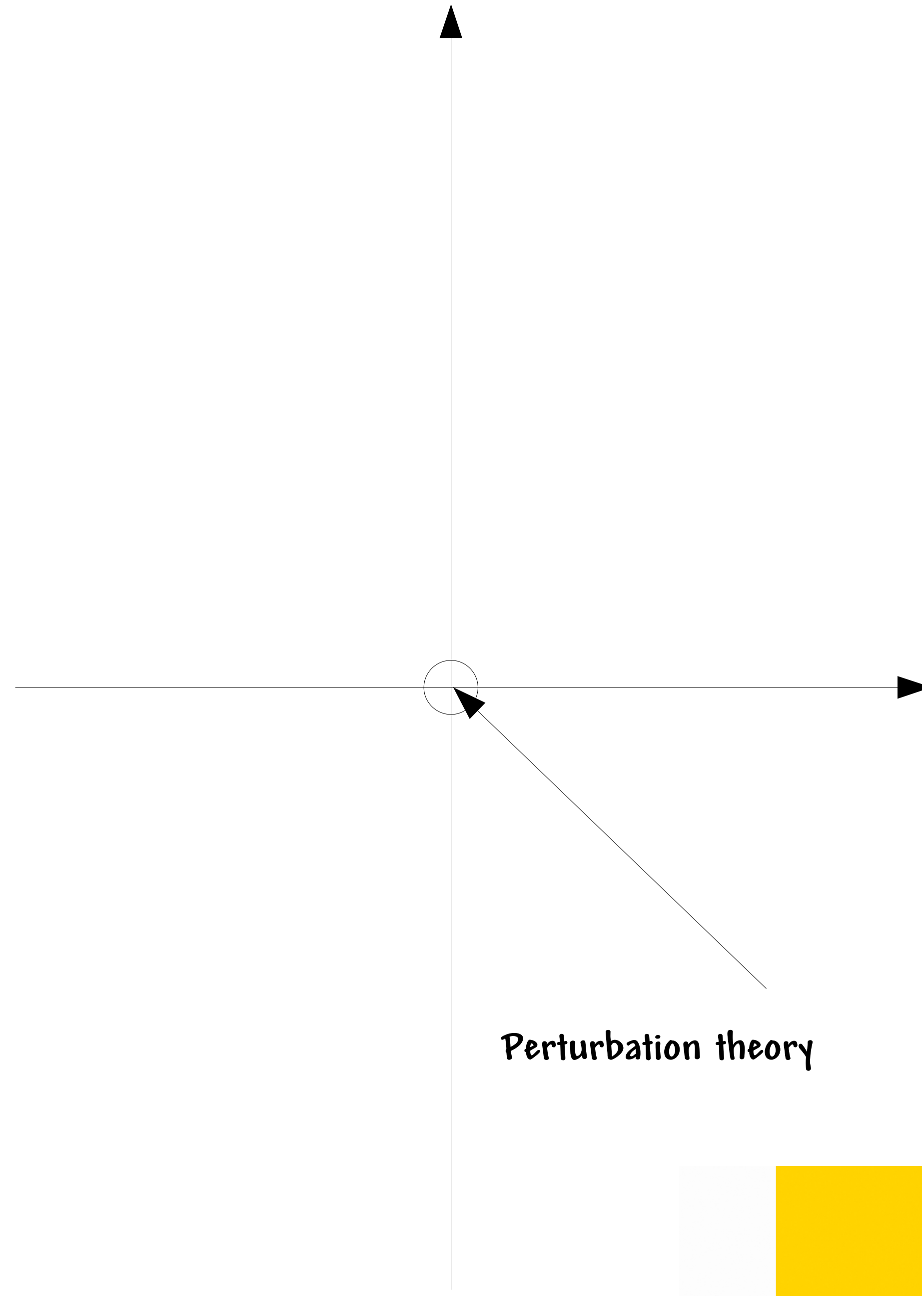
- Perturbation theory is applicable close to the origin



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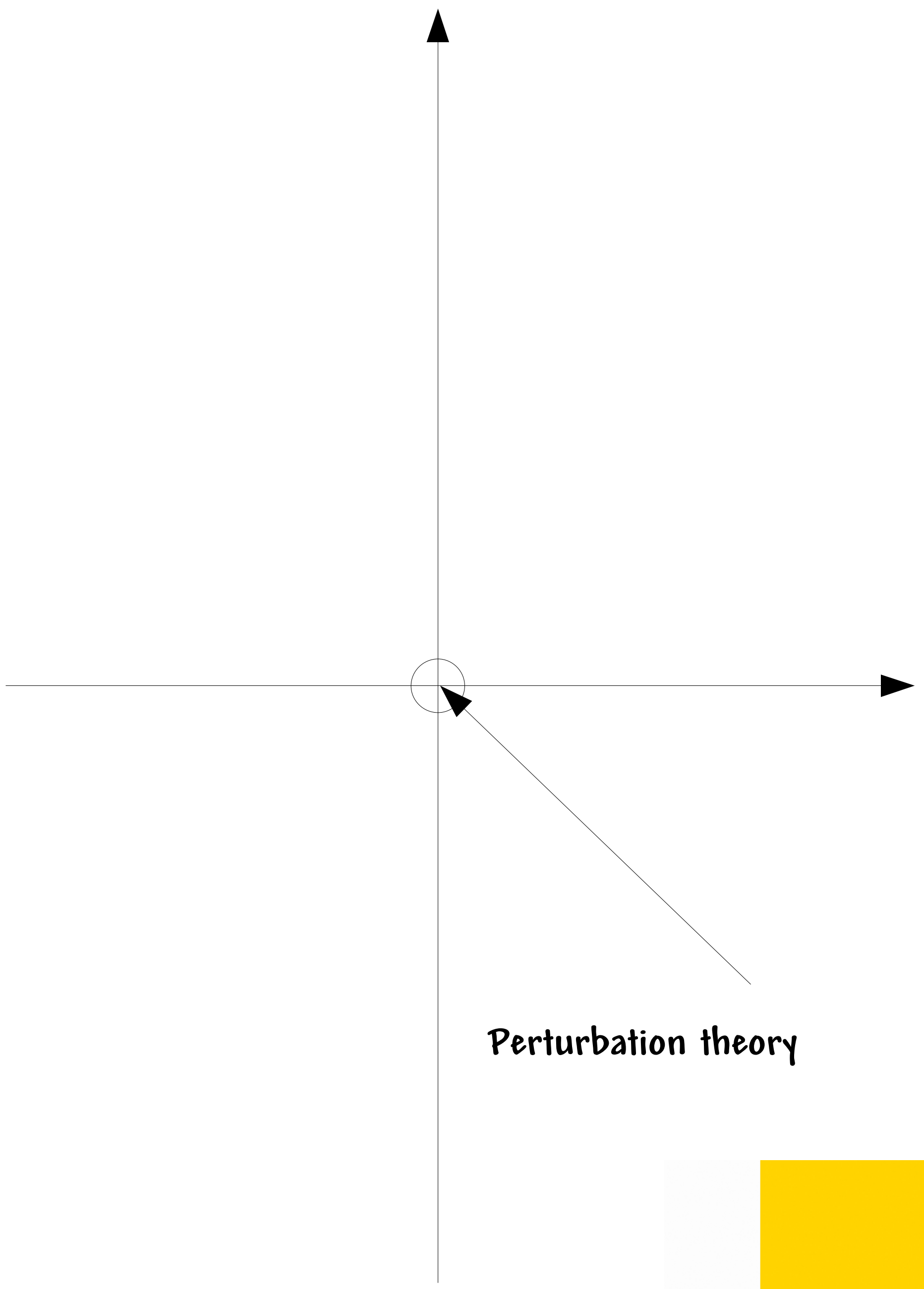
- Perturbation theory is applicable close to the origin
- Non-perturbative physics probes the complete hypersurface



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- Local gauge condition
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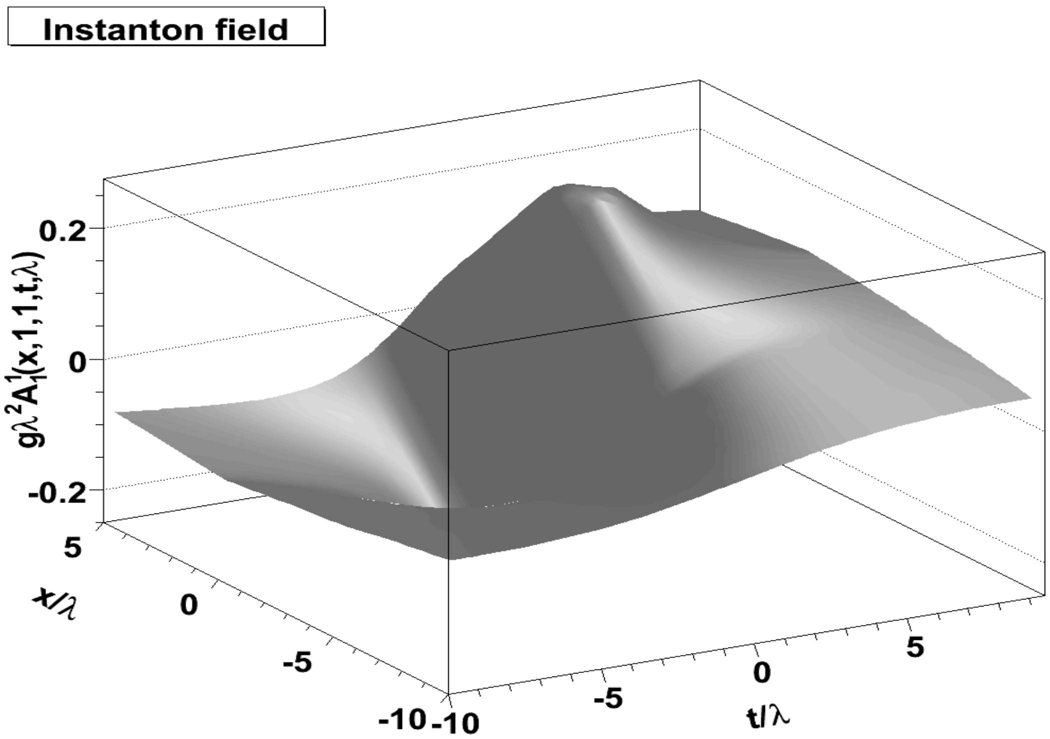
- There are no local gauge conditions known, which select a unique gauge field configuration [Singer 1978]
  - Non-local conditions possible

Example: Instanton [Maas, EPJC 2006]



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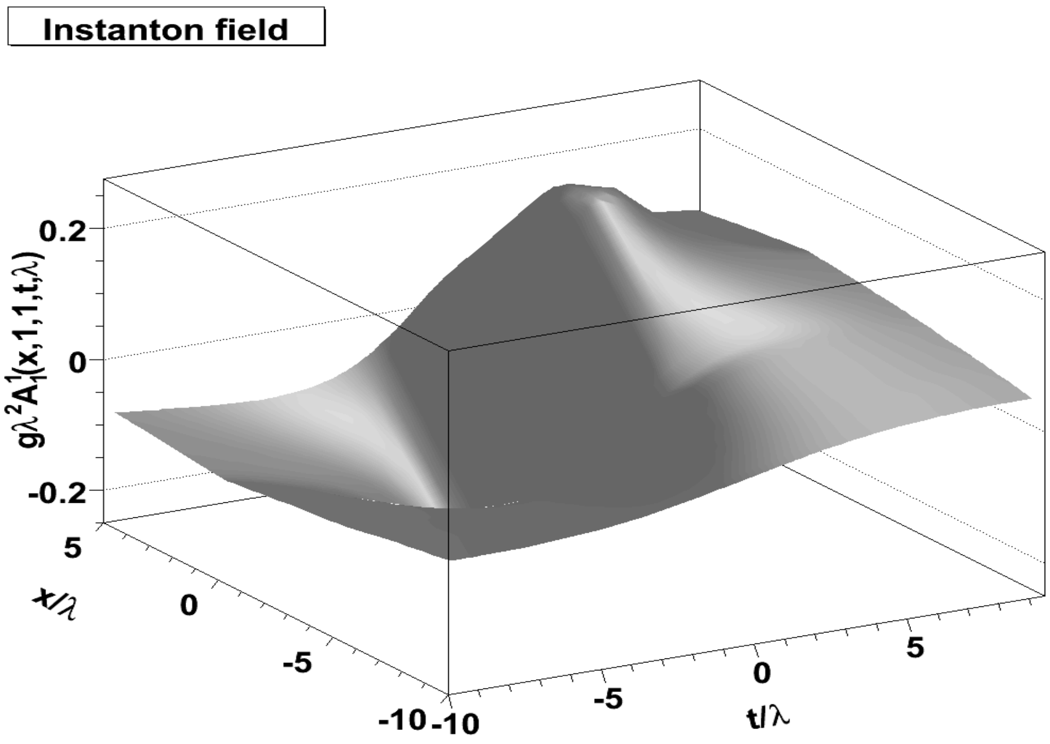

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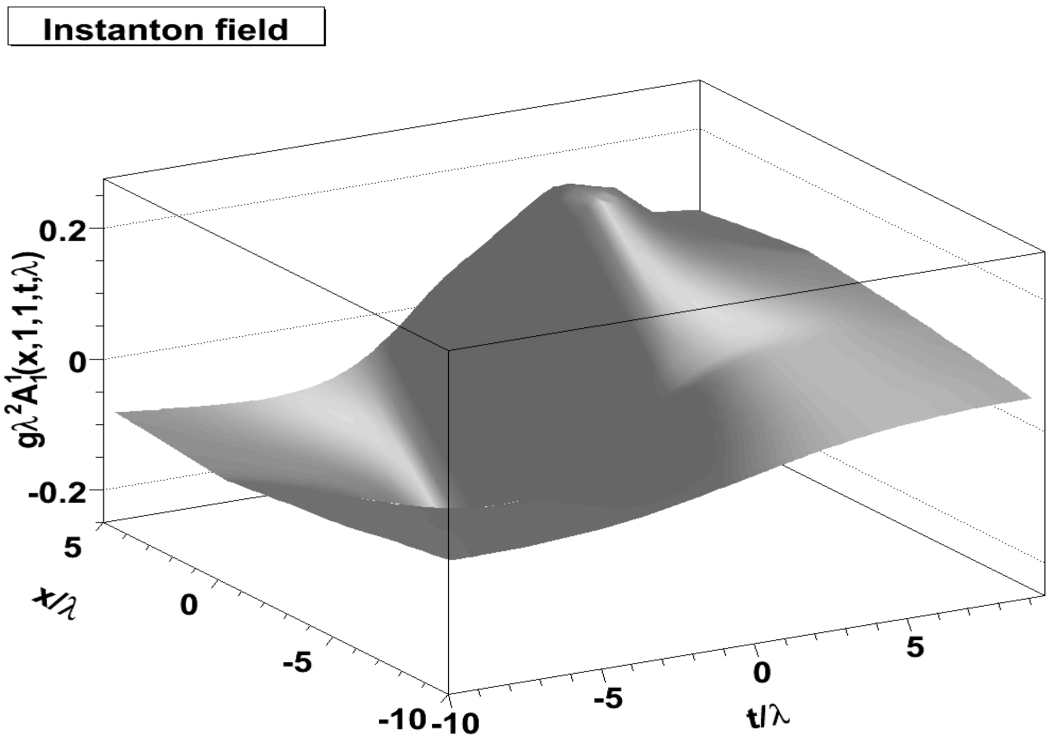

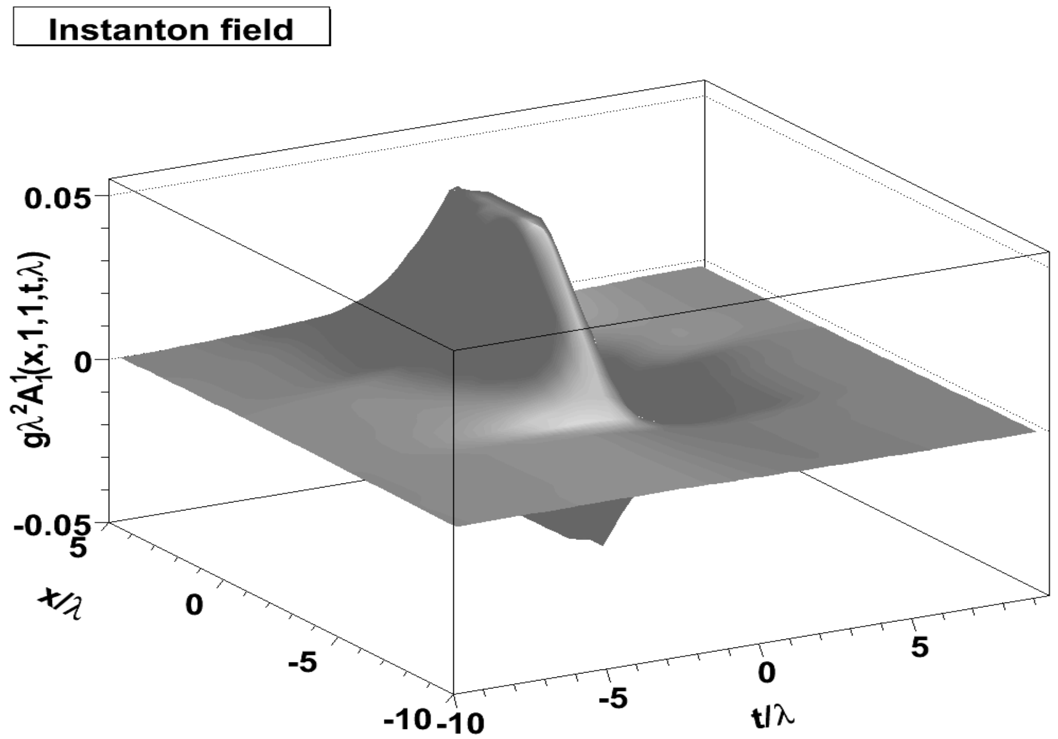

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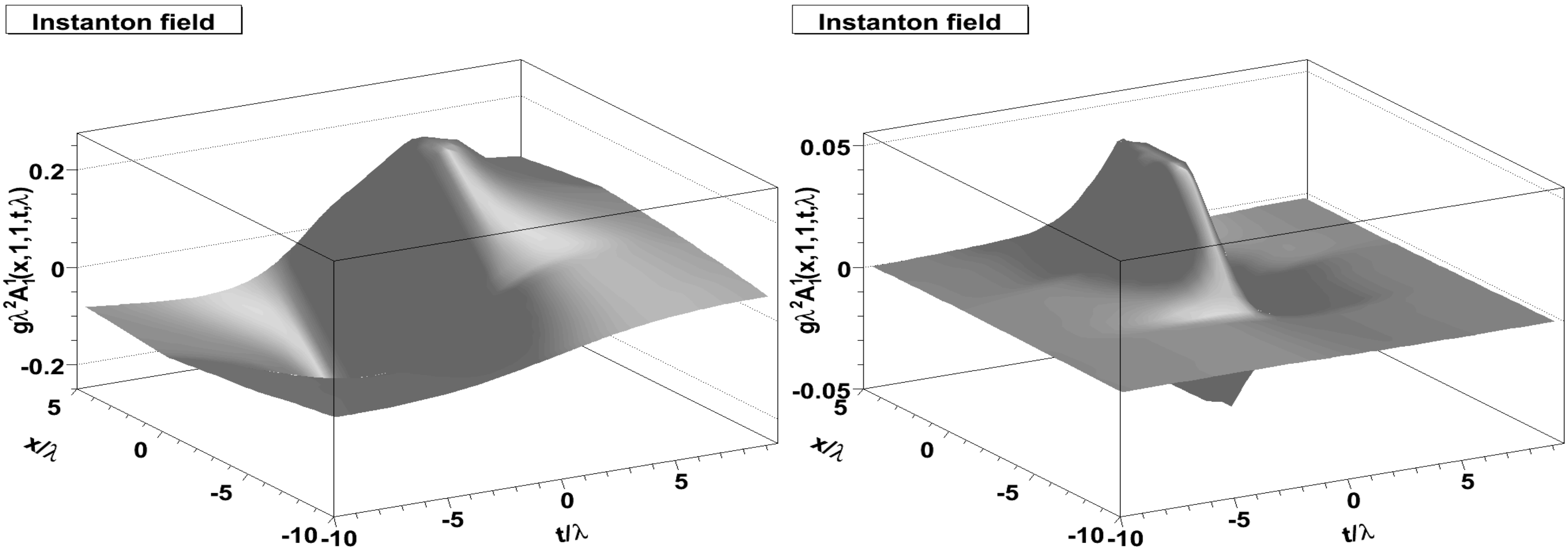

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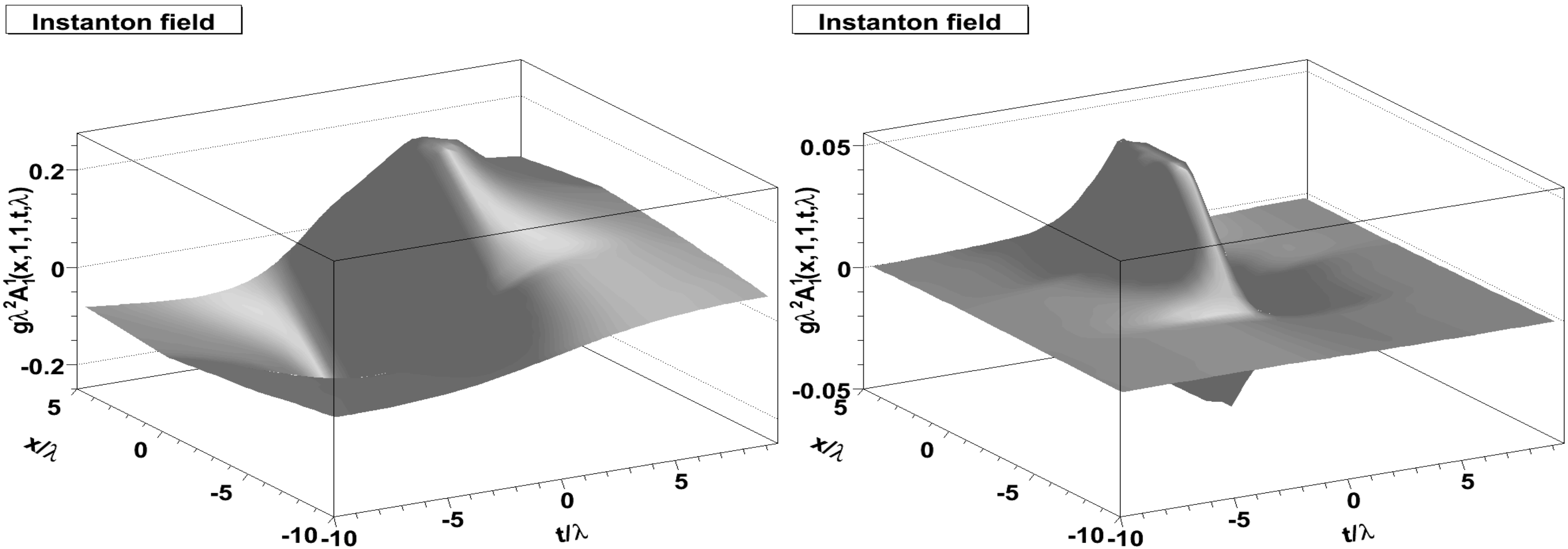

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  - Gribov copy
  - Non-perturbative: Depends on $1/g$

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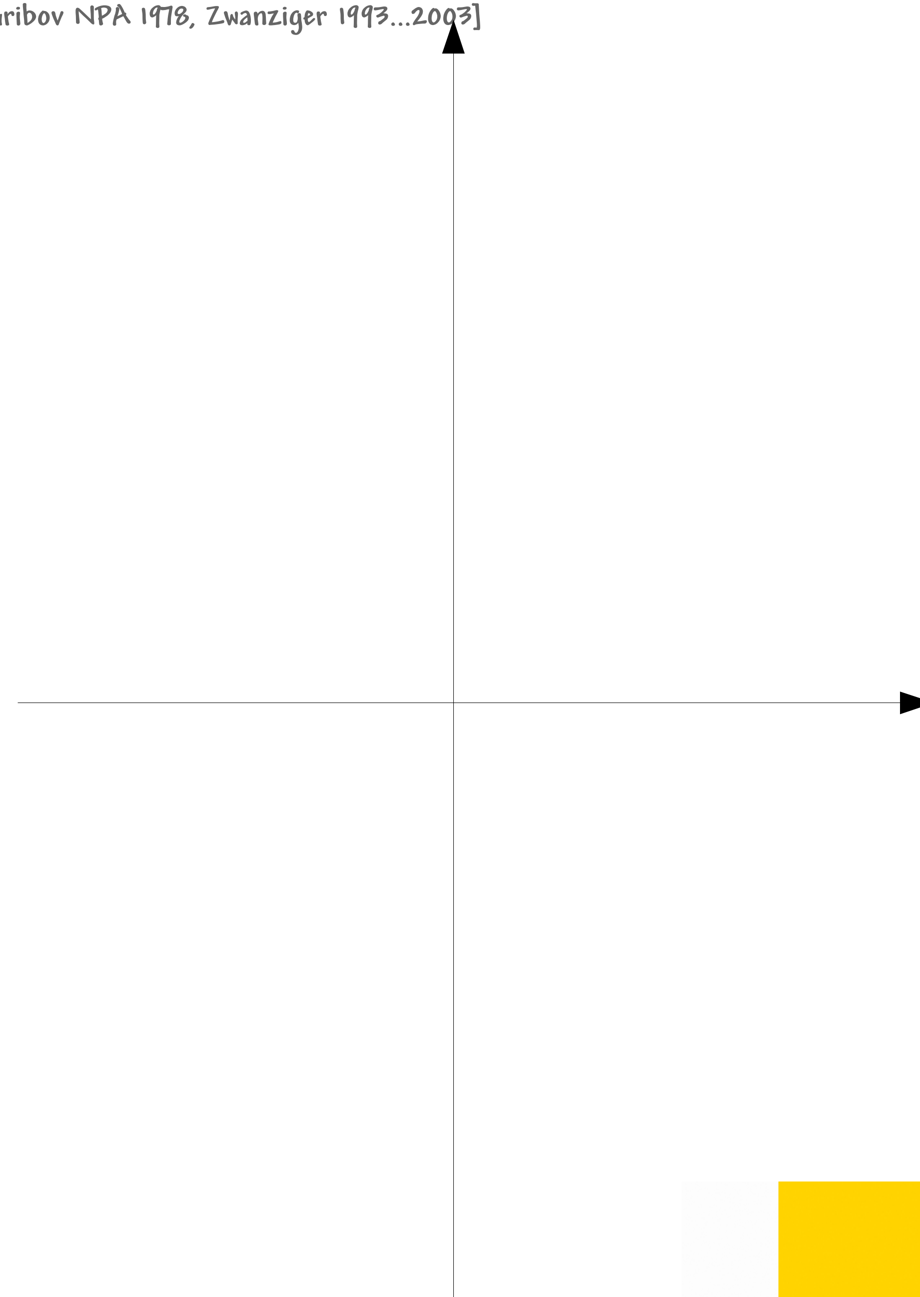
- Construct a non-local condition instead to solve the problem
- Choice: Leave the global color symmetry unfixed

Configuration space (artist's view) [Gribov NPA 1978, Zwanziger 1993...2003]

- Absolute Landau gauge



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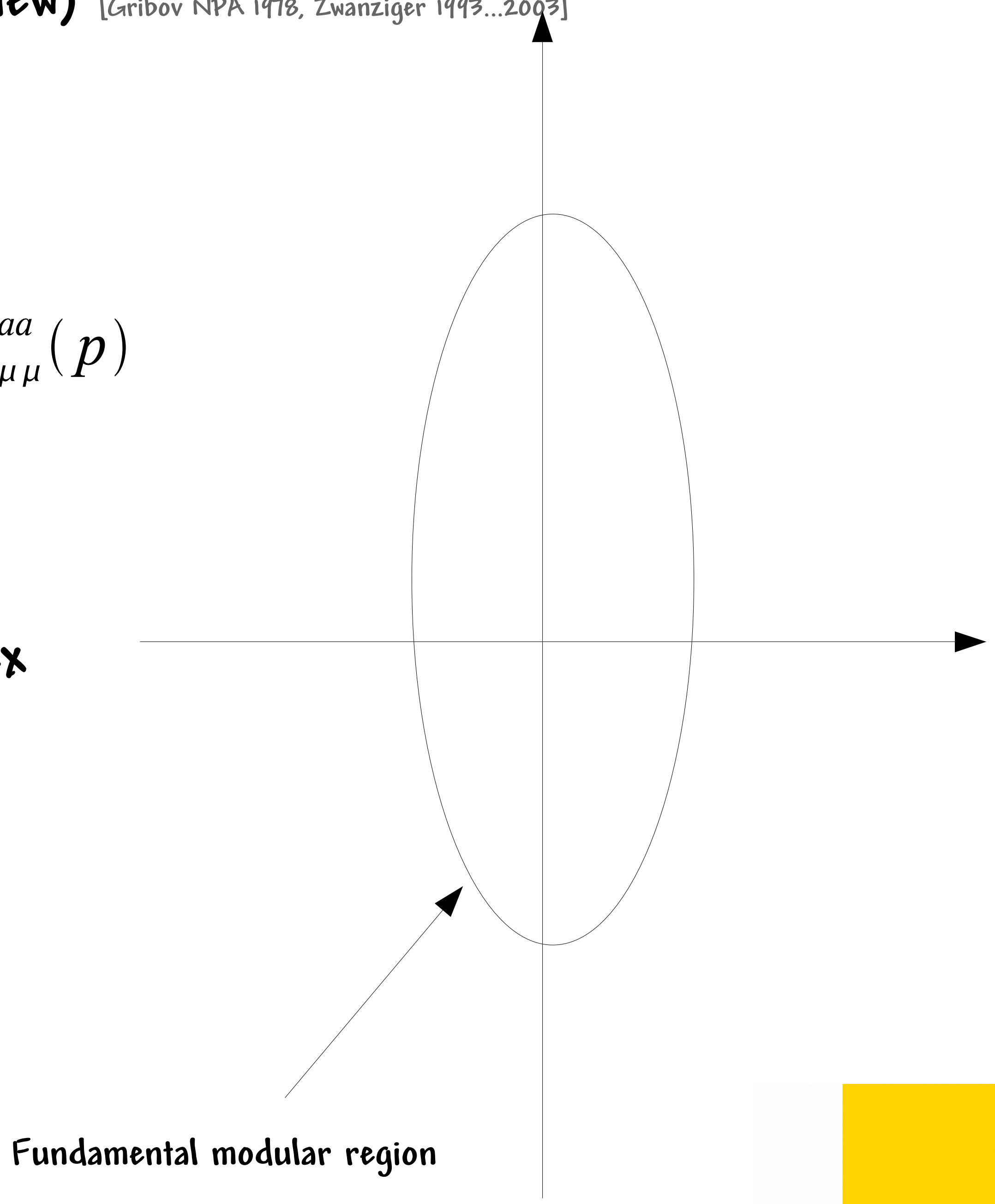
- Absolute Landau gauge
- A global minimum of

$$-\int d^d x\, A_\mu^a(x)\, A_\mu^a(x) \sim -\int d^d p\, D_{\mu\mu}^{aa}(p)$$

defines the fundamental modular region

- This region is bounded and convex



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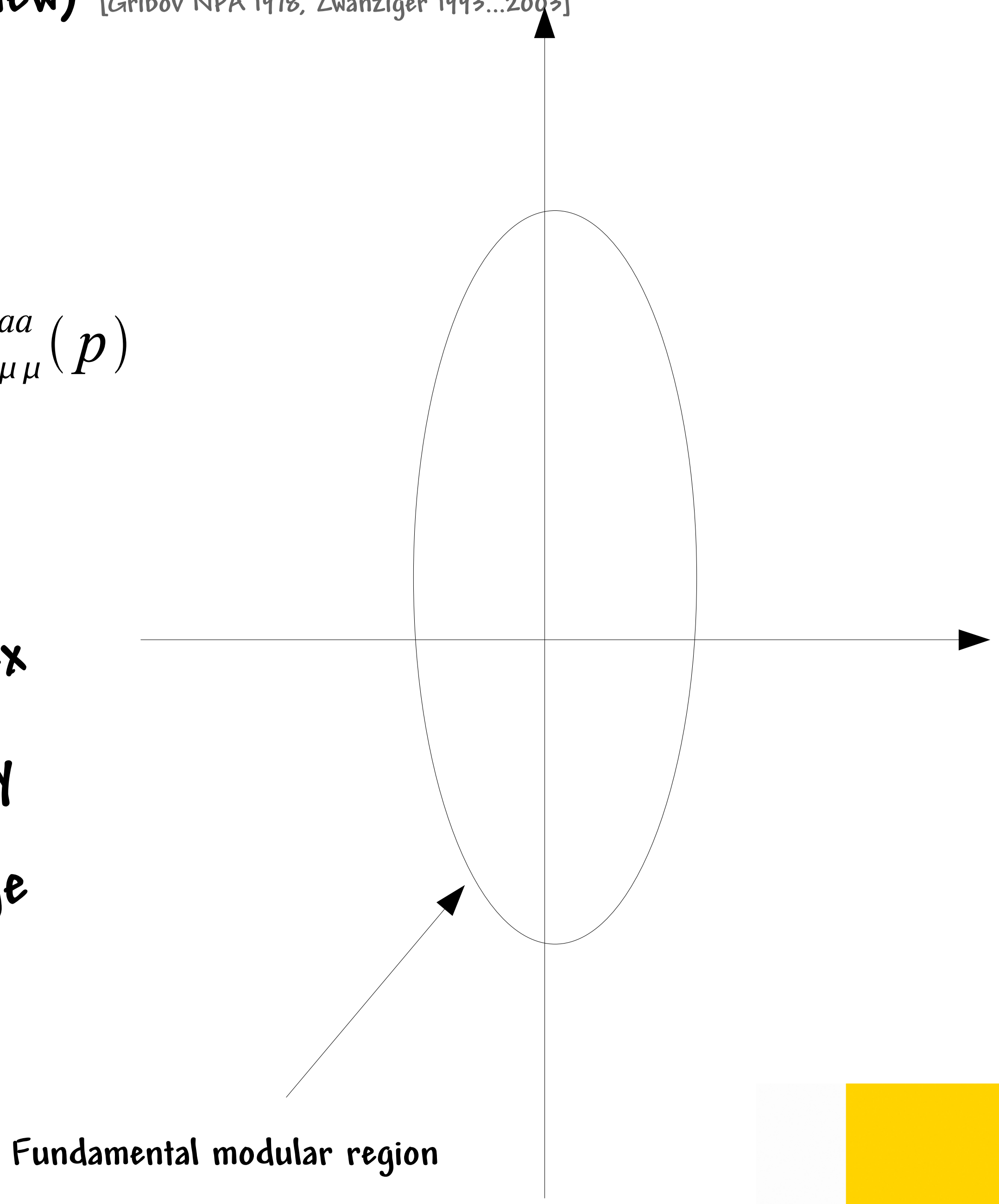

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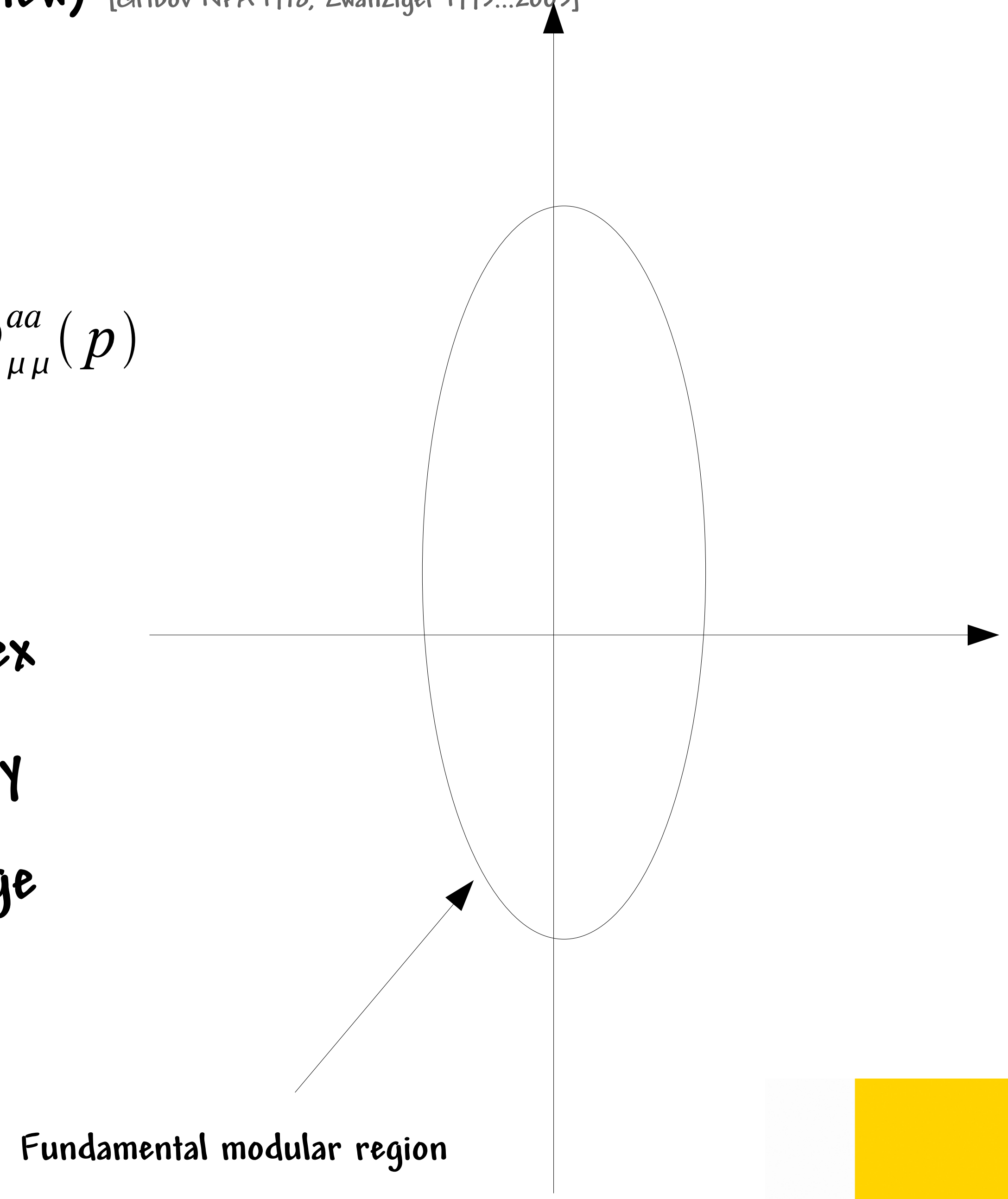
- Drawback: Badly divergent and regularization may introduce degeneracies



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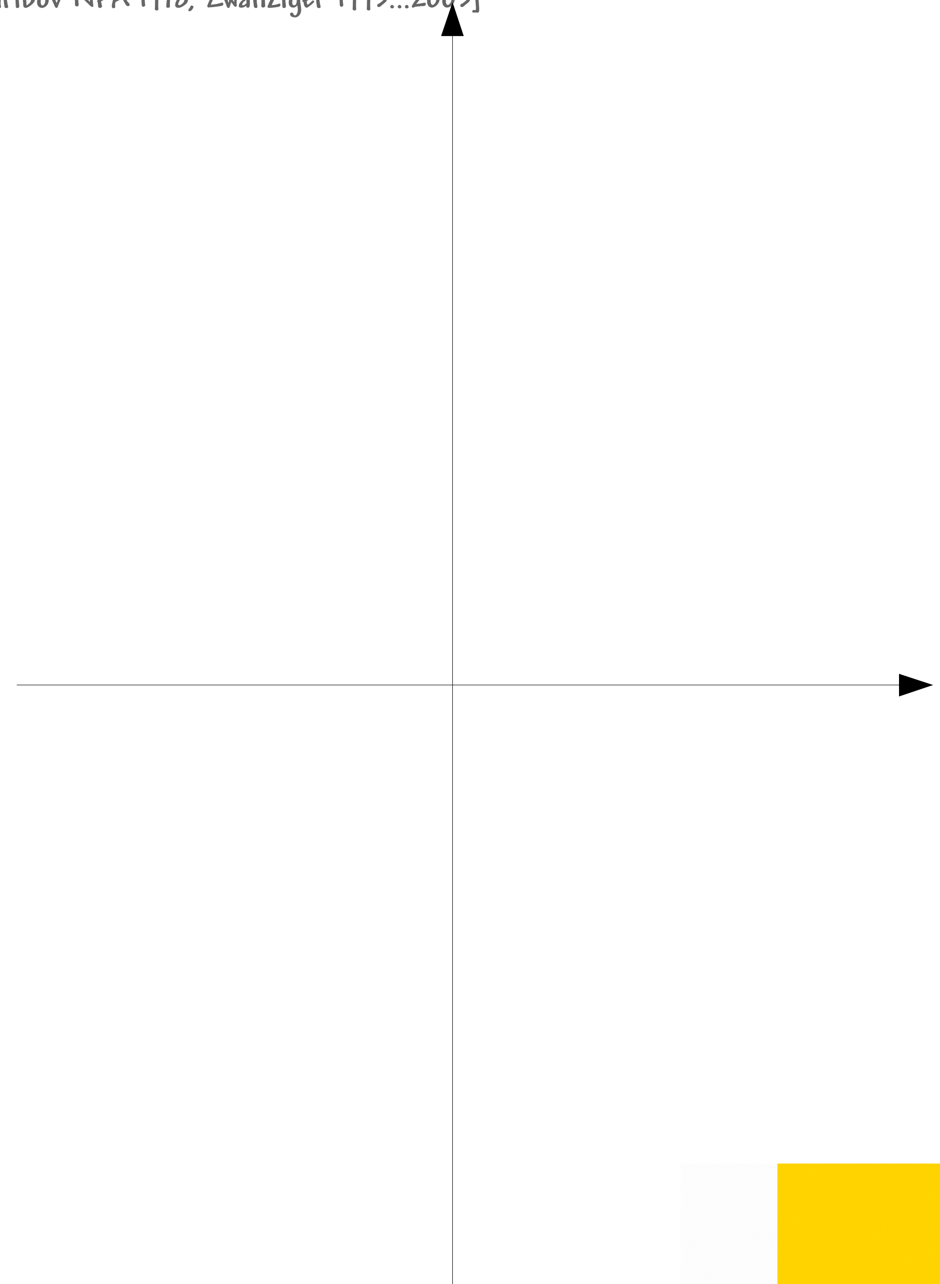

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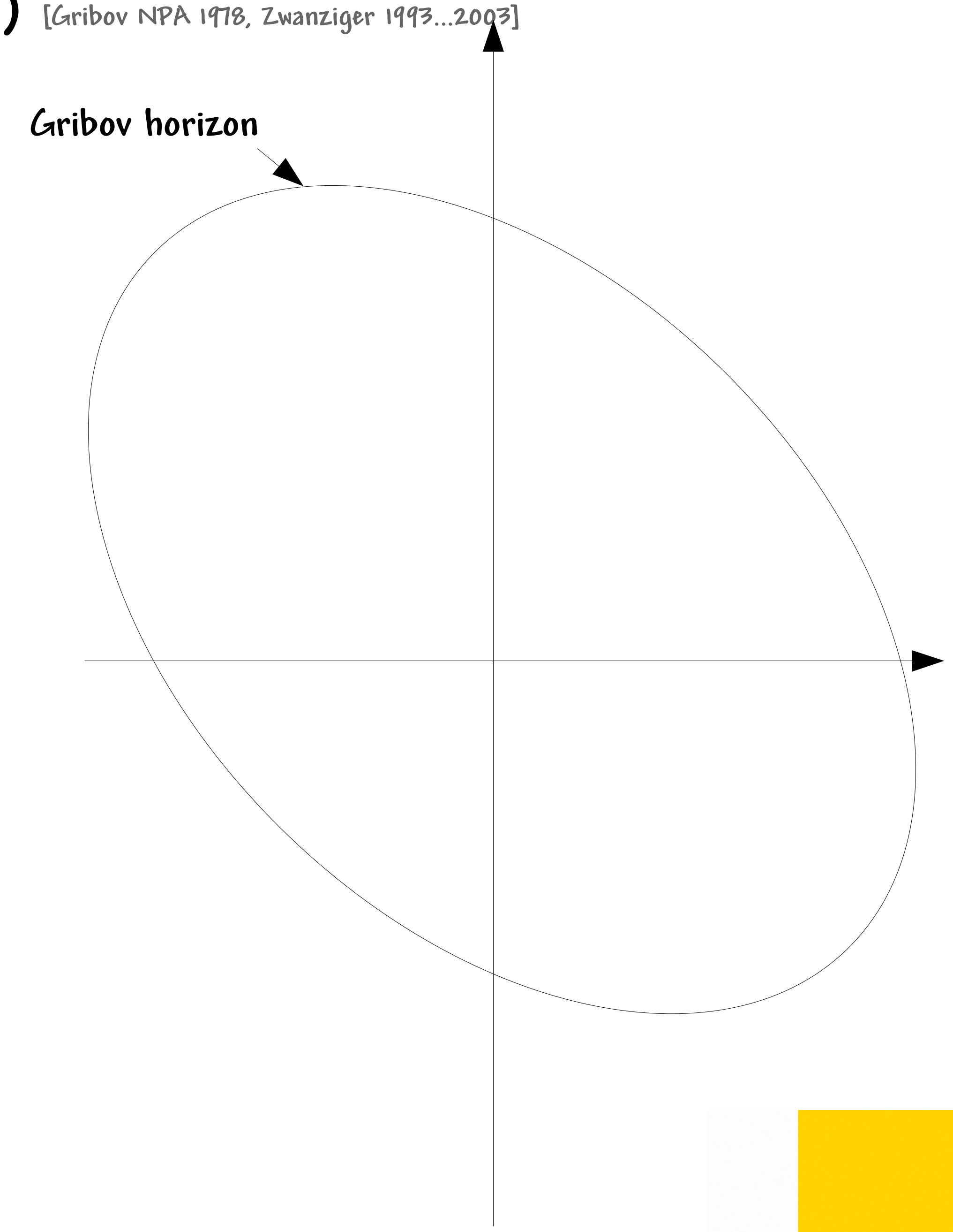

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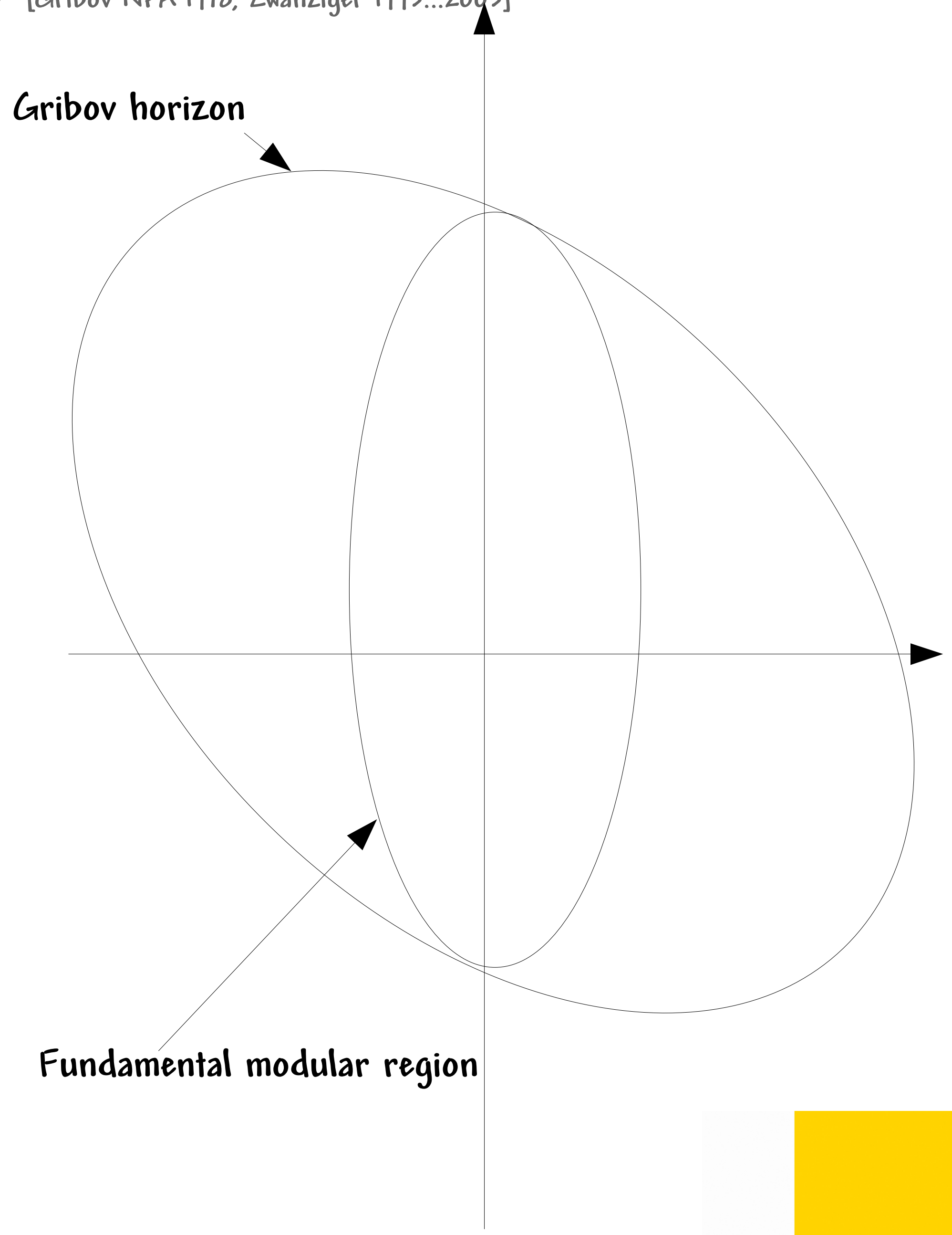

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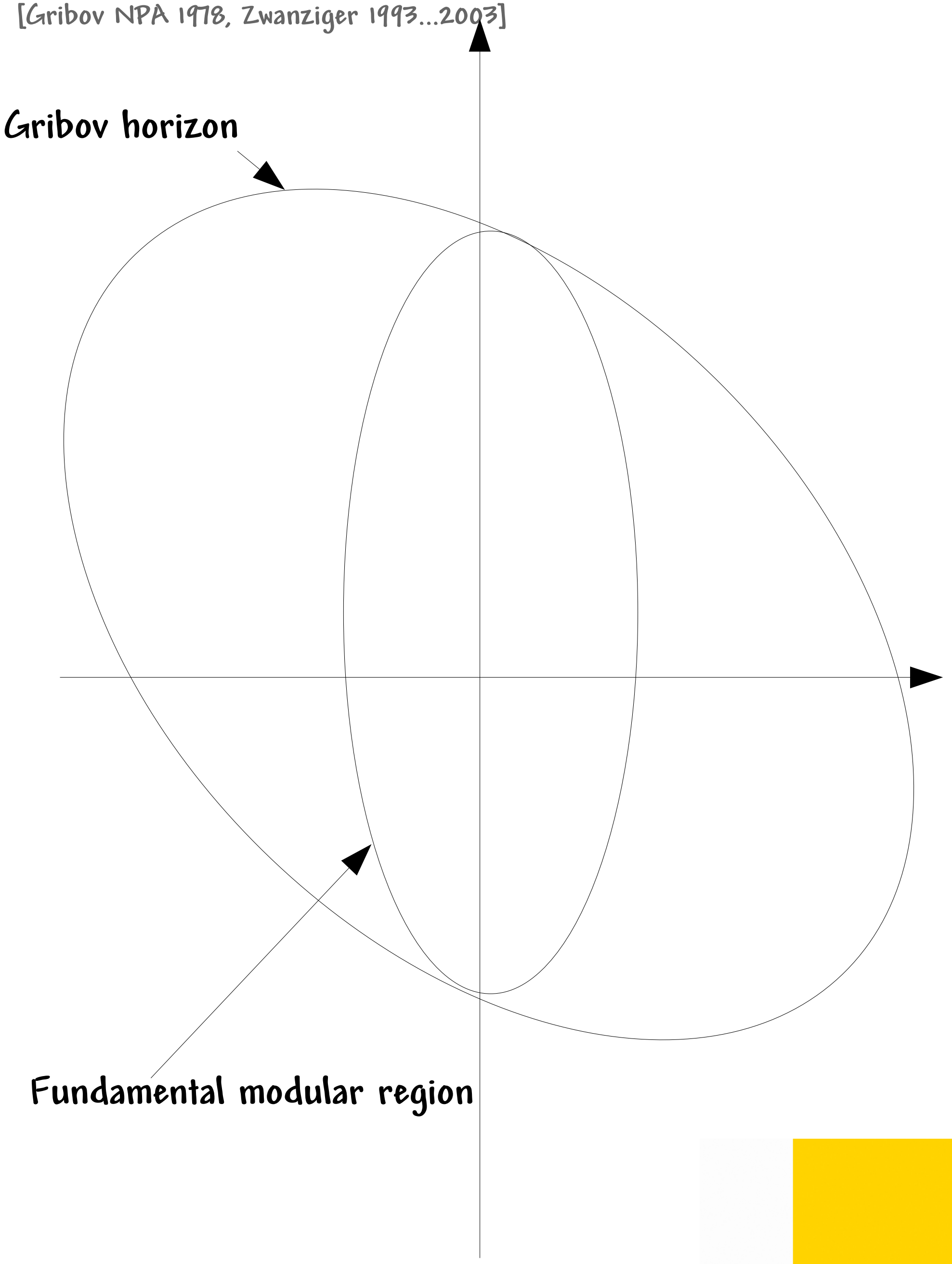

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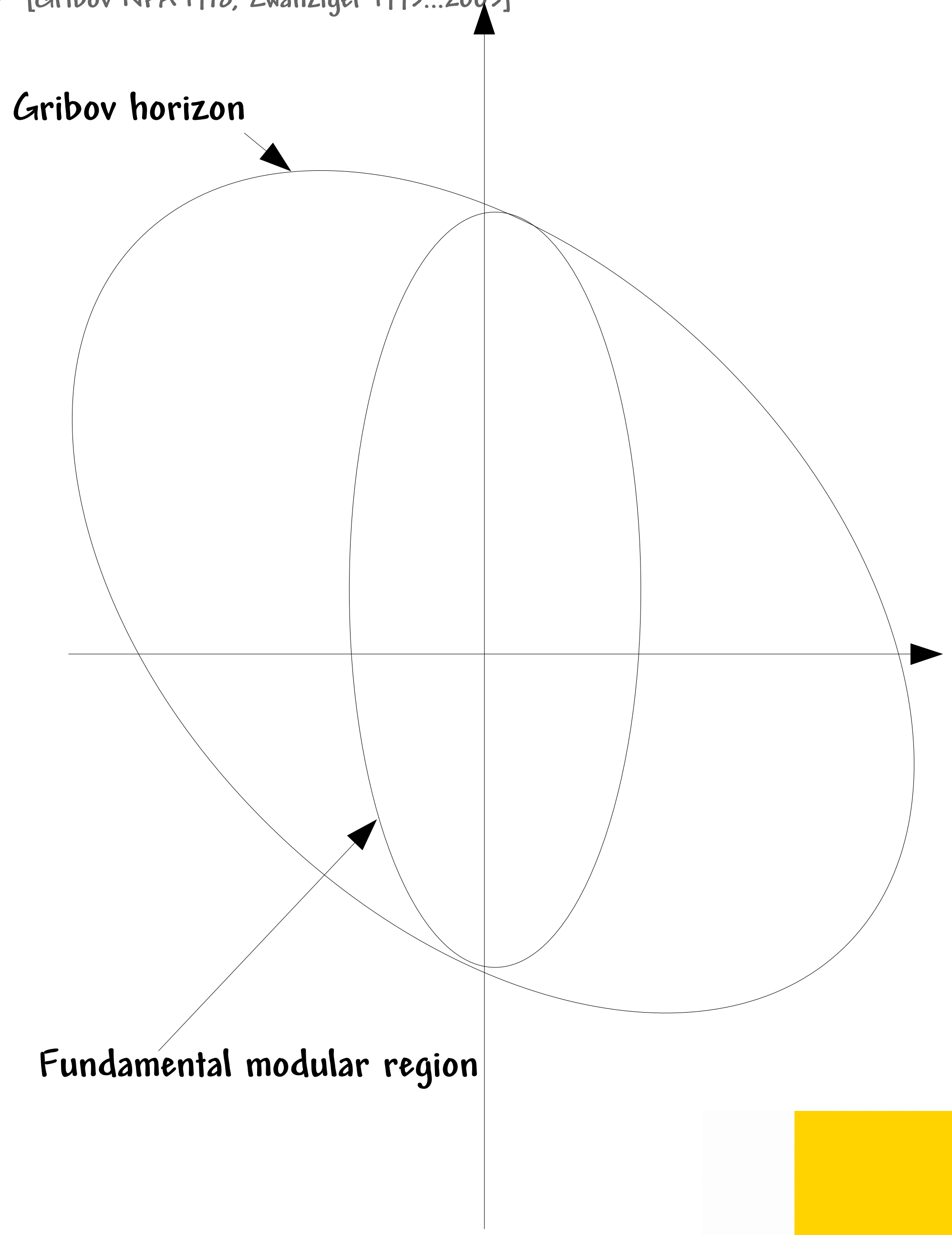

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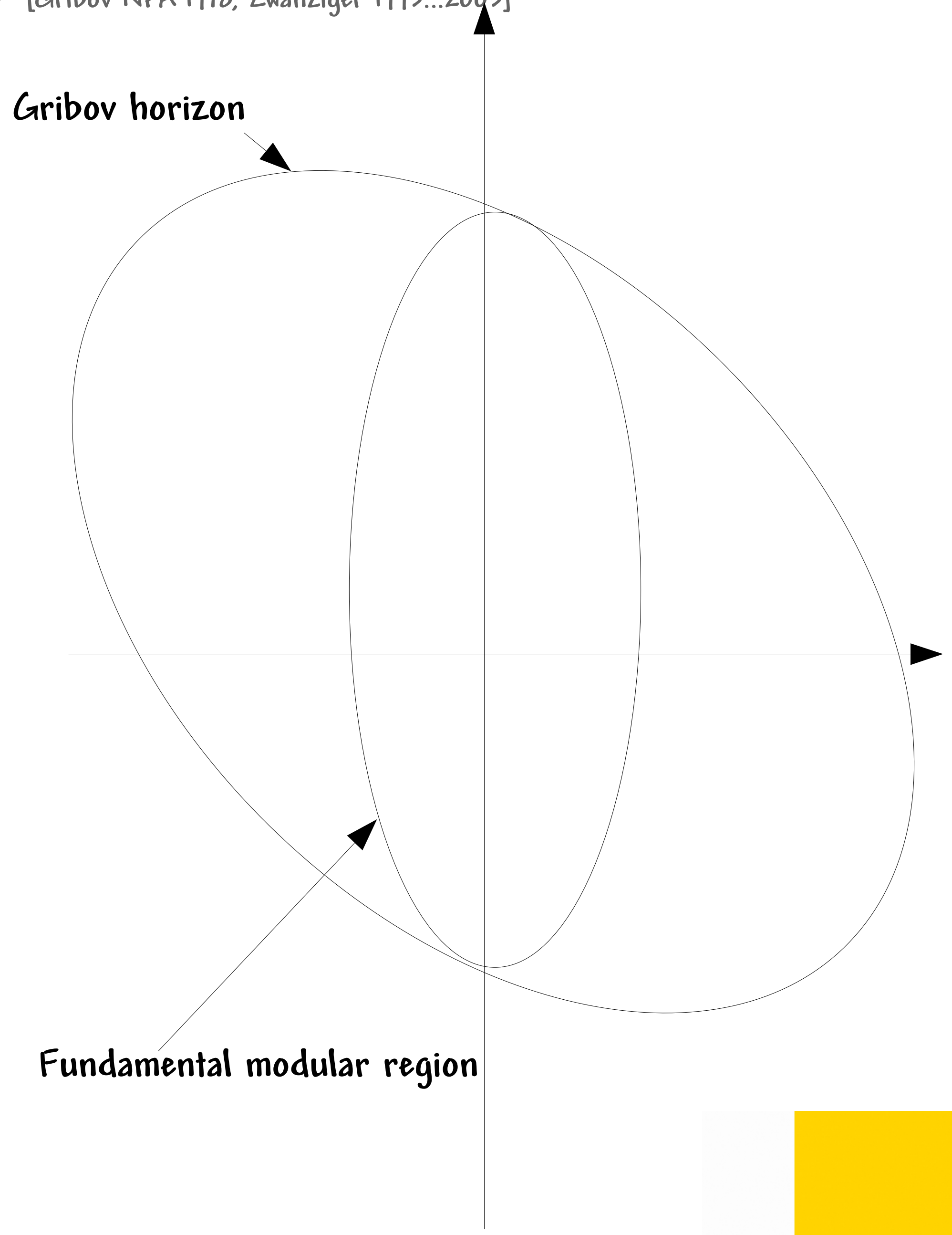
  - Requires a tool for investigations



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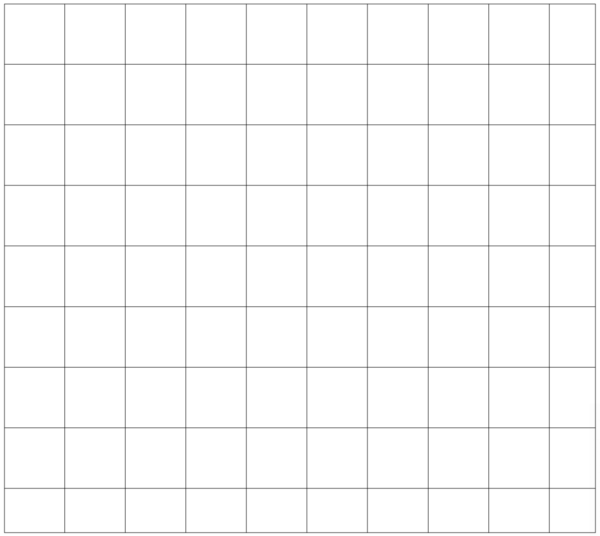

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- Calculate observables using path integration
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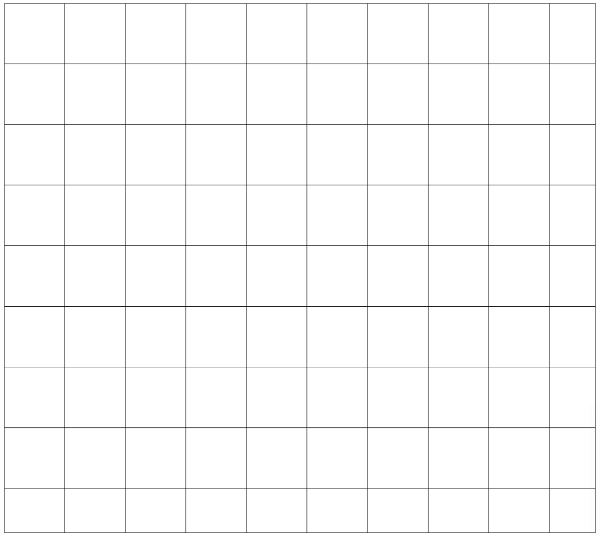

Lattice calculations

- Take a finite volume – usually a hypercube
- Discretize it, and get a finite, hypercubic lattice
- Calculate observables using path integration
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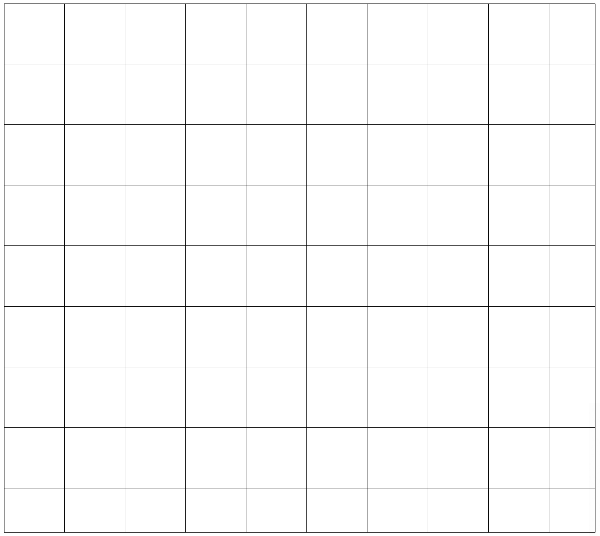
- Artifacts
  - Finite volume/discretization
  - Zero momentum problematic



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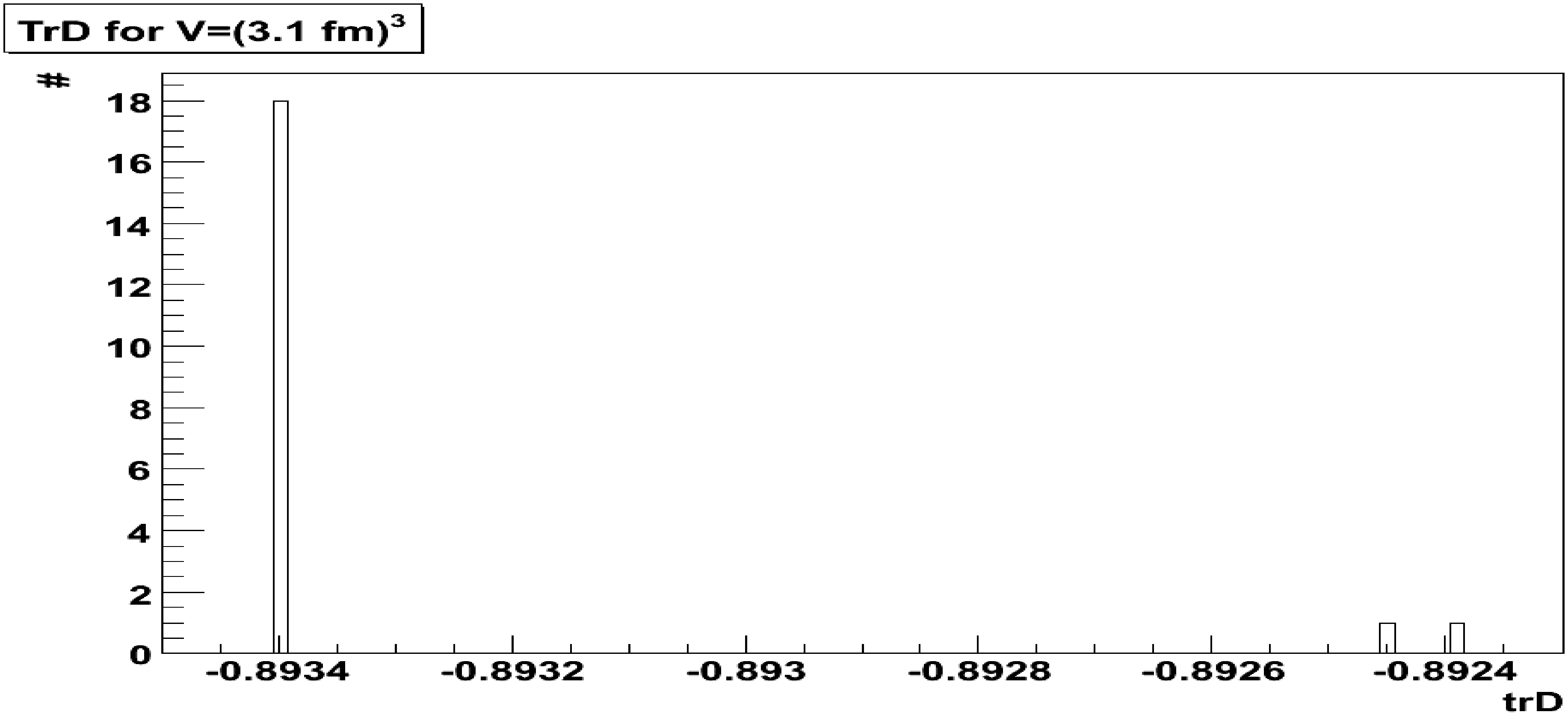
    - How to decide whether two copies are different?
      - Non-trivial problem. Requires in principle to show a connection by a non-trivial gauge transformation
      - Practical solution: Require sufficiently different value of TrD
      - Other are dismissed as numerical/lattice artifacts
      - Is this good? Will possibly work only on a lattice!

TrD on the residual gauge orbit [3d, beta=3.46, Maas, unpublished]



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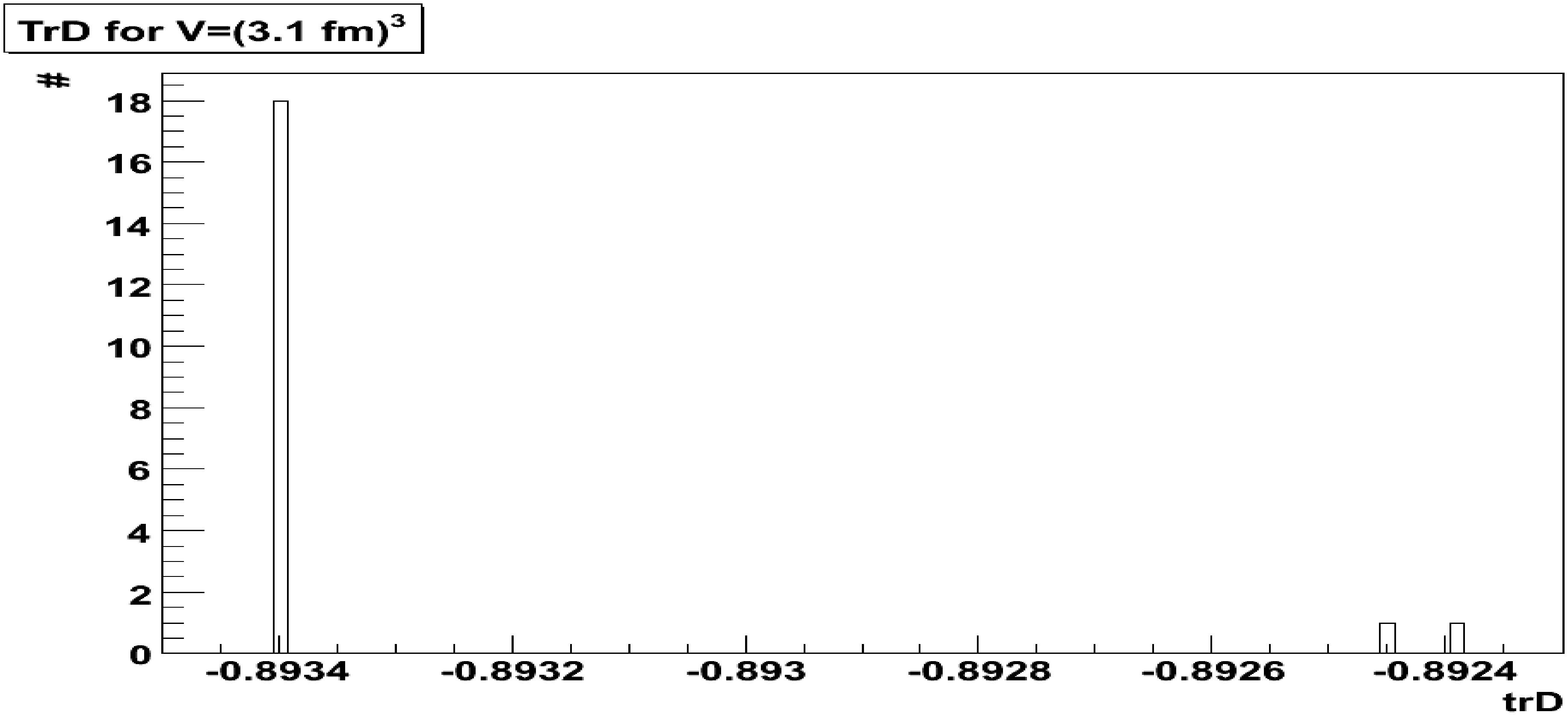

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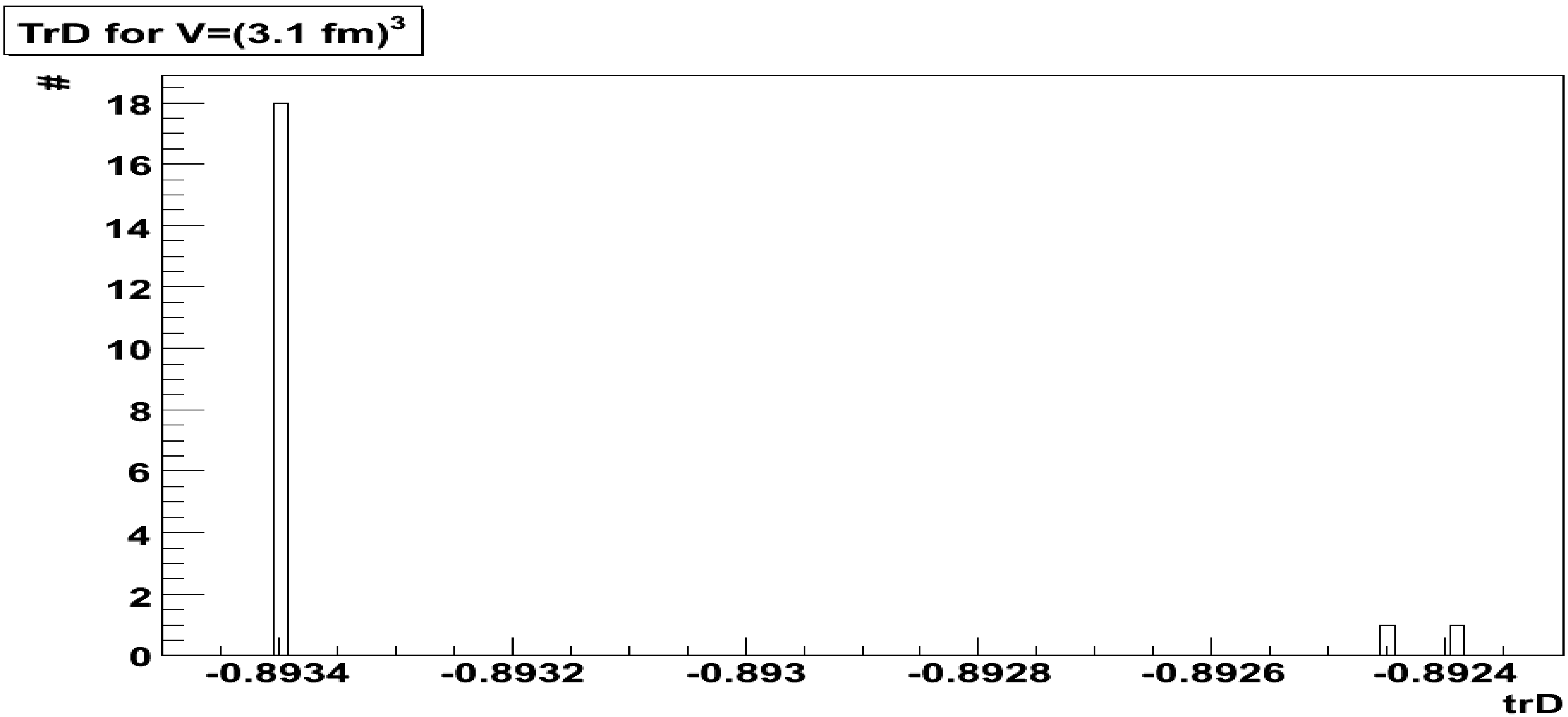
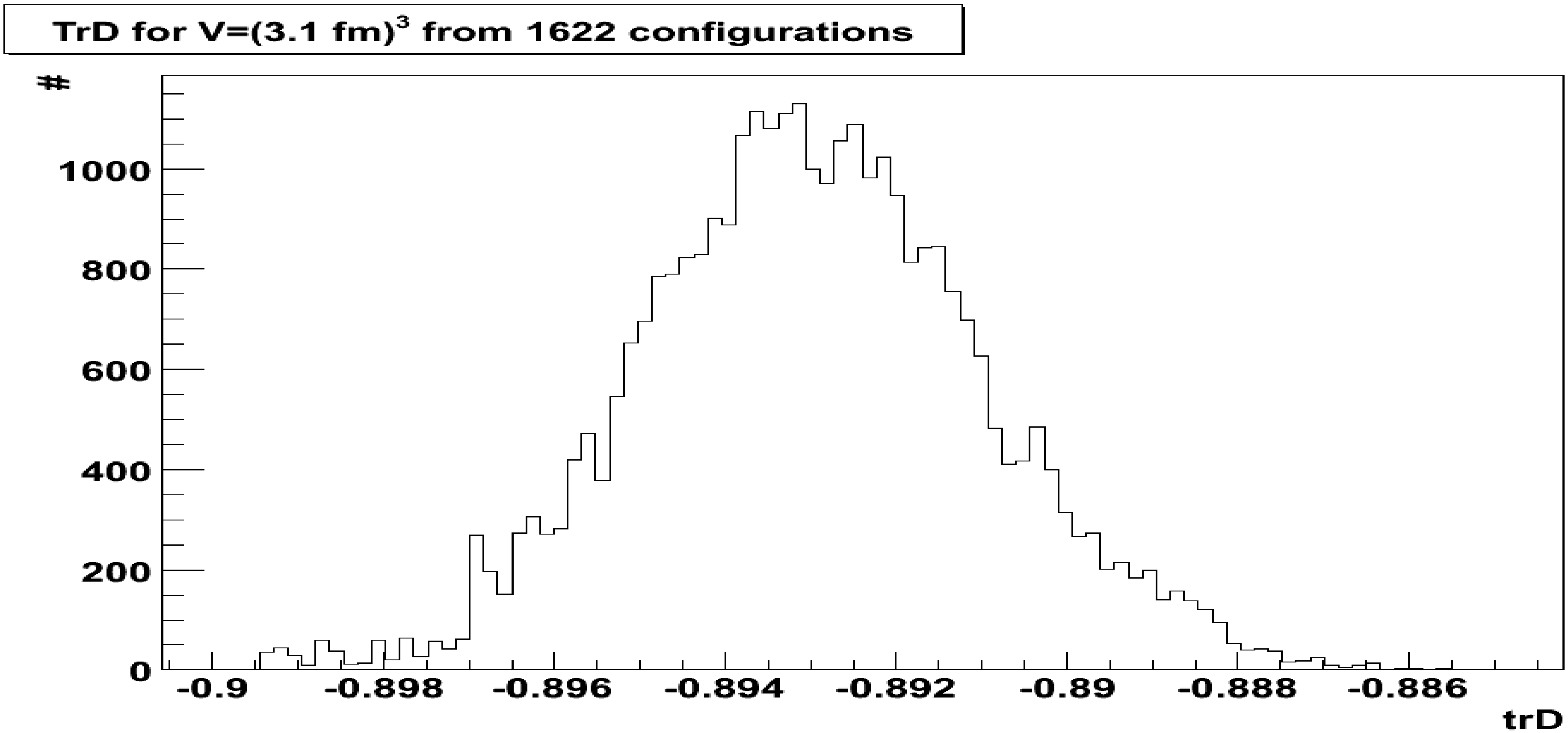


- Rather different values of TrD for different Gribov copies
- Many artifical copies of one particular copy: Attractive basin in TrD
- Difference in peak is below 10^{-10}: Numerical/lattice artifacts
- Pragmatic defintion: Copy requires a separation of 10^{-5} in TrD

Distribution of TrD [3d, beta=3.46, Maas, unpublished]



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  - Never a real Gaussian

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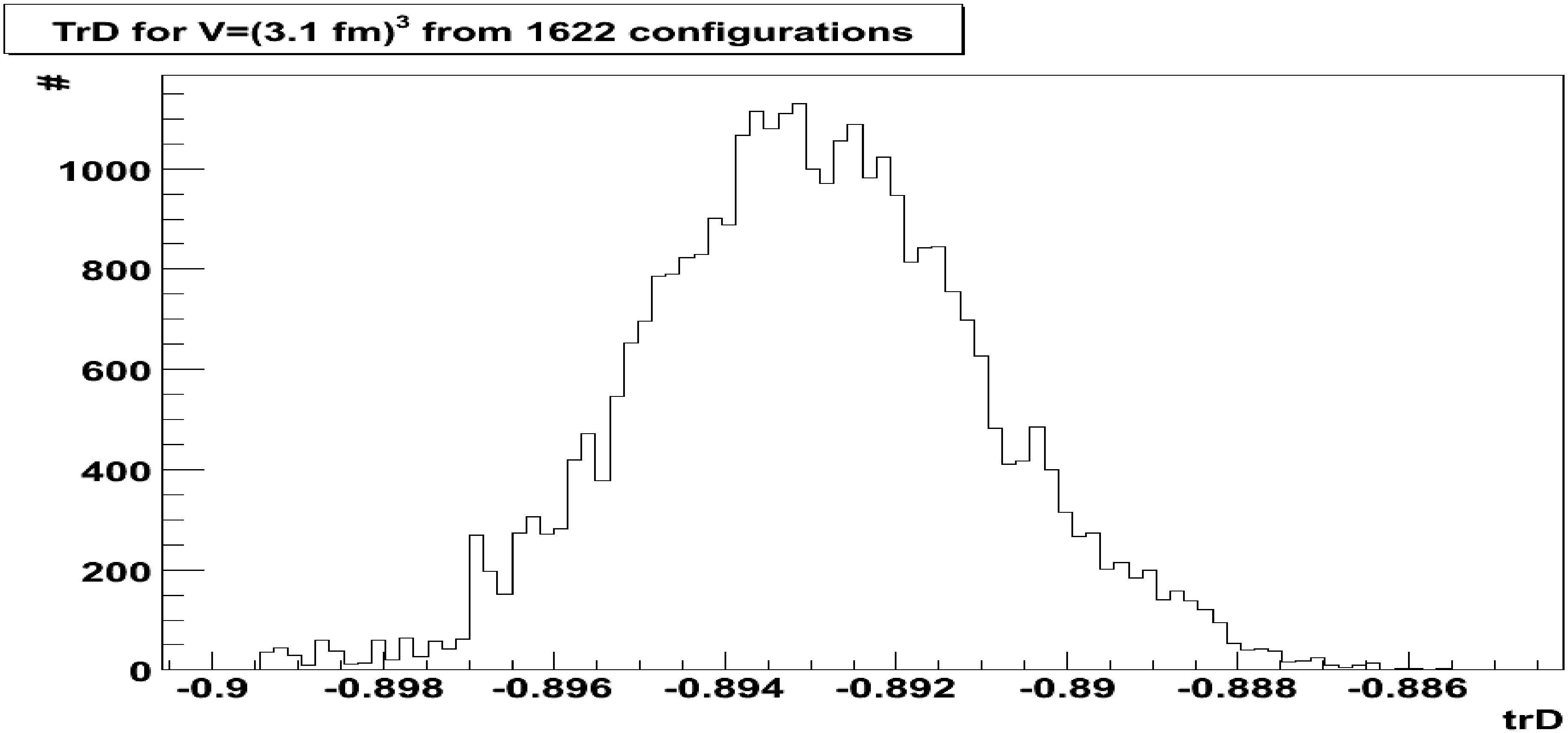

- Distribution is almost Gaussian
  - Never a real Gaussian
- Variation of TrD corresponds to varying gluon propagator

TrD as a gauge parameter [3d, beta=3.45, Maas, unpublished]

- Define gauge corridor as [max TrD,min TrD]

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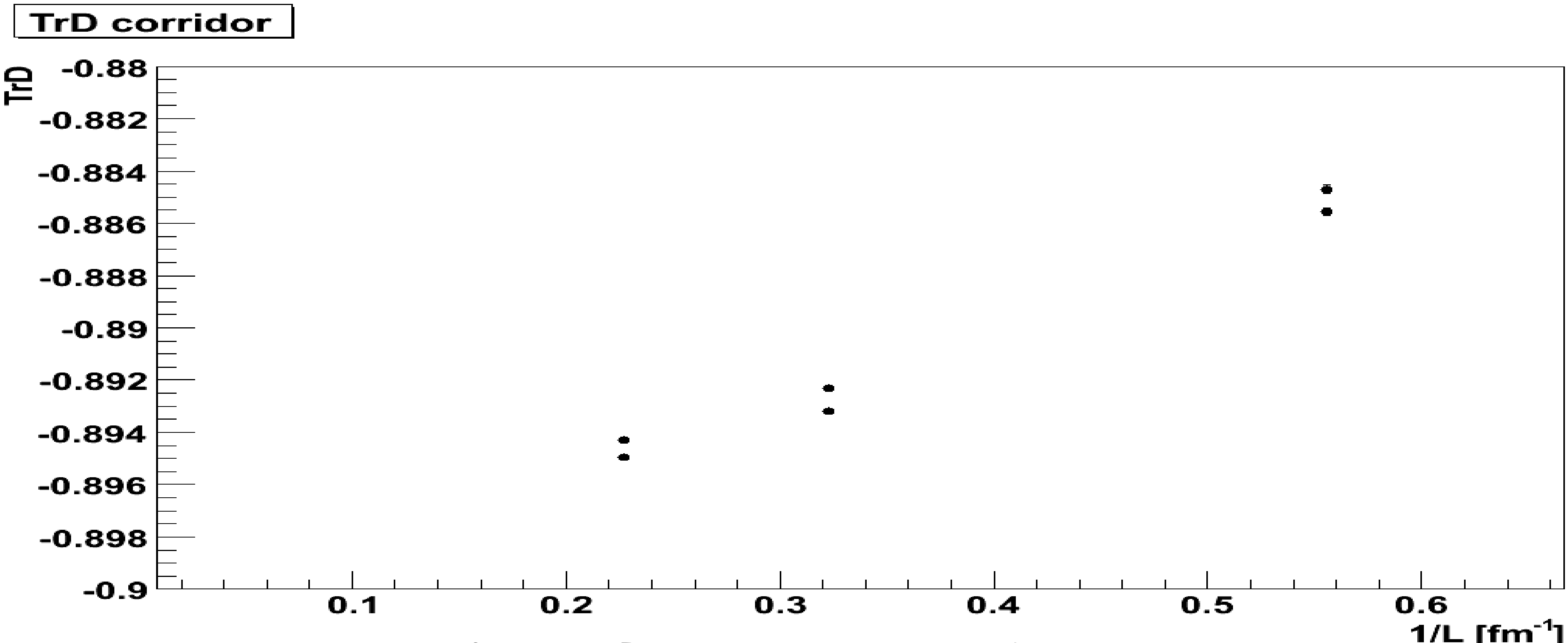

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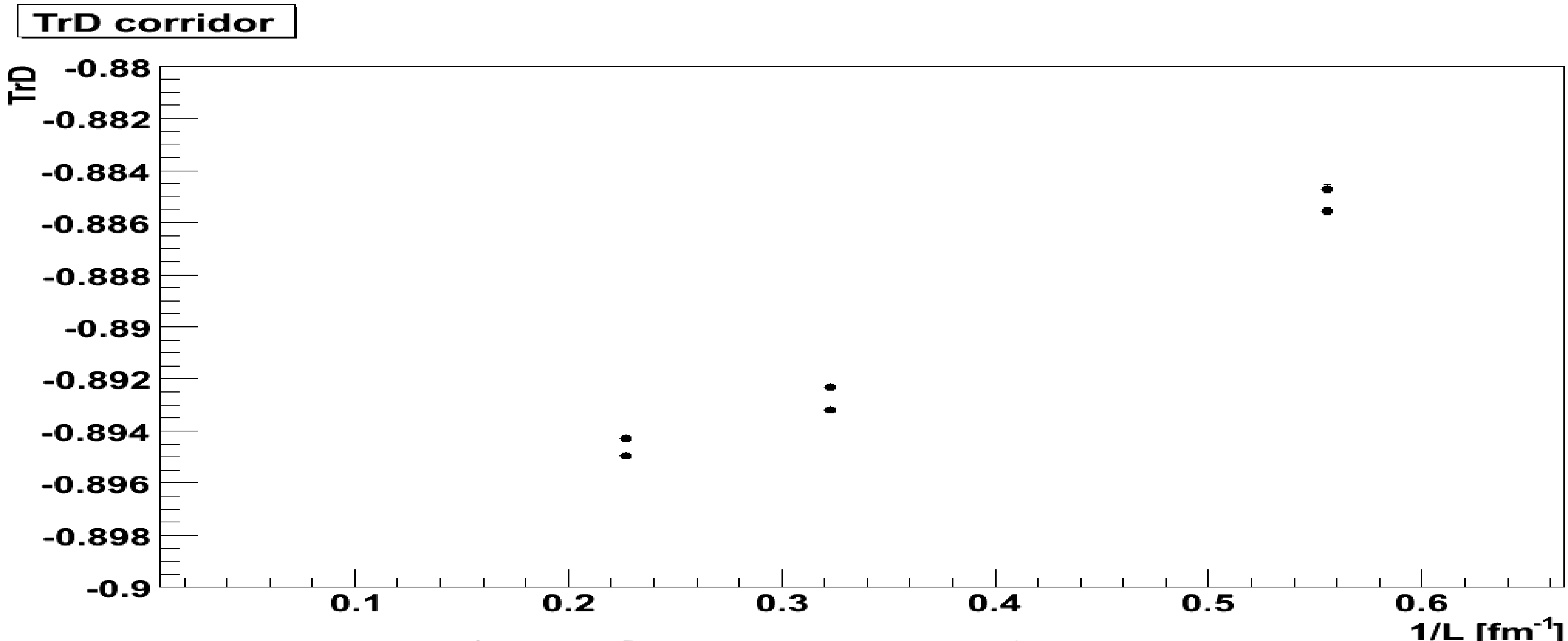
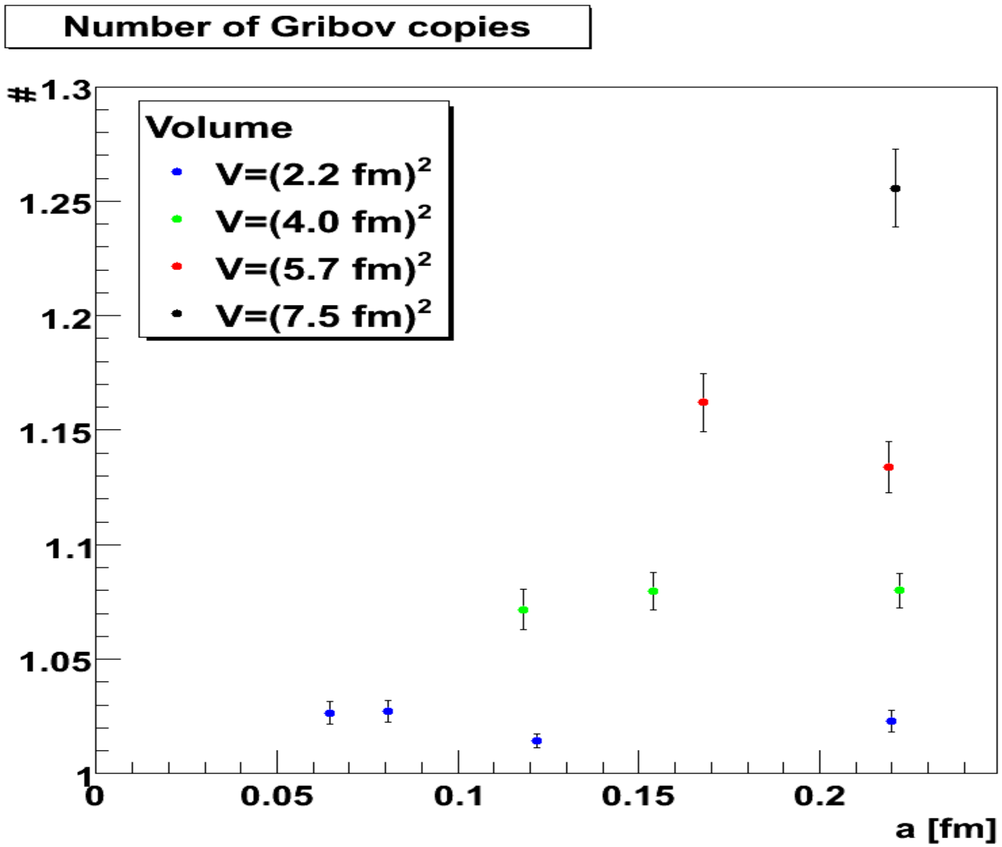
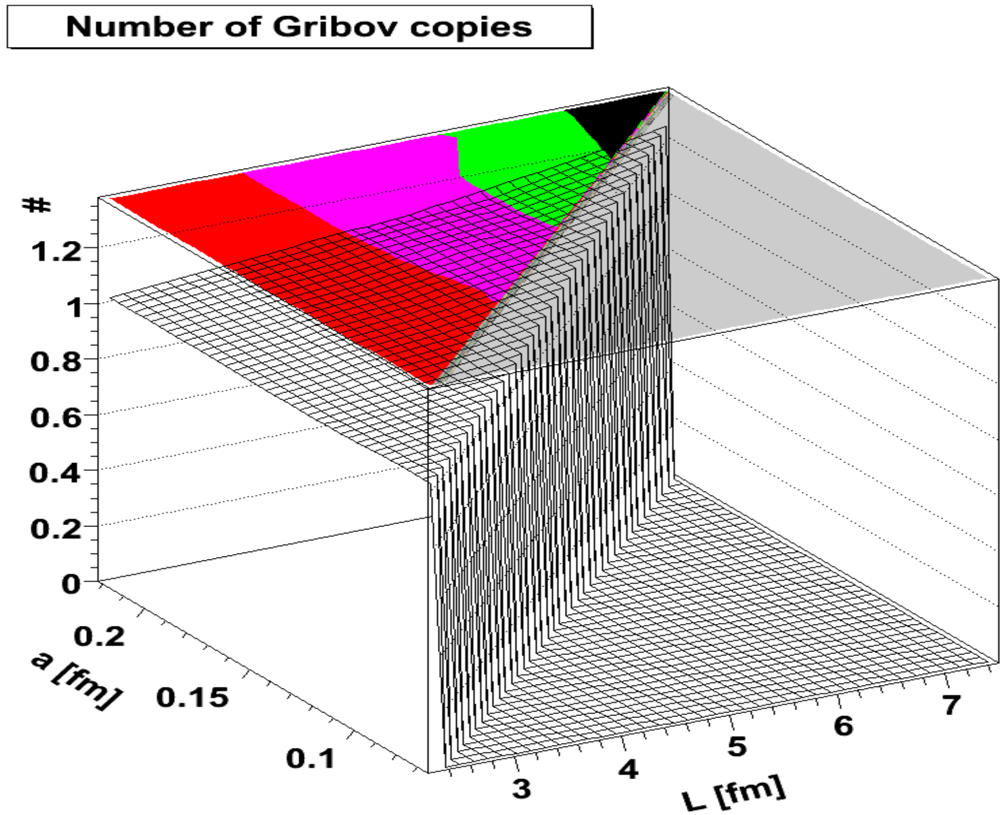


- Define gauge corridor as [max TrD,min TrD]
- TrD is not showing a strong volume dependence
  - For some theories known to degenerate [de Forcrand, 1994]
  - Degeneracies would support the Zwanziger entropy conjecture [Zwanziger, 2003]

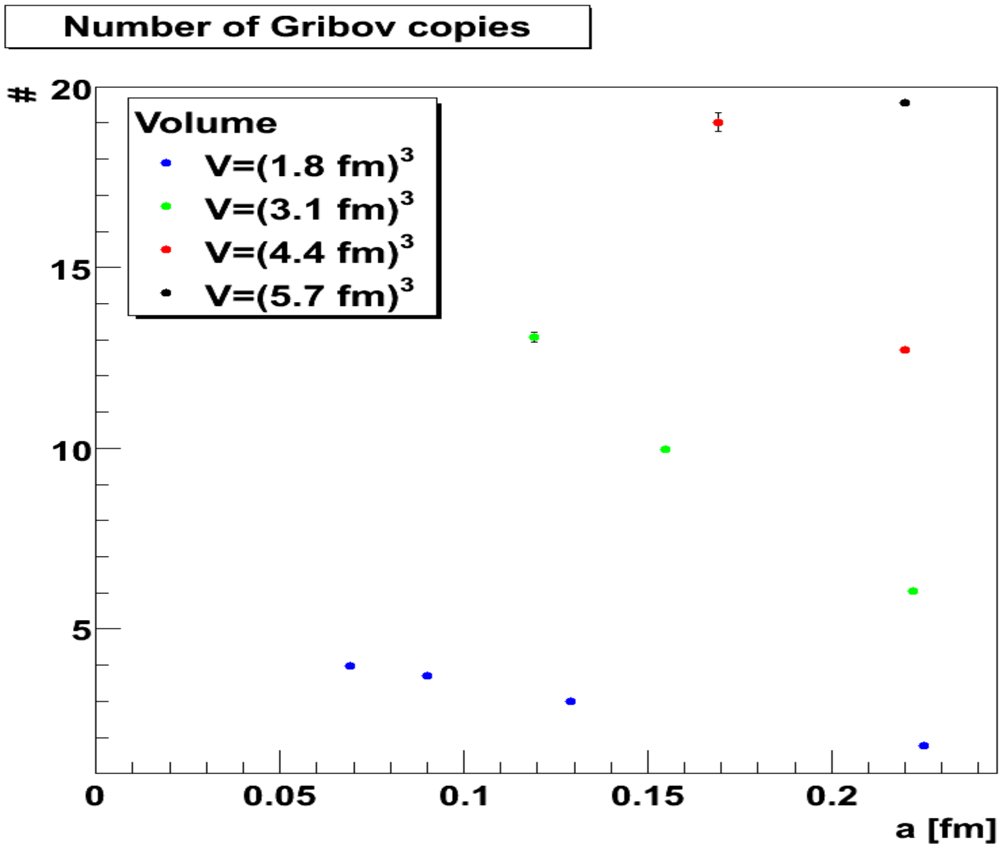
Number of Gribov copies in two dimensions [Maas, unpublished]

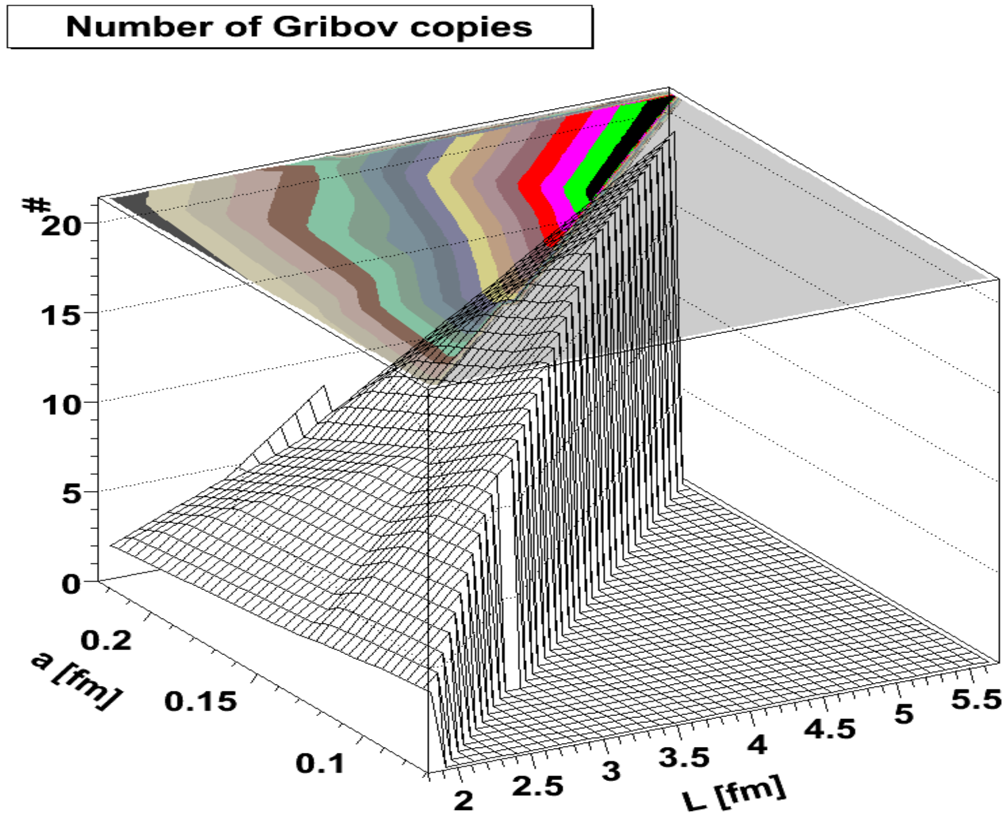




- Very slow evolution with volume and discretization
- Very large volumes will be needed to see an effect

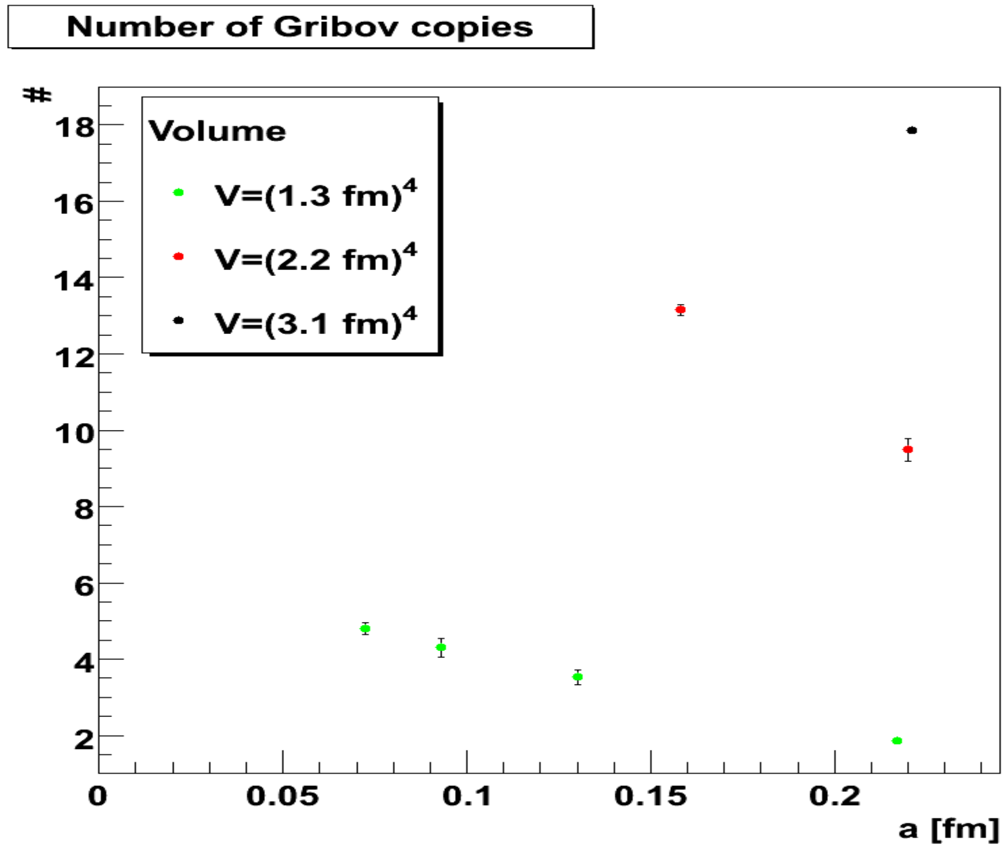
Number of Gribov copies in three dimensions [Maas, unpublished]

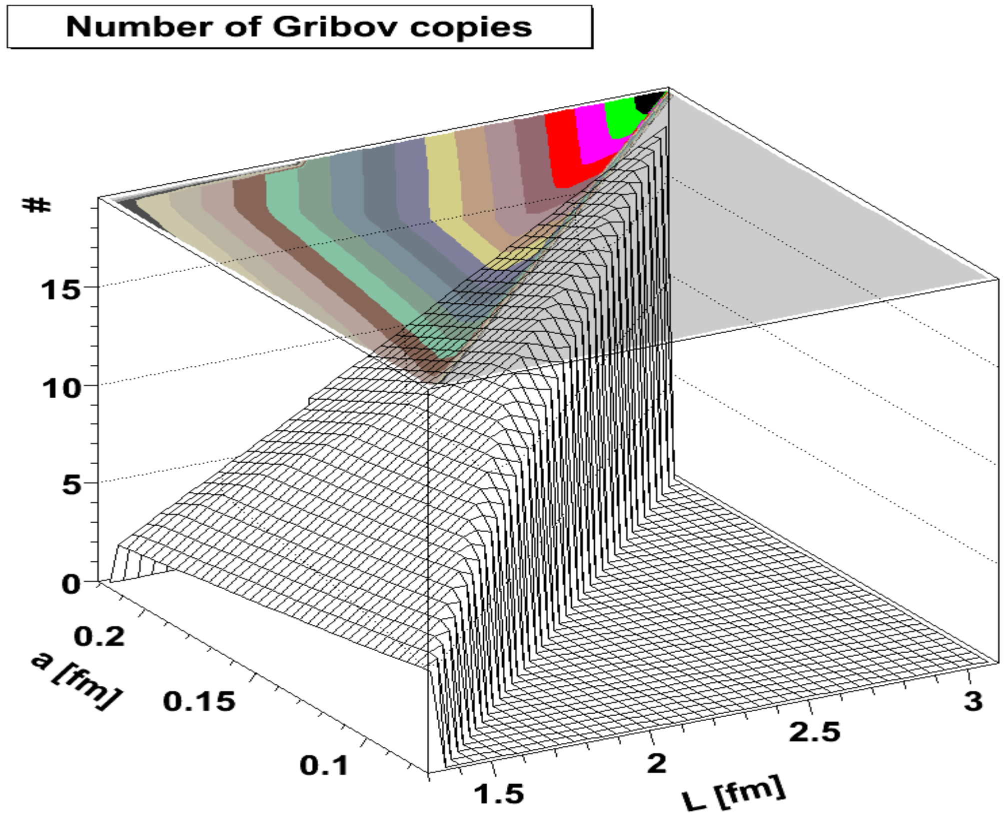




- Evolution with volume and discretization
- Already rather small volumes have a large number of copies

Number of Gribov copies in four dimensions [Maas, unpublished]

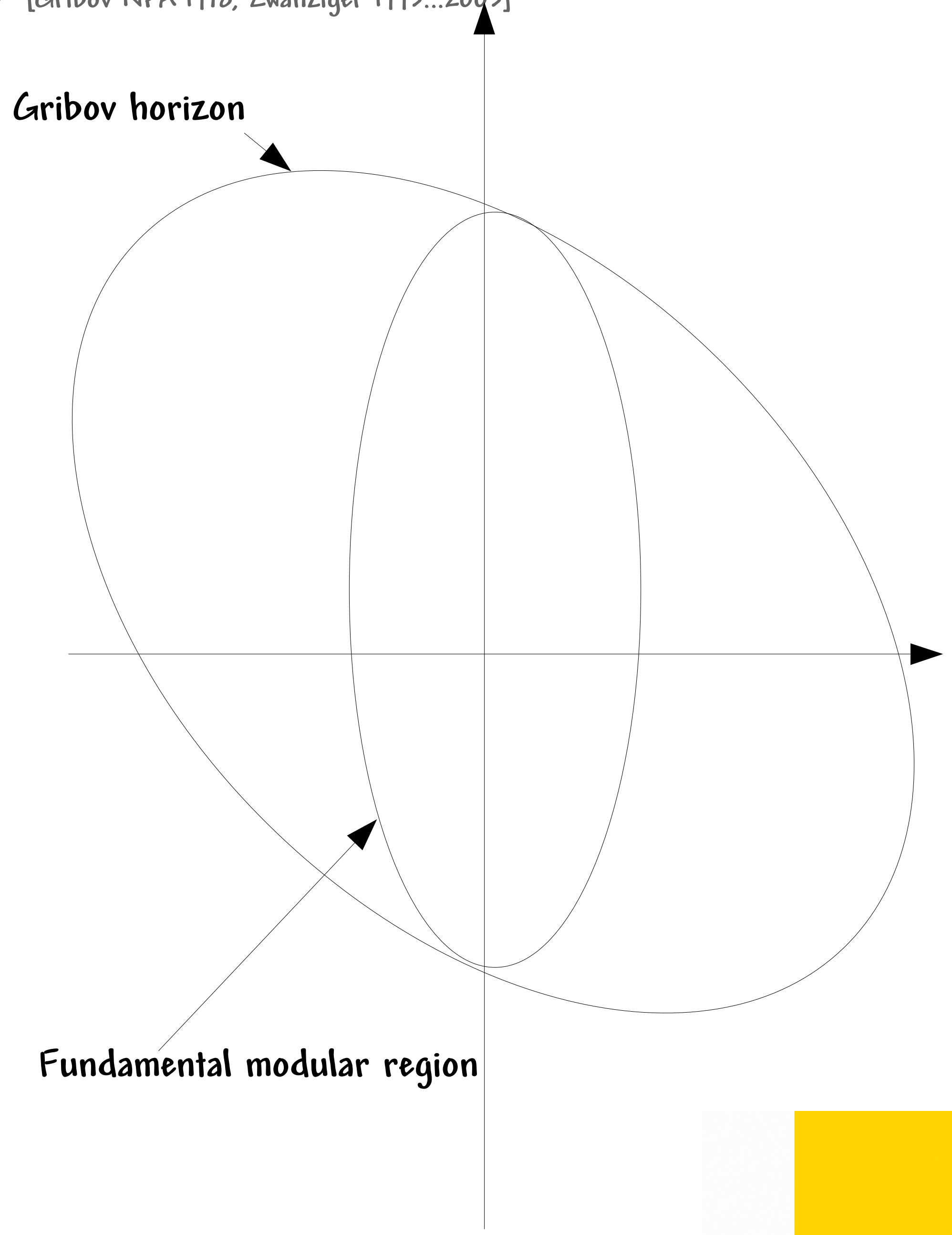




- Large number of copies even on small volumes
- Big effects can be expected
- Number of Gribov copies rises quickly with the number of dimensions

Configuration space (artist's view) [Gribov NPA 1978, Zwanziger 1993...2003]

- Different approach: Enlarge the search space
- Gribov horizon encloses all field configurations with positive Faddeev-Popov operator $(-\partial_\mu D_\mu)$
- Includes the fundamental modular region
- All gauge orbits pass through this region
  - Many Gribov copies for each
- How to select a unique representative?



Constructing a global gauge condition

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- Not directly. But the ghost propagator is influenced by the eigenspectrum:

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- G(p) candidate for a characterization of a copy

Constructing a global gauge condition [Maas, unpublished, Maas, 2008, Fischer et al. 2008]

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- Is the renormalization-group invariant $b=G(0)/G(P)$ with $P>0$ fixed a possible second gauge parameter?
  - Fourier-transform at low momentum: Highly non-local
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- Investigate using the lattice
  - Not possible to access $G(0)$ on the lattice
  - Take lowest momentum instead: Finite-volume Artifact

Investigating the residual gauge orbit

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- There are many copies along the residual gauge orbit
- Is b different for each copy?
- If, how is b distributed?
- How is b correlated with the quantity TrD?
- How evolves b with volume and discretization?
- Is there a possibility to use b as a gauge parameter?

Distribution of b [3d, beta=3.46, Maas, unpublished]

- At small volumes: Small number of Gribov copies

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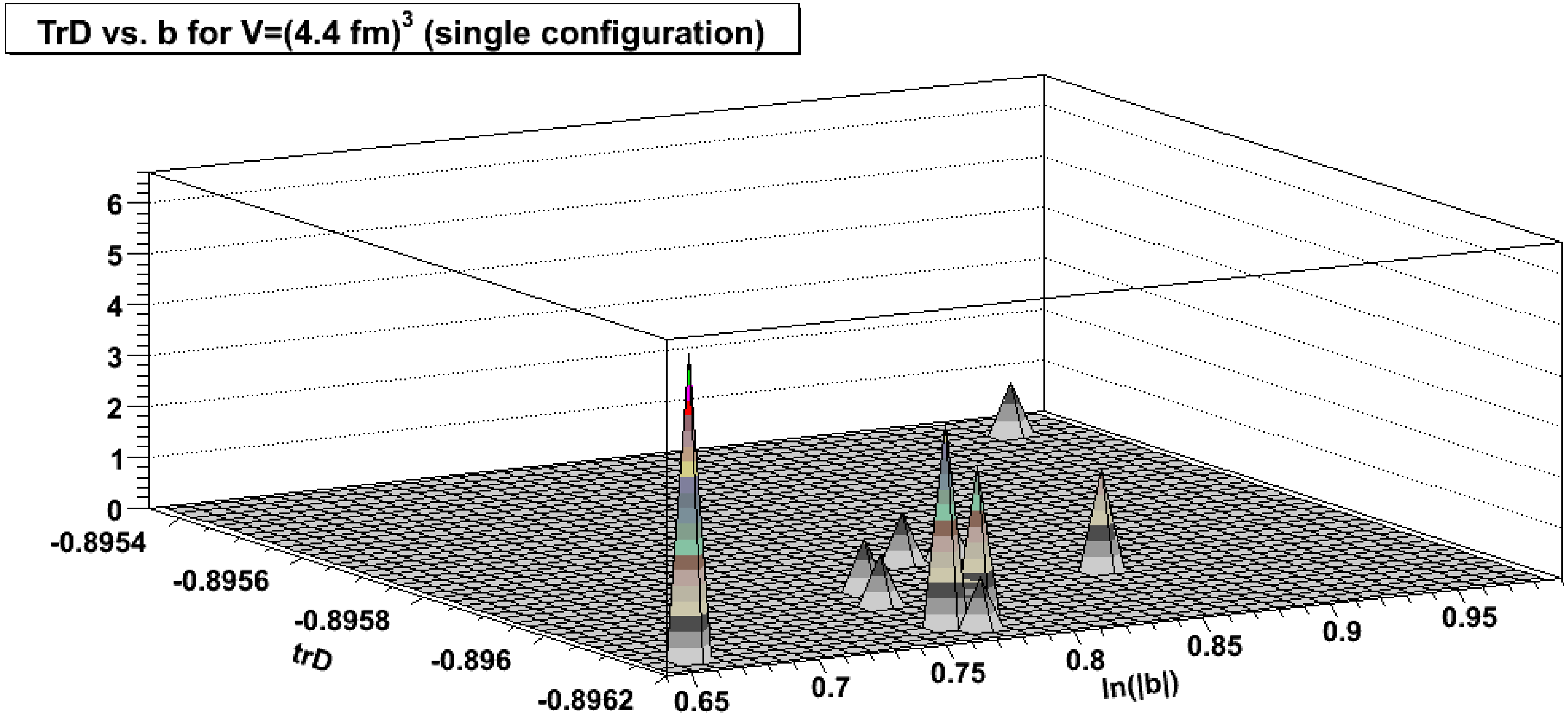
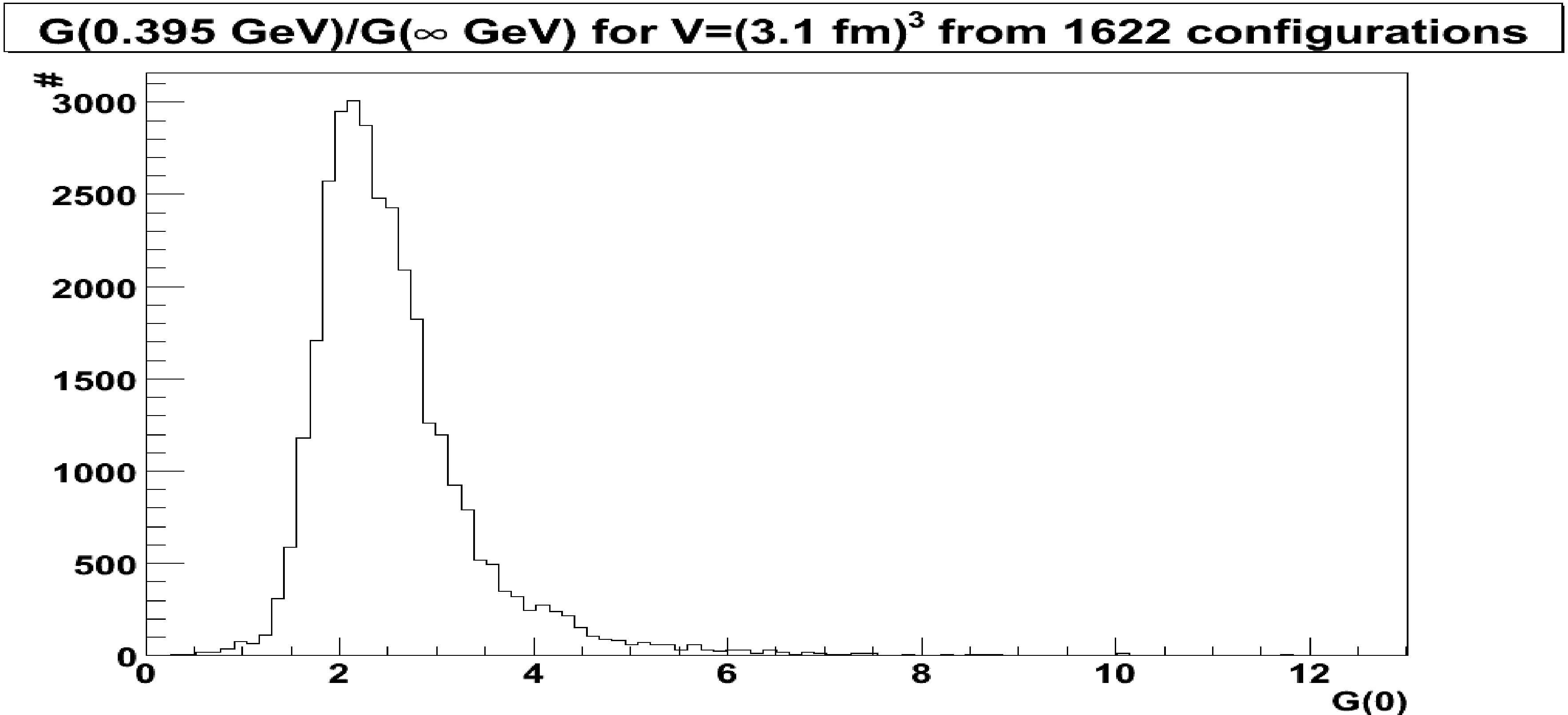


- At small volumes: Small number of Gribov copies
- b significantly different, if TrD is significantly different
  - Correspond to different continuum Gribov copies
  - b is relatively more different than TrD

Distribution of b on average [3d, beta=3.46, Maas, unpublished]



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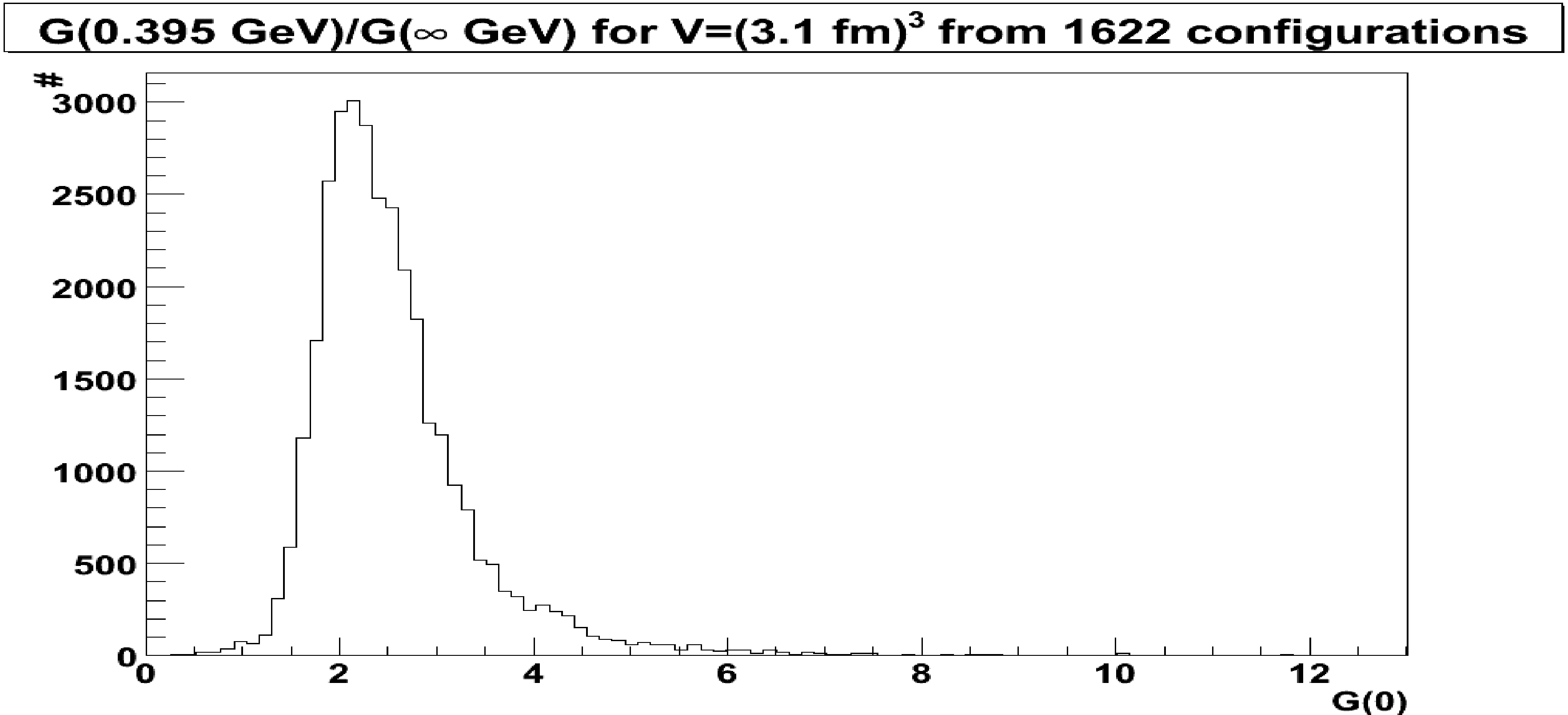

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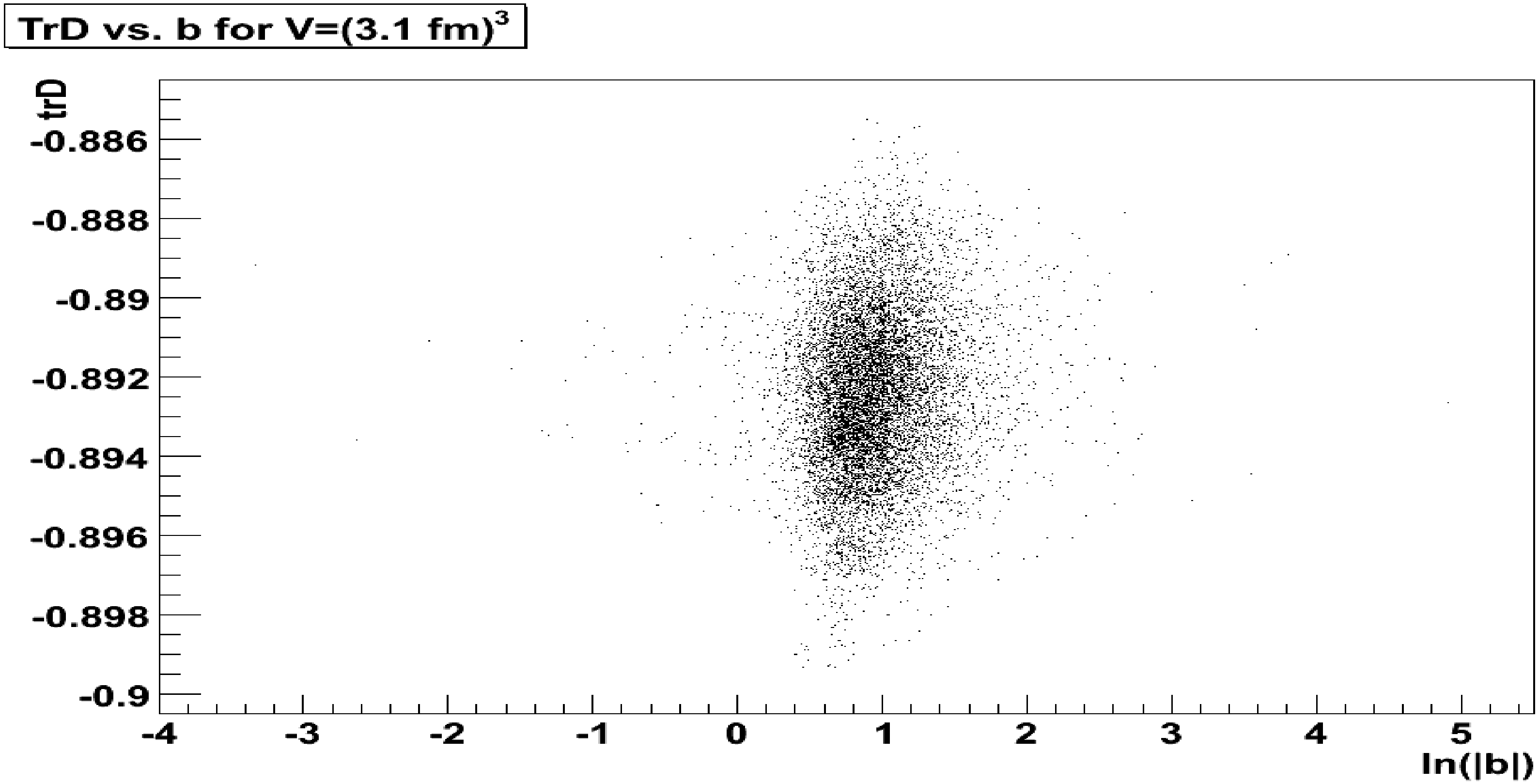

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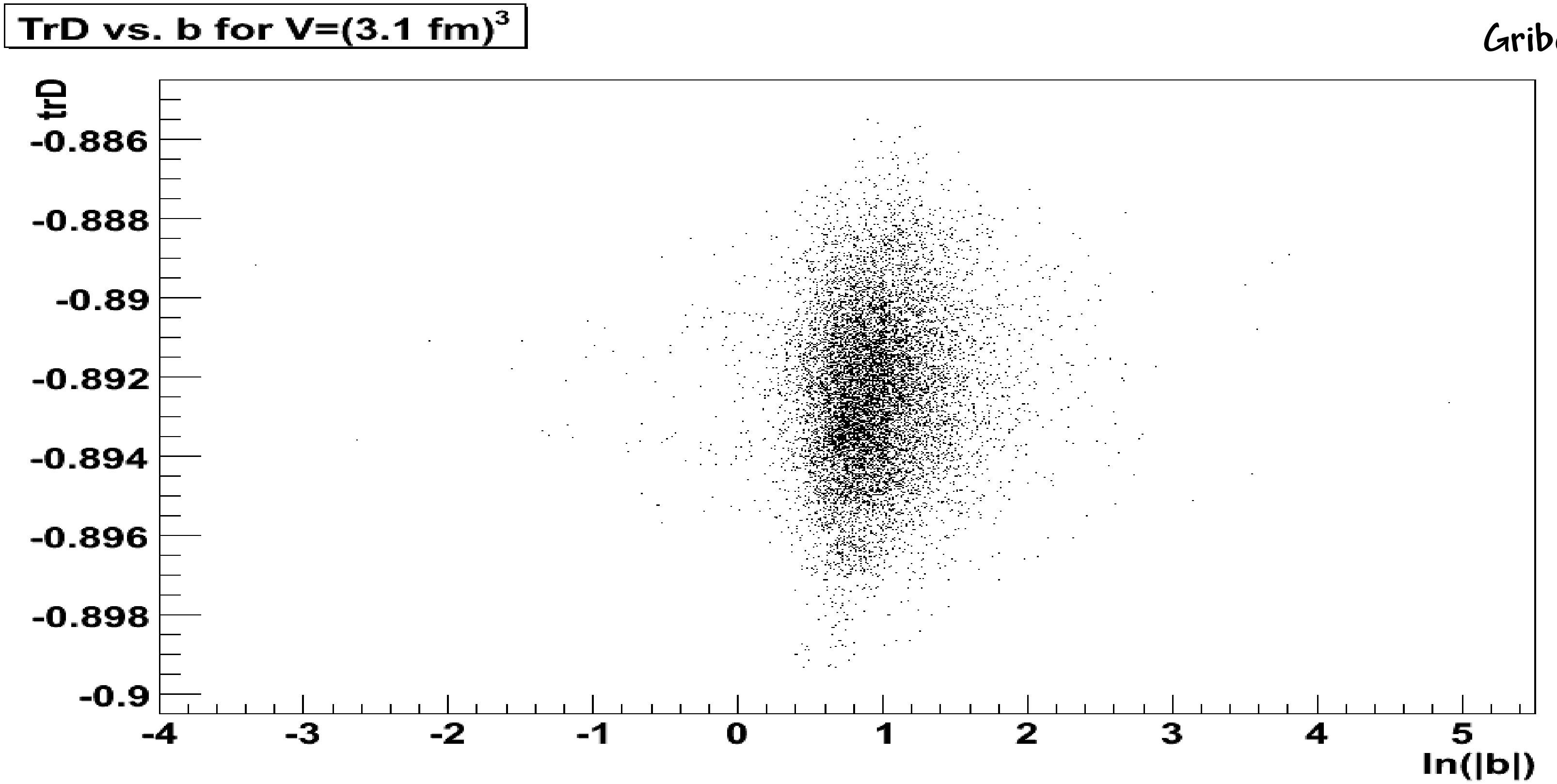
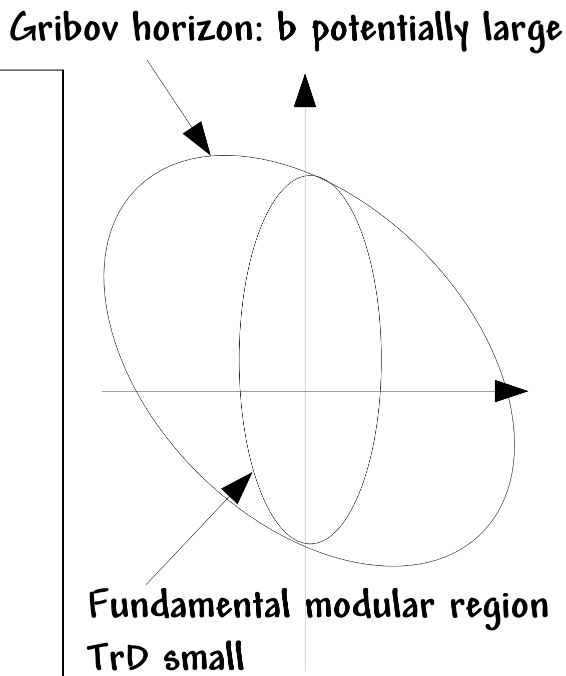
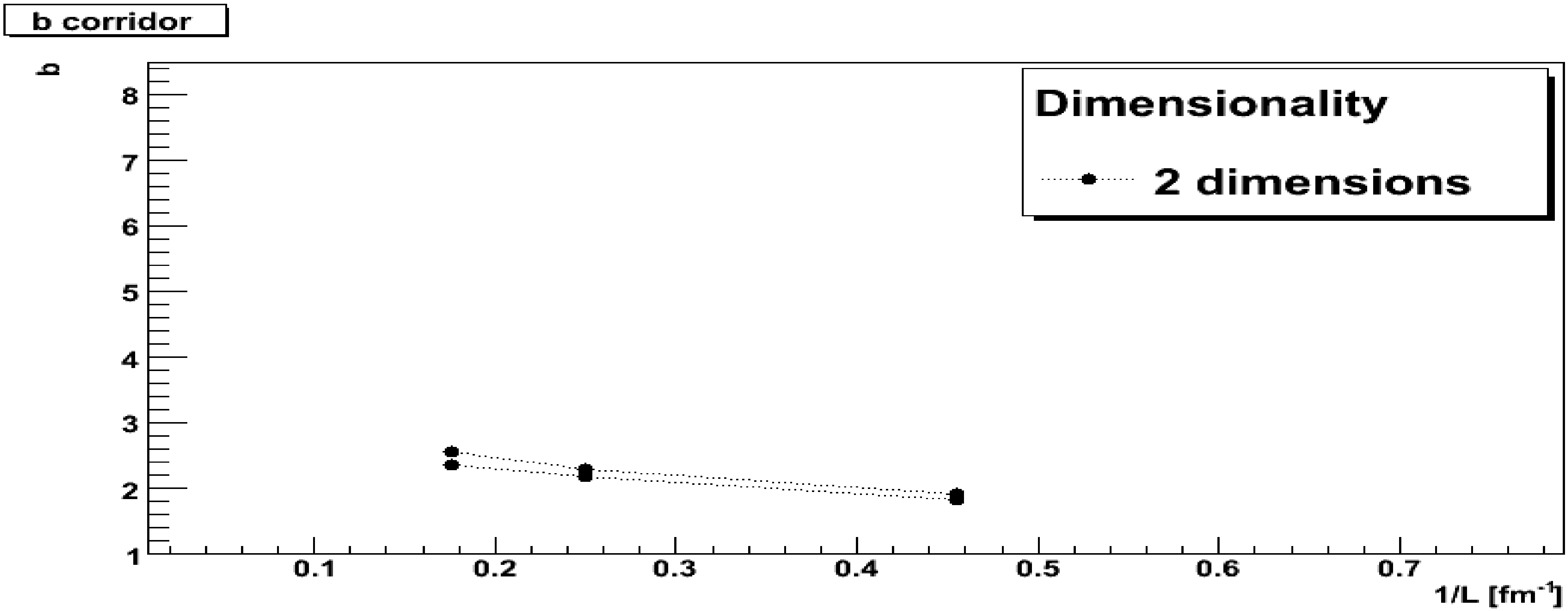




- No obvious correlation between both quantities
    - Only a small tendency that small b (far away from the Gribov horizon) correlate with small TrD (fundamental modular region)

The b gauge corridor [Maas, unpublished]



- The corridor [min b,max b] opens with volume
  - Overall scale partly a finite-volume effect
  - max b/min b not: Possible range of b values increases
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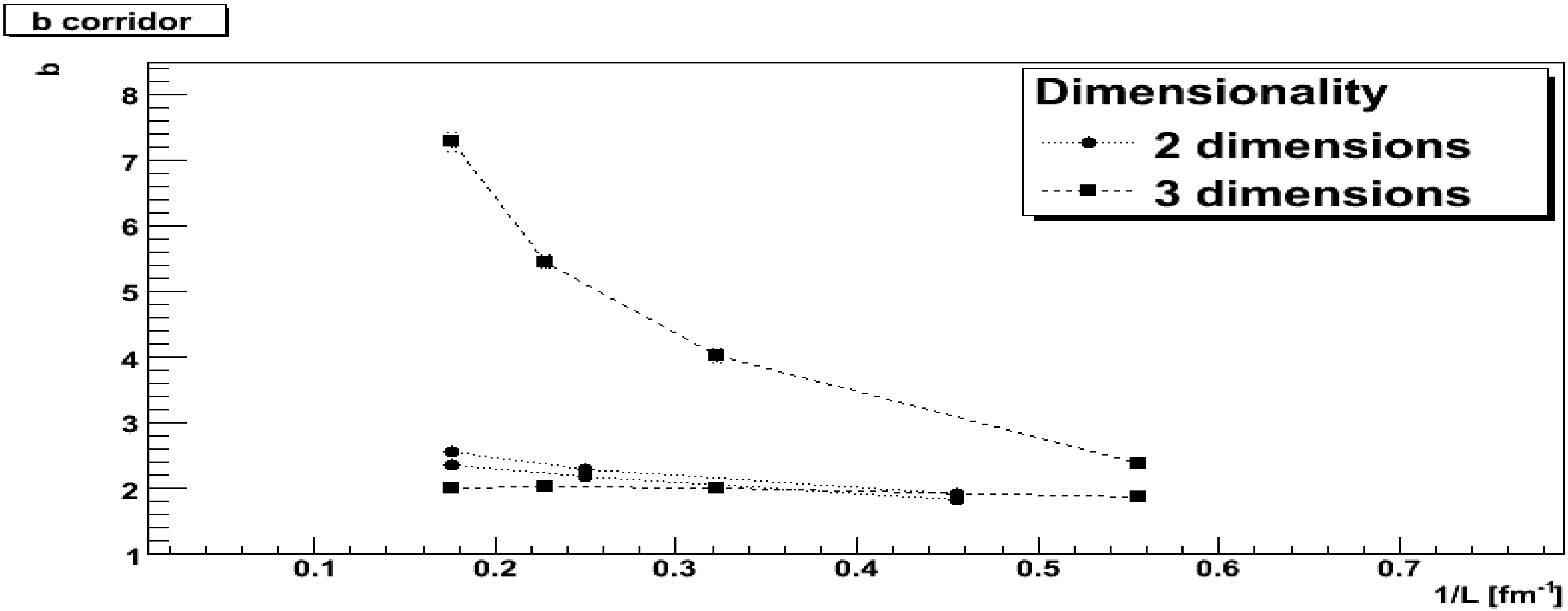

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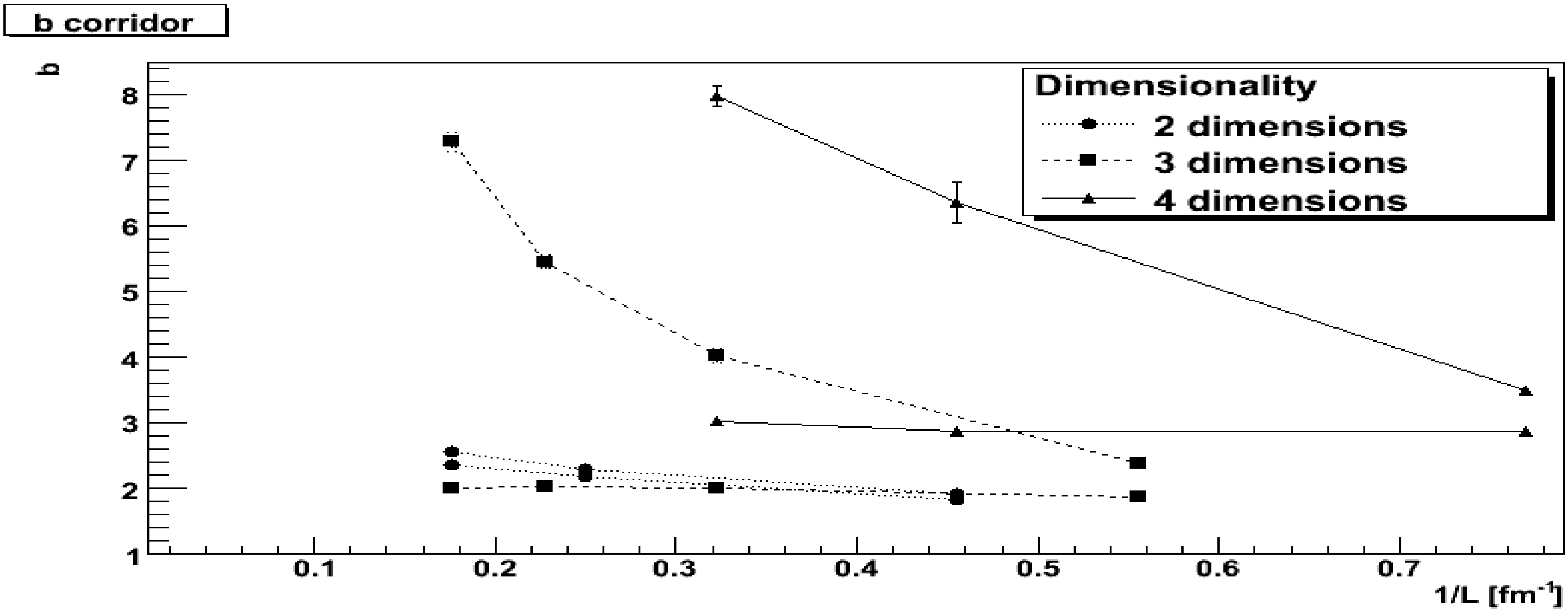

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  - In addition: min B and max B gauges, selecting the extremal values of b on each residual gauge orbit

Compare correlation functions between Landau gauges

- Landau-B gauges
  - Here: minB und maxB-gauges

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Compare correlation functions between Landau gauges

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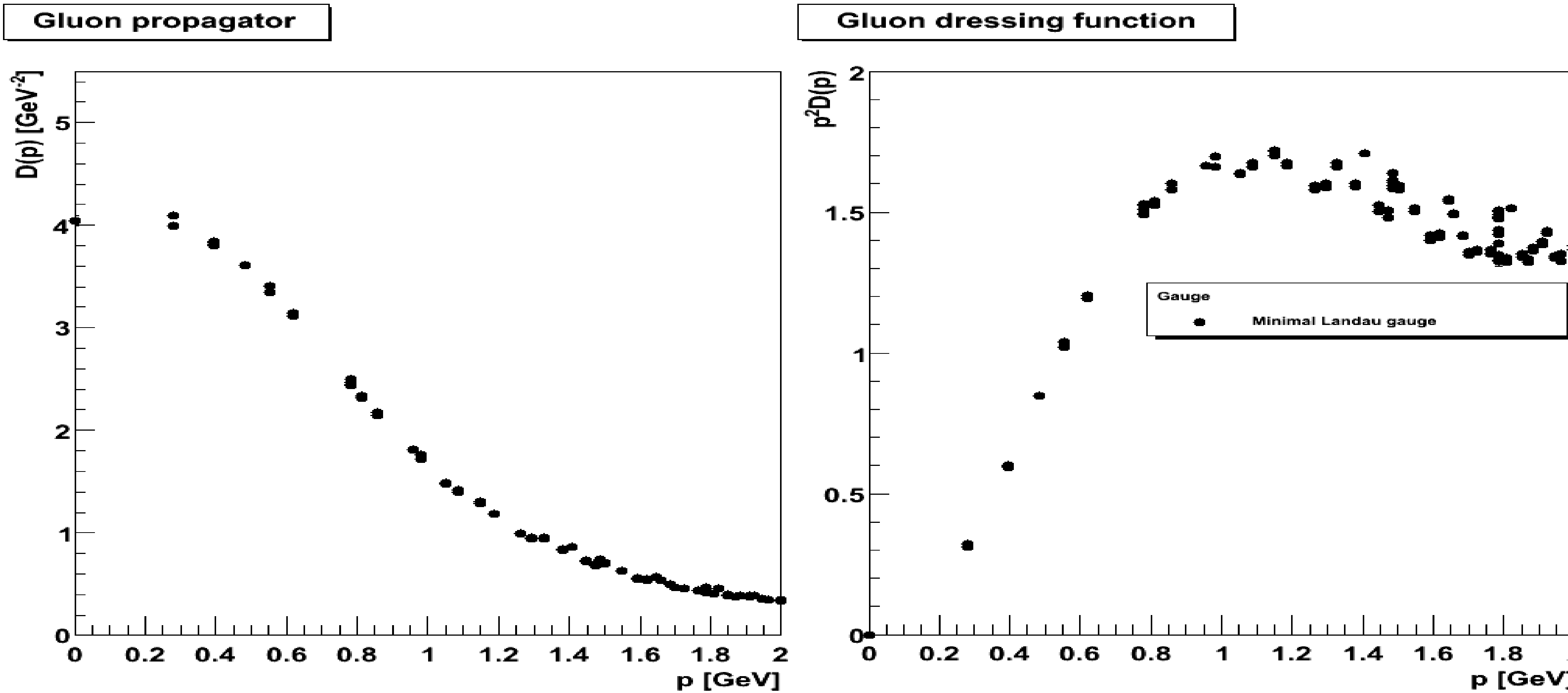
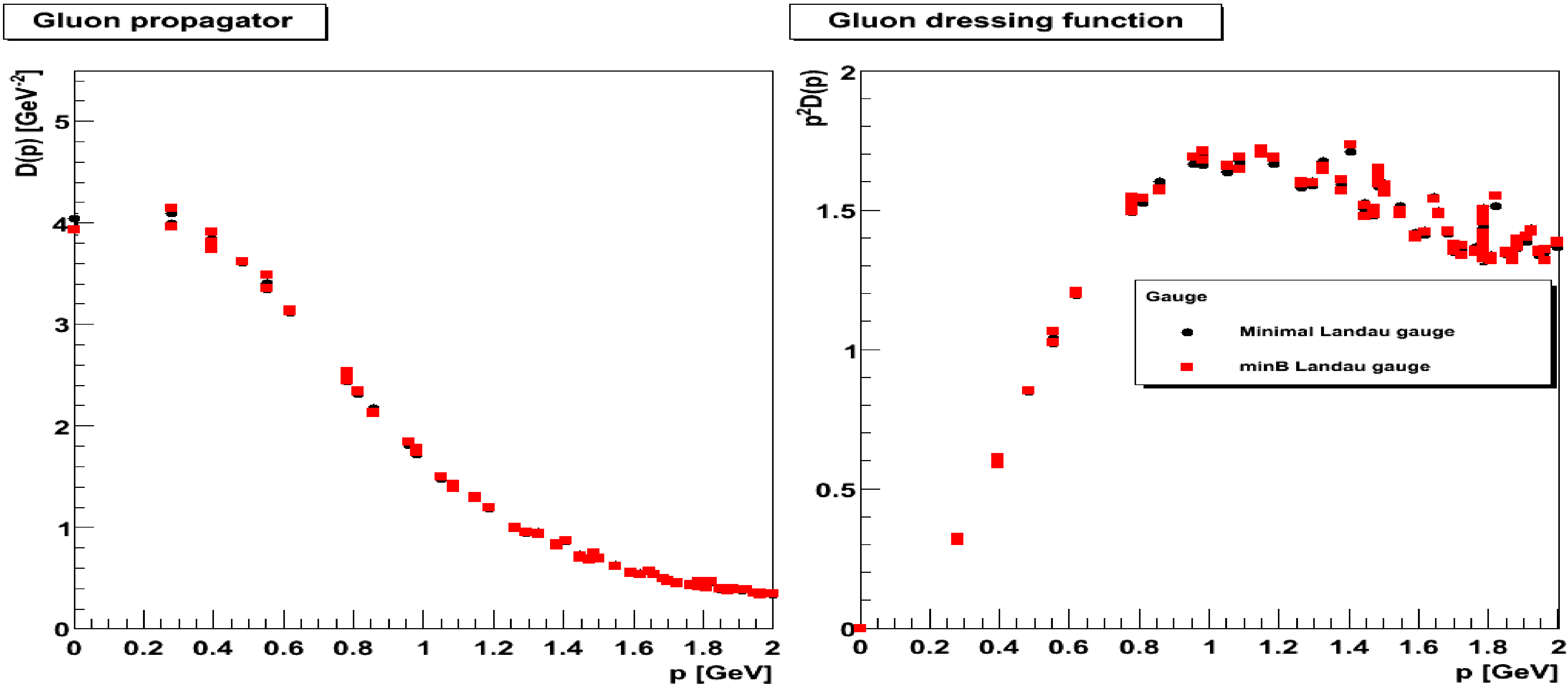
  - Here: minB und maxB-gauges
- Absolute Landau gauge
- Inverse Landau gauge
  - Maximize TrD
- Minimal Landau gauge
  - Select a random copy on each residual gauge orbit
  - Probability distribution is determined by the underlying algorithm
    - Differ for different algorithms, but cheap compared to all other gauges
  - No direct analogue in or translation to continuum field theory
  - Current standard in lattice calculations

Gluon propagator in the minimal Landau gauge [3d, Maas, unpublished]



- Mild intrared suppression in 3d (strong in 2d, none in 4d)
  - Infrared constant in the infinite-volume/continuum limit [Cucchieri et al. 2007/8, von Smekal et al. 2008]
  - Possibly vanishing in 2d [Maas, 2007, Cucchieri 2007/8]

Gluon propagator in the minB Landau gauge [3d, Maas, unpublished]



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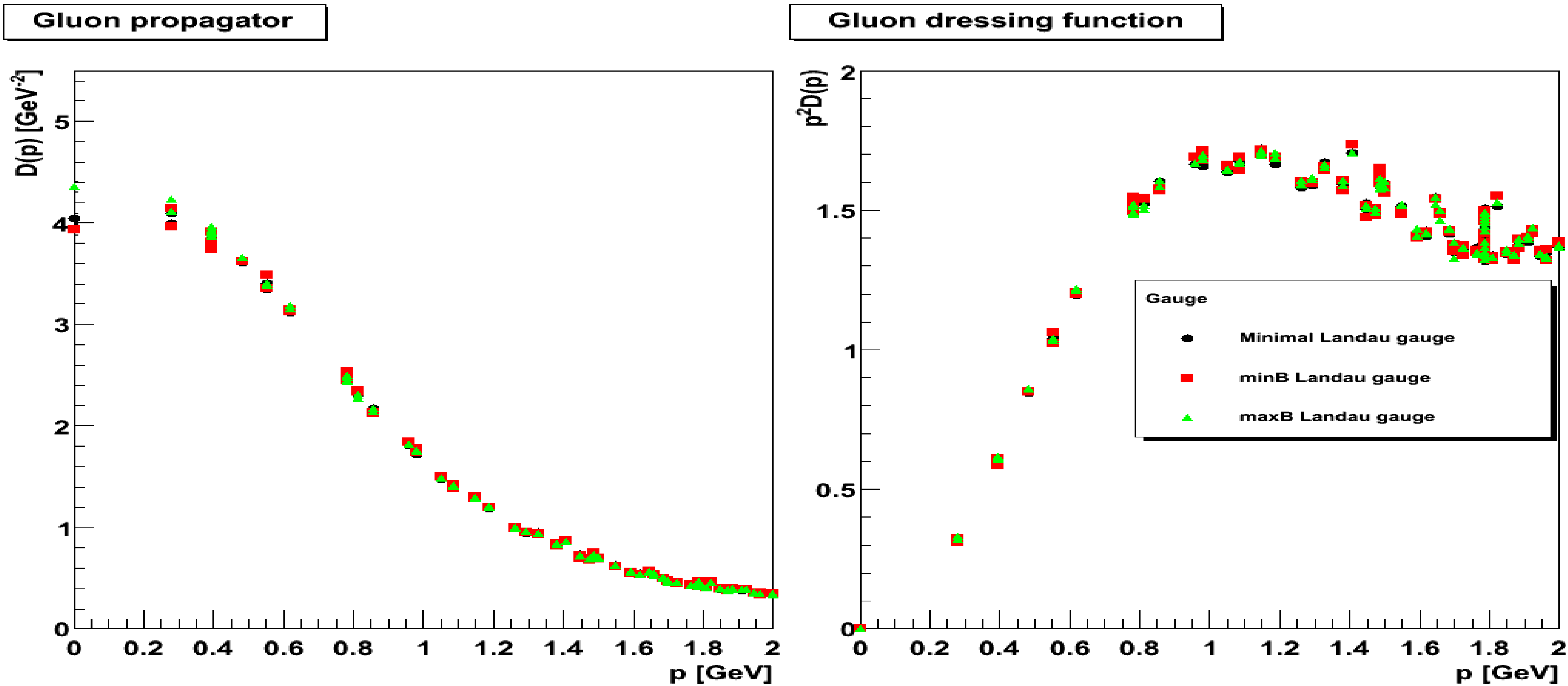
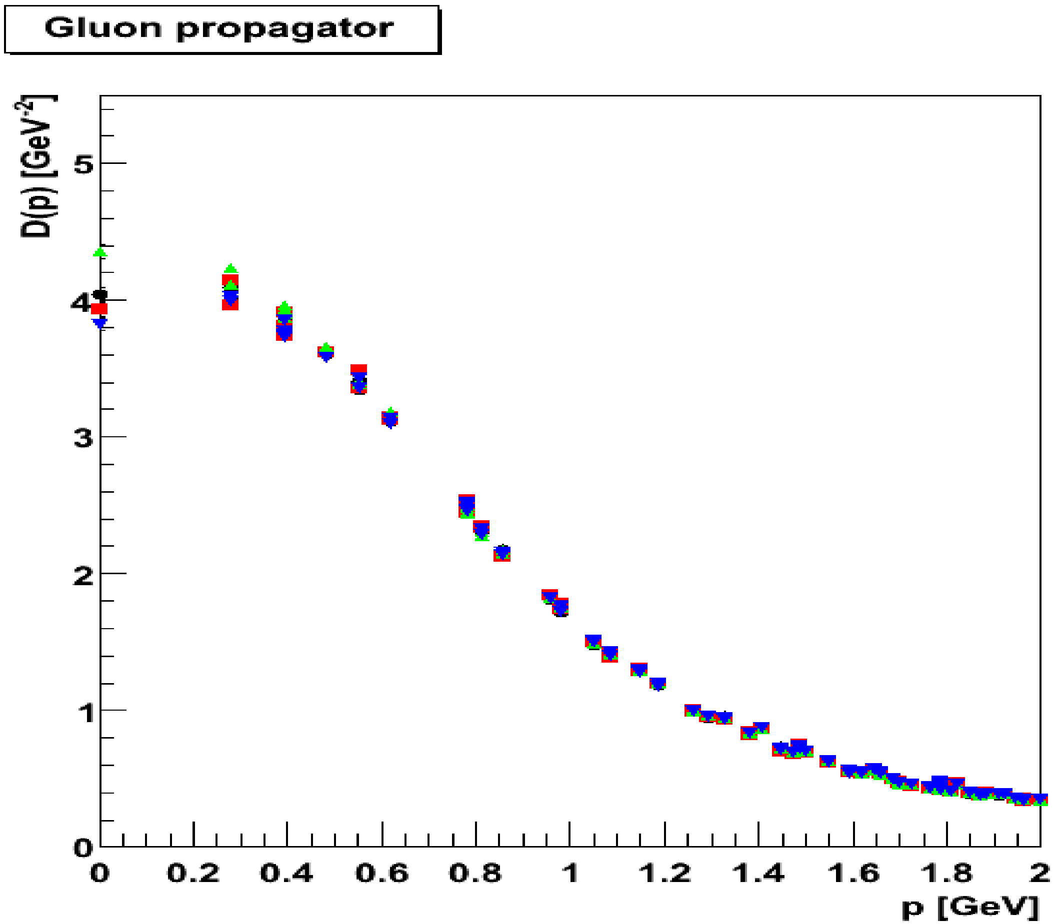
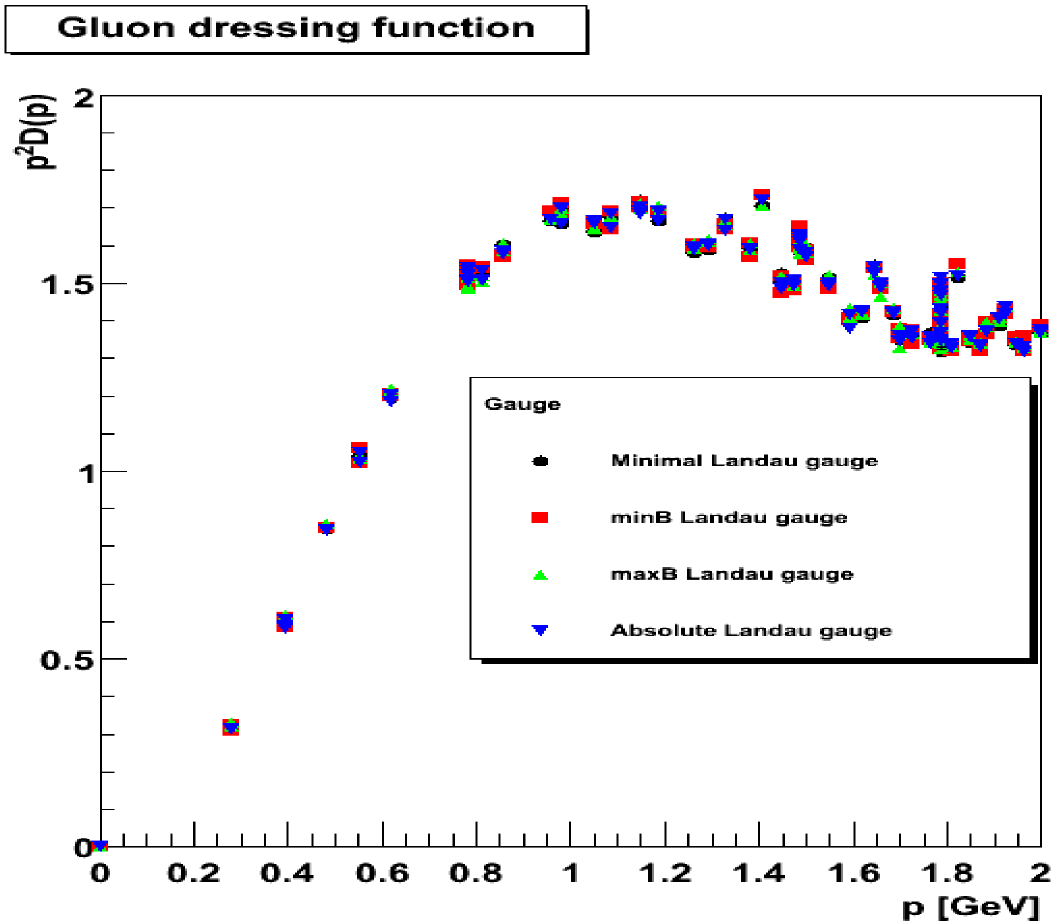


- At current volumes little difference to minimal Landau gauge
  - A little less infrared suppressed

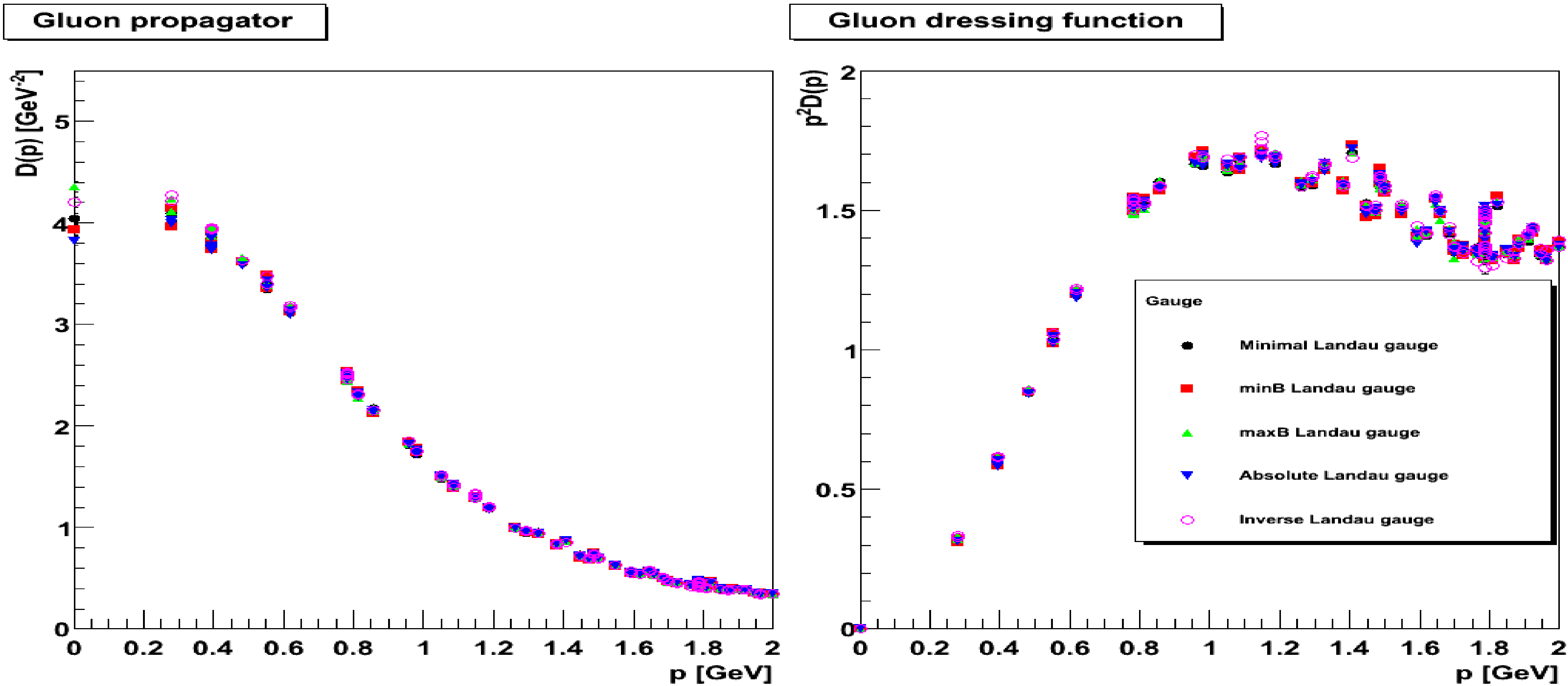
Gluon propagator in the absolute Landau gauge [3d, Maas, unpublished]





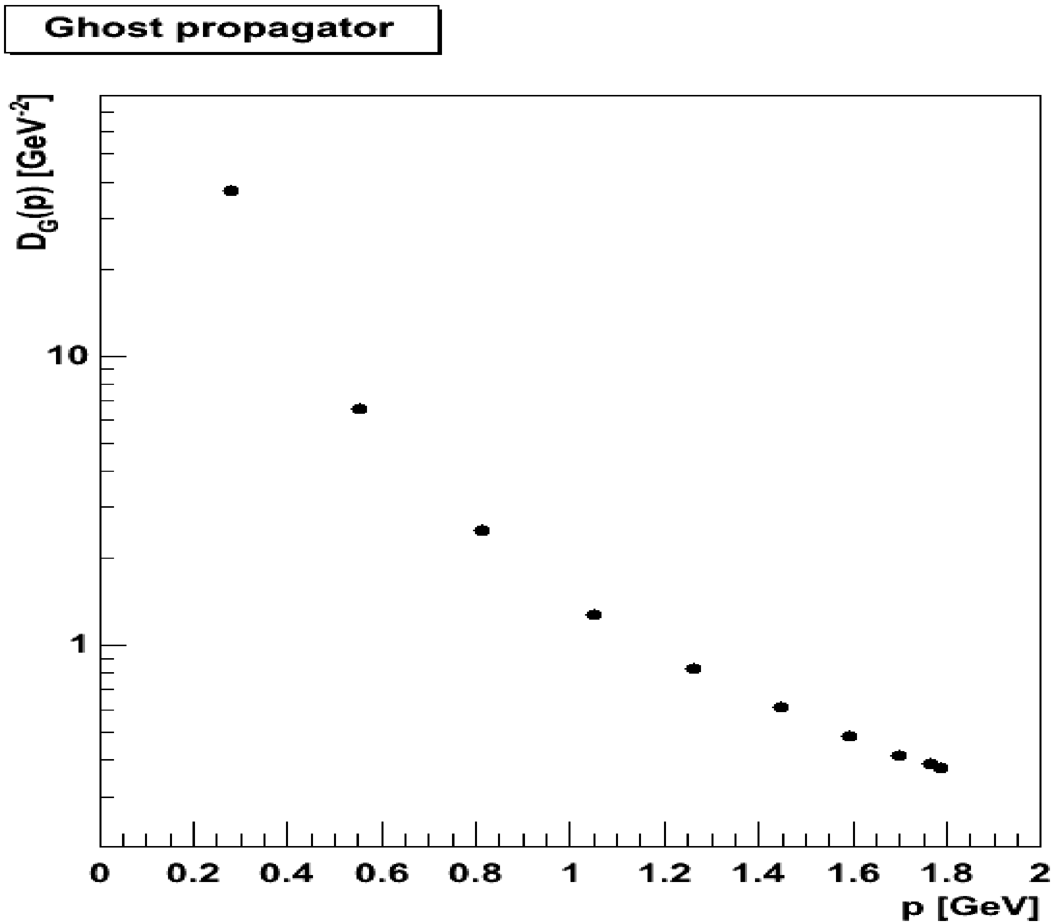
- At current volumes little difference to minimal Landau gauge
  - Somewhat stronger infrared suppressed
  - Unclear whether it vanishes

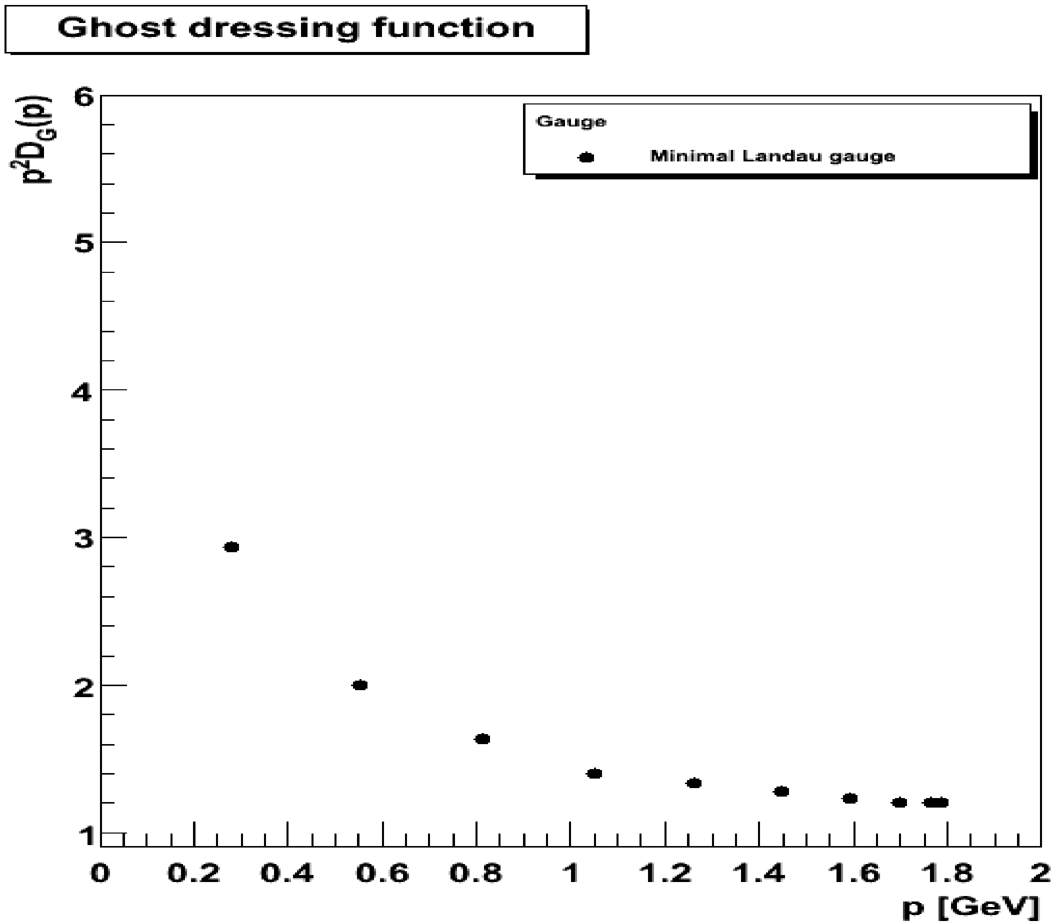
Gluon propagator in the inverse Landau gauge [3d, Maas, unpublished]



- At current volumes little difference to minimal Landau gauge

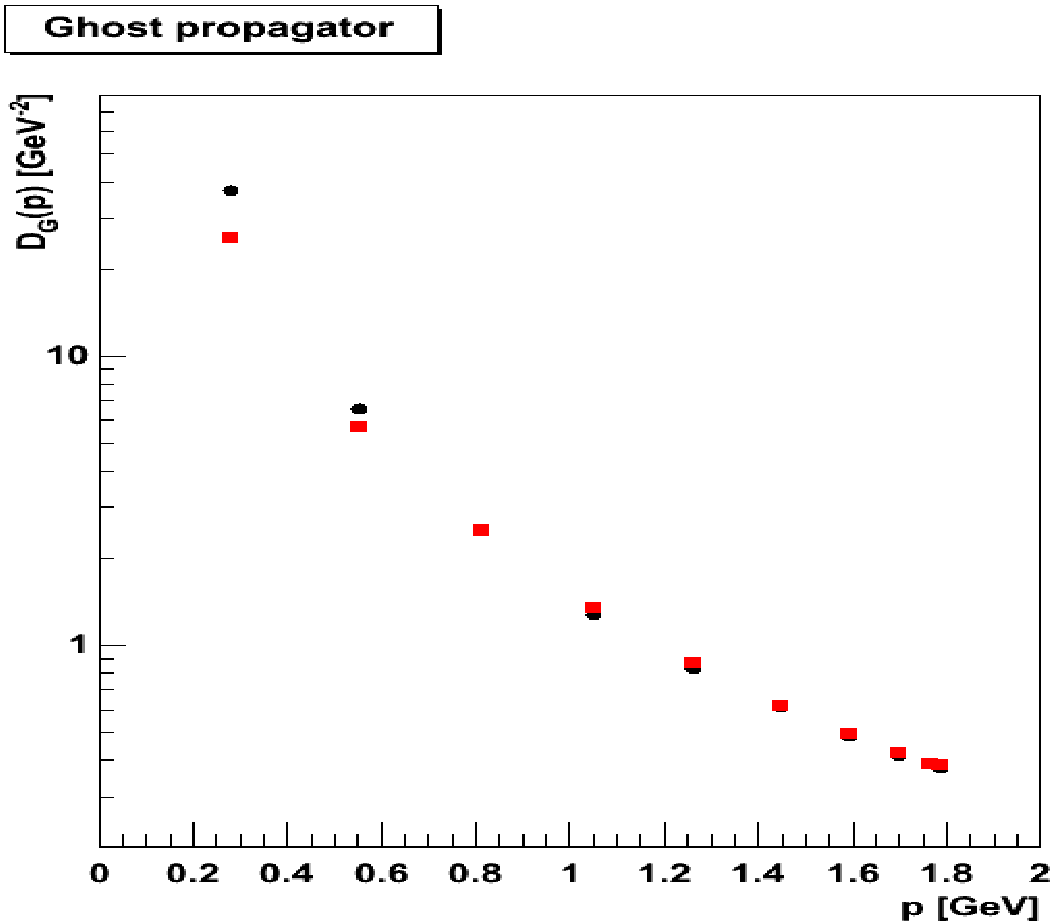
Ghost propagator in the minimal Landau gauge [3d, Maas, unpublished]

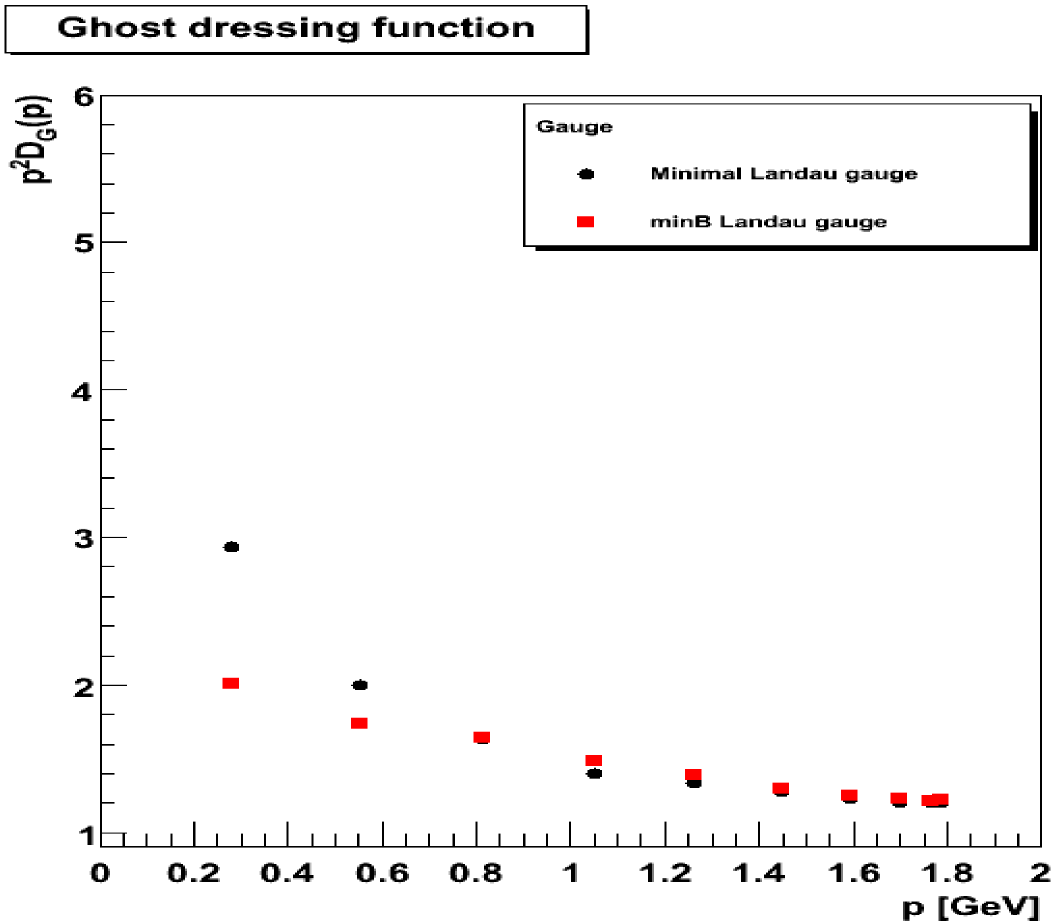




- Infrared enhanced (in all dimensions)
  - In 3d and 4d infrared finite dressing function [Cucchieri et al. 2007/8, von Smekal 2007]
  - In 2d possibly diverging [Maas 2007, Cucchieri et al. 2007/8]

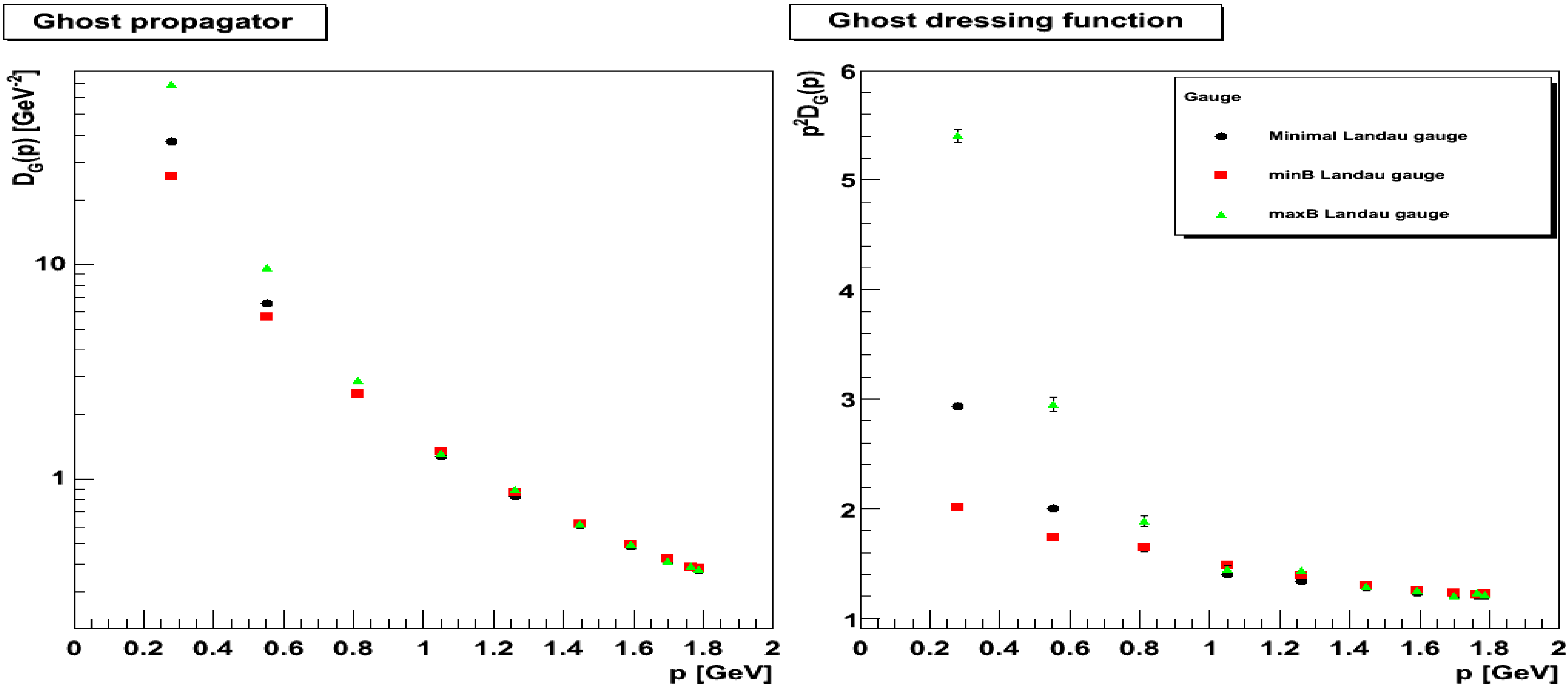
Ghost propagator in the minB Landau gauge [3d, Maas, unpublished]





- Significantly less enhanced
  - Will have in 3d and 4d infrared finite dressing function
  - Fate in 2d yet unclear

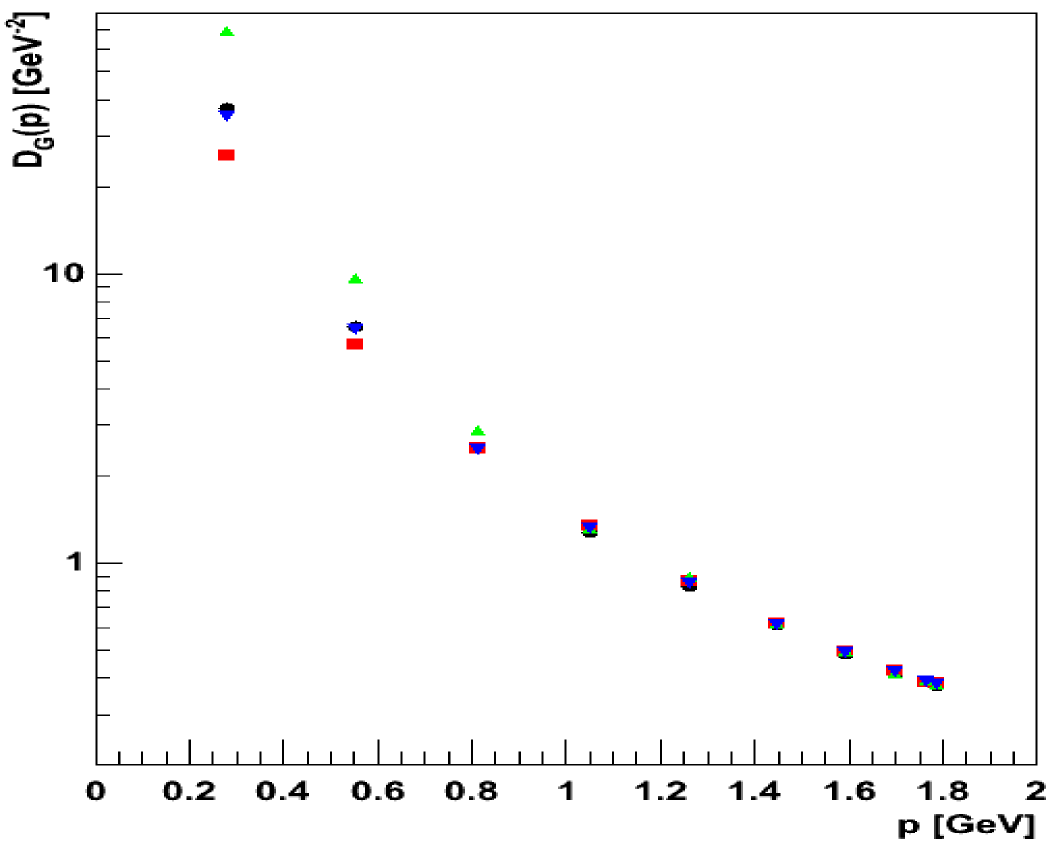
Ghost propagator in the minimal Landau gauge [3d, Maas, unpublished]



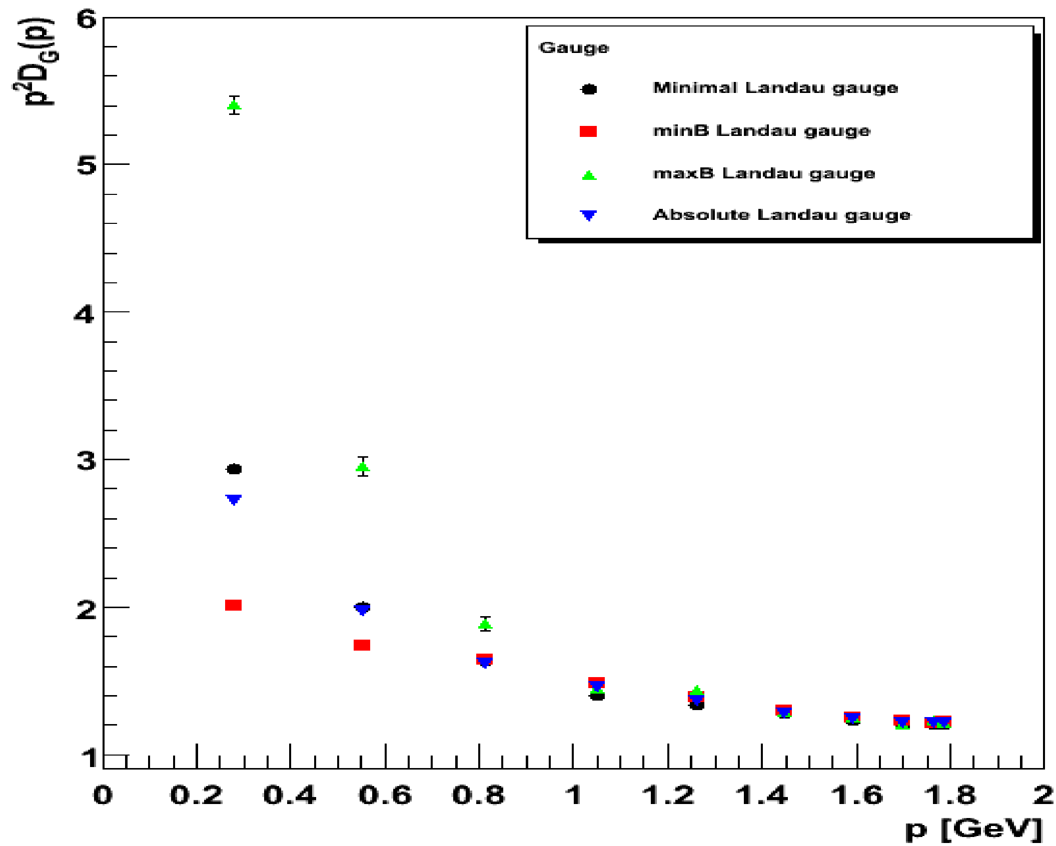
- Strongly enhanced
  - Could have infrared divergent dressing function in all d
  - Effects up to almost 1 GeV

Ghost propagator in the minimal Landau gauge [3d, Maas, unpublished]

Ghost propagator



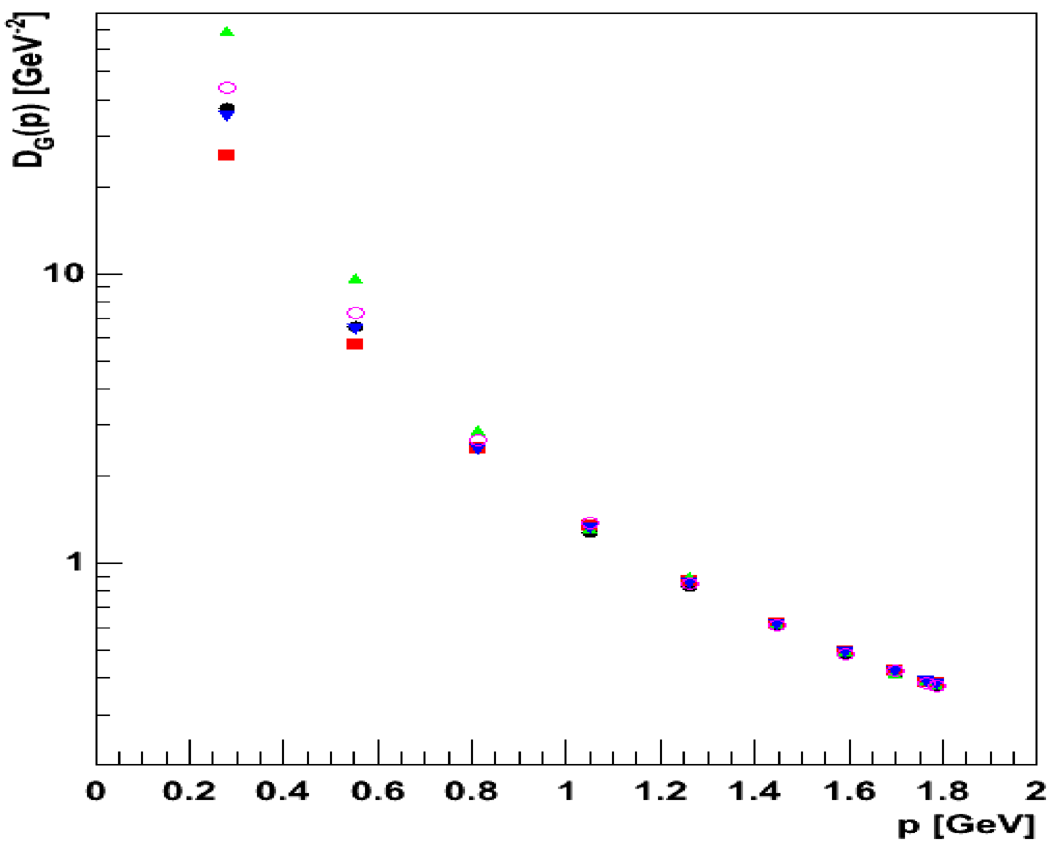
Ghost dressing function



- Less strongly enhanced than minimal Landau gauge
  - But has different finite volume evolution [Maas, 2008]

Ghost propagator in the minimal Landau gauge [3d, Maas, unpublished]

Ghost propagator



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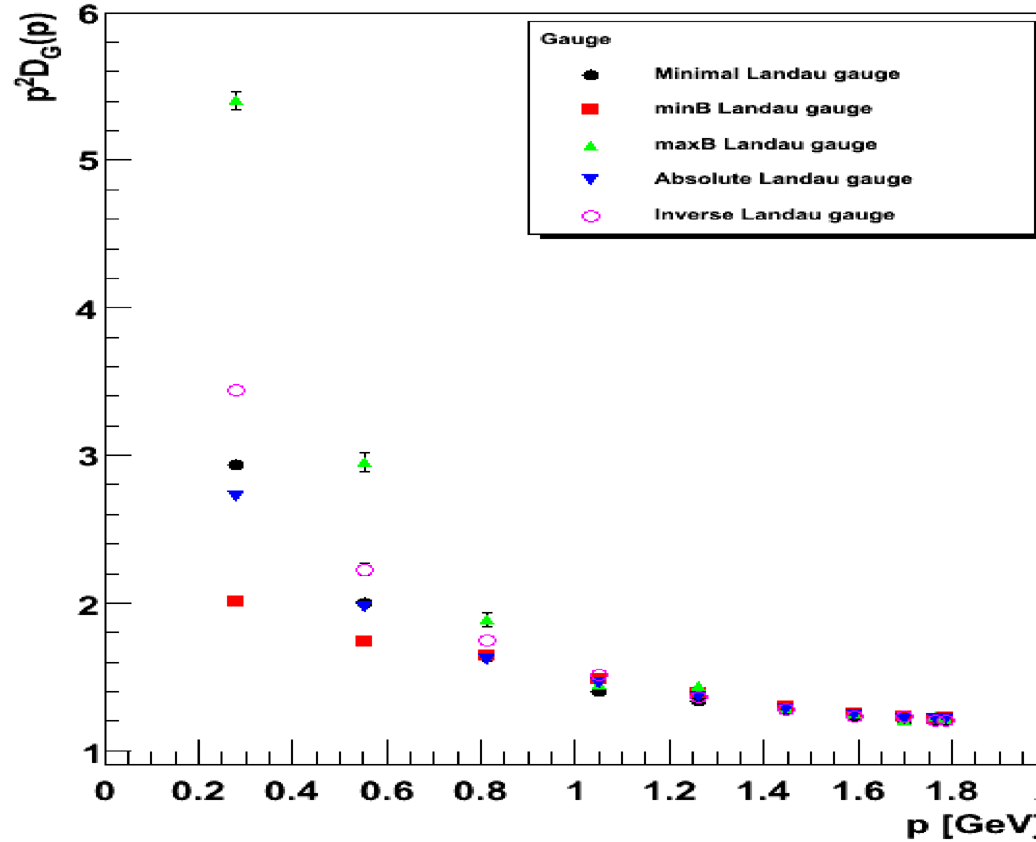

- Mildy stronger enhanced than minimal Landau gauge
  - Infinite volume fate is unclear

B gauges

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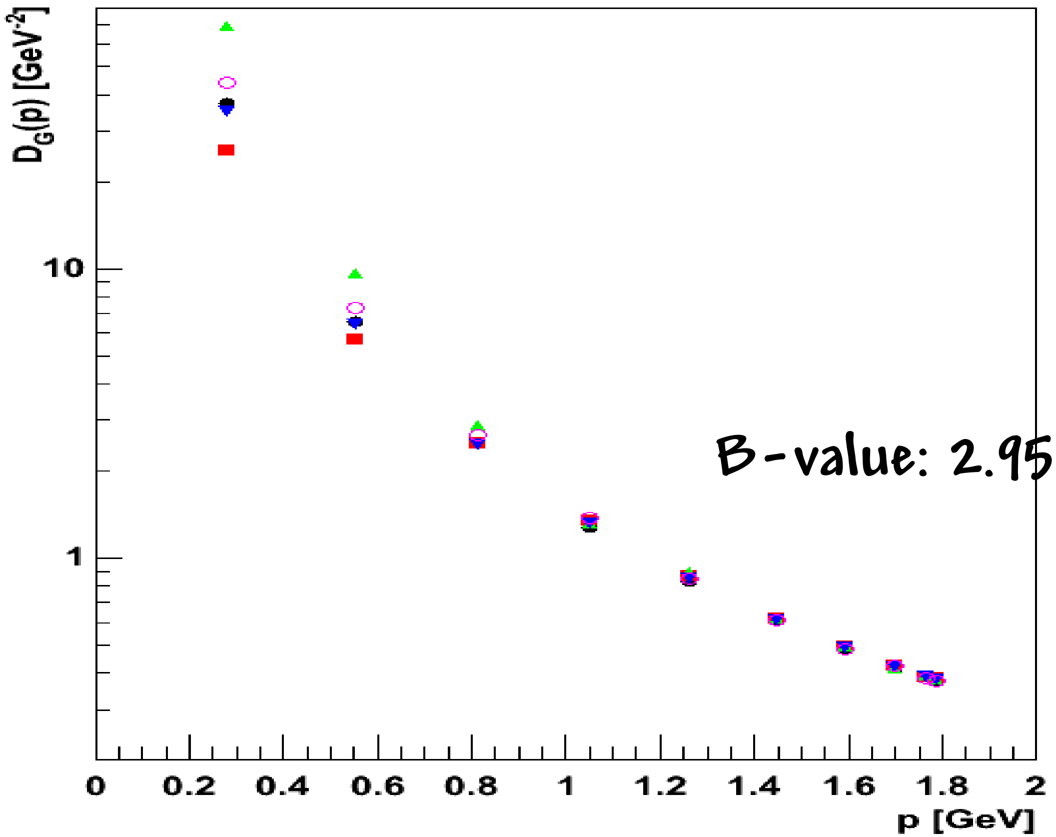
- B-Landau gauges permit unique gauge-fixing prescription
  - Are correlation functions in a B gauge identical to e.g. the minimal Landau gauge with its effective b value?
    - If it is a faithful map of the probability distribution

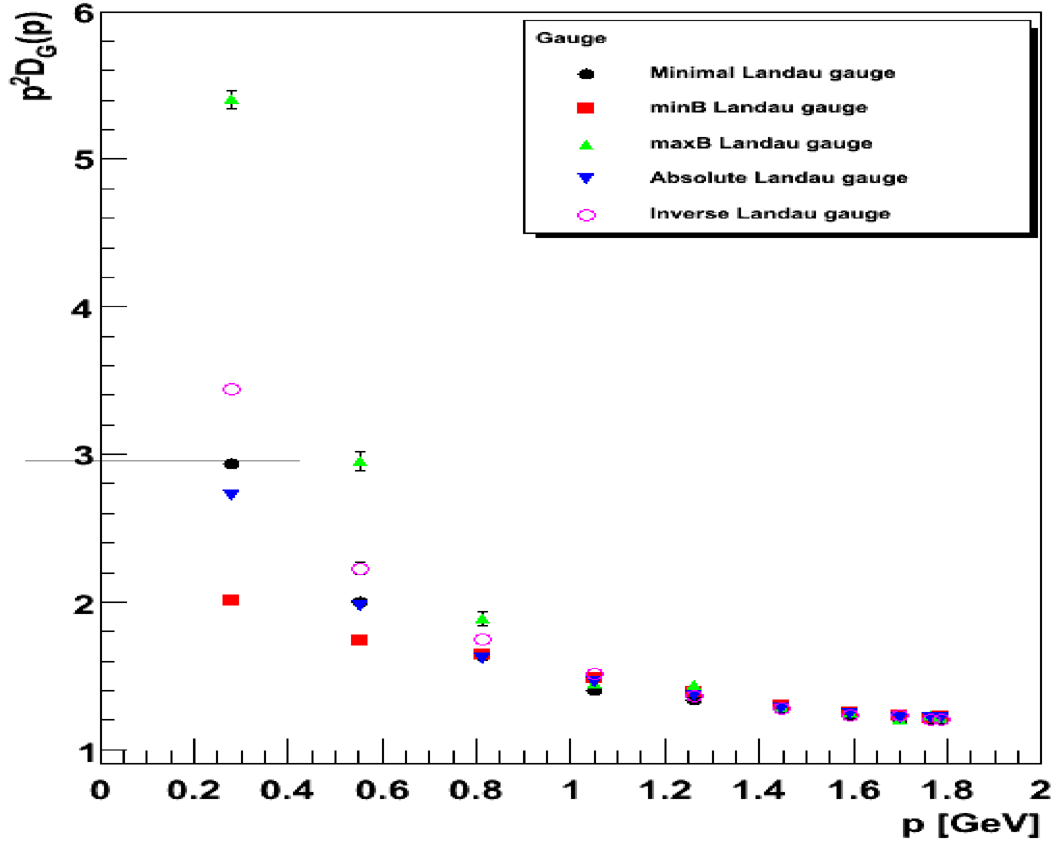
Mapping gauge descriptions [3d, Maas, unpublished]

Ghost propagator



B-value: 2.95

Ghost dressing function



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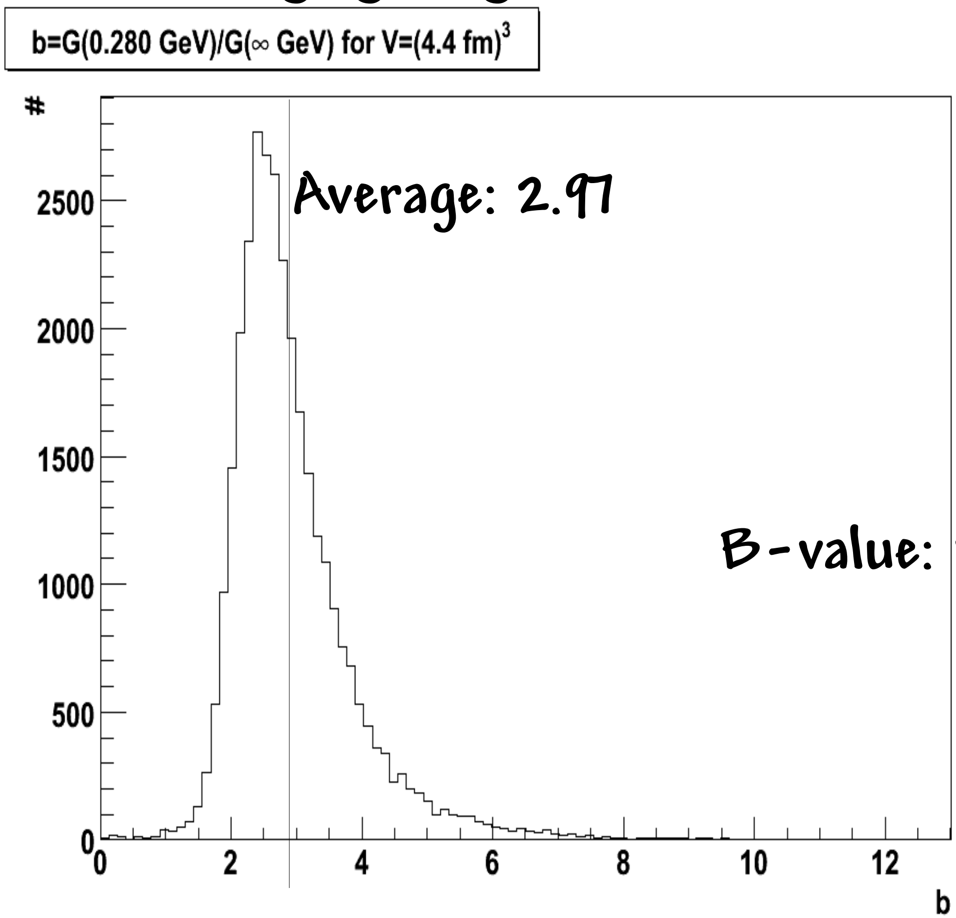

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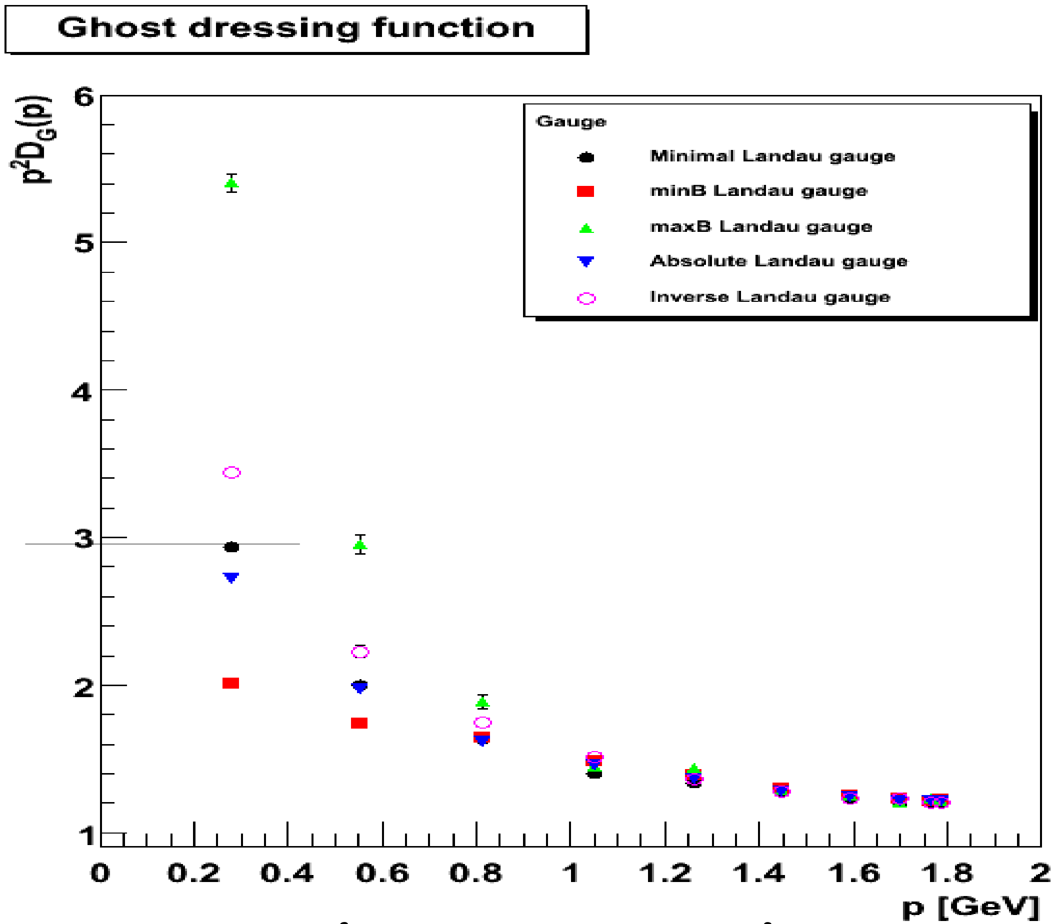

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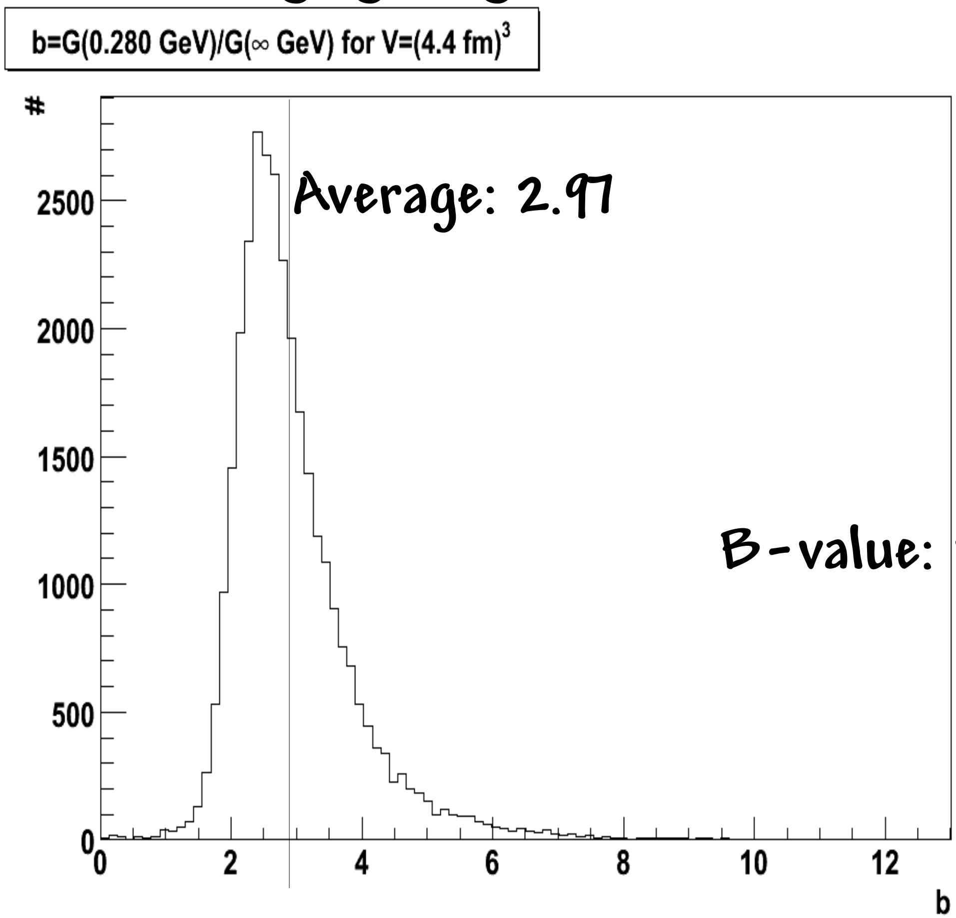

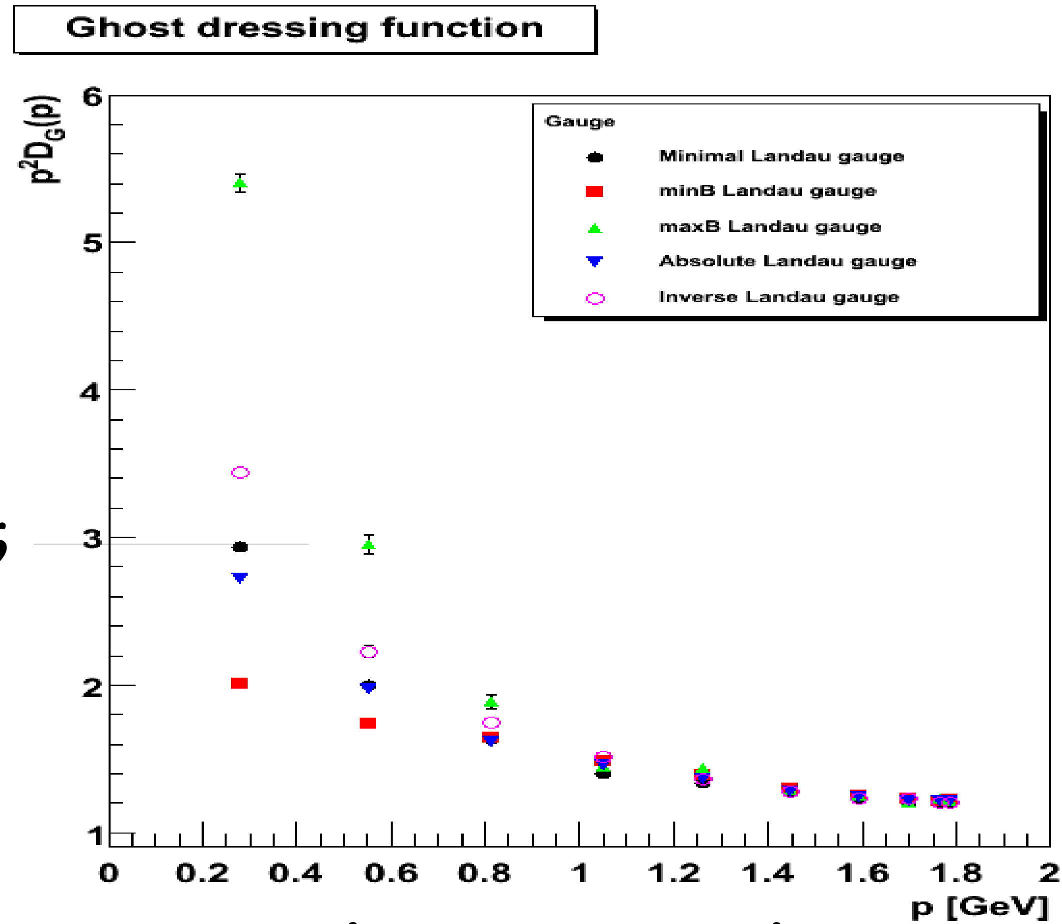

- Minimal Landau gauge is a Landau-B gauge at the average B value
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  - Also for absolute or inverse Landau gauge, but with a deformed map
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  - But how to identify the first Gribov horizon outside the lattice?
    - Positivity and montonicity of the ghost propagator? Assume

[Reinhardt et al. 2008]

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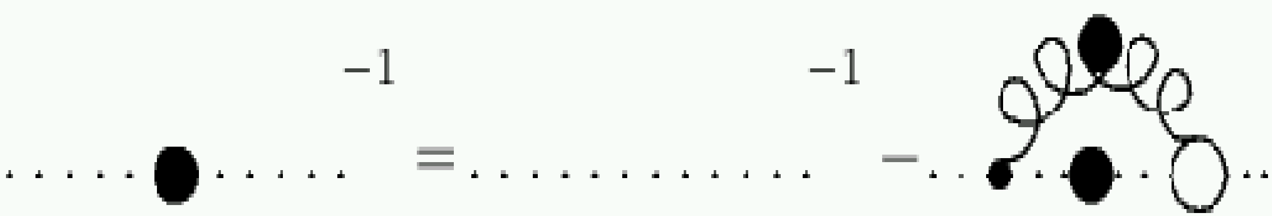

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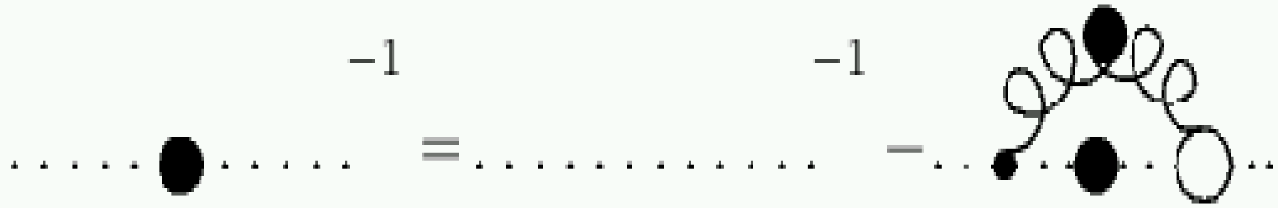

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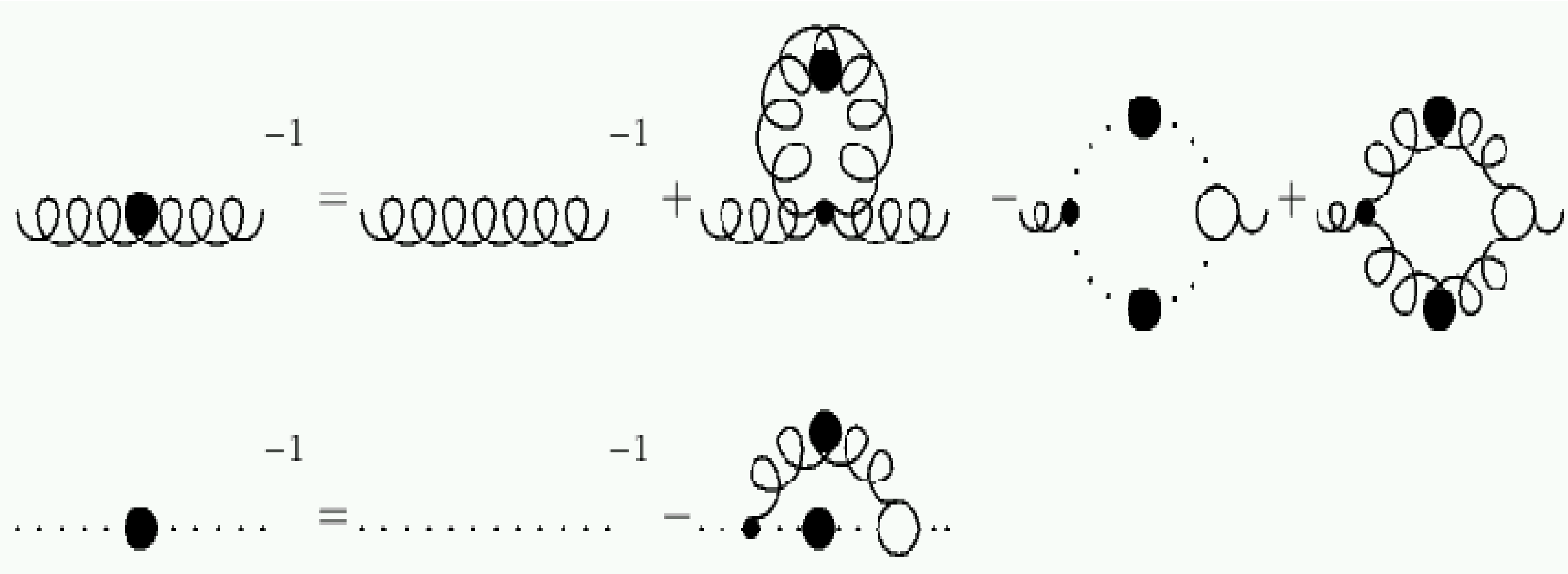

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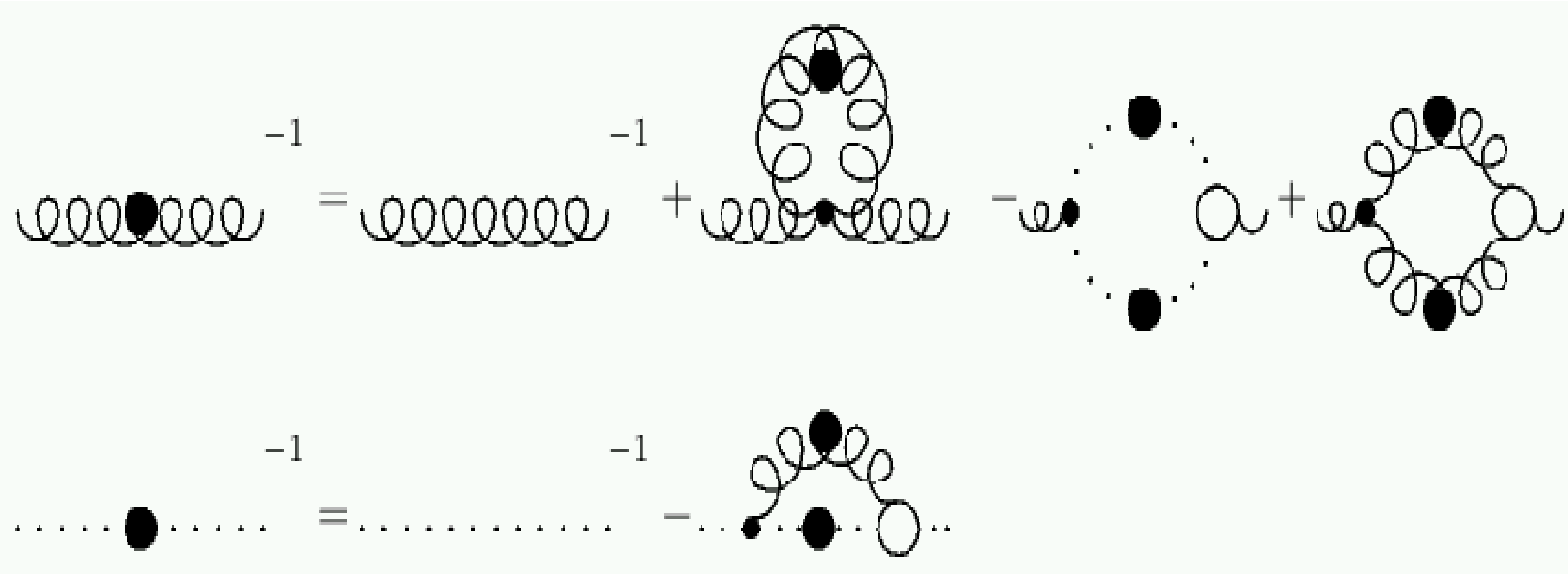

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- Consistent and self-consistent truncation schemes can be developed

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- Combination of all methods most successful!

Implementing B Landau gauges in the continuum

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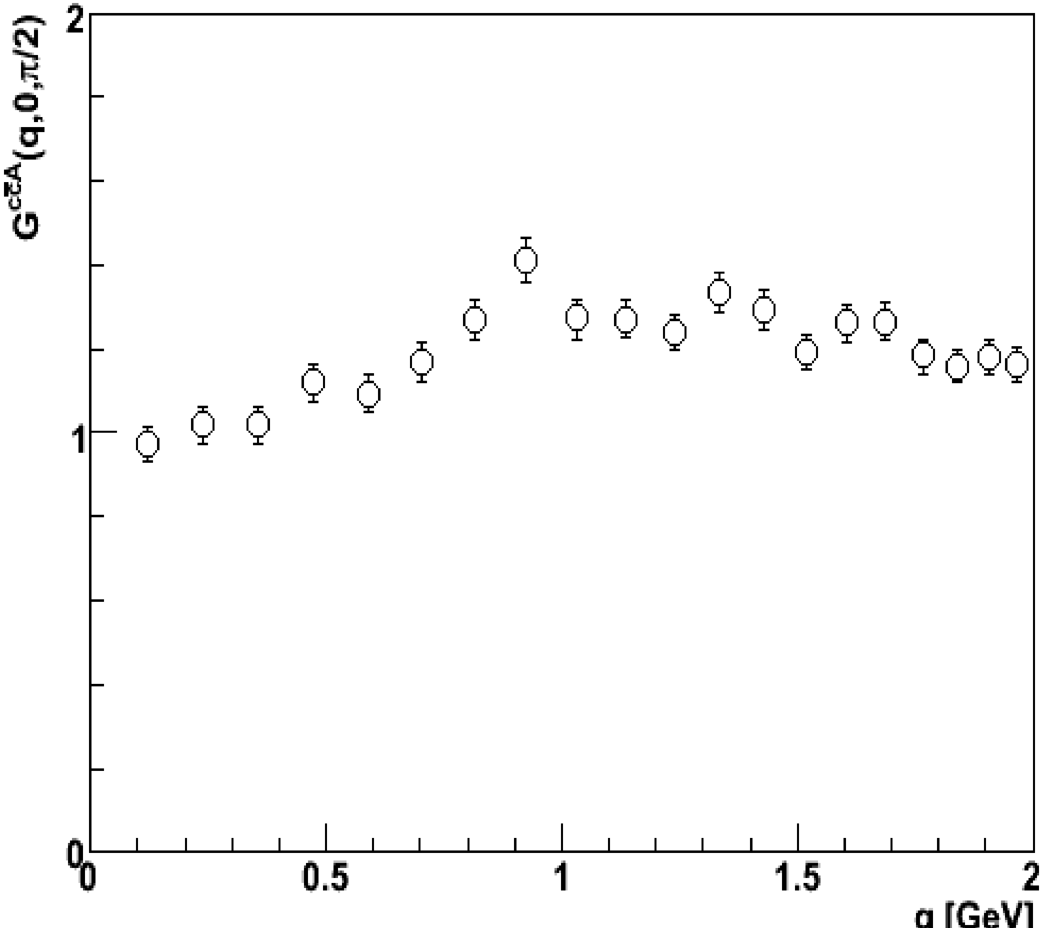
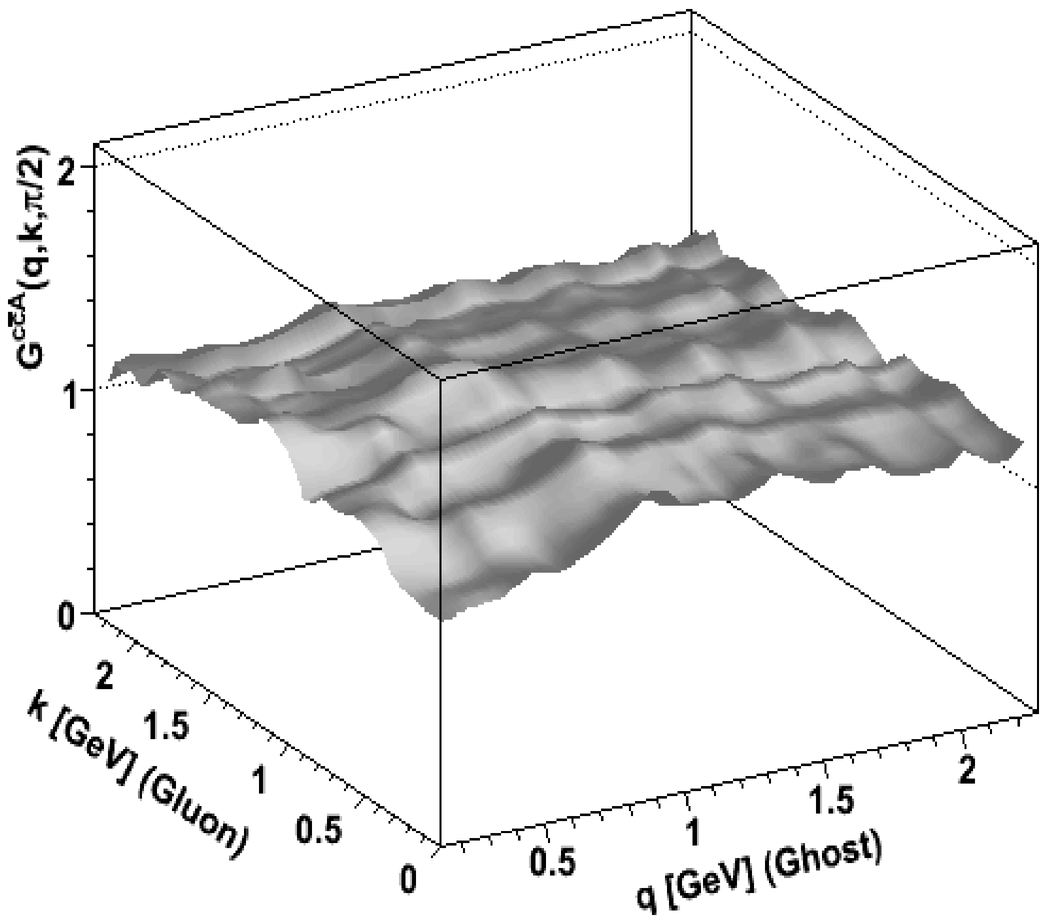
Ghost-gluon vertex in 3d

[60³, beta=4.2, Cucchieri et al., 2008]

Ghost-gluon vertex, one momentum vanishing	Ghost-gluon vertex, orthogonal momenta





- Essentially constant, only some structure at 1 GeV
- Same in 2d and 4d
- In agreement with DSE predictions [Schleifenbaum et al., PRD 2005]

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- Select wave-function renormalization Z such as to implement a particular B-value B=1/(Z+A) and thus B-Landau gauge

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- Continuous 1-parameter family of solutions

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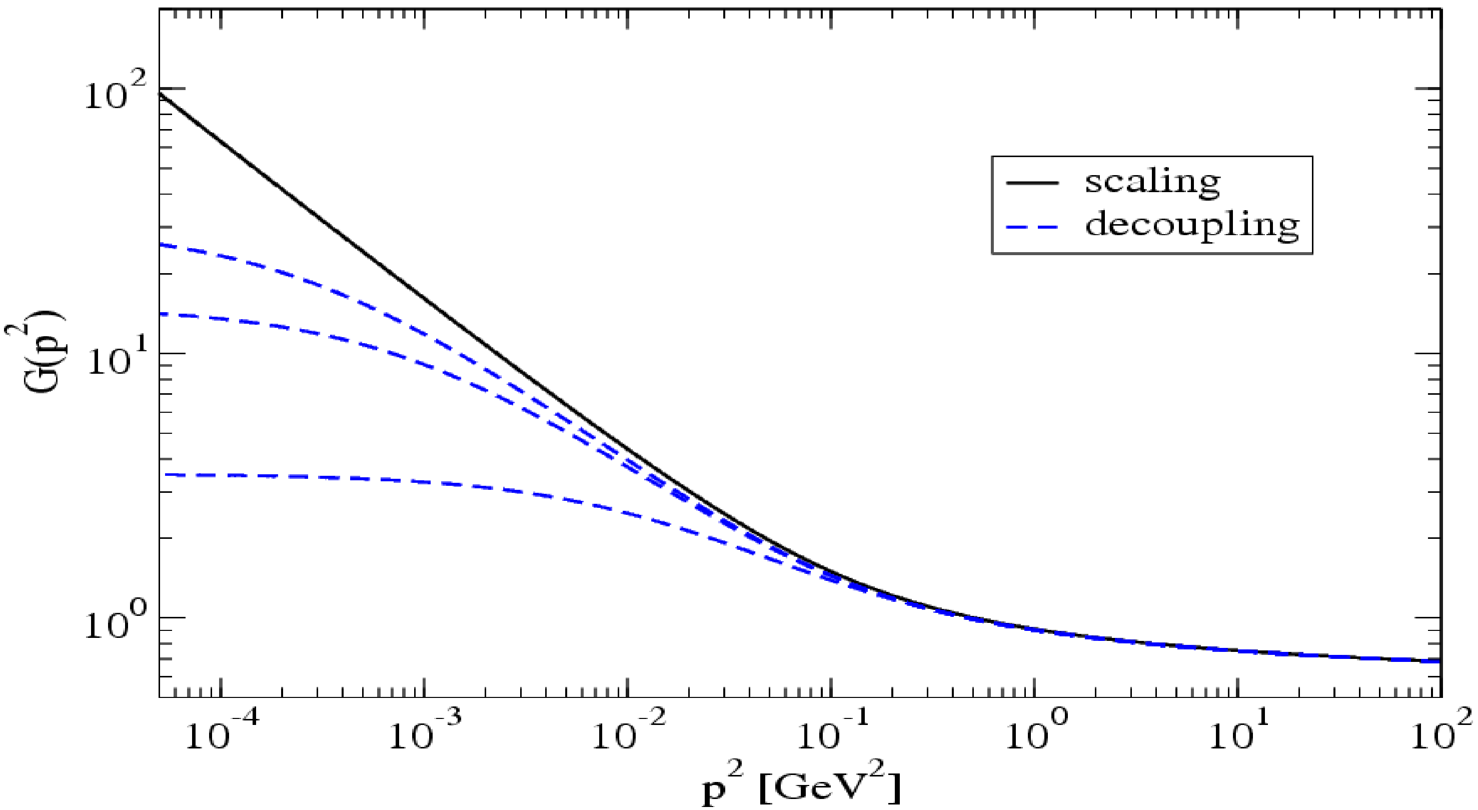

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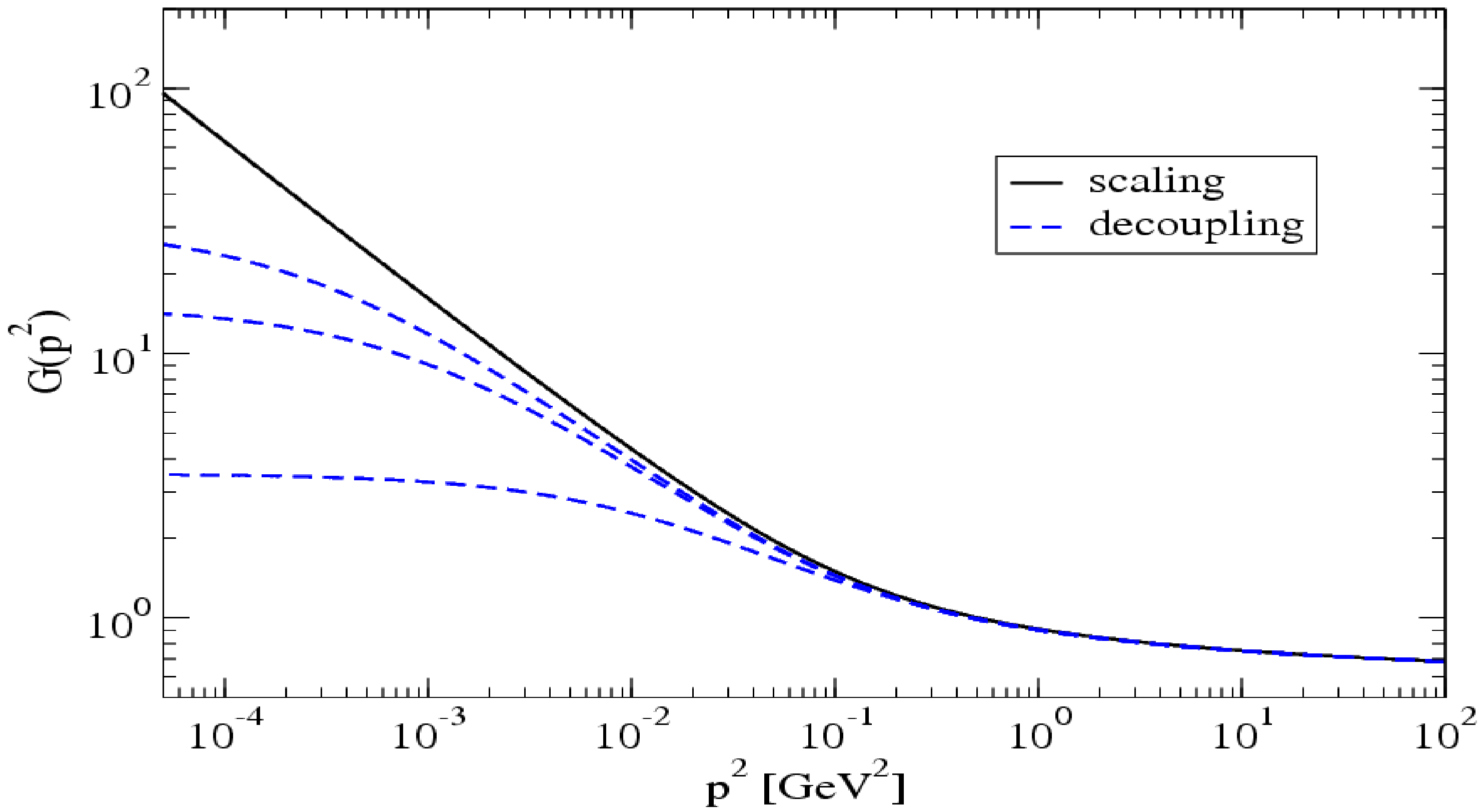

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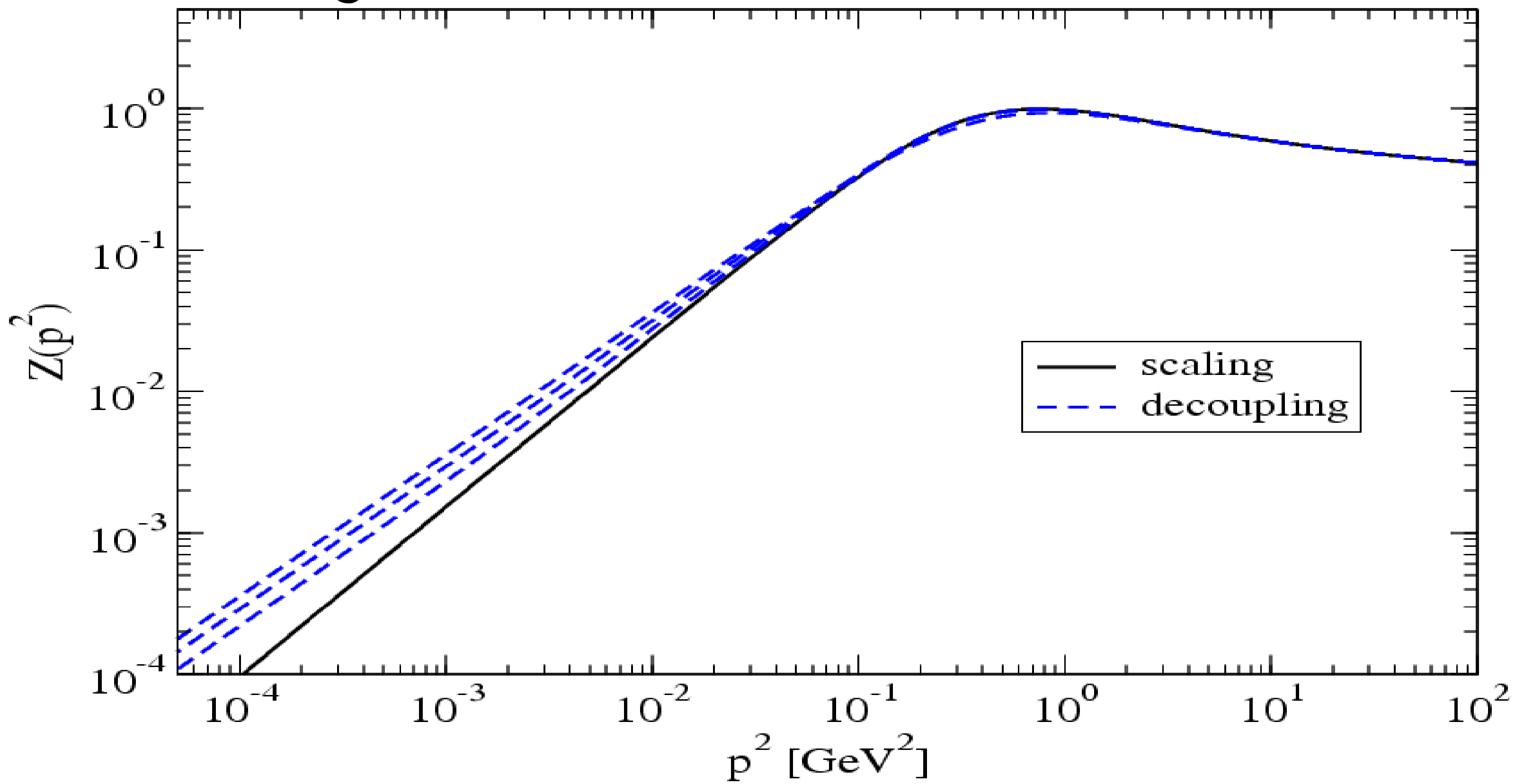
Gluon dressing function in the continuum [Fischer et al., 2008]



- Decoupling gauges yield a decoupling (infrared massive) gluon propagator – consistent with the lattice result
- The scaling gauge yields an infrared vanishing gluon propagator

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  - This needs a more formal formulation, volume studies...etc.

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- Can be expanded to the case with matter fields [Alkofer et al., 2007/8]

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  - Can be extended to Wilson criteria for quarks [Pawlowski et al., 2008]

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- But a formal formulation and more details are still required...