

What ghosts (may) have to do with Confinement

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Overview

- **Confinement**

[hep-th/0511307, EPJC in press]

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- Confinement
 - Quarks and topological configurations

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- **Confinement**
 - **Quarks and topological configurations**
 - **Gluons, ghosts and the Faddeev-Popov operator**

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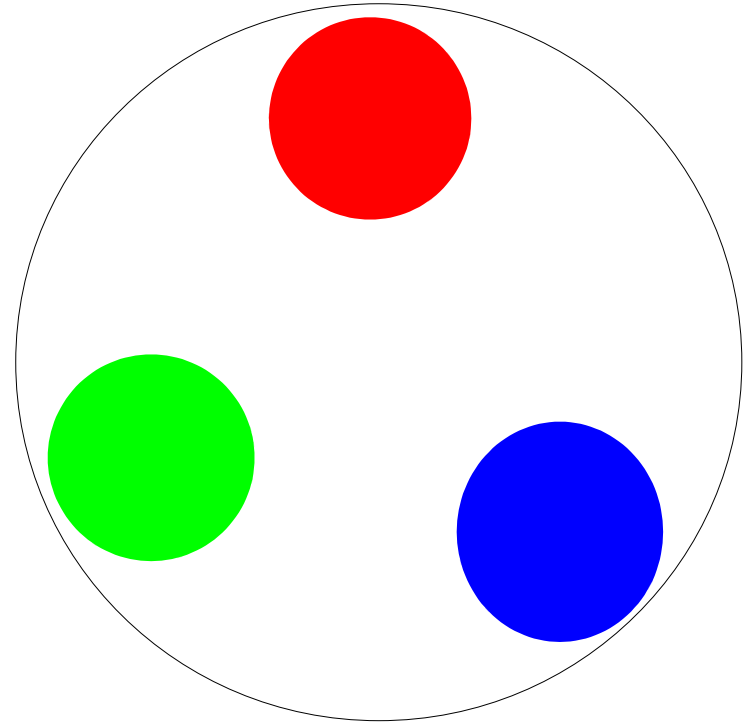
Overview

- **Confinement**
 - **Quarks** and **topological configurations**
 - **Gluons, ghosts** and the **Faddeev-Popov operator**
- **Eigenspectrum** of the **Faddeev-Popov operator** in **topological fields**
- **Summary**

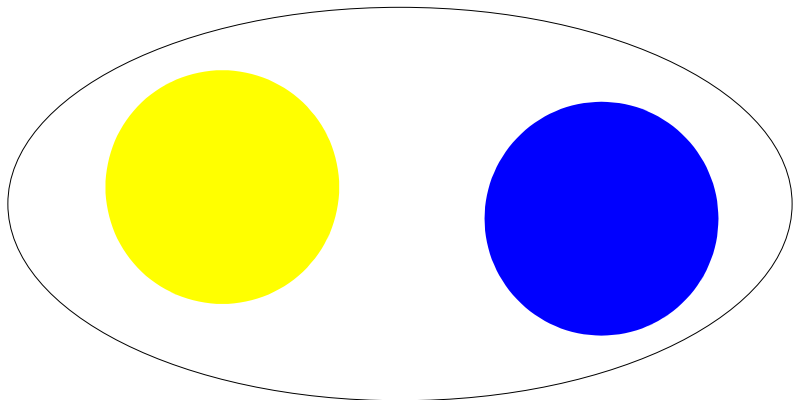
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QCD degrees of freedom

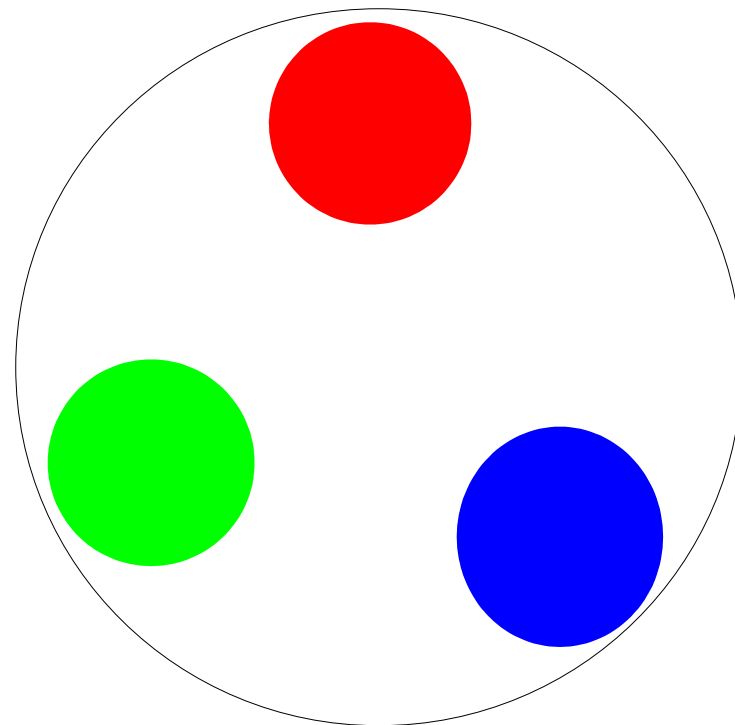
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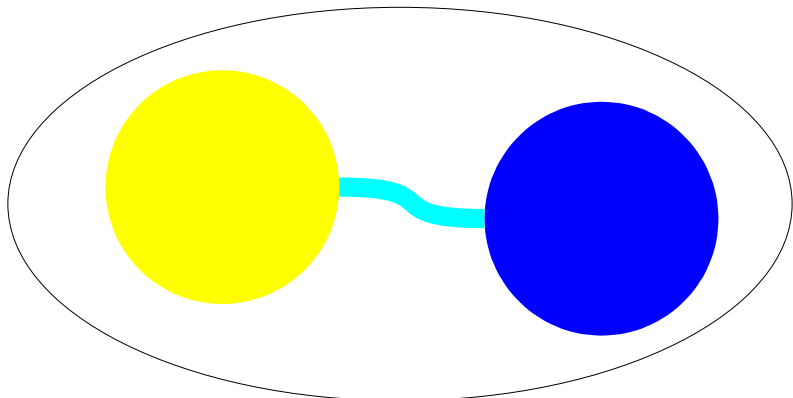
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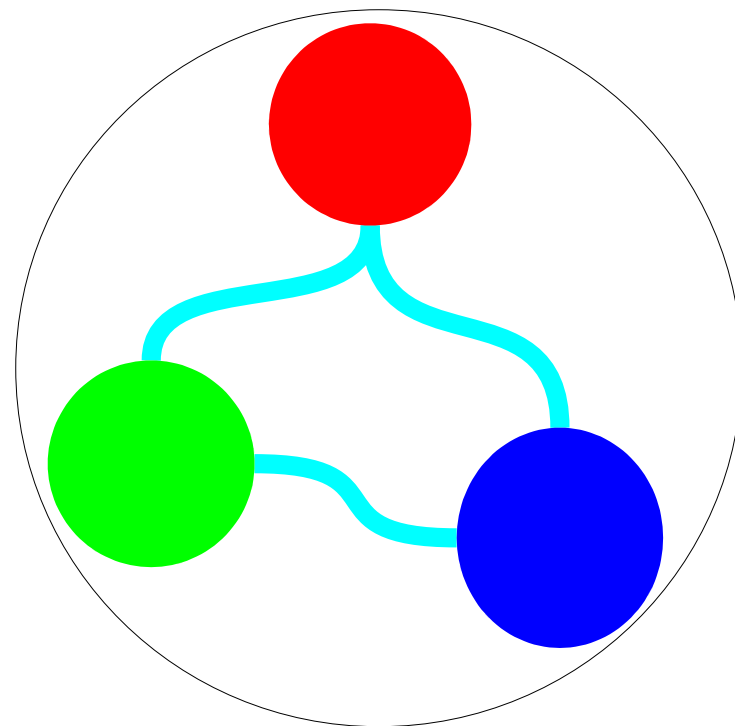
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- Meson: 1 quark and 1 antiquark



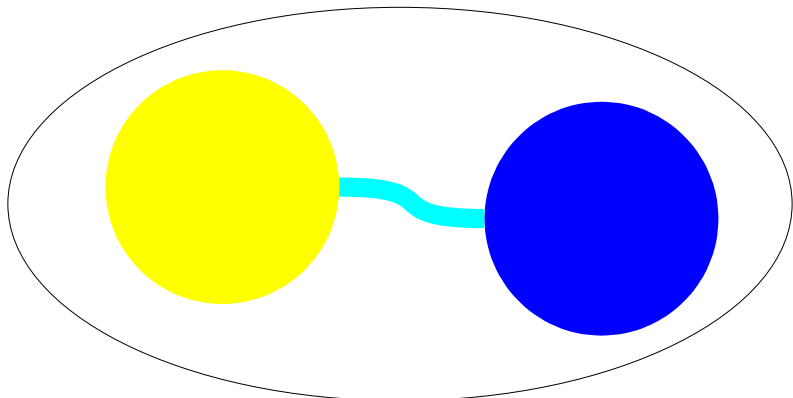
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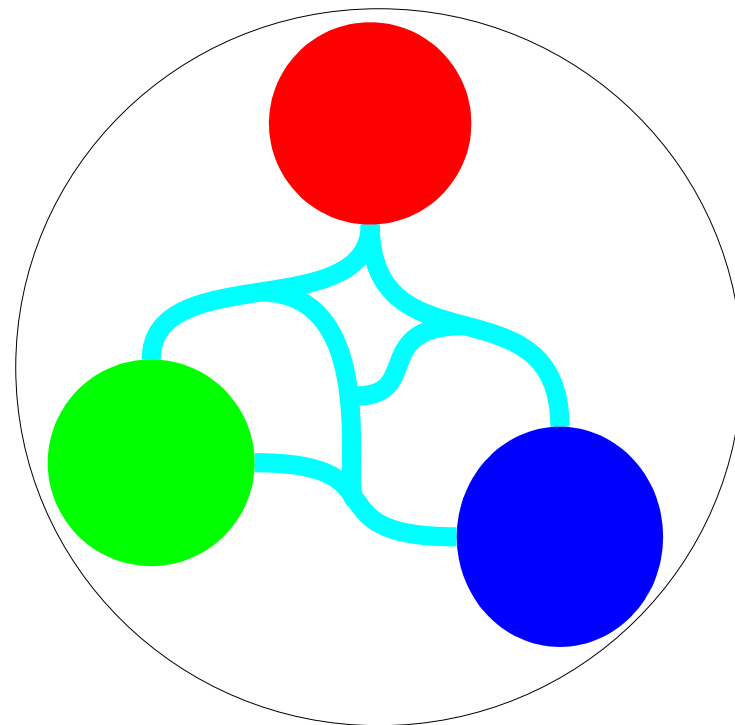
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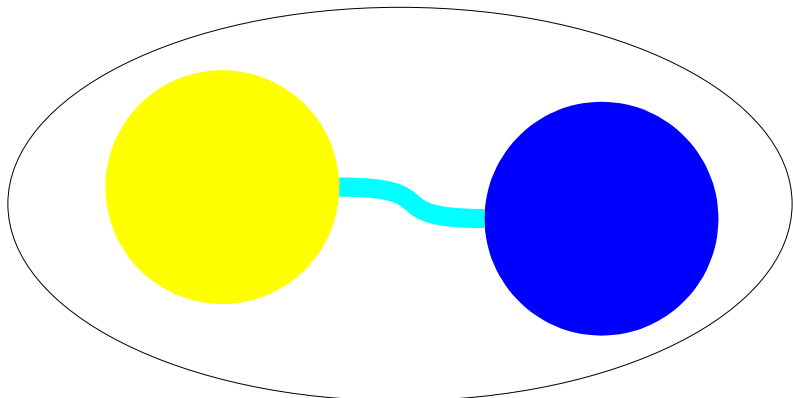
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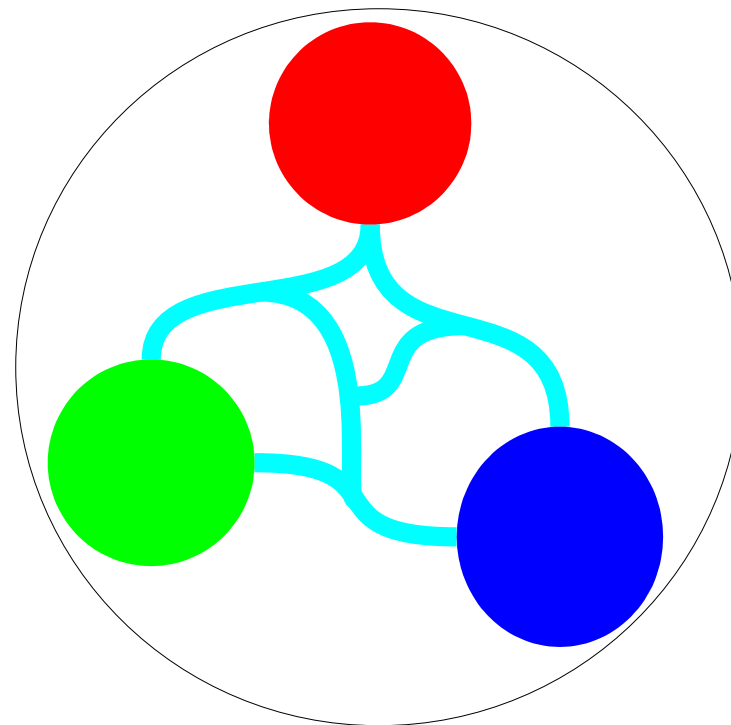
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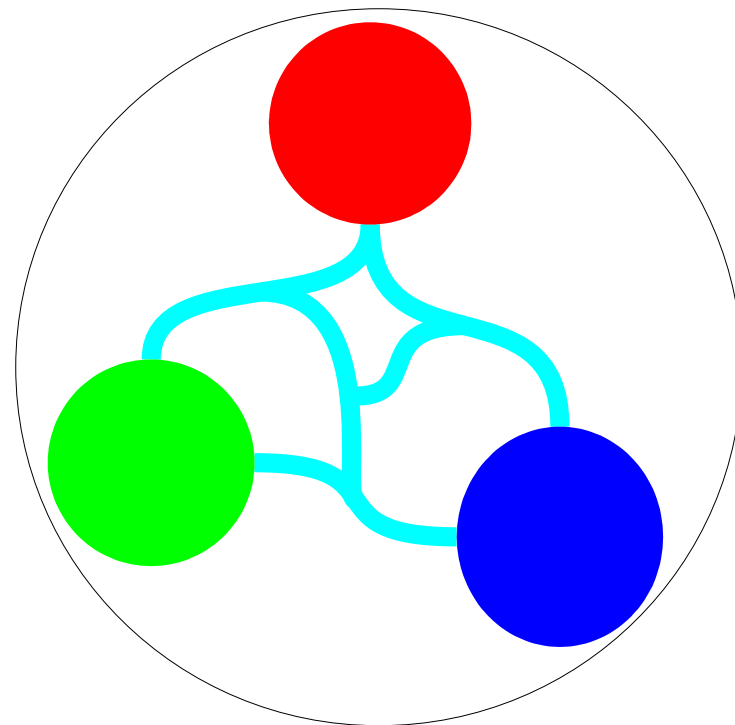
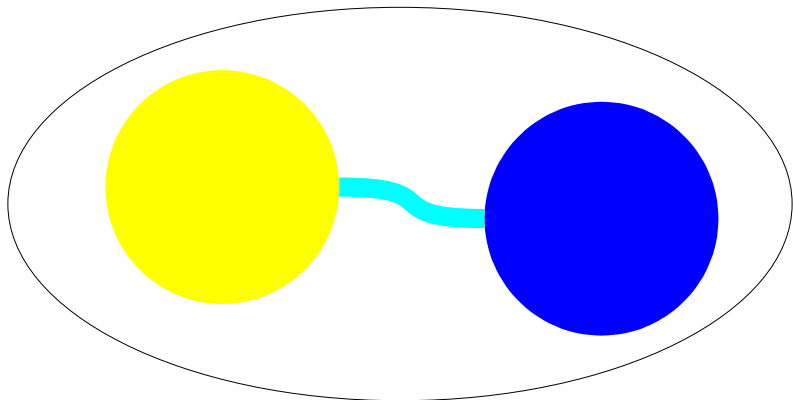
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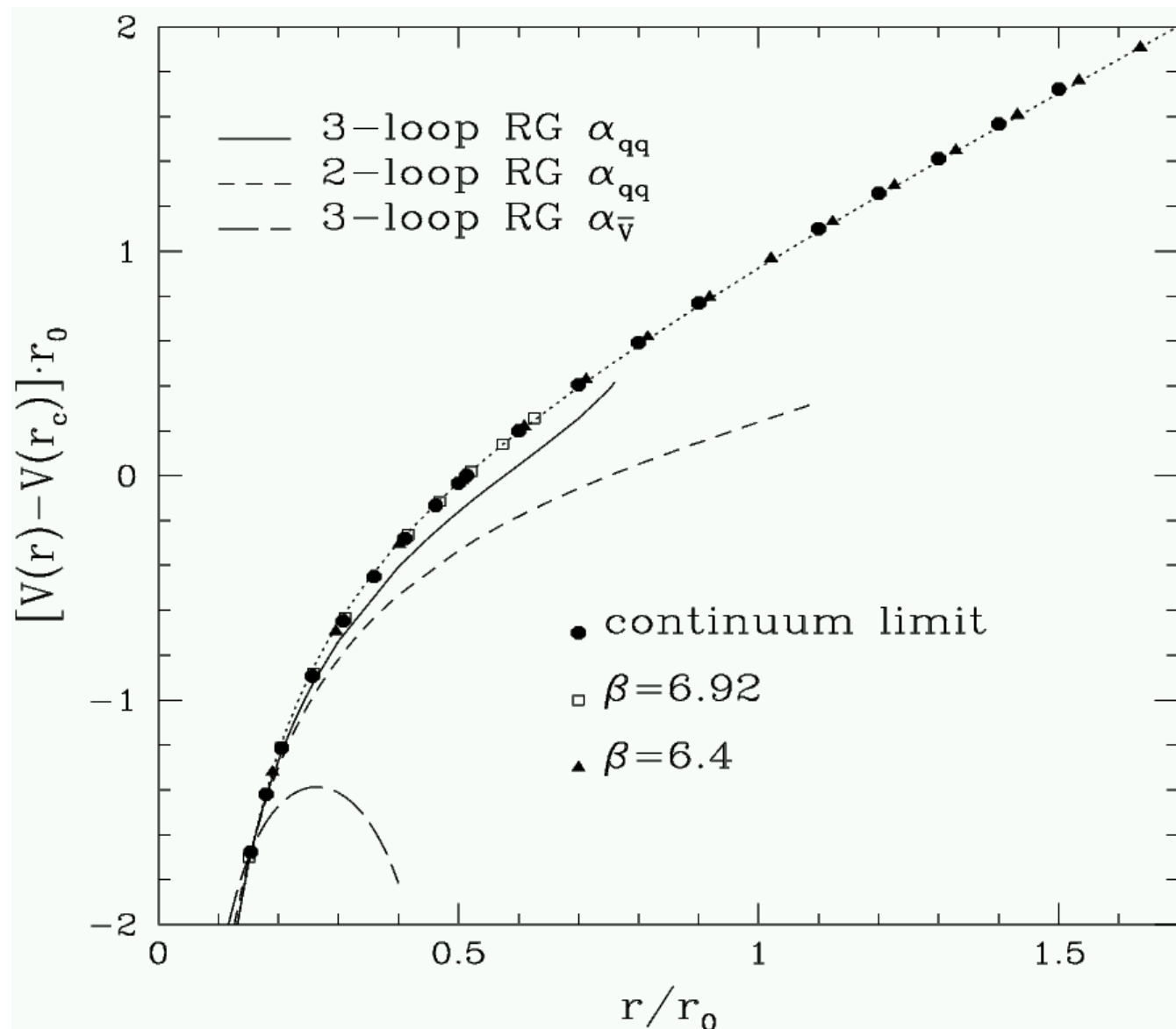
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 - Measured to very high precision for quarks

Confinement of quarks

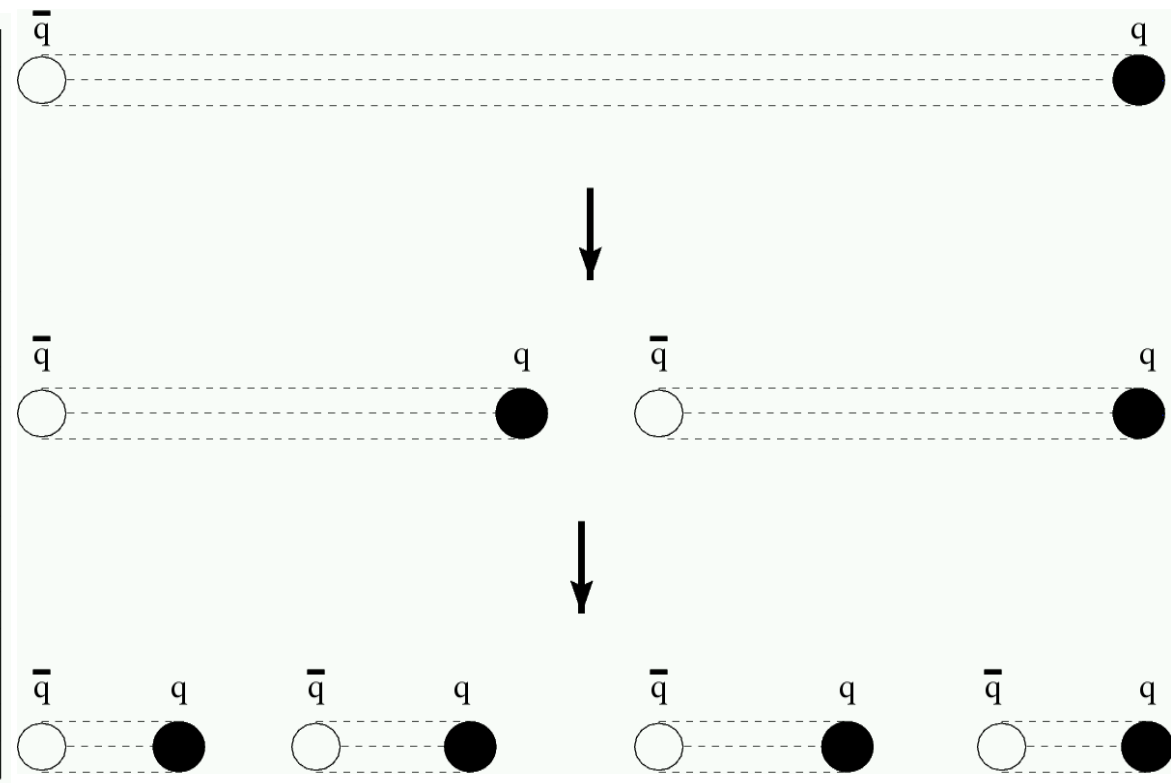
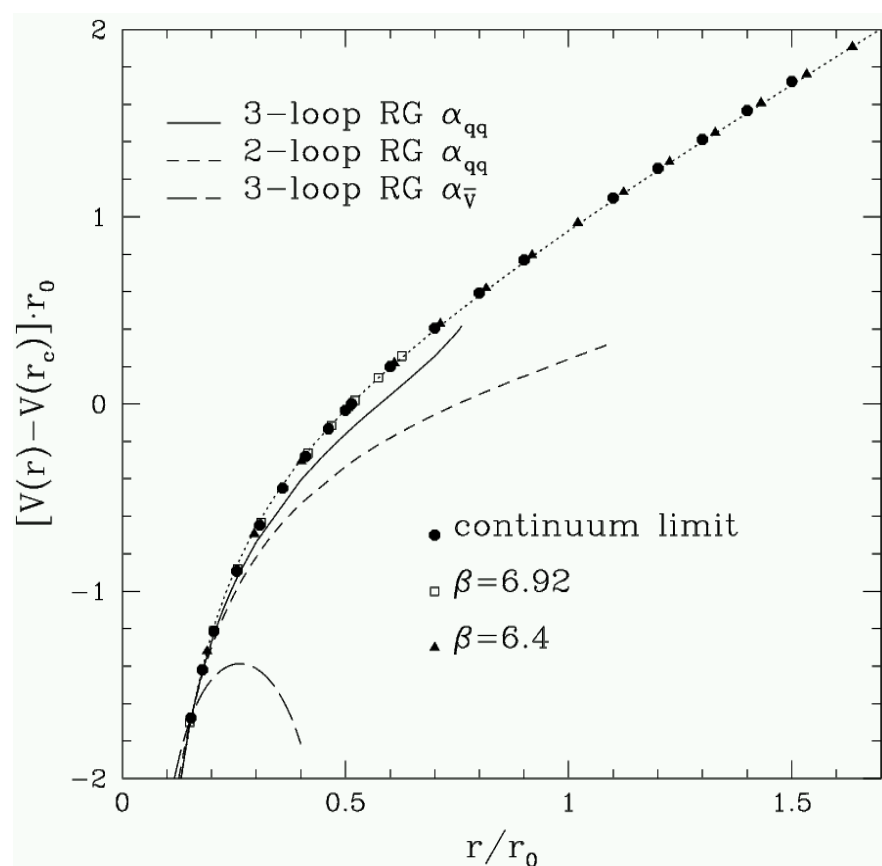
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[Figures from Sommer et al., 2001]

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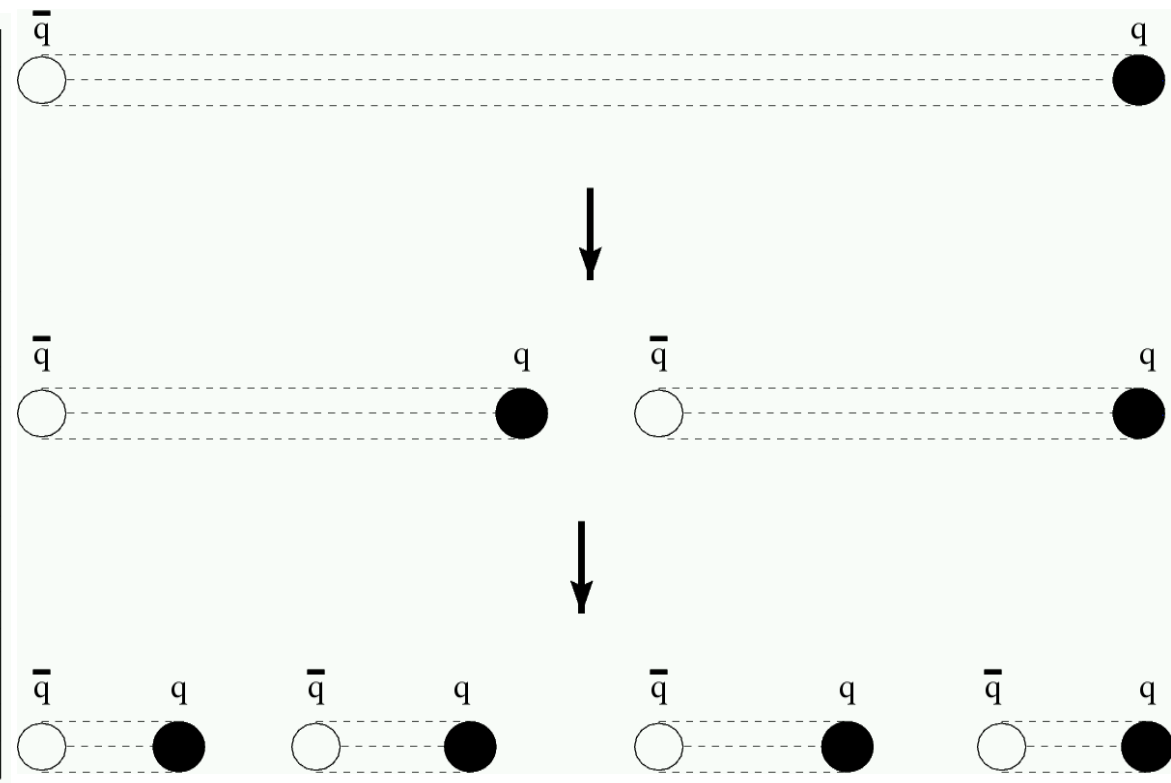
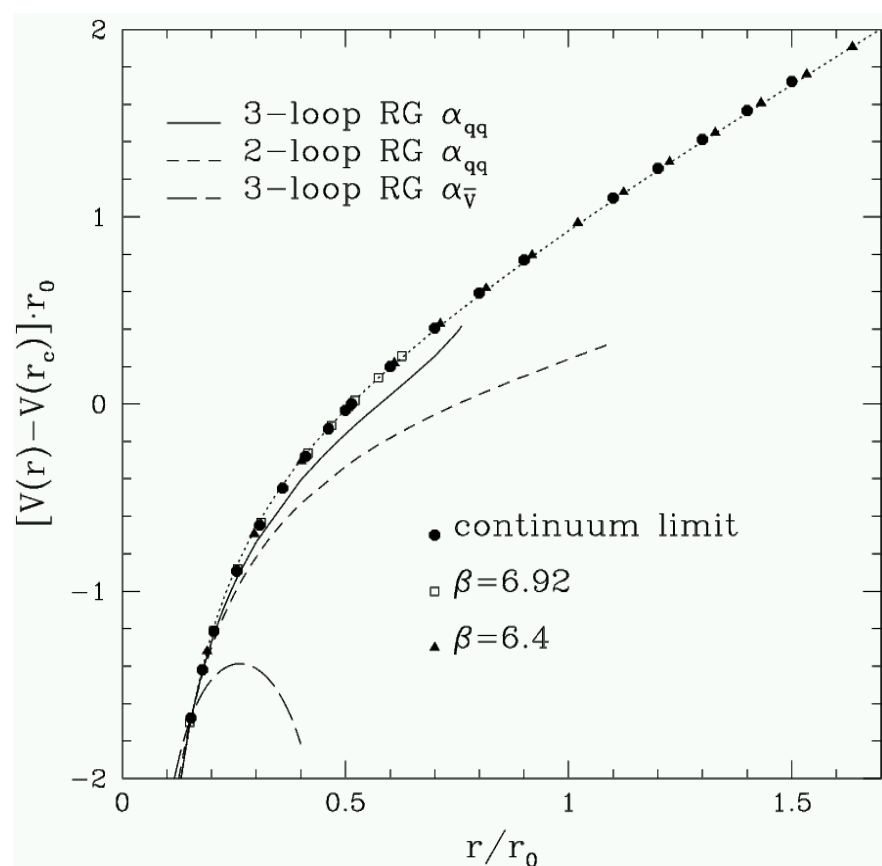
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[Figures from Sommer et al., 2001 (left) and from Greensite, 2003 (right)]

Confinement of quarks

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- Origin of linear potential?

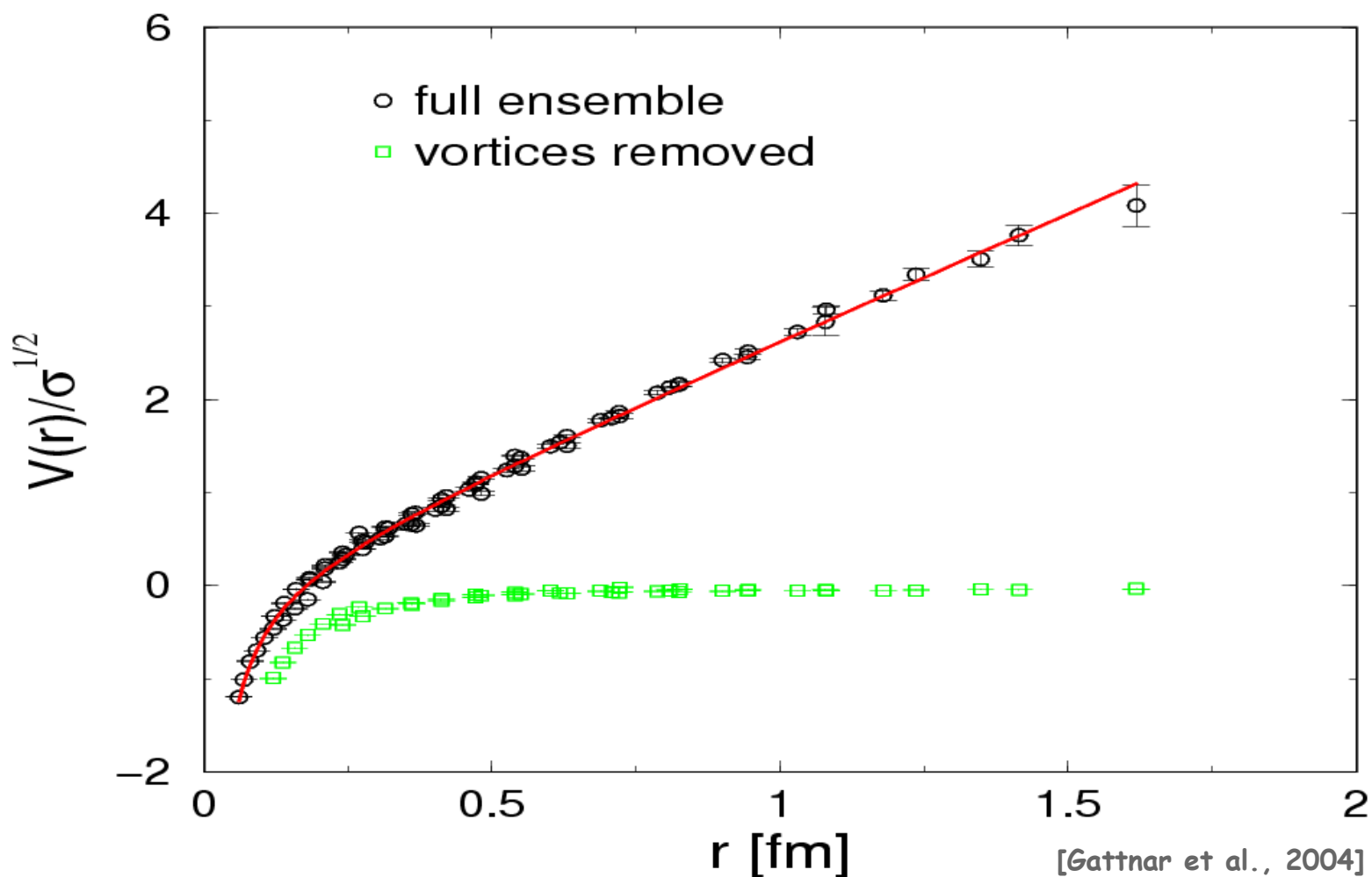


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SU(2), 12^4



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- Complicated structure - many questions open

Gluon confinement

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 - **Gribov-Zwanziger** [Gribov, 1977, Zwanziger, 1994,...,2003]
 - **Kugo-Ojima** [Kugo & Ojima, 1979]
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 - Predictions for correlation functions in various gauges

Landau-gauge and ghosts

- Gauge sector: Choose **Landau gauge**

$$L = -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu,a} - \bar{c}^a \partial^\mu D_\mu^{ab} c^b$$

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(Intermediate states - not observable)

Propagators

[Introduction: Alkofer & von Smekal, 2001]

- **Ghost:**

$$D_G^{ab}(x-y) = \langle \bar{c}^a(x) c^b(y) \rangle$$

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- **Criterion** for $p^2 \rightarrow 0$

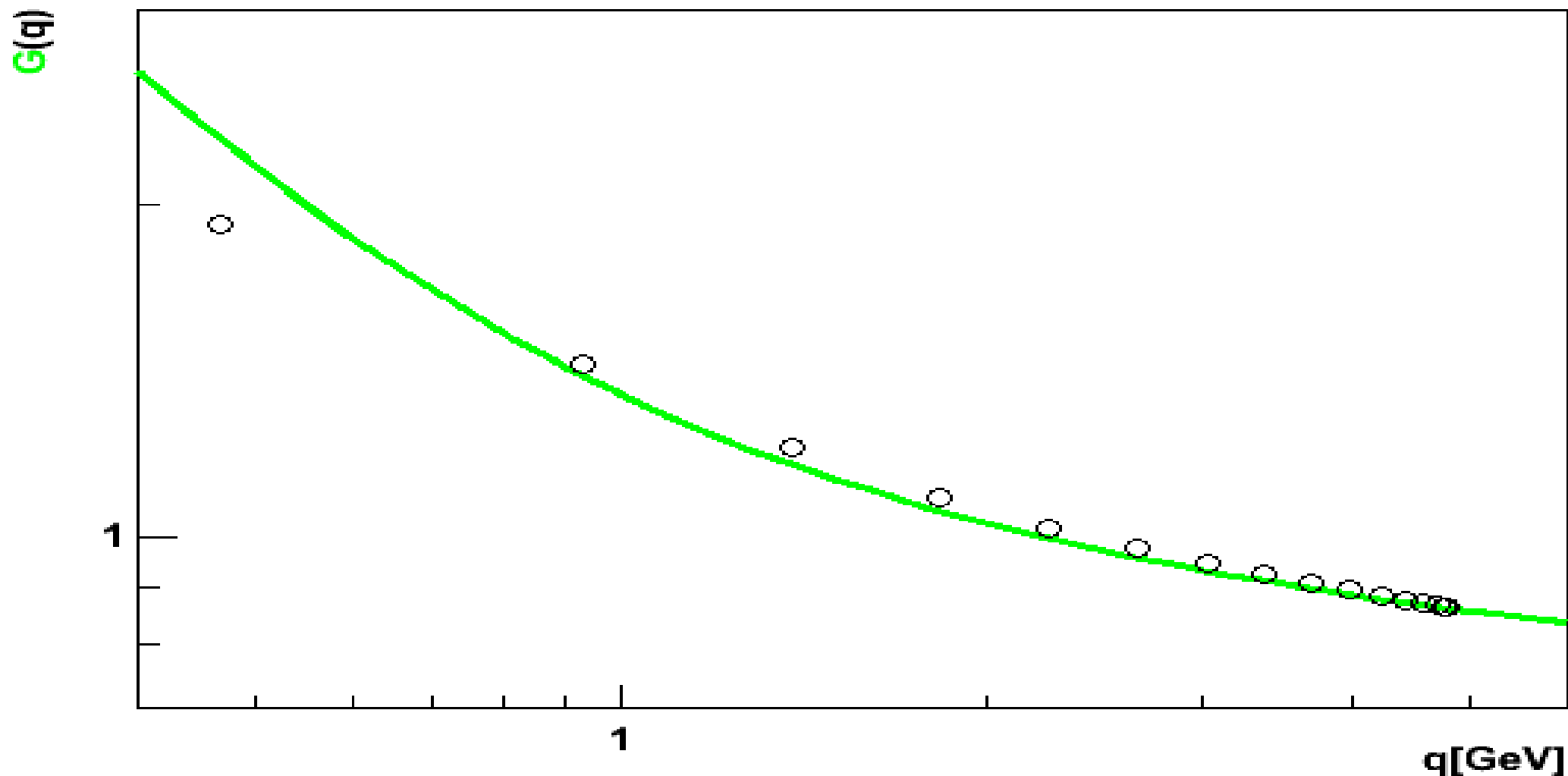
- $G(p^2 \rightarrow 0) \rightarrow \infty$

Results on the **ghost**

[DSE: Fischer et al., PLB 2002,
Lattice 32⁴: Cucchieri et al., unpublished]

- **IR divergence** in accordance with the scenario

Ghost Dressing Function



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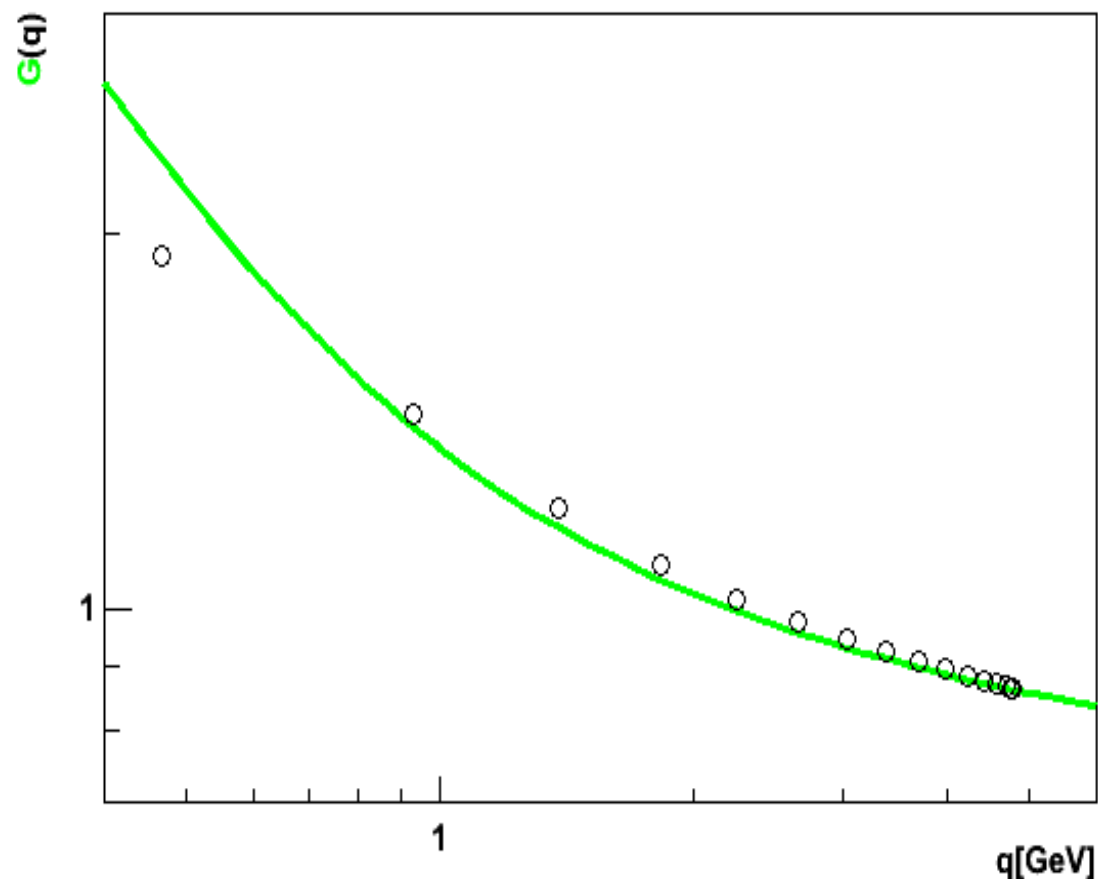
- Lattice calculations

- Dyson-Schwinger equations

- Renormalization group

[Pawlowski et al., 2004]

Ghost Dressing Function



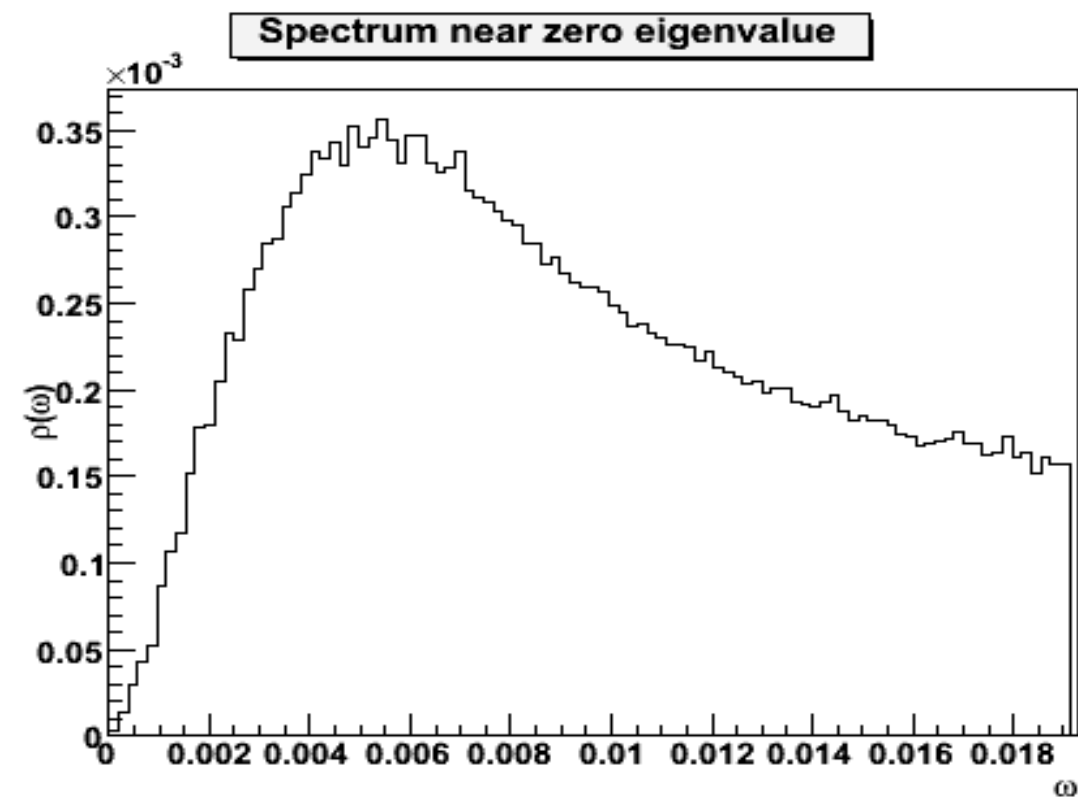
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- Corresponding results in other gauges
 - Coulomb [Feuchter et al., 2004, Cucchieri et al. 2002]
 - Interpolating gauges [Fischer et al., 2005, Cucchieri et al. 2001]

Faddeev-Popov operator eigenspectrum

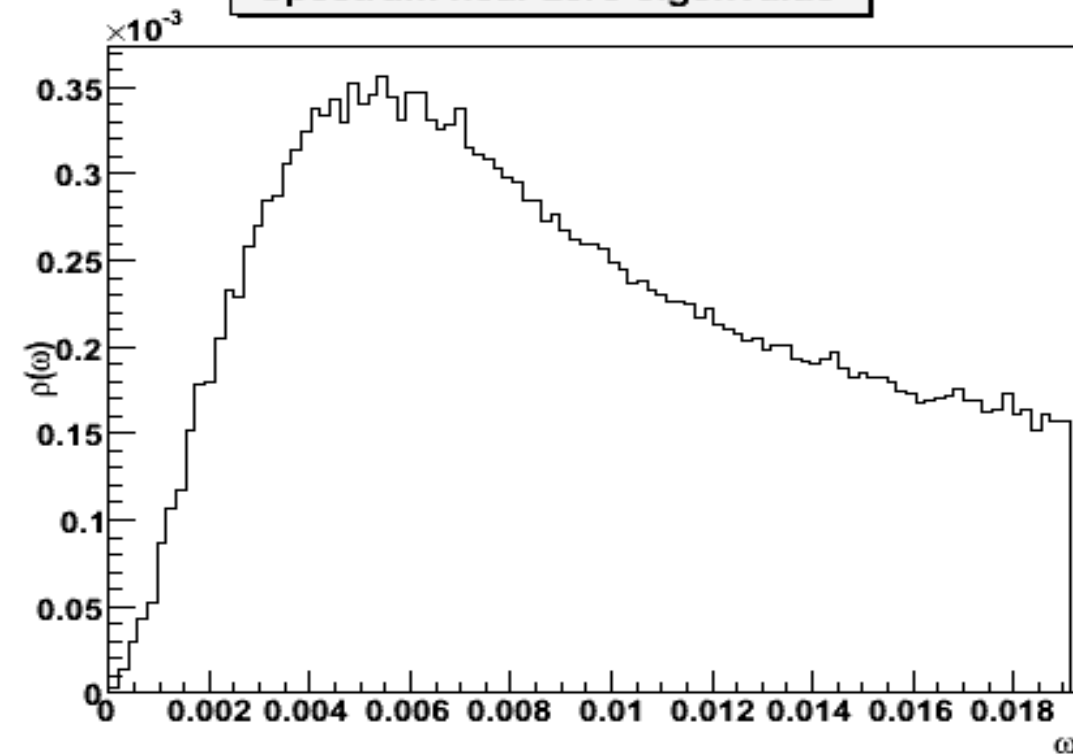


[Cucchieri et al., PRD 2006]

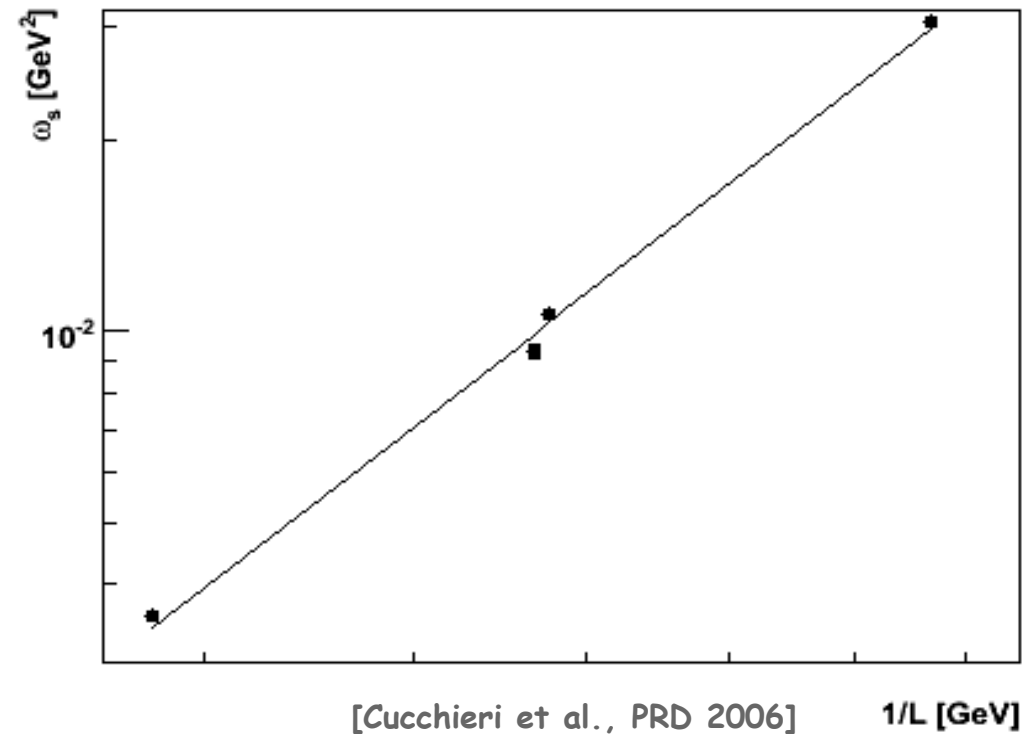
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Faddeev-Popov operator eigenspectrum

Spectrum near zero eigenvalue



Smallest eigenvalue of the Faddeev-Popov operator

[Cucchieri et al., PRD 2006] $1/L$ [GeV]

- Near zero enhanced
- Average configuration in the continuum limit on the Gribov horizon
- Agrees with Gribov-Zwanziger scenario

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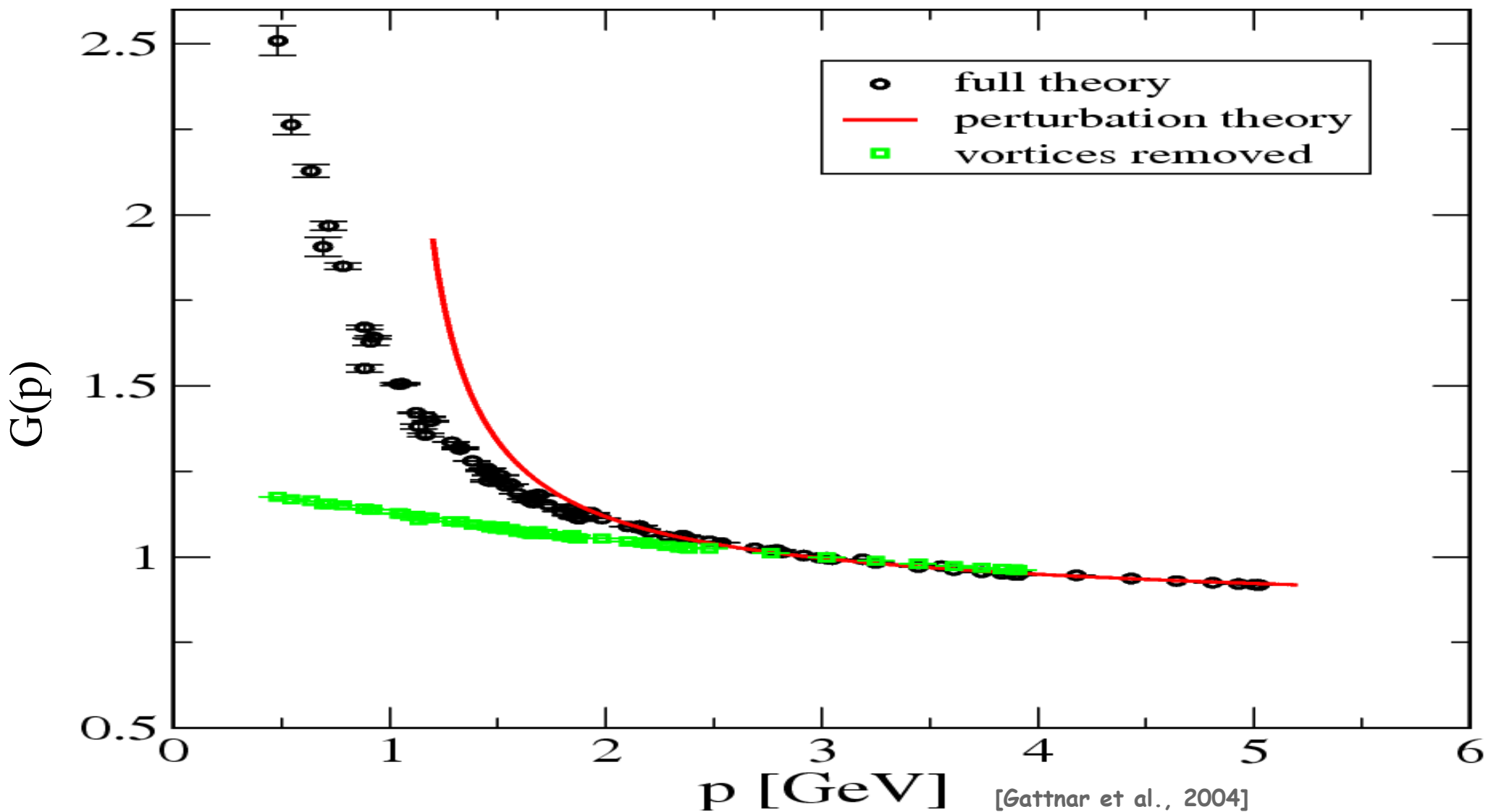
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- Some results from lattice calculations

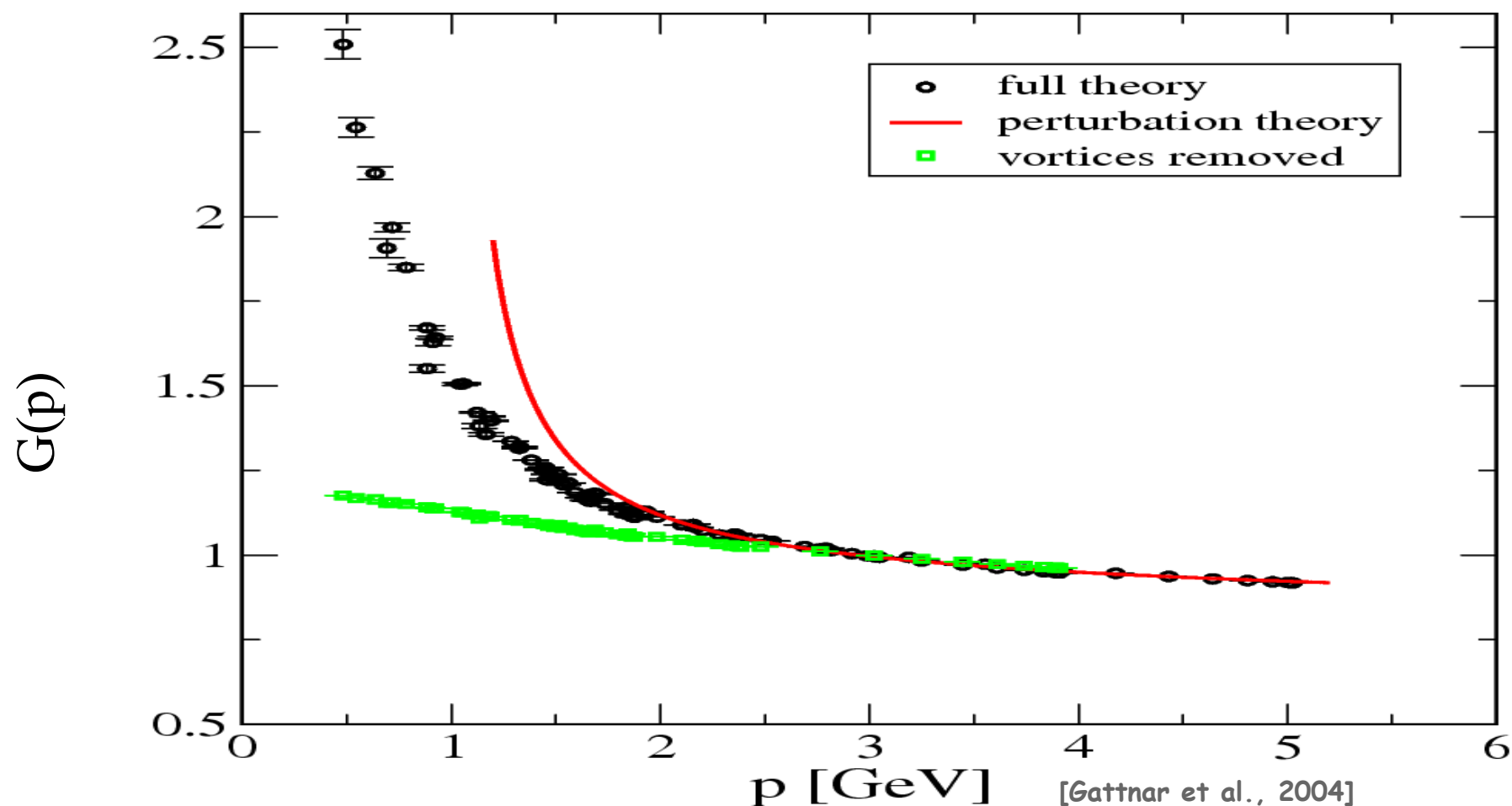
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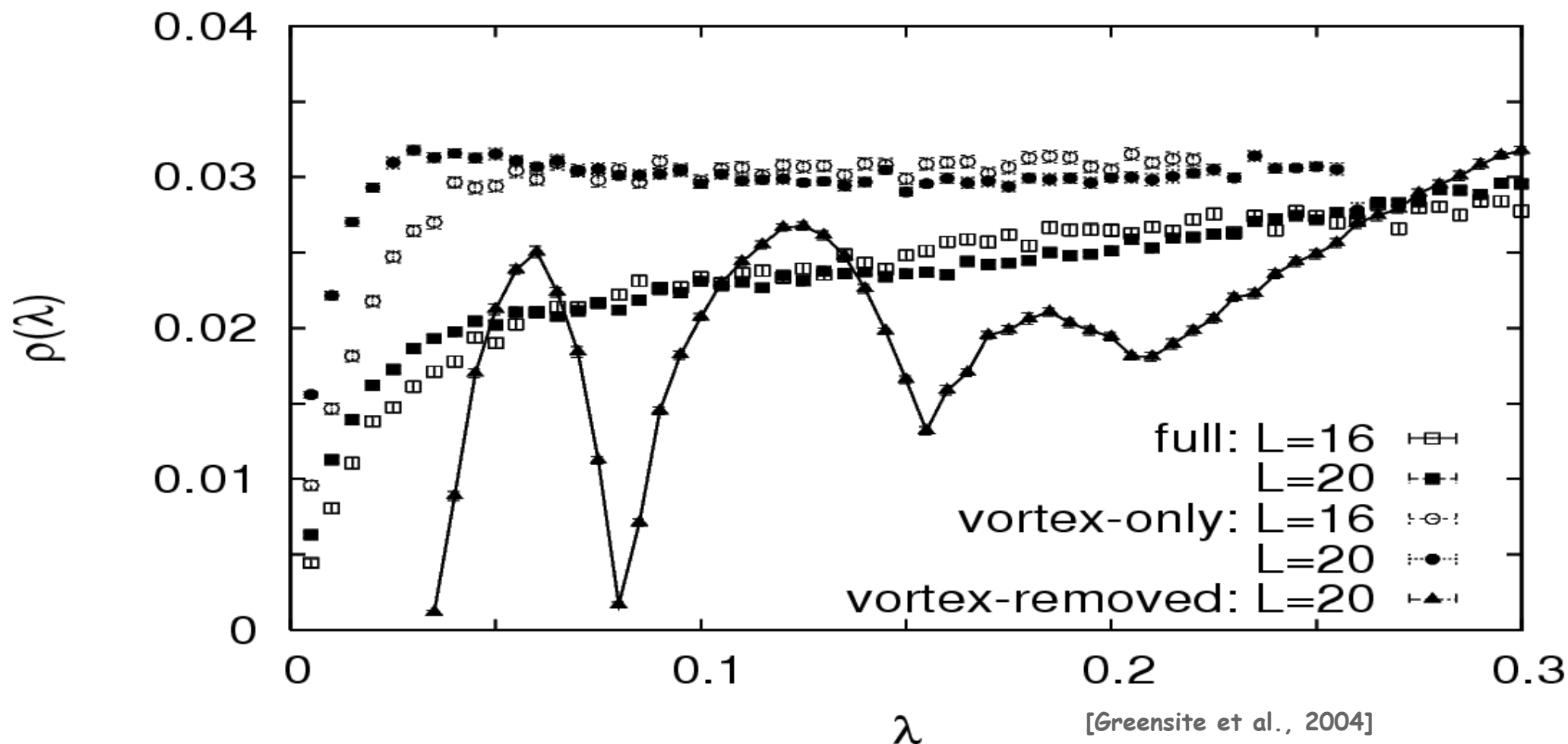
- Removal of vortices tames infrared divergence
- Hints to a change of the FP-eigenspectrum



Vortices and the eigenspectrum of the FPO

- Removal of vortices in Coulomb gauge reduces enhancement of near-zero modes

Eigenvalue density, $\beta=2.3$



Analytical approach

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- **(Center) Vortices** [Maas, EPJC 2006 in press]
 - Thick, string-like configurations

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- General form (for transverse fields)

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- Solutions are series of hypergeometric type

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- **Non-localized**: Correspond to scattering states in quantum mechanics

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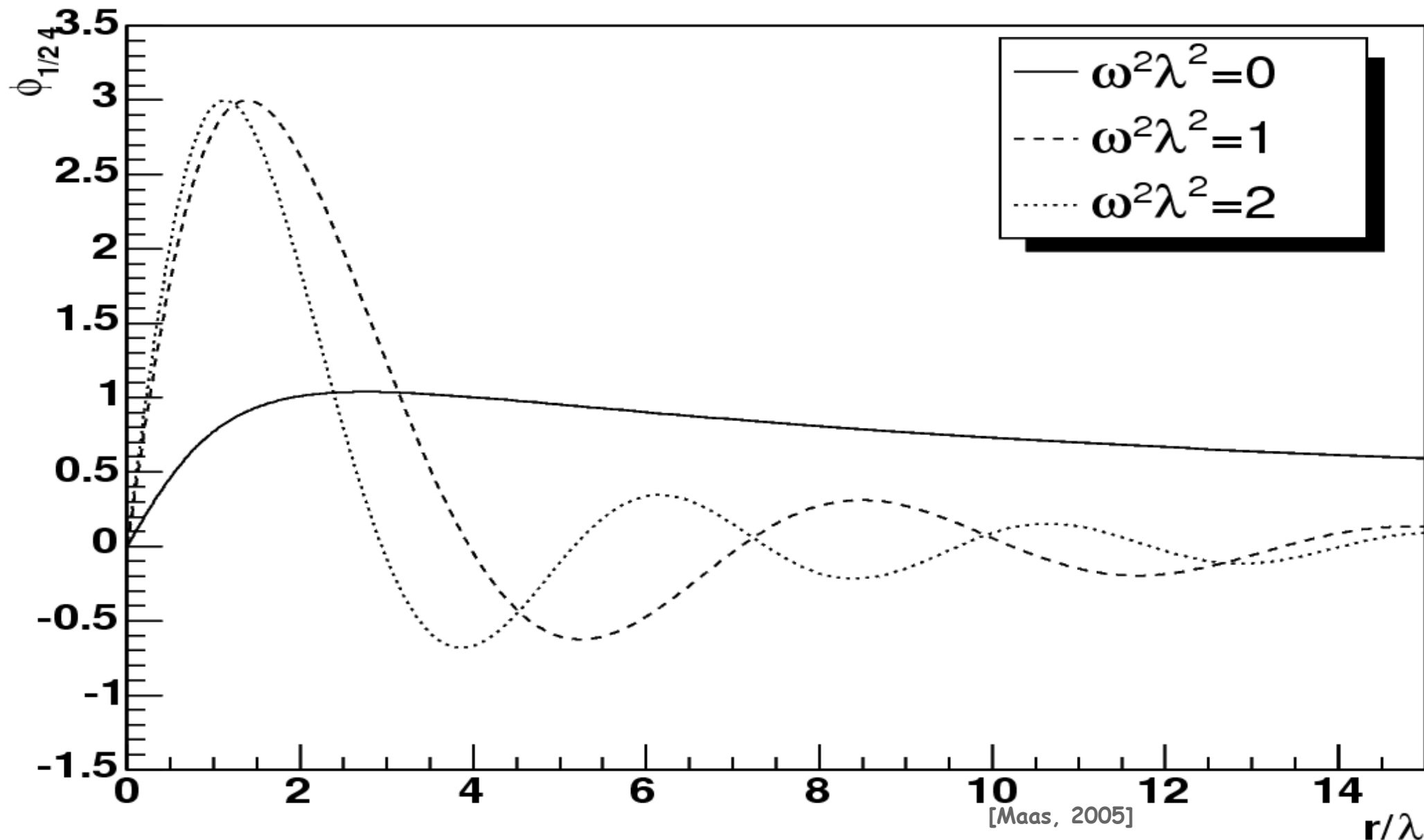
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 - Vortex: $2(\text{flux}-1)$
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 - Zero-modes are non-localized
 - Instanton and vortices: Non-trivial radial behavior
 - Monopole: Non-trivial angular behavior

Example: Radial part of an instanton zero-mode

Radial eigenfunction at $l=1/2$ and $c=4$



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 - Possibility to remove non-confined 'diagonal' gluon in the dual superconductor picture by a Gribov-Zwanziger mechanism
 - Can be done by monopoles or vortices

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- Connection between the confinement of quarks and gluons likely by **topological configurations**
- Vortices, monopoles, instantons all contribute
 - At least the vortex fulfills a necessary confinement criterion
- A Gribov-Zwanziger effect could make the dual color-superconductor picture consistent

Summary

- Confinement of **quarks** and **gluons** still challenging
- Connection between the confinement of quarks and gluons likely by **topological configurations**
- Vortices, monopoles, instantons all contribute
 - At least the vortex fulfills a necessary confinement criterion
- A Gribov-Zwanziger effect could make the dual color-superconductor picture consistent
- **Origin or symptom?**