Propagators and vertices in Landau-gauge Yang-Mills theory

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Overview

- Introduction
  - Confinement
  - Green's functions
  - Landau-gauge Yang-Mills theory

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- Propagators
- Vertices
- Summary

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- Substructure cannot be isolated: Confinement

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  - Degrees of freedom are quarks and gluons
  - Color charge carried by both
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  - Non-perturbative effect
  - How to establish confinement?

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  - Where to look?

Yang-Mills-Theory - QCD without quarks

- Gauge theory: Choose Landau gauge

$$L=-\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu,a}-\bar{c}^a\partial^\mu D^{ab}_\mu c^b$$

$$F^a_{\mu\nu}=\partial_\mu A^a_\nu-\partial_\nu A^a_\mu+g f^{abc}A^b_\mu A^c_\nu$$

$$D^{ab}_\mu=\delta^{ab}\partial_\mu-g f^{abc}A^c_\mu$$

- Degrees of freedom:

Gluons: A^a_μ

Ghosts: $c^a,\bar{c}^a$

(Intermediate states - not observable)

Propagators

[Introduction: Alkofer & von Smekal, 2001]

- 2-point Green's functions are the propagators
- Gluon:

$$D_{\mu\nu}^{ab}(x-y) = \langle A_\mu^a(x) A_\nu^b(y) \rangle$$

$$D_{\mu\nu}^{ab}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \frac{Z(p)}{p^2}$$

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- Ghost:

$$D_G^{ab}(x-y) = \langle \bar{c}^a(x) c^b(y) \rangle$$

$$D_G^{ab}(p) = \frac{-G(p)}{p^2}$$

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- (Euclidean) Infrared: $q^2=0$
- Gluons and ghosts 'massless': $q^2=0$ is on-shell condition
- Vanishing propagator at $q^2=0$: No propagating physical particle - confined!
- Divergence at $q^2=0$: Mediates long range forces

Lattice and functional methods

- Lattice gauge theory
  - (Nearly) no approximations involved
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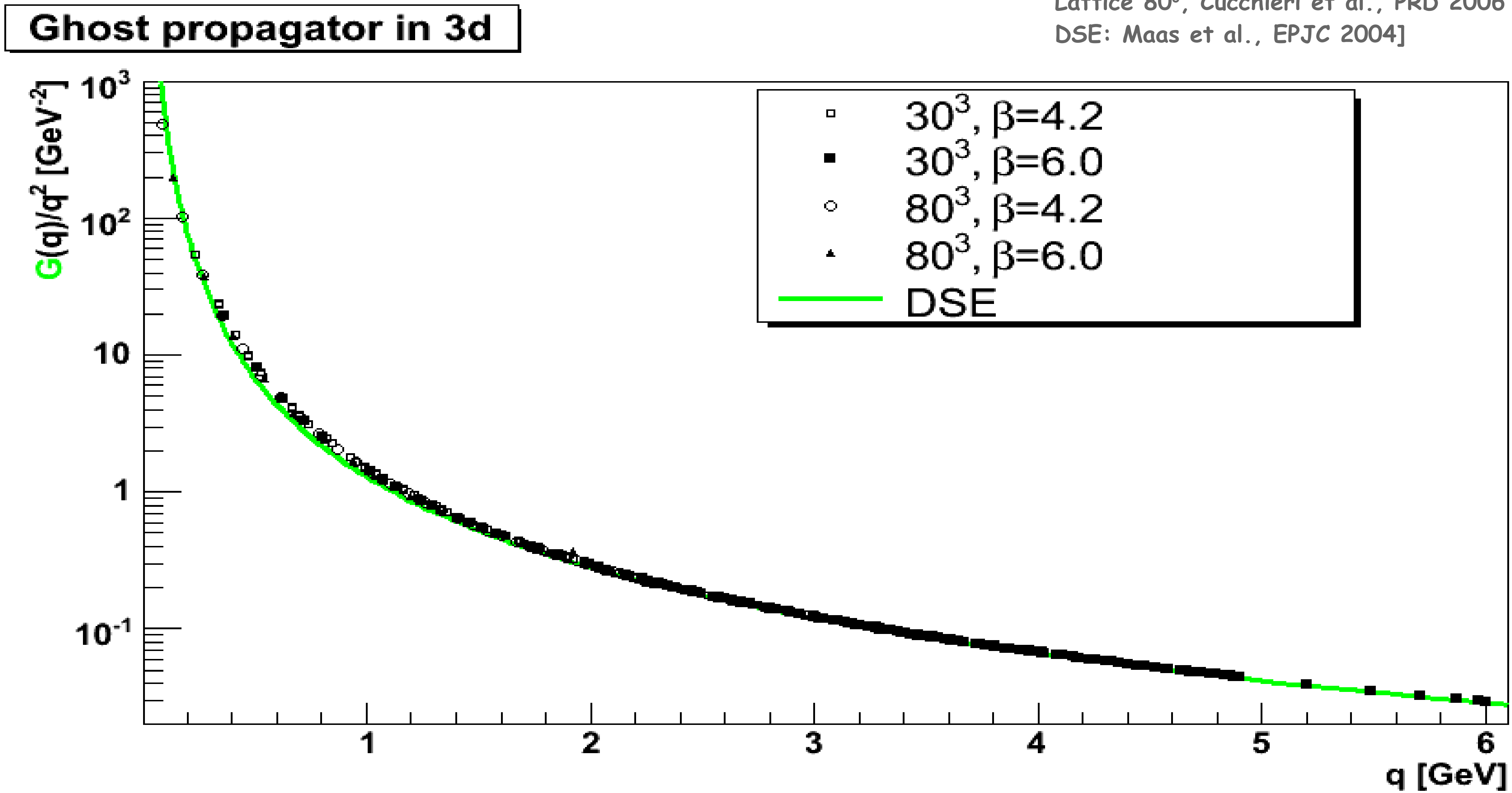
- Close cooperation between both approaches
- Lattice needs large volume – go to 3d
  - 4d results similar, but with smaller volumes

Ghost propagator

[Lattice 30^3, Cucchieri et al., 2006
Lattice 80^3, Cucchieri et al., PRD 2006
DSE: Maas et al., EPJC 2004]

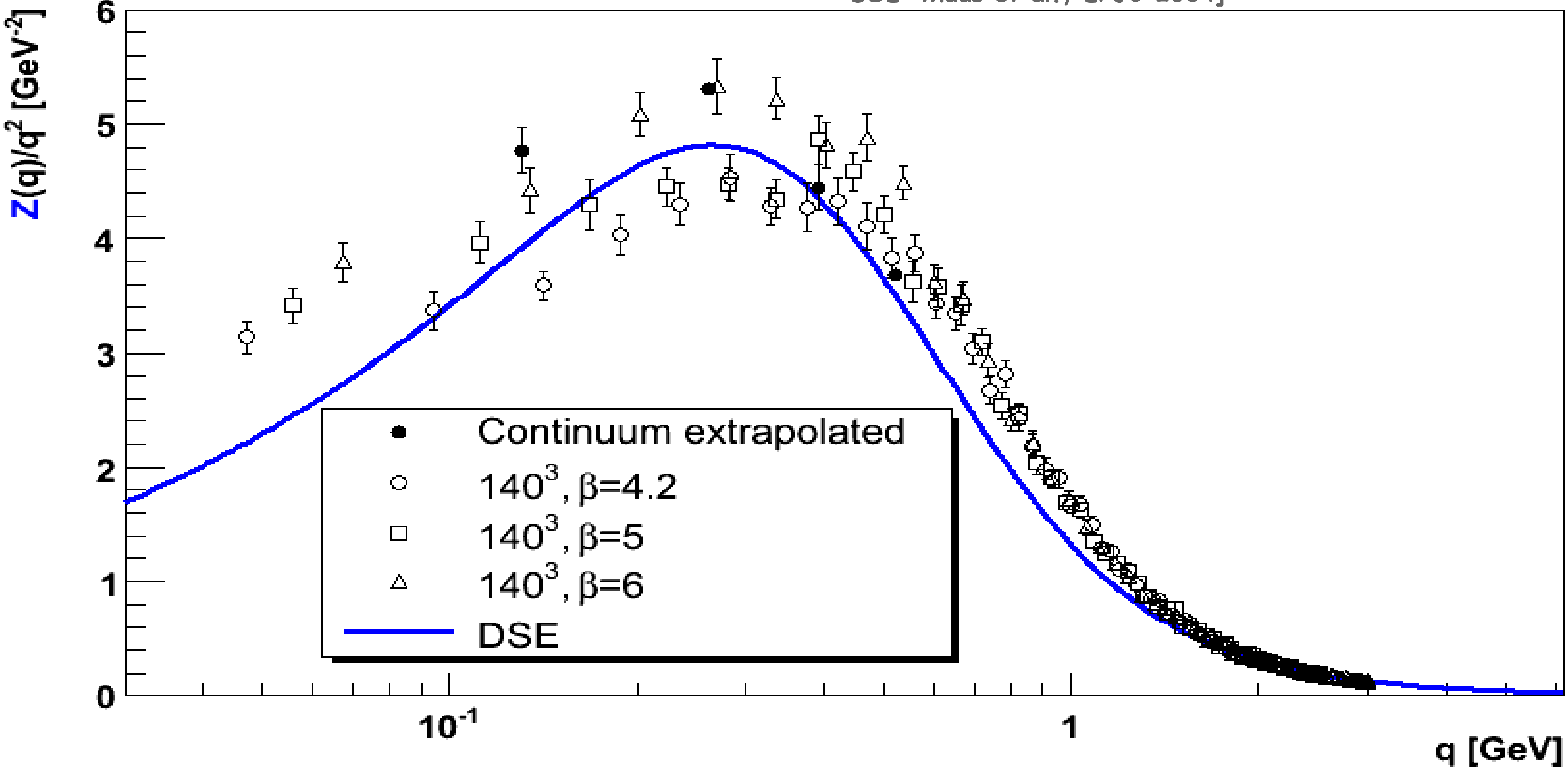


- Strong divergence – already on small lattices
- DSEs: Diverges, $G(p) \sim (p^2)^{-0.39}$ [Zwanziger, PRD 2002, Maas et al., EPJC 2004]

Gluon – large lattice volumes needed

Gluon propagator in 3d

[Lattice continuum extrapolated: Cucchieri et al., PRD 2001
Lattice 140³: Cucchieri et al., PRD 2003
DSE: Maas et al., EPJC 2004]



- Much larger lattices needed to see bend over
- DSEs: Goes to zero, $Z(p) \sim (p^2)^{1.28}$ [Zwanziger, PRD 2002, Maas et al., EPJC 2004]

Confinement scenario

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- How to understand? Alternative scenarios?
  - Kugo-Ojima scenario based on BRST [Kugo et al., Prog.Theo.Phys., 1979]
  - Gribov-Zwanziger scenario [Gribov, Nucl.Phys.,1977, Zwanziger, PRD 2002, PRD 2004]
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    - Well supported by detailed investigations
- Behavior of vertices important for consistency

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$$G^{Ac\bar{c}} = \Gamma^{tl} < A c \bar{c} > \ / (\Gamma^{tl} D D_G D_G \Gamma^{tl})$$

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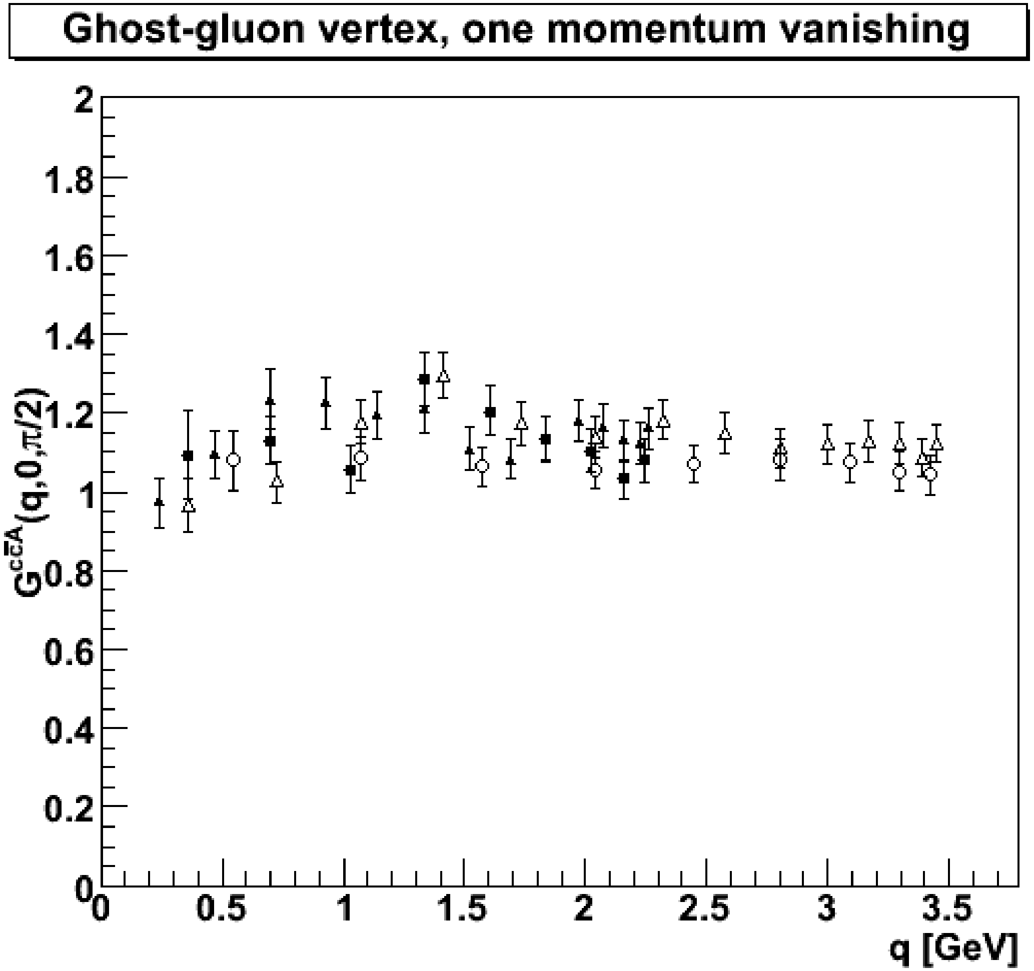
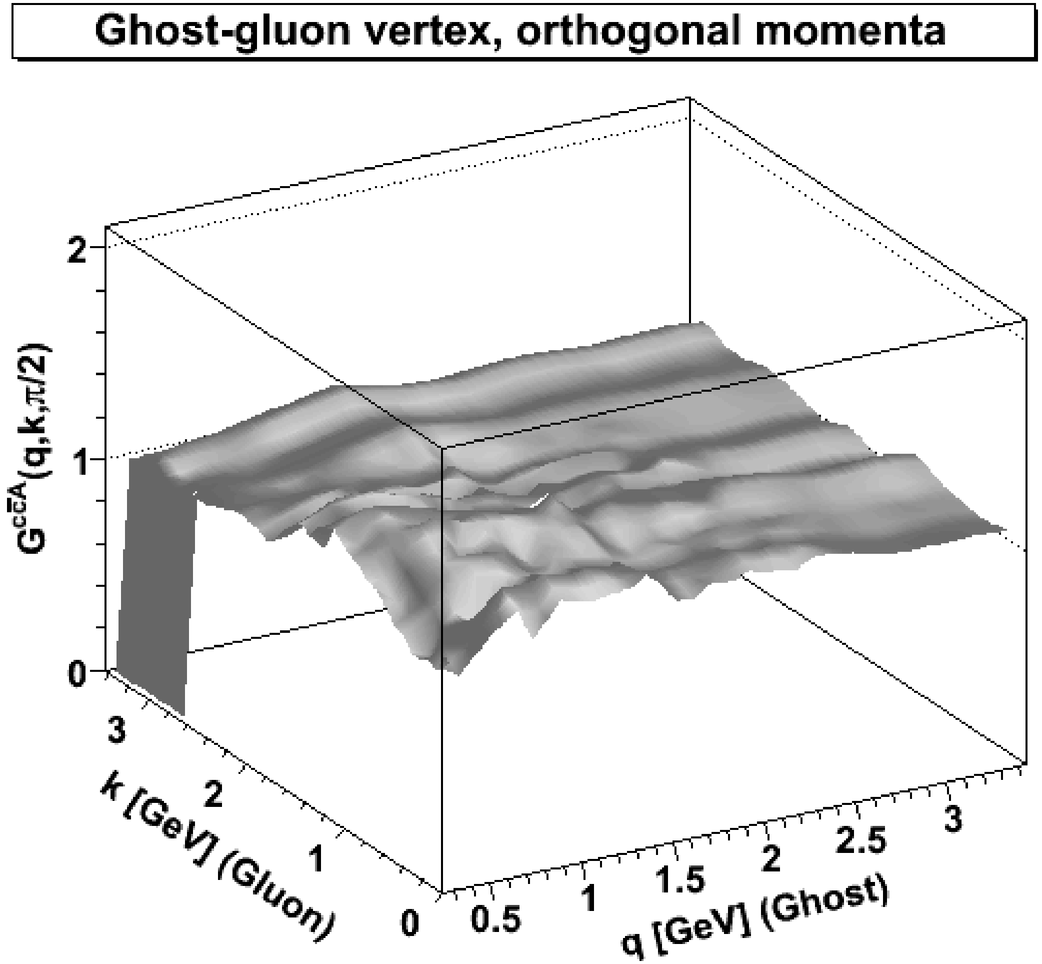
  - Three-gluon vertex

$$<A_\mu^a A_\nu^b A_\rho^c> \;= \; D_{\mu\alpha}^{ad} D_{\nu\beta}^{be} D_{\rho\gamma}^{cf} \Gamma_{\alpha\beta\gamma}^{def}$$

$$G^{A^3} = \Gamma^{tl} < AAA > \; / (\Gamma^{tl} DDD \Gamma^{tl})$$

Ghost-gluon vertex





- Lattice data (20^3 and 30^3): Constant [Cucchieri et al. 2006]
  - also in 4d [Cucchieri et al. JHEP 2004]
- Consistent with DSE predictions [Schleifenbaum et al., PRD 2005, 2006, Alkofer et al. PLB 2005]

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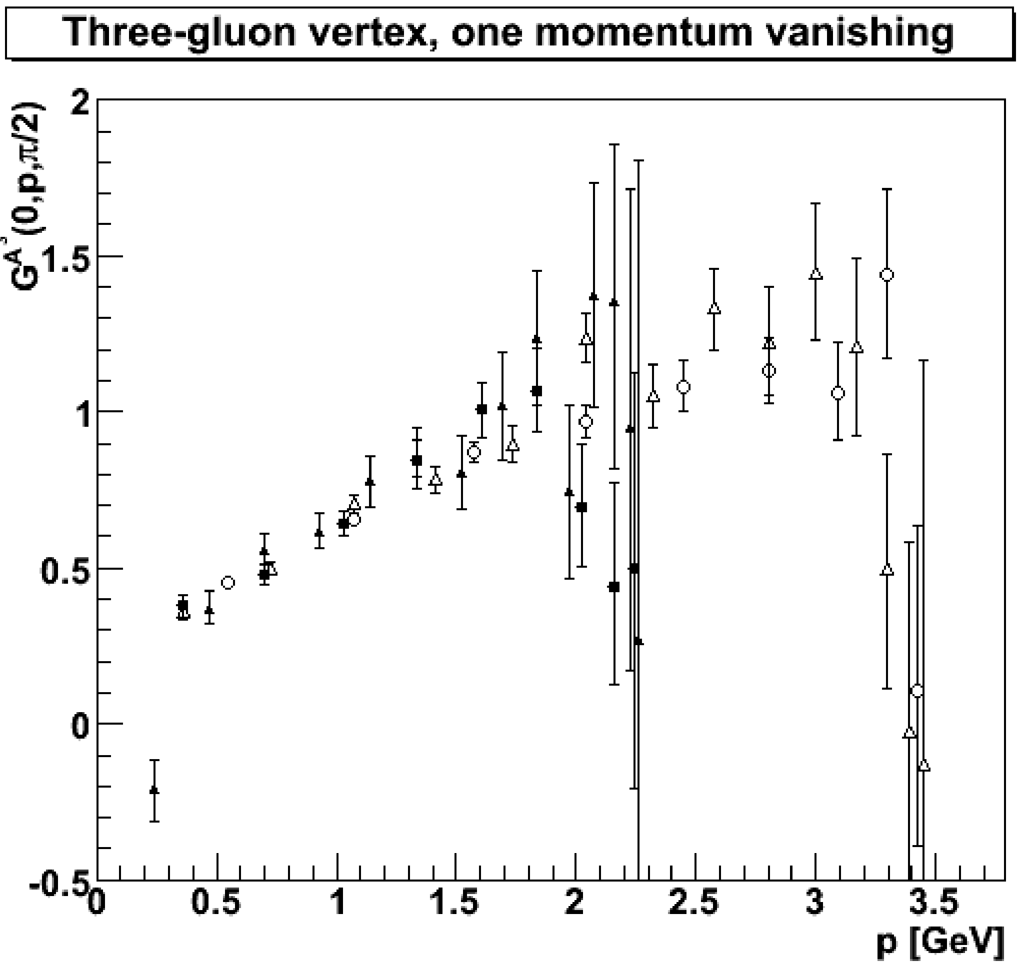

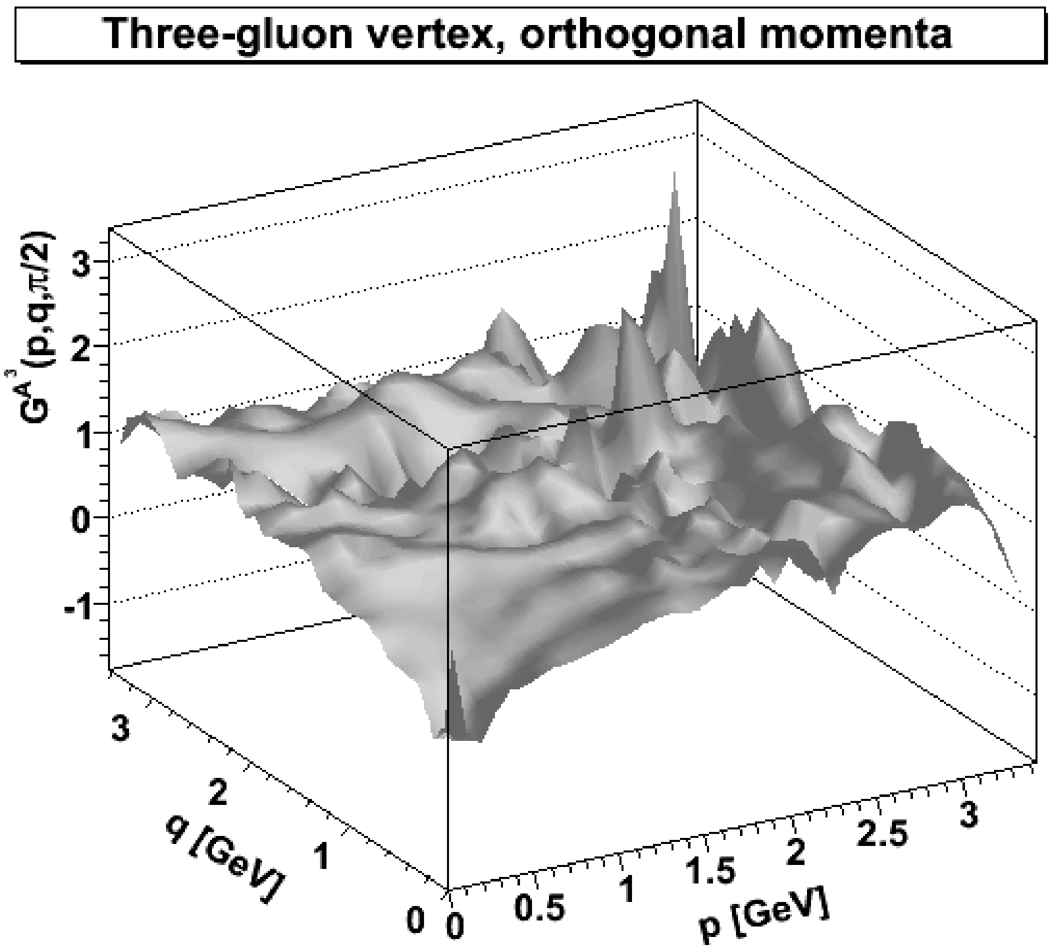

- Lattice data (20^3 and 30^3): Vanishes or reverses sign [Cucchieri et al. 2006]
  - Strong statistical fluctuations at large momenta
- Different from DSE predictions [Schleifenbaum et al., 2006, Alkofer et al. PLB 2005]
- Does not affect propagators in DSE calculations

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- Green's functions key to confinement
- Functional and lattice methods cooperate closely
- Results comply with Gribov-Zwanziger scenario
- Lattice results on vertices verify assumptions in functional calculations
- Consistent picture of confinement starts to emerge

IFSC-USP lattice team

- Attilio Cucchieri[a]
- Axel Maas[b]
- Tereza Mendes[c]
- Antonio Mihara[d]
  - Recent publications (as e-prints) on this topic:

    hep-lat/0605011[abc], hep-lat/0602012[ac], hep-lat/0408034[acd], hep-lat/0312036[ac+], hep-lat/0302022[ac+]
  - Supported by FAPESP, CNPq, and DFG