

3-point vertices in Landau-gauge Yang-Mills theory

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Confinement: Connecting the light- and heavy-quark domains
Trento, Italy



Overview

- Vertices in the continuum and on the lattice

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- The infinite-temperature case: **Three dimensions**
- Going far infrared: **Two dimensions**
- Conclusions

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- Interactions in QCD are gauge-dependent

Landau-gauge and ghosts

- Ignore quarks and choose Landau gauge

$$L = -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu,a} - \bar{c}^a \partial^\mu D_\mu^{ab} c^b$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f^{abc} A_\mu^b A_\nu^c$$

$$D_\mu^{ab} = \delta^{ab} \partial_\mu - g f^{abc} A_\mu^c$$

- Degrees of freedom:

Gluons: A_μ^a

Ghosts: c^a, \bar{c}^a

(Intermediate states - not observable)

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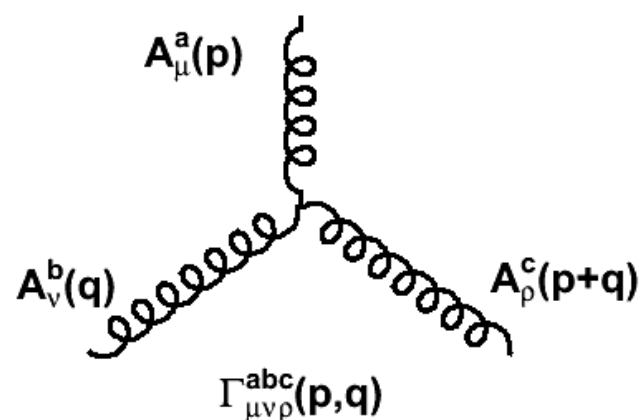
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- Practical calculations **limited to a selection**

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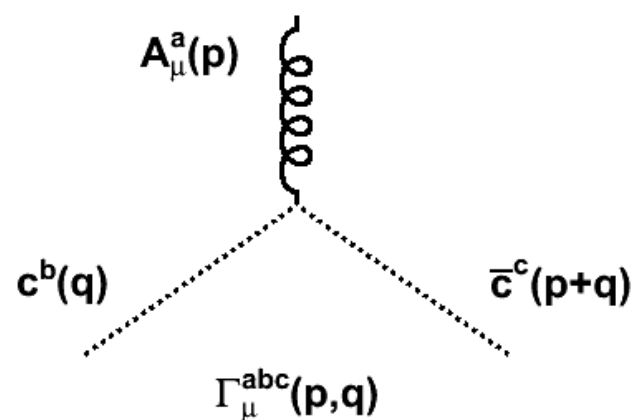
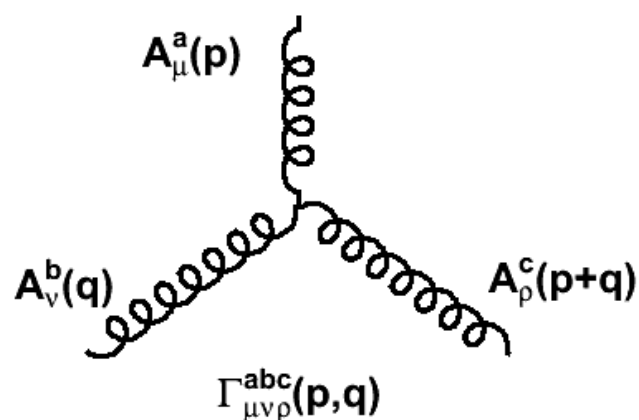
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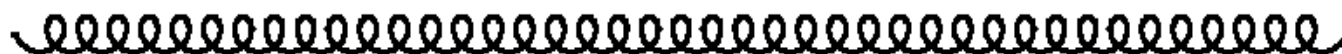
Vertices in Landau gauge

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 - Describe gluon self-interaction and ghost-gluon self-interaction



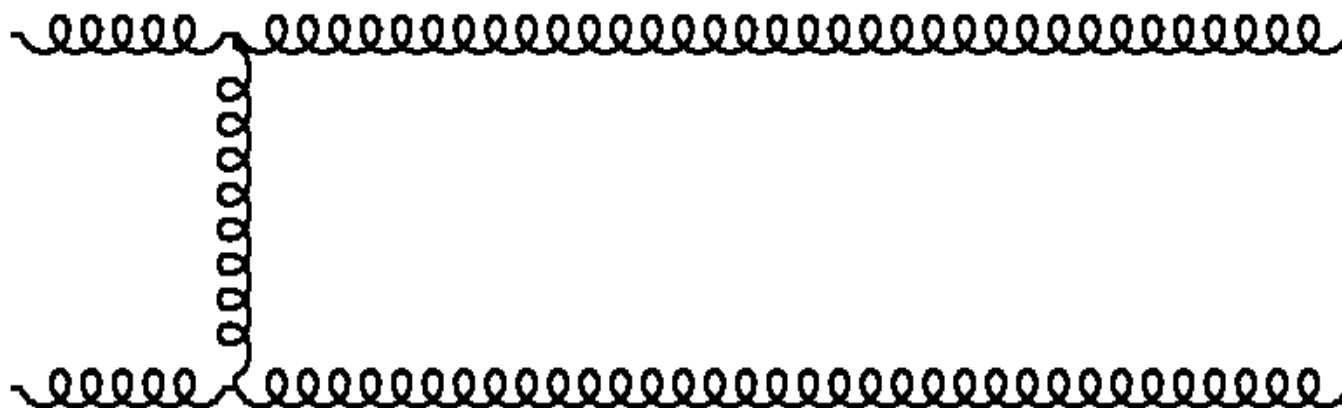
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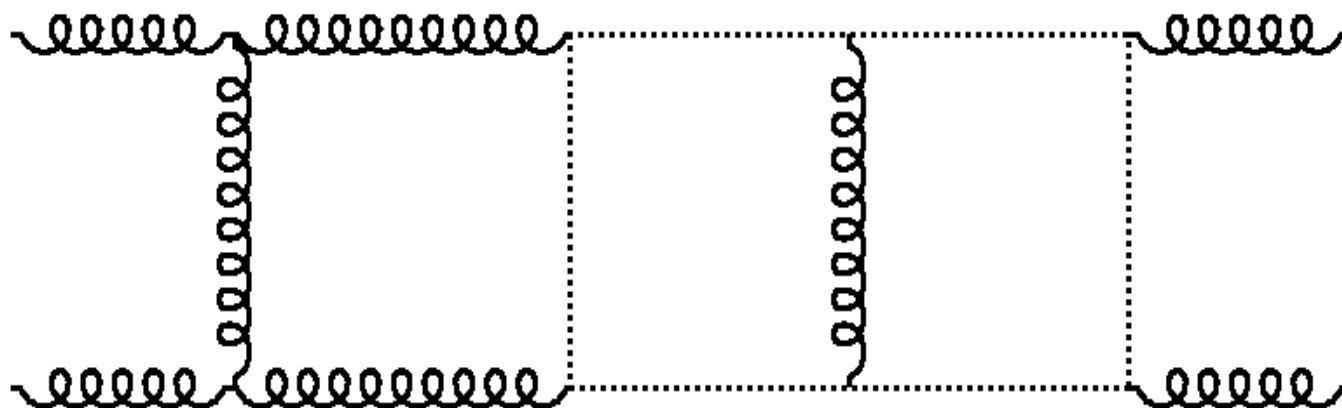
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- **Binding relies on vertices**
- Also lead eg to a **ghost-content of glueballs** in Landau gauge



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- Truncated DSEs give **quantitative predictions**, in $d=2$ $K = 1/5$ or 0 [Zwanziger, PRD 2002, Maas et al., EPJC 2004]
- In particular: IR-constant ghost-gluon vertex, infrared divergent three-gluon vertex for all d

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 - Except when non-local quantities are involved, eg inverse Faddeev-Popov or fermion operators

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- Mixes in general tensor structures

- Here only in case of the three-gluon vertex

Kinematic configurations

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 - Eg Popular symmetric configuration only possible in three or more dimensions
 - May limit momentum range

Kinematic configurations

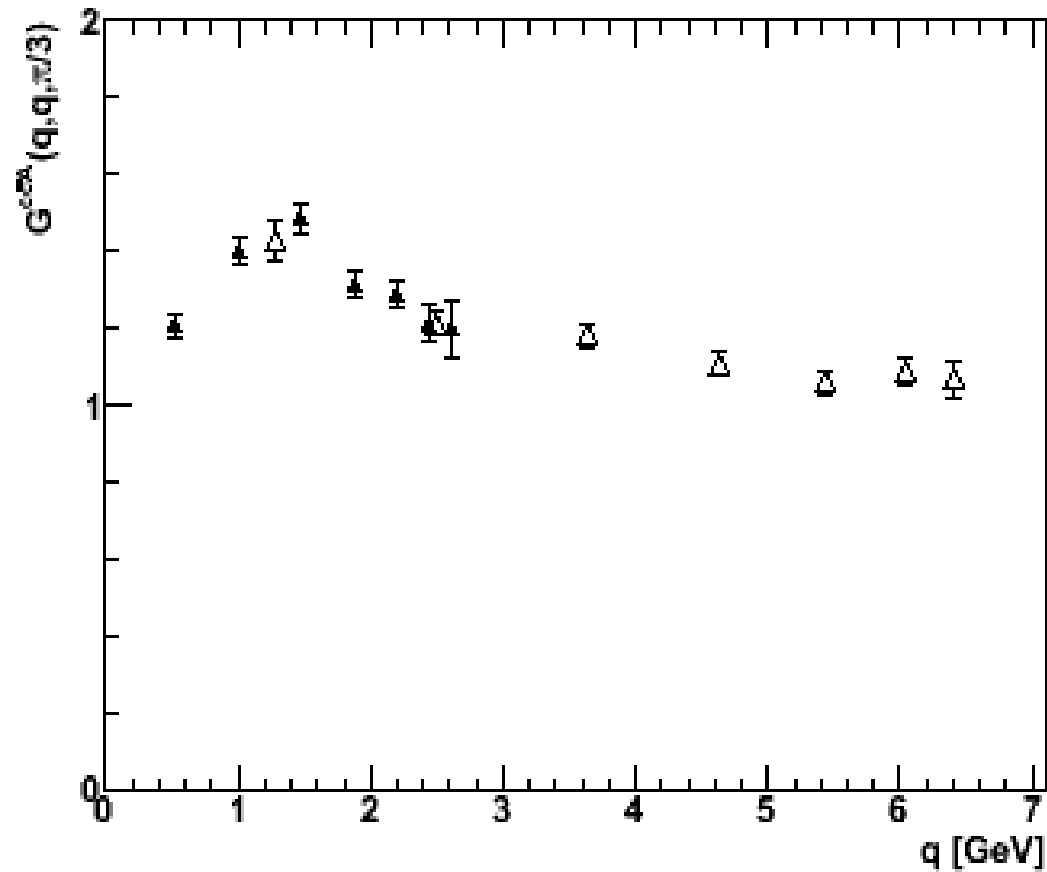
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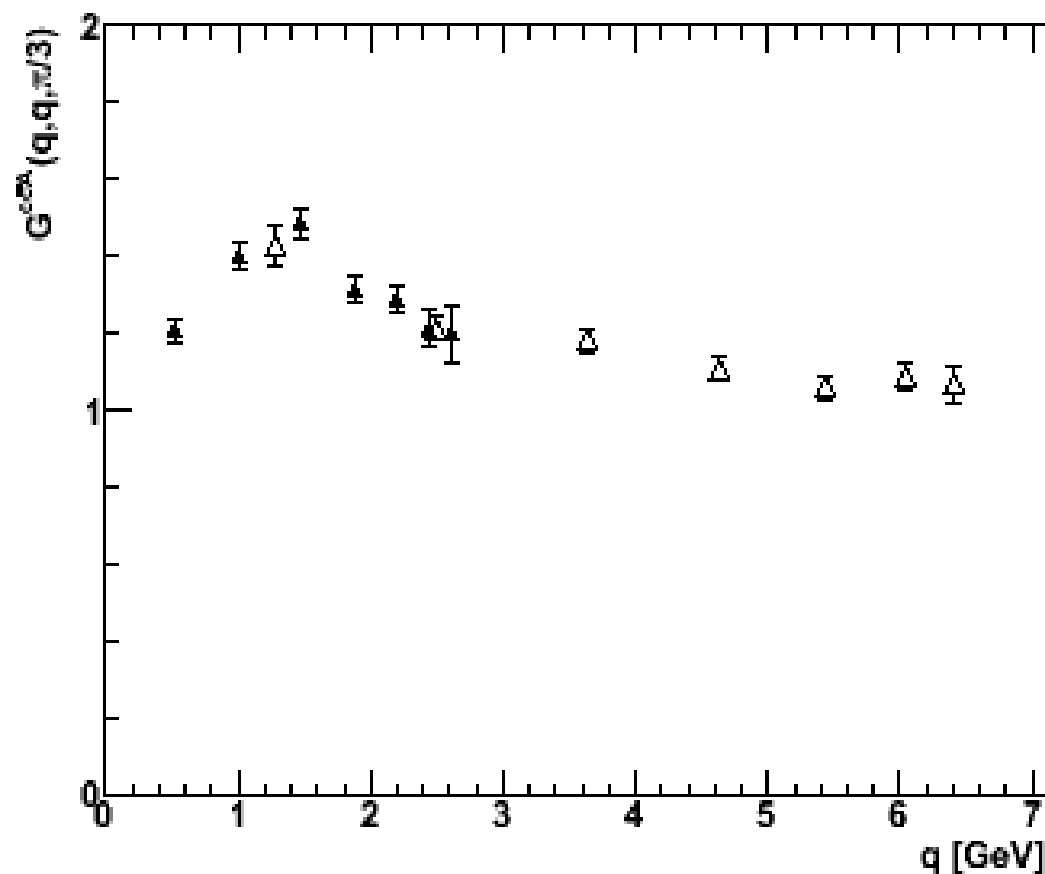
Ghost-gluon vertex in 4d

Ghost-gluon vertex, all momenta equal

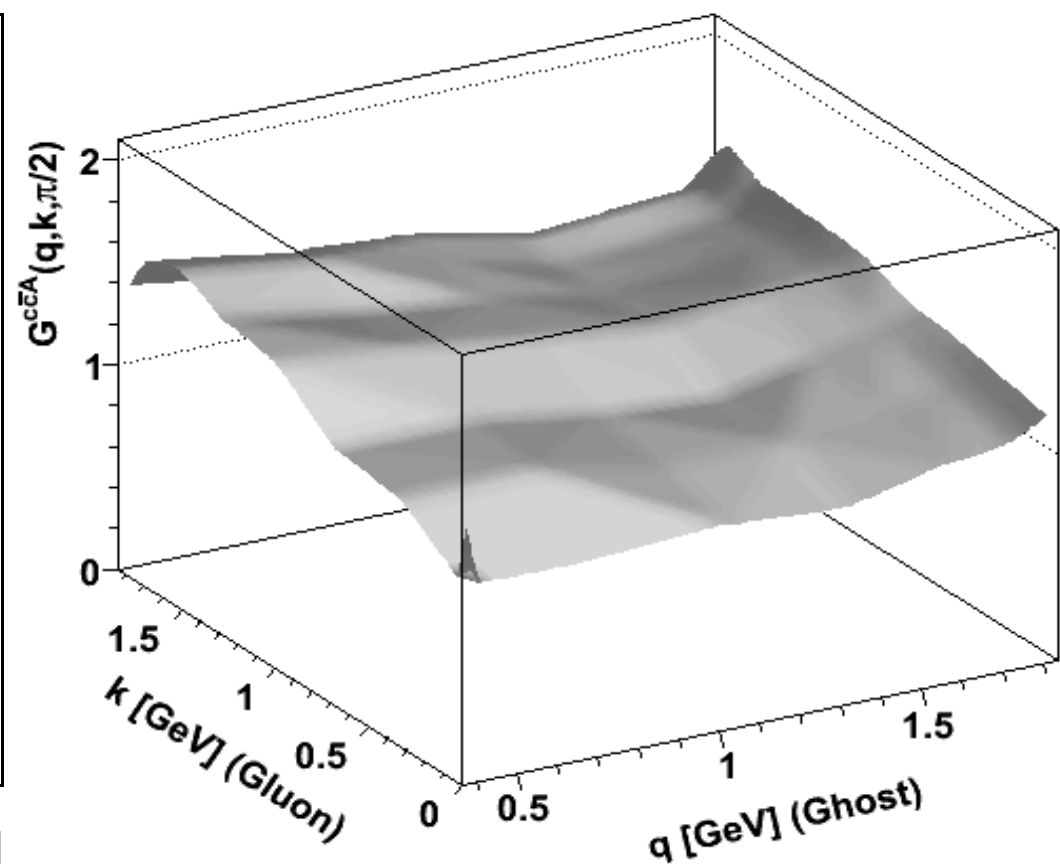


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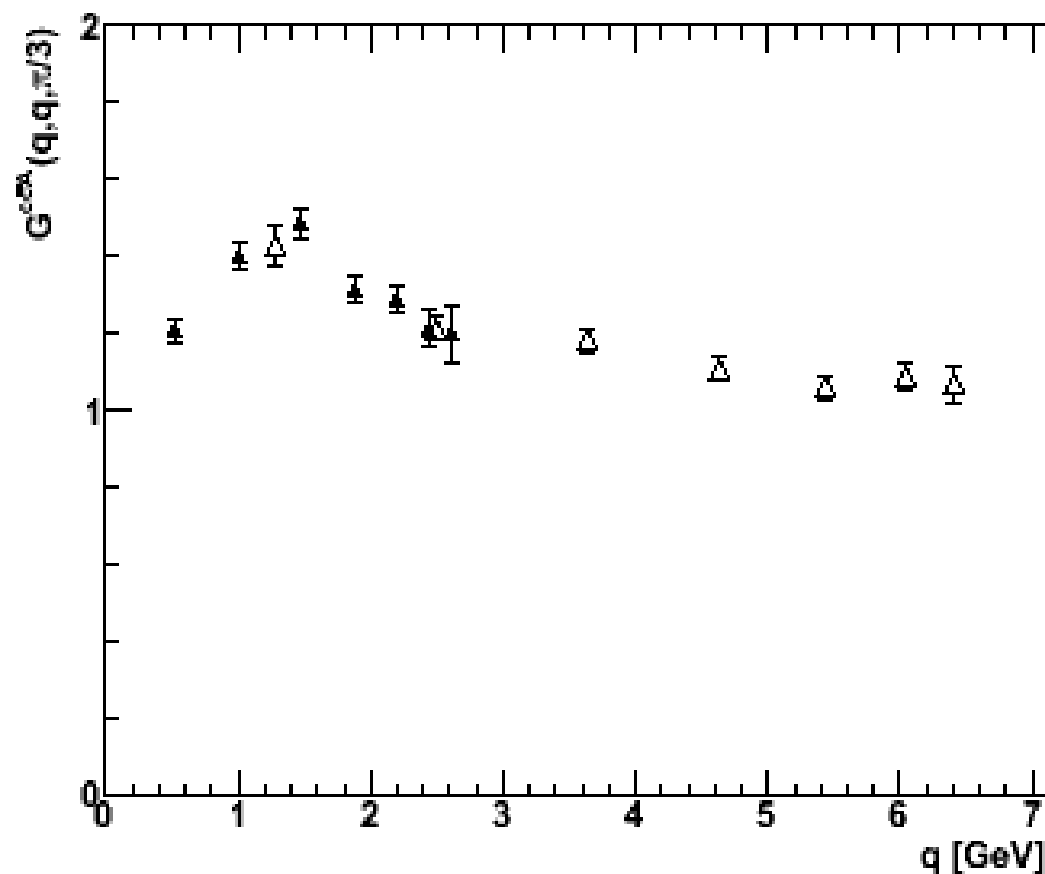


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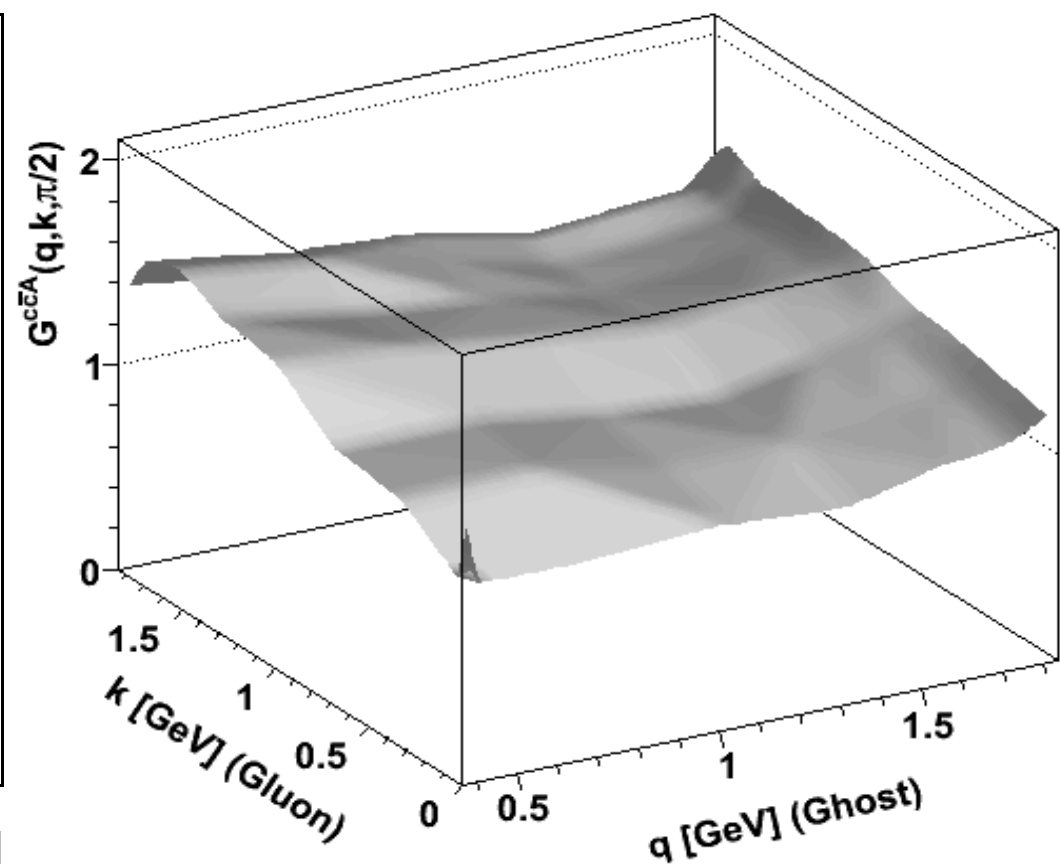


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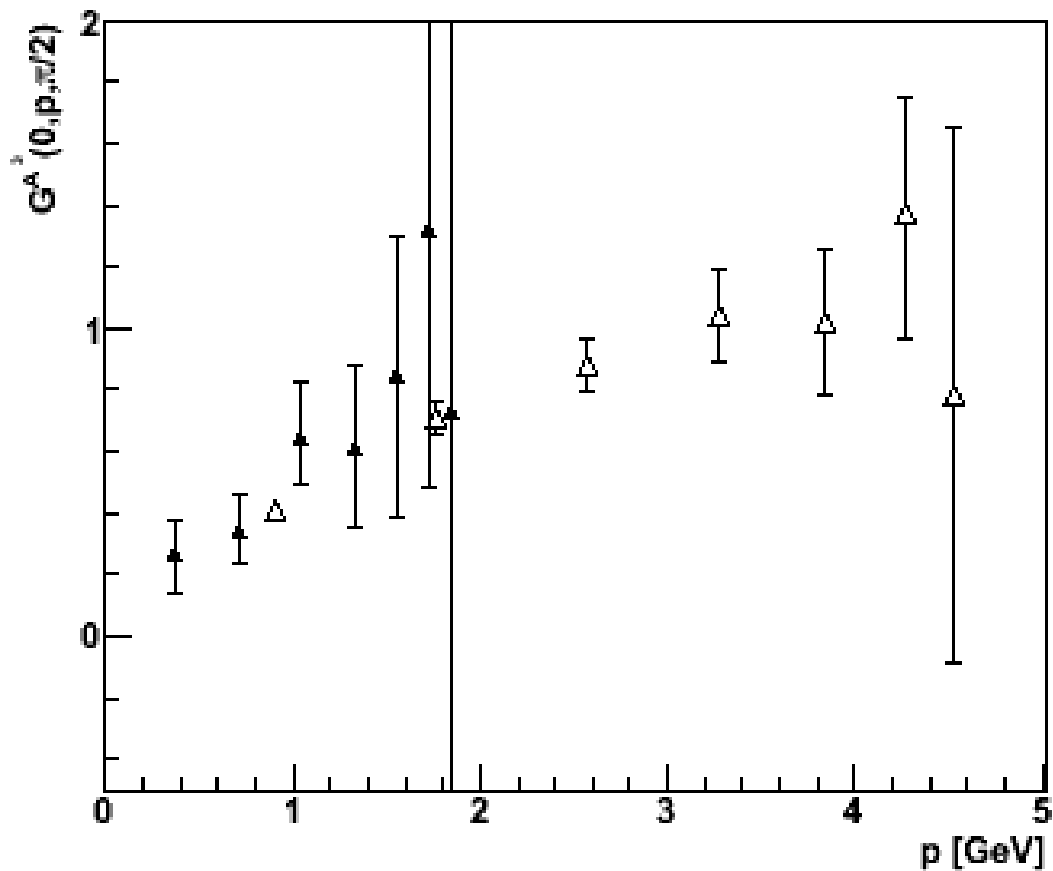
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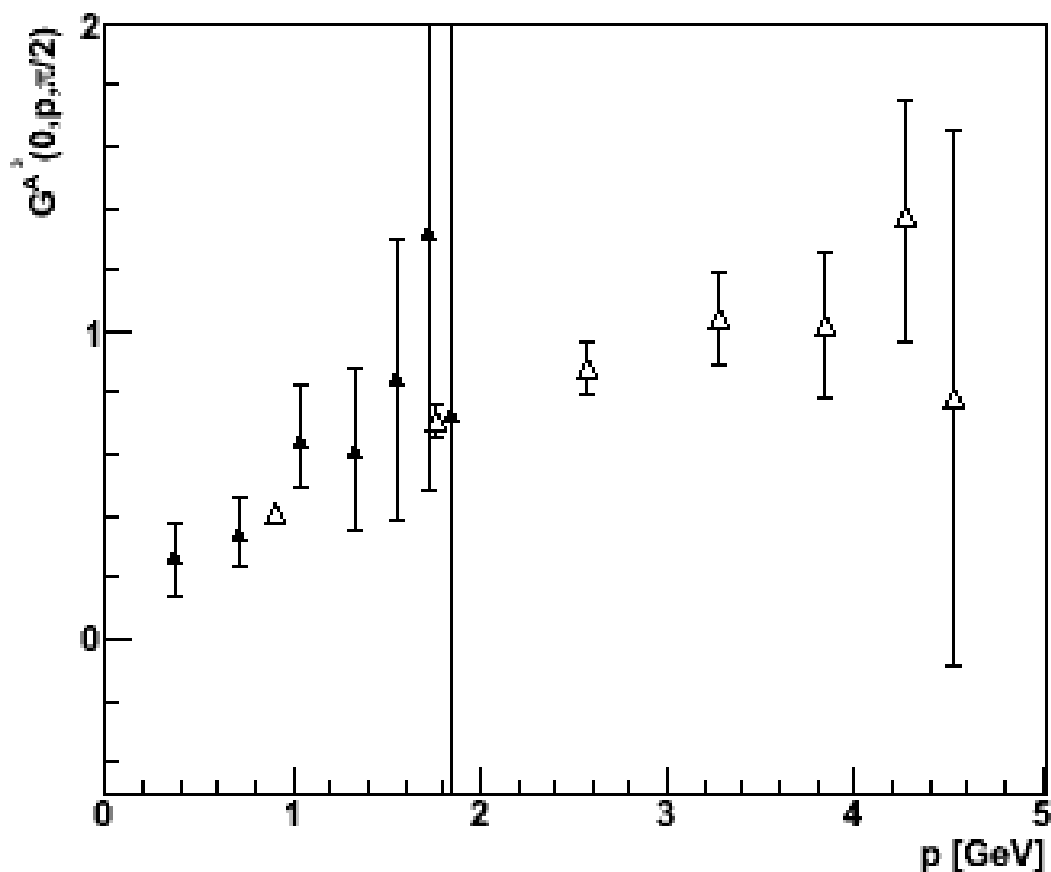
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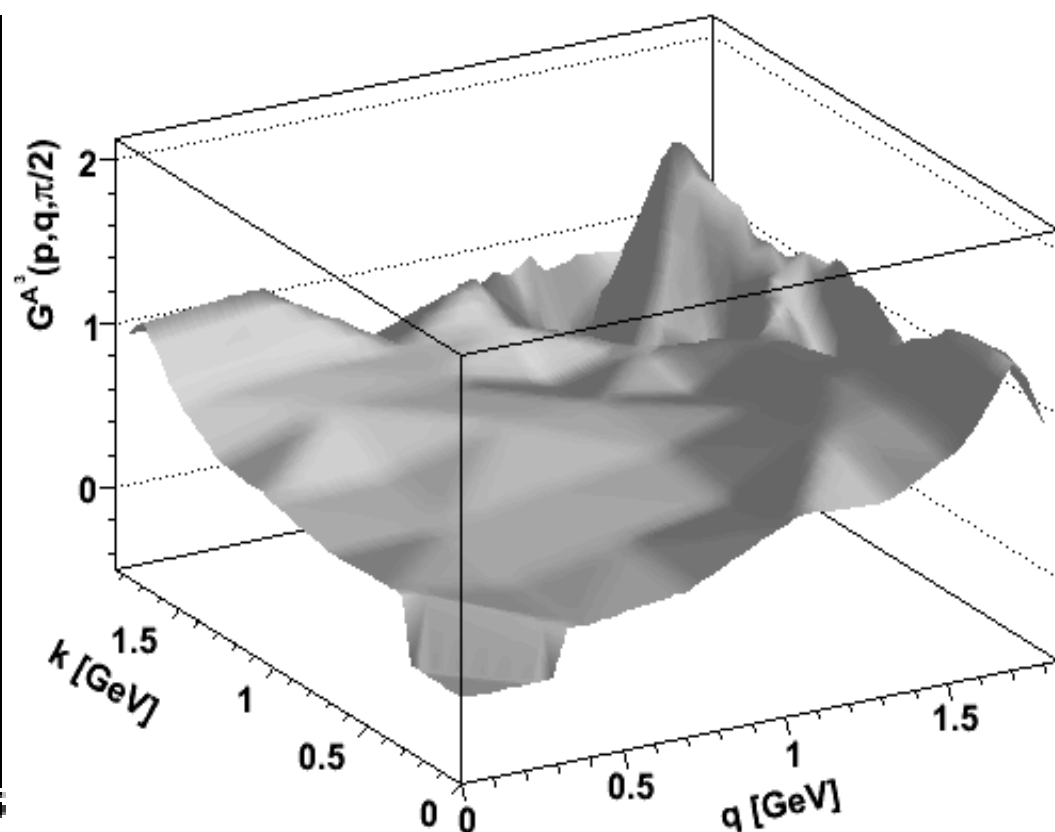


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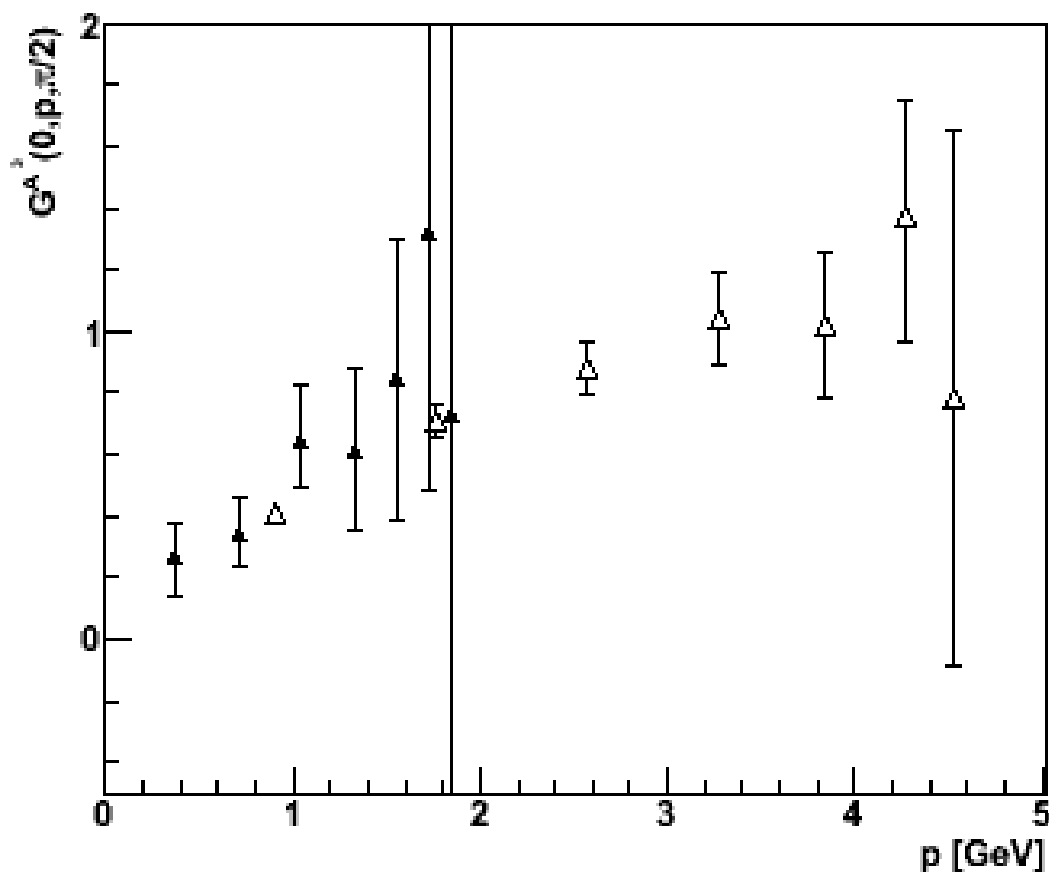


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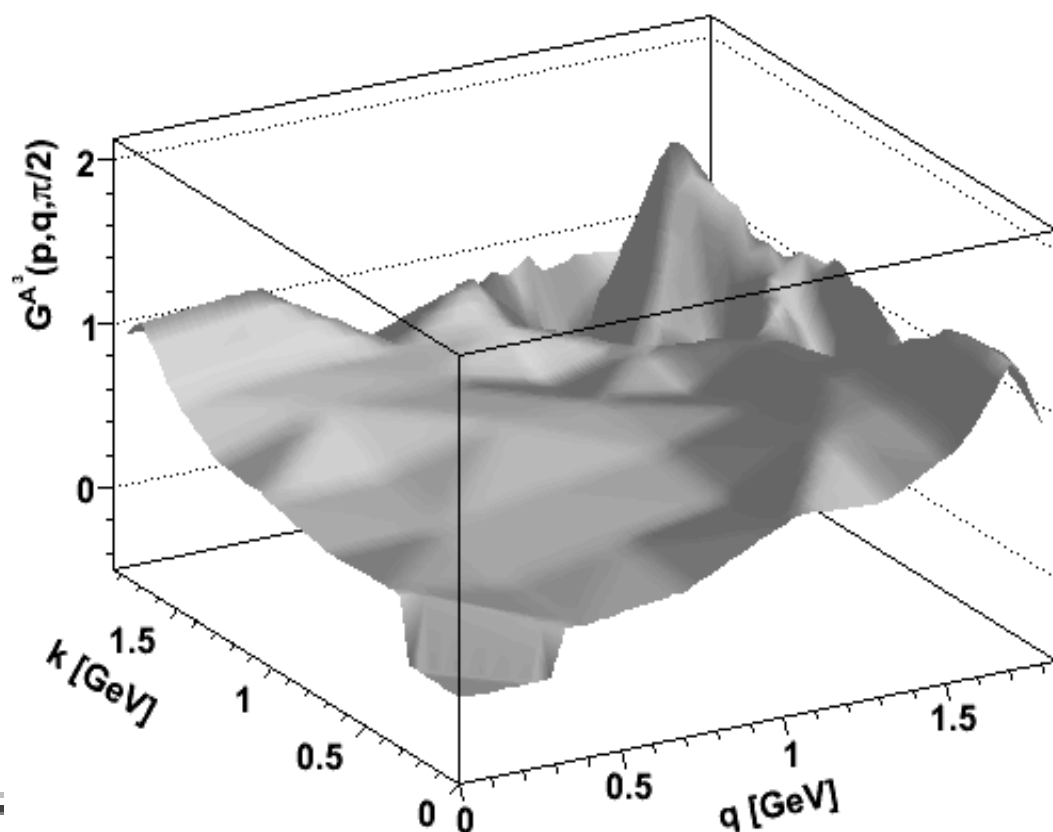


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- Strong statistical fluctuations
- Suppressed at low momenta - momenta too large for comparison to DSEs which predict enhancement?

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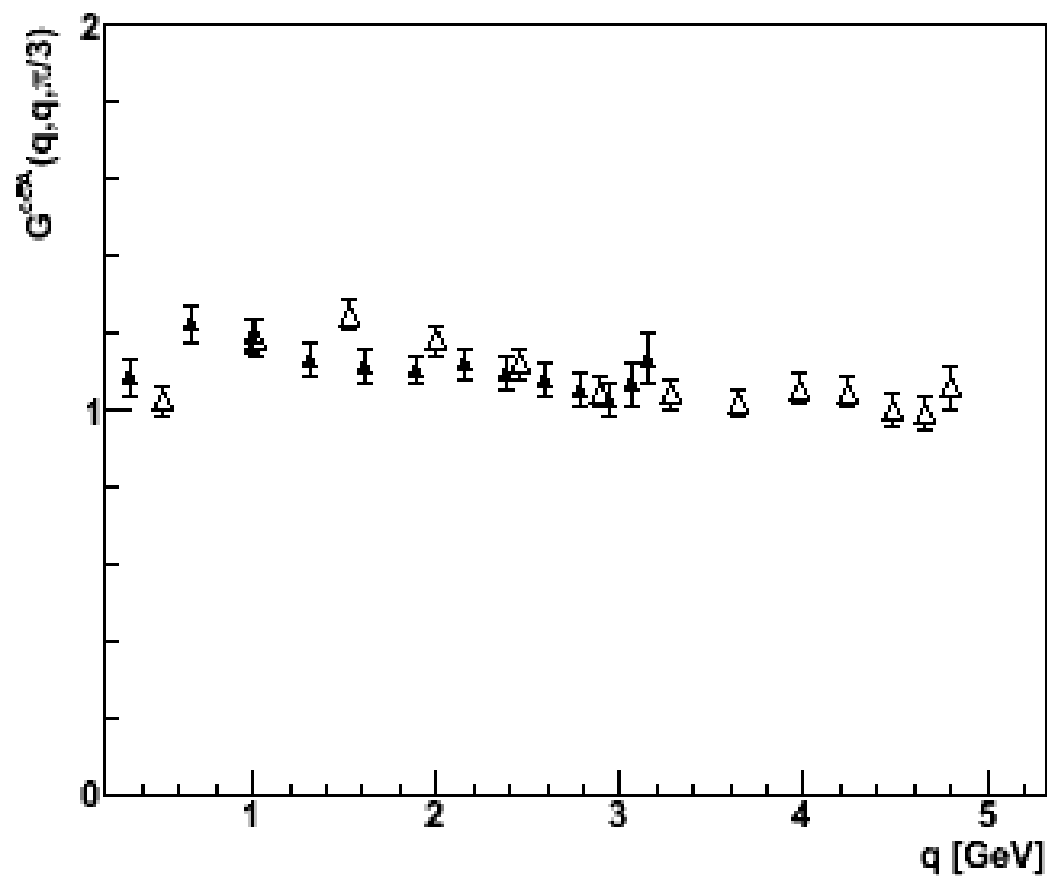
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- Interesting also because larger volumes can be accessed: Model lab
 - There is a connection to Coulomb gauge in 4d

[Zwanziger, PRD 2004]

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[30³@beta=4.2, 6.0, Cucchieri et al., PRD 2006]

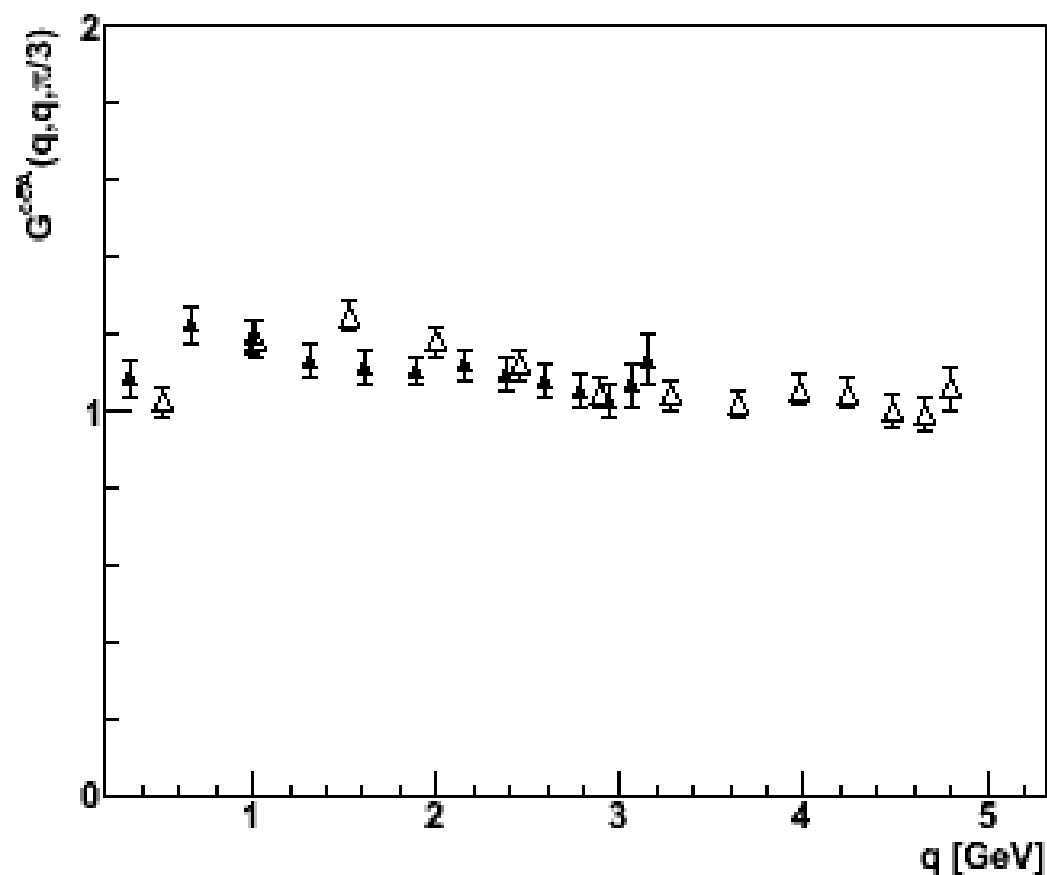
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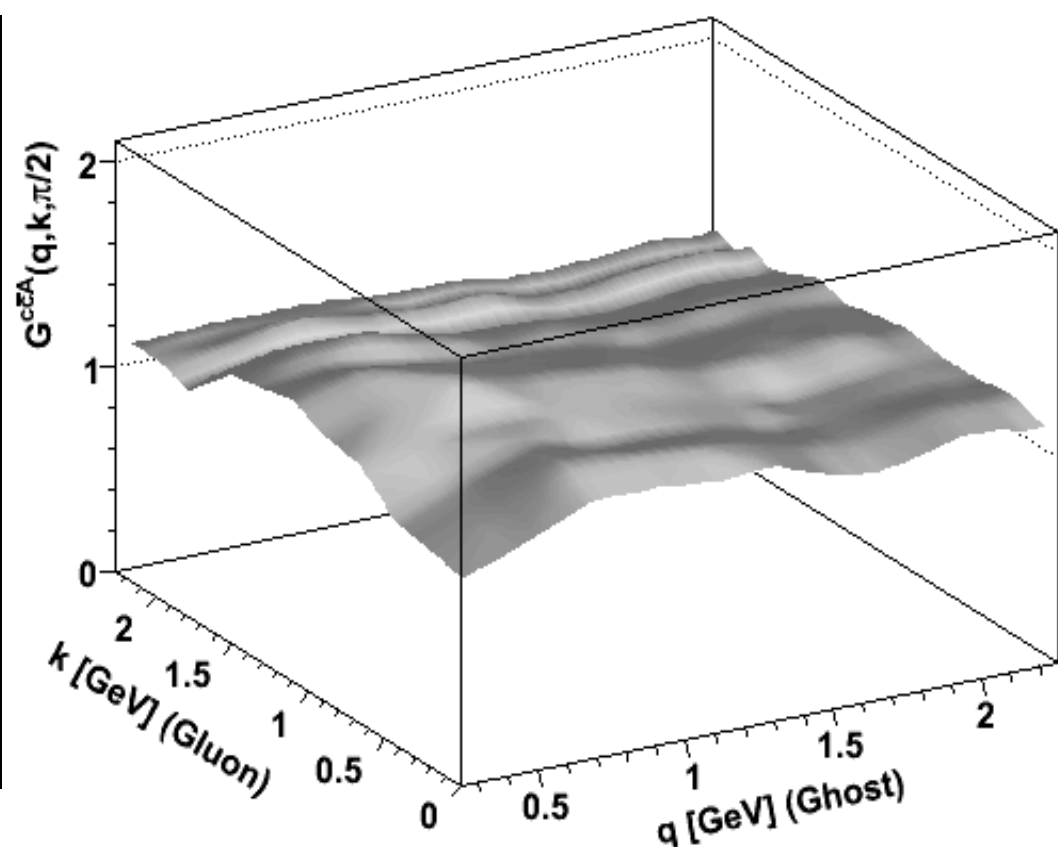
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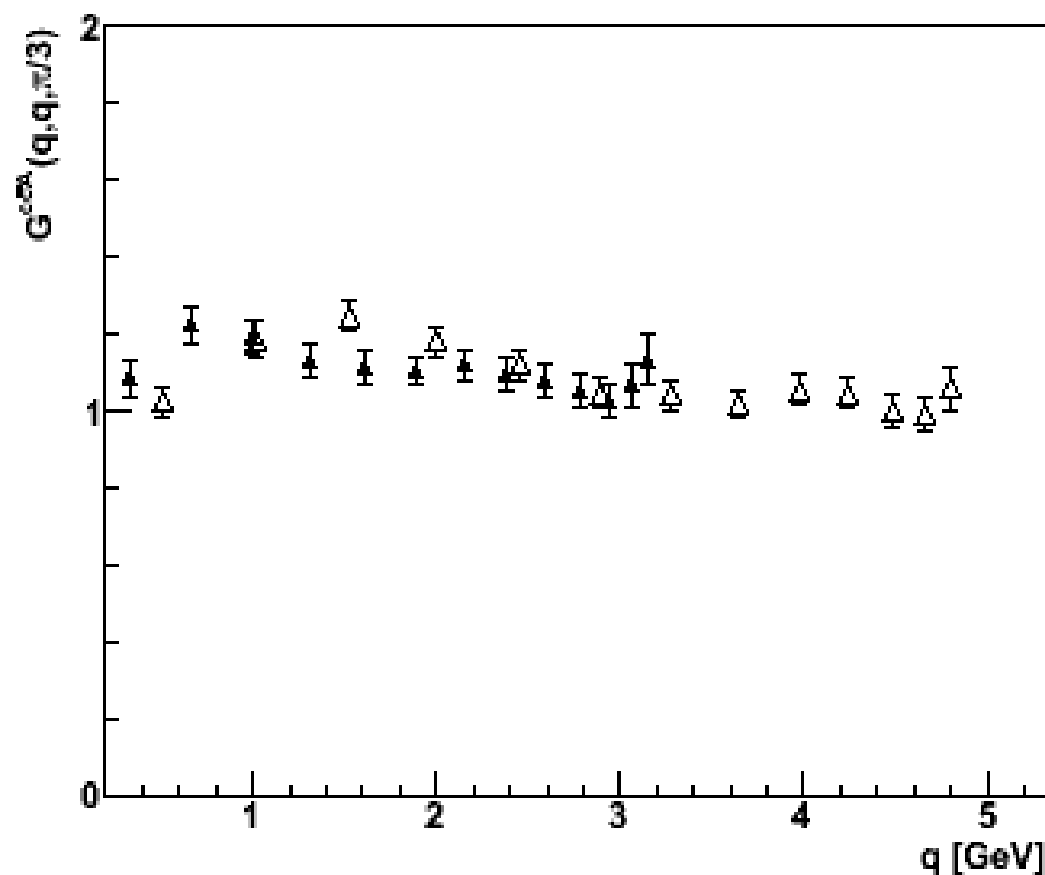
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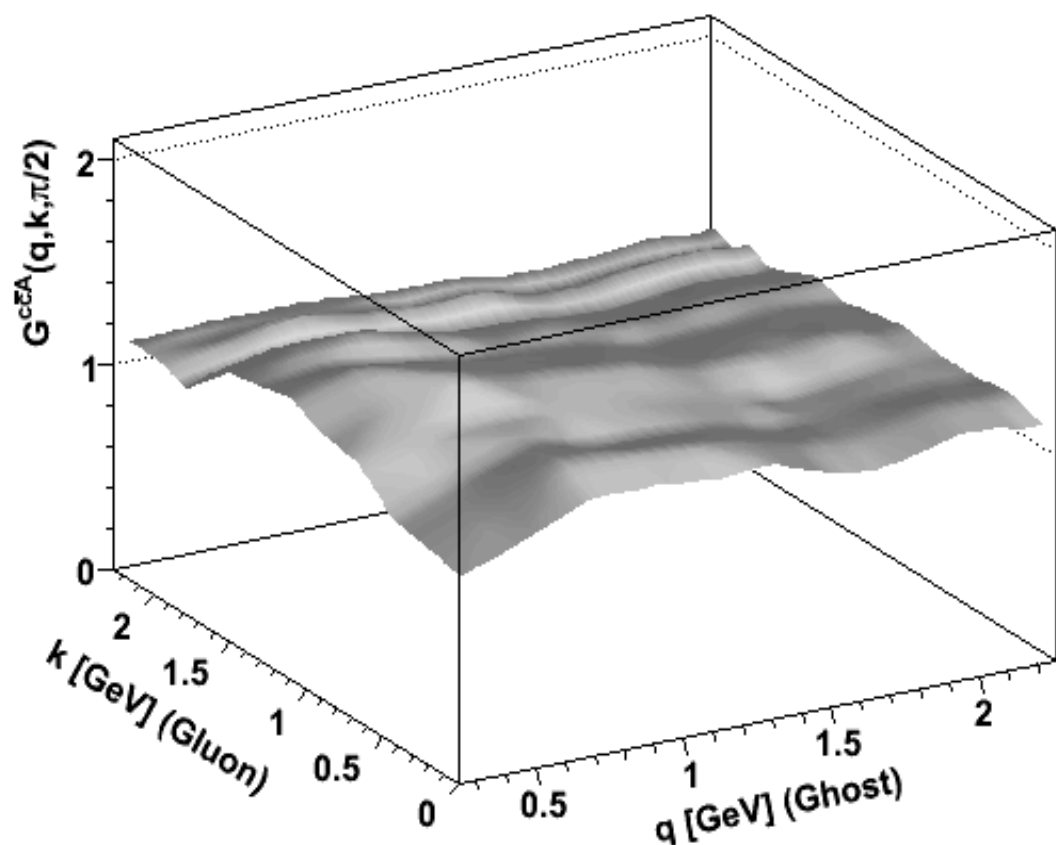
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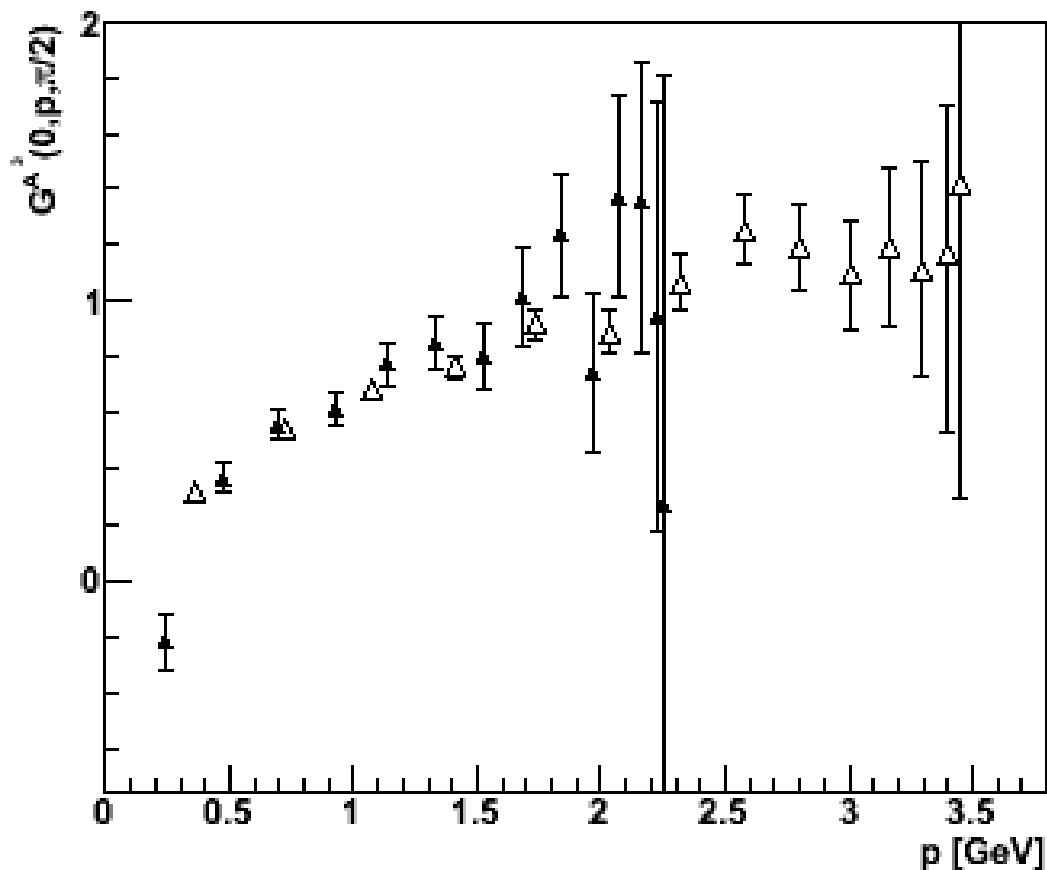


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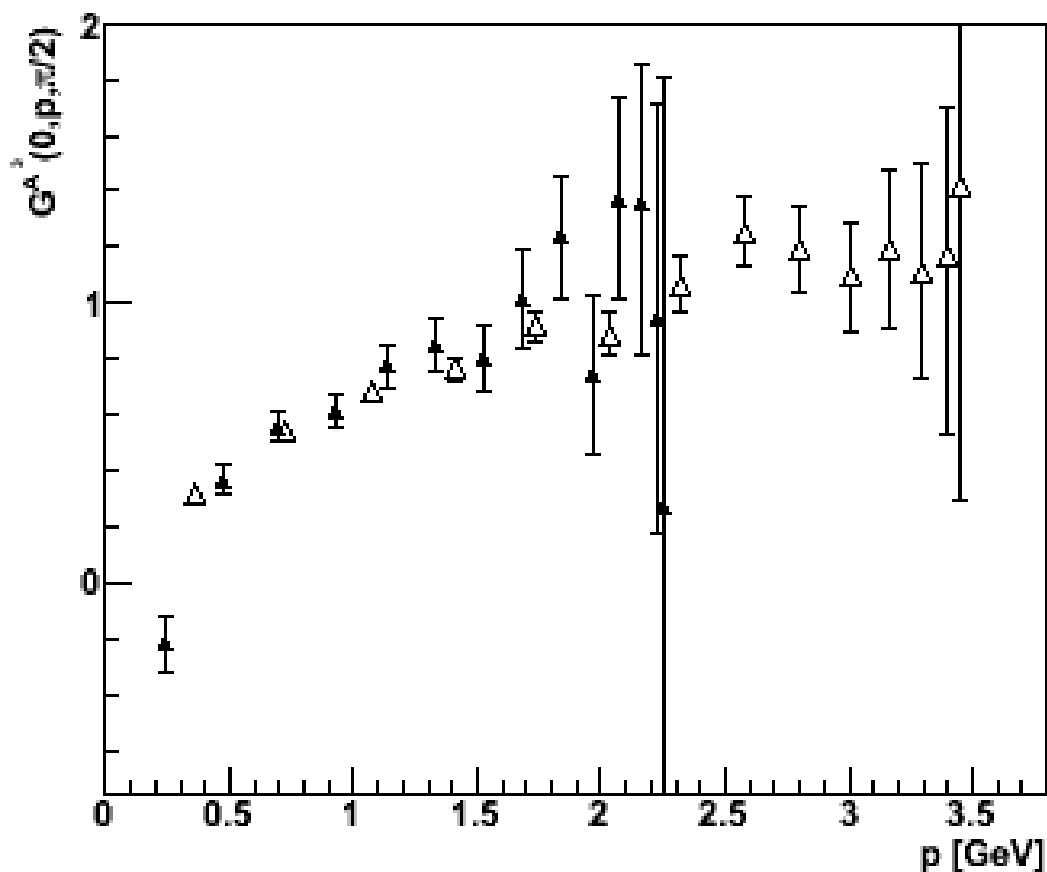
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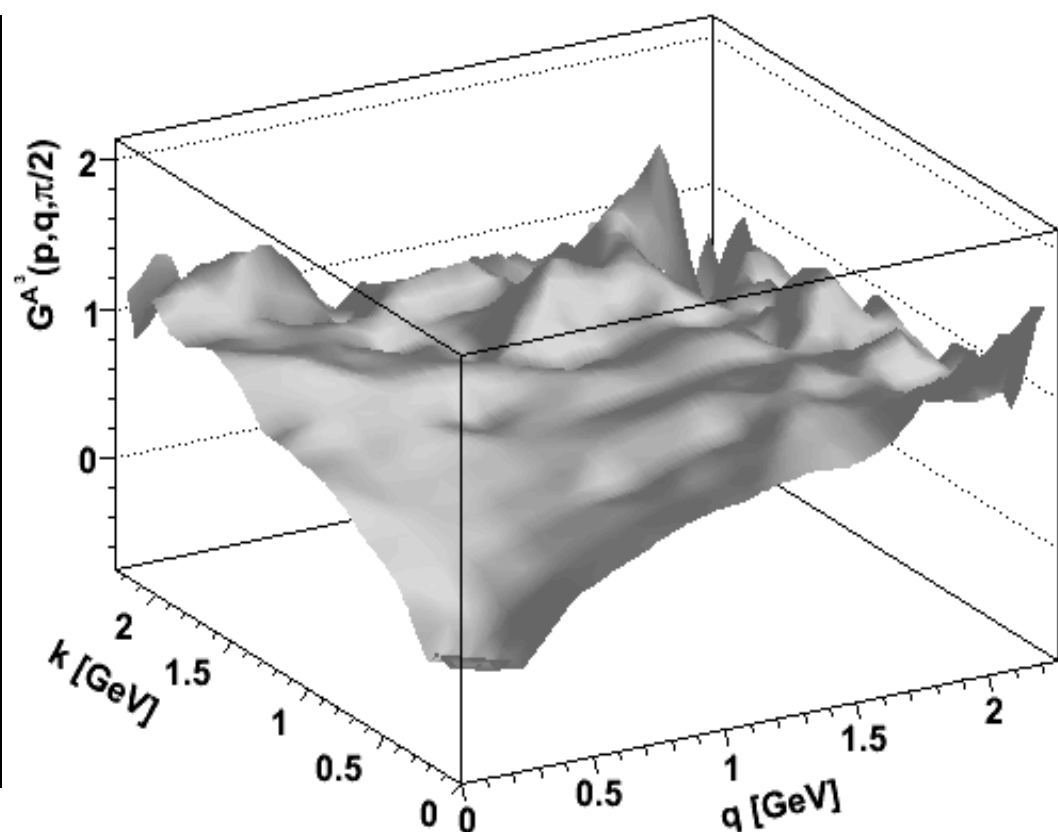
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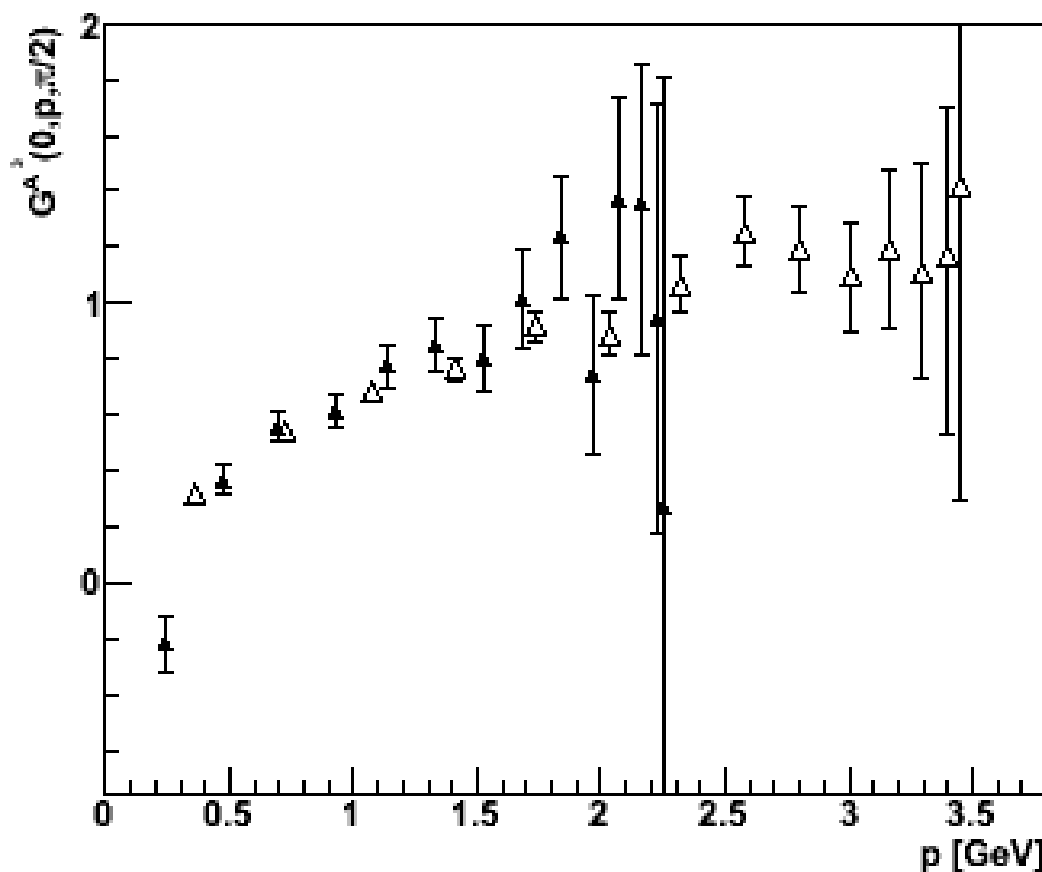
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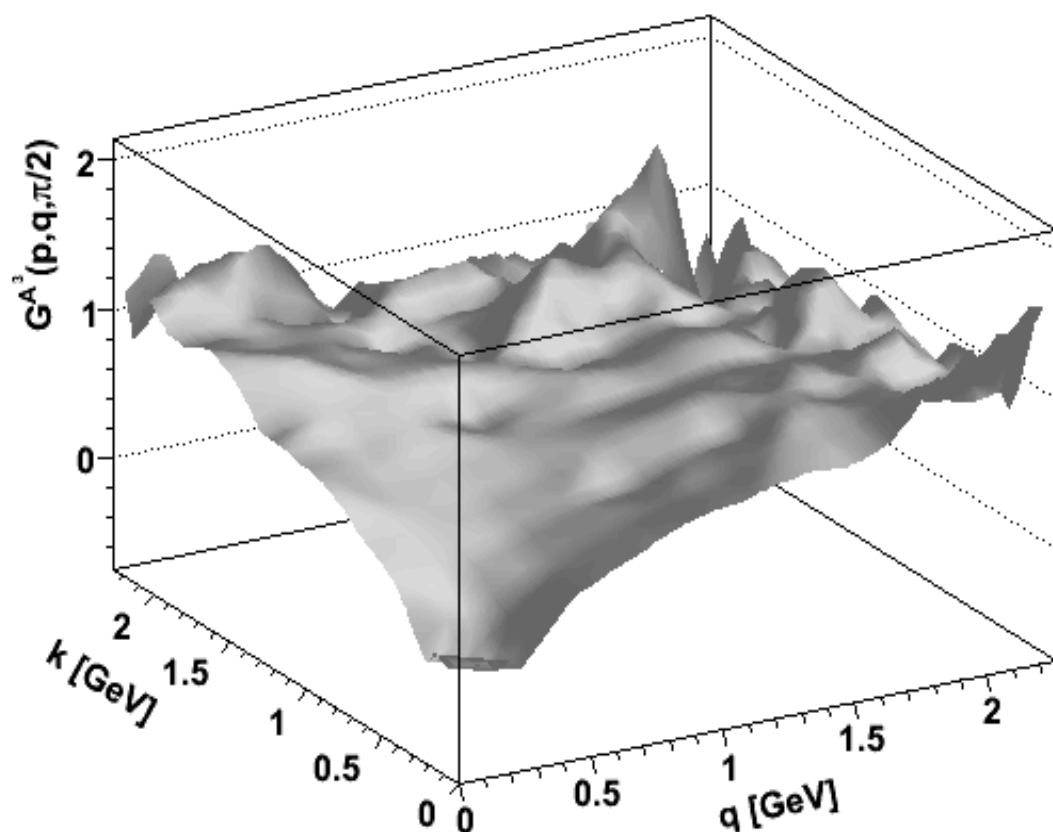
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- Has possibly a zero at momenta of order of the gluon propagator maximum - requires larger volumes
- Comparison to DSEs: Divergence? Sign of pre-factor?

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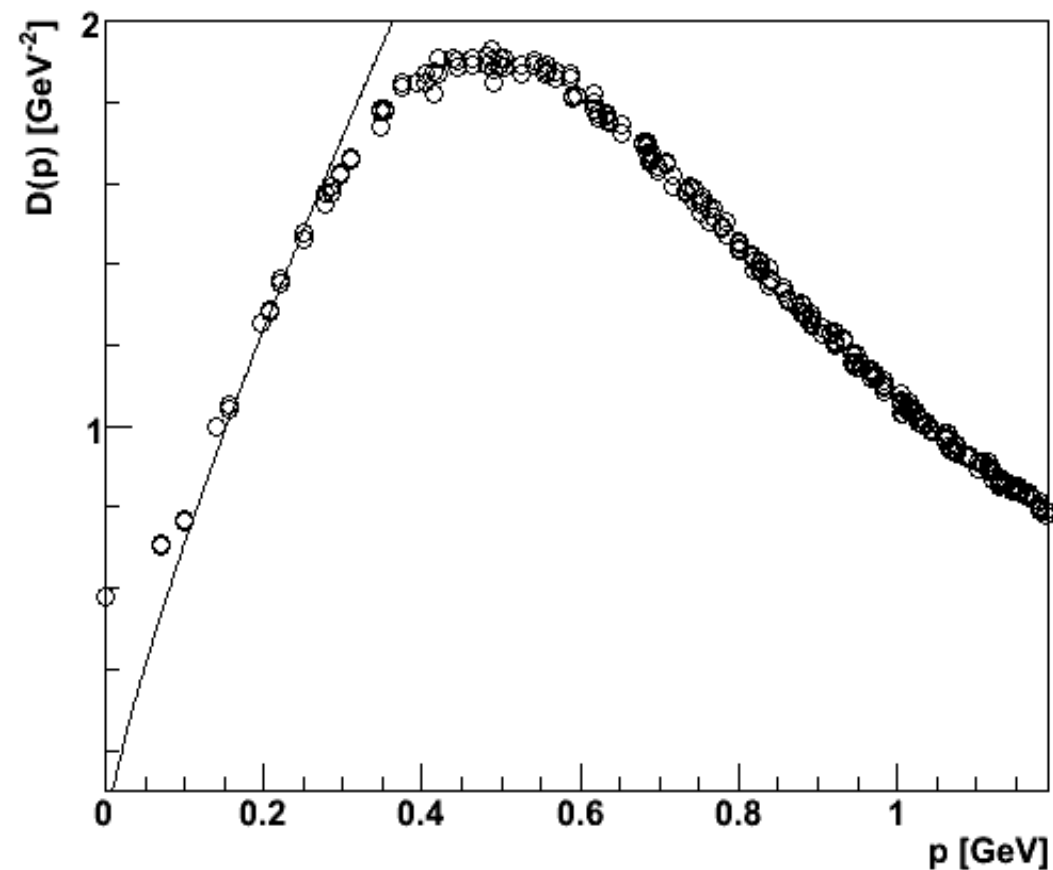
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- **Very large volumes** ($\sim 40 \text{ fm}^2$) possible

Similarities: Propagators

[Gluon 100²@beta=10,
Maas, unpublished]

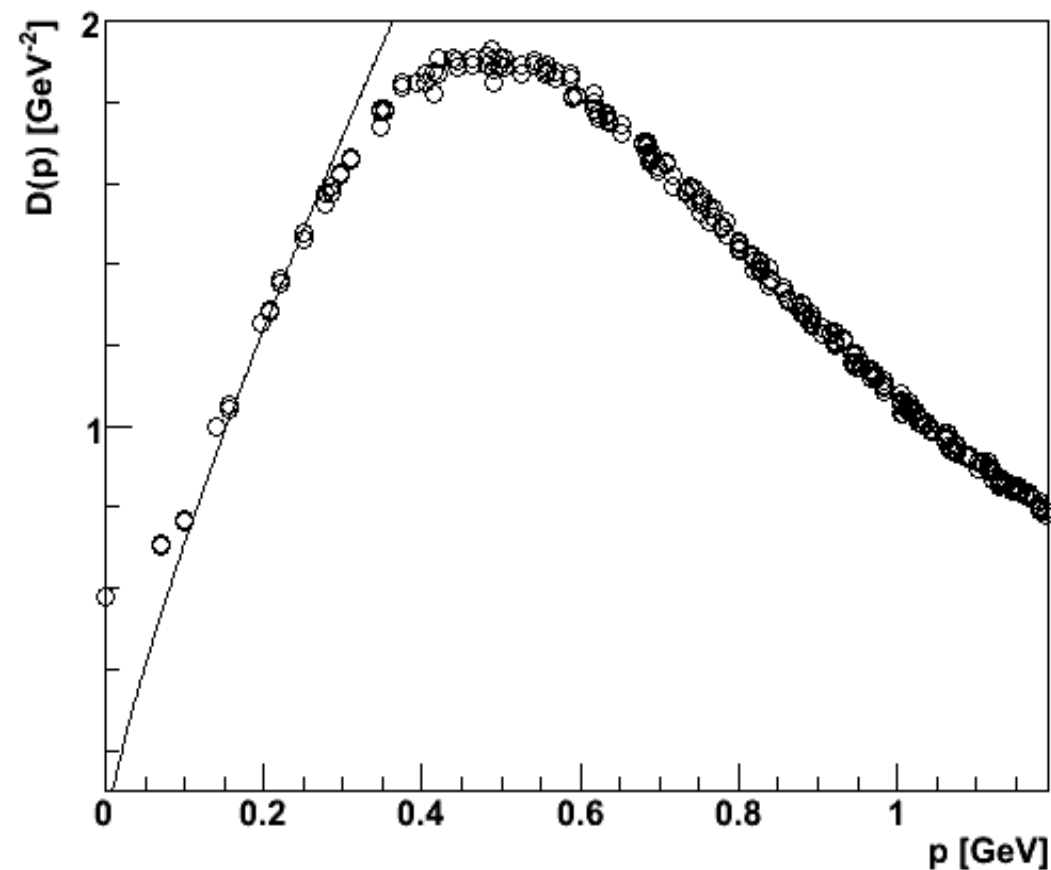
Gluon propagator



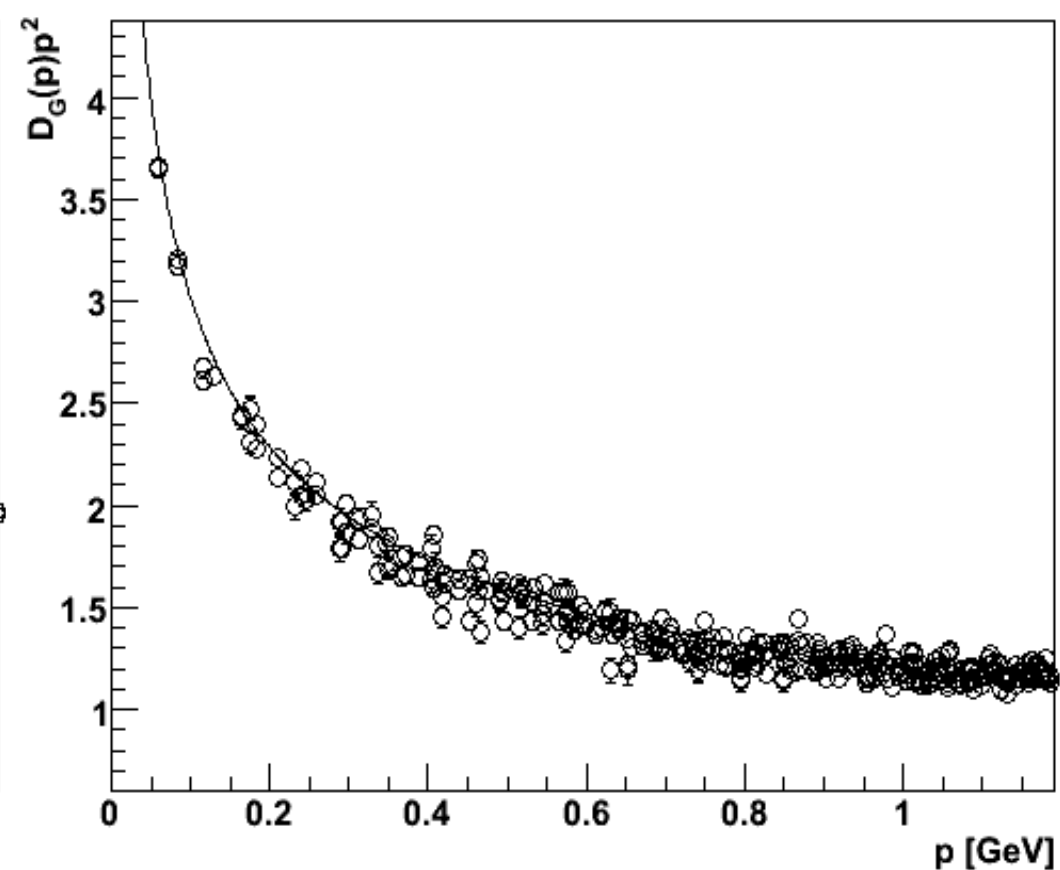
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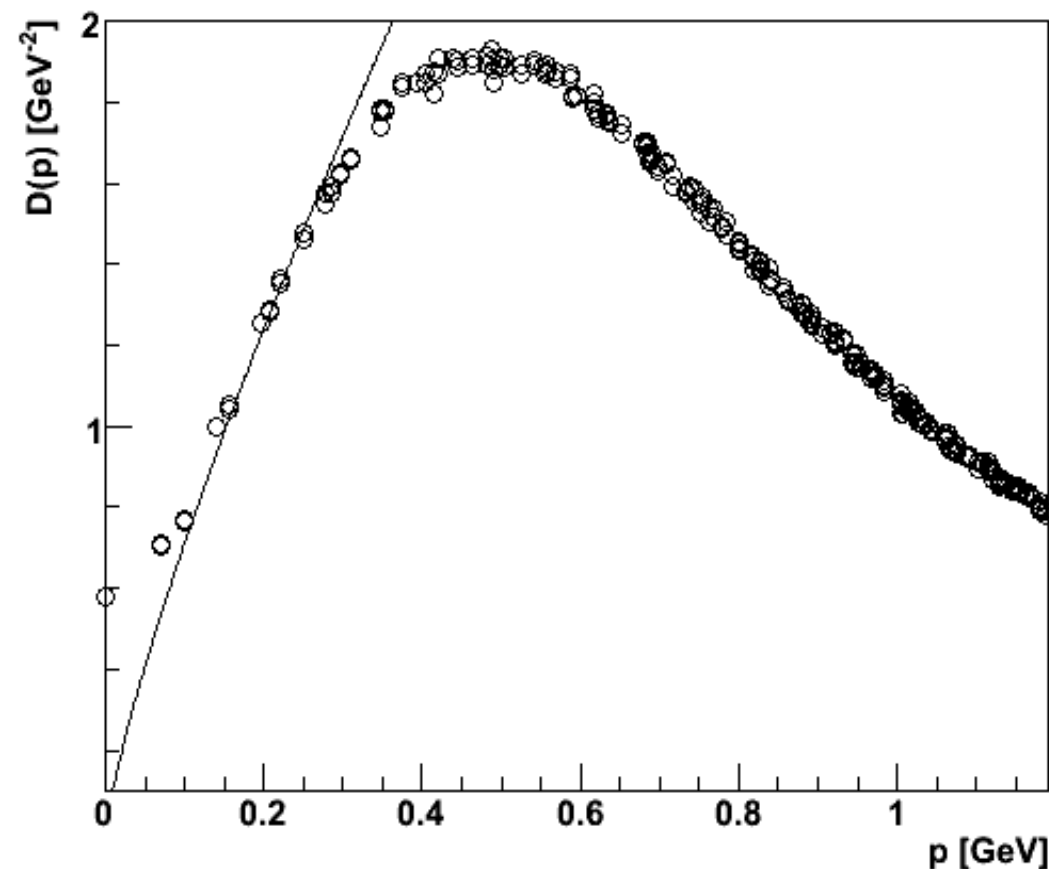
Ghost dressing function



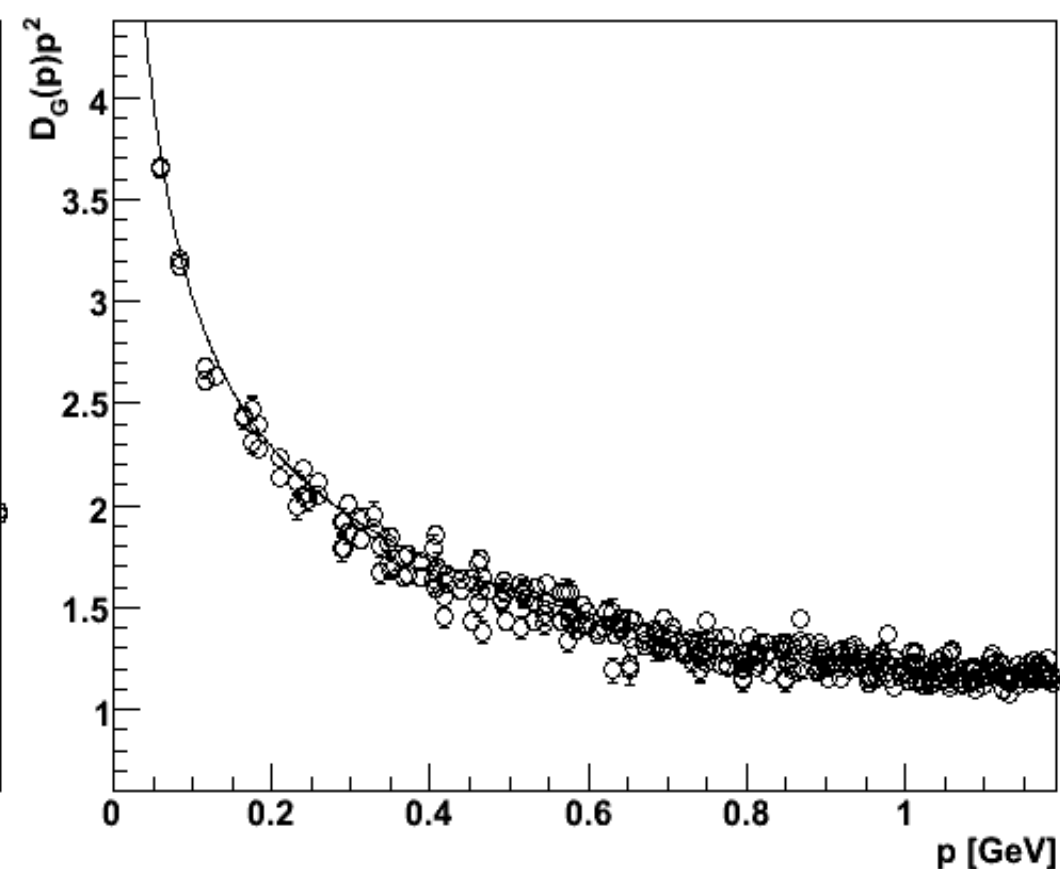
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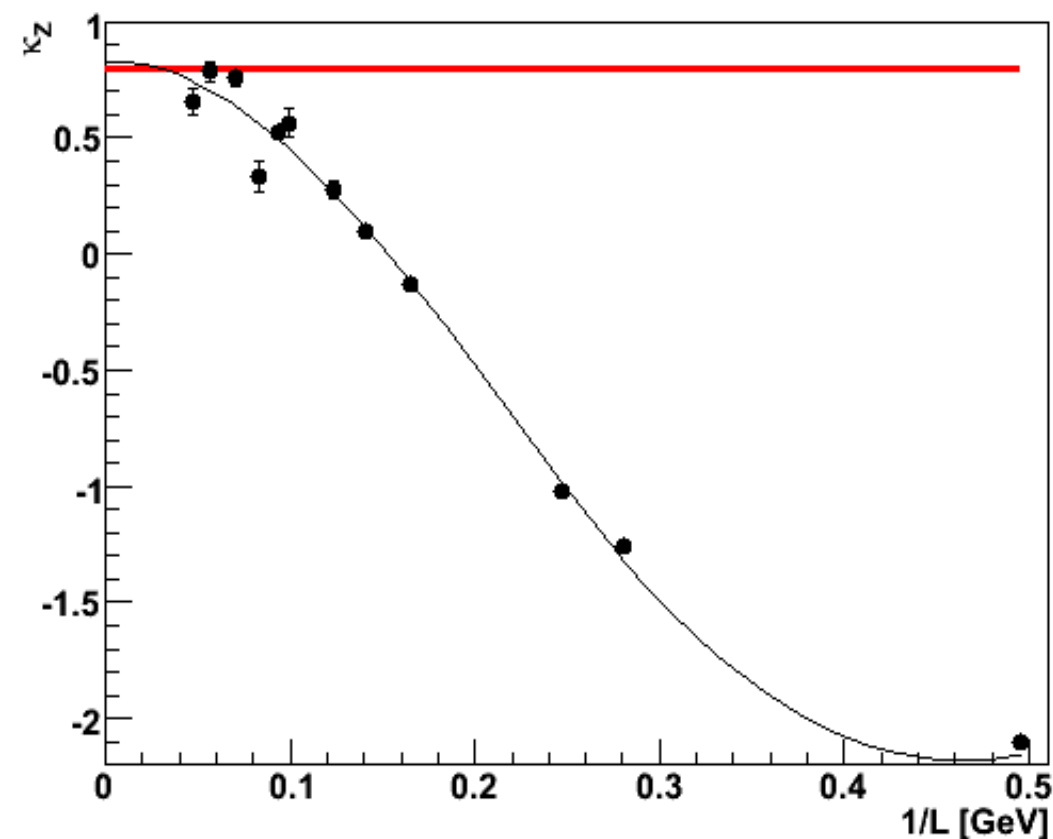


- **Qualitatively no change** compared to 3d, and to 4d (up to renormalization)
- Up to finite volume effects in agreement with predictions

Possibility to extract exponents quantitatively

Gluon infrared exponent

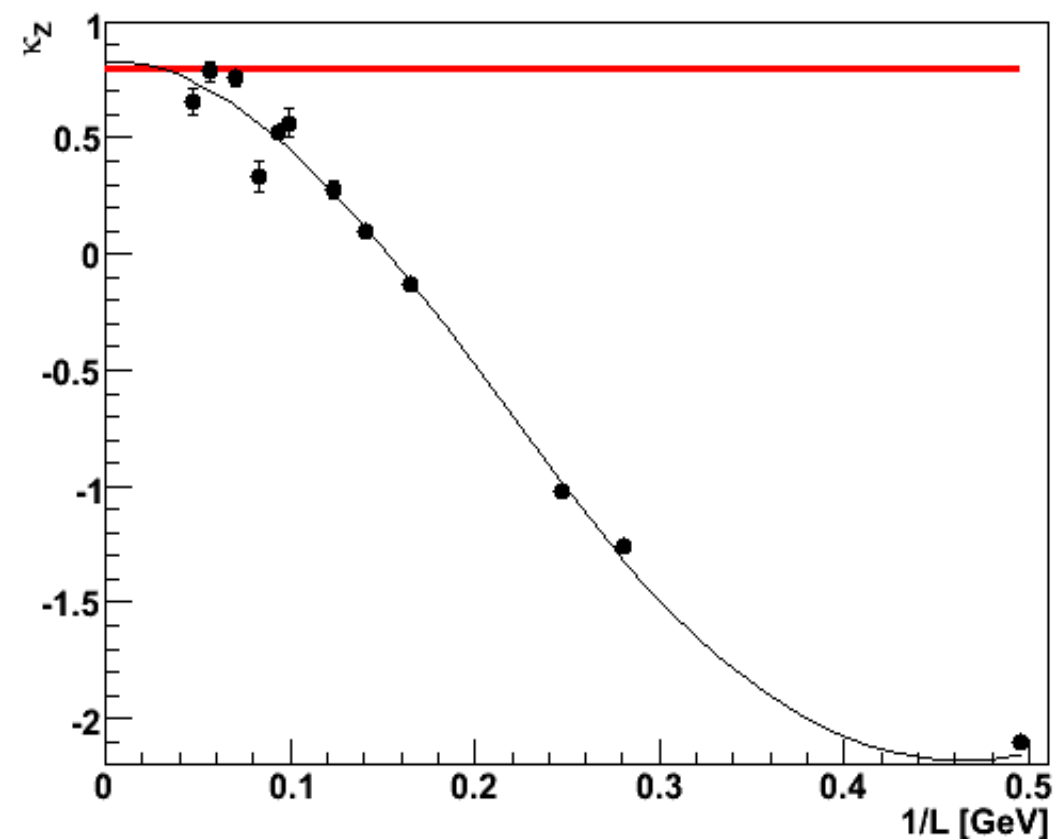
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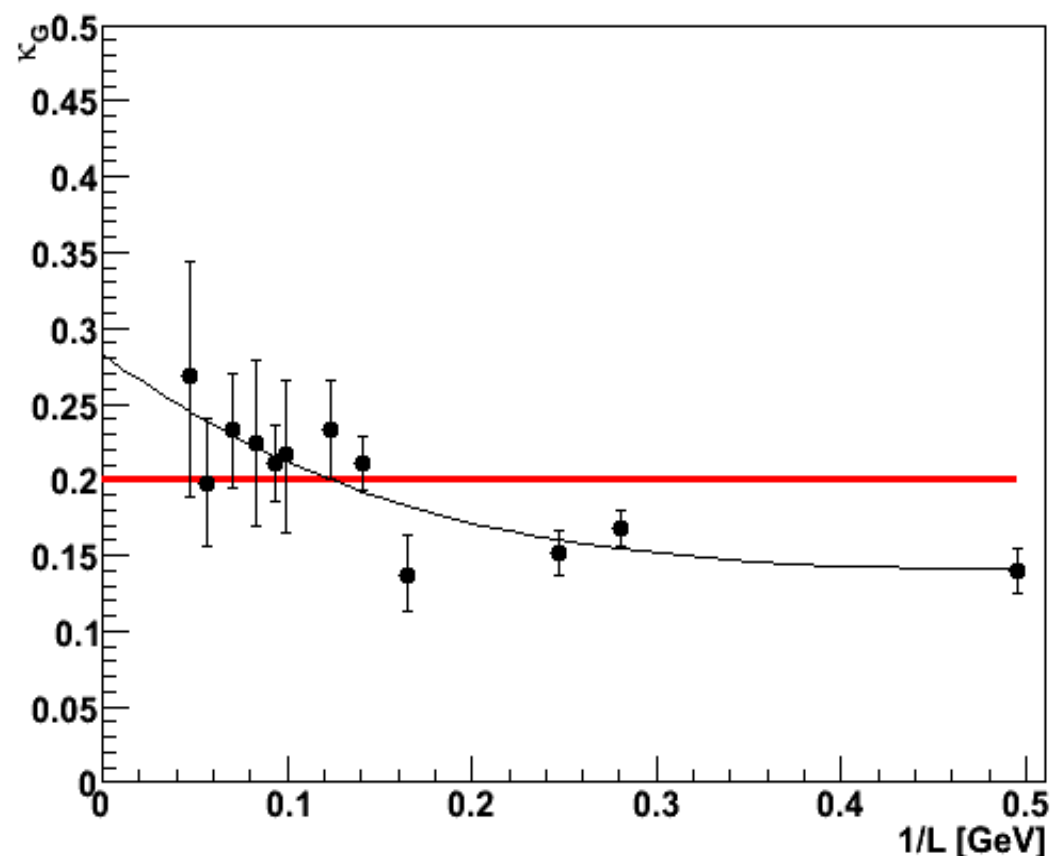
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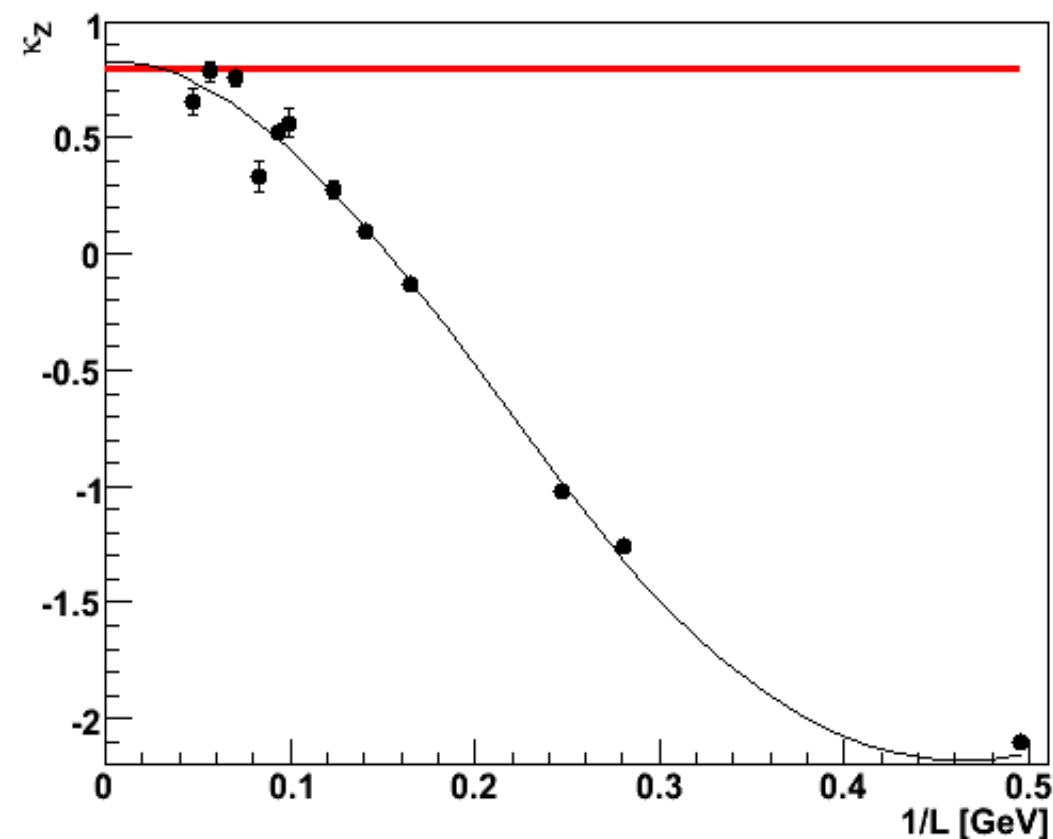
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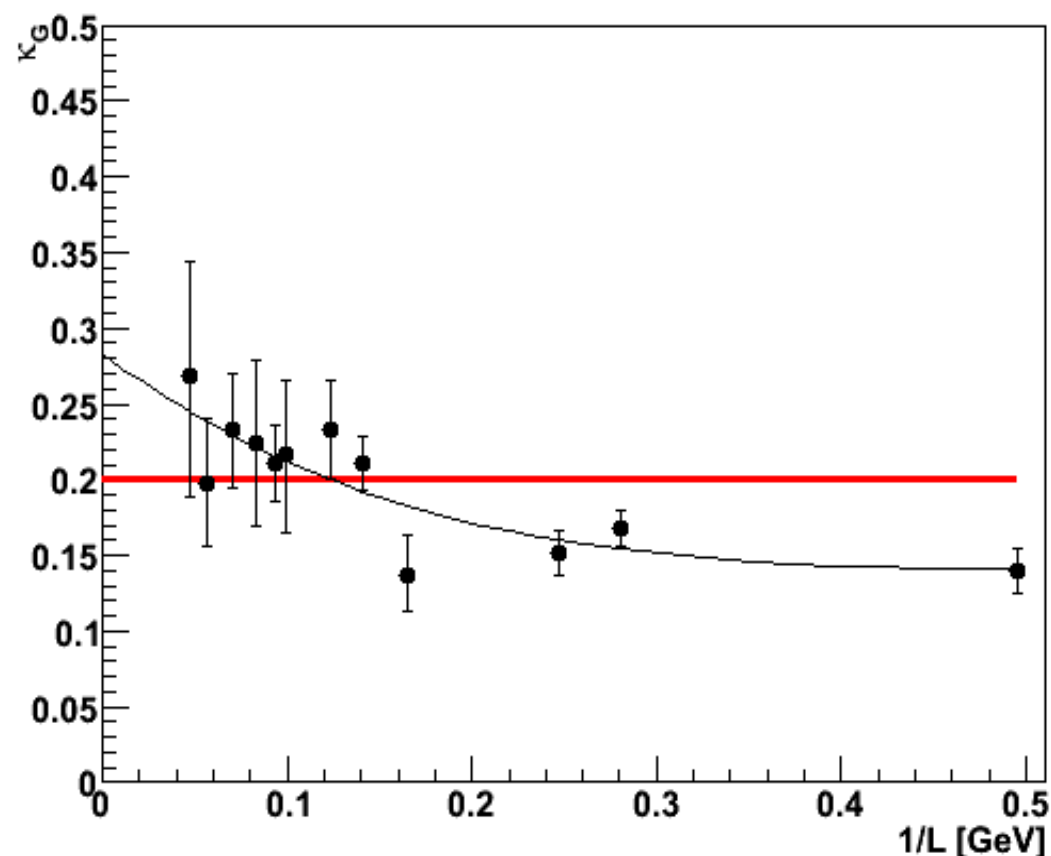
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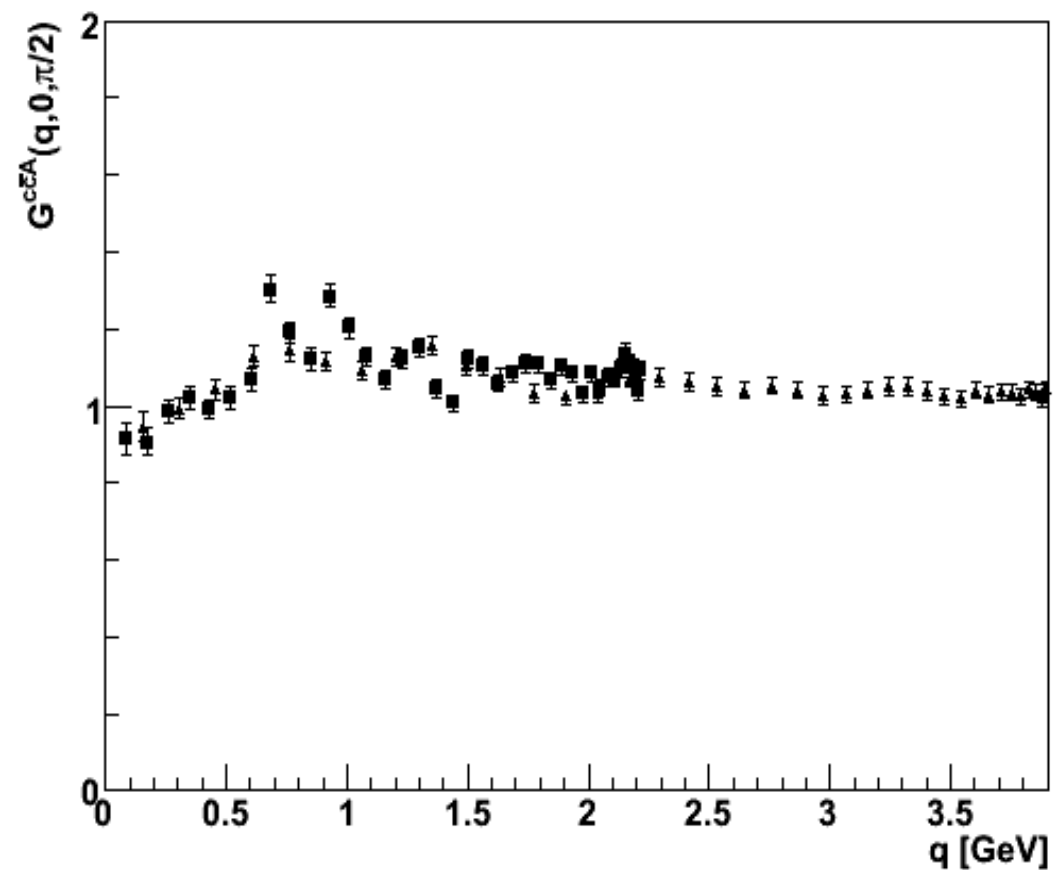


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- Requires better statistics and larger volumes

Ghost-gluon vertex in 2d

[80²@beta=10, 30, Maas, unpublished]

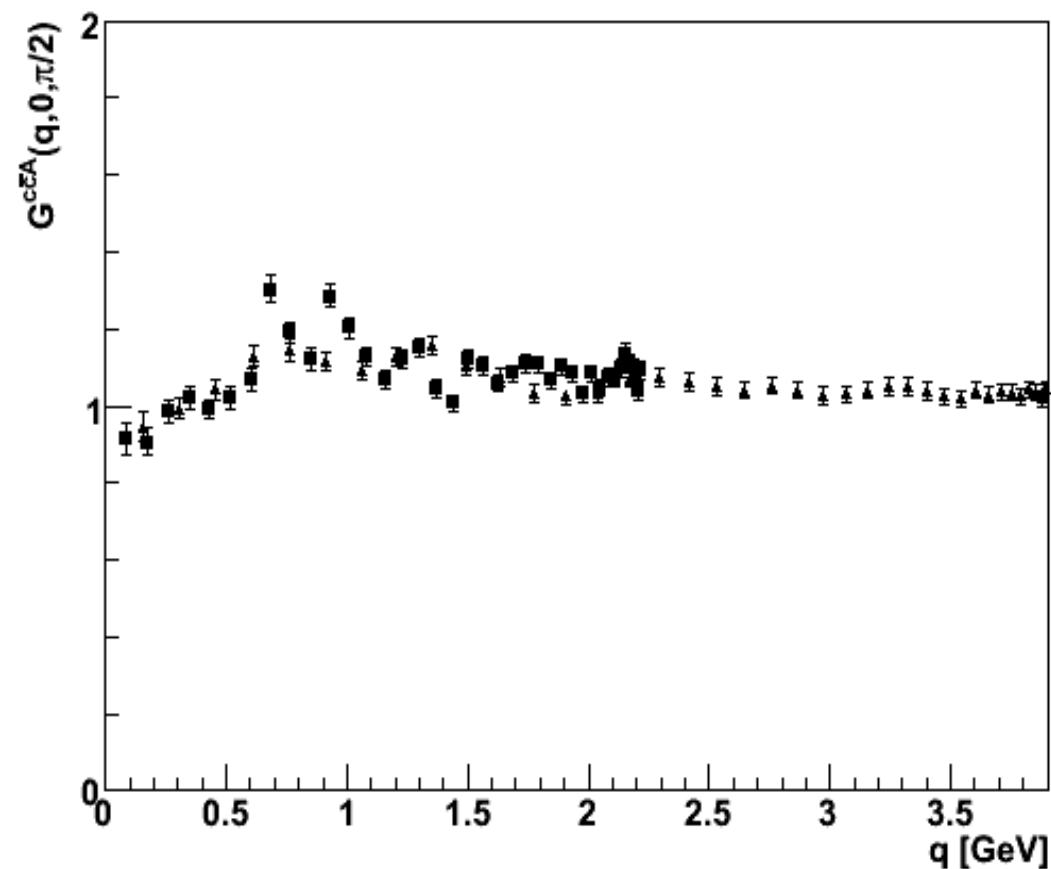
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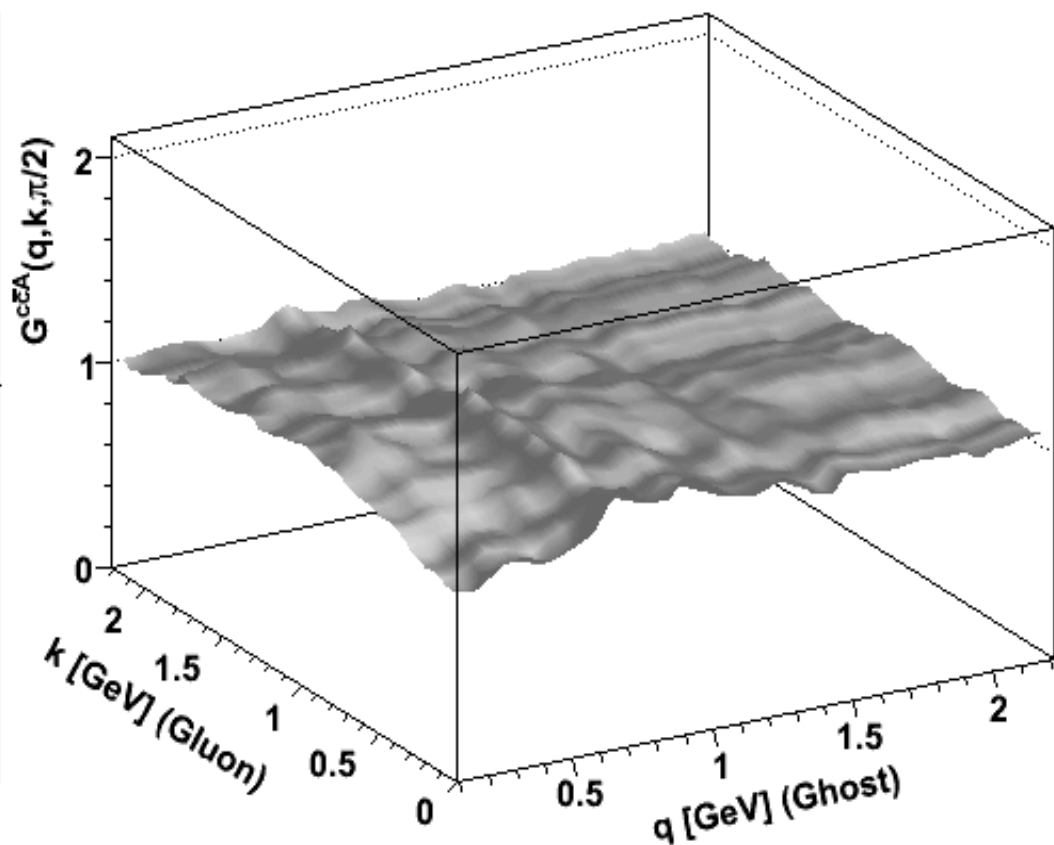
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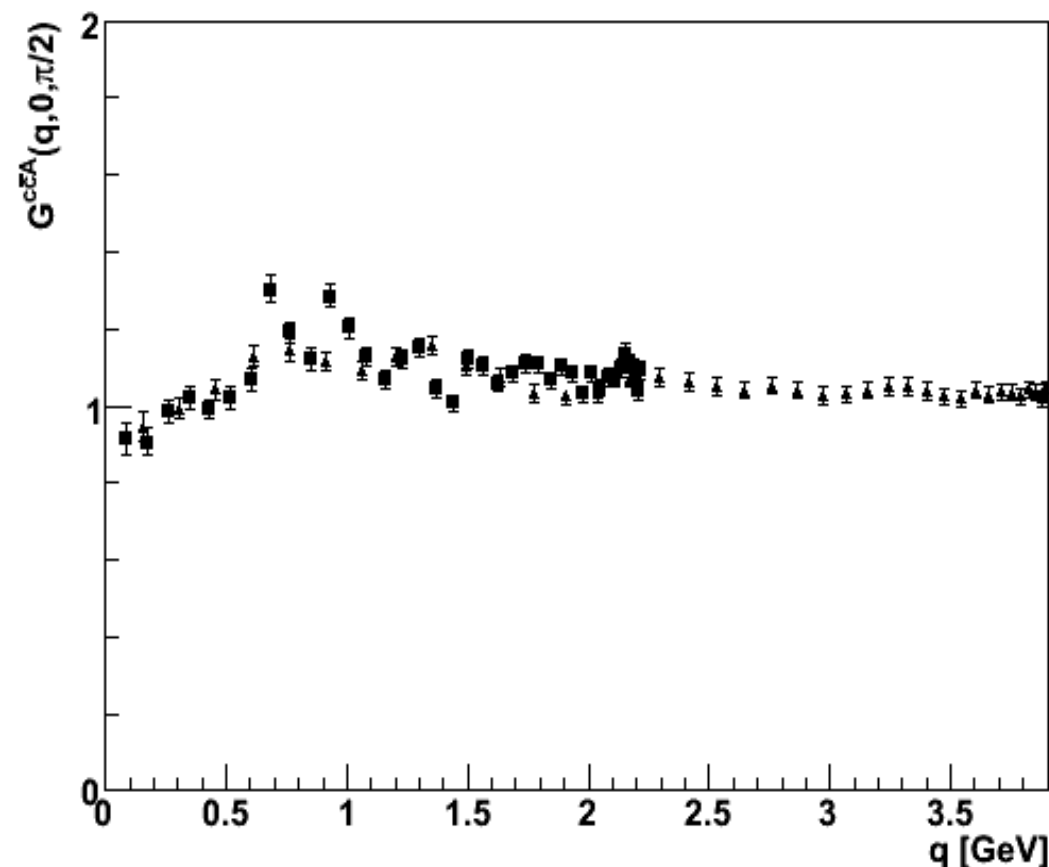
Ghost-gluon vertex, orthogonal momenta



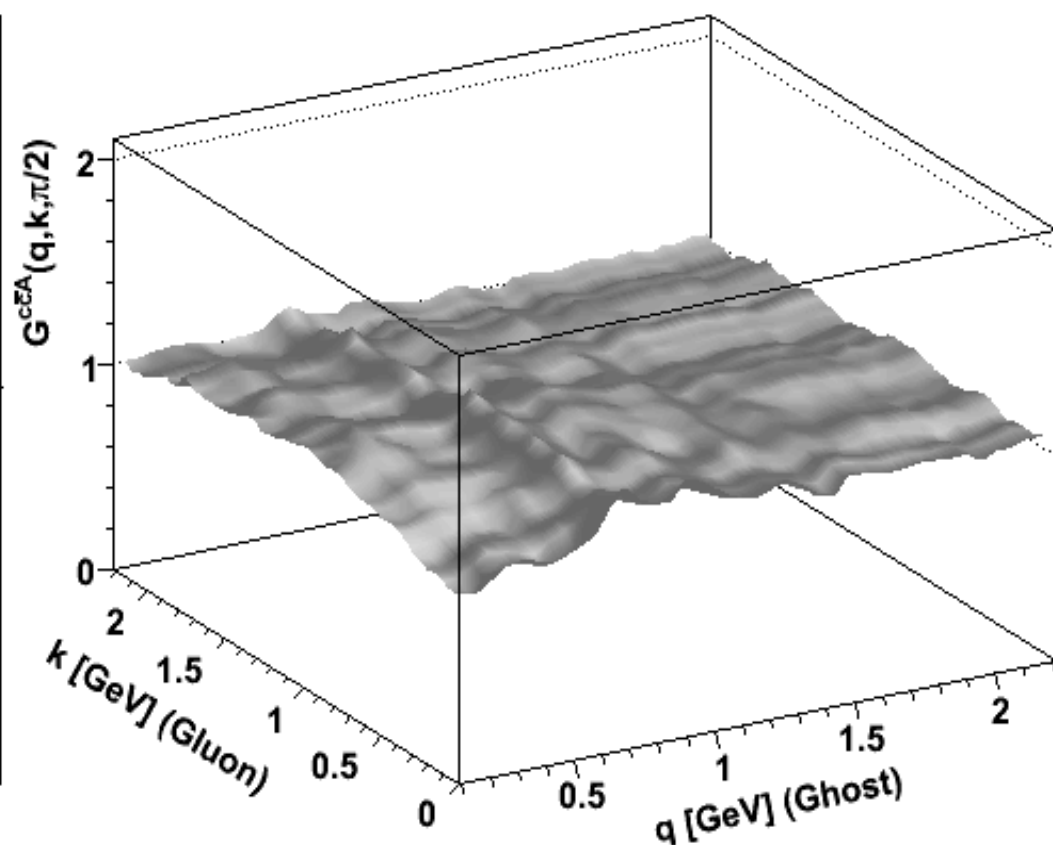
Ghost-gluon vertex in 2d

[80²@beta=10, 30, Maas, unpublished]

Ghost-gluon vertex, one momentum vanishing



Ghost-gluon vertex, orthogonal momenta

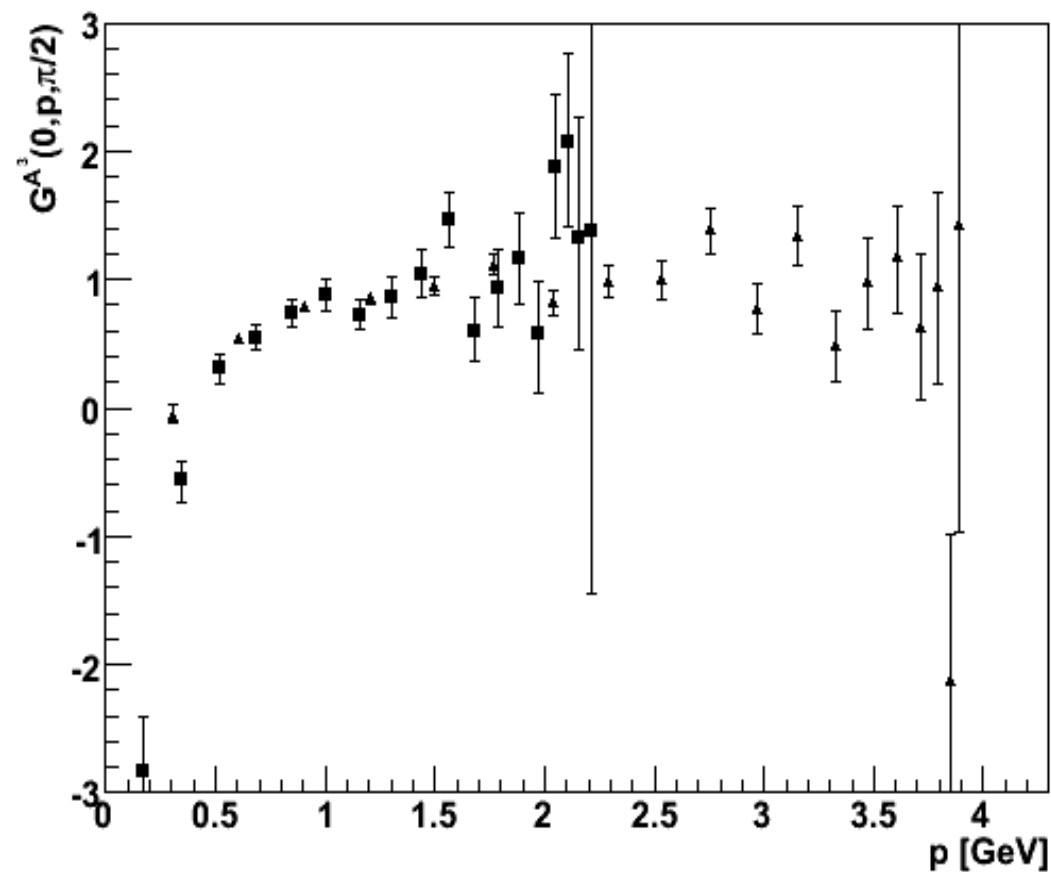


- Essentially constant, only some structure at 1 GeV
- Finite constant smaller than one in the far infrared

Three-gluon vertex in 2d

[40²@beta=10, 30, Maas, unpublished]

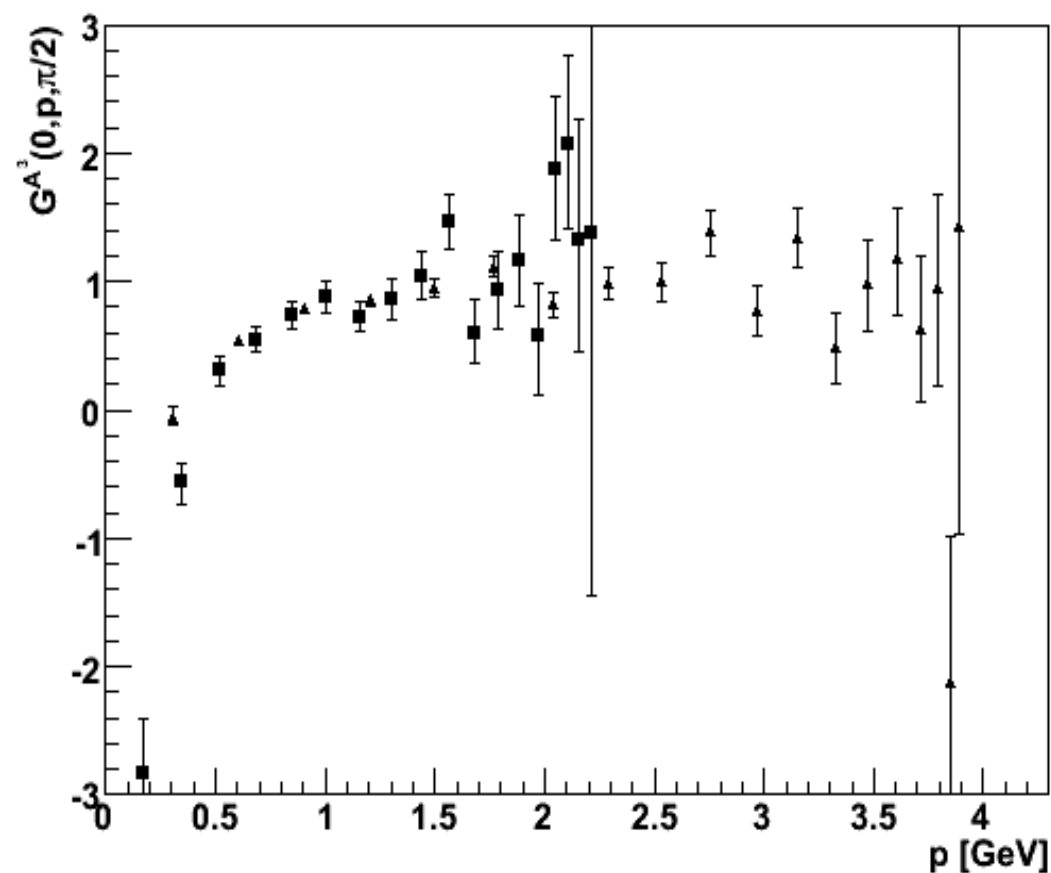
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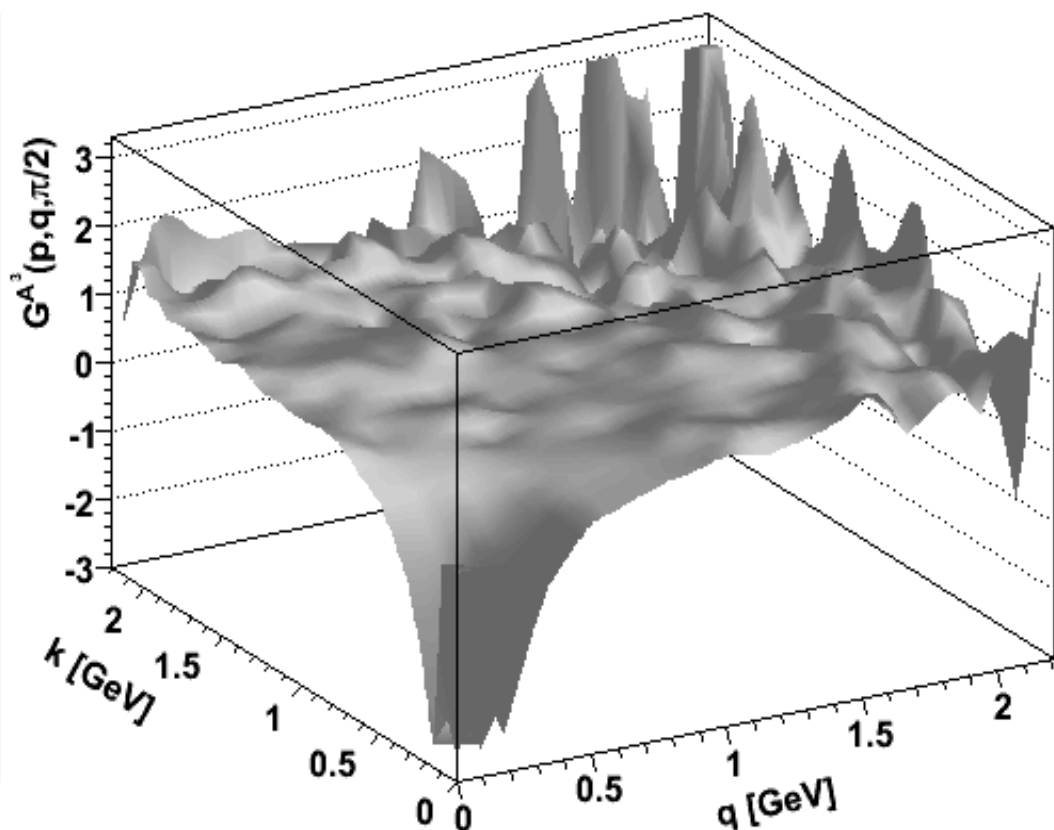
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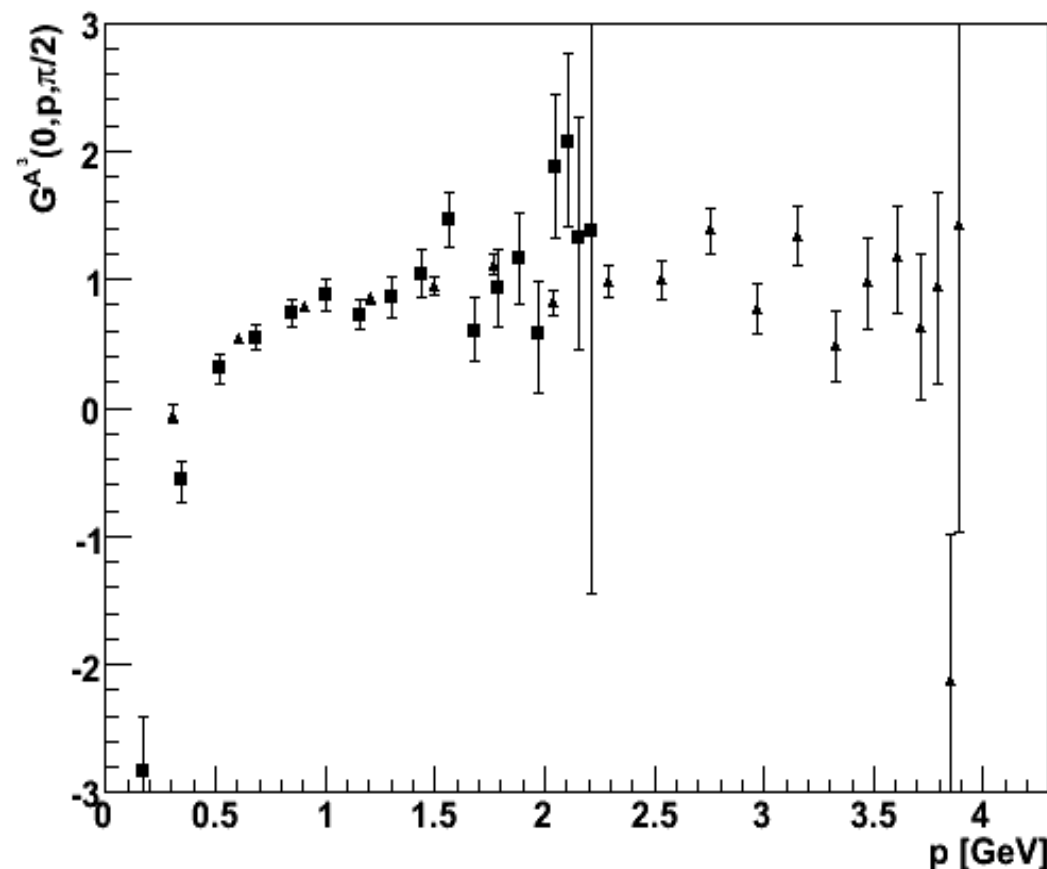
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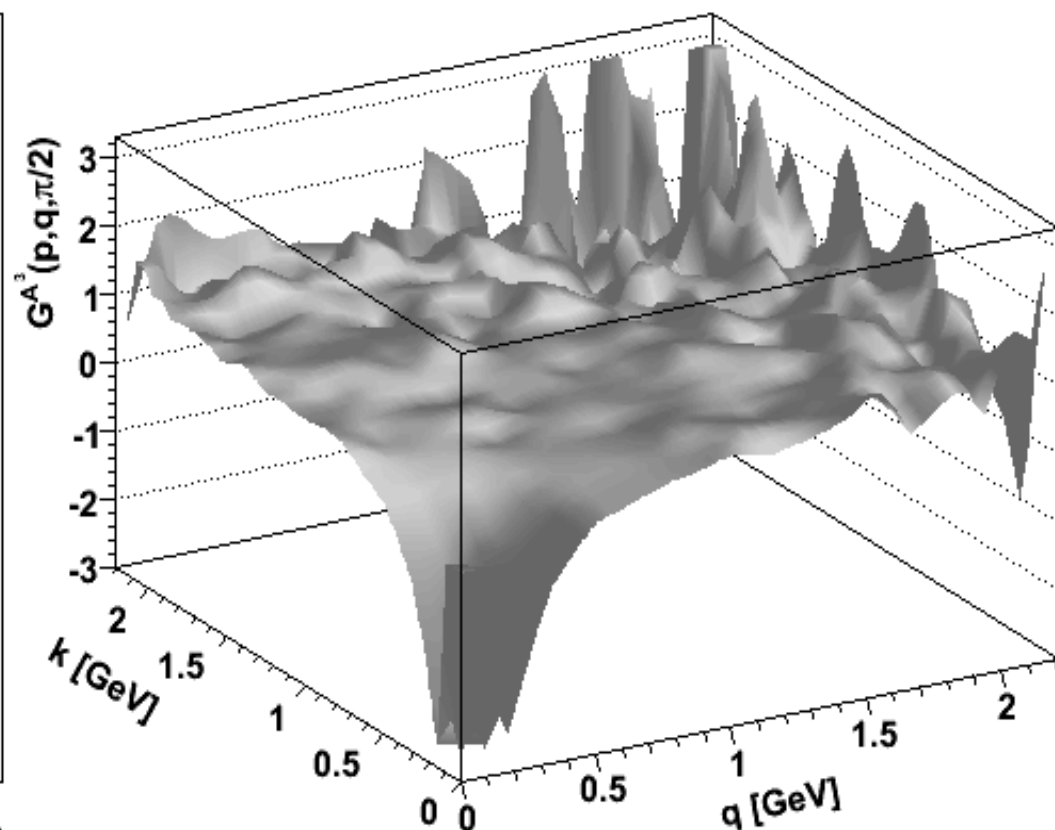
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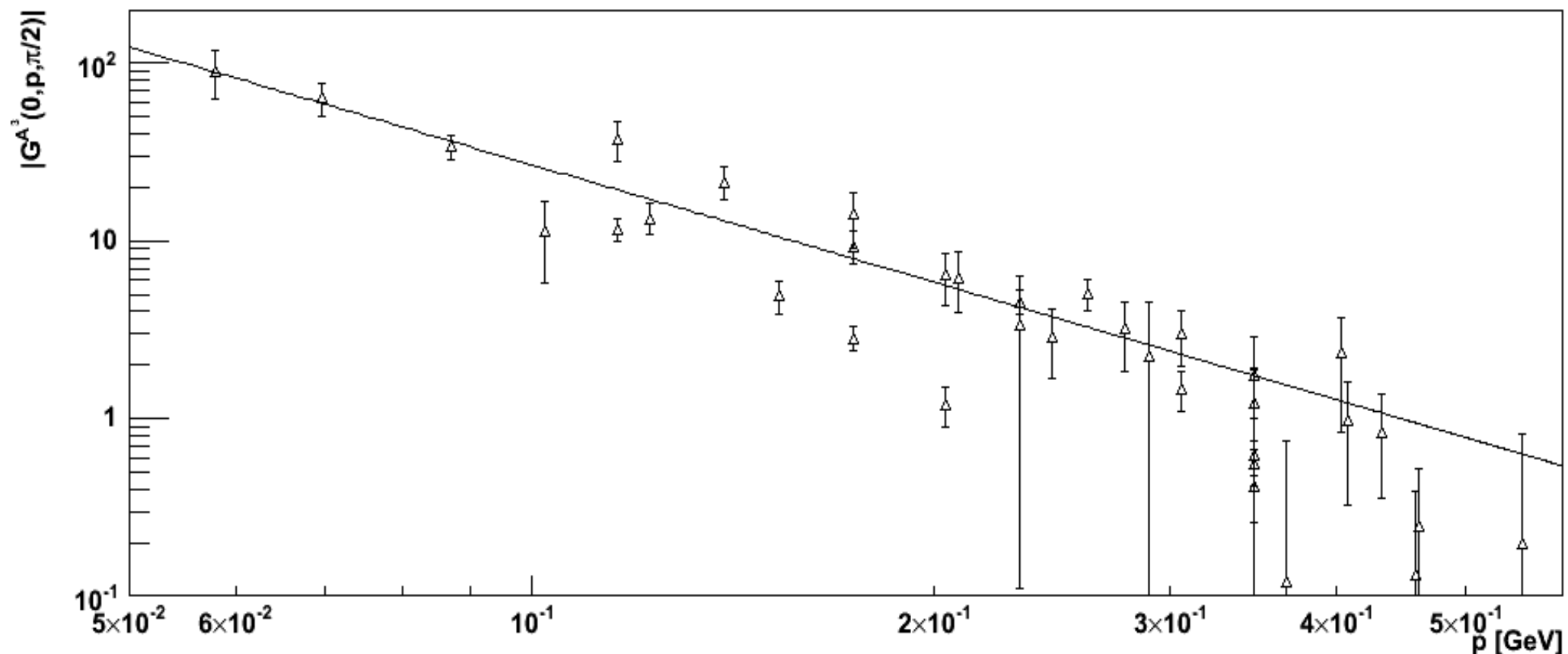


- **Clear zero crossing** – diverges to negative infinity
- **Relevant scale:** Maximum in the gluon propagator
- **Strong statistical noise**

Far infrared of ghost gluon vertex

Three-gluon vertex, one momentum vanishing

[Maas, unpublished]

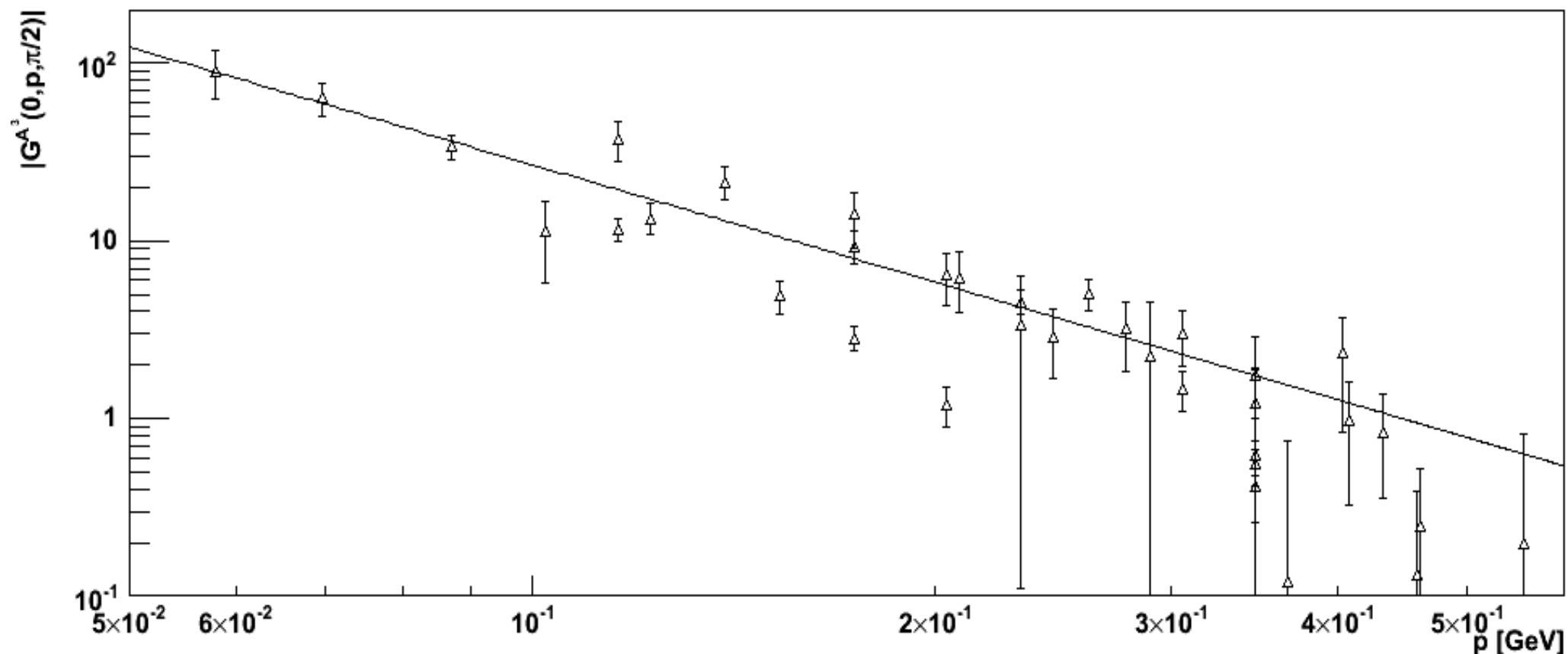


- Results agree well with predicted infrared exponent

Far infrared of ghost gluon vertex

Three-gluon vertex, one momentum vanishing

[Maas, unpublished]



- Results agree well with predicted infrared exponent
- But different kinematics: Scale dependence and angular dependence separable?

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- Relevance to bound state calculations?
- What is going on beyond $SU(2)$ Landau gauge Yang-Mills?

Collaboration

- **A. Cucchieri, T. Mendes (Uni. of São Paulo, Brazil)**
 - Results in three and four dimensions
 - hep-lat/0605011, hep-lat/0610006 and more upcoming
- **Supported by the DFG and VEGA**