From gauge-fields to observables

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A bottom-up construction in Landau-gauge QCD and beyond

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15th of February 2010
University of Sussex
UK

Overview

- Odds and ends: The necessity for non-perturbative physics

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  - Gluons
  - Scalar matter

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UNI GRAZ

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- Properties of elementary particles
  - Gluons
  - Scalar matter
- Extended outlook
  - Fermionic matter and hadrons
  - Gluons at finite temperature
- Summary

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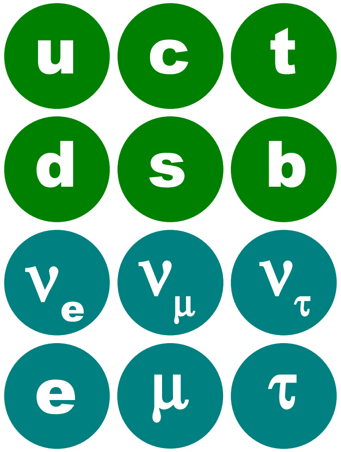

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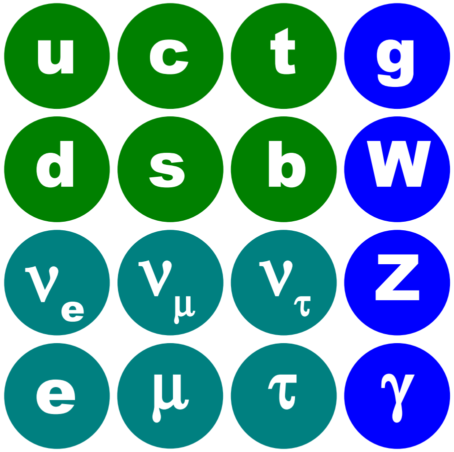

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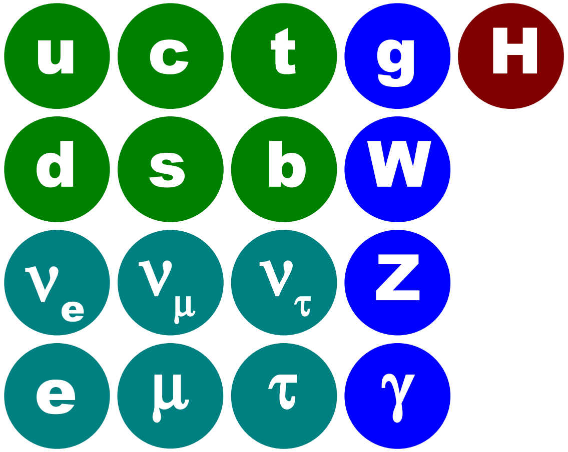

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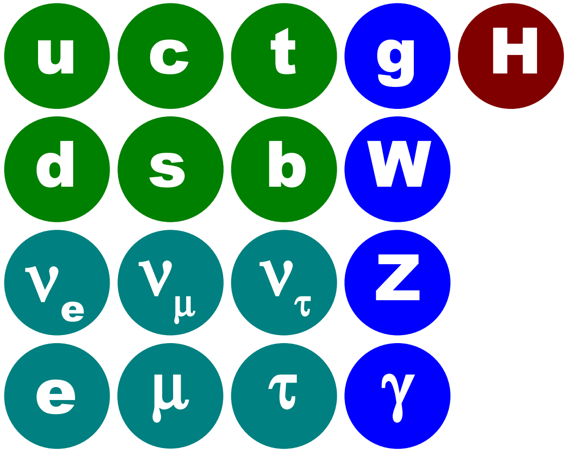

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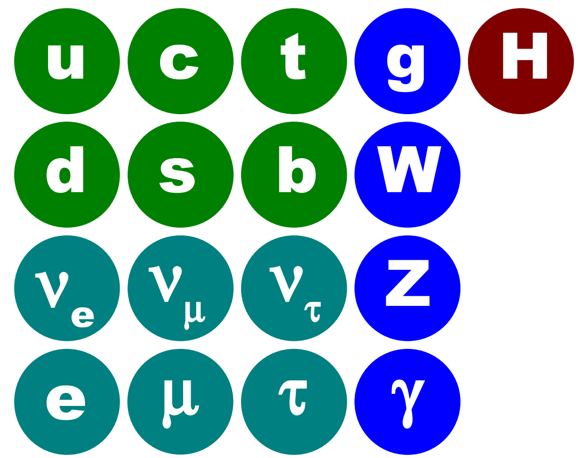

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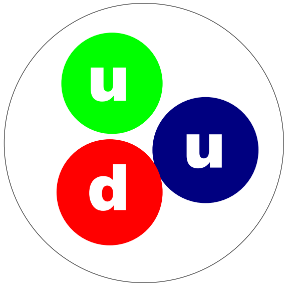

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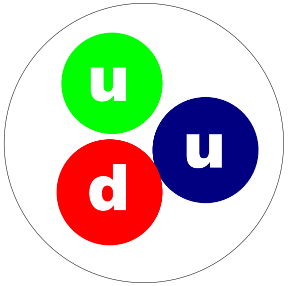

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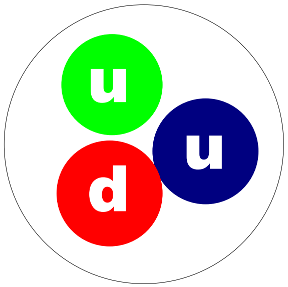

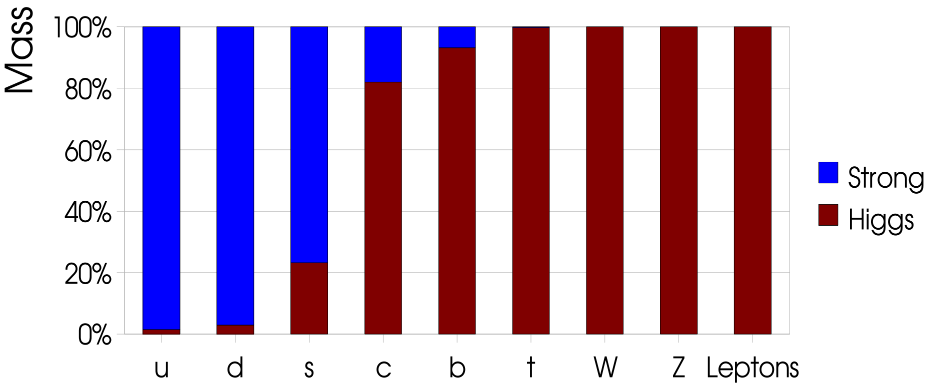

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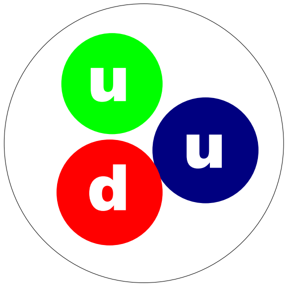

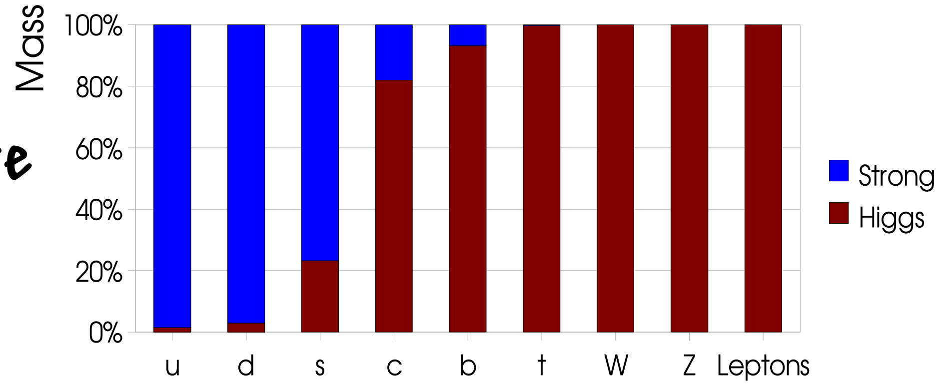

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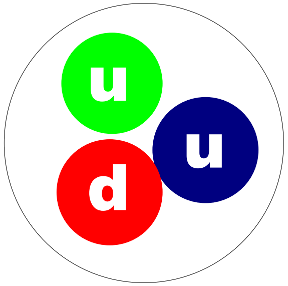

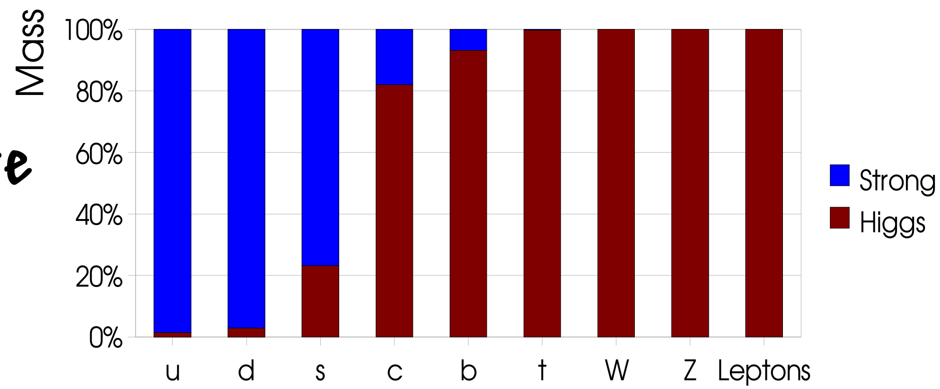

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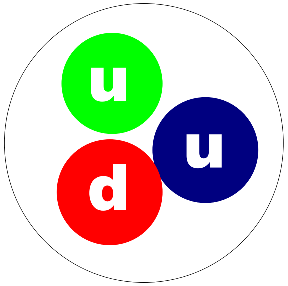

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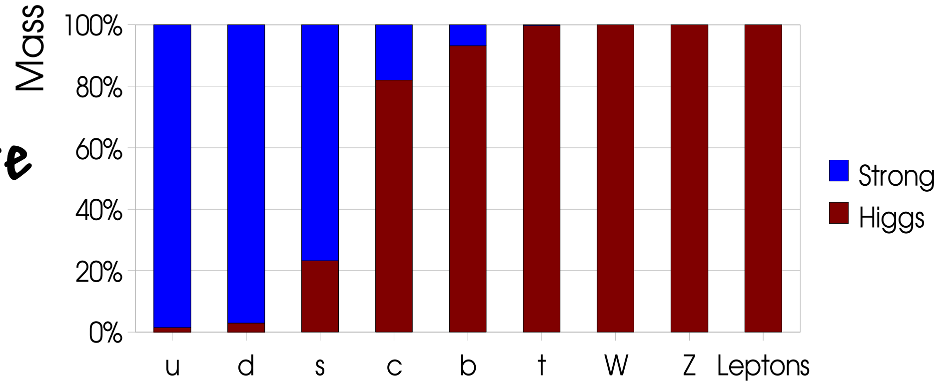

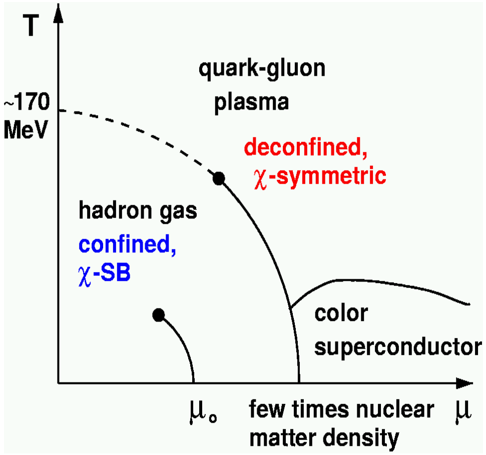

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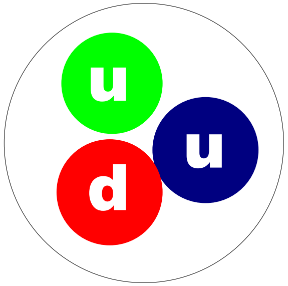

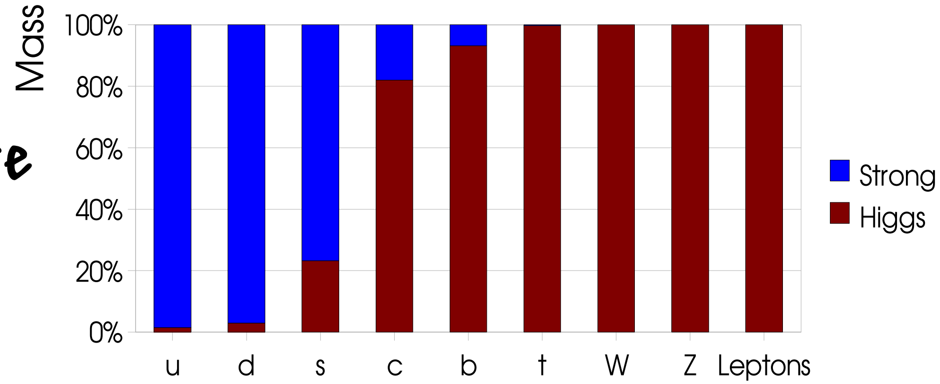

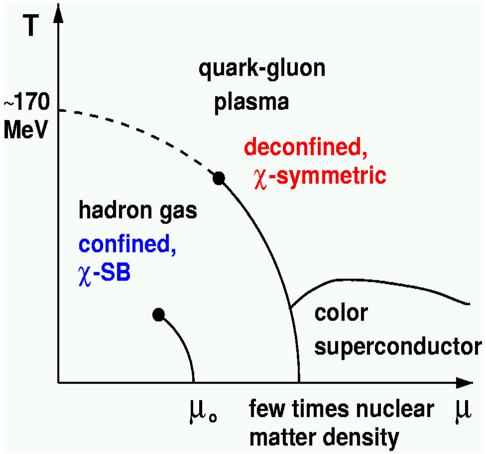

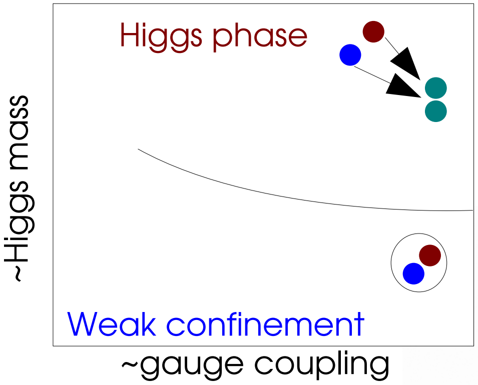

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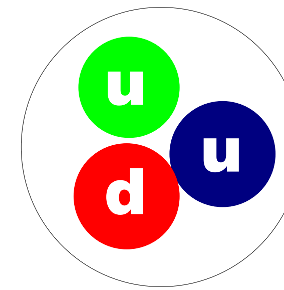

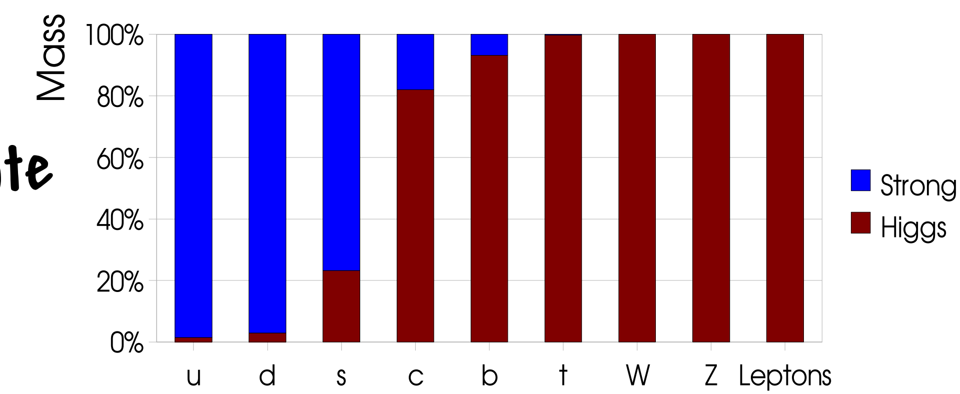

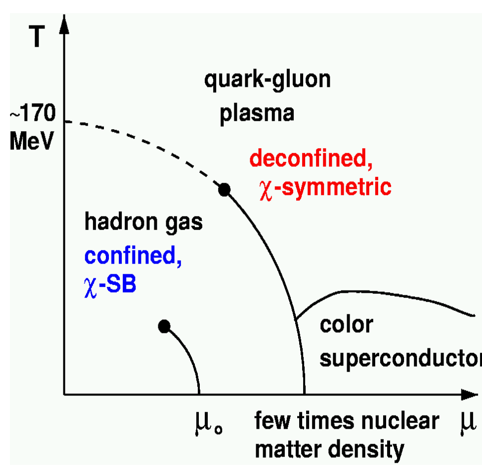

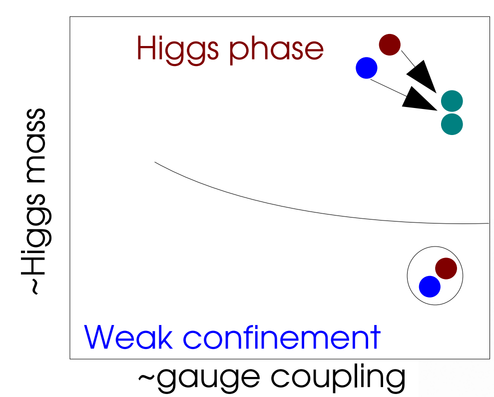

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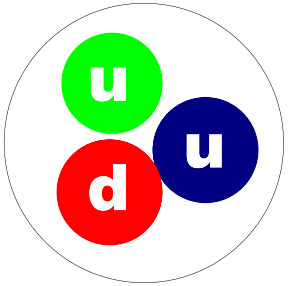

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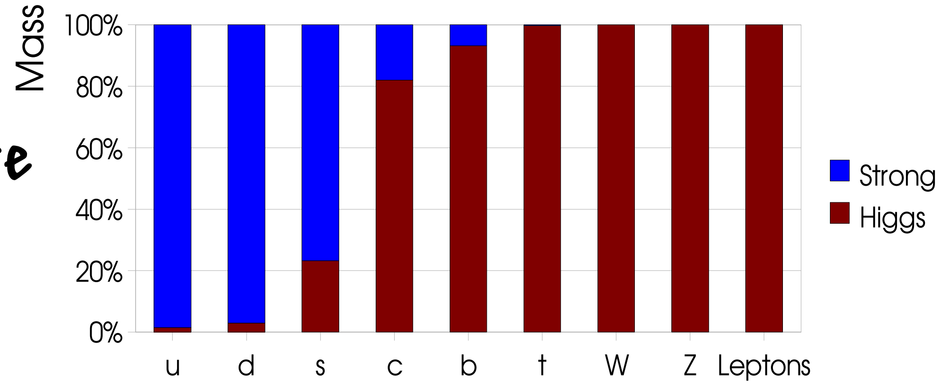

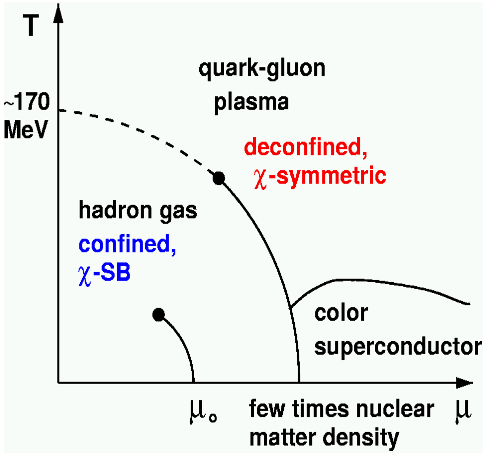

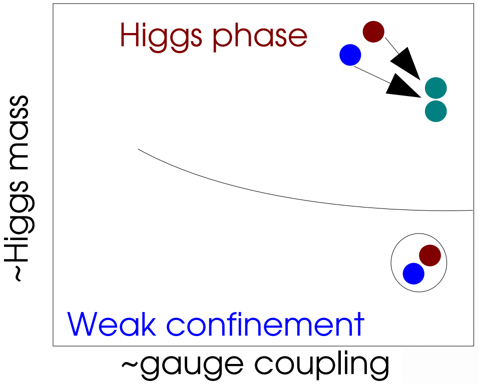

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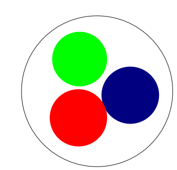

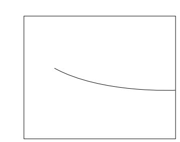

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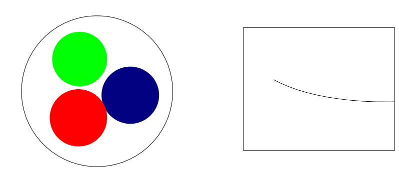

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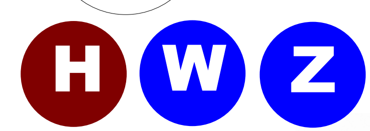

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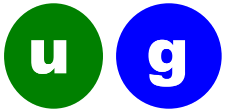

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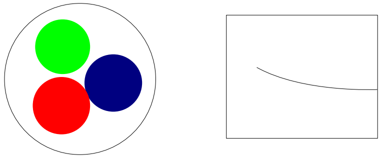

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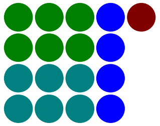

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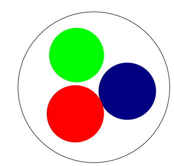

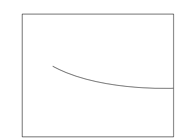

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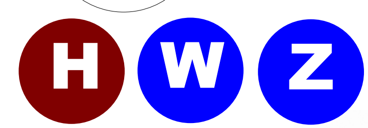

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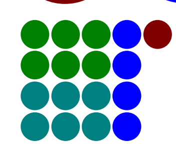

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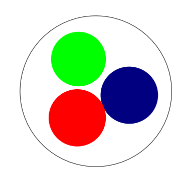

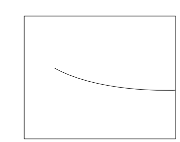

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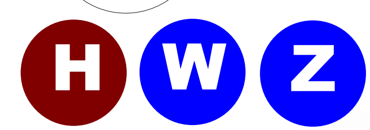

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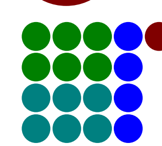

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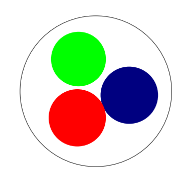

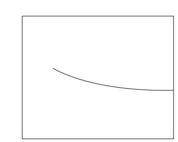

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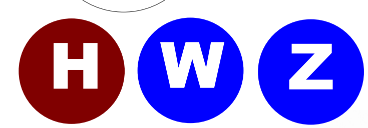

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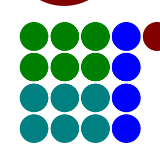

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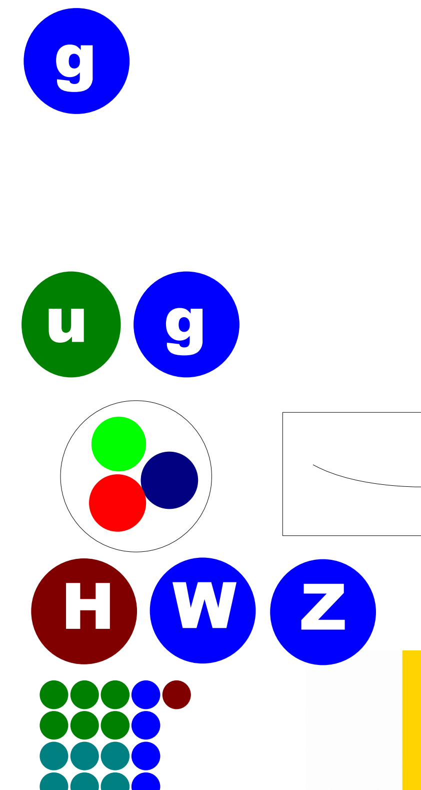

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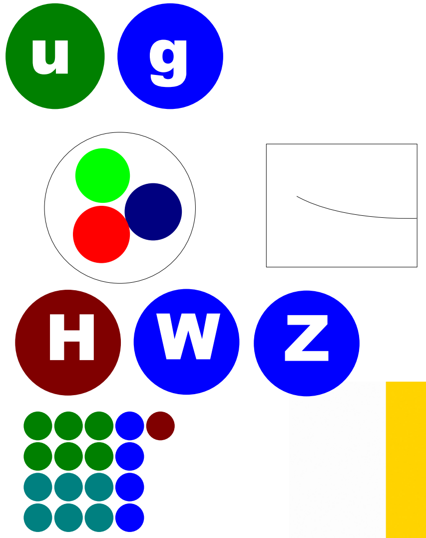

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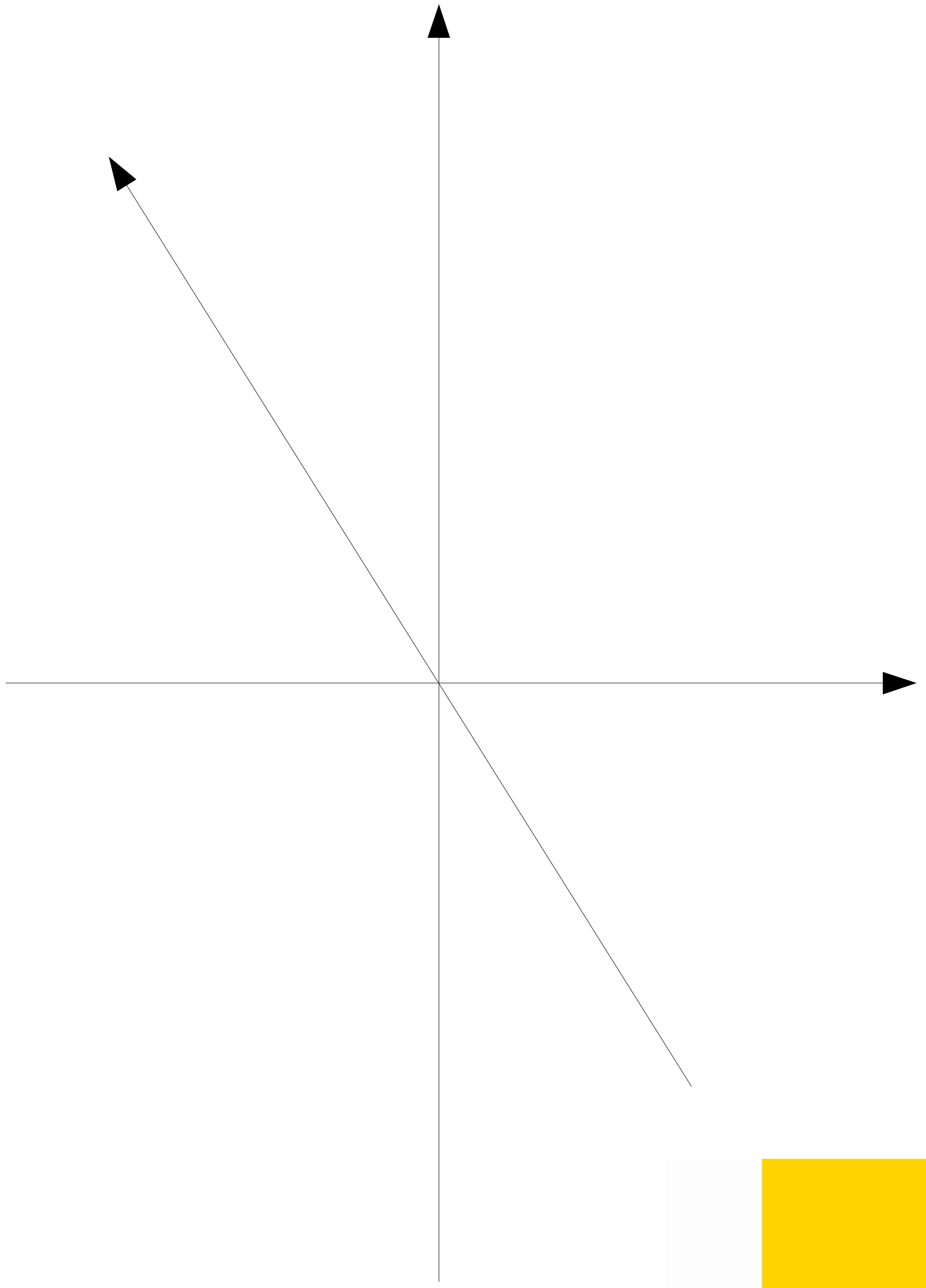

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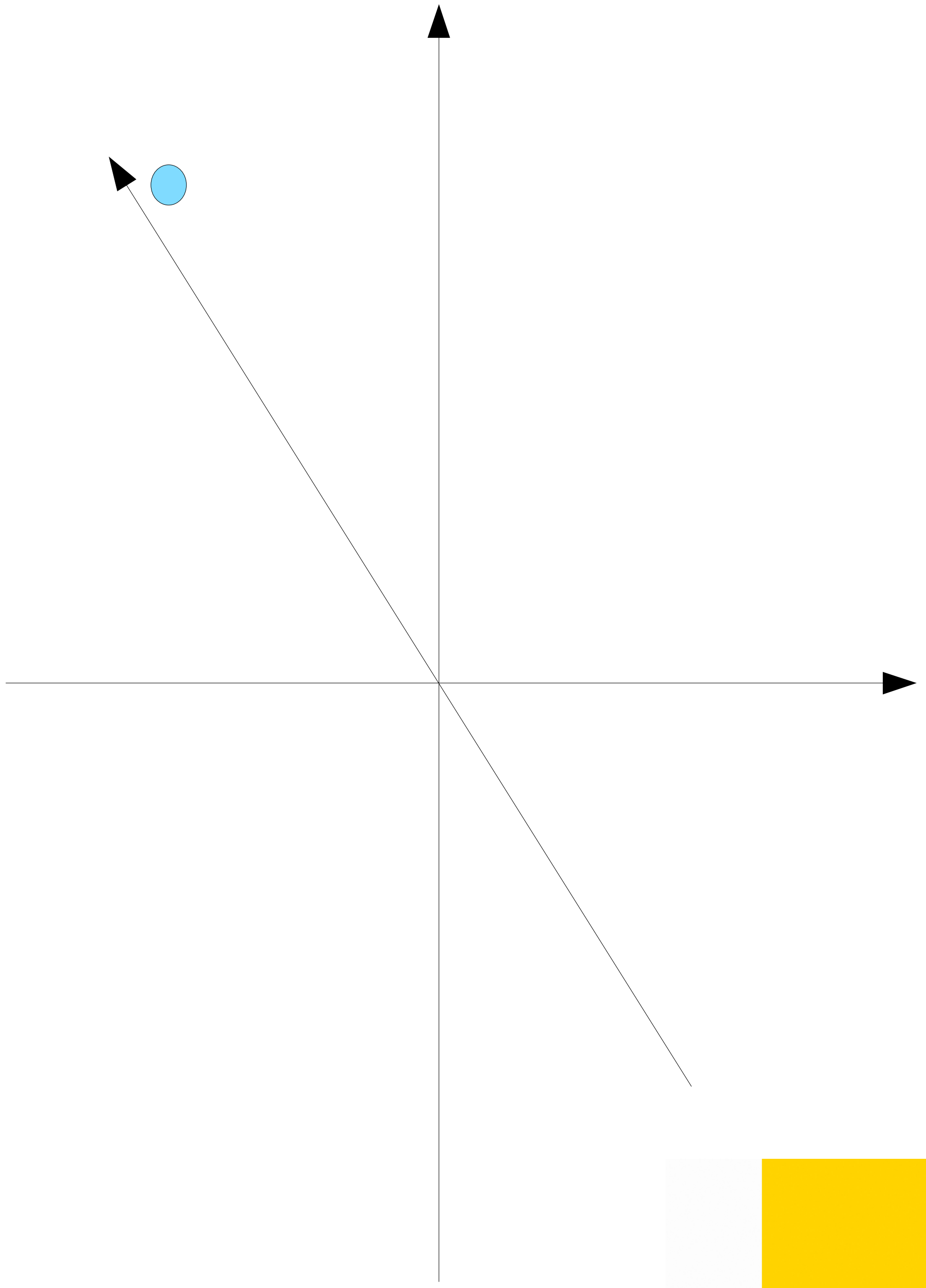

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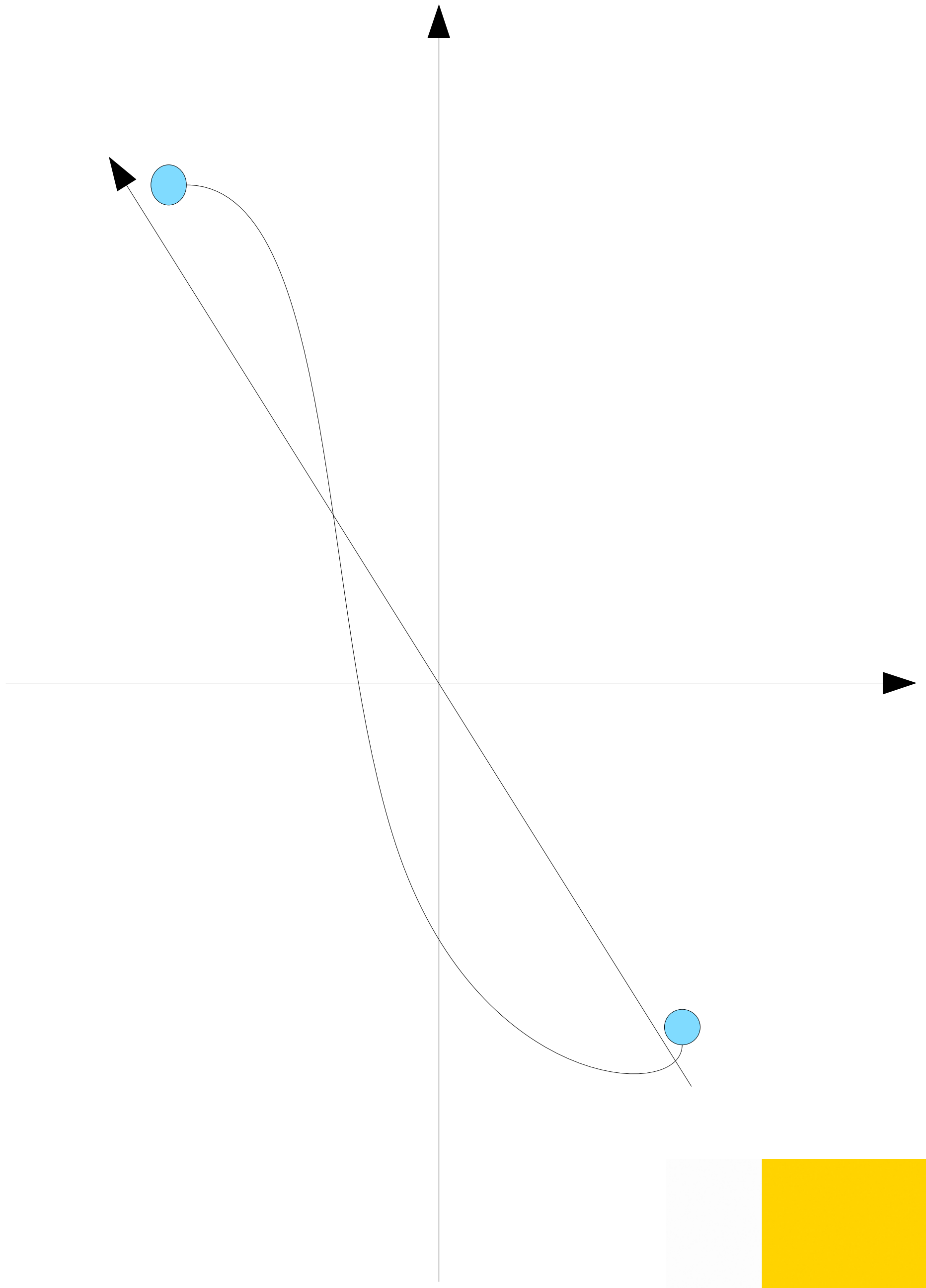

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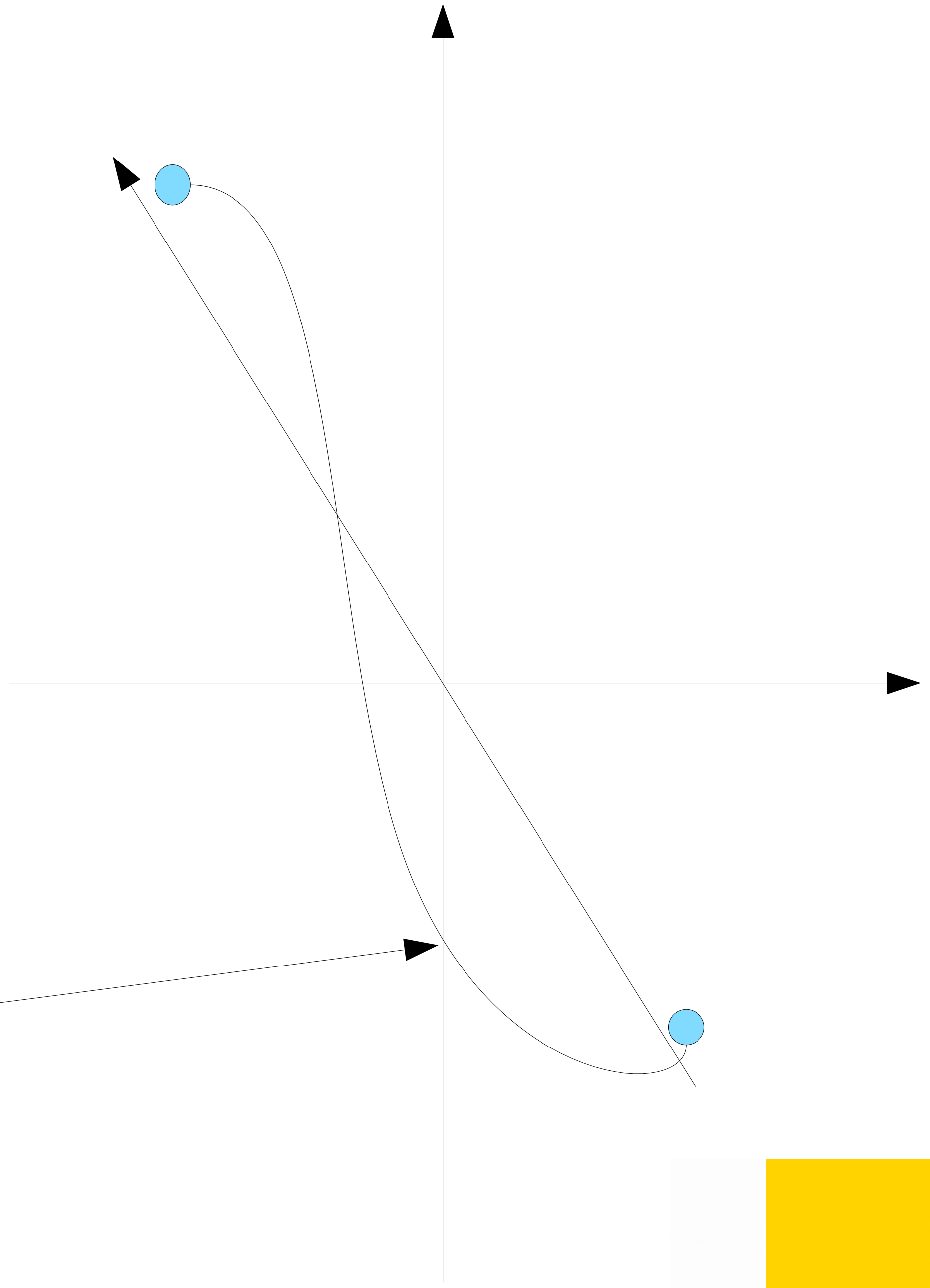
- Each configuration related by a gauge transformation provides the same observables
  - Gauge orbit



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  - Even the charges are not gauge-invariant
    - Except for electric charges
    - No concept of a local gauge-invariant charge distribution
      - Similar to energy density in general relativity

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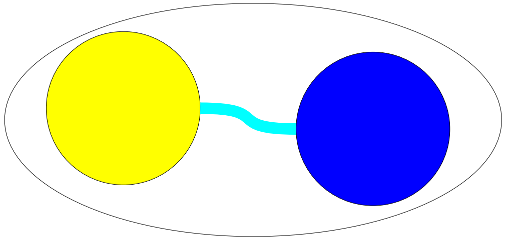

    - Except for electric charges
    - No concept of a local gauge-invariant charge distribution
      - Similar to energy density in general relativity
  - Only bound states can be gauge-invariant, and thus physical

Prototype: Yang-Mills Theory

- Lagrangian:

$$L = -\frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^a_{bc} A^b_\mu A^c_\nu$$

- Degrees of freedom:

Gluons: A^a_μ

- g is the coupling constant, giving the strength of coupling
- f^{abc} are numbers, depending on the gauge group, SU(3) for QCD: gluons are organized in multiplets, just as with spin

Gauge-fixing

- Yang-Mills theory is a gauge theory
  - Gauge transformations $A_\mu^a \to A_\mu^a + (\delta_b^a \partial_\mu - g f_{bc}^a A_\mu^c)\phi^b(x)$

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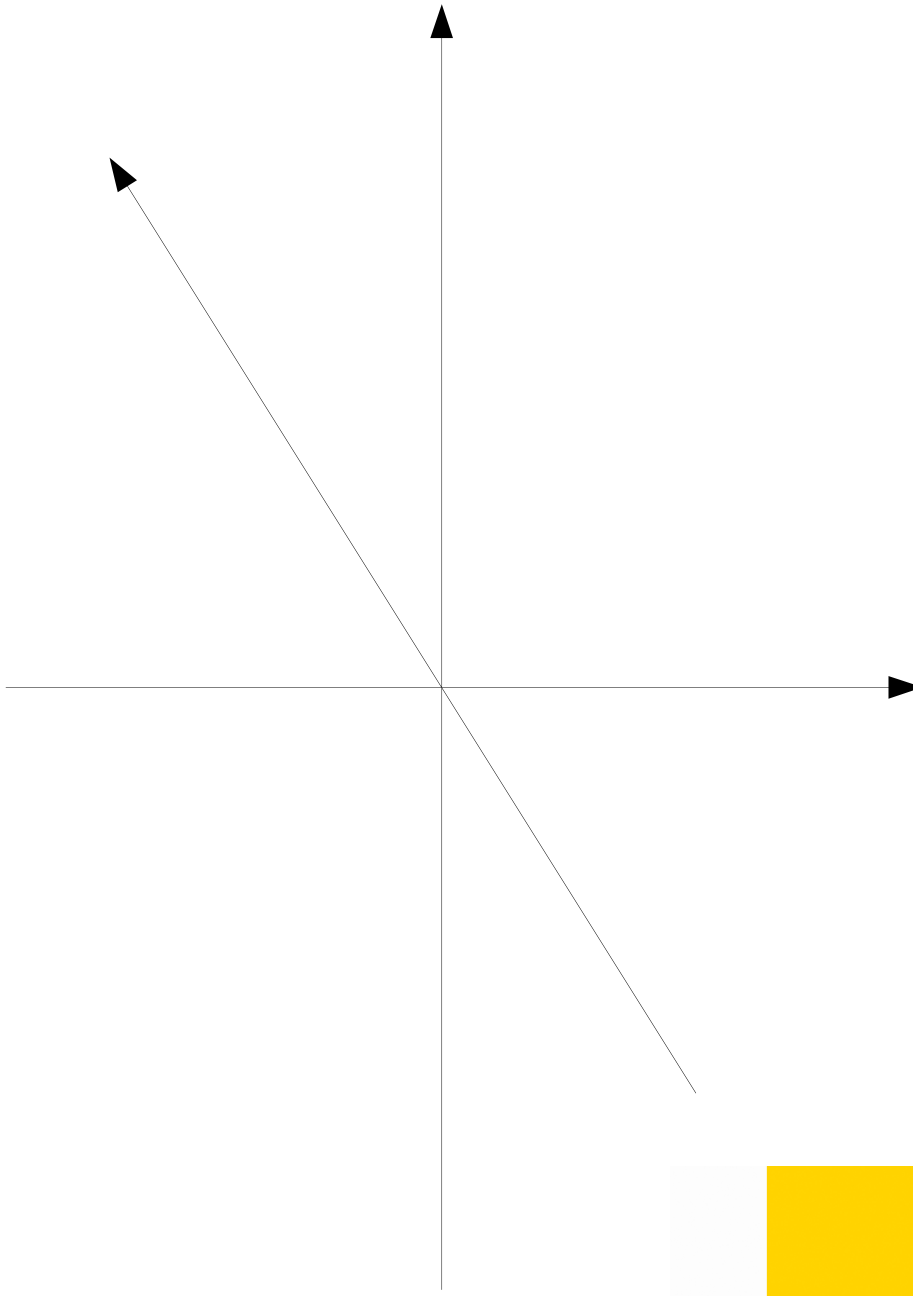
  - Requires a prescription to make comparisons or obtain properties
- Example: Landau gauge condition $\partial^\mu A_\mu^a = 0$
  - Here only Landau gauge results
  - Many other gauges have been studied

Configuration space (artist's view)



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- Impose Landau gauge condition
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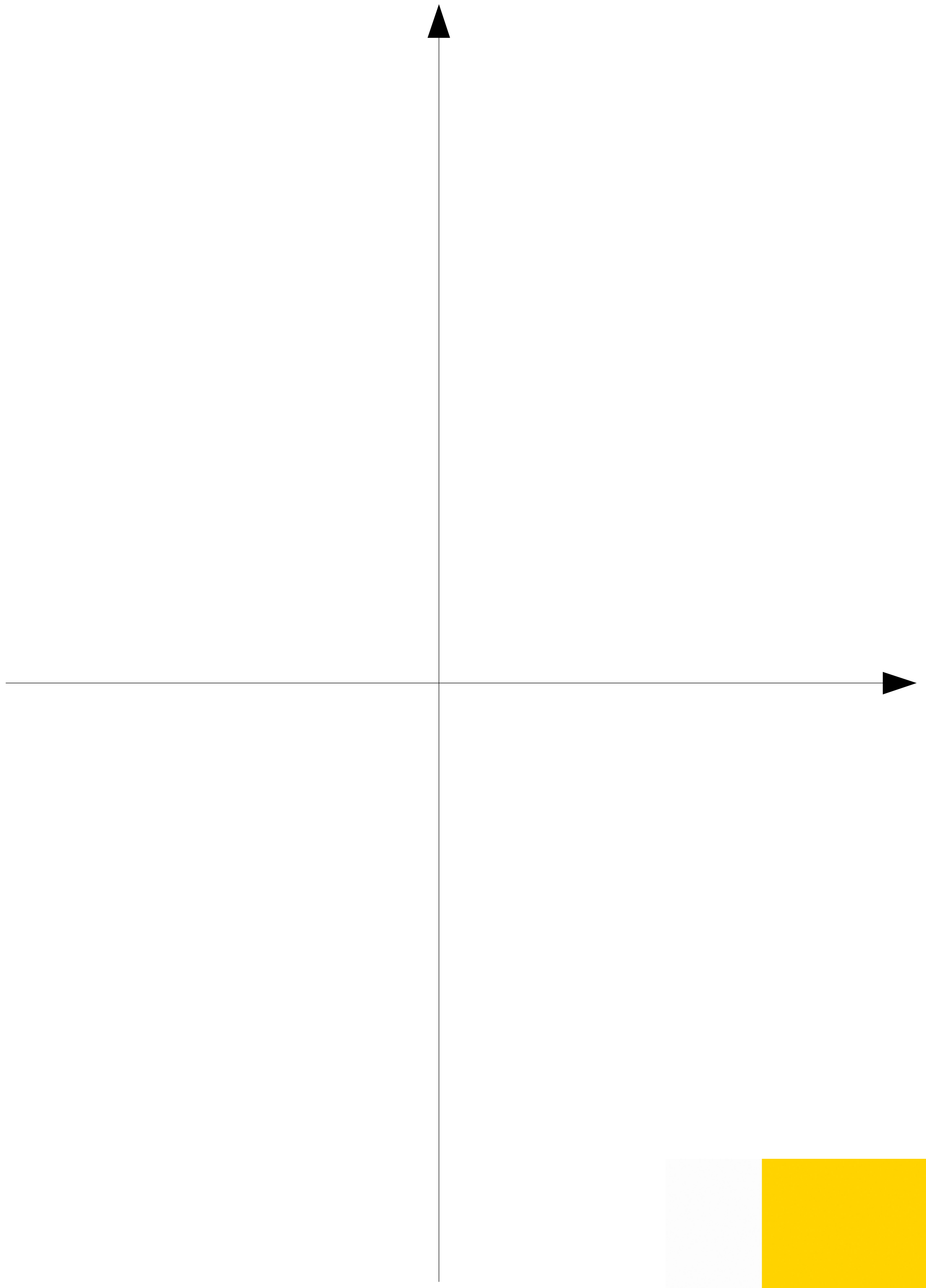

Unambiguous gauge-fixing [For an introduction: Sobreiro & Sorella, 2005]

- Local gauge condition
  - Landau gauge: $\partial_\mu A_\mu^a = 0$
- Can be implemented using auxiliary fields, the so-called ghost fields
  - No physical objects: Pure mathematical convenience

(Perturbative) Landau gauge

- Lagrangian:

$$L=-\frac{1}{4}F^a_{\mu\nu}F_a^{\mu\nu}-\bar{c}^a\partial_\mu D^\mu_{ab}c^b$$

$$D_\mu^{ab}=\delta^{ab}\partial_\mu-igf^{ab}_c A^c_\mu$$

- Degrees of freedom:

Gluons: A^a_μ

Ghosts: $\bar{c}^a, c^a$

- Ghosts interact with gluons: They have to be included

- **Here: Euclidean version**

Unambiguous gauge-fixing [For an introduction: Sobreiro & Sorella, 2005]

- Local gauge condition
  - Landau gauge: $\partial_\mu A_\mu^a = 0$
- Sufficient for perturbation theory

Proceeding

- Once the gauge is fixed, all kind of (perturbative) calculations can be done
- Use correlation functions as basic entities

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- If having a non-vanishing color charge they change under gauge transformation
- Simplest non-zero correlation functions: 2-point functions or propagators
  - Expectation values of products of two field operators
  - 1-point functions vanish

Propagators

- In Landau gauge: Gluon and one auxiliary field: Ghost
- Gluon:

$$D_{\mu\nu}^{ab}(x-y) = \langle A_\mu^a(x) A_\nu^b(y) \rangle$$

$$D_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\frac{Z(p)}{p^2}$$

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- Ghost propagator can be expressed as a gluon operator, the inverse Faddeev-Popov operator

$$D^{ab}_G(x-y) \sim \langle(\partial_\mu D^{ab}_\mu)^{-1}\rangle = \langle(\partial_\mu(\delta^{ab}\partial_\mu - g f^{abc} A^c_\mu))^{-1}\rangle$$

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- Combination of gauge-variant correlation functions yield gauge-invariant results
  - E.g. scattering cross-sections
- Almost all perturbative calculations proceed via gauge-variant correlation functions

Also non-perturbatively?

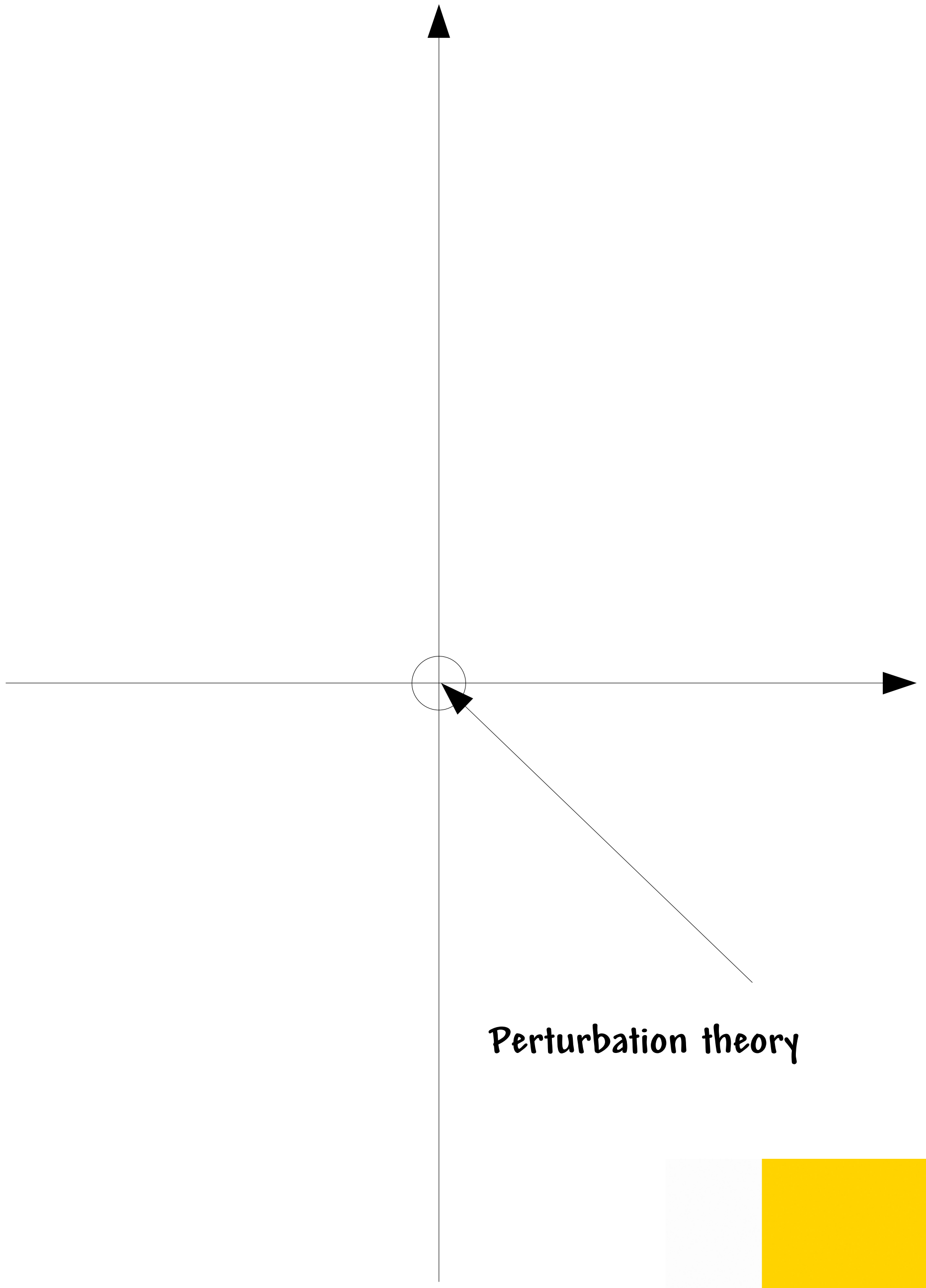
Would be nice:

Same entities and concepts as in perturbation theory

Direct connection to perturbation theory

Configuration space (artist's view)

- Perturbation theory is applicable close to the origin



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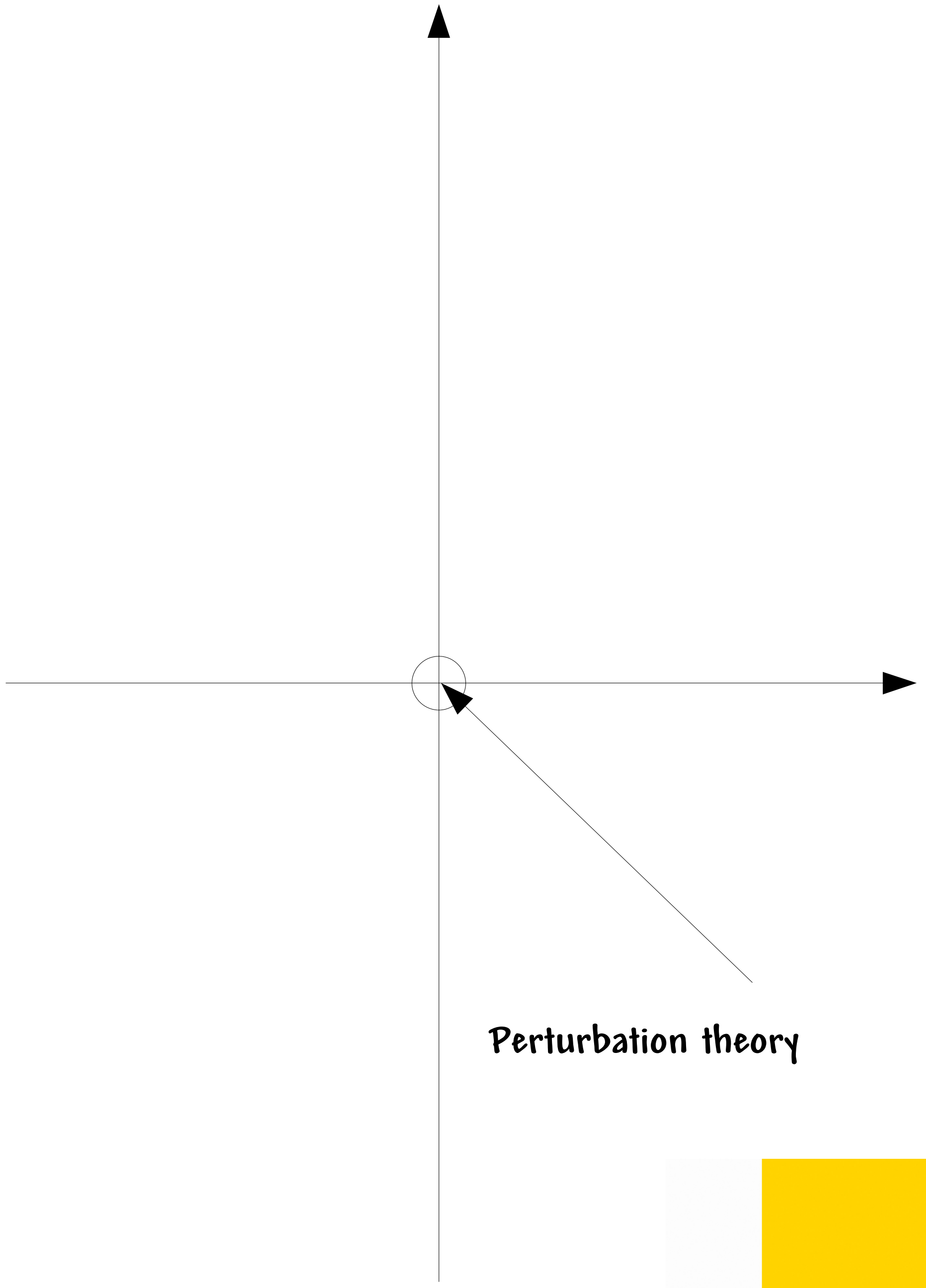
- Perturbation theory is applicable close to the origin
- Non-perturbative physics probes the complete hypersurface



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- Local gauge condition
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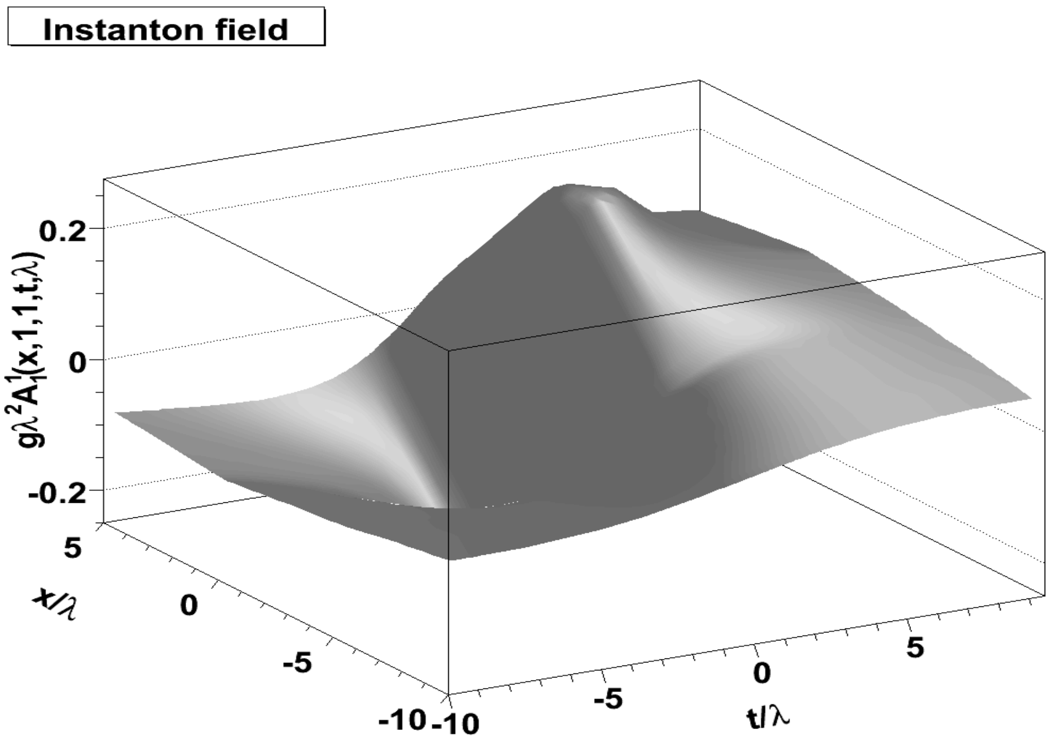
- There are no local gauge conditions known, which select a unique gauge field configuration [Singer 1978]
  - Non-local conditions possible

Example: Instanton [Maas, 2005]



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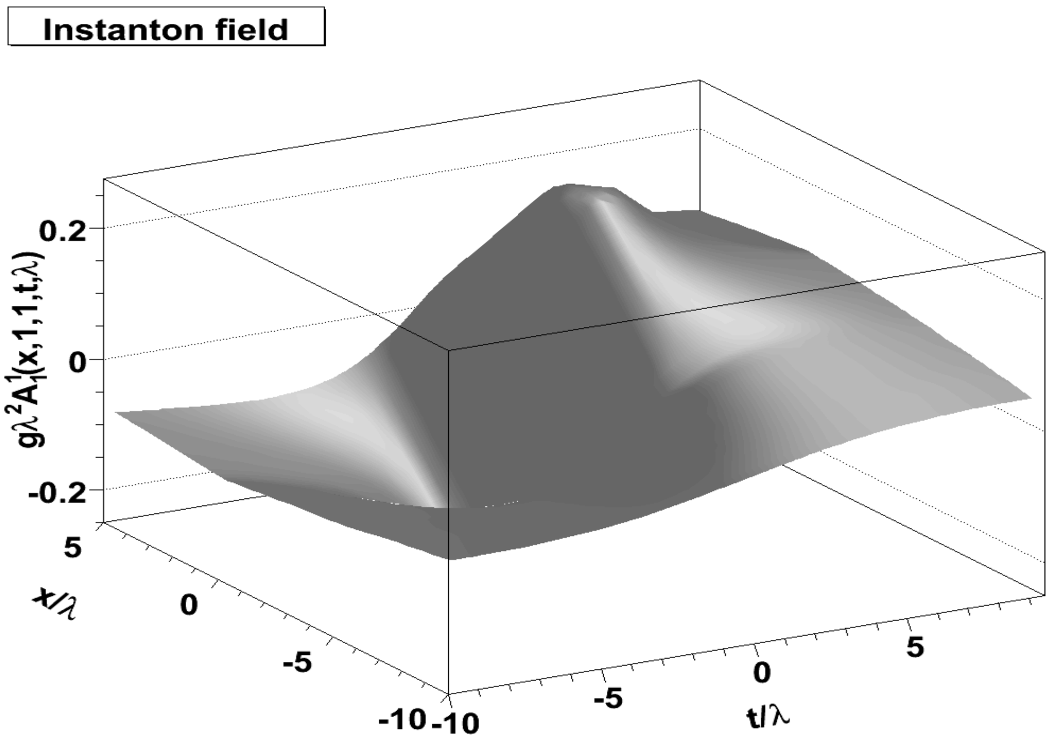

- Instanton field configuration is $A_\mu^a(r,\lambda)=2\mathrm{r}_\nu \eta_{\nu\mu}^a/(g(r^2+\lambda^2))$
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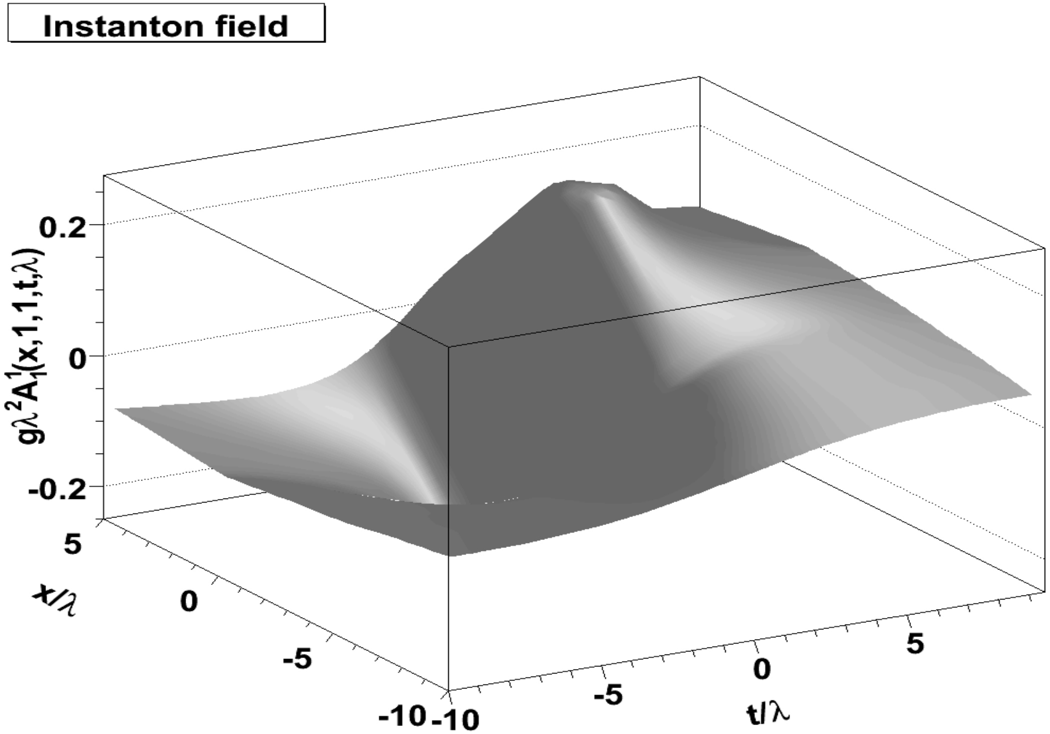

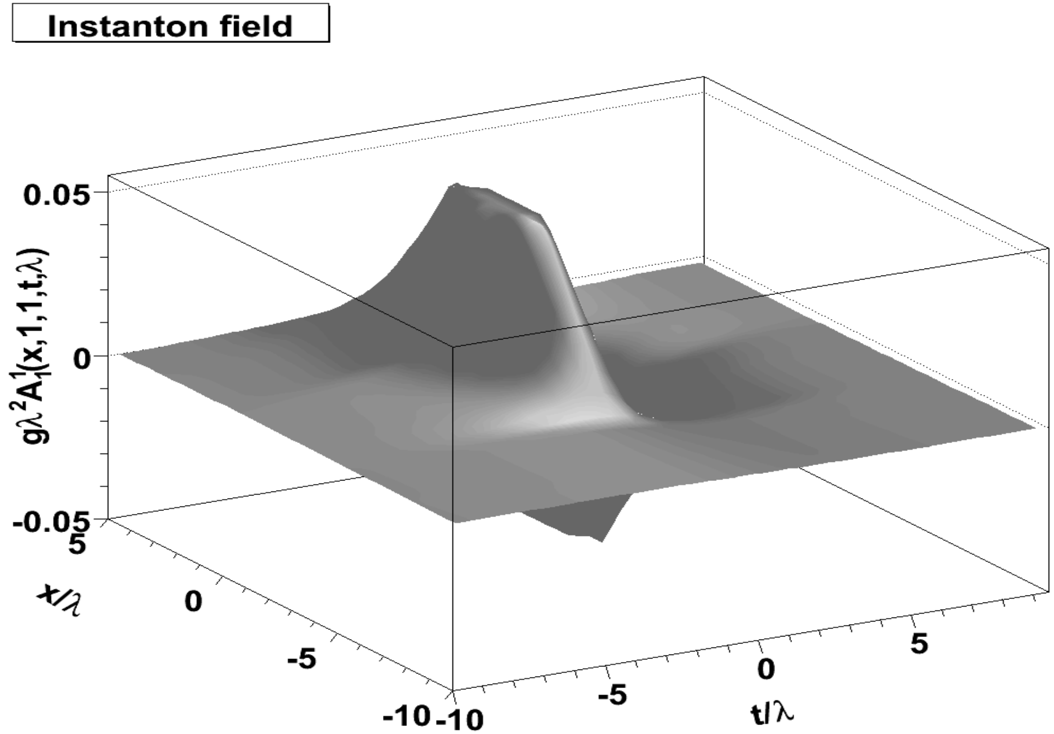

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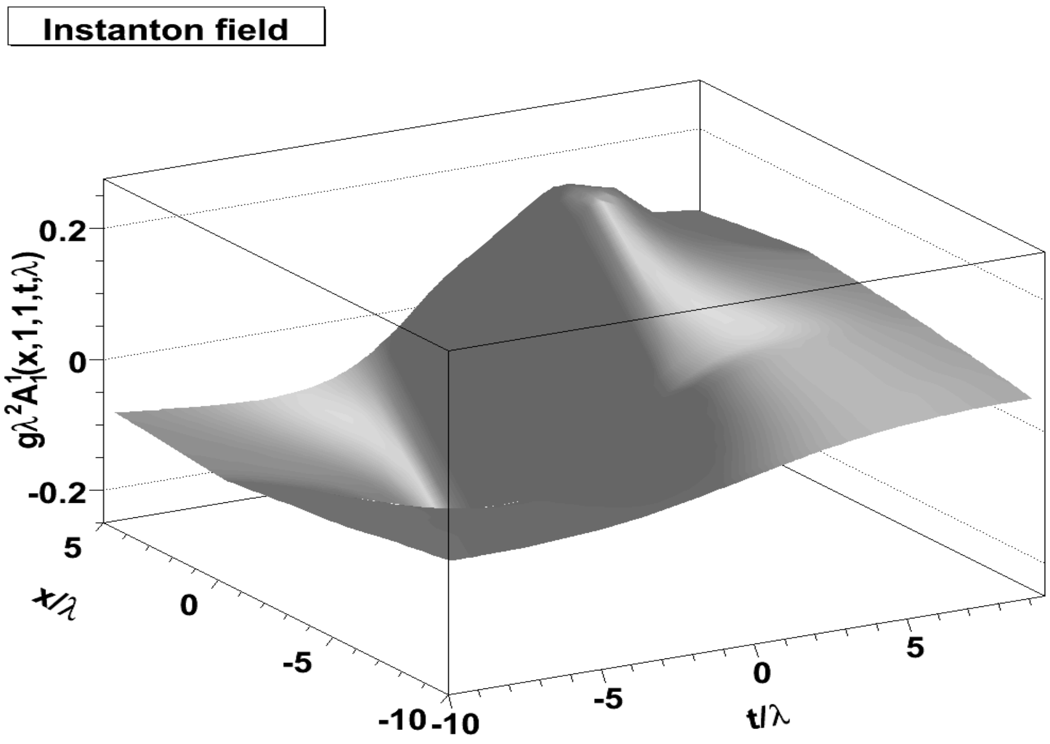

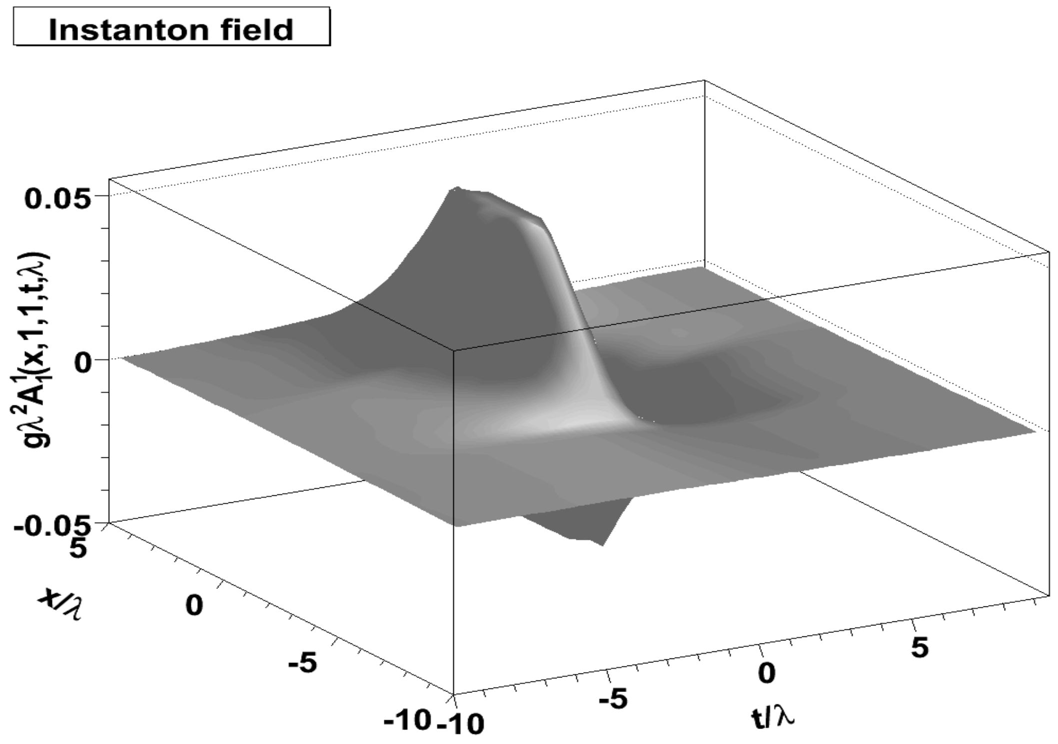

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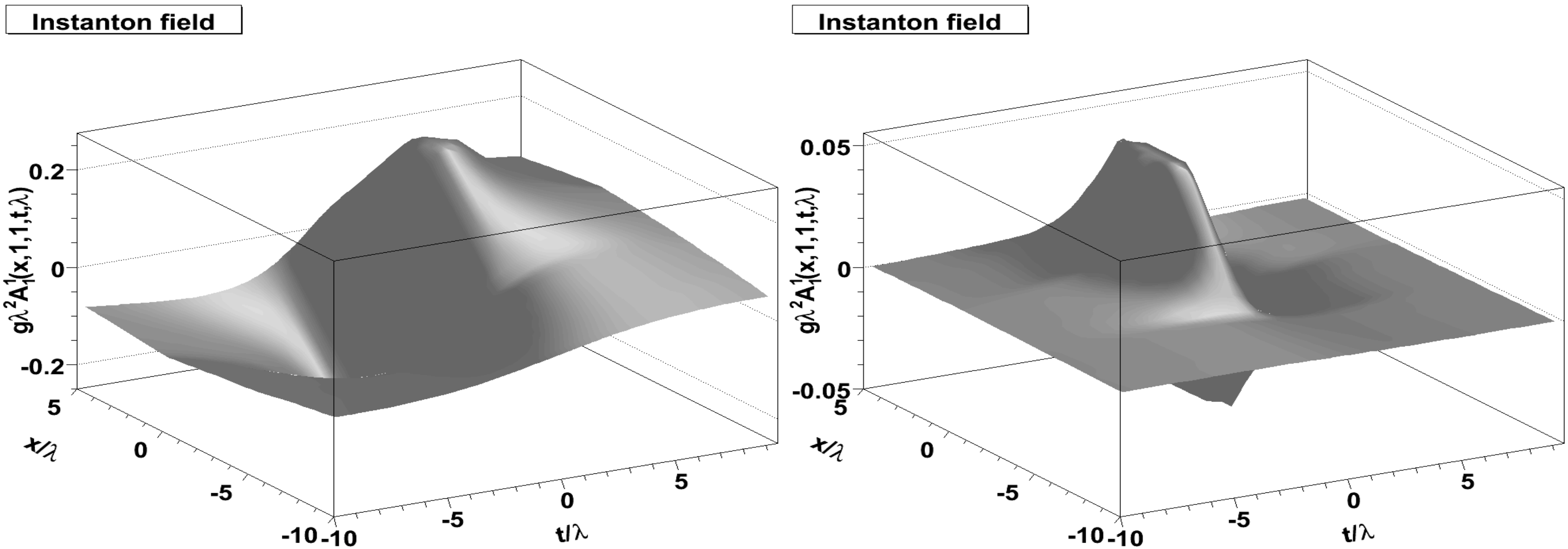

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  - Gribov copy
  - Non-perturbative: Depends on $1/g$

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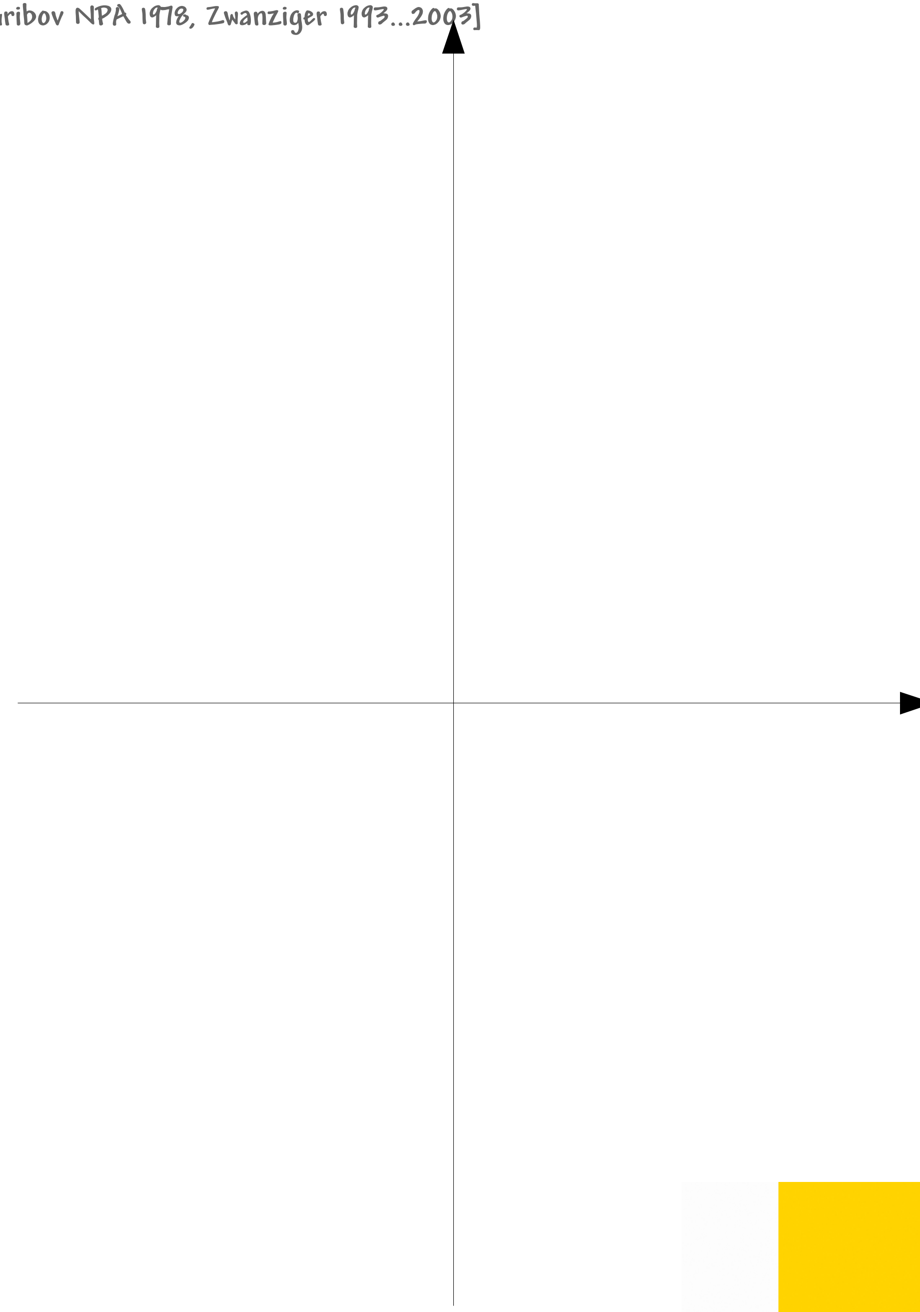
- Construct a non-local condition instead to solve the problem

Configuration space (artist's view) [Gribov NPA 1978, Zwanziger 1993...2003]



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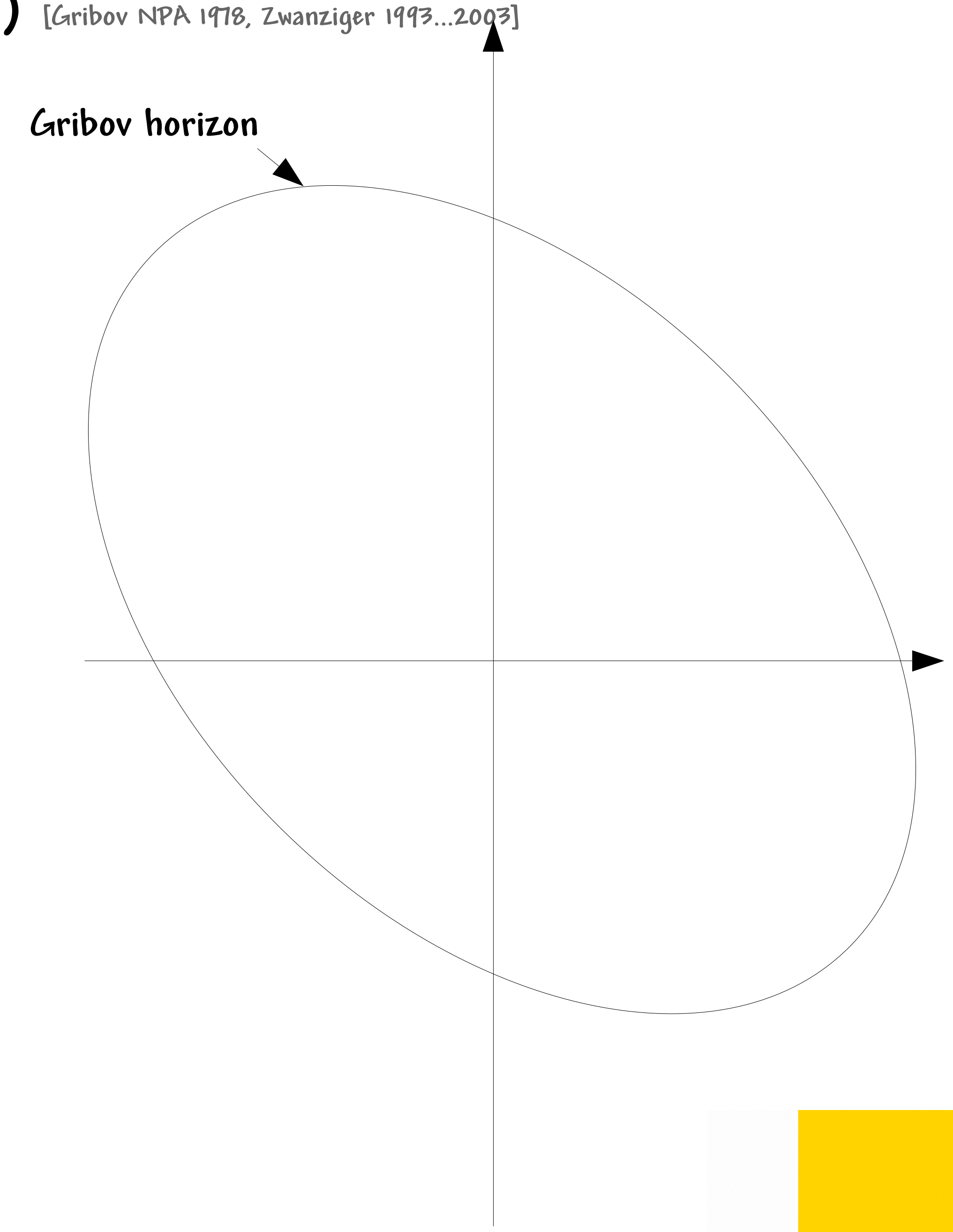

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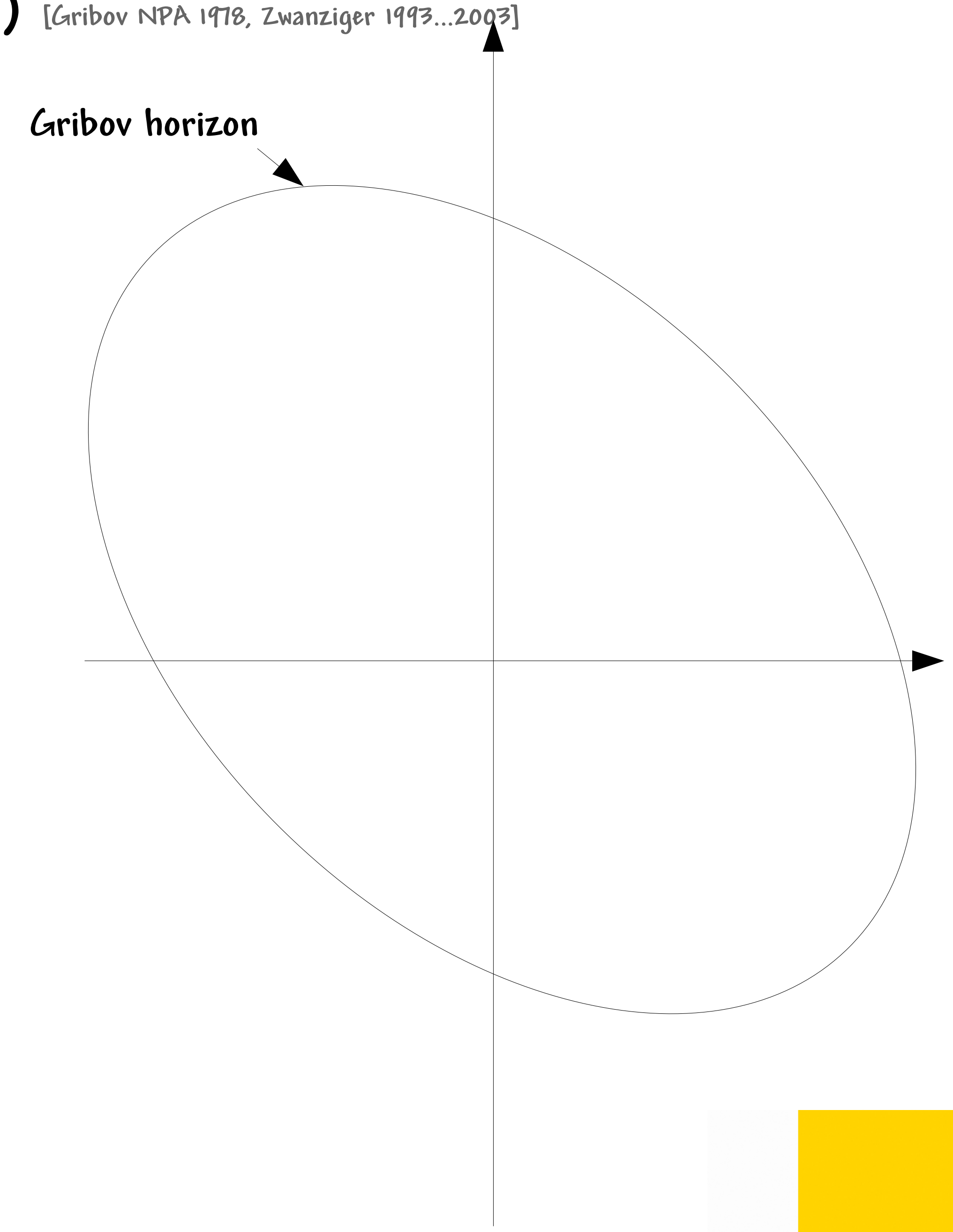

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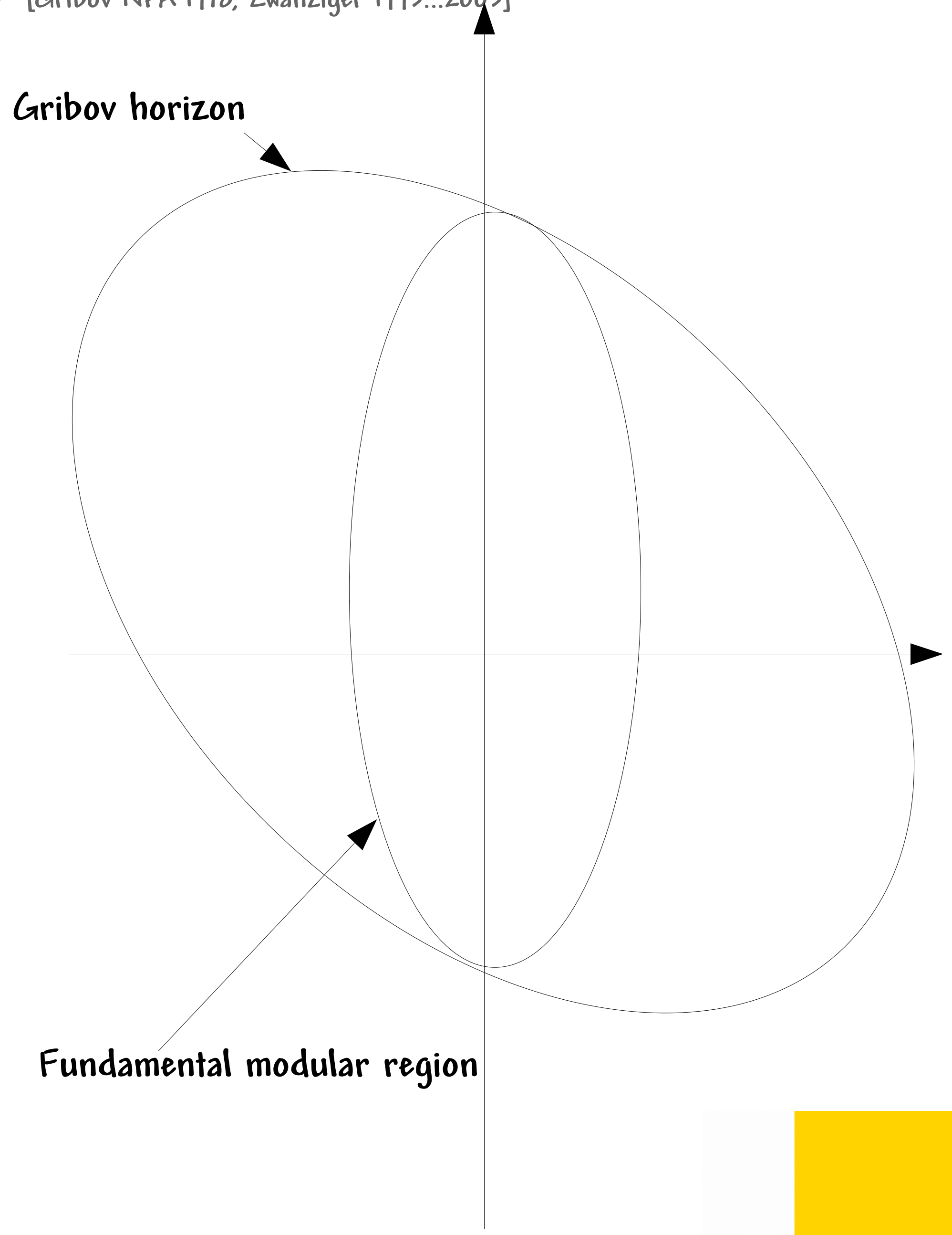

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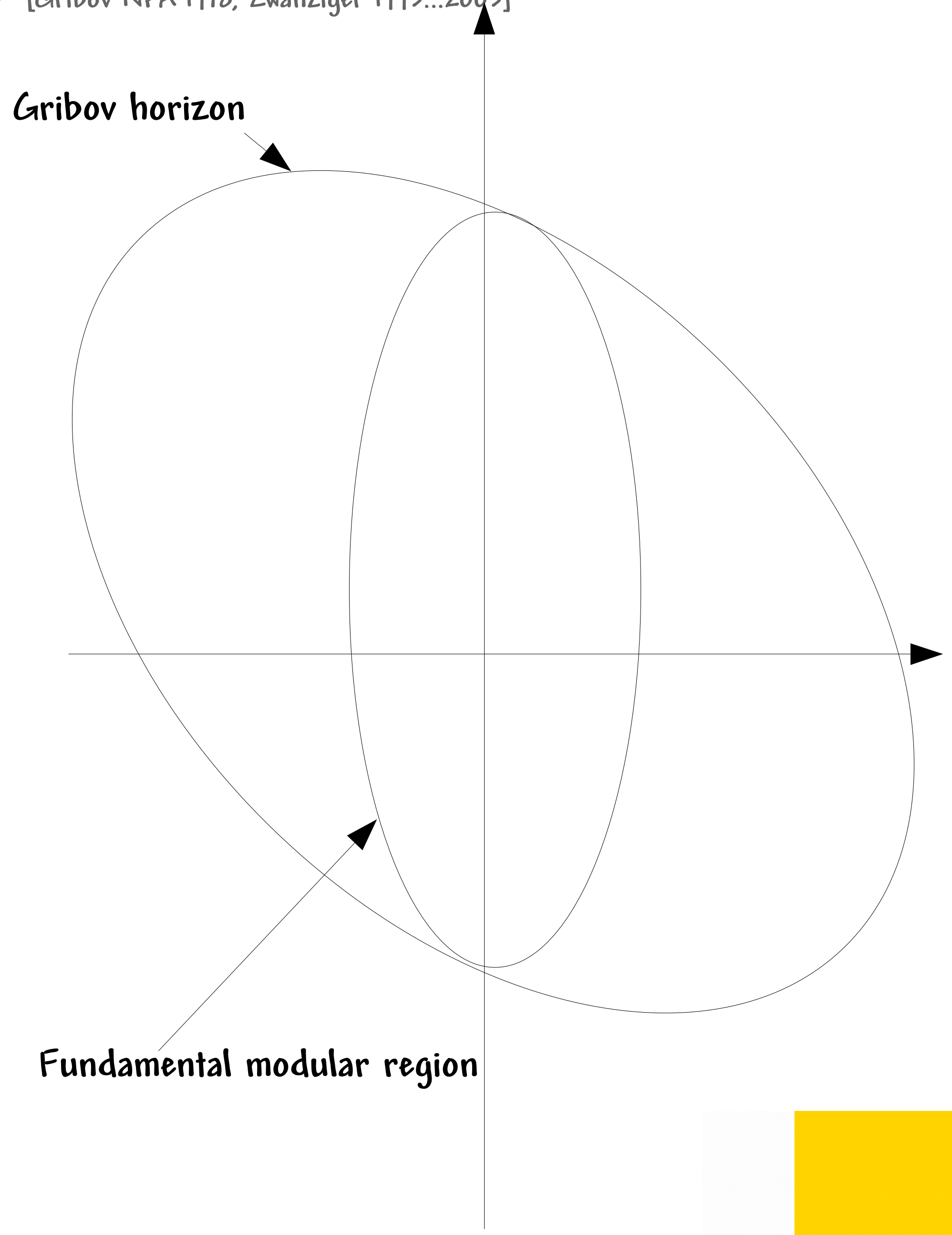

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- G(p) candidate for a characterization of a copy

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  - Similar to define perturbatively the Landau gauge by $p_\mu p_\nu D^{\mu\nu}=0$
- Would provide an unambiguous definition of the gauge
  - Resolves the Gribov ambiguity

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- Provides the basis to calculate non-perturbatively correlation functions

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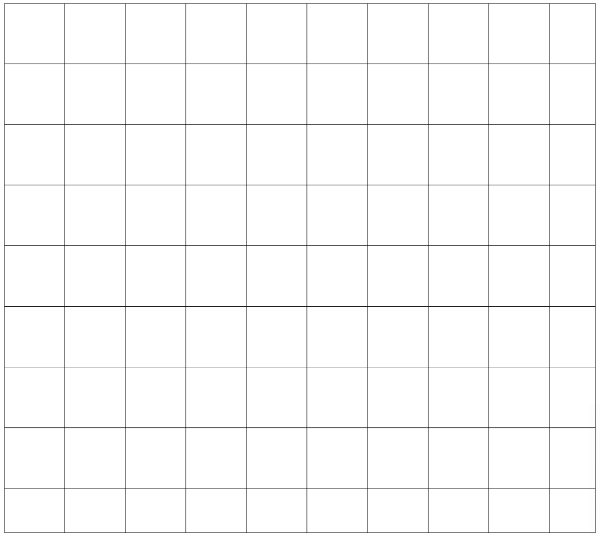

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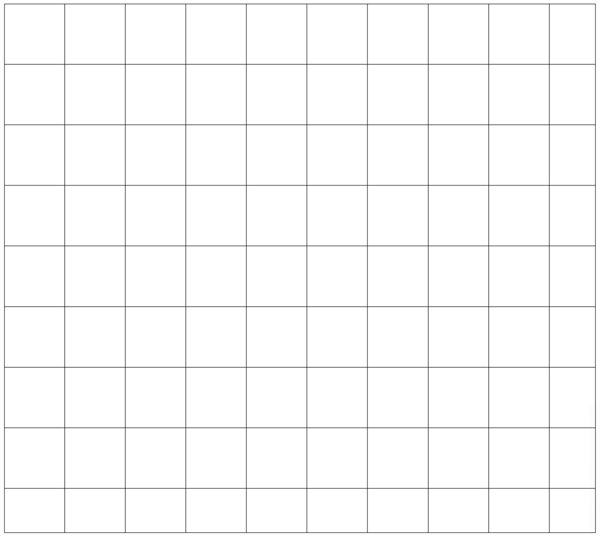

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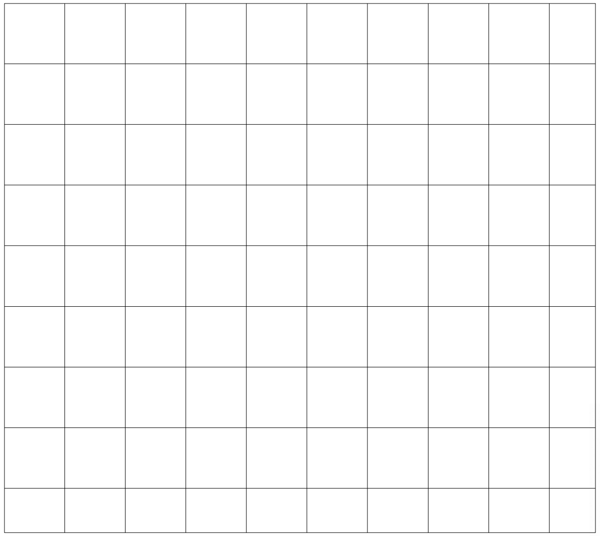

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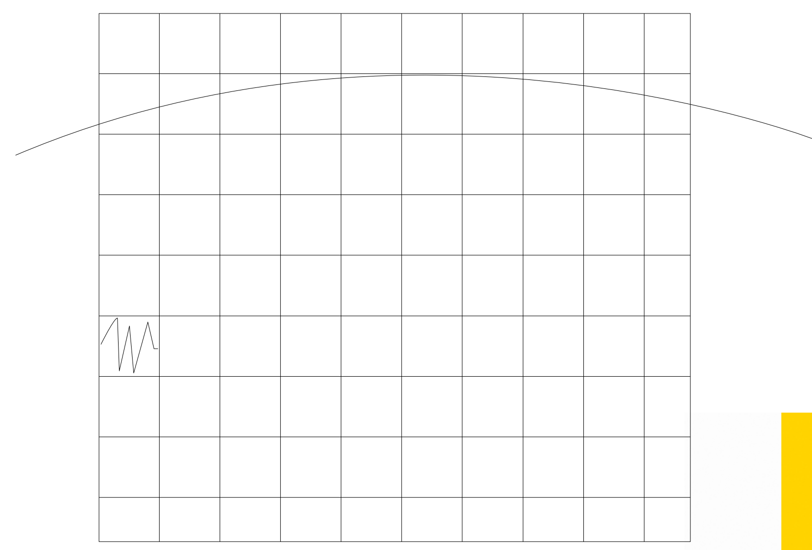

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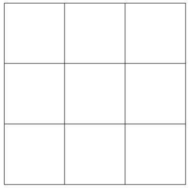

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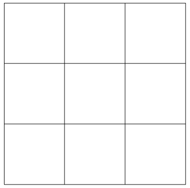

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- Functional methods (DSE, RGE...)

(Truncated) Dyson-Schwinger Equations (DSEs)

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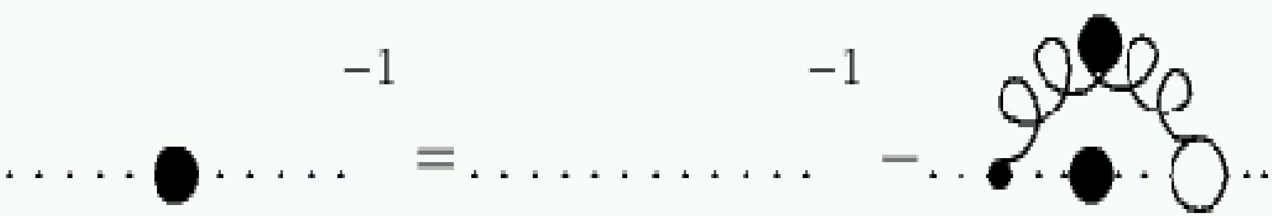

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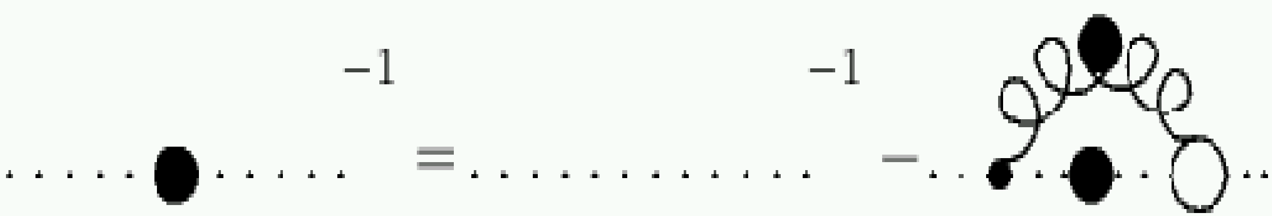

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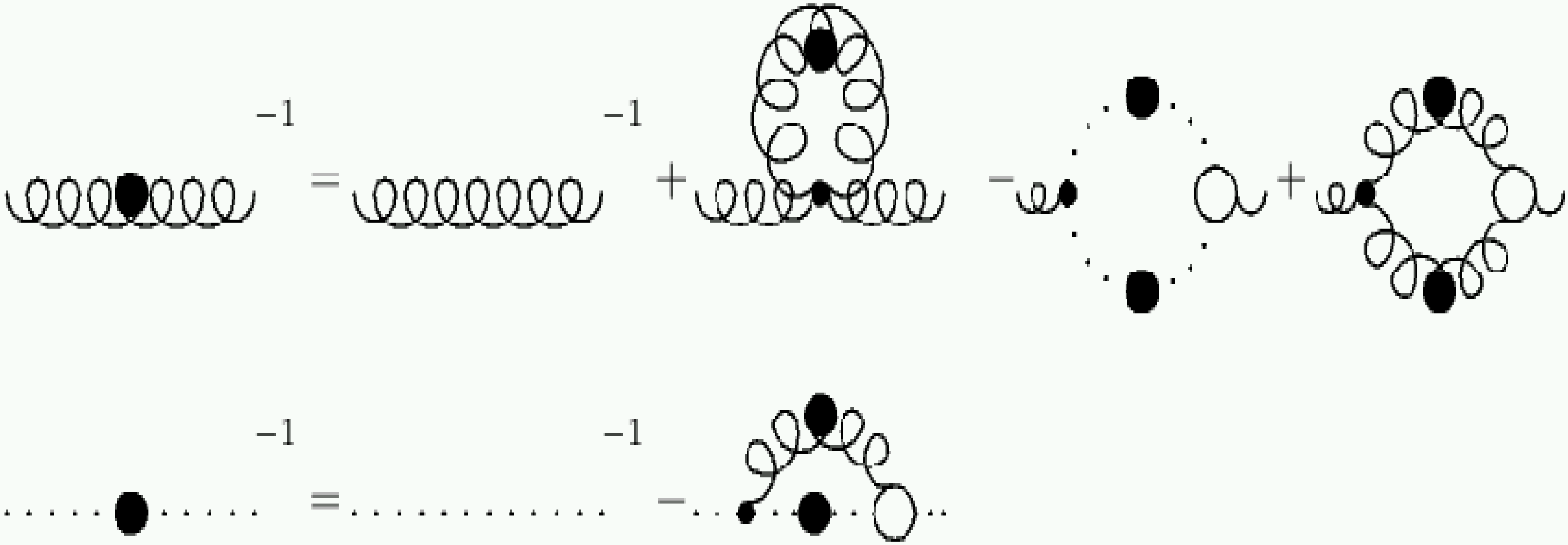

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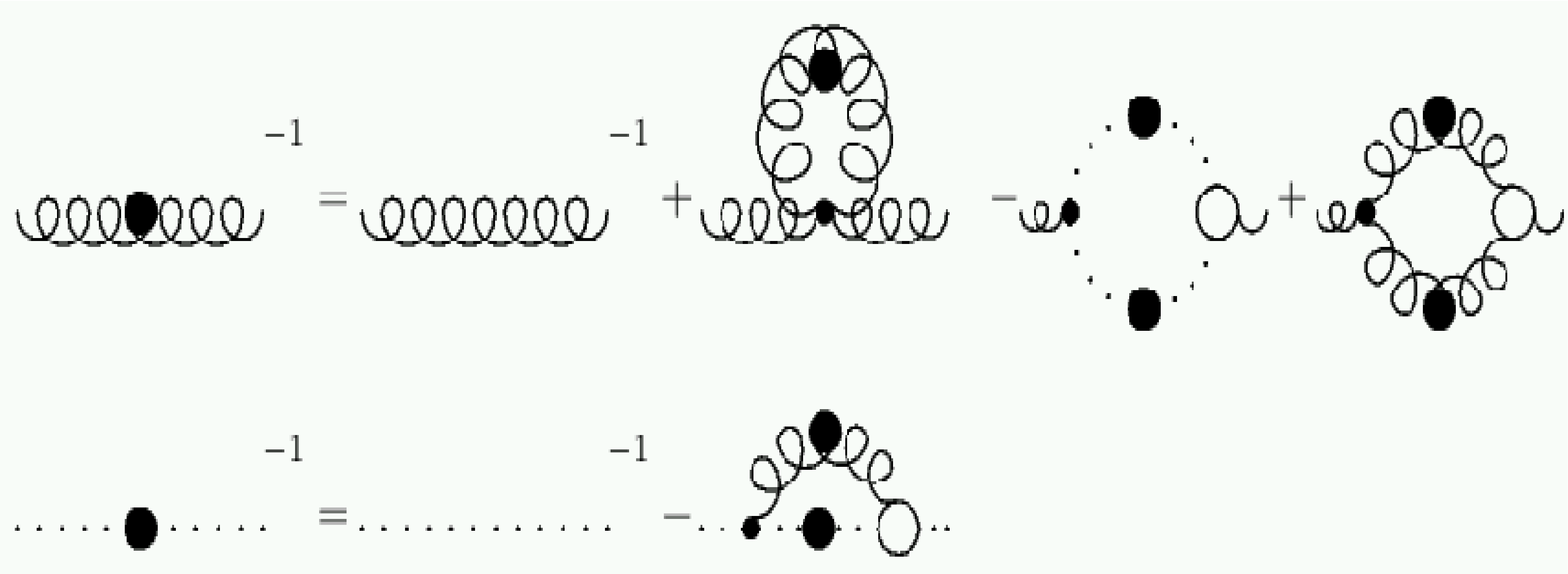

- Infinite set of coupled non-linear integral equations
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- Similar: Renormalization group equations (RGEs)

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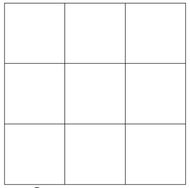
Summary of methods



- Lattice
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  - Finite volume artifacts, disparate scales most severe obstacles
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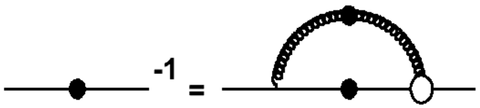

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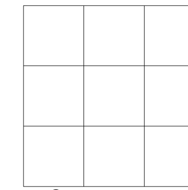

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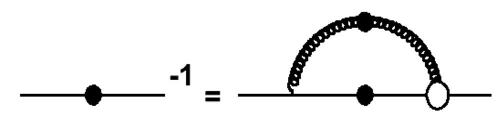

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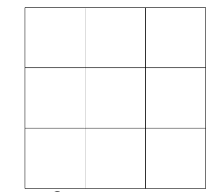

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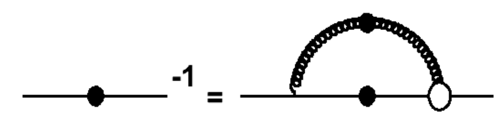


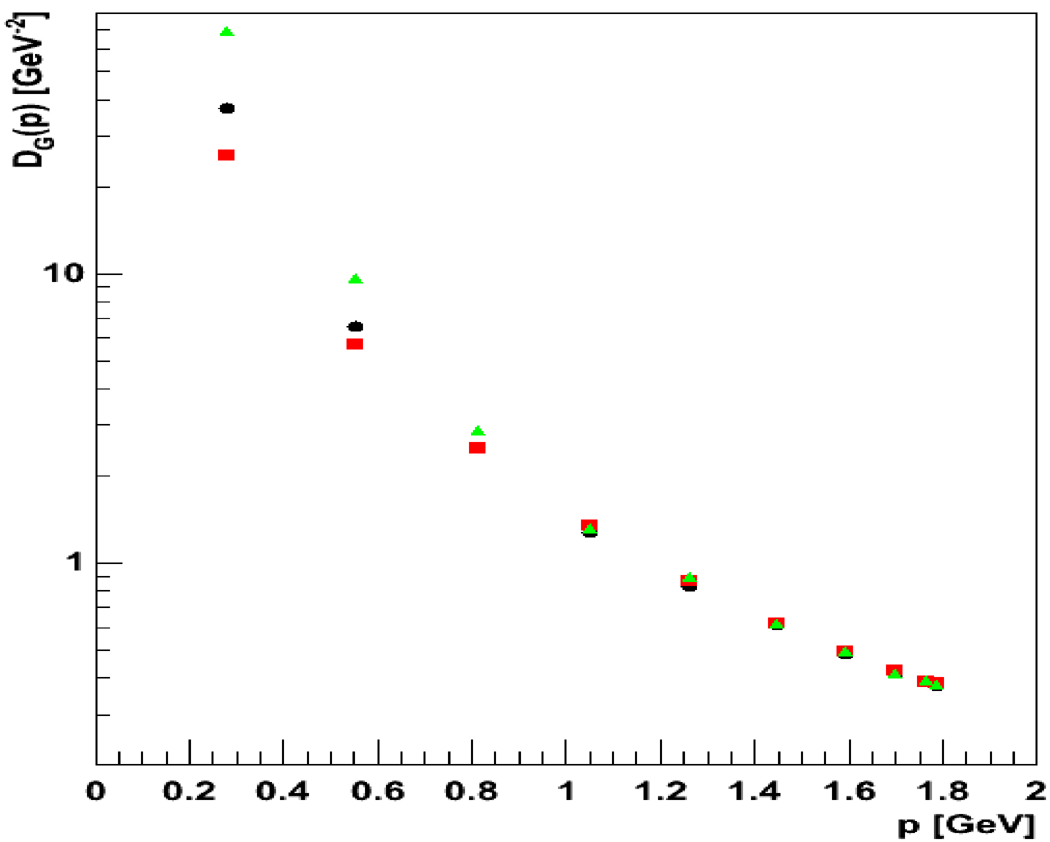
  - Coupled non-linear integral equations must be solved
  - Requires (often completely uncontrolled) approximations
  - Continuum, partly analytical in the far infrared
- Perturbation theory included as limiting case at high energies
- Combination of all methods most successful!

Properties of gluons

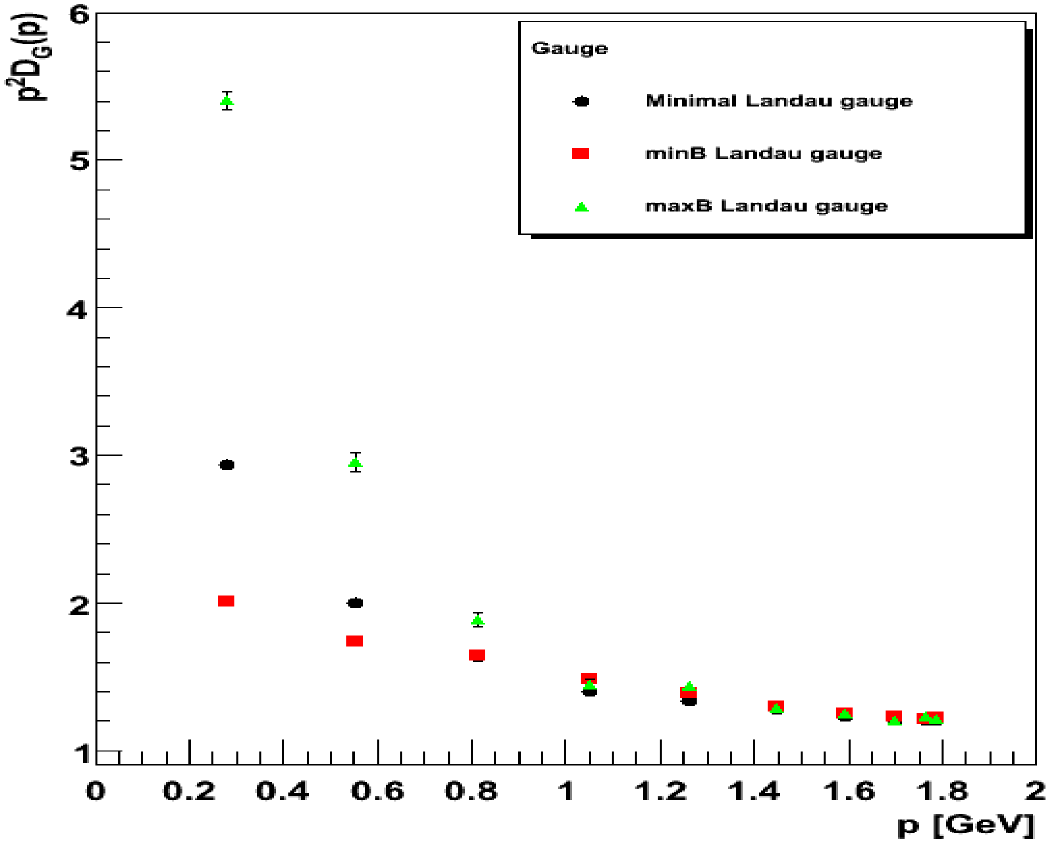
Nb: The results depend little on the number of dimensions
but the lattice can reach larger volumes in lower dimensions
Results shown are therefore mixed from
3 and 4 dimensions, but are qualitatively very similar in both

Ghost propagator from the lattice [Maas, 2009]

Ghost propagator

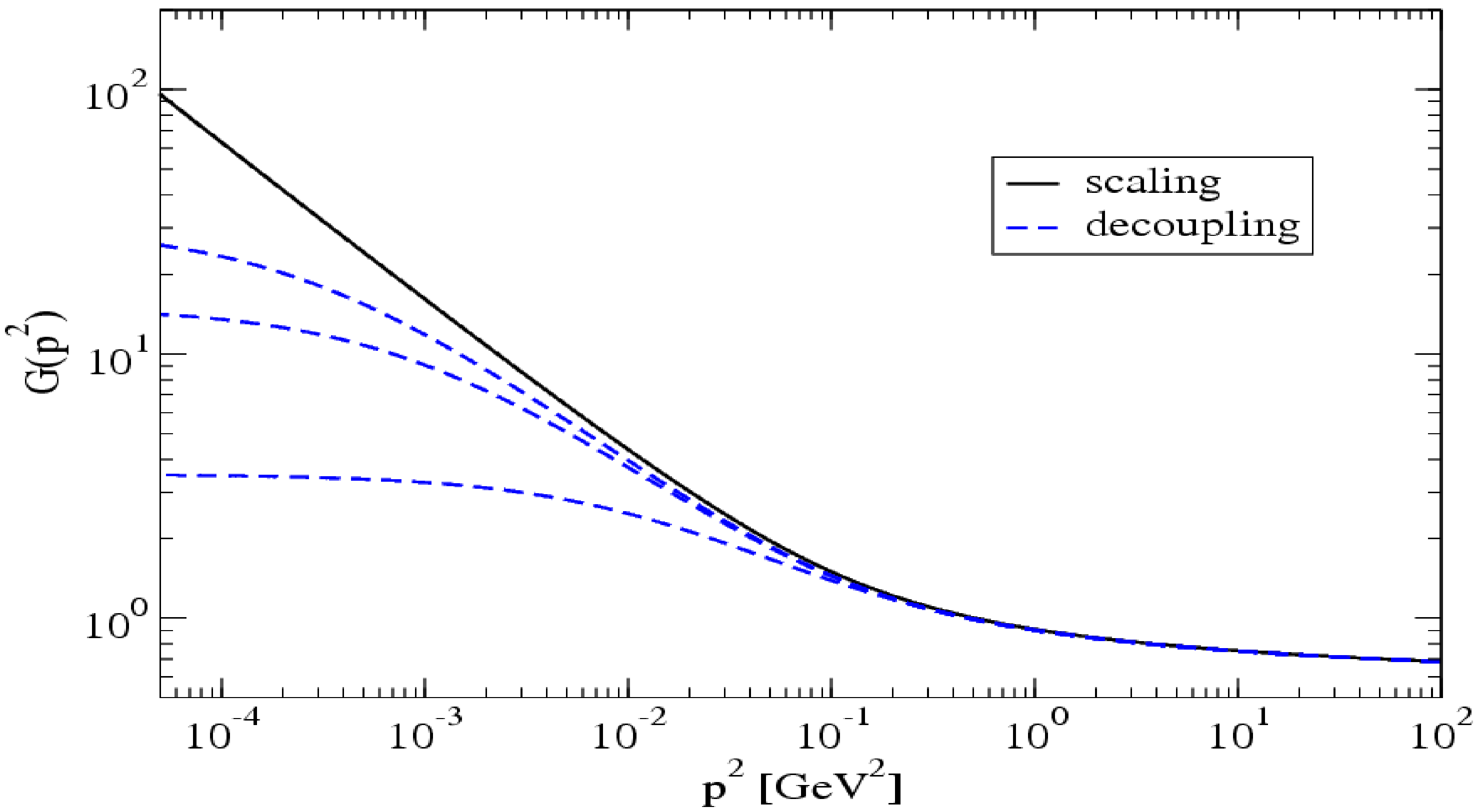


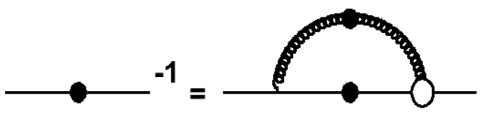
Ghost dressing function



- Results from lattice calculations
- Different gauge choices yield different propagators
- Lattice artifacts still to be studied

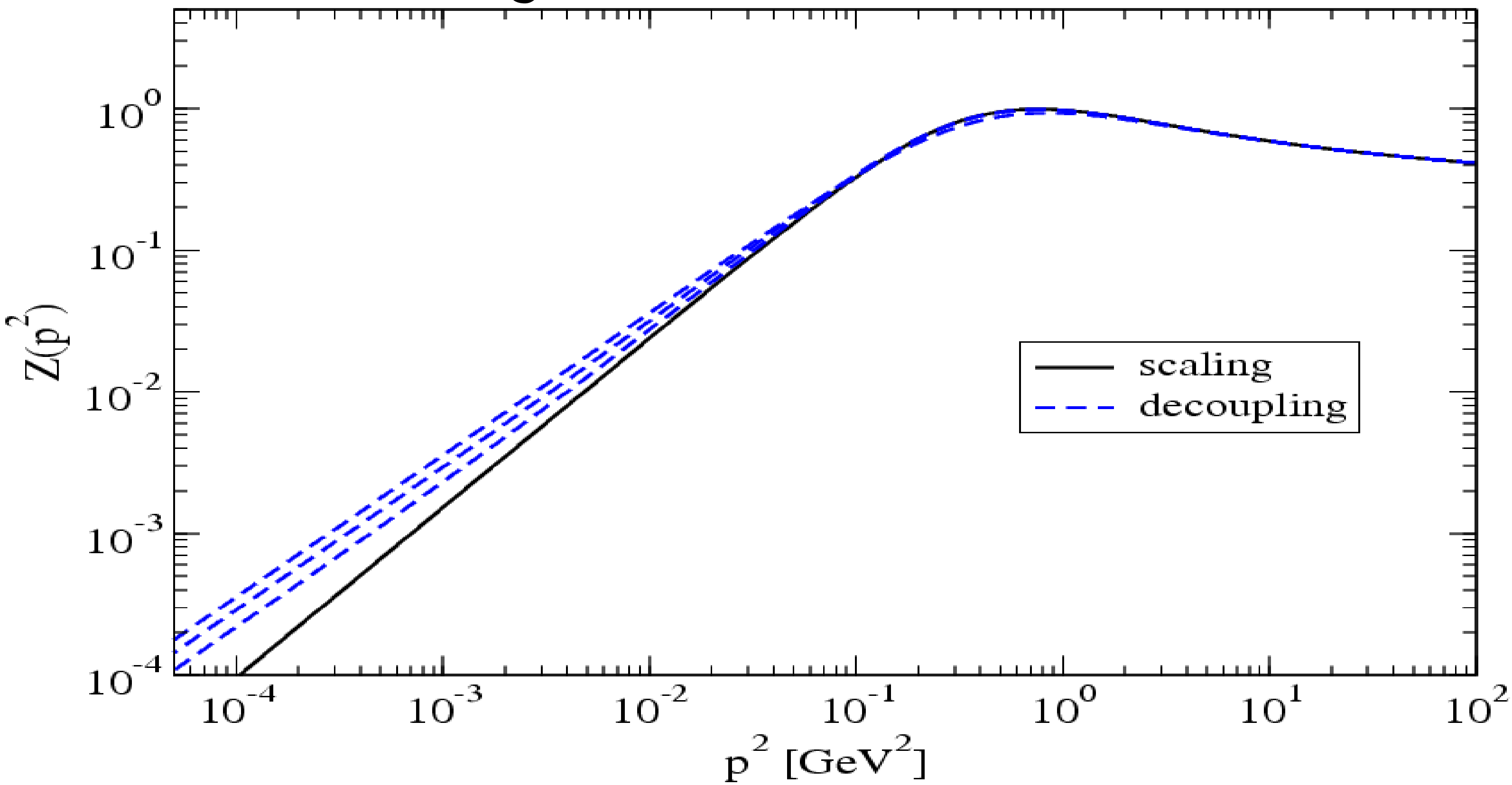
Ghost dressing function in the continuum [Fischer, et al.i, 2008]

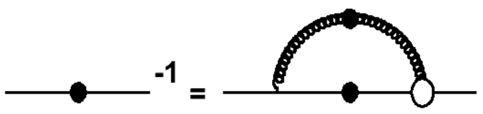




- Corresponding results from functional methods (Dyson-Schwinger equations (DSEs))
- One-to-one-correspondence of lattice and continuum methods
- Scaling: Divergent, Decoupling: Finite dressing function

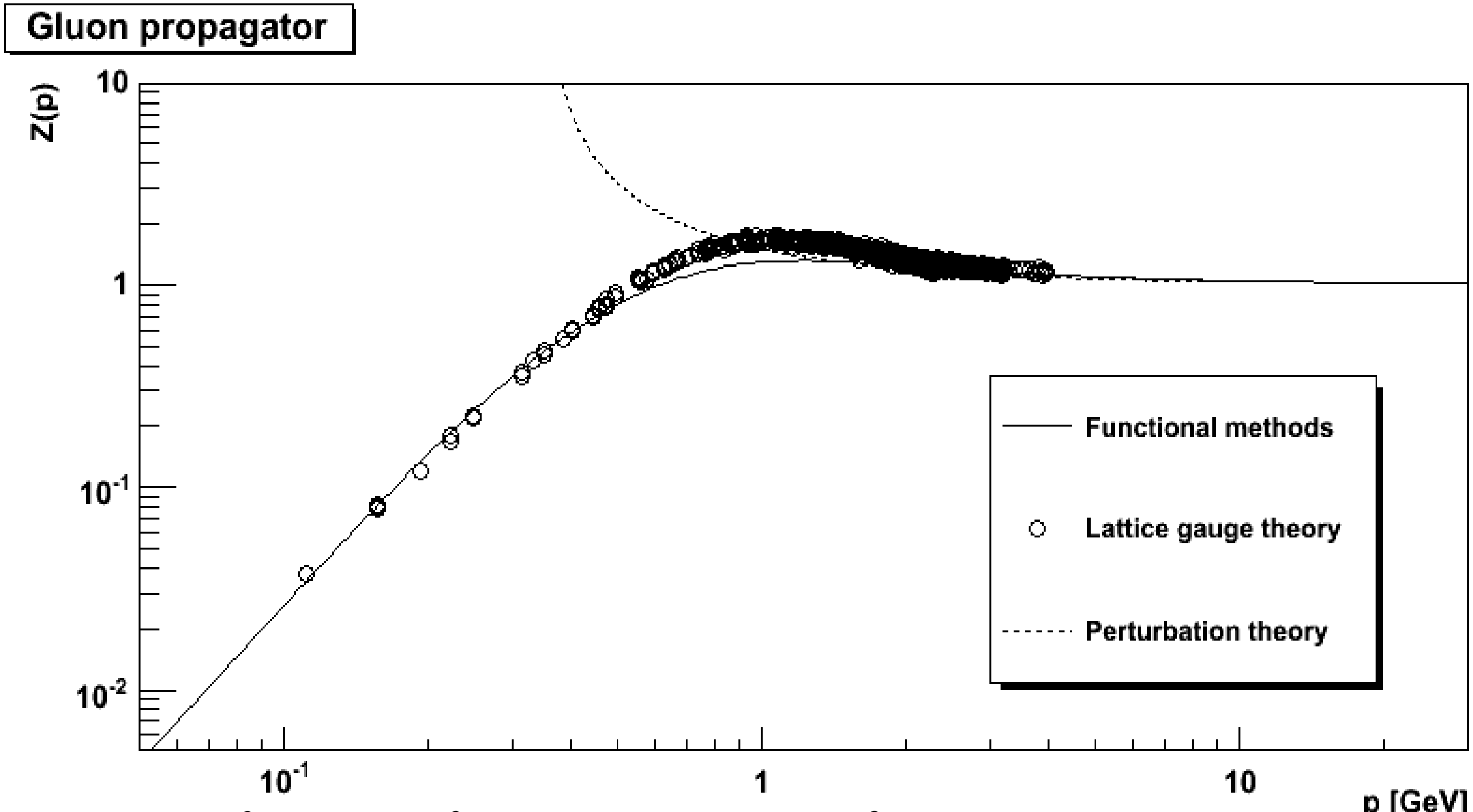
Gluon dressing function in the continuum [Fischer et al., 2008]

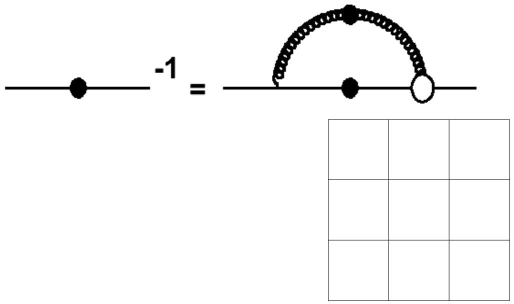




- Decoupling gauges yield a decoupling (infrared massive) gluon propagator
- The scaling gauge yields an infrared vanishing gluon propagator

Method comparison [Maas et al. 2004, Maas 2008]





- Perturbation theory recovered
- Combination confirm assumption for functional equations
- Extrapolation of lattice results by functional methods
- Disadvantage cancellation

Absence of the gluon from the physical spectrum [Zwanziger, 1994-2009]

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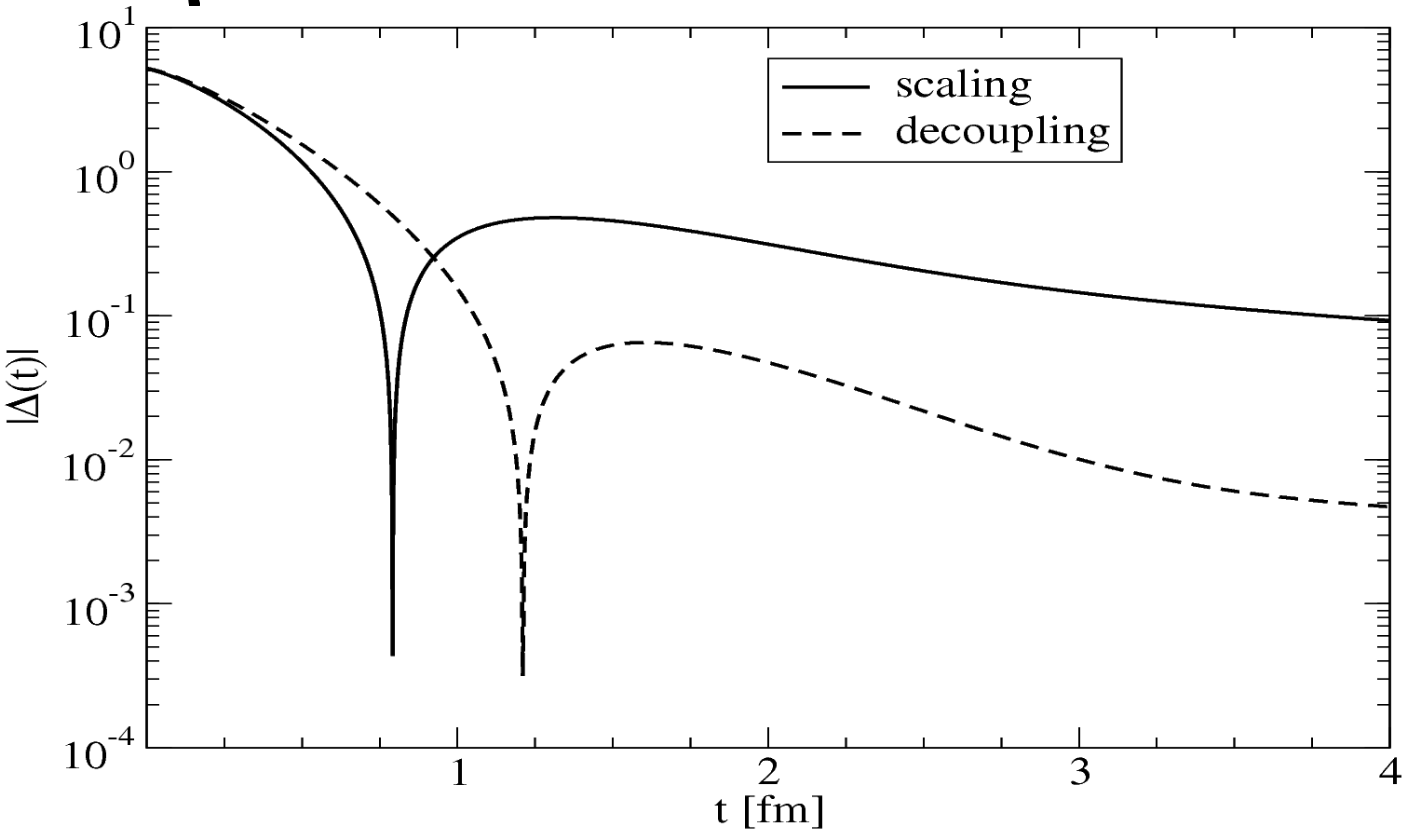
  - (Possibly) to all colored states using Kugo-Ojima construction
    - Uses the Neuberger-von Smekal BRST construction

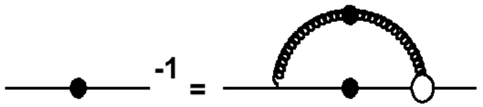
      [Neuberger 1986, von Smekal 2006-2009, Pawlowski et al., 2008]

Analytic structure

- Vanishing D(0): Gluons do not propagate on the light-cone

Analytic structure [Alkofer et al. 2003, Fischer et al. 2008]





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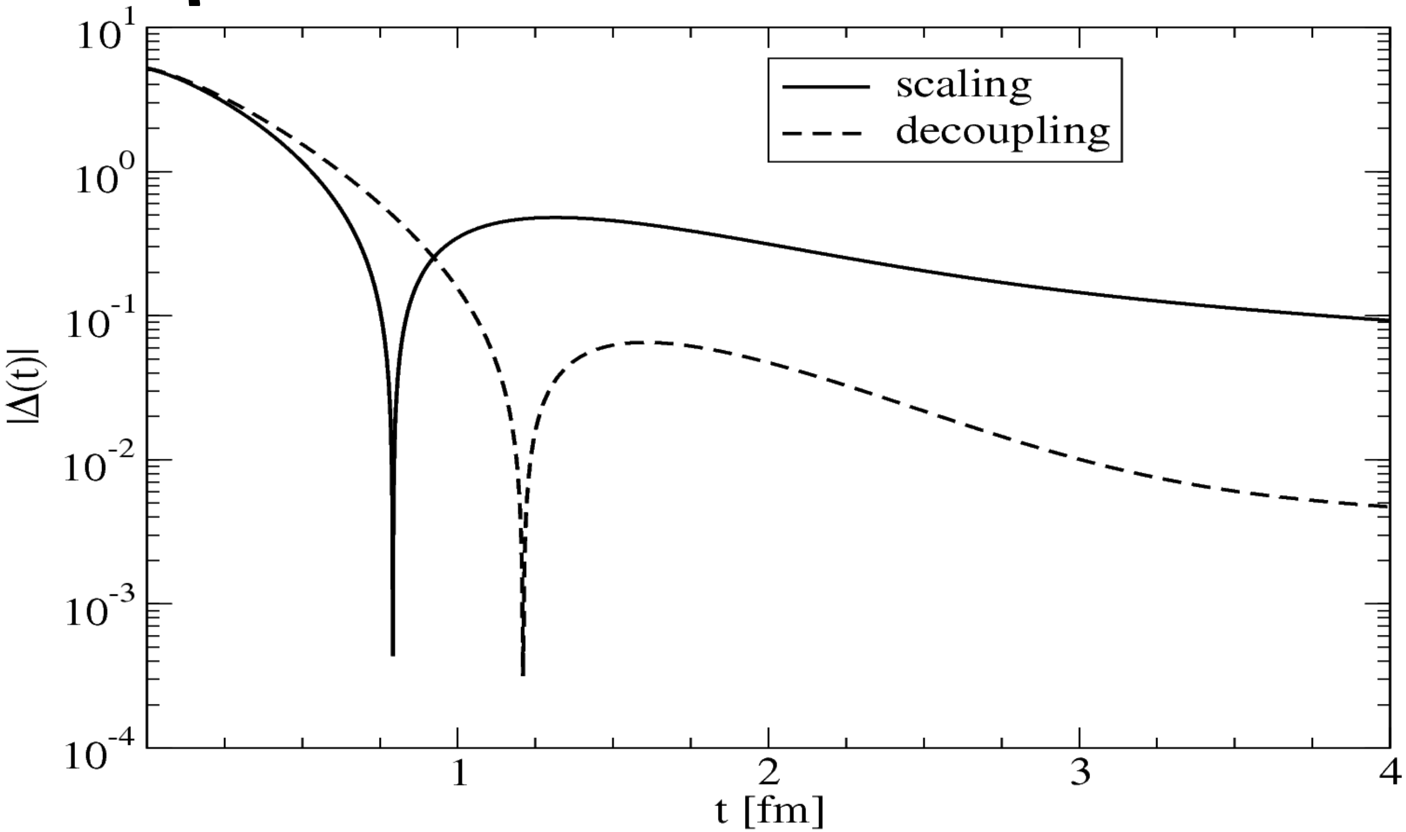

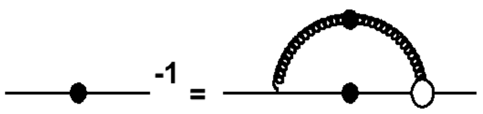

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- Analytic structure: Cut along the time-like axis from 0

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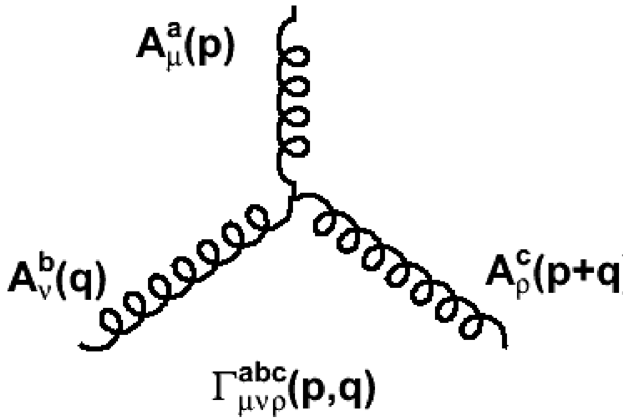
  - Other: Ghost-gluon vertex, 4-gluon vertex, scattering kernels...



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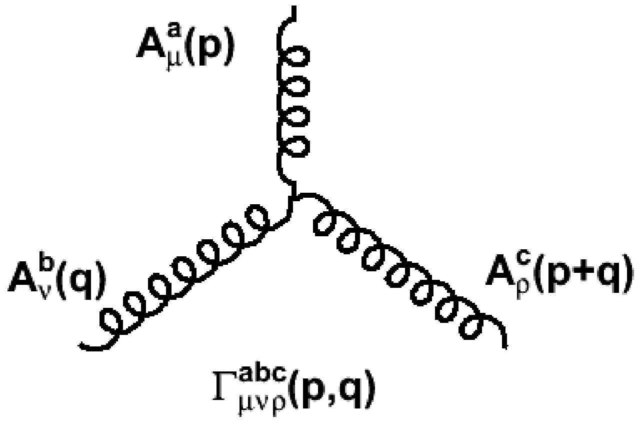

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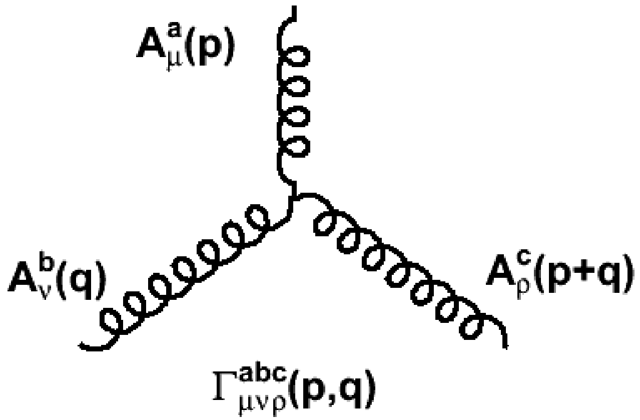

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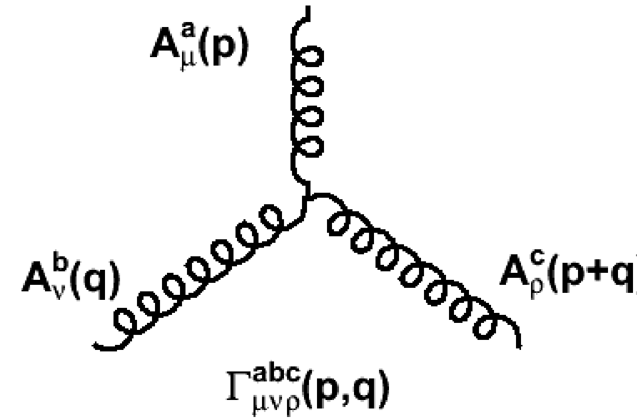

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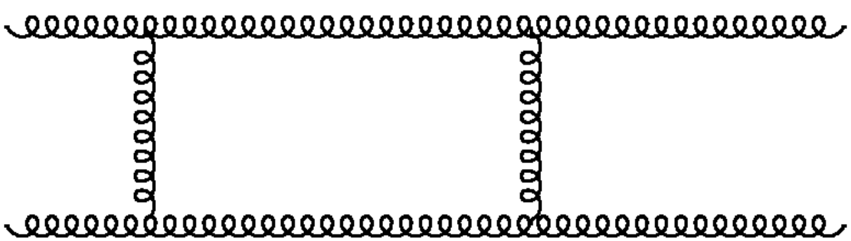

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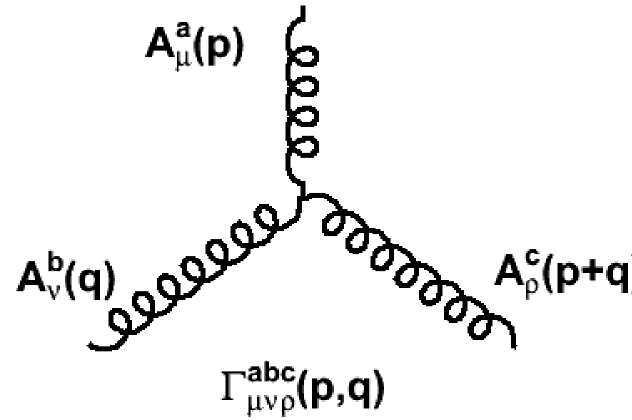

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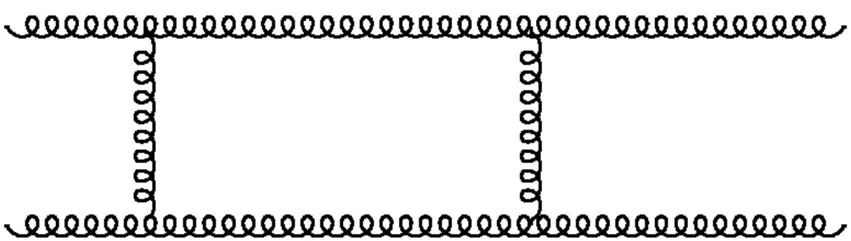
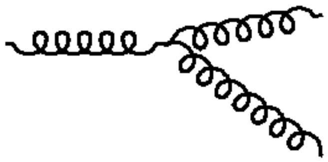


  - Splitting functions



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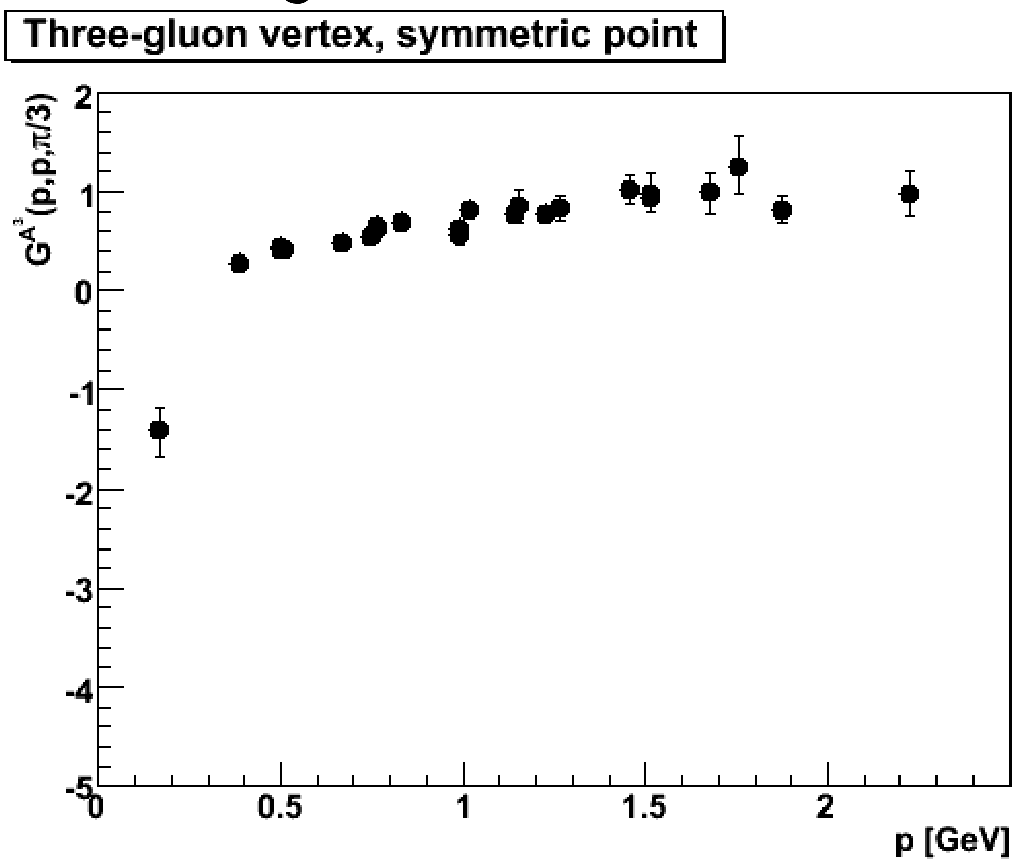
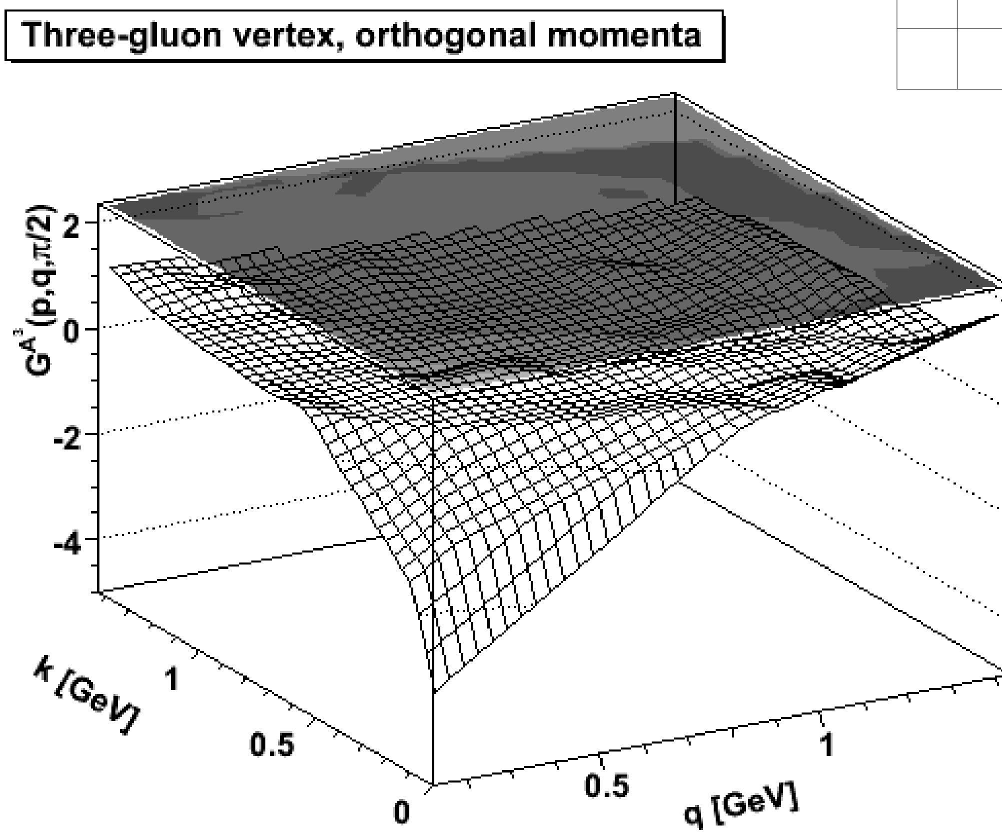
- Measures the deviation from the tree-level vertex
- Appears in the gluon one-loop self-energy

Three-gluon vertex [Cucchieri et al. 2008]



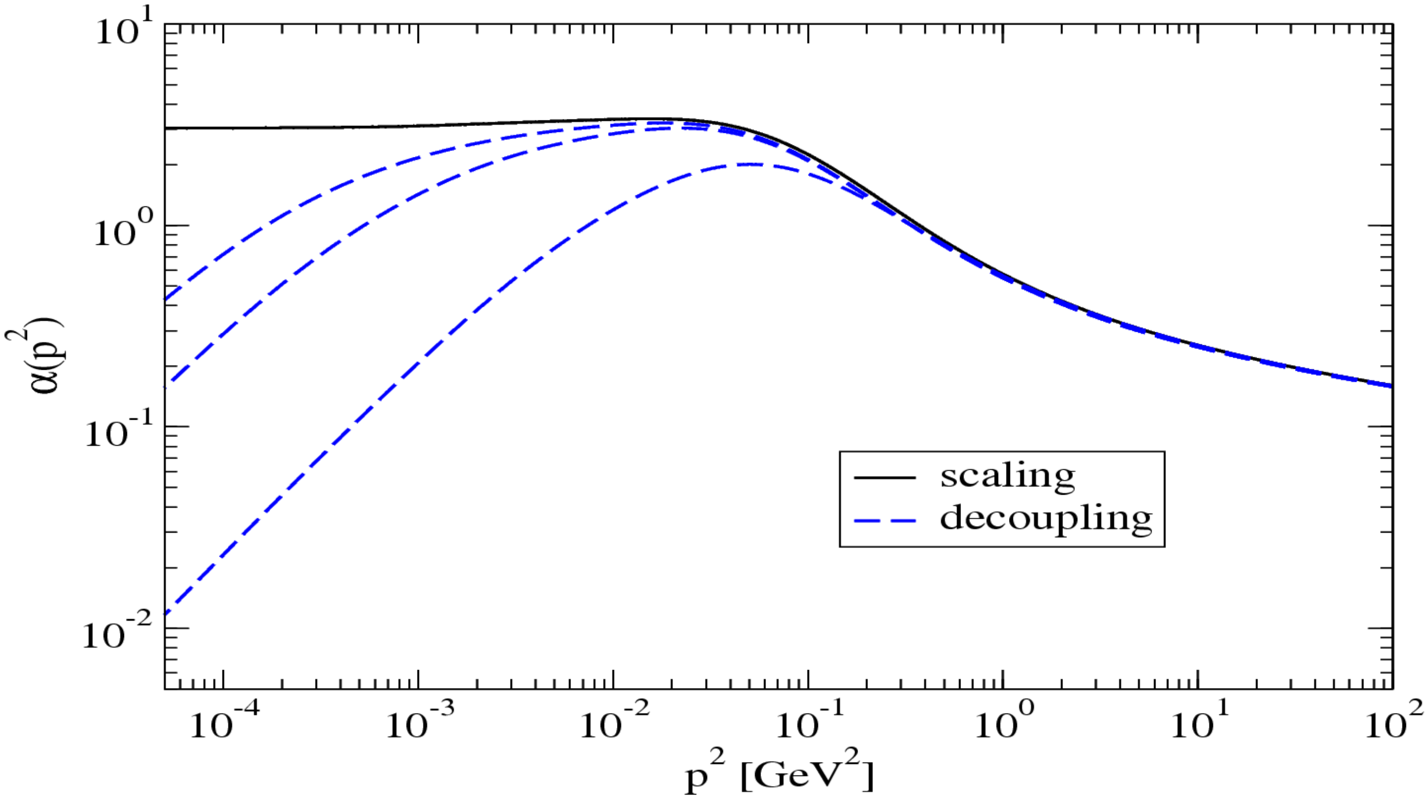


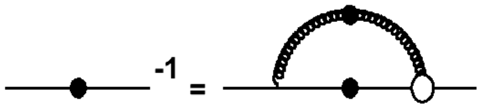
- No emission around hadronic energy scales!
- Infrared enhanced: Strong emission of (non-propagating) gluons on the light-cone

Running coupling

- Possible to extract a running coupling

Running coupling [Alkofer et al. 1997, Fischer et al. 2008]





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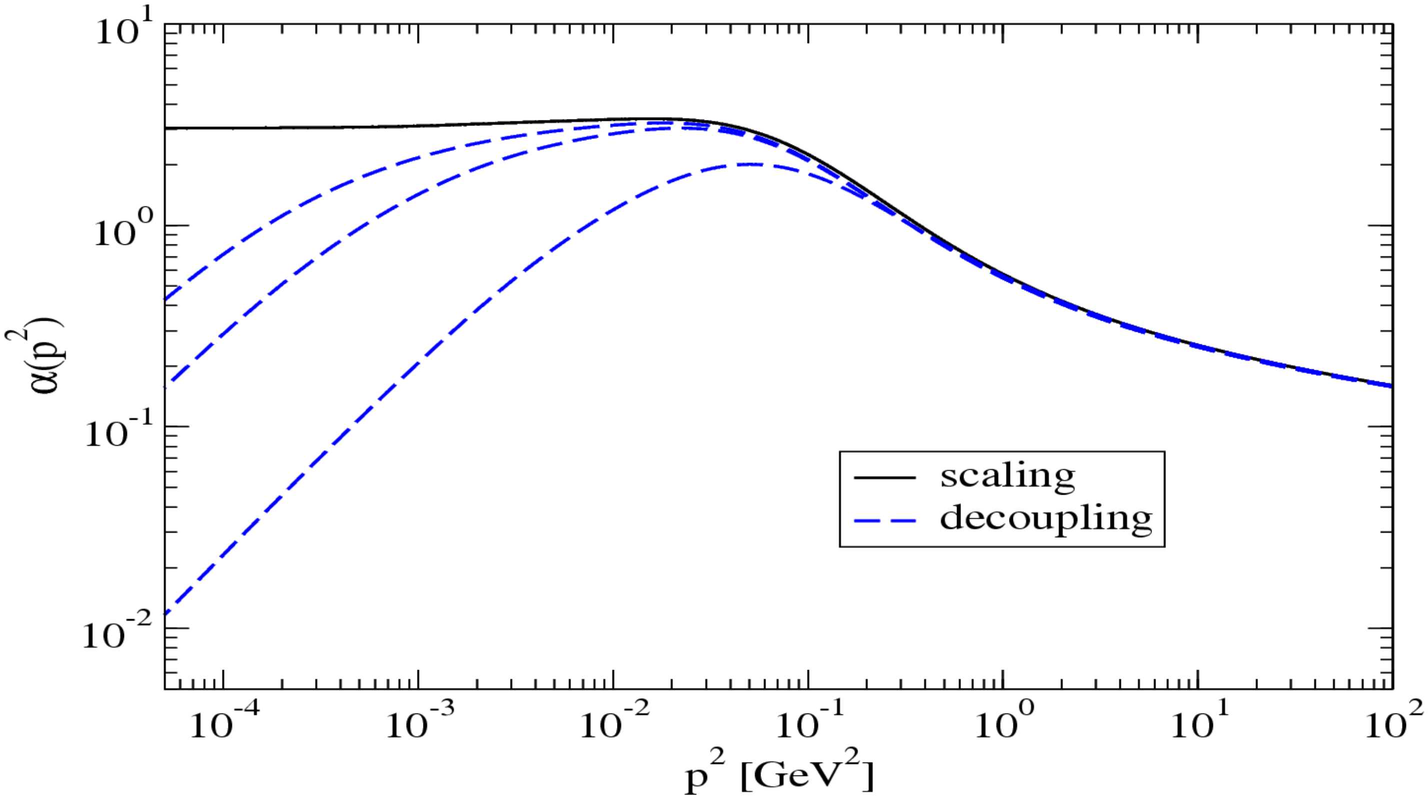

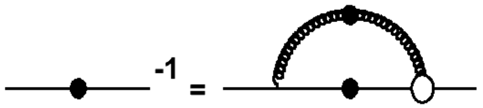

- Possible to extract a running coupling
  - IR fixed point for scaling gauge
  - IR vanishing for decoupling gauge
- Known in the perturbative domain up to four loops

[Sternbeck et al. 2008]

Infrared properties in the scaling gauge

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- In particular: IR-vanishing gluon propagator, IR-enhanced ghost propagator

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- DSEs and RGEs deliver a qualitative asymptotic infrared solution for all correlation functions and all dimensions [Alkofer et al., 2005, Fischer et al. 2007/9]
  - Power-laws. Simplest case: All momenta equal and m gluon and n ghost legs in d dimensions:
    - $p^{\left(\frac{n_{ghost}}{2}-m_{gluon}\right)\kappa+\left(1-\frac{n_{ghost}}{2}\right)\left(\frac{d}{2}-2\right)}$ [Alkofer et al. 2006, Huber at al. 2008]
  - Analytic solution without truncation
  - It is the only power-law-like solution [Fischer et al. 2007/9]
- Truncated DSEs give quantitative predictions for the exponent κ [Zwanziger, 2002, Lerche et al. 2002]
- In particular: IR-vanishing gluon propagator, IR-enhanced ghost propagator
- Can be expanded to the case with matter fields [Alkofer et al., 2007/8]

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- Gluon correlation functions depend on the gauge choice
- There is no pole mass for the gluon
  - Non-trivial analytic structure
- Gluon splitting and propagation together suppress gluon emission at low energies
- Confinement of gluons is manifest

Matter

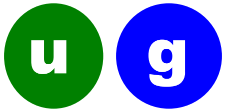
What is required?

- A unified framework covering all aspects
  - Must include perturbation theory for systematic control
- Construct it step-by-step

✔ Basic entities: Force particles (gluons,...)

✔ Yang-Mills theory as a prototype – very technical

➔ Simple matter particles – scalar



Matter fields

- Scalar matter

$$L=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\phi^{+}\left(\frac{1}{2}\;D_{\mu}D^{\mu}+m^{2}\right)\phi-\bar{c}\,\partial^{\mu}D_{\mu}c$$

$$D_{\mu}=\partial_{\mu}-ieA_{\mu}^{a}\tau_{R}^{a}$$

- R denotes the representation
  - Fundamental: Like quarks - Adjoint: Like gluons

Scalar matter

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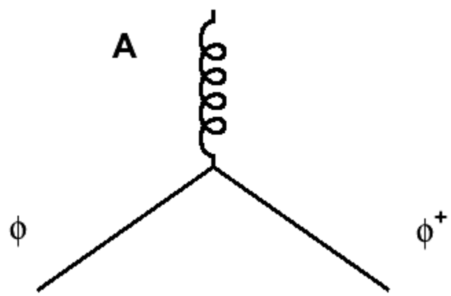
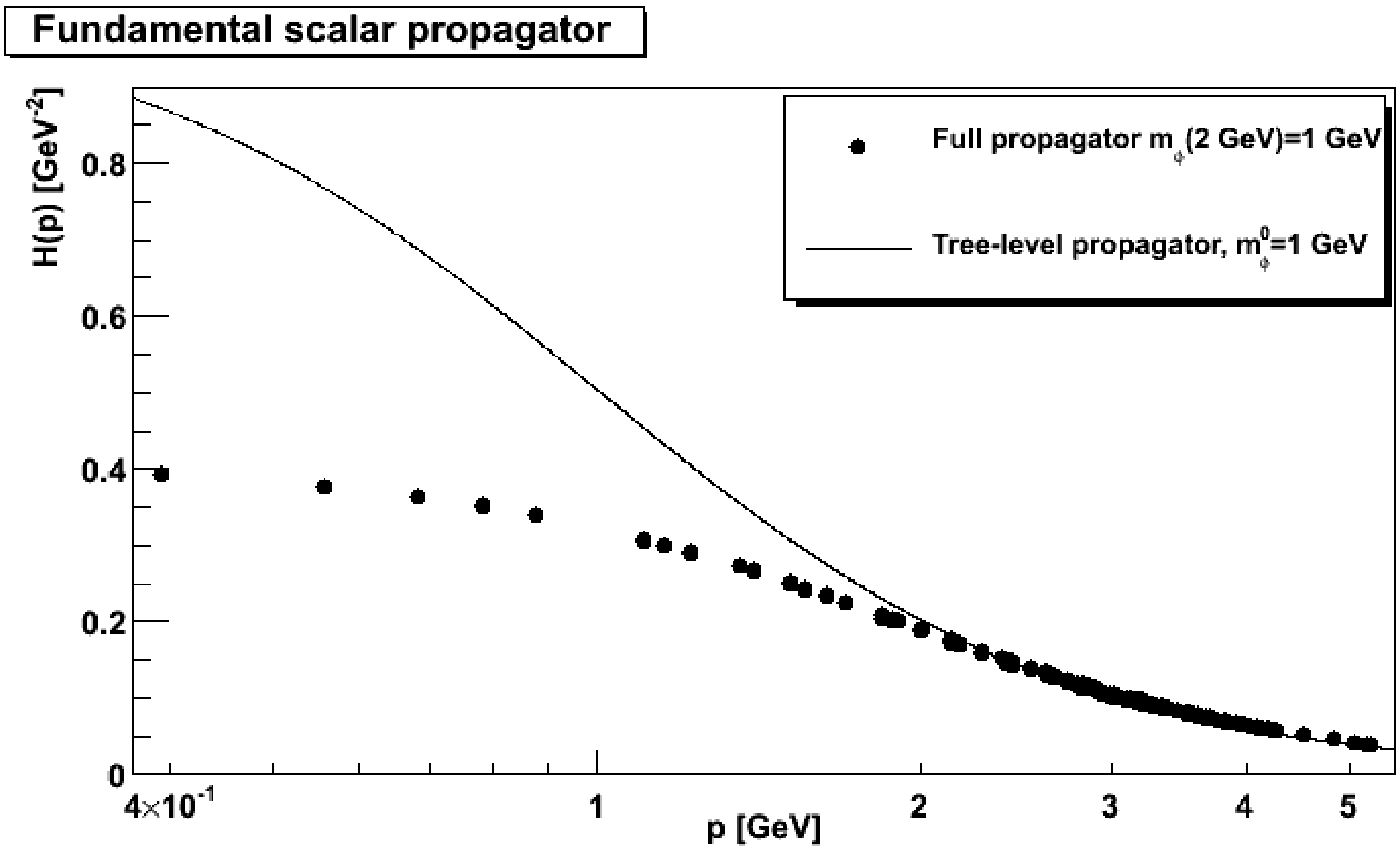
- (Simplest) Vertex



$$<A_\mu^a \phi_i^+ \phi_j>$$

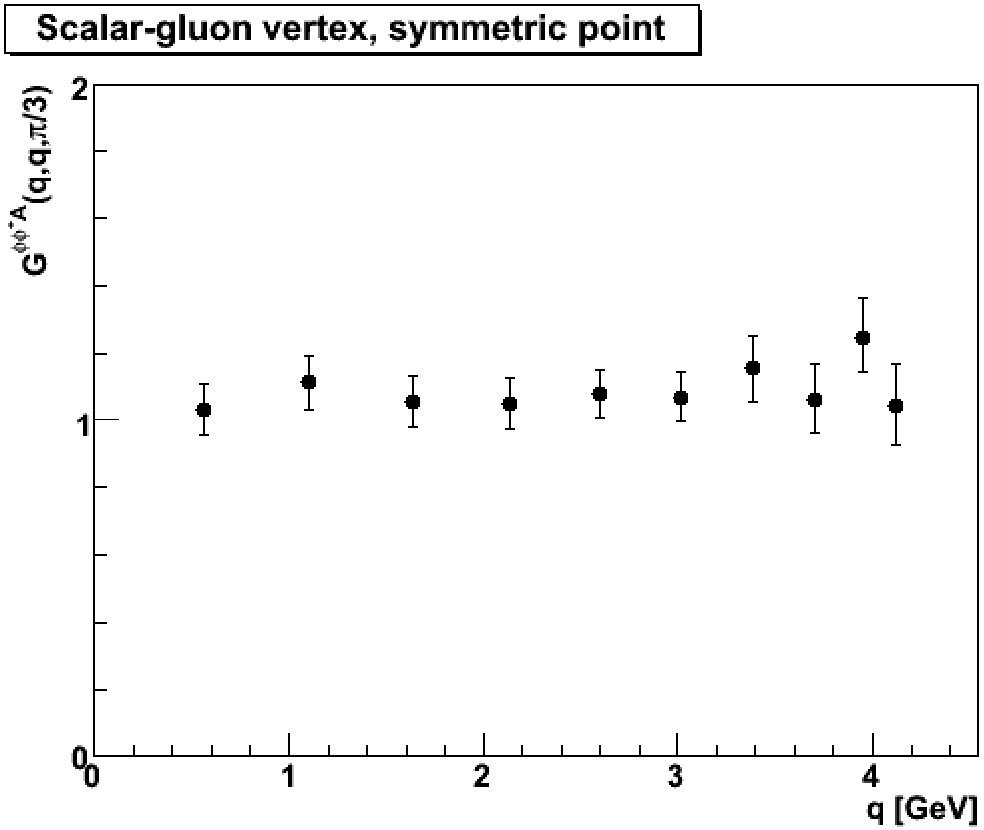
$$G^{A\phi^+\phi} = \Gamma^{tl} <A\phi^+\phi> / (\Gamma^{tl} DHH \Gamma^{tl})$$

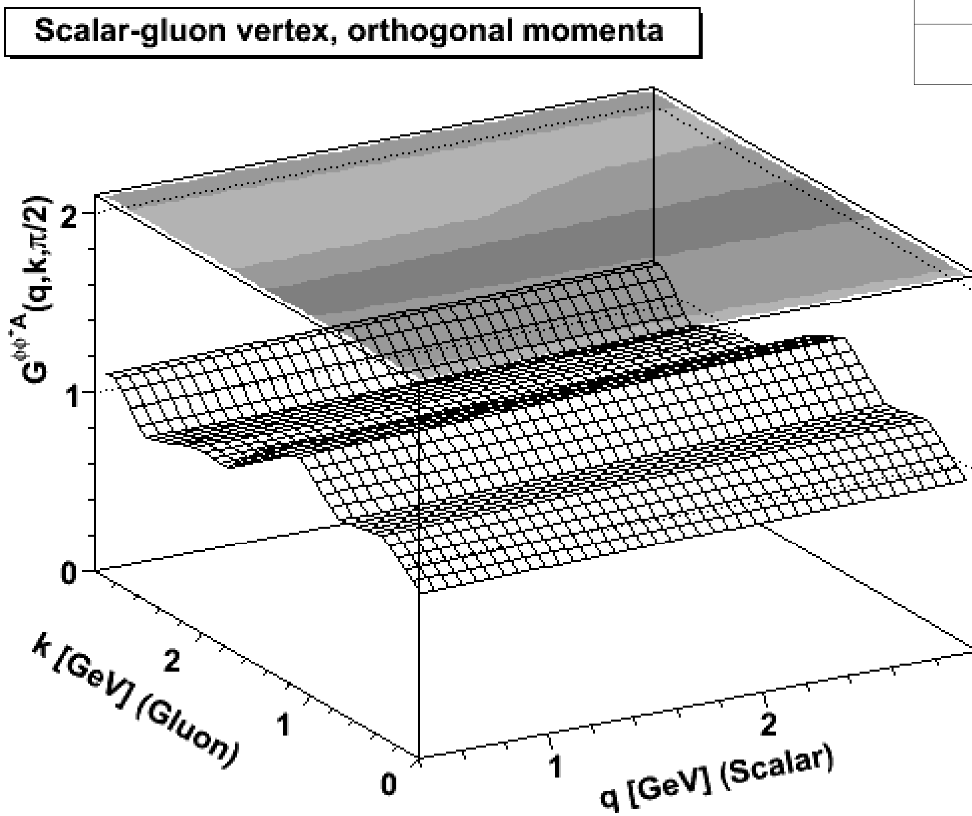
Scalar proagator [Maas, unpublished]



- 1 GeV tree-level mass – effective mass is 1.6 GeV
  - Dynamical mass generation, independent of tree-level mass
- Analytic structure requires more data

Scalar-gluon vertex [Maas, unpublished]





- Almost no difference to tree-level
- Low-momentum behavior mass-dependent
  - Suppression for small masses

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Outlook

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$$L=-\frac{1}{4}F_{\mu\nu}^{a}F^{\mu\nu,a}+\overline{\psi}(iD_{\mu}\gamma^{\mu}-m)\psi-\bar{c}^{a}\partial^{\mu}D_{\mu}^{ab}c^{b}$$ ¿

Fermionic matter

- Quark propagator

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  - M mass function
  - A^{-1} wave function renormalization

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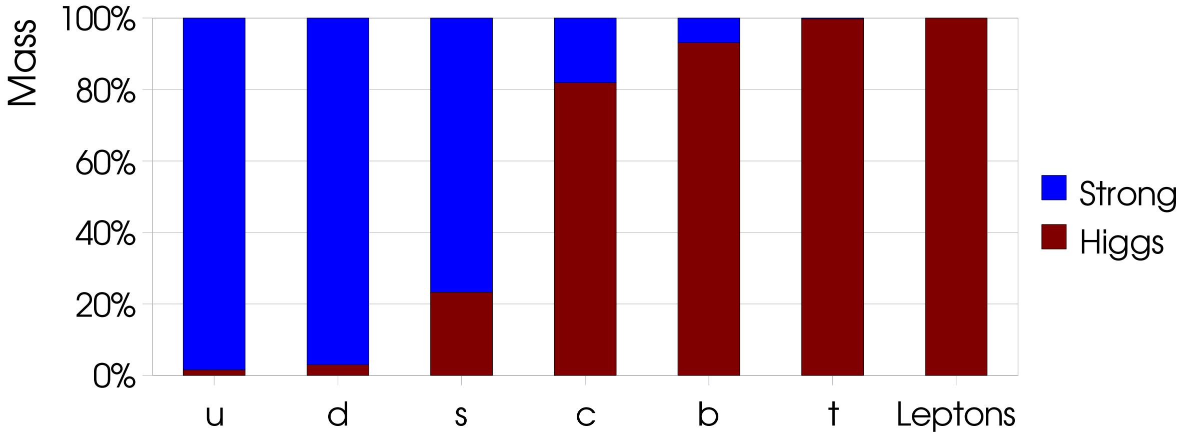

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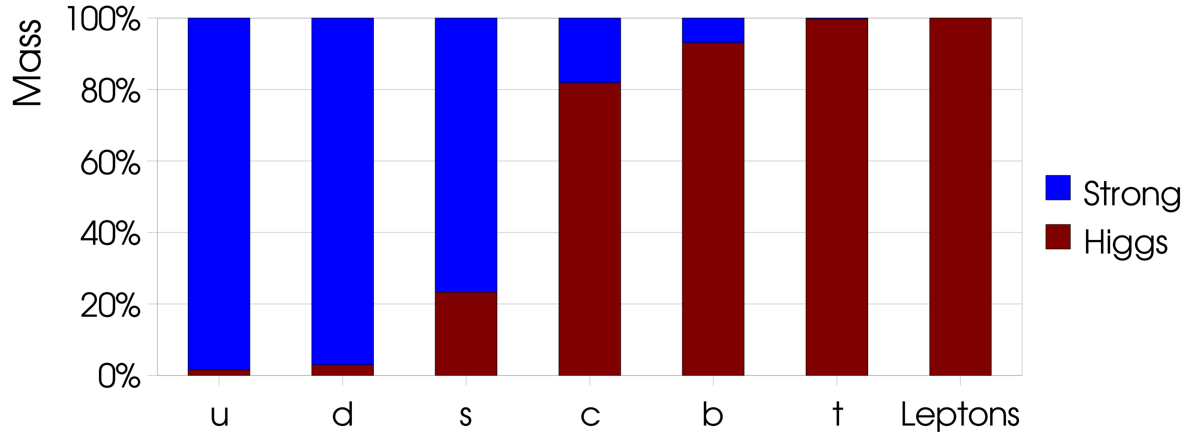
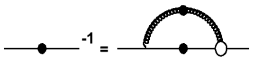


    - Measure: trS

Quarks

[DSE: Fischer et al., 2003
Lattice: Overlap: Bonnet et al 2003, Asqtad: Bowman et al., 2005]



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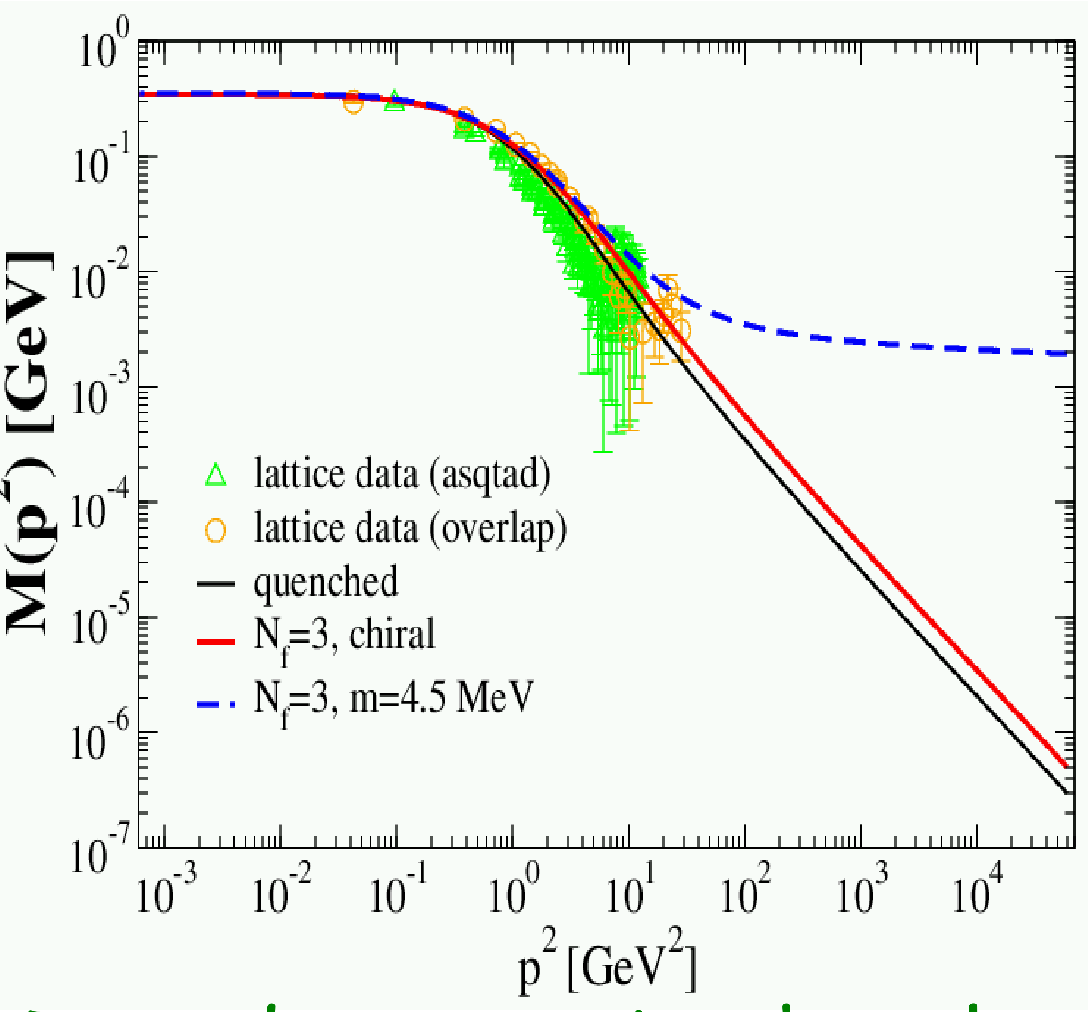

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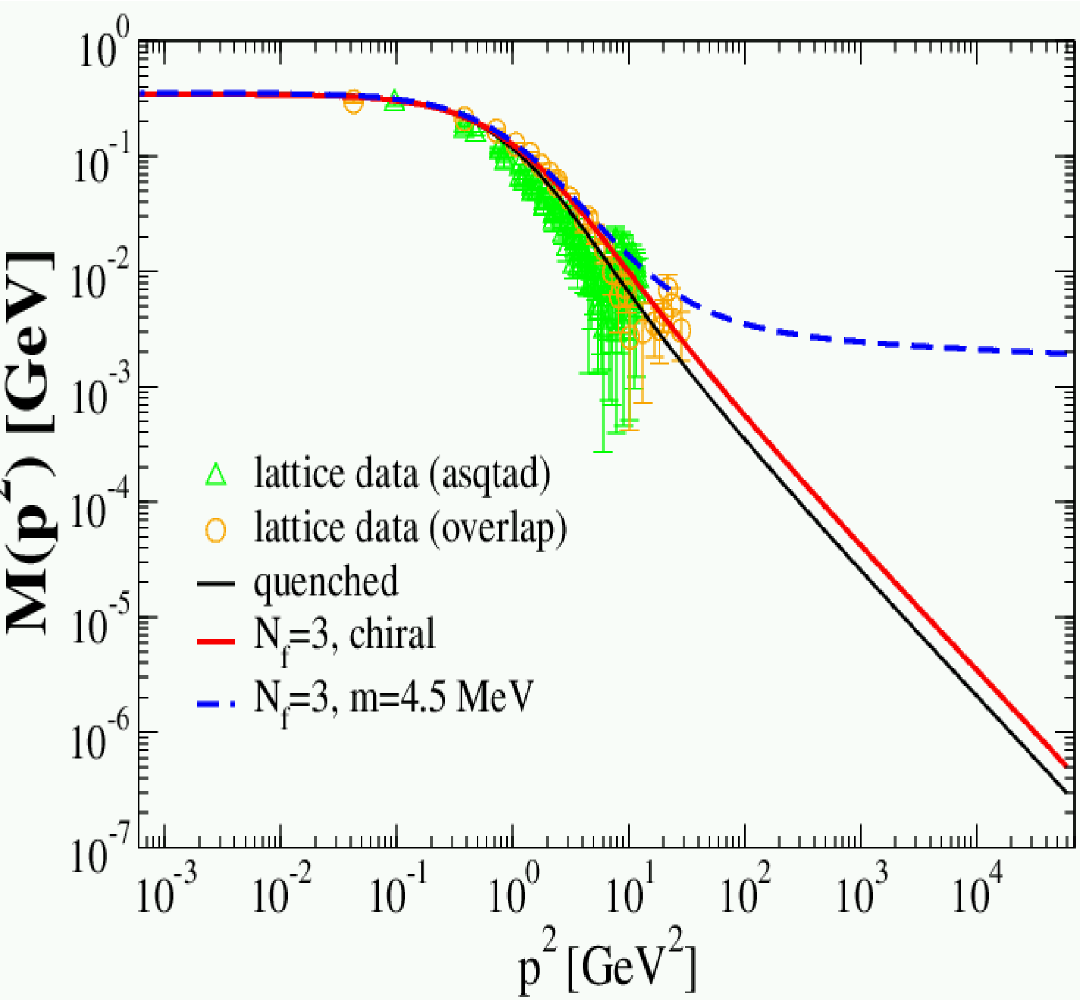
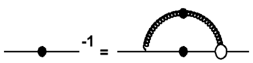
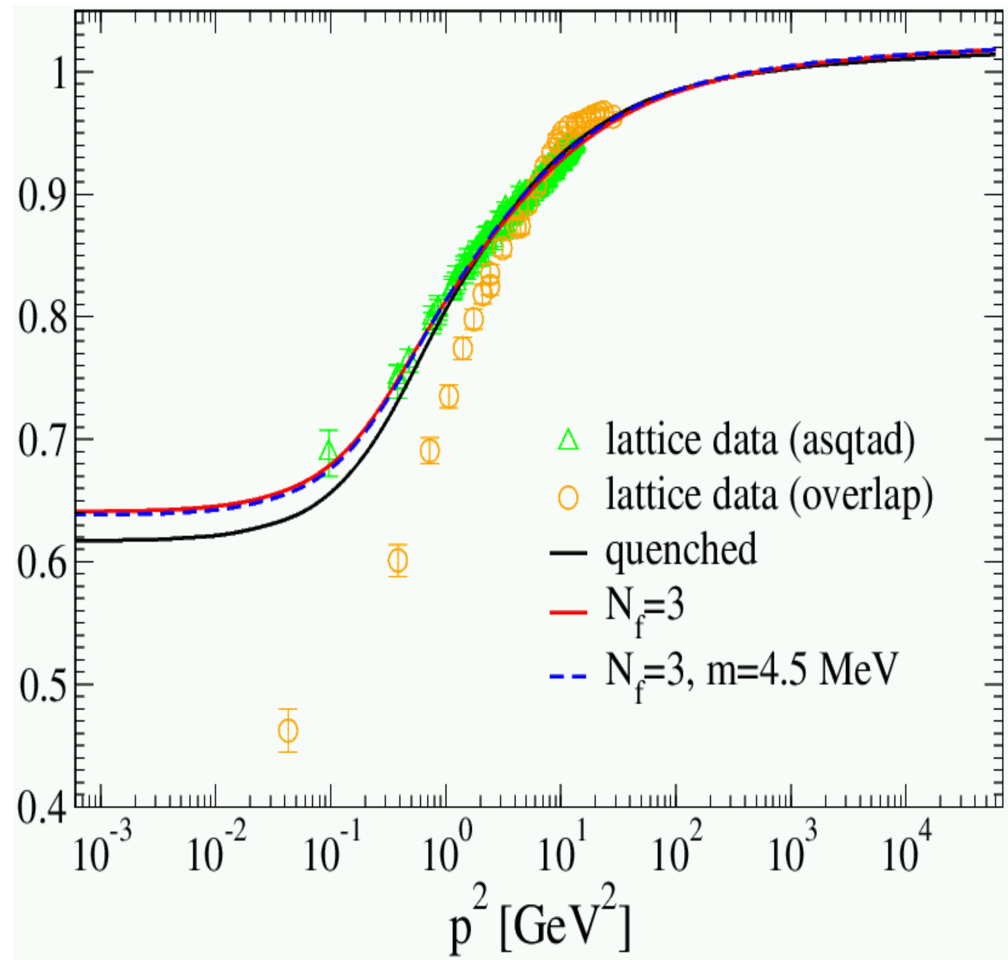


Quark wave function renormalization





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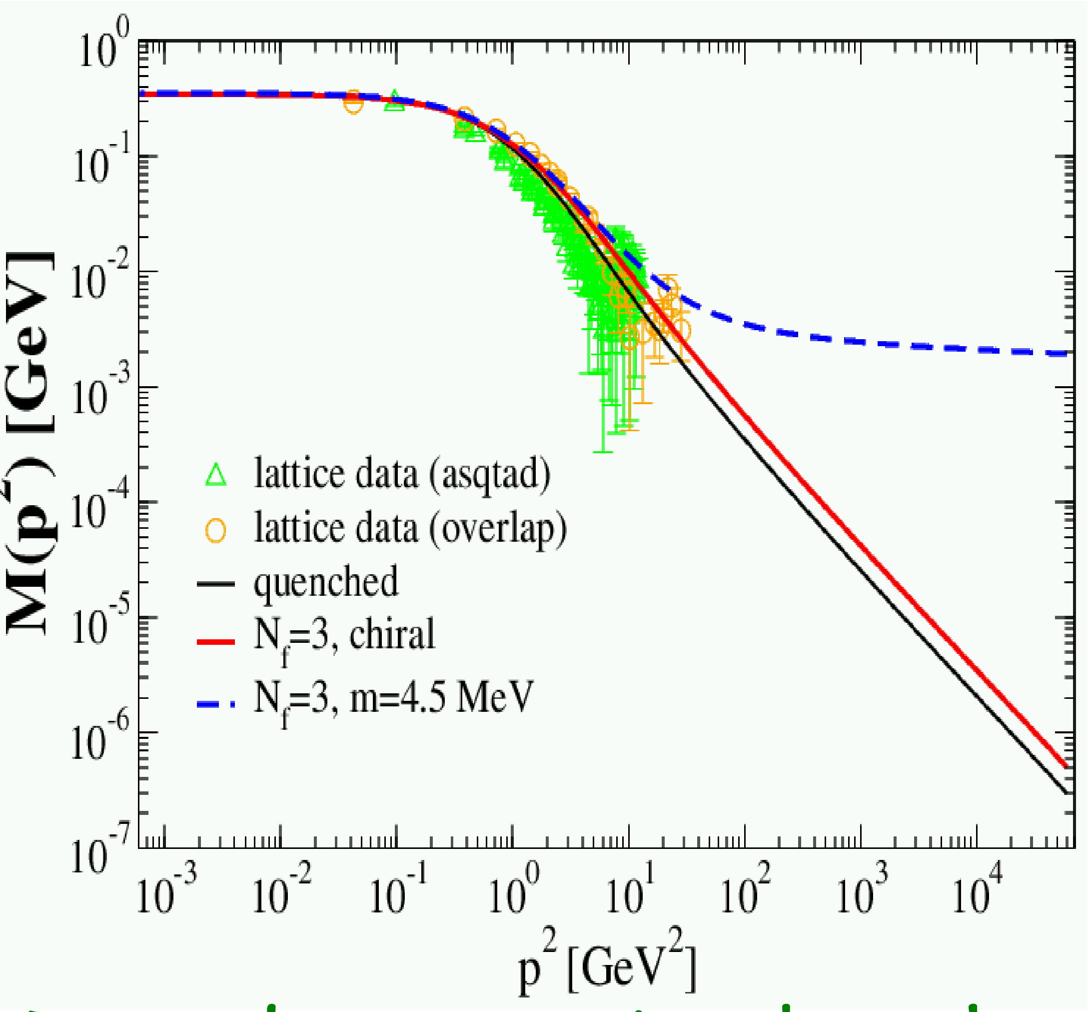

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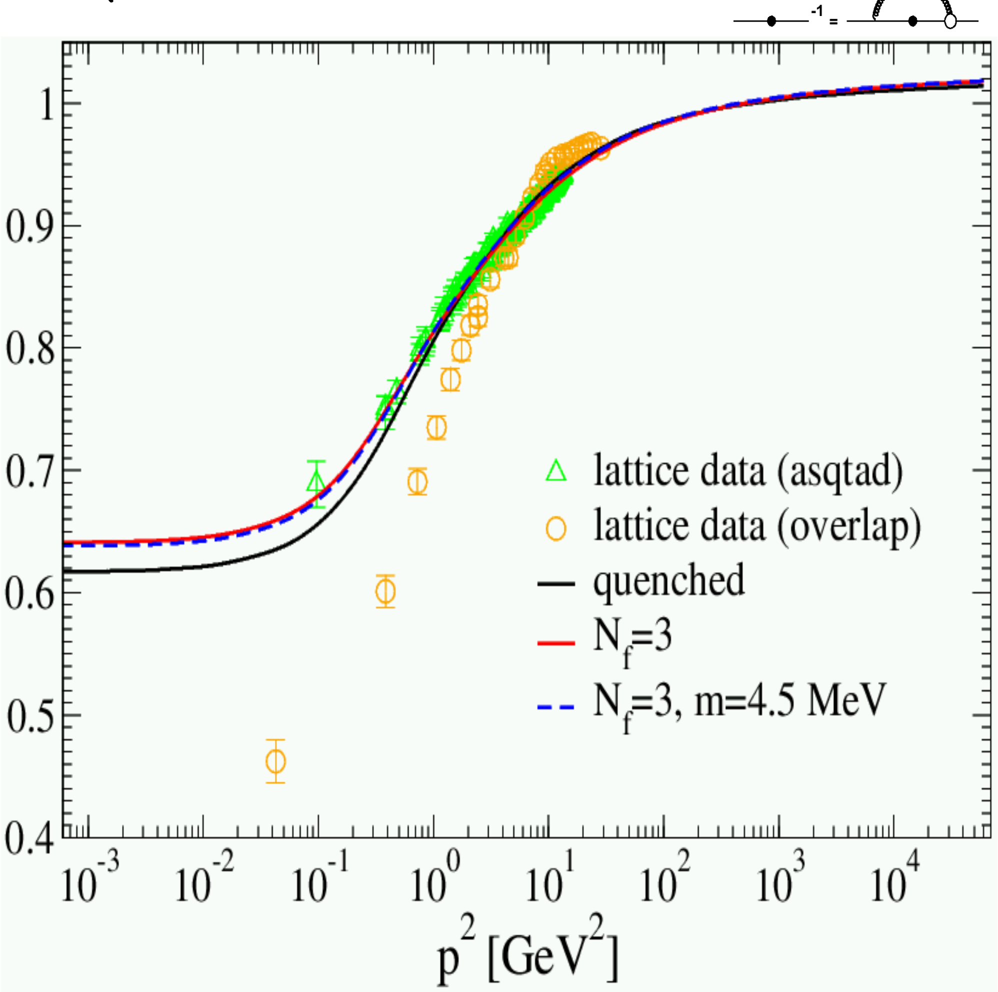

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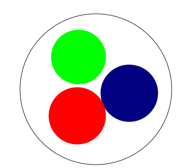

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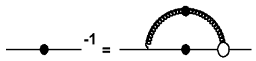
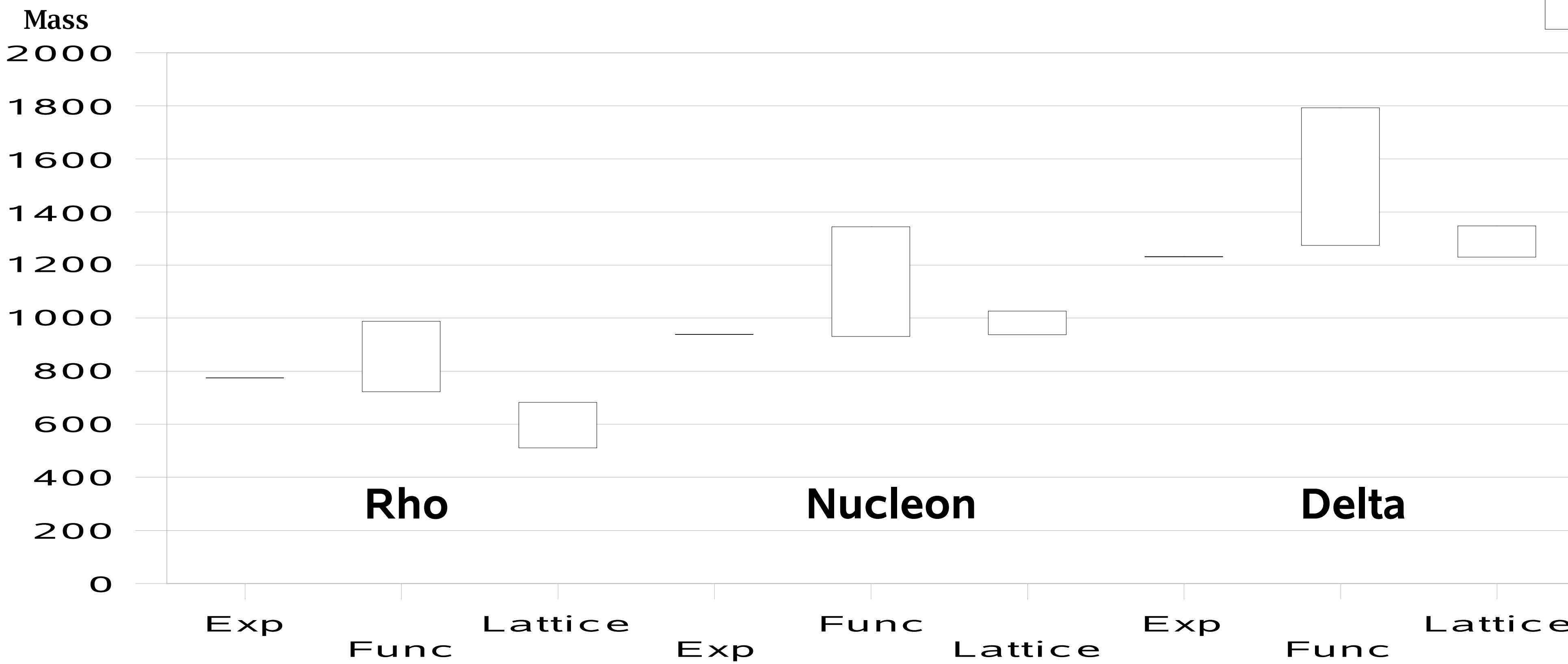
Bound states

[Eichmann et al., 2009, PDG 2009, Aoki et al . 2009]





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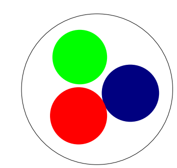

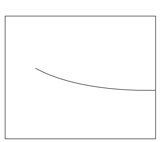

Finite temperature

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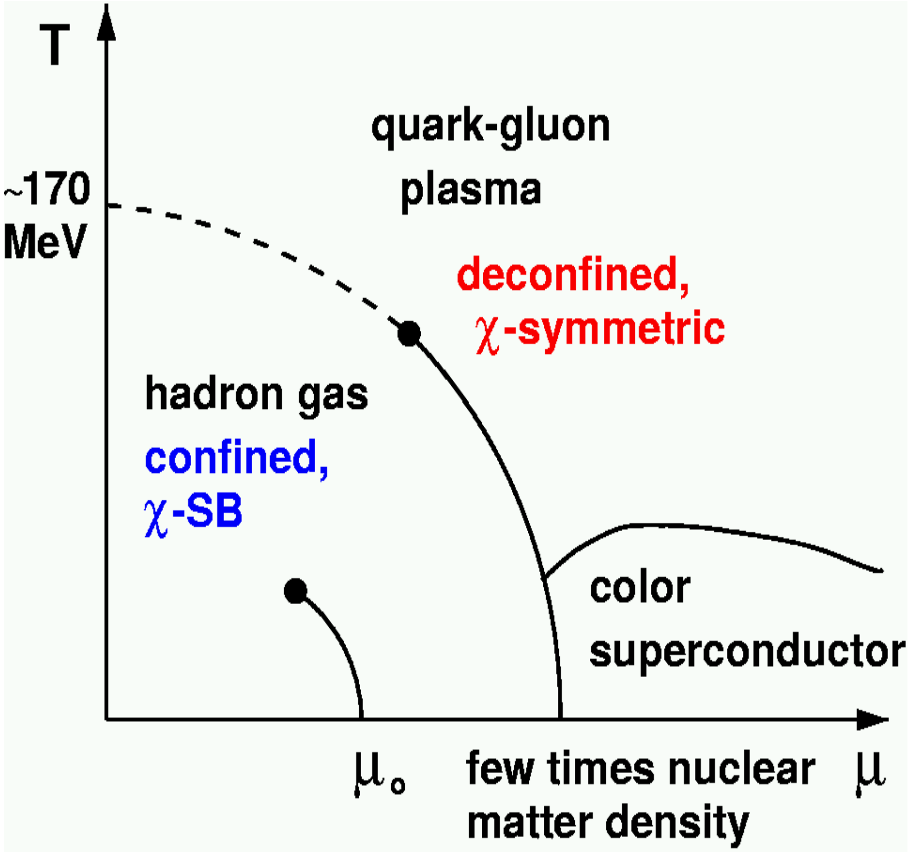
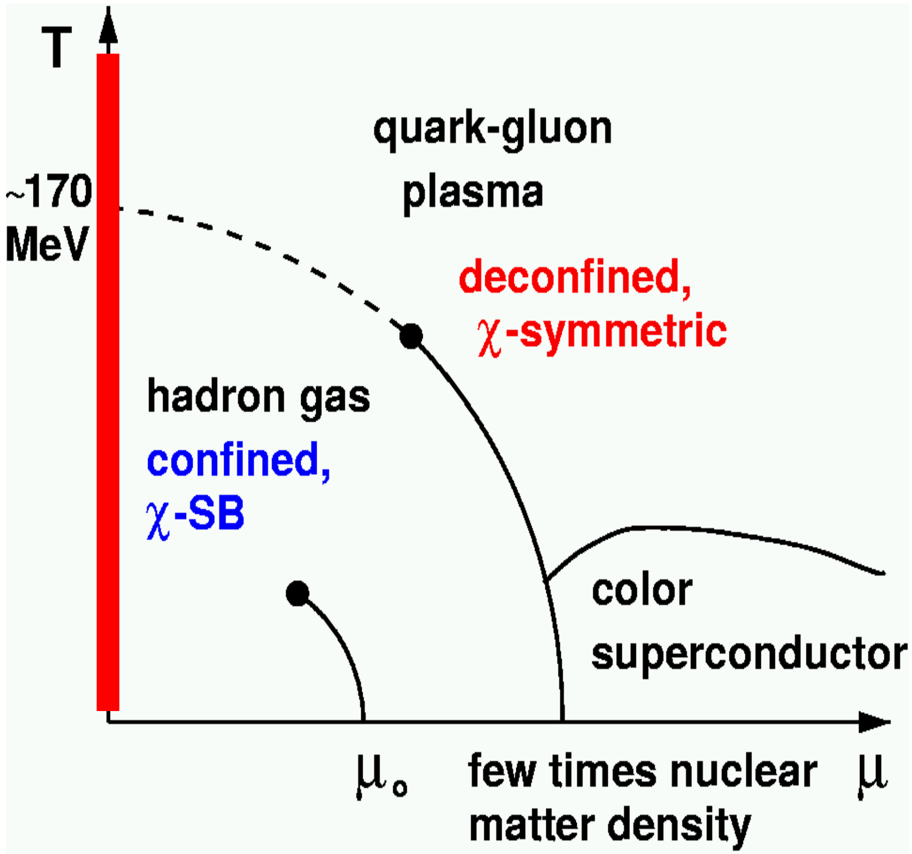


- QCD Phase transition

Finite temperature



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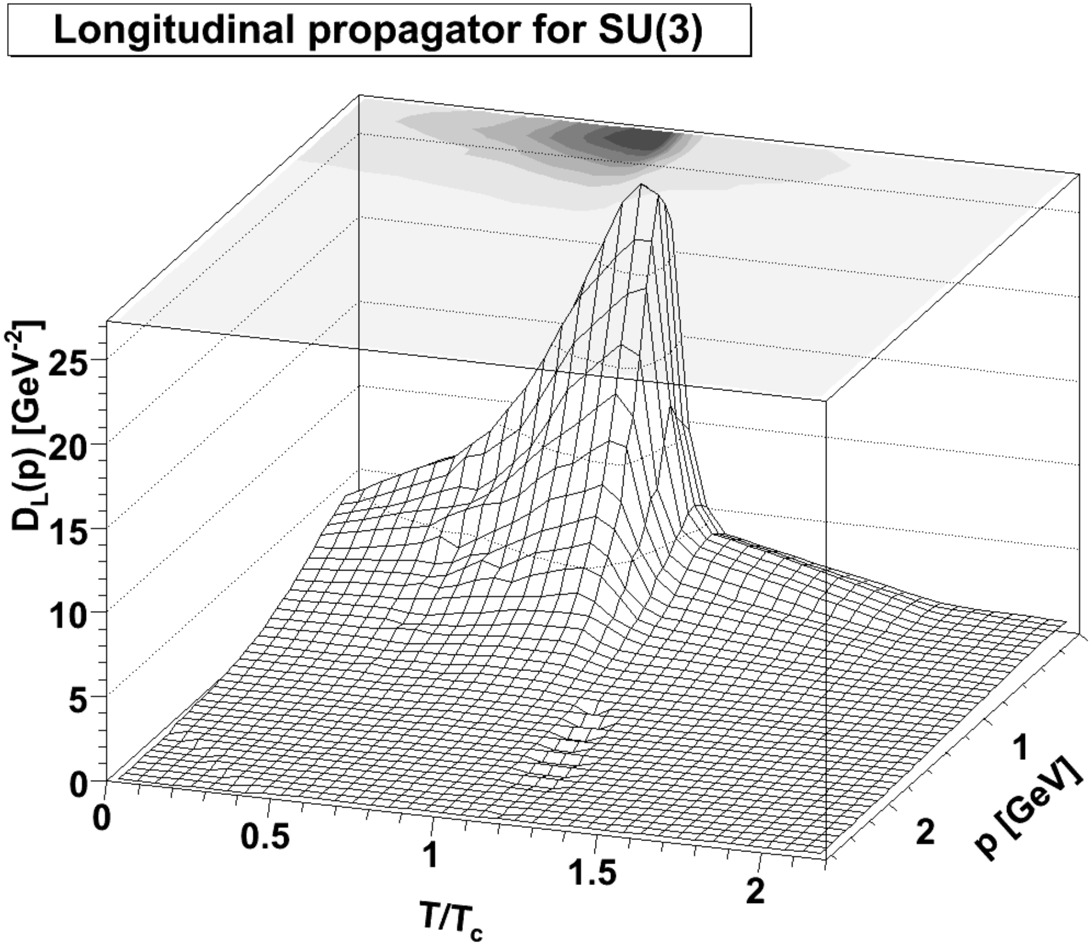
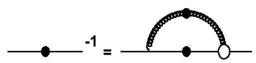
[Maas, 2009]

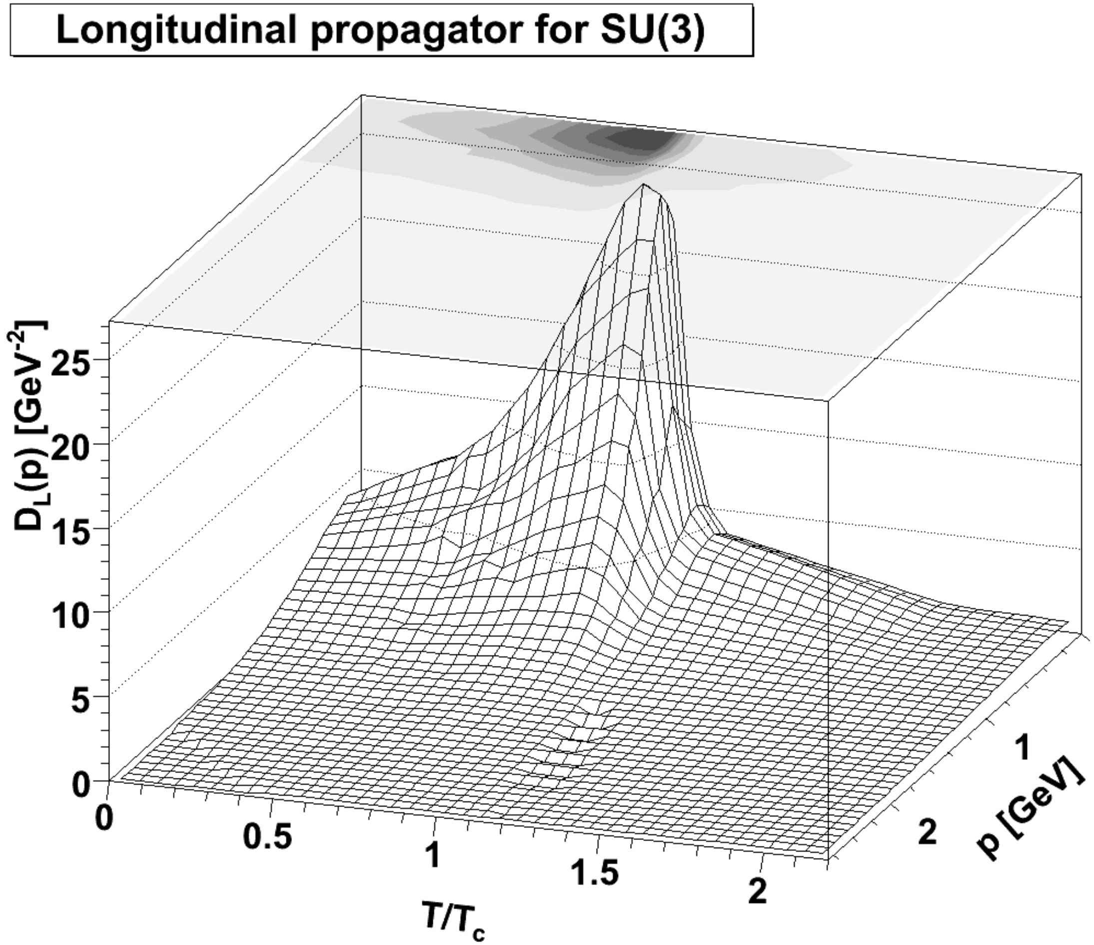


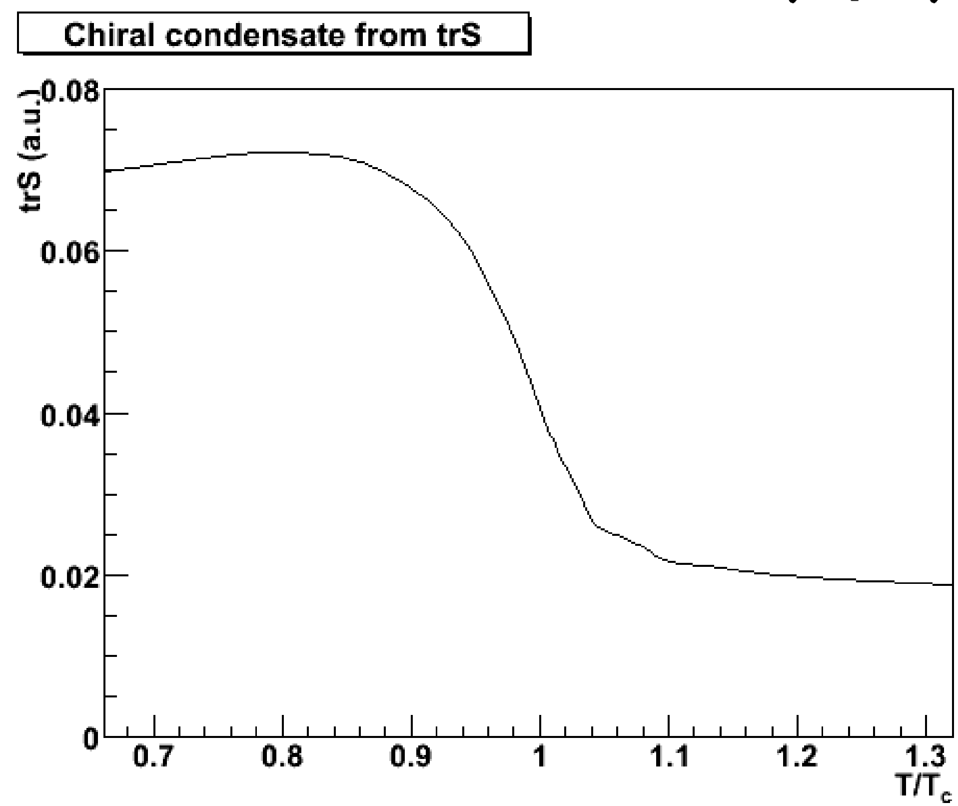
- QCD Phase transition can be observed in some of the gluon propagator tensor components

Finite temperature

[Left: Maas, 2009
Right: Fischer et al., 2009]







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- Reflected in trS of the quark propagator

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- Going to the standard model...and beyond
  - Applicable to any field theory
  - Applications eg to supersymmetry [Wipf et al., 2009]