QUANTUM CHROMODYNAMICS

a selective overview

Rudolf Baier

Faculty of Physics, University Bielefeld
Quantum Chromodynamics (QCD)
is *the* well established relativistic quantum field theory of
strong interactions:

the microscopic theory of hadrons

this force is responsible for binding nucleons, i.e. protons
and neutrons inside the atomic nucleus
nucleons, in turn, are composed of quarks, interacting with
gluons according to the laws of QCD

along with the electro-weak theory
QCD is part of the Standard Model of particle physics
QCD is already in its **fourth decade** - one talk is not much to cover its very rich dynamical content with a wide variety of sometimes totally unexpected phenomena.

ongoing efficient interactions between theory and experimental confirmations

in the 60’s of the last century it was NOT obvious that field theories are the proper description of the forces in nature, **BUT ..**
long field theory tradition - mainly on QED - of the GRAZ Institute

thanks to PAUL URBAN

e.g. at least one third of Schladming Internationale Universitätswochen have been devoted to QCD - including the last - the 47. one in 2009 on:

Fundamental Challenges of QCD

- Hadrons and chiral symmetry
- QCD and supersymmetry
- Topology and confinement
- Challenges on the lattice
- QCD phase diagram
- QCD at high temperature
- String theory and holography for QCD
motif of

“Strange Beauty”

Murray Gell-Mann and the Revolution in Twentieth-Century Physics
(by George Johnson, New York 2000)

“In our work we are always between Scylla and Charybdis; we may fail to abstract enough, and miss important physics, or we may abstract too much and end up with fictitious objects in our models turning into real monsters that devour us”

— Murray Gell-Mann, in the 1972 lecture on “QUARKS” at the XI. Internationale Universitätswochen für Kernphysik in Schladming

“In connection with the written version of these lectures, I should like to thank Dr. Heimo Latal and his collaborators for the excellent lecture notes that they provided me.”
historical remarks
basic structure: color symmetry, Lagrangian
anti-screening and asymptotic freedom - dimensional transmutation
hadron masses
maximal color screening - confinement
hard processes
dee inelastic scattering (DIS) - saturation
heavy ion collisions (HIC) - quark gluon plasma (QGP)
gauge/gravity duality: black holes and QGP
perspectives - outlook

QCD could not have been developed without the work which predominantly comes from many scientists: ESCAPE DILEMMA of ATTRIBUTION - almost NO REFERENCES!
as a reminder a few key historical dates -
NOT complete at all! -
[“The rise of the Standard Model” eds. by L. Hoddeson et al. (1997)]
- and introducing crucial notions:

- 1964: M. Gell-Mann and G. Zweig quark/aces model
- 1965 - 1966: Y. Nambu notion of color degree of freedom
color octet gluon picture
1973: H. D. Politzer and D. J. Gross - F. Wilczek
asymptotic freedom
1974: K. G. Wilson lattice QCD - confinement of quarks
1975: J. C. Collins - M. J. Perry high temperature QCD
1978: QCD - name invented by M. Gell-Mann (W. Marciano and H. Pagels)
1979: A. M. Polyakov and L. Susskind QCD phase transition: confinement - deconfinement
1983: L. V. Gribov et al. gluon saturation
1973: “hard” large $p_{T}$ photons and pions at CERN ISR
1974: $J/\Psi$ - charm at BNL and SLAC
1977: $\Upsilon$ - beauty at Fermilab

1979: evidence for gluons at DESY in $e^{+}e^{-}$ collisions
1989 - 2000: QCD at LEP(CERN)
history, cont.

- 1967 - 2007: MIT-SLAC and HERA-DESY experiments on DIS
- 1995: discovery of top quark ($m_{top} = 180.1 \pm 5.5$ GeV)
- 1986 - today: CERN and BNL experiments on HIC

2009: start at the Large Hadron Collider (LHC) at CERN (Alice, Atlas, CMS)
quarks

PERIODIC TABLE OF HADRONS

<table>
<thead>
<tr>
<th>$Q = +\frac{2}{3}$</th>
<th>u</th>
<th>c</th>
<th>t</th>
</tr>
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<tbody>
<tr>
<td>$Q = -\frac{1}{3}$</td>
<td>d</td>
<td>s</td>
<td>b</td>
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flavors: u .. up, d .. down, c .. charm
s .. strange, t .. top, b .. beauty/bottom

existence of a new - conserved - quantum number

**color**

each quark flavor has $N_C = 3$ different colors:

$q^\alpha, \alpha = 1, 2, 3$ (red, yellow, violet)
evidence for color

*dimensionless ratio*: \( R_{e^+e^-} \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \)

Feynman diagram:

\[
\begin{align*}
e^- & \quad \gamma, \, Z \quad q \\
e^+ & \quad \bar{q}
\end{align*}
\]

\( e^+e^- \to \text{quark} + \text{antiquark} \to \text{hadrons} \)

\[
R_{e^+e^-} \approx NC \sum_{f=1}^{N_f} Q_f^2 = \begin{cases} 
\frac{2}{3} NC = 2, & (N_f = 3 : u, d, s) \\
\frac{10}{9} NC = \frac{10}{3}, & (N_f = 4 : u, d, s, c) \\
\frac{11}{9} NC = \frac{11}{3}, & (N_f = 5 : u, d, s, c, b)
\end{cases}
\]
electron-positron annihilation

measurements of $R_{e^+e^-} -$ continuous curves are QCD fits: requires interactions!

\[
R_{e^+e^-} \approx NC \sum_{f=1}^{N_f} Q_f^2 = \frac{11}{9} NC = \frac{11}{3}, \quad (N_f = 5 : u, d, s, c, b)
\]
$SU(3)_C$ invariant QCD Lagrangian density: 

basic objects are the fields $q_f(x)$ and $G^a_\mu$

$$\mathcal{L}_{QCD} \equiv -\frac{1}{4} G_a^{\mu\nu}(x) G^a_{\mu\nu}(x) + \sum_f \bar{q}_f(x) \left( i\gamma^\mu D_\mu - m_f \right) q_f(x)$$

$G_a^{\mu\nu}$ ... gluon field strength tensor, \( D_\mu \) ... covariant derivative

i.e. decomposed into its different pieces: kinetic terms for the different fields,

color interaction between spin \( \frac{1}{2} \) quarks $q_f$ and $a = 1, \ldots, 8$ colored spin 1 gluon fields $G^a_\mu$,

quarks emit gluons, cubic and quartic gluon self-interactions at a given space-time point $x^\mu$

$$\mathcal{L}_{QCD} = -\frac{1}{4} (\partial^\mu G^\nu_a - \partial^\nu G^\mu_a)(\partial_\mu G^a_\nu - \partial_\nu G^a_\mu) + \sum_f \bar{q}_f^\alpha \left( i\gamma^\mu \partial_\mu - m_f \right) q_f^\alpha$$

$$+ g_s G^\mu_a \sum_f \bar{q}_f^\alpha \gamma_\mu \left( \frac{\lambda^a}{2} \right)_{\alpha\beta} q_f^\beta$$

$$- \frac{g_s}{2} f^{abc} (\partial^\mu G^\nu_a - \partial^\nu G^\mu_a) G^b_\mu G^c_\nu - \frac{g_s^2}{4} f^{abc} f_{ade} G^b_\mu G^c_\nu G^d_\mu G^e_\nu$$
interaction vertices of the QCD Lagrangian:

\[ \lambda^a \ldots \text{Gell-Mann matrices}, \quad [\lambda^a, \lambda^b] = 2f^{abc}\lambda^c \text{ i.e. non-Abelian structure constants of } SU(3)_C \]

\[ f^{abc} \ldots \text{dynamics from exact non-Abelian gauge symmetry} \]

\[ \mathcal{L}_{QCD} \text{ remains invariant under } \text{local } SU(3)_C \text{ transformations:} \]

\[ U = \exp \left\{ -ig_s \frac{\lambda^a}{2} \theta_a(x) \right\} \]

\[ \theta_a(x) \text{ are 8 functions of space-time} \]
asymptotic freedom

running coupling - general phenomenon in quantum field theory - dynamical vacuum effects

- gluons carry charge and spin carries magnetic moment
- important electric and magnetic forces

\[ \epsilon . \text{ dielectric constant, } \mu . \text{ magnetic permeability: } \epsilon \mu = 1 \]

QCD: paramagnetic effect dominates \( \mu > 1, \epsilon < 1 \): *anti-screening*

(QED: diamagnetic \( \mu < 1, \epsilon > 1 \) \( \rightarrow \) *screening*)
asymptotic freedom, cont.

momentum space \((Q \sim \frac{1}{r})\)

\[
\alpha_s(Q^2) = \frac{g_s^2}{4\pi} = \frac{\alpha_s(Q_0^2)}{1 - \frac{\beta_1 \alpha_s(Q_0^2)}{2\pi} \ln \left(\frac{Q^2}{Q_0^2}\right)}
\]

first one-loop \(\beta\) function coefficient \(\beta_1 = \frac{2N_f - 11}{6} N_c\) : positive contribution proportional to the number of quark flavours \(N_f\) is generated by the \(q-\bar{q}\) loops and corresponds to the QED result; gluonic-magnetic self-interactions introduce the additional \textit{negative} contribution proportional to \(N_c\)

\[
\longrightarrow \beta_1 < 0 \text{ if } N_f \leq 16, N_c = 3:
\]

QCD running coupling, \(\alpha_s(Q^2)\), decreases at short distances: \textit{asymptotically approaching free-field theory}

\[
\lim_{Q^2 \to \infty} \alpha_s(Q^2) = 0
\]
success of perturbative QCD (pQCD)

\[
\alpha_s(Q) = \frac{\alpha_s(M_Z)}{Q^2} = 0.1176 \pm 0.002 \sim 1/8.5
\]
**dimensional transmutation**

- Consider dimensionless quantity $R(Q)$, depending on dimensionful variable, e.g. energy $Q$

- $\mathcal{L}_{QCD-Lite}$ scale invariant (for all $m_f = 0$)
  
  under $x \rightarrow \lambda x$:

  no dimensionful parameter (coupling $g_s$ is dimensionless): dimensional analysis

  $\rightarrow R(Q) = \text{const}$

- QCD requires renormalization $\rightarrow$ dimensionful parameter ("effective" scale: breaking of scale invariance by UV cut-off!) enters into predictions, e.g.

  $$R_{e^+e^-}(Q) = \frac{11}{3} \left\{ 1 + \frac{\alpha_s(Q^2)}{\pi} + 1.411 \left( \frac{\alpha_s(Q^2)}{\pi} \right)^2 - 12.80 \left( \frac{\alpha_s(Q^2)}{\pi} \right)^3 + ... \right\},$$

  i.e. non-trivial energy $Q$—dependence
transmutation, cont.

introduce scale $\Lambda_{QCD}$:

$$\alpha_s(Q_0^2) = \frac{2\pi}{-\beta_1 \ln \frac{Q_0^2}{\Lambda_{QCD}^2}} \rightarrow \alpha_s(Q^2) = \frac{2\pi}{-\beta_1 \ln \frac{Q^2}{\Lambda_{QCD}^2}}$$

(at leading order)

dimensionless $\alpha_s(Q^2) = \alpha_s(Q^2/\Lambda_{QCD}^2)$ and dimensionful $\Lambda_{QCD}$ can be traded for each other ($\Lambda_{QCD}^{(N_f=5)} \simeq 210 \text{ MeV}$):

$$\Lambda_{QCD} = Q \exp \left[ \frac{\pi}{\beta_1 \alpha_s(Q^2)} \right], \quad Q \rightarrow \infty$$

all dimensionful quantities are then related to scale $\Lambda_{QCD}$
mass problem - "weight of the world"

the masses of particles, e.g. nucleons, vectormesons,.. are expressed as *pure numbers* times the scale parameter $\Lambda_{QCD}$ (up the small corrections due to quark masses $m_{u,d}$):

bulk of the mass of ordinary matter is accounted for by QCD dynamics using *no* mass unit at all, arising from pure energy!

Dürr et al. (2008): non-perturbative lattice QCD mass spectrum vs. the observed one
chiral symmetry and spontaneous breaking (SBS)

non-trivial vacuum structure leading to the formation of eight
Nambu-Goldstone bosons: $\pi, K, \eta$

$$H_{QCD}|0\rangle = 0, \ [H_{QCD}, Q_5]|0\rangle = 0,$$

state $Q_5|0\rangle \neq 0 \rightarrow H_{QCD}Q_5|0\rangle = 0,$

i.e. $Q_5|0\rangle$ massless state degenerate with non-trivial
asymmetric vacuum state $|0\rangle$ - although $H_{QCD}$ is symmetric!

in QCD chiral condensate:

$$< 0|Q_5|0\rangle \equiv < 0|\bar{u}_L u_R|0\rangle = < 0|\bar{d}_L d_R|0\rangle \propto \Lambda_{QCD}^3 \neq 0$$

anticipated in "five-men paper" by
H. P. Dürr, W. Heisenberg, H. Mitter, S. Schlieder and
K. Yamazaki (1959) by introducing SBS in particle theory
global symmetry $SU(3)_L \otimes SU(3)_R$ of $\mathcal{L}_{QCD}$ for $m_{u,d,s} = 0$

$$q_{L,R}(x) \equiv \frac{(1 \pm \gamma_5)}{2} q(x) \to \exp \{ \pm i\vec{\theta} \cdot \vec{\lambda} \} q_{L,R}(x)$$

chiral condensate: $< 0 | \bar{u}_L u_R | 0 > = < 0 | \bar{d}_L d_R | 0 > \propto \Lambda_{QCD}^3 \neq 0$

for $m_{u,d,s} \neq 0$, e.g. light pion

$$m_\pi^2 \propto (m_u + m_d) \Lambda_{QCD} \ll m_\rho^2, m_N^2$$

and $m_d \sim 8.2 \text{ MeV} \sim 1.8 m_u$ \Rightarrow proton ($\sim |uud>$) is stable, but neutron ($\sim |udd>$) decays!
maximal color screening

NO one has ever seen a quark sitting by itself somewhere!
real-world QCD has light $u$ and $d$ quarks ($m_{u,d} < \Lambda_{QCD}$)

strong chromoelectric fields in between quarks at distance $r \simeq 1/\Lambda_{QCD}$ become instable:
decay by pair-producing light quarks and antiquarks and (dynamical) screening:
\[ \text{quark} \rightarrow \text{meson} + \text{quark} \]
is possible since energy density $\epsilon \simeq \Lambda^4_{QCD}$

"true" confinement only when all the quarks are very heavy, $m_q \rightarrow \infty$ - mass gap in four dimensional non-Abelian gauge theory

[E. Witten: "hope that it will be solved one day"!]
analogous model: Schwinger mechanism

cf. super critical QED phenomena: \( A_Z(Z > Z_{crit}) \rightarrow A_{Z-1} + e^+ \)

\[ Z > Z_{crit} \rightarrow A_{Z-1} + e^+ \]

pair production of \( e^+ e^- \) in a strong electromagnetic field - pair production of \( q\bar{q} \) as a quark tries to escape from a proton

QED: \( Z > Z_{crit} \simeq 137 \), QCD: \( \alpha_{QCD}(r \simeq 1/\Lambda_{QCD}) \simeq \mathcal{O}(1) \)

QCD is hard to solve because \( \alpha_s \) becomes strong at low energies

[R. Alkofer and coworkers]
jets: "here we see a quark, here a gluon"

collimated jets are NOT single particles (like photons, ..)
jets are sets of hadrons moving rapidly in nearly the same direction, following the nominal paths of the original "hard" energetic quarks/gluons due to copious "soft" (low momentum) radiation creating new particles, which do **not** disrupt the flow of energy

e.g. jets produced in $e^+e^-$ collisions - e.g. rare "hard" 3-jets: $q\bar{q}G$ at $O(\alpha_s)$
high energy jet cross section for $p + \bar{p} \rightarrow \text{jet} \ (y, p_T) + \text{anything in three angular bins}$
Sketch of dijet production and pQCD collinear factorisation in hadronic collisions: $f_{a/A}(x)$ are the parton distribution functions (PDFs), $D_{i \rightarrow h}(z)$ the fragmentation functions (FFs). ISR (FSR) represents initial (final)-state radiation.
nucleon structure function

deep inelastic electron-proton/deuteron scattering (DIS) at SLAC and DESY
[continuation of form factor measurements by (in-)elastic scattering in the early 60’s:
in Graz theoretical activities by H. Latal, P. Kocevar and colleagues]

proton/deuteron is an extended object of the order of $1_{fm} = 10^{-13} cm$ - investigated through inelastic electromagnetic scattering for $Q \gg m_{proton} : Q^2 = -q^2$ ,
dimensionless Feynman scaling variable $x = \frac{Q^2}{2P\cdot q}$; pQCD evolution by gluon radiation
DESY-HERA results for dimensionless structure function $F_2(x, Q^2) \sim Q^2 \sigma_{\gamma^* \text{proton}}$ illustrating the effects of the $Q$–evolution with increasing $Q^2$ for different fixed values of $x$ including pQCD fit

$$F_2(x, t) \simeq q_f^2 q(x, t) + O(\alpha_s(t)) \ , \ t \sim \ln Q^2$$

pQCD $Q$–evolution of PDF’s - quark $q$ and gluon $G$ distributions in the nucleon according to DGLAP - "scaling violation" by logarithmic deviations:

$$\frac{d}{dt} q(x, t) = \frac{\alpha_s(t)}{2\pi} [q \otimes P_{qq}] + \frac{\alpha_s(t)}{2\pi} [G \otimes P_{qG}]$$
nucleus A and small $x$ gluons: saturation

gluons dominate at small Feynman $x$, i.e. at high energies

color glas condensate (CGC)

Gluons split ($\sim \alpha_s$) and start to overlap at saturation scale $Q = Q_s \sim A^{1/6}$:

$$\alpha_s \frac{1}{Q_s^2(x)} A x G(x, Q_s^2) \simeq R_A^2 \simeq A^{2/3}$$

$$\alpha_s \ll 1 \Rightarrow \text{large - but limited - gluon density at high energy } (x \to 0): n \sim \frac{Q_s^2(x)}{\alpha_s}$$
forward physics in $d + Au$ collisions

suppression pattern at large angles/rapidities
via nonlinear high density small $x$ gluons in $Au$ nucleus

ratio of central over peripheral $N_{coll} \sim R_A \sim A^{1/3}$-scaled yields for charged hadrons

at $\eta = 0$ (dots) and very forward $\eta = 3.2$ (open circles) at RHIC

gluon evolution for $\alpha_s \ln \frac{1}{x} \gg 1$ - resummed:

$$n \sim \frac{Q_s^2}{\alpha_s} \rightarrow \frac{Q_s^2}{\alpha_s} \left( \frac{Q_s^2(x)}{Q_0^2} \right)^{\gamma_s} \rightarrow R_{cp} \sim A^{-(1-\gamma_s)/3} \sim 0.5, \quad \gamma_s \simeq 0.63$$
heavy ion collision: space-time evolution

CGC
Initial Singularity
Glasma
sQGP
Hadron Gas

\[ z = -t \quad t \quad z = t \]

Quark Gluon Plasma → Hadronization
\( \tau \sim 1 \text{ - } 10 \text{ fm/c} \)

Topological Excitations
Density Fluctuations, Thermalization
\( \tau \sim 0.1 \text{ - } 1 \text{ fm/c} \)

Event Horizon
Quantum Fluctuations
\( \tau \sim 0 \text{ - } 0.1 \text{ fm/c} \)

Initial Singularity
Initial Nuclei as CGC

Coherent, High-density Gluons
QCD phase diagram

lattice QCD - $2 + 1$ flavors:
deconfinement and chiral symmetry restoration
happen in the same narrow temperature interval/"cross over"

$180 \text{ MeV} \leq T \leq 200 \text{ MeV}$

[project: Ch. Gattringer et al.]
jet quenching: a true pQCD prediction at RHIC

suppression of $\pi^0$ vs. photons $R_{AA}^{\pi^0} \approx 0.2 - 0.4 < 1$; $R_{AA}$ = ratio of the measured yield to the point-like scaled $pp$ cross section

FINAL STATE EFFECT: pQCD medium-induced radiative energy loss - dominant LPM gluon radiation due to multiple scatterings, mean free path $\lambda >$ range of screened interactions, $E_{parton} \rightarrow \infty$, energy loss $\Delta E$
jet quenching in a head-on $Au - Au$ collisions, cont.

two quarks suffer a hard scattering: one goes out directly to the vacuum, radiates a few gluons and hadronises, the other goes through the dense plasma created and suffers energy loss due to medium-induced gluonstrahlung and finally fragments outside into a (quenched) jet. Note: same-side jet is not modified in $Au - Au$ vs. $p - p$ collisions

\[ \text{energy density : plasma } \epsilon > 15 \text{ GeV/fm}^3 \quad \text{--- \ --- nuclear matter } \simeq 0.15 \text{ GeV/fm}^3 \]
properties of colliding nuclei are explained by black hole theory

duality $\sim$ equality

is between

a special Quantum Field Theory (but NOT QCD !)
in $d = 4$ space-time dimensions (for colors $N_c \gg 1$, $g_{YM}^2 N_c \gg 1$)

and

Classical $AdS_5$ Gravity in $d = 5$

via holographic property

at finite temperature $\equiv$ Hawking temperature
strong coupling result:

Energy density of lattice QCD and $\mathcal{N} = 4$ SYM via black hole Bekenstein-Hawking ($AdS_5$) entropy

\[ S_{BH} = \frac{3}{4} S_{Boltzmann} = \frac{\pi^2 N_c^2}{2} T^3 \]
viscosity/entropy density $\eta/s$

pQCD (transport/Boltzmann equation)
[cf. Boltzmann equation, Graz 1873]

$$\frac{\eta}{s} \sim \frac{\#}{g_s^4(T) \ln 1/g_s} \sim \mathcal{O}(1)$$

strong coupling AdS/CFT universal lower bound:

$$\frac{\eta}{s} \geq \frac{1}{4\pi} \sim 0.1$$

RHIC: $\eta/s < 0.5 \rightarrow$ almost a “PERFECT FLUID” (sQGP)
QCD is a perfect QFT

- many important topics are not covered, nevertheless:

- overwhelming experimental and theoretical evidence for describing correctly the hadronic world

- no indication within the theory of a distance scale at which it must break down

- it has no free, adjustable parameters (neglecting the irrelevant quark masses), and dimensional observables are calculable in terms of the dynamically produced scale $\Lambda_{QCD}$
it shows no diseases when extrapolated to infinitely high energies. To the contrary, asymptotic freedom means that at high energies/temperatures QCD becomes simple and perturbation theory is a better and better approximation

non-perturbative nature of low-energy behaviour is still challenging the theoretical capabilities

STILL A LOT TO LEARN

relations to the electro-weak sector of the STANDARD MODEL
tomorrow: Higgs production

quantum-mechanical process ("loops"), where the color coupling of QCD and the gluon structure in the proton enter:

possible mechanism through glueon fusion from colliding protons
for production and observation of the Higgs particle $H$ by the decay into two photons
HERZLICHE GLÜCKWÜNSCHE

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