Mesonic Baryon Resonance Decays
in Relativistic Constituent Quark Models

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Betreuer:
Univ.-Prof. Dr. Willibald Plessas
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Für Gert
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Chapter 1

Introduction

1.1 The Issue

A challenging problem of present-day physics is the description of baryon resonances on the basis of quantum chromodynamics (QCD), which is generally accepted as the fundamental theory of strong interactions and was developed already more than thirty years ago [1]. QCD is a non-linear quantum field theory with an infinite number of degrees of freedom and a running (energy-dependent) coupling constant. Whereas its structure is well-known, a comprehensive solution at all energies is not at hand, and especially in the low-energy regime one is left with tremendous problems. The fundamental particles of QCD are the coloured quarks and gluons, which have never been observed as free states. Furthermore, the quarks appear in six flavors, namely up, down, strange, charm, bottom, and top, where in the light sector (up, down) the QCD Lagrangian exhibits an approximate chiral symmetry. This chiral symmetry is explicitly broken due to the non-vanishing up- and down-quark masses of about $1.5 - 3.0$ and $3 - 7$ MeV (see the Particle Data Group (PDG) [2]), respectively. In addition, in the low-energy regime chiral symmetry is also broken spontaneously leading to phenomena such as constituent quarks (with significantly higher masses than the current quark masses) and Goldstone bosons.

Over the recent years lattice gauge theory has turned out as a promising tool to solve QCD, and one has achieved a number of important results. However, the computer facilities currently available pose restrictions with respect to the size of the lattice and the magnitudes of the input quark masses. The latter ones are still far too high compared to the physical values for up and down quarks. Nevertheless, at least within the quenched approximations, one approaches the understanding of ground-state spectroscopy of baryons by producing masses whose extrapolations lead to the right values (see, e.g., the contribution of F. X. Lee in Ref. [3]). With respect to the spectroscopy of excited baryons the situation is much more intricate,
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and even within quenched lattice calculations a comprehensive description of excited baryons is still out of reach [4,5].

In the low-energy regime of QCD, constituent quark models (CQMs) provide interesting alternative approaches. They can be designed such as to incorporate the most essential properties of QCD relevant for this energy regime. Usually, CQMs are designed in a framework with a finite number of degrees of freedom, i.e. they rely on a Hamiltonian that contains a confining interaction and also includes a hyperfine interaction deriving from a specific dynamical concept.

It is generally assumed that the properties of baryon resonances should be calculated within a relativistic approach. In this respect, modern CQMs are set up along a relativistic formalism, i.e. one works within Poincaré-invariant quantum mechanics (see, e.g., Ref. [6]). Clearly, due to the limitation to a finite number of degrees of freedom, this approach is a priori different from a quantum field theory. It is based on a relativistically invariant mass operator including the interaction between the involved particles.

In 1953, Bakamjian and Thomas [7] designed a construction that provides a practical way for the inclusion of interactions in all of the three major forms of relativistic dynamics, namely the instant, front, and point forms already introduced in 1949 by Dirac [8].

This work is dedicated to the study of mesonic baryon resonance decays. There exists a large amount of experimental data accumulated over the years (see, e.g., the PDG [2]), and currently high-quality measurements are performed at several facilities, e.g., at JLAB, MAMI, etc. However, a comprehensive theoretical description of strong decay processes turns out to be extremely involved. Early attempts to describe the strong decays of hadrons date back to the sixties and early seventies of the past century: One has studied the mesonic decay processes advocating the so-called elementary emission model (see, e.g., Refs. [9–13]) or, alternatively, the pair-creation model (see, e.g., Ref. [14]). Over the years a number of attempts to describe mesonic decays have been undertaken, e.g., in Refs. [15–19]. The corresponding calculations have been carried out within a non-relativistic or at most semi-relativistic framework, and in addition one essentially restricted the investigations to the non-strange ($\pi$ and $\eta$) decays of non-strange baryons (nucleon and $\Delta$).

Relativistic CQMs provide a natural starting point for the investigation of baryon properties beyond spectroscopy such as, e.g., the electromagnetic structure or the strong decays. With respect to the strong decays, the Graz group recently started to investigate the mesonic $\pi$ and $\eta$ decays of light (nucleon and $\Delta$) baryons within the point form of RQM [20]. The aim of the present thesis is to extend the study of hadronic decays to the non-strange decays of strange ($\Lambda$, $\Sigma$, and $\Xi$) baryons as well as the strange ($K$) decays of light and strange baryons. We will thus follow the approach as described in Ref. [20] and analyze the mesonic baryon resonance decays
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specifically within the point form of RQM. By the study of the various decays of excited baryon resonances one hopes to learn about the advantages and drawbacks of relativistic CQMs. Also one is interested in clarifying the particular dynamics of the decay mechanism.

1.2 Structure of this Work

After this introductory remarks on the issue of baryon resonance decays, in Chapter 2, we provide a short historical survey of various CQMs elaborated over the past decades. In particular, we discuss the Goldstone-boson-exchange and the one-gluon-exchange CQMs used in this work, and we point out the main characteristics of the corresponding baryon spectra. Subsequently, for the solution of the eigenvalue problem of the CQM Hamiltonian, we introduce the stochastic variational method and show, how one can apply it specifically to three-quark baryon systems.

In Chapter 3 we discuss the concept of relativistic (Poincare-invariant) quantum mechanics (RQM). We start with the introduction of the Poincaré group for a relativistic one-particle system. Then we extend the discussion to systems comprising several particles. As baryons are considered to be composed of three quarks interacting via strong forces, we then deal with the problem of implementing interactions in a proper way. This leads us to the different forms of dynamics along with the Bakamjian-Thomas construction.

Chapter 4 is devoted to the general definition of the transition amplitude and the decay width. We start with the introduction of the $S$-matrix, and find a proper expression for the decay width via the transition probability. There is no unique theory available for the decay of a baryon resonance by the emission of a meson. In the past several models were developed for the decay operator, mostly in non-relativistic approaches. In our covariant framework we cannot deal with the full (many-body) decay operator but we have to resort to an approximation. The specific decay operator we assume consists in a spectator model, and the corresponding formalism is outlined in Chapter 5. In Chapter 6 we then take the non-relativistic limit of the model defined in Chapter 5. The various results obtained for the decay widths are discussed in Chapter 7. In particular, we present the relativistic results pertaining to a pseudovector as well as a pseudoscalar coupling for the quark-meson vertex. A comparison is given also with the non-relativistic predictions according to Chapter 6. Chapter 8 contains our conclusions together with a short outlook to possible future investigations.

The appendices are set up in the following way: In Appendix A we provide some additional information concerning the baryon wave functions that are used as an input for the decay calculations. Appendix B covers some useful notations in RQM. In Appendix C we detail the spin and flavor matrix elements contained in the transition amplitude, and we provide some
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insight into the numerical calculations. We continue in Appendix D with the detailed comparison of the experimental as well as theoretical baryon masses. In Appendix E we discuss the flavor multiplet structure of the baryon resonances and the connection to the baryon wave functions. Appendix F contains detailed listings of all decay widths calculated in the present thesis.
Chapter 2

Constituent Quark Models

Nowadays QCD is generally accepted as the fundamental theory of strong interactions. Unfortunately, it is not yet possible to solve QCD comprehensively at all energies. Therefore, depending on the energy domain, one has to find suitable approaches complying with the characteristics that govern the corresponding regime. Especially at low and intermediate energies, where a perturbative treatment is not possible, one has to resort to numerical approaches with restricted actions or to effective (field) theories and models. Recently, it has been supported by lattice studies (see, e.g., Refs. [21, 22]) that low-energy QCD is characterized by the appearance of constituent quarks, which have a dynamically generated effective mass much bigger than the one of current quarks. This effect can be ascribed to the spontaneous breaking of chiral symmetry (SB\textbackslash S) below a certain energy scale. Thus, it seems reasonable to treat baryons in terms of these quasiparticles. In this light, CQMs provide an effective tool for the description of hadron properties and hadronic reactions. Such models have already been introduced long ago and passed through a vivid development over the years. They permit to introduce the essential features of non-perturbative QCD, and moreover, they can be formulated to allow for a relativistic treatment of hadrons.

In this thesis we study the hadronic decays of all light and strange baryons within the framework of CQMs. We shall first give a review of the various CQMs, then we detail the two specific CQMs employed in this work, namely the Goldstone-boson exchange (GBE) and the one-gluon exchange (OGE) CQM, respectively. Subsequently, we discuss the stochastic variational method (SVM), which is employed for the solution of the corresponding eigenvalue problems of the baryon states. The mass spectra as well as the associated wave functions of the baryon states then provide the input for the calculations of the various decay widths.
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2.1 History

CQMs already have a long history. Starting from the naive quark model, originally proposed by Gell-Mann and Zweig [23, 24], much progress has been made. A typical CQM is based on a Hamiltonian with a kinetic energy part and a quark-quark interaction consisting of a confinement and a hyperfine interaction. It is quite evident that the kinematics of constituent quarks confined to a volume inside hadrons should be treated in a relativistic manner (i.e., one has to employ a relativistic mass operator). However, the first CQMs have been performed within a non-relativistic framework, and only recently relativistic formalisms have become available (see the discussion below).

Concerning the confinement and hyperfine interactions intensive investigations have been conducted over the years. Whereas one first assumed simple confining forces like, e.g., a harmonic-oscillator potential, nowadays the confining interaction is substantiated by the results of lattice calculations [25, 26]. The question after the hyperfine interaction is more intricate, i.e., one has elaborated several models, but up to now it is not entirely clear, which kind of hyperfine interaction describes hadronic properties most adequately in all respects. Originally, one assumed that the hyperfine interaction derives from one-gluon exchange [27]. Several CQMs for baryons - initially set up in a non-relativistic formalism - are based on this kind of interaction (see, e.g., Refs. [28–30] or [31]1). Shortly after, one developed a relativized quark model for mesons [33], and as a generalization from $q\bar{q}$ to $qqq$ states a relativized quark model for baryons [34]. However, one soon became aware that the OGE potential encounters difficulties in describing the light baryon spectra with the proper level orderings. Also beyond nucleon and $\Delta$ baryons, due to the missing flavor dependence the light and strange baryon spectra cannot be reproduced simultaneously in an OGE CQM. Similar problems have been found in hybrid CQMs, whose hyperfine interaction is furnished by a superposition of OGE and meson exchange (see the discussion in Ref. [35]). An alternative way to the hyperfine interaction of constituent quarks consists in instanton-induced forces, where one utilizes a t’Hooft interaction [36]. The corresponding instanton-induced (II) CQM has been designed a few years ago by the Bonn group [37–39]. It is based on the solution of the Bethe-Salpeter equation [40] for bound-state problems, hence the model is covariant from the beginning. However, the II CQM is also not able to describe the light and strange baryon spectra with the right level orderings, especially in the case of the nucleon and $\Delta$ excitations. Several years ago the Graz group [41] proposed a CQM whose hyperfine interaction derives from the exchange of Goldstone bosons [42].

1The latter one is based on a modified OGE potential proposed by Bhaduri et al. (see Ref. [32]).
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This type of dynamics is considered as a consequence of the SBχS of QCD at low energies. It generates a spin-flavor dependence just as demanded by the phenomenological spectra. As a result one has arrived at a comprehensive relativistic description of the excitation spectra of all light and strange baryons \[35,41,43,44\]. Recently, this model has been extended with respect to the inclusion of all force components from pseudoscalar exchange and the introduction of vector and scalar exchange \[45\].

Beyond baryon spectroscopy one is also interested in the other properties of baryon states, e.g., the electroweak form factors and the charge radii as well as the magnetic moments. Such results obtained within a fully relativistic approach have been found, e.g., in Refs. \[46–52\]. Beyond that, the mesonic resonance decays (studied in this thesis) still represent a big challenge. In this regard one has already investigated the decays of light (and strange) baryons within relativized frameworks \[19,53,54\]. Recently, the Graz group performed a covariant calculation in the light baryon sector \[20\]. Following the same approach the investigations are now being extended to the strange sector, i.e. we calculate the non-strange (π and η) decays of the lowest light and strange baryon resonances, and in addition we also consider strange (K) decays.

2.2 Goldstone-Boson-Exchange CQM

In this section we first discuss the physical arguments for assuming GBE dynamics for the hyperfine interaction of light and strange baryons. Then we present the specific pseudoscalar GBE CQM used in this work \[41,55\].

2.2.1 GBE Dynamics

In the limit of vanishing quark masses the QCD Lagrangian for three quark flavors exhibits a chiral symmetry. This can be attributed to the fact that the mass term is the only one that mixes left- and right-handed quarks. The chiral symmetry is explicitly broken by the occurrence of any mass term. Furthermore it is spontaneously broken by dynamical mass generation. The current quark-masses of the \(u\) and \(d\) quarks are rather small \(m_{u,d}^{\text{curr}} \lesssim 10\ \text{MeV}\), and the chiral symmetry of the original QCD Lagrangian is almost exact in the \(u\) and \(d\) sectors. On the other hand, the current quark mass of the \(s\) quark is considerably larger \(m_{s}^{\text{curr}} \approx 150\ \text{MeV}\), and the \(SU(3)\) QCD Lagrangian with the three flavors \(u\), \(d\), and \(s\) fulfills only an approximate chiral symmetry.

In the low-energy regime the (approximate) chiral symmetry of the QCD Lagrangian gets spontaneously broken by the behaviour of the QCD vacuum \((SU(3)_{L} \times SU(3)_{R} \rightarrow SU(3)_{V})\). A phenomenological evidence for the SBχS of the QCD vacuum is provided by the fact that the value of the quark condensate amounts to about \(<\bar{q}q> \simeq -(240 - 250\ \text{MeV})^{3}\). From model
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studies, such as the σ-model [56] or the Nambu-Jona-Lasinio model [57,58] it is suggested that the SBXS has two important consequences:

First, the quarks acquire a dynamical mass much larger \((m_{u,d} \simeq 300 \text{ MeV})\) than the current quark mass. In principle, the dynamical quark masses are momentum dependent with increasing values towards low momenta.

Second, an octet of pseudoscalar Goldstone bosons appears. It is realized by the pseudoscalar mesons \((\pi^-, \pi^0, \pi^+, K^-, K^0, K^+, \bar{K}^0, \eta)\). In the chiral limit (with vanishing current-quark masses), these Goldstone bosons would have zero mass. However, since the original chiral symmetry of the QCD Lagrangian is only approximate, the Goldstone bosons also acquire a mass. This is reflected in the Gell-Mann-Oakes-Renner relations (see Ref. [59]). It is seen that the masses of the pseudoscalar mesons are directly related to the quark condensate. In addition, the flavor singlet \(\eta'\) decouples from the original octet due to the axial \(U(1)\) anomaly [36,60]. The \(\eta'\) meson has a large mass of about 958 MeV, which makes it impossible to consider it as the ninth Goldstone boson. Nevertheless, in the large-\(N_C\) limit this axial \(U(1)\) anomaly would vanish and the \(\eta'\) would become a Goldstone boson. The SBXS would then imply a nonet of Goldstone bosons [61].

In the light of the observed consequences of SBXS, Glozman and Riska [42] suggested that light and strange baryons should be considered as systems of three constituent quarks. Moreover, the effective quark-quark interaction should consist of a phenomenological confinement and a chiral interaction mediated by the exchange of pseudoscalar mesons.

2.2.2 Hamiltonian of the GBE CQM

In order to achieve a realistic description of the light and strange baryon spectra the Graz group has suggested the Hamiltonian

\[
H = H_{\text{free}} + \sum_{i<j=1}^3 V_{ij}
\]  

(2.1)

with a relativistic kinetic-energy operator \(H_{\text{free}}\) and a quark-quark interaction

\[
V_{ij} = V_{\text{conf}}(r_{ij}) + V_{hf}(r_{ij}),
\]  

(2.2)

where \(V_{\text{conf}}(r_{ij})\) confines the three quarks to a finite volume, and \(V_{hf}(r_{ij})\) is the hyperfine interaction. The corresponding eigenvalue equation of the three-quark system can be solved with the SVM discussed below\(^2\). With such a model one can describe the spectra of all low-lying light and strange baryons in a unified framework.

\(^2\)Alternatively, one can apply a completely different approach, i.e. solving the three-body Faddeev integral equations. For more details concerning this approach, see Refs. [62, 63].
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Relativistic Kinetic Energy

The kinetic-energy operator is taken in the relativistic form

\[ H_{\text{free}} = \sum_{i=1}^{3} \sqrt{m_i^2 + k_i^2}, \] (2.3)

where \( m_i \) and \( k_i \) represent the masses and the three-momenta of the constituent quarks in the center-of-momentum frame, respectively. Non-relativistic approaches always face some disturbing problems as, e.g., \( v/c > 1 \). Above all, the implementation of a relativistic kinetic energy operator is required for setting up a Poincaré-invariant mass operator in relativistic (Hamiltonian) quantum mechanics. Thus, the relativistic treatment of the kinetic-energy operator allows for the inclusion of relativistic kinematical effects, and it is needed with regard to the further application of the CQM wave functions, here, especially for the covariant calculation of the decay widths.

Confinement

The confinement potential is taken in the linear form

\[ V_{\text{conf}}(r_{ij}) = V_0 + C r_{ij}, \] (2.4)

with a strength \( C \) of about the magnitude of the string tension of QCD (see, e.g., Ref. [64]). \( V_0 \) is an additional constant in order to fix the nucleon ground state to 939 MeV.

GBE Potential

For the derivation of the pseudoscalar GBE potential one starts out from an effective Lagrangian [42] coupling the constituent quark fields to Goldstone boson fields

\[ \mathcal{L}_\chi \propto ig\bar{\psi}\gamma_5 \chi^F \cdot \bar{\psi}. \] (2.5)

Here one employs a pseudoscalar coupling, where \( \psi \) denote the fermion (constituent-quark) fields, \( \phi \) the pseudoscalar boson field, \( g \) represents the coupling constant, and \( \chi^F \) are the \( SU(3)_F \) Gell-Mann flavor matrices (see Appendix B). It should be noted, that a chirally symmetric Lagrangian would additionally require a scalar meson field to complete the chiral multiplet. However, it can effectively be incorporated in the (phenomenological) confining potential. Performing a non-relativistic reduction in the constituent-quark spinors one can (in lowest order) deduce a potential between two constituent quarks \( i \) and \( j \). The corresponding octet potential includes a spin-spin and a tensor part

\[ V^\text{octet}_\chi(r_{ij}) = \left[ \sum_{a=1}^{3} V^S_m(r_{ij}) \lambda^a_i \lambda^a_j + \sum_{a=4}^{7} V^S_K(r_{ij}) \lambda^a_i \lambda^a_j + V^S_\eta(r_{ij}) \lambda^8_i \lambda^8_j \right] \sigma(i) \cdot \sigma(j). \]
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\[ + \left[ \sum_{a=1}^{3} V_{\pi}^T(\mathbf{r}_{ij}) \lambda^a_i \lambda^a_j + \sum_{a=4}^{7} V_{K}^T(\mathbf{r}_{ij}) \lambda^a_i \lambda^a_j + V_{\eta}^T(\mathbf{r}_{ij}) \lambda^S_i \lambda^S_j \right] \hat{S}_{ij}, \quad (2.6) \]

where \( \sigma(i) \) represents the spin of constituent quark \( i \) and \( \hat{S}_{ij} \) is the tensor operator given by

\[ \hat{S}_{ij} = 3\sigma(i) \cdot \hat{r}_{ij} \sigma(j) \cdot \hat{r}_{ij} - \sigma(i) \cdot \sigma(j). \quad (2.7) \]

The spin-spin part represents the most dominant contribution to the hyperfine interaction in baryons. Therefore, one also may neglect the tensor component. The pseudoscalar GBE CQM of Ref. \[41\] includes only a spin-spin dependent potential. In an extension of the model \[45\] one also assumed the pseudoscalar tensor force as well as further potentials deriving from scalar- and vector-meson exchanges.

In Eq. (2.6) one assumes isospin symmetry, i.e. one neglects small differences in the \( u \) and \( d \) constituent-quark masses and in the masses within the pion and kaon multiplets (e.g., \( V_{\pi^+} = V_{\pi^0} = V_{\pi^-} \)). However, we expect the \( SU(3)_F \) symmetry to be broken, i.e. the pion, kaon, and \( \eta \) exchange interactions are not the same (\( V_{\pi} \neq V_{K} \neq V_{\eta} \)) due to the different masses of light and strange quarks (\( m_u = m_d \neq m_s \)) and the mesons (\( m_{\pi} \neq m_{K} \neq m_{\eta} \)), respectively. Pion exchange takes place for pairs of light quarks, kaon exchange is only possible for a pair of quarks with different masses, and \( \eta \) exchange is possible for every type of quark pairs.

Since in the large-\( N_C \) limit the \( \eta' \) would become the ninth Goldstone boson, it is advantageous to consider a certain contribution of the flavor-singlet exchange

\[ V_{\chi^{\text{singlet}}}(\mathbf{r}_{ij}) = \left[ V_{\eta}^S(\mathbf{r}_{ij}) \sigma(i) \cdot \sigma(j) + V_{\eta'}^T(\mathbf{r}_{ij}) \hat{S}_{ij} \right] \lambda^0_i \lambda^0_j. \quad (2.8) \]

Here, \( \lambda^0 = \sqrt{\frac{2}{3}} \mathbf{1}_3 \), with the \((3 \times 3)\) identity matrix \( \mathbf{1}_3 \), is the ninth generator stemming from \( U(1) \), in addition to the eight generators of \( SU(3) \). The product \( \lambda^0_i \lambda^0_j \) thus leads to a factor \( \sqrt{\frac{2}{3}} \). Within the pseudoscalar GBE CQM one also neglects the tensor part of the singlet potential.

Next, we briefly consider the radial dependence of the meson-exchange potentials. Assuming that the boson fields satisfy the linear Klein-Gordon equation and that mesons and quarks are point-like particles one obtains for the spin-spin part in an instantaneous approximation

\[ V_{\gamma}^S(\mathbf{r}_{ij}) = \frac{g_{\gamma}^2}{4 \pi} \frac{1}{12 m_i m_j} \left\{ m_{\gamma}^2 e^{-m_{\gamma} r_{ij}} - 4 \pi \delta(\mathbf{r}_{ij}) \right\}, \quad (2.9) \]

for \( \gamma = \pi, K, \eta, \eta' \). Here, \( g_{\gamma}^2 \) represent the meson-quark coupling constants, \( m_{\gamma} \) are the phenomenological meson masses, and \( m_i \) are the constituent quark masses. The potential exhibits a typical Yukawa behaviour at long
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range, and the short-range contact interaction is given by the $\delta$-function. Since the baryon wave function is localized within a relatively narrow area, the latter part is of crucial importance. At small distances one has to take into account that mesons and most probably also the constituent quarks are structured particles with a finite size. Furthermore, boson fields cannot be described by a linear equation near their source. Hence, the contact term of the potential has to be modified accordingly, i.e. the $\delta$-function has to be smeared out. This is also required for practical reasons: In case of an attractive $\delta$-function contact interaction the baryon spectrum would not be bounded from below. One can apply a Yukawa-type smearing

$$4\pi\delta(r_{ij}) \rightarrow \Lambda^2_\gamma e^{-\Lambda_\gamma r_{ij}}/r_{ij}, \quad \gamma = \pi, K, \eta, \eta'$$ (2.10)

corresponding to a form factor in the momentum-space representation of $V^S_\gamma$

$$F(q^2) = \sqrt{\Lambda^2_\gamma - m^2_\gamma}/\Lambda^2_\gamma + q^2$$ (2.11)

with $q$ being the three-momentum transfer. The parameters $\Lambda_\gamma$ act as cut-off parameters for large momenta and may depend on the individual exchanged mesons. Adopting the Yukawa-type smearing (2.10), Eq. (2.9) finally becomes

$$V^S_\gamma(r_{ij}) = \frac{g^2_\gamma}{4\pi} \frac{1}{12m_im_j} \left\{ m^2_\gamma e^{-m_\gamma r_{ij}}/r_{ij} - \Lambda^2_\gamma e^{-\Lambda_\gamma r_{ij}}/r_{ij} \right\},$$ (2.12)

for $\gamma = \pi, K, \eta, \eta'$.

Parameters in the GBE CQM

The parameterization of the pseudoscalar GBE CQM [41] results from a fit to the baryon spectra. In order to keep the number of free parameters as small as possible one assumed for the cut-off parameter $\Lambda_\gamma$ a linear scaling prescription

$$\Lambda_\gamma = \Lambda_0 + \kappa m_\gamma, \quad \gamma = \pi, K, \eta, \eta',$$ (2.13)

where only two free parameters $\Lambda_0$ and $\kappa$ are involved. Here, the cut-off parameter increases with the meson mass $m_\gamma$ in order to provide proper relative contributions of the various meson-exchange potentials.

Since the chiral symmetry of QCD is explicitly broken, the various meson-quark coupling constants could naturally be different. Again trying to keep the number of free parameters as small as possible one assumed only one single octet-quark coupling for all octet mesons $\pi, K$, and $\eta$

$$\frac{g^2_{\pi\pi\eta}}{4\pi} = \frac{g^2_\pi}{4\pi} = \frac{g^2_K}{4\pi} = \frac{g^2_\eta}{4\pi}.$$ (2.14)
The value of the pion-quark coupling constant \( g_2^2 \) can be deduced from the phenomenological pion-nucleon coupling (see, e.g., Ref. [65]) using the Goldberger-Treiman relations for both the pion-quark and the pion-nucleon vertex. Inserting the light quark mass \( m_u = 340 \text{ MeV} \) and 939 MeV for the nucleon ground state into the corresponding relation, one finds the coupling constant to be 0.67. The \( \eta' \) meson decouples from the octet, and therefore the flavor-singlet coupling constant may be different from the octet one. This is taken into account by treating the ratio \( \left( \frac{g_0}{g_{qqm}} \right)^2 \) as a free parameter.

The confinement potential brings about two further parameters \( V_0 \) and \( C \), and consequently the total quark-quark potential of the three-quark Hamiltonian derived in [41] includes altogether five free parameters. The other parameters, namely the constituent quark masses as well as the meson masses, and the already mentioned pion-quark coupling constant, are pre-determined by other sources. These ”fixed” parameters together with the free parameters are listed in Table 2.1. More details concerning the parameterization, and moreover the fit to the baryon spectra, can be found in Ref. [55].

### 2.2.3 Mass Spectra of the GBE CQM

Applying the discussed parameterization, one obtains the baryon spectra as shown in Fig. 2.1. The corresponding mass values of the light and strange baryons are listed in Tables D.1 - D.6. One can see immediately that all light and strange baryon ground states can generally be reproduced in good agreement with the phenomenological data (compare with the PDG [2]). In particular, the level orderings of the lowest positive- and negative-parity states in the nucleon spectrum are reproduced correctly. Namely, the \( \frac{1}{2}^- \) Roper resonance \( N(1440) \) lies well below the negative-parity \( \frac{1}{2}^- \) and \( \frac{3}{2}^- \) states \( N(1535) \) and \( N(1520) \), respectively. Simultaneously, in the strange \( \Lambda \) spectrum the positive-parity \( \frac{1}{2}^+ \) excitation \( \Lambda(1600) \) falls below the negative-
parity $\frac{1}{2}^-$ and $\frac{3}{2}^-$ states $\Lambda(1670)$ and $\Lambda(1690)$; and the same is true for the $\frac{1}{2}^+$ excitation $\Sigma(1660)$, which falls below the $\frac{1}{2}^-$ state $\Sigma(1750)$. At the same time, the negative-parity $\frac{1}{2}^-$ and $\frac{3}{2}^-$ states $\Lambda(1405)$ and $\Lambda(1520)$ are the lowest excitations above the $\Lambda$ ground state just as demanded from phenomenology. However, it should be noted, that in all CQMs the $\Lambda(1405)$ cannot be reproduced satisfactorily, i.e. the mass value is by far too large. Since this resonance lies close to the $KN$ threshold \cite{66}, a coupling to the corresponding decay channel should be explicitly included. It may be expected that the resonance mass will then be shifted down.

The overall good agreement of the GBE CQM with the experimental data can be attributed to the explicit spin-flavor dependence of the chiral interaction. Here, the flavor-dependent factor $\lambda_i^f \lambda_j^f$ in the spin-spin interaction leads to the proper orderings of the positive and negative parity states in the baryon spectra.

### 2.3 One-Gluon-Exchange CQM

In order to compare the influences of different kinds of quark-quark dynamics on the decay widths we employ in this thesis also a typical OGE CQM, namely, the one based on the model of Bhaduri, Cohler and Nogami \cite{67}. While this model was originally set up in a non-relativistic framework, we use here a relativized version as parametrized in Ref. \cite{19}. The corresponding three-quark Hamiltonian is of the form

$$H = H_{\text{free}} + \sum_{i<j=1}^{3} V_{ij}, \quad (2.15)$$

where $H_{\text{free}}$ is the relativistic kinetic energy as in Eq. (2.3), and the quark-quark potential $V_{ij}$ contains a confinement plus hyperfine potential. It is explicitly given as

$$V_{ij} = V_0 + C r_{ij} - \frac{2b}{3r_{ij}} + \frac{\alpha_S}{9m_i m_j} \Lambda^2 e^{-\Lambda r_{ij}} \frac{s_i \cdot s_j}{r_{ij}}, \quad (2.16)$$

i.e. it consists of a linear confinement with the parameters $V_0$ and $C$, a short-range Coloumb term including a strength parameter $b$, and a flavor-independent spin-spin interaction including an effective coupling constant $\alpha_S$ as well as a cut-off parameter $\Lambda$. In the original work, the parameters were determined from a fit to the meson spectra \cite{67}. For the version used here, the parameters have been obtained by a fit to the baryon spectra, and their values are summarized in Table 2.2.
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Figure 2.1: Energy levels of the lowest light and strange baryon states for the GBE CQM of Ref. [41]. The mass of the nucleon ground state is 939 MeV. The shadowed boxes represent the experimental values together with their uncertainties [2].
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\begin{table}[h]
\centering
\begin{tabular}{lcc}
\hline
 & $C$ & 3.12 [fm$^{-2}$] \\
& $V_0$ & $-409$ [MeV] \\
& $b$ & 0.57 \\
\hline
$m_u = m_d$ & 337 [MeV] \\
m$_s$ & 600 [MeV] \\
$\alpha_s$ & 0.57 \\
$\Lambda$ & 2.7 [fm$^{-1}$] \\
\hline
\end{tabular}
\caption{Parameters of the OGE CQM after Ref. [19].}
\end{table}

Again, the eigenvalue problem is solved with the SVM, and the corresponding mass spectra of the lowest light and strange baryons are shown in Fig. 2.2. The explicit mass values of the various baryon states can be found in Tables D.1 - D.6. With respect to the baryon ground states it is immediately evident that the OGE CQM exhibits the same qualitative picture as the GBE CQM. However, it is seen that the right level ordering of positive- and negative-parity nucleon and $\Delta$ excitations cannot be reproduced. Furthermore, also the simultaneous description of the light and strange baryon states cannot be provided satisfactorily in all respects. This can primarily be attributed to the missing of an explicit flavor dependence in the quark-quark interaction.

2.4 The Stochastic Variational Method

Variational methods (VM) are standard techniques to solve the eigenvalue problems of quantum-mechanical Hamiltonians. For the solution of the associated bound-state problem the choice of suitable trial functions is important. Here we will outline the idea of a specific type of VM, namely the stochastic VM (SVM) introduced by Kukulin et al. [68] and extensively examined in Ref. [69]. In recent years, the Graz group adopted this method for the calculation of the light and strange baryon spectra. In the following we shortly outline the main characteristics of the SVM, where we closely follow the formalism as outlined in my diploma thesis (see Ref. [70]).

It was shown by Suzuki and Varga [69] that suitable basis functions for few-body systems can be constructed with so-called correlated Gaussians. They are expressed in terms of a set of Jacobi coordinates $x=\{x_1, \ldots, x_{N-1}\}$. Accounting for the ingredients needed in the baryon wave function, the complete three-particle basis functions then become

$$\psi_{(LS)\Sigma M_\Sigma TM_T}(x, A) = \mathcal{S} \cdot \left\{ e^{-\frac{1}{2} k^\dagger A x} [\Theta_{LM_L}(\tilde{x}) \chi_S]_{\Sigma M_\Sigma} \phi_{TM_T} \right\}, \quad (2.17)$$

where $\Sigma$ and $M_\Sigma$ denote the intrinsic baryon spin and its $z$-projection. The functions $\chi$ as well as $\phi$ describe the spin and the flavor parts of the full
Figure 2.2: Energy levels of the lowest light and strange baryon states for the OGE CQM of Ref. [19]. The mass of the nucleon ground state is 939 MeV. The shadowed boxes represent the experimental values and their uncertainties [2].
basis function, respectively. The function $\Theta_{LM_L}(\mathbf{x})$ represents the angular part of the center-of-momentum wave function, which depends on the total orbital angular momentum $L$ and its projection $M_L$ as well as on several variational parameters. The $\mathbf{x}$ stands for the row vector comprising the Jacobi coordinates, and $\mathbf{\tilde{x}}$ represents its transpose. The $((N-1) \times (N-1))$ dimensional matrix $A$ in the exponential function is positive-definite and symmetric and contains nonlinear parameters $\alpha$, specific for each basis element. Finally, the operator $S$ acting on the right-hand side of Eq. (2.17) symmetrizes the wave function of the three-particle system with respect to the interchange of identical particles. Generally, this operator can be expressed by a combination of the permutations $P$ with suitable phases.

Baryons are considered as composites of three quarks obeying Fermi statistics, therefore the resulting baryon wave function has to be totally antisymmetric. Furthermore, baryons are colorless objects building a color-singlet state. In our calculations we split up the baryon wave function in two parts, namely the color part and the ”rest”, where the latter can be expanded with the trial wave functions (2.17). Then, in order to comply with the generalized Pauli exclusion principle the baryons are symmetric with respect to the interchange of any two particles concerning the product of the spin-, flavor-, and space parts of the wave function.

2.4.1 Solution of Eigenvalue Problems with the Variational Method

For a given Hamiltonian $H$ of a few-body system, which is time-independent and bounded from below, one is generally interested in finding the discrete eigenvalues as well as the corresponding eigenstates. Thus one has to solve the eigenvalue problem

$$H\Psi_n = E_n\Psi_n, \quad n = 1, 2, ..., \quad (2.18)$$

where the energies $E_n$ are real and we assume that the ground state is non-degenerate. Depending on the potential included in the Hamiltonian, one obtains a specific eigenvalue spectrum, e.g., a confining interaction leads in principle to an infinite number of eigenvalues. Although we know the Hamiltonian, it is generally difficult to solve the eigenvalue equation (2.18).

The VM used in this work provides the approximate determination of the baryon energy spectra, and it is based on the Rayleigh-Ritz principle. For the ground state energy it says the following: For an arbitrary function $\Psi$ of the state space the expectation value of $H$ in the state $\Psi$ is such that

$$E \equiv \frac{\langle \Psi \vert H \vert \Psi \rangle}{\langle \Psi \vert \Psi \rangle} \geq E_1, \quad (2.19)$$

where the equality holds if and only if $\Psi$ is an eigenstate of $H$ with eigenvalue $E_1$. An arbitrary function $\Psi \in \mathcal{H}$ is given as a linear combination of (a
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probably infinite number of) basis functions spanning the Hilbert space \( \mathcal{H} \).

Now within the VM one tries to minimize the energy functional with respect to a set of variational parameters \( A \), i.e. one calculates

\[
\langle \Psi(A)|H|\Psi(A) \rangle = E(A) \langle \Psi(A)|\Psi(A) \rangle,
\]

where \( \Psi(A) \) is an arbitrary function in \( \mathcal{H} \) normalized to \( \langle \Psi(A)|\Psi(A) \rangle = 1 \). The aim is to find an optimized set \( A_{opt} \) leading to an energy \( E(A_{opt}) \) that approximates the true ground state energy as accurately as possible.

In our calculations we apply the generalized Ritz theorem [71], which allows also for the calculation of the excited states of the Hamiltonian. For the moment let us assume for simplicity that the parameter set \( A \) contains \( N \) parameters \( c_i \), i.e. \( A = \{c_i, i = 1, 2 \ldots N\} \), which are the coefficients of a linear combination of \( N \) linear independent basis functions \( \psi_i \). The variational wave functions \( \Psi(A) \) are expanded in the form

\[
\Psi(A) = \sum_{i=1}^{N} c_i \psi_i.
\]

Obviously, the test functions \( \psi_i \) do not span the complete Hilbert space. Consequently the true eigenfunctions \( \Psi_E \), corresponding to the true eigenvalues \( E \) of a given Hamiltonian, are only approximated by \( \Psi(A) \). Now in order to find the lowest possible upper bounds for the true ground-state energy as well as for the energies of the excited states, one inserts the expression (2.21) into Eq. (2.18) and obtains

\[
\sum_{i=1}^{N} \sum_{j=1}^{N} c_i c_j^* \langle \psi_j|H|\psi_i \rangle = E_N \sum_{i=1}^{N} \sum_{j=1}^{N} c_i c_j^* \langle \psi_j|\psi_i \rangle,
\]

which then reduces to a generalized matrix eigenvalue problem inside an \( N \)-dimensional state space spanned by the set \( \{\psi_i, i = 1, 2 \ldots N\} \)

\[
\sum_{i=1}^{N} c_i H_{ji} = E_N \sum_{i=1}^{N} c_i B_{ji}.
\]

Here \( H_{ji} \) as well as \( B_{ji} \) are the matrix elements of the \((N \times N)\) matrices \( \mathbf{H} \) and \( \mathbf{B} \)

\[
H_{ji} = \langle \psi_j|H|\psi_i \rangle, \quad B_{ji} = \langle \psi_j|\psi_i \rangle.
\]

In case of an orthonormal basis the eigenvalue problem would reduce to the standard eigenvalue problem with \( B_{ji} = \delta_{ji} \). By means of linear algebra methods for solving the eigenvalue problem Eq. (2.23) an accurate numerical solution can be found for the \( N \) eigenvalues \( E_{N,k} \). Here, the lowest eigenvalue of Eq. (2.23) corresponds to the upper bound of the true ground-state energy, and similarly, the eigenvalues \( E_{N,k} \) for \((k = 2, \ldots N)\) represent the upper
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bounds for the true energies of the \((N-1)\) excited states of the Hamiltonian. The variational eigenfunctions for the ground as well as for the excited states can be written as

\[
\Psi_k = \sum_{i=1}^{N} c_{i,k} \psi_i, \quad (k = 1, \ldots N),
\]

(2.25)

where \(c_{i,k}\) are the corresponding variational parameters. The normalization of the states is given by

\[
\sum_{j,i=1}^{N} c_{j,k}^* B_{ji} c_{i,k} = 1.
\]

(2.26)

Clearly, the energies will most probably not have reached their optimal values at once. Thus in order to reduce the error of the upper bounds one aims at an improvement. One would first think of an enhancement of the number of basis functions. Unfortunately, by systematically extending the set of states from a complete basis of the Hilbert space the time needed to solve the eigenvalue problem will grow extensively.

2.4.2 Basis Optimization via the Stochastic VM

Some basis states contained in a huge basis may not be really important for the solution of a specific problem, and therefore one better finds an alternative way to decrease the errors of the upper bounds. Demanding the basis size to be as small as possible one tries a refined ansatz. The basis functions are now assumed to depend in addition on a set of nonlinear variational parameters\(^3\) \(\alpha_i\):

\[
\Psi(A) = \sum_{i=1}^{N} c_i \psi(\alpha_i).
\]

(2.27)

Here, \(A\) denotes the set \(\{(c_i, \alpha_i), \ i = 1, 2, \ldots N\}\) containing both the linear and nonlinear variational parameters.

Within the concept of the SVM one searches now only for basis states that are important for a given problem, where the essential point is the random selection of pertinent parameters. Over the years this method has been taken up and elaborated for applications in various few-body systems, especially by Varga and Suzuki [69,71,72]. For the proper description of the correlations between quarks building up baryons they used in their calculations a correlated Gaussian “basis” [69]. In this sense, the CQMs being the basis of this work are based on the suggested approach.

\(^3\)The set \(\alpha_i\) may contain discrete (such as angular momentum, flavor content, etc.) as well as continuous parameters included in the spatial wave function.
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There exists no universal prescription how to realize the random selection to improve the upper bounds, rather one can follow various strategies with equal quality. Here we will outline one specific concept that is based on a trial and error procedure, combined with an admittance test.

Our starting point is a single basis state. We generate a number of random parameter sets $\alpha_i^{\text{rand}}$ and calculate the matrix elements $\langle \psi(\alpha_i^{\text{rand}}) | H | \psi(\alpha_i^{\text{rand}}) \rangle$ of the Hamiltonian for all of the corresponding basis states. We then adopt as the first basis state the function $\psi(\alpha_1)$ with the property

$$\langle \psi(\alpha_1) | H | \psi(\alpha_1) \rangle = \min_{i} \langle \psi(\alpha_i^{\text{rand}}) | H | \psi(\alpha_i^{\text{rand}}) \rangle.$$  \hfill (2.28)

Now we add step by step new basis states according to the following criteria:

a.) Create a number of random parameter sets $\alpha_i^{\text{rand}}$.

b.) Add the corresponding basis states separately to the already existing basis and solve the resulting $(N+1)$-dimensional eigenvalue problems. This leads to $(N+1)$ eigenvalues $E_{N+1,n}(\alpha_i^{\text{rand}})$ in each case.

c.) As the new basis state $\psi(\alpha_{N+1})$ choose the one that satisfies

$$E_{N+1,1}(\alpha_{N+1}) = \min_{i} E_{N+1,1}(\alpha_i^{\text{rand}}),$$  \hfill (2.29)

i.e. try to decrease the upper bound for the ground state energy in a maximal manner.

In order to achieve the desired accuracy of the ground state energy, one has to control the convergence of the eigenvalue $E_{N+\nu,1}$ by measuring the shift during the extension of the basis size from $N$ to $N + \nu$ basis states. Embarking on this strategy one can build a stochastically optimized basis of any desired final size.

If one is interested in keeping the dimension of the basis fixed, one can simply generate a random basis state and replace a single, stochastically chosen state of the already existing basis by it. If the replacement decreases the lowest upper bound for the ground state it is accepted, otherwise one keeps the original basis state.

As we are not only interested in the lowest eigenvalue but also in some excited baryon states, it is necessary to minimize also a certain number of higher states. Thus one has to modify the random selection condition for a trial state. In order to obtain good upper bounds for the energies of the lowest $n$ states one selects those basis states $\psi(\alpha_j^{\text{rand}})$, which lead to a maximal sum of decreases of the lowest $n$ eigenvalues

$$\sum_{k=1}^{n} \omega_k (E_{N,k} - E_{N+1,k}(\alpha_j^{\text{rand}})).$$  \hfill (2.30)
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Here one can introduce weights $\omega_k > 0$ determining the relative importance of some states as compared to the others. The energy shifts $E_{N,k} - E_{N+1,k} (\alpha_j^{\text{rand}})$ always have to be positive. Consequently every new basis state added to the already existing $N$ basis states has to improve all upper bounds. For the additional eigenstate in each step the inequality $E_{N+1,N+1} (\alpha_j^{\text{rand}}) \geq E_{N,N}$ has to be satisfied. Using the known solution of the $N$-dimensional generalized eigenvalue problem one can reduce the $(N + 1)$-dimensional problem. One has to find the $(N + 1)$ roots of the secular equation [69]

$$
\lambda(E) = \prod_{i=1}^{N} (E_{N,i} - E) \left\{ (a - E) - \sum_{j=1}^{N} q_j^2 \frac{j}{E_{N,j} - E} \right\} = 0.
$$

(2.31)

This procedure drastically reduces the time needed to perform the diagonalization since some profitable properties can be used (see, e.g., Ref. [55]).

To measure the accuracy one considers the shifts of the $n$ lowest eigenstates during the steps from basis size $(N - \nu)$ to the basis size $N$

$$
\delta E_{n,\nu} = \sum_{k=1}^{n} (E_{N-\nu,k} - E_{N,k}),
$$

(2.32)

where we take $\nu > 1$ to average over statistical fluctuations, which appear by enlarging the basis.

Recapitulating, one can say that the concept of the SVM, i.e. using a stochastic selection of basis parameters, leads to energy convergence independently of different random paths. This is of utmost importance to avoid falling into local minima. Furthermore, the results for the eigenenergies of the ground as well as the excited states can be made rather accurate. It is not necessary to compute the whole matrix elements of the Hamiltonian and therefore the full diagonalizations [71]. An advantage lies in the economy of time and the lower computational effort as compared with other optimization strategies, such as the ones in Refs. [73,74].

### 2.5 Application of the SVM to Three-Quark Systems

In CQMs baryons are considered as systems of three constituent quarks. In the previous section we introduced the SVM as a viable approach to tackle many-body problems. Here we detail the SVM for the three-quark system and discuss the resulting baryon wave function.

#### 2.5.1 Baryon Wave Functions

Generally, the baryon wave function $\Psi_{XSC}$ can be expressed as a combination of spatial ($X$), spin ($S$), flavor ($F$), and color ($C$) degrees of freedom.
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Constituent quarks are fermions, therefore the baryon wave function has to be totally antisymmetric due to the Pauli exclusion principle. Since all baryons have to be color singlets, one can separate out the color part and hence the baryon wave function can be written as

\[ \Psi_{XSF} = \Psi_{XSF} \Psi^{\text{singlet}}_C, \]  

(2.33)

where \( \Psi^{\text{singlet}}_C \) is the totally antisymmetric color part and \( \Psi_{XSF} \) has then to be totally symmetric. Any operator \( \mathcal{O} = \mathcal{O}_{XSF} \mathcal{O}_C \) acting on the whole space can thus also be split into an operator \( \mathcal{O}_{XSF} \) (acting in the subspace of the spatial, spin, and flavor coordinates) times an operator \( \mathcal{O}_C \), which always contributes in the same way to the matrix elements of the full operator. The contribution of the color part \( \Psi^{\text{singlet}}_C \) can thus be implemented by restricting the calculation to the symmetric part \( \Psi_{XSF} \) of the baryon wave function. In the following, we denote this part of the baryon wave function as the total wave function.

2.5.2 Basis Functions

In our approach, the baryon wave functions are produced via the SVM, i.e. one performs an expansion into a (finite) number of basis functions, satisfying the specific symmetry conditions of the particular baryon states. Thus, in order to obtain a totally symmetric wave function one employs basis functions \( \psi^S(\alpha_i) \), where the superscript \( S \) emphasizes that the functions are properly symmetrized. The baryon wave function then reads

\[ \Psi_{XSF} = \sum_i c_i \psi^S(\alpha_i), \]  

(2.34)

where \( \alpha_i \) denotes the sets of nonlinear variational parameters defining the basis functions, and the linear coefficients \( c_i \) represent the corresponding amplitudes. For the construction of the basis functions\(^4\) \( \psi^S_\alpha \) we first introduce functions \( \psi^{(k, pq)}_\alpha \) and demand

\[ \psi^{(k, pq)}_\alpha = \psi^{(k, qp)}_\alpha, \]  

(2.35)

where \((k, p, q)\) is a permutation of \((1, 2, 3)\). This means that the functions \( \psi^{(k, pq)}_\alpha \) are symmetric with respect to the exchange of particles \( q \) and \( p \), which are considered to be the components of the basis function of a certain configuration \((k, pq)\). The third particle \( k \) specifies the configuration and no symmetry with respect to the exchange of particles involving particle \( k \) is required. For the construction of the full basis function \( \psi^S_\alpha \) one then

\(^4\)For simplicity we neglect in the following the index \( i \) specifying the current basis function in Eq. (2.34), and we keep the \( \alpha \) specifying the nonlinear variational parameters as a subscript.
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builds the sum over basis functions of all three possible configurations \((k,pq)\), \((p,qk)\), and \((q,kp)\) for the three-particle system. This results in the following expression:

\[
\psi^S_\alpha = \psi^{(k,pq)}_\alpha + \psi^{(p,qk)}_\alpha + \psi^{(q,kp)}_\alpha.
\]  

(2.36)

Evidently, Eq. (2.36) is totally symmetric under interchange of any of the three particles as required for the total wave function.

**Introduction of Jacobi Coordinates**

For the spatial part of the wave function in the center-of-momentum frame we introduce so-called Jacobi coordinates in order to describe the internal motion of the three-quark system in a proper way. Let us first restrict ourselves to a basis function in configuration\(^5\) \((k)\). The corresponding set of Jacobi coordinates comprises two relative coordinates defined by

\[
\xi^{(k)}_k = x_p - x_q,
\]

\[
\eta^{(k)}_k = x_k - \frac{m_p x_p + m_q x_q}{m_p + m_q},
\]  

(2.37)

with \(x_i\) and \(m_i\) the individual particle coordinates and masses, respectively. Note, that both the configuration and the so-called Jacobi partition are labelled by indices \(k\), \(p\), and \(q\). In order to distinguish between them we denote the configuration numbers as superscripts in brackets and use subscripts for the Jacobi coordinates given in a specific partition. An illustration of Eq. (2.37) is given in the middle picture of Fig. 2.3. The coordinate \(\xi^{(k)}_k\) defines the relative coordinate between the quarks \(p\) and \(q\), whereas the \(\eta^{(k)}_k\) represents the relative coordinate among the remaining quark \(k\) and the center of mass of the first two quarks \(p\) and \(q\). Eq. (2.37) represents one out of three possible partitions for the Jacobi coordinates, namely partition \(k\). With \(k\), \(p\), and \(q\) corresponding to an even permutation of \((1,2,3)\), one

---

\(^5\)For abbreviation we introduce the odd-man-out notation \(\psi^{(k,pq)}_\alpha = \psi^{(k)}_\alpha\) for the basis function of the configuration \((k,pq)=(k)\). In the same way we use the notations \(\psi^{(p,qk)}_\alpha = \psi^{(p)}_\alpha\) and \(\psi^{(q,kp)}_\alpha = \psi^{(q)}_\alpha\) for \((p,qk)=(p)\) and \((q,kp)=(q)\), respectively.
The three different partitions \( k, p, \) and \( q \) for three particles in a given configuration \((k)\) are shown in Fig. 2.4.

Up to now we have not specified the dependence of the full basis function \( \psi_\alpha^S \) on any set of coordinates in configuration space. With the introduction of the Jacobi coordinates we are now in the position to rewrite Eq. (2.36).

In order to keep the right symmetries one would first think of

\[
\psi_\alpha^S = \psi_\alpha^{(k)}(\xi_k^{(k)}, \eta_k^{(k)}) + \psi_\alpha^{(p)}(\xi_p^{(p)}, \eta_p^{(p)}) + \psi_\alpha^{(q)}(\xi_q^{(q)}, \eta_q^{(q)}),
\]

where the partition is always the same as the configuration. Of course, it is also possible to construct a full basis function \( \psi_\alpha^S \) depending only on one specific pair of Jacobi coordinates, e.g., \( \xi_k \) and \( \eta_k \),

\[
\psi_\alpha^S(\xi_k, \eta_k) = \psi_\alpha^{(k)}(\xi_k^{(k)}, \eta_k^{(k)}) + \tilde{\psi}_\alpha^{(p)}(\xi_p^{(p)}, \eta_p^{(p)}) + \tilde{\psi}_\alpha^{(q)}(\xi_q^{(q)}, \eta_q^{(q)}),
\]

where

\[
\tilde{\psi}_\alpha^{(p)}(\xi_k^{(p)}, \eta_k^{(p)}) = \psi_\alpha^{(p)}(\xi_p^{(p)}, \eta_k^{(p)}), \quad \tilde{\psi}_\alpha^{(q)}(\xi_k^{(q)}, \eta_k^{(q)}) = \psi_\alpha^{(q)}(\xi_k^{(q)}, \eta_q^{(q)}).
\]

Clearly, in either configuration-space representation the basis functions in a given configuration have to be symmetric under the exchange of the clustered particles.

Let us first concentrate on the basis function \( \psi_\alpha^{(k)}(\xi_k^{(k)}, \eta_k^{(k)}) \). It represents the simplest case one can imagine, namely, partition and configuration...
concoincide. We also assume, that in configuration \((k)\), the corresponding particle is the "different" one (see Fig. 2.3, middle), and the subsystem of the particles \(p\) and \(q\) is specially marked due to the specific symmetry between these two particles\(^6\). For the construction of the full basis function of a specific configuration \((k)\) it is necessary to combine the spatial, spin, and flavor parts. With regard to the analysis of the respective functional dependences and the appropriate transformation properties, an extensive discussion can be found in Appendix A.

In the center-of-momentum frame\(^7\) we couple the spatial wave function \(\varphi_{(\beta,\gamma=0,\delta,\nu,n,\lambda,l,L,M_L)}(k)\) of total orbital angular momentum \(L\) with the spin basis function of total spin \(S\) to a space-spin basis function, which leads to the intrinsic baryon spin \(\Sigma\) and its \(z\)-projection \(M_{\Sigma}\). Further coupling with a flavor basis state yields the complete basis function corresponding to a state characterized by the intrinsic spin \(\Sigma\), \(z\)-component \(M_{\Sigma}\), hypercharge \(Y\), and total isospin \(T\) as well as isospin \(z\)-component \(M_T\).

\[
\psi_{\Delta}(\xi^{(k)}_k, \eta^{(k)}_k) = \psi_{(\beta,\delta,\nu,n,\lambda,l,L,s,S,F}{\Sigma}_{\Sigma = Y,T}{M_T}(\xi^{(k)}_k, \eta^{(k)}_k) \\
= \left\{ \varphi_{(\beta,\delta,\nu,n,\lambda,l,L,M_L)}(\xi^{(k)}_k, \eta^{(k)}_k) \otimes \chi_{(s,S,M_S),k} \right\}_{\Sigma_{\Sigma = Y,T}{M_T}} \psi_{\Delta}. \tag{2.42}
\]

In this case, the basis function has already a definite symmetry with respect to the exchange of the particles \(p\) and \(q\). The flavor and spin parts have an obvious symmetry under the exchange of particles. Here, also the spatial part has a definite symmetry, because the exchanged particles are the clustered ones (see Appendix A.1). Such a basis function is called a "zero" basis function.

If we construct a basis function, where the configuration \((k)\) does not coincide with the partition, the spatial part does not have a definite symmetry under the exchange of the clustered particles \(p\) and \(q\) (unless, all three are identical). However, it is possible to restore the required symmetry using suitable linear combinations, yielding "plus" and "minus" basis functions.

\[
\psi^{(k)}_{(\beta,\delta,\nu,n,\lambda,l,L,s,S,F){\Sigma}_{\Sigma = Y,T}{M_T}(\xi^{(k)}_p, \eta^{(k)}_p)} \\
\pm \psi^{(k)}_{(\beta,\delta,\nu,n,\lambda,l,L,s,S,F){\Sigma}_{\Sigma = Y,T}{M_T}(\xi^{(k)}_q, \eta^{(k)}_q)}. \tag{2.43}
\]

Therefore, also with "plus" or "minus" basis functions it is possible to construct full basis functions \(\psi_{\Delta}\). The concept of "plus" and "minus" has to be introduced also for the spin part as it depends on the partition, but it does

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\(^6\)The concept "different" concerns in particular the quark masses appearing in the definition of the Jacobi coordinates. In the sector of light and strange baryons we deal with light \((m_u = m_d)\) and strange quarks \((m_s \neq m_u)\). Consequently, the particle with the different mass has to be at the stressed position. More details can be found in Appendix A.

\(^7\)Note that all baryon wave functions are calculated in the center-of-momentum frame. Only in this specific frame the total angular momentum \(J\) coincides with the intrinsic spin \(\Sigma\) of the respective baryons.
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not directly influence the flavor part, where the concept of a partition is not needed.

In order to distinguish between the three possible basis functions we introduce an extra parameter $b = 0, 1$ or $-1$ for "zero", "plus" and "minus". It allows us to classify the type of the basis function according to this construction. Thus the set of parameters

$$\alpha = \{\beta, \delta, \nu, n, \lambda, l, S, F, b\} \quad (2.44)$$

characterizes the complete basis function of configuration $(k)$

$$\psi_{(\alpha)\Sigma M_\Sigma Y TM_T}^{(k)} \quad (2.45)$$

It is symmetric under the exchange of particles $p$ and $q$ if

$$(-1)^\lambda(-1)^{s+1}(-1)^{P_F} = \begin{cases} 1, & \text{for } b = 0, 1, \\ -1, & \text{for } b = -1. \end{cases} \quad (2.46)$$

Here, the parameters $\lambda$, $s$, and $P_F$ correspond to the spatial, spin and flavor parts of the wave function, respectively. More details can be found in Appendix A.
Chapter 3

Relativistic Quantum Mechanics

The principle of relativity states that the laws of physics do not distinguish between different inertial coordinate systems. In this respect, Einstein’s principle of special relativity demands that a relativistic theory is invariant under the inhomogeneous Lorentz-transformations. The corresponding group is called inhomogenous Lorentz group or Poincaré group. In this sense, we denote relativistic quantum mechanics (RQM) to be a quantum mechanical theory that is invariant under the transformations of the Poincaré group.

Following closely the formalism outlined in Ref. [6] we first introduce the Poincaré group and discuss its implications on a free single massive particle with spin. However, within CQMs baryons are composed of three constituent quarks. Hence, we extend the considerations and investigate the impact of Poincaré invariance on the description of composite particles. The introduction of interactions into the baryon system then leads us to inherent difficulties that are overcome by the Bakamjian-Thomas (BT) construction. The BT construction can be implemented in a variety of schemes according to Dirac’s forms of RQM. In this thesis we concentrate on the so-called point form.

3.1 Introduction of the Poincaré Group

Einstein’s principle of special relativity states that the transformations relating different inertial systems preserve the proper time \( \tau = t^2 - x^2 \). A classical theory is relativistically invariant, if the solutions of the original dynamical equations are identical to the transformed ones; the solutions are the observables. However, in quantum physics the measurable quantities are not the solutions of the dynamical equation. Instead, the observables are the expectation values, which are constructed via scalar products of state vectors on a Hilbert space \( \mathcal{H} \). Therefore, a quantum mechanical symmetry
transformation has the property that its action on a state $|\psi\rangle$ resulting in a state $|\psi'\rangle$ leaves the probabilities unchanged. In RQM the symmetry transformations connecting different inertial systems form the Poincaré group. The general requirement of Poincaré invariance in quantum mechanics was studied originally by Wigner [75] and refined by Bargmann [76]. It can be stated as:

A quantum mechanical model formulated on a Hilbert space preserves probabilities in all coordinate systems if and only if the correspondence between states in different inertial coordinate systems can be realized by a single-valued unitary representation of the covering group of the Poincaré group.

The Poincaré transformations are given by the continuous four-dimensional (space-time) translations denoted by $a^\mu$, and the Lorentz-transformations $A_\nu^\mu$ containing the three-dimensional (space) rotations and the three-dimensional rotationless Lorentz-transformations (canonical boosts). Moreover, the Poincaré group has four disconnected components that are related by the discrete transformations of space inversion (parity transformation $P$) and/or time reversal $T$. Together with the identity $\mathbf{1}$ these discrete transformations build a subgroup of the Poincaré group. Only transformations connected to the identity preserve the direction of time. The remaining Poincaré transformations involving space and/or time reflections relate inertial frames that are not physically accessible from a given inertial frame. Hence, in the present context only Poincaré transformations continuously connected to the identity are of interest. They are characterized by the conditions $\det(\Lambda) = +1$ and $\Lambda_0^0 \geq 1$ and correspond to the proper, orthochronous Lorentz-transformations. The following discussion will therefore always refer to the proper, orthochronous Poincaré group, and its elements will be given as ordered pairs $(\Lambda, a)$ consisting of Lorentz-transformations $\Lambda$ and space-time translations $a$.

The covering group of the inhomogenous Lorentz group is the inhomogeneous special linear group $ISL(2, C)$. Its group elements are given by ordered pairs of $(2 \times 2)$ matrices $(\hat{\Lambda}, \hat{a})$, where $\hat{\Lambda}$ has determinant $+1$ and $\hat{a}$ is hermitian $(\hat{a} = \hat{a}^\dagger)$. Any space-time four-vector $x^\mu$ can be represented as $(2 \times 2)$ hermitian matrix

$$\hat{X} = x^\mu \sigma_\mu = \begin{pmatrix} x^0 + x^3 & x^1 - i x^2 \\ x^1 + i x^2 & x^0 - x^3 \end{pmatrix}, \quad x^\mu = \frac{1}{2} Tr \left( \sigma_\mu \hat{X} \right),$$

with the usual definitions for the Pauli matrices $\sigma_i$ (see Appendix B) and $\sigma_0 = \mathbf{1}_2$. The most general transformation that preserves proper time, hermiticity of $\hat{X}$, handedness of space and direction of time is given by

$$\hat{X} \rightarrow \hat{X}' = \hat{\Lambda} \hat{X} \hat{\Lambda}^\dagger + \hat{a}.$$

We mark the $(2 \times 2)$ matrices with a hat $(\hat{\Lambda}, \hat{a})$ in order to distinguish them from the $(4 \times 4)$ matrices $\Lambda$ and the four-vector $a$ to be used subsequently.
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Defining the composition
\[
\left( \hat{\Lambda}_2, \hat{a}_2 \right) \circ \left( \hat{\Lambda}_1, \hat{a}_1 \right) = \left( \hat{\Lambda}_2 \hat{\Lambda}_1, \hat{\Lambda}_2 \hat{a}_1 \hat{\Lambda}_1^\dagger + \hat{a}_2 \right),
\]
the ordered pairs of matrices \((\hat{\Lambda}, \hat{a})\) form a group with the inverse
\[
\left( \hat{\Lambda}, \hat{a} \right)^{-1} = \left( \hat{\Lambda}^{-1}, -\hat{\Lambda}^{-1} a \left( \hat{\Lambda}^\dagger \right)^{-1} \right),
\]
and the identity
\[
\hat{1} = (\hat{1}, 0).
\]
The proper, orthochronous Poincaré group specified by the constraints \(\det(\hat{\Lambda}) = +1\) and \(\Lambda_0 \geq 1\) on the Lorentz-transformations is isomorphic to the particular group obtained by identifying the ordered pairs \((\hat{\Lambda}, \hat{a})\) and \((-\hat{\Lambda}, \hat{a})\) contained in \(ISL(2, C)\). Therefore, each of the pairs \((\hat{\Lambda}, \hat{a})\) and \((-\hat{\Lambda}, \hat{a})\) corresponds to the same proper, orthochronous Poincaré transformation \((\Lambda, a)\), and the connection is given by
\[
\Lambda^\mu _\nu = \frac{1}{2} Tr \left( \sigma_\mu \hat{\Lambda} \sigma_\nu \hat{\Lambda}^\dagger \right), \quad a^\mu = \frac{1}{2} Tr (\sigma_\mu \hat{a}) .
\]
A single-valued unitary representation of \(ISL(2, C)\) is a function from the group \(ISL(2, C)\) to the space of linear operators on the model Hilbert space \(H\). The corresponding unitary operators satisfy the following conditions:
\[
U \left( \hat{\Lambda}_2, \hat{a}_2 \right) U \left( \hat{\Lambda}_1, \hat{a}_1 \right) = U \left[ \left( \hat{\Lambda}_2, \hat{a}_2 \right) \circ \left( \hat{\Lambda}_1, \hat{a}_1 \right) \right] = U \left( \hat{\Lambda}_2 \hat{\Lambda}_1, \hat{\Lambda}_2 \hat{a}_1 \hat{\Lambda}_1^\dagger + \hat{a}_2 \right),
\]
\[
U \left( \hat{\Lambda}, \hat{a} \right)^\dagger = U \left( \hat{\Lambda}, \hat{a} \right)^{-1} = U \left[ \left( \hat{\Lambda}, \hat{a} \right)^{-1} \right] .
\]

In order to find a parameterization of the Poincaré transformations we have to write down the most general \((2 \times 2)\) matrices \(\hat{\Lambda}\) and \(\hat{a}\) with the restrictions \(\det(\hat{\Lambda}) = +1\) and \(\hat{a} = \hat{a}^\dagger\), respectively. They can be given by the expressions
\[
\hat{\Lambda} = \hat{\Lambda} (\Theta, \rho) = \exp \left[ -\frac{i}{2} (\Theta + i\rho) \cdot \sigma \right] \quad \text{and} \quad \hat{a} = a^\mu \sigma_\mu .
\]
Here, \(\Theta\) denotes the angle and direction of a rotation, \(\rho\) the direction and rapidity of rotationless Lorentz-transformations (canonical boosts), and \(a^\mu\) are real coefficients. It is noteworthy that the set \(\Theta, \rho, a^\mu\) has the property that if any 9 of these are set to zero, the remaining one is a one-parameter Abelian subgroup. These are the 10 parameters of \(ISL(2, C)\), and the corresponding infinitesimal generators are defined by
\[
U \left( \hat{\Lambda} (\Theta, \rho), \hat{a} \right) = e^{-iP^\mu a^\mu} e^{-i(J^\Theta + K^\rho)}
\]
with

\[ P^\mu = ig^{\mu\nu} \frac{\partial}{\partial a^\nu} U \left( \Lambda, \dot{\Lambda} \right) \bigg|_{\Theta = \rho = a^\mu = 0} , \]  
(3.11)

\[ K^j = i \frac{\partial}{\partial \rho^j} U \left( \Lambda, \dot{\Lambda} \right) \bigg|_{\Theta = \rho = a^\mu = 0} , \]  
(3.12)

\[ J^j = i \frac{\partial}{\partial \Theta^j} U \left( \Lambda, \dot{\Lambda} \right) \bigg|_{\Theta = \rho = a^\mu = 0} . \]  
(3.13)

Here, \( P^0 \) is the infinitesimal generator of time translations leading to the operator for the total energy of the system, \( P \) is the total linear momentum and \( J \) the total angular momentum of the system, respectively. Furthermore, \( K \) are the generators of Lorentz boosts. It is straightforward to show that these operators satisfy the following set of commutation relations

\[ [J^j, J^k] = i \varepsilon^{jkl} J^l, \quad [K^j, K^k] = -i \varepsilon^{jkl} J^l, \quad [J^j, K^k] = i \varepsilon^{jkl} K^l, \]  
(3.14)

\[ [P^\mu, P^{\nu}] = 0, \]  
(3.15)

\[ [K^j, P^0] = -i P^j, \quad [J^j, P^0] = 0, \quad [K^k, P^j] = -i \delta^{jk} P^0, \quad [J^j, P^k] = i \varepsilon^{jkl} P^l. \]  
(3.16)

The first three commutation relations (3.14) show that the boost and rotation operators form a closed group, namely the Lorentz group. The fourth commutation relation (3.15) defines the invariance under space-time translations. The last four commutation relations given in Eq. (3.16) guarantee that \( P^\mu \) transforms as a four-vector under Lorentz transformations. By Stone’s theorem, it follows that all these generators are self-adjoint operators.

Note that from the set of the generators \( \{ H, P, J, K \} \) one can construct a set of commuting self-adjoint operators. The corresponding eigenvalues can be used to label different irreducible representations of the Poincaré group. In our context \( H = P^0 \) is the Hamiltonian of the system.

### 3.1.1 Commuting Self-Adjoint Operators

In RQM it is useful to define the mass operator through the invariant operator

\[ M^2 = P_\mu P^\mu = H^2 - P^2. \]  
(3.17)

The operator \( M^2 \) is a polynomial function of the generators and has the property that it commutes with all 10 generators of the Poincaré group, i.e. it is a Casimir operator. For the physical reason that the mass of a system is always positive we demand that all eigenvalues of \( M^2 \) are greater than zero. This is called the spectral condition on \( M^2 \). Now, any self-adjoint
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operator with strictly non-negative eigenvalues has a unique non-negative square root

\[ M = \sqrt{H^2 - P^2}, \]

and we define the physical mass of the system by the eigenvalues of the operator \( M \). The mass of a single particle is then given by the eigenvalue of the free mass operator.

The Hamiltonian of the system can now be given by the expression

\[ H = \sqrt{M^2 + P^2}. \]

In RQM the general spin operator can be related to the Pauli-Lubanski operator, which is defined by

\[ W^\mu = -\frac{1}{2} \epsilon^{\mu\alpha\beta\gamma} P_\alpha J_{\beta\gamma}, \]

where \( \epsilon^{\mu\alpha\beta\gamma} \) is the completely anti-symmetric tensor in space-time with \( \epsilon^{0123} = 1 \), and \( J_{\beta\gamma} \) is an antisymmetric tensor of rank 2 with the components \( J^{0j} = K^j \) and \( J^{jk} = \epsilon^{jkl} J^l \). It can be seen that \( W^\mu \) is a pseudo-four-vector specified by

\[ W^0 = P \cdot J, \quad W = P^0 J - P \times K, \]

\[ W^0 = (W^0)^\dagger, \quad W = (W)^\dagger. \]

The commutation relations with the 10 generators are given by

\[ [P^\mu, W^\nu] = 0, \quad [J^j, W^0] = 0, \quad [J^j, W^k] = i\epsilon^{jkl} W^l, \]

\[ [K^j, W^0] = -iW, \quad [K^j, W^k] = -i\delta^{jk} W^0, \]

\[ [W^\mu, W^\nu] = i\epsilon_{\mu\nu\sigma\tau} W^\sigma P^\tau. \]

The four commutation relations (3.23) show that the Pauli-Lubanski four-vector \( W^\mu \) transforms in the same way as the four-momentum vector \( P^\mu \) under Lorentz-transformations and consequently is itself a four-vector. Therefore, the Pauli-Lubanski operator squared is an invariant under Lorentz-transformations and we denote it by

\[ W^2 = W_\mu W^\mu = -M^2 \Sigma^2, \]

where \( \Sigma^2 \) is the intrinsic spin operator of the system. For the case that the particle is in its rest frame, \( \Sigma^2 = J^2 \) and Eq. (3.25) simplifies to

\[ W^2 = W_\mu W^\mu = -M^2 J^2. \]
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It can be shown that $W^2$ and $M^2$ are the only independent polynomials of generators that commute with all 10 generators. The commutation relations show that the scalar product of the Pauli-Lubanski operator with the four-momentum vector gives an invariant equal to zero ($W_\mu P^\mu = 0$). Since the four-momentum is naturally a time-like vector, this can only happen if the Pauli-Lubanski vector is space-like, i.e. it is orthogonal to $P^\mu$. Any time-like vector can be Lorentz-transformed to the rest frame, where the spatial components vanish. On the other hand, any space-like vector can be Lorentz-transformed to a vector, where the time component vanishes. In the current case of $W^\mu$ and $P^\mu$, both transformations can be realized with one and the same boost operator $L_g(v)_\nu^\mu$, namely

$$
(1, 0) = M^{-1} L_{g(v)}^{-1} (v)_\nu^\mu P^\nu, \quad (3.27)
$$

$$
(0, \Sigma_g) = M^{-1} L_{g(v)}^{-1} (v)_\nu^\mu W^\nu. \quad (3.28)
$$

Eq. (3.27) defines the general type of boost denoted by the subscript $g$, whereupon $v = \frac{P}{M}$ is the four-velocity of the system. Clearly, applying the boost to a simultaneous eigenstate of $M$ and $P$, the rest-frame state $(M, 0)$ is transformed to the state $P_\mu = (P^0, P)$. It should be noted that the class of valid operators satisfying Eqs. (3.27) and (3.28) is larger than just the rotationless Lorentz-transformations. The latter are also called canonical boosts. The explicit form of the canonical boost ($g = c$) is given by the $(2 \times 2)$ matrix representation

$$
\hat{L}_c(v) = \exp \left( \frac{1}{2} \omega \cdot \sigma \right)
$$

with $\omega = \frac{v}{|v|} \sinh^{-1} |v|$. The spin operator $\Sigma_g$ on the left-hand side of Eq. (3.28) depends on the general type of boost denoted by the subscript $g$ and looks like a three-vector under rotations, whereas the right-hand side of Eq. (3.28) looks formally like a four-vector. However, only for canonical boosts the spin operator $\Sigma_g$ transforms like a three-vector, but it never transforms like a four-vector. Generally, the spin vector operators transform under Poincaré transformations as follows:

$$
U^\dagger \left( \hat{\Lambda}, \hat{a} \right) \Sigma_g U \left( \hat{\Lambda}, \hat{a} \right) = W_g(v; \Lambda) \Sigma_g.
$$

(3.30)

Here, we introduced the Wigner rotation

$$
W_g(v; \Lambda) = L_{g(v)}^{-1} (\Lambda v) \Lambda L_g(v).
$$

(3.31)

This clearly shows that the spin operator does not transform as a four-vector. Since for canonical boosts the Wigner rotation $W_g(R)$ of a rotation $R$ is the rotation $R$ itself, it is seen that for canonical boosts the corresponding spin operator $\Sigma_c$ transforms like a three-vector. Now it remains to be shown
that the spin vector satisfies the required angular momentum commutation
relations. As the squared of the Pauli-Lubanski operator is invariant under
Lorentz-transformations, it follows that

\[ \Sigma_g \cdot \Sigma_g = -\frac{W^2}{M^2} = \Sigma^2, \quad (3.32) \]

where the second equality follows directly from the definition of the total-
spin operator \( \Sigma^2 \). The commutation relations can also be given directly
using the commutation relations of the Pauli-Lubanski operator, namely

\[ M^2 \left[ \Sigma^k, \Sigma^l \right] = i \varepsilon^{kln} M^2 \Sigma^n, \quad (3.33) \]

If the mass eigenvalue is not equal to zero, we immediately recover the com-
motion relations typical for angular-momentum operators. It also follows
that the spectrum of the intrinsic spin operator \( \Sigma^2 \) is of the form \( \Sigma (\Sigma + 1) \),
with \( \Sigma \) being an integer or half-integer, and the spectrum of any component
of the spin operator has the form \( \{ -\Sigma, -\Sigma + 1, ..., \Sigma - 1, \Sigma \} \). Consequently,
the definition of the spin operator is indeed appropriate.

The complete set of commuting self-adjoint operators \( \{ M, P, \Sigma^2, \Sigma^3 \} \)
constructed from the generators \( \{ H, P, J, K \} \) describes a single massive par-
ticle with spin. The eigenvalues of these operators fix the representations of
the model Hilbert space and the eigenvalues of the mass and spin operator
are used to label the representation. For composite particles the eigenvalues
for the spin and mass operators are not a priori unique. One has to advocate
additional conditions (beyond mere Poincaré invariance) in order to arrive
at a complete set of commuting operators defining the representation of the
system.

3.1.2 Transformation Properties of the Four-Momentum
Operator

Here, we briefly discuss the transformation properties of the four-momentum
operator \( P^\mu \) of an arbitrary system containing one or more particles. The
unitary representation of a space-time translation is given by

\[ U \left( \hat{1}, \hat{a} \right) = \exp (-i P^\mu a^\mu). \quad (3.34) \]

Together with the commutation relations

\[ [P^\mu, P^\nu] = 0 \quad (3.35) \]

it can be shown immediately that the unitary representation of the space-
time translation operator commutes with the self-adjoint four-momentum
operator

\[ \left[ U \left( \hat{1}, \hat{a} \right), P^\mu \right] = 0. \quad (3.36) \]
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In other words, the four-momentum operator is itself invariant under space-time translations

\[ U \left( \hat{1}, \hat{a} \right) P^\mu U \left( \hat{1}, \hat{a} \right) = U \left( \hat{1}, -\hat{a} \right) P^\mu U \left( \hat{1}, \hat{a} \right) = P^\mu. \] (3.37)

The transformation properties of the four-momentum operator under Lorentz-transformations are given by the operator equation

\[ U \left( \Lambda, \hat{0} \right) P^\mu U \left( \Lambda, \hat{0} \right) = \Lambda^\mu_\nu P^\nu. \] (3.38)

This means the momentum operator indeed transforms like a four-vector under Lorentz-transformations and is invariant under space-time translations. The spectrum of the four-vector operator \( P \) consists of four-vectors \( p \), which form a four-dimensional space called the momentum space. Since the Hamiltonian, represented by the in\( \times\)finitesimal time generator \( P^0 \), commutes with the four-momentum operator, i.e.

\[ [H, P^\mu] = 0, \] (3.39)

an eigenstate with eigenvalue \( p^\mu \) remains an eigenstate with the same eigenvalue for all times. Its behaviour under Lorentz-transformations is

\[ P^\mu \left( U \left( \Lambda, \hat{0} \right) |p\rangle \right) = U \left( \Lambda, \hat{0} \right) U^\dagger \left( \tilde{\Lambda}, \hat{0} \right) P^\nu U \left( \tilde{\Lambda}, \hat{0} \right) |p\rangle \]

\[ = U \left( \tilde{\Lambda}, \hat{0} \right) \Lambda^\mu_\nu p^\nu |p\rangle = p^\mu U \left( \tilde{\Lambda}, \hat{0} \right) |p\rangle, \] (3.40)

with

\[ p^\mu = \Lambda^\mu_\nu p^\nu. \] (3.41)

Therefore, \( U \left( \Lambda, \hat{0} \right) |p\rangle \) is an eigenvector of \( P^\mu \) with eigenvalue \( p^\mu \), and the eigenvalues of the momentum operator have the expected transformation properties. If the physical system consists of only one particle with mass \( m \), then the eigenvalues have to satisfy

\[ p_0 p^\mu = m^2. \] (3.42)

In the case of two particles, each with the mass \( m \), the following relation will hold:

\[ p_\mu p^\mu \geq 4m^2. \] (3.43)

For any physical system the mass operator \( M^2 \) has an eigenvalue \( \lambda = m^2 \), and the mass of the system is then \( m = |\lambda|^{\frac{1}{2}} \). The set of state vectors one gets when applying all Lorentz-transformations \( U \left( \Lambda, \hat{0} \right) \) to a specific four-momentum state \( |p\rangle \) is called an orbit. Since the mass-operator \( M^2 \) commutes with all Lorentz-transformations, all states in a given orbit have the same eigenvalue \( m^2 \). Consequently, the orbit is closed under all operations in the restricted inhomogenous Lorentz group and a general representation can be decomposed into the direct sum (integral) of representations on orbits. For physical states one restricts the investigations to orbits with \( m^2 \geq 0 \) and \( p_0 \geq 0 \).
3.2 Composite Particles and Poincaré Invariance

So far we have sketched the relativistic description of free massive particles with spin. Up to this point, the irreducible representations of the Poincaré group have not depended on any dynamical theory. However, in multi-particle systems the underlying dynamics can lead to different implementations of RQM. Before we introduce the dynamics, we first establish a suitable description of a free three-particle system without any interactions. For this purpose we define a free three-particle momentum state with spin given as the tensor product of three one-particle states

$$|p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3\rangle = |p_1; \sigma_1\rangle \otimes |p_2; \sigma_2\rangle \otimes |p_3; \sigma_3\rangle. \quad (3.44)$$

Here, the $p_i$ and $\sigma_i$ denote the individual particle four-momenta and spin projections. Since we assume that all particles are on their respective mass-shells ($p_i p_i^\mu = m^2$), the zero components of the individual four-momentum operators are given by

$$p_{i0} = \sqrt{m_i^2 + p_i^2}. \quad (3.45)$$

The free total four-momentum $P^\mu$ of the system is then given by the sum of the individual four-momenta

$$P^\mu = \sum_{i=1}^{3} p_i^\mu. \quad (3.46)$$

The properties of these free three-particle states can be derived from the corresponding properties of the single-particle states, and under Lorentz-transformations $U(\Lambda)$ one obtains

$$U(\Lambda) |p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3\rangle = \sum_{\sigma'_i = -\frac{1}{2}}^{\frac{1}{2}} |\Lambda p_1, \Lambda p_2, \Lambda p_3; \sigma'_1, \sigma'_2, \sigma'_3\rangle \prod_{i=1}^{3} D_{\sigma'_i \sigma_i}^{\frac{1}{2}} [R_{W_g}(v_i; \Lambda)]. \quad (3.47)$$

The functions $D_{\sigma'_i \sigma_i}^{\frac{1}{2}} [R_{W_g}(v_i; \Lambda)]$ are called Wigner D-functions [77].

Here, the superscript $\frac{1}{2}$ refers to the intrinsic spin of the constituent quarks, and $\sigma_i(\sigma'_i)$ are the corresponding z-projections. The Wigner D-functions depend on the Wigner rotations defined in Eq. (3.31), where $v_i$ are the velocities defining the boosts. In general each one of the three Wigner D-functions depends on a different single-particle four-velocity, and the spins of such states cannot be coupled in the standard way (like in non-relativistic quantum mechanics).

Any composite particle has to be defined with respect to its total four-momentum $P^\mu$ as well as intrinsic spin $\Sigma$. The free total four-momentum
operator is just given by the sum over the individual four-momentum operators, whereas the intrinsic spins of the individual particles and the relative orbital angular momenta also need to be coupled to the intrinsic spin of the system. In general the tensor product of the three irreducible representations of the single-particle systems does not lead to an irreducible representation of the total three-particle system. In this respect, the transformation from a product of single-particle eigenstates to the linear combination of multi-particle eigenstates is equivalent to the problem of defining appropriate Clebsch-Gordan coefficients (see the discussion in Ref. [6]).

Since the three individual four-momenta \( p_i^\mu \) are time-like vectors, the total four-momentum \( P^\mu \) will also be a time-like vector. Hence, a Lorentz-transformation has to exist that transforms the total four-momentum eigenstate to its rest frame, where its spatial components vanish. Then, it is possible to find a corresponding space-like Pauli-Lubanski vector operator that transforms to a spin-like vector operator with zero time-component under the same Lorentz-transformation. Any rest-frame eigenstate has to transform under rotations like

\[
U(R) | (M, 0); \Sigma M_\Sigma \rangle = \sum_{\Sigma M_\Sigma'} \int (M, 0) \Sigma M_\Sigma' D^{\Sigma}_{\Sigma \Sigma'} [R] \]  \hspace{1cm} (3.48)

where \( M \) is the rest-mass of the system, \( \Sigma \) the intrinsic spin and \( M_\Sigma \) the corresponding spin projection. In general, the Clebsch-Gordan coefficients depend on the boost properties of the system, but in case of canonical boosts the transformation properties of the rest-frame states under rotations are equivalent to the standard non-relativistic ones. This can be attributed to the fact that for canonical boosts the Wigner rotation of a rotation \( R_W(R) \) is the rotation \( R \) itself. Consequently, in this particular case all the non-relativistic angular-momentum coupling rules also apply in the relativistic framework. Therefore, it is natural to perform the spin couplings in the rest-frame eigenstates of the total four-momentum and subsequently boost the system to the appropriate total four-momentum eigenstates. As in the following we restrict ourselves to canonical boosts \( (g = c) \), we drop the corresponding index and as a special case of Eq. (3.47) we immediately get the relation

\[
U(B(v)) | k_1, k_2, k_3; \mu_1, \mu_2, \mu_3 \rangle = \sum_{\sigma_i = -\frac{1}{2}}^{\frac{1}{2}} | p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 \rangle \prod_{i=1}^{3} D^{\frac{1}{2}}_{\sigma_i \mu_i} [R_W(k_i; B(v))], \]  \hspace{1cm} (3.49)

where we have specified the Wigner \( D \)-functions with respect to the individual four-momenta \( k_i = m_i v_{k_i} \) instead of the four-velocities \( v_{k_i} \). The expression (3.49) defines a specific Lorentz-transformation, namely a boost
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$B^{-1}(v)$ that transforms the four-momentum state to its rest frame. The four-component vectors $k_i$ correspond to the three-particle system in its own rest frame; they are constrained by

$$\sum_{i=1}^{3} k_i = 0, \quad (3.50)$$

and are related to a general three-particle state through the boost via

$$B(v)\mu_i k_i' = p_i'^\mu. \quad (3.51)$$

The boost $B(v)$ cannot change the character of a four-vector in the transformation. Therefore, the $k_i$ define one particular four-vector, namely the one where the spatial components add up to zero. Now the question arises, whether it is possible to perform a basis transformation on the boosted eigenstates in such a way that the new basis does not depend anymore on the individual four-momenta $p_i$, but on relative four-momenta plus a vector denoting the overall momentum of the system. At this point we introduce the so-called velocity states, given by

$$|v, k_1, k_2, k_3; \mu_1, \mu_2, \mu_3⟩ = U(B(v))|k_1, k_2, k_3; \mu_1, \mu_2, \mu_3⟩. \quad (3.52)$$

The velocity states are eigenstates of the four-velocity operator

$$V^\mu = \frac{P^\mu}{M}, \quad (3.53)$$

with $P^\mu$ the total four-momentum operator and $M$ the mass operator. They are also eigenstates of the four-vector operators $k_i$. The transformation properties for the velocity operator are those of a four-vector, i.e.

$$U^\dagger(\Lambda, a)V^\mu U(\Lambda, a) = \Lambda^\mu_\nu V^\nu. \quad (3.54)$$

On the other hand, the operators $k_i$ do not transform as four-vectors. Since they represent the four-momentum vectors this might appear strange at first sight. However, one has to realize that the boost connects a constrained system represented in terms of $k_i$ with an unconstrained system expressed in terms of $p_i$. The spatial components of the $k_i$ are always constrained to add up to zero, while the overall motion is given by the additional four-velocity operator. Under a Lorentz-transformation the most general type of transformation for the $k_i$, constraint as $k_1 + k_2 + k_3 = 0$, is that of a spatial rotation. Furthermore, the transformation property still has to lead to a four-vector behaviour for the $p_i$. We will show that the Wigner rotations fulfill exactly these conditions. Therefore, the vector operators $k_i$ are defined to transform under Lorentz-transformations in the following way:

$$U^\dagger(\Lambda, a)k_iU(\Lambda, a) = R_W(v; \Lambda)k_i. \quad (3.55)$$
In order to show that the corresponding $p_i$ transform as four-vectors we have to show that
\[
U^\dagger (\Lambda, a) p_i^\mu U (\Lambda, a) = \Lambda^\mu_{\nu} p_i^\nu. \quad (3.56)
\]
We insert the definition of the boosted momenta, and finally obtain:
\[
U^\dagger (\Lambda, a) p_i U (\Lambda, a) = U^\dagger (\Lambda, a) B (v) k_i U (\Lambda, a)
= U^\dagger (\Lambda, a) B (v) U (\Lambda, a) U^\dagger (\Lambda, a) k_i U (\Lambda, a)
= U^\dagger (\Lambda, a) B (v) U (\Lambda, a) R_W (v; \Lambda) k_i
= B (\Lambda v) R_W (v; \Lambda) k_i
= B (\Lambda v) B^{-1} (\Lambda v) \Lambda B (v) k_i
= \Lambda p_i. \quad (3.57)
\]
It is noteworthy that the $k_i$ vector operators have exactly the behaviour under Lorentz-transformation as the previously defined spin operators. Next we investigate the transformation properties of the velocity states. Here, one should be aware of the fact that the three $k_i$ operators do not give a complete set of commuting operators, but the $v$ and two of the $k_i$ do.

A general velocity state is defined by a boost on an eigenstate of the four-momentum operators $k_i$ referring to the rest frame (see Eq. (3.52)). A Lorentz-transformation on these states is expressed by
\[
U (\Lambda) |v, k_1, k_2, k_3; \mu_1, \mu_2, \mu_3\rangle = U (\Lambda B (v)) |k_1, k_2, k_3; \mu_1, \mu_2, \mu_3\rangle. \quad (3.58)
\]
Now we can replace the argument in the unitary representation utilizing the definition of the Wigner rotation (3.31) and get
\[
U (\Lambda) |v, k_1, k_2, k_3; \mu_1, \mu_2, \mu_3\rangle = U (B (\Lambda v) R_W (v; \Lambda)) |k_1, k_2, k_3; \mu_1, \mu_2, \mu_3\rangle. \quad (3.59)
\]
Next we apply the group properties of the unitary transformations again, which yields
\[
U (\Lambda) |v, k_1, k_2, k_3; \mu_1, \mu_2, \mu_3\rangle = U (B (\Lambda v)) U (R_W (v; \Lambda))
\times |k_1, k_2, k_3; \mu_1, \mu_2, \mu_3\rangle. \quad (3.60)
\]
However, in Eq. (3.48) we have shown that the rest-frame states transform under rotations like a spin-1 irreducible representation, and since the Wigner rotation is a rotation, we can write
\[
U (\Lambda) |v, k_1, k_2, k_3; \mu_1, \mu_2, \mu_3\rangle
= U (B (\Lambda v)) \sum_{\mu_i' = -\frac{1}{2}}^{\frac{3}{2}} |R_W k_1, R_W k_2, R_W k_3; \mu_1', \mu_2', \mu_3'\rangle \prod_{i=1}^{3} D_{\mu_i' \mu_i}^{\frac{3}{2}} [R_W], \quad (3.61)
\]
where we have omitted the argument of the Wigner rotation for simplicity. Now we can use the definition of the velocity states (3.52) again, and finally Eq. (3.58) becomes

\[
U(v) |v, k_1, k_2, k_3; \mu_1, \mu_2, \mu_3\rangle = \sum_{\mu'_1, \mu'_2, \mu'_3} |v, R_W k_1, R_W k_2, R_W k_3; \mu'_1, \mu'_2, \mu'_3\rangle \prod_{i=1}^{3} D^{\frac{1}{2}}_{\mu'_i \mu_i} [R_W]. \tag{3.62}
\]

Obviously, the Wigner D-functions and the states depend on the same Wigner rotations and therefore this basis yields a natural starting point for the spin coupling procedure.

At this point we have given different representations of the Poincaré algebra for the free system. We also defined complete sets of commuting observables, their corresponding eigenstates and the transformation properties of both. The next step will be to include the interaction into the system.

### 3.3 Implementation of Interactions: Forms of RQM and the BT Construction

The introduction of interaction into a relativistic system proves to be a non-trivial challenge. The reason is that the interaction-dependent generators of the Poincaré group have to satisfy the Poincaré algebra given by the Eqs. (3.14) - (3.16). The problem can be seen most easily already in the commutator relation

\[
\left[ P^j_{\text{free}}, K^k_{\text{free}} \right] = iP^0_{\text{free}} \delta^{jk} = iH_{\text{free}} \delta^{jk}, \tag{3.63}
\]

where the subscript ”\text{free}” indicates that the system does not yet include dynamics. If an interaction is added to the free Hamiltonian \(H_{\text{free}}\) on the right-hand side of Eq. (3.63), it must also appear on the left-hand side. There exist several possibilities for the inclusion of the interaction. In each case the interaction has to satisfy certain constraints in order to guarantee the commutation relations in the Poincaré algebra. This presents the main challenge in the derivation of an interacting relativistic system, because in general the constraints on the interaction are of non-linear nature.

Already in 1949, Dirac found three specific forms of relativistic dynamics\(^2\), which minimally affect an originally free system by the introduction of interaction [8]. These forms are defined by the existence of a non-trivial subgroup of the Poincaré group, whose unitary representations are the same for the free and interacting systems. The subgroup is called the kinematical

\(^2\text{It should be noted that Dirac’s forms of relativistic dynamics originally referred to classical systems. Only several years later the forms of relativistic dynamics have been applied to quantum mechanics.}\)
subgroup for the form of dynamics, and one refers to it as "stability group". Dirac called his three forms of relativistic dynamics the point, instant, and front form, respectively. Years later, Leutwyler and Stern [78] have shown that there exist two more forms. However, the dimensions of the corresponding stability groups are low ($d = 5$), and as so far there has been no physical application of these forms, we will not discuss them further.

The stability group has the property that it leaves the hypersurfaces (specific for each form of dynamics) in Minkowski space invariant even under the implementation of an interaction. In the point form the hypersurface has the shape of a hyperboloid satisfying $x^2 - \tau^2 = 0$ with $\tau = \text{const}$. The stability group comprises the full homogeneous Lorentz group, while the four components of the four-momentum operator $P^\mu$ are dynamical. In the instant form, the hypersurface, which is invariant under the stability group of this form is the instant $t = 0$. The stability group contains the three spatial components $\mathbf{P}$ of the four-momentum operator and the three components of the angular momentum $\mathbf{J}$, while the generators of boosts $\mathbf{K}$ become interaction-dependent. Finally, the hypersurface of the front form is a hyperplane tangent to the light cone. In this form of relativistic dynamics it is useful to introduce so-called light-front coordinates. Here, the stability group comprises seven out of the 10 generators of the Poincaré group, and only three generators include interactions. These are the "front-form Hamiltonian" $P^-$ and two other combinations of rotations around as well as boosts along the $x$- and $y$-directions (see, e.g., Ref. [79]). In order to visualize the three forms of relativistic dynamics, we plot the respective hypersurfaces in Fig. 3.1. More details concerning the the point, instant, and front forms can be found, e.g., in Ref. [80].

![Figure 3.1: Invariant hypersurfaces characterizing point, instant, and front forms (left to right).](image)

As mentioned above, the general constraints on the interaction are of non-linear nature. In this respect, Dirac stated in his paper [8]: "These conditions are not easily fulfilled and provide the real difficulty in the problem of constructing a theory of a relativistic dynamical system..." However, in 1953 Bakamjian and Thomas [7] developed a prescription arriving only
at linear constraints. In each one of the above forms of RQM one constructs auxiliary operators out of the original generators of the Poincaré group, where one of them is the mass operator. Then one includes the interaction via corresponding potential terms into the mass operator, and finally reconstructs the Poincaré generators from the auxiliary operators. A priori, this construction has a problem with macroscopic locality. However, for a system of confined quarks the clusterization of the individual quarks into non-interacting clusters is not allowed, i.e. it is not possible to observe asymptotically free quarks and consequently we make the assumption that the neglect of cluster properties is a reasonable approximation. In more detail the procedure to include interactions into a relativistic theory according to Bakamjian and Thomas [7] is described as follows:

- From the set of free generators in the Poincaré algebra, one builds a set of auxiliary operators containing the mass operator and fulfilling commutation relations related to the ones of the original set of generators.

- One then adds the interaction to the mass operator leaving the other auxiliary operators unaffected. This implies only linear conditions on the potential in order to guarantee that the algebra of the auxiliary operators does not change.

- Finally, one reconstructs the original set of generators from the auxiliary operators. Via the mass operator the interaction is now implemented in the generators while the Poincaré algebra has not been changed.

As in this thesis we are concerned with the point form, let us briefly discuss the implementation of the BT construction in this specific form of RQM: We have already seen, that the free generators \( \{H_{\text{free}}, P_{\text{free}}, K_{\text{free}}, J_{\text{free}} \} \) of the Poincaré group satisfy the Poincaré algebra. For the introduction of interaction, one first replaces the Hamiltonian \( H_{\text{free}} \) by the mass operator \( M_{\text{free}} \), and instead of the three spatial components of \( P_{\text{free}}^\mu \) one can work with the corresponding velocity:

\[
M_{\text{free}} = \sqrt{H_{\text{free}}^2 - P_{\text{free}}^2}, \quad V_{\text{free}}^\mu = \frac{P_{\text{free}}^\mu}{M_{\text{free}}}. \quad (3.64)
\]

Then, one introduces the generalized position operator \( X_{V_{\text{free}}} \) corresponding to the three-velocity \( V_{\text{free}} \)

\[
X_{V_{\text{free}}} = \left[ -\frac{1}{2} \left( \frac{M}{H} \cdot K \right) - \frac{MV \times (HJ - MV \times K)}{H(M + H)} \right]_{\text{free}}, \quad (3.65)
\]
where \( \cdot \) denotes an anti-commutator. Furthermore, the canonical spin is given by

\[
\Sigma_{c,\text{free}} = \left[ \frac{1}{M} (H \mathbf{J} - M \mathbf{V} \times \mathbf{K}) - \frac{M^2 \mathbf{V} \cdot (\mathbf{V} \cdot \mathbf{J})}{M + H} \right]_{\text{free}}.
\] (3.66)

Then the operators \( \{ M_{\text{free}}, P_{\text{free}}, X_{V_{\text{free}}}, \Sigma_{c,\text{free}} \} \) also give a representation of the Poincaré group, provided that following commutation relations are satisfied:

\[
\left[ X_{V_{\text{free}}}^j, V_{\text{free}}^k \right] = i\delta^{jk}, \quad \left[ \Sigma_{c,\text{free}}^j, \Sigma_{c,\text{free}}^k \right] = i\epsilon^{jkl}\Sigma_{c,\text{free}}^l.
\] (3.67)

In the BT construction one starts with the set of free auxiliary operators \( \{ M_{\text{free}}, P_{\text{free}}, X_{V_{\text{free}}}, \Sigma_{c,\text{free}} \} \), where the free mass operator commutes with all the remaining nine operators. Now one can add an interaction operator \( M_{\text{int}} \) to the free mass operator

\[
M = M_{\text{free}} + M_{\text{int}},
\] (3.68)

with the condition that the interaction operator \( M_{\text{int}} \) also commutes with the remaining nine operators

\[
\left[ V_{\text{free}}^j, M_{\text{int}} \right] = \left[ X_{V_{\text{free}}}^j, M_{\text{int}} \right] = \left[ \Sigma_{c,\text{free}}^j, M_{\text{int}} \right] = 0.
\] (3.69)

From this, it follows that the sets of operators for the free and interacting systems satisfy the same commutation relations. Finally, one can reconstruct the original set \( \{ H, P, K_{\text{free}}, J_{\text{free}} \} \), and the interaction will only occur in the four-momentum operator

\[
P^\mu = MV^\mu_{\text{free}},
\] (3.70)

while keeping the homogenous Lorentz group free from interactions.

Baryons are now properly described as simultaneous eigenstates of a complete set of commuting self-adjoint operators. Of particular interest are the two equivalent sets of states \( |E, P, \Sigma, M_\Sigma, T, M_T \rangle \) and \( |M, V, \Sigma, M_\Sigma, T, M_T \rangle \), where \( \Sigma \) denotes the intrinsic spin of the baryon with its projection \( M_\Sigma \). In addition, baryons are also specified by the isospin \( T \) and its projection \( M_T \). Some technical details concerning the point form of RQM can be found in Appendix B.6.
Chapter 4

Decay Widths of Baryon Resonances

The description of mesonic baryon resonance decays represents a major challenge of strong interaction physics (see, e.g., the recent NSTAR workshops [81–84]). In particular, it is not yet possible to derive the mesonic decay widths of light and strange baryon resonances entirely on a microscopic level within a relativistic framework. Several difficulties complicate the situation; let us point out a few of them: The emitted meson of a strong decay process is clearly not an elementary particle, but by itself made up of constituents. Furthermore, within the framework of CQMs excited states of light and strange baryons are described as bound states rather than as resonances. In addition, final-state interactions are usually neglected.

In this chapter we provide insight into the calculation of the decay widths of baryon resonances as performed within relativistic CQMs. For this purpose, we first define the \( S \)-matrix, which leads to the transition amplitudes and decay rates. We then arrive at decay widths depending on a phase-space factor as well as on a transition amplitude, both complying with the requirements of a Poincaré-invariant theory. Concerning the transition amplitude, the main challenge is the Poincaré-invariant definition of the decay operator. This topic will be discussed in Chapter 5.

4.1 General Definition of the Decay Width

Generally, in an experimental setup one observes asymptotic states of non-interacting particles. In reactions, such states can be identified as the incoming \( \phi^{\text{in}} \) and outgoing \( \phi^{\text{out}} \) states, which are transferred into each other through an interaction region of finite size. These asymptotically free states develop with the free time-evolution operator \( U_{\text{free}}(t) = e^{-iH_{\text{free}}t} \). On the other hand the states \( \Psi \), which are subjected to the interaction \( H = H_{\text{free}} + H_{\text{int}} \), develop with the operator \( U(t) = e^{-iHt} \). Now the
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asymptotic condition claims that

$$\lim_{t \to +\infty} \| e^{-iHt}\Psi - e^{-iH_{\text{free}}t}\phi_{\text{in}} \| = 0. \quad (4.1)$$

The $S$-matrix $S$ is identified as the operator that maps the states in the remote past (incoming states) to the ones in the remote future (outgoing states). The eigenstates of the free Hamiltonian $H_{\text{free}}$ span the whole Hilbert space $\mathcal{H}$ connected with the system under investigation. Consequently, the $S$-matrix simply defines the transition amplitudes given by the representation of the outgoing with respect to the incoming eigenstates. As the $S$-matrix connects two sets of orthonormal states it must be unitary [85]

$$S^\dagger S = 1, \quad (4.2)$$

and it is invariant under Poincaré transformations

$$U(\Lambda, a)SU^{-1}(\Lambda, a) = S, \quad (4.3)$$

where $U(\Lambda, a)$ is the operator corresponding to the four-translation $a$ and the Lorentz transformation $\Lambda$. Separating off a trivial unity from $S$ leads to

$$S = 1 + iT, \quad (4.4)$$

with $T$ the usual transition operator.

Next we consider the matrix elements of $S$ between initial (incoming) states $|i\rangle$ and final (outgoing) states $|f\rangle$:

$$\langle f|S|i\rangle = \langle f|i\rangle + i\langle f|T|i\rangle = \langle f|i\rangle + i\delta^4(p_i - p_f)F_{i\to f}. \quad (4.5)$$

Here, $p_i$ and $p_f$ are the total four-momenta of the incoming and outgoing states and $F_{i\to f}$ is known as transition amplitude. We have extracted the four-momentum conservation $\delta^4(p_i - p_f) = \delta^4(p_f - p_i)$ from the transition matrix element leaving the amplitude $F_{i\to f}$, which is - due to the invariance of $S$ and the four-delta function - also an invariant function.

In order to describe the decay of a baryon resonance into a baryon ground state and a meson, one has to derive the decay rate from the transition probability corresponding to the decay width. The transition probability is given as

$$P_{i\to f} \propto |S_{i\to f}|^2 = |\langle f|S|i\rangle|^2. \quad (4.6)$$

Here, we meet the problem that the matrix elements of $S$ comprise a delta-function. In order to find a proper expression for the transition probability, we proceed as in Ref. [85], i.e. we switch to a finite volume $V$, where $\delta^3(p - p')$ is replaced by

$$\delta^3_{V}(p - p') = \frac{1}{(2\pi)^3} \int_V d^3x \exp[i(p - p') \cdot x] = \frac{V}{(2\pi)^3} \delta_{pp'} \quad (4.7)$$
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with discrete momenta

\[ \mathbf{p} = \frac{2\pi}{L} (n_1, n_2, n_3), \]

(4.8)

where \( n_i \) are integers and \( L^3 = V \). In this respect, the expression \( \delta_{\mathbf{p}\mathbf{p}'} \) in Eq. (4.7) is an abbreviation for

\[ \delta_{\mathbf{p}\mathbf{p}'} = \delta_{n_1 n_1'} \delta_{n_2 n_2'} \delta_{n_3 n_3'}. \]

(4.9)

We also take a finite time interval \( T \) with

\[ \delta_T(p_0 - p_0') = \frac{1}{2\pi} \int_{-T/2}^{T/2} dt \exp[i(p_0 - p_0')t]. \]

(4.10)

The initial state pertaining the decaying resonance and the final state containing a baryon and a meson are defined by

\[ |i\rangle = |P\rangle \quad \text{and} \quad |f\rangle = |P', q\rangle, \]

(4.11)

where \( P \) defines the four-momentum of the decaying resonance, and \( P' \) and \( q \) are the four-momenta of the final baryon and meson, respectively. The normalization of the states is done according to Ref. [86], which means for the incoming one-body state

\[ \langle \tilde{P} | P \rangle = 2P_0 \delta^3(\mathbf{P} - \tilde{\mathbf{P}}), \]

(4.12)

where \( P_0 = E \) is the energy of the decaying resonance, and for the outgoing two-body state

\[ \langle \tilde{P}', \tilde{q} | P', q \rangle = 2P_0' \delta^3(\mathbf{P}' - \tilde{\mathbf{P}}') 2q_0 \delta^3(\mathbf{q} - \tilde{\mathbf{q}}), \]

(4.13)

with \( P_0' = E' \) and \( q_0 = \omega_m \) the energies of the final particles. For the moment we neglect the internal quantum numbers of the states. In the finite box we introduce states

\[ |i\rangle_{\text{box}} = \left( \frac{(2\pi)^3}{V} \right)^{1/2} |i\rangle, \quad |f\rangle_{\text{box}} = \left( \frac{(2\pi)^3}{V} \right)^{1/2} |f\rangle, \]

(4.14)

leading to normalizations

\[ \langle \tilde{P} | P \rangle_{\text{box}} = 2E \delta_{\mathbf{P}\tilde{\mathbf{P}}}, \]

(4.15)

and

\[ \langle \tilde{P}', \tilde{q} | P', q \rangle_{\text{box}} = 2E' \delta_{\mathbf{P}'\tilde{\mathbf{P}}'} 2\omega_m \delta_{\mathbf{q}\tilde{\mathbf{q}}}, \]

(4.16)

respectively. The relation between the \( S \)-matrix elements is then

\[ \langle f | S | i \rangle = \left( \frac{V}{(2\pi)^3} \right)^{1/2} \langle f | S | i \rangle_{\text{box}}. \]

(4.17)
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The transition probability in the box is given by

\[ P_{i \rightarrow f}^{\text{box}} = \frac{\left| \langle f | S | i \rangle \right|_{\text{box}}^2}{\langle f | f \rangle_{\text{box}} \langle i | i \rangle_{\text{box}}} = \frac{2}{2E' 2\omega_m 2E} \left( \frac{(2\pi)^3}{V} \right)^3 |\langle f | S | i \rangle|^2. \]  

(4.18)

When going to arbitrarily large volumes the discrete momenta will become continuous and sums will be replaced by integrals

\[ \sum_{P',q} \rightarrow \int d^3P' d^3q \left( \frac{V}{(2\pi)^3} \right)^2. \]  

(4.19)

The transition probability density is thus

\[ dP_{i \rightarrow f}^{\text{box}} = \frac{2}{2E' 2\omega_m 2E} \left( \frac{(2\pi)^3}{V} \right)^3 |\langle f | S | i \rangle|^2 d^3P' d^3q. \]  

(4.20)

Since in case of a decay \( \langle f | i \rangle = 0 \) the S-matrix element is given as

\[ \langle f | S | i \rangle = i\delta_{V,T}(P - P' - q) F_{i \rightarrow f}. \]  

(4.21)

Due to the introduction of the box we can interpret the squares of delta functions in Eq. (4.20) as

\[ [\delta^3_P(P - P' - q)]^2 = \delta^3_P(P - P' - q) \frac{V}{(2\pi)^3}, \]

\[ [\delta_T(E - E' - \omega_m)]^2 = \delta_T(E - E' - \omega_m) \frac{T}{(2\pi)}. \]  

(4.22)

So we obtain

\[ |\langle f | S | i \rangle|^2 = |F_{i \rightarrow f}|^2 \delta^3_{V,T}(P - P' - q) \frac{VT}{(2\pi)^4}, \]  

(4.23)

and the transition probability density becomes

\[ dP_{i \rightarrow f} = \frac{T}{2\pi 2E' 2\omega_m 2E} \delta^3_P(P - P' - q) \delta_T(E - E' - \omega_m) d^3P' d^3q, \]  

(4.24)

where now the limit \( V \rightarrow \infty \) has been taken. Finally, the connection between the transition probability \( P_{i \rightarrow f} \) and the decay width \( \Gamma_{i \rightarrow f} \) is given by the corresponding differentials

\[ d\Gamma_{i \rightarrow f} = \frac{dP_{i \rightarrow f}}{T} = \frac{1}{4\pi E} |F_{i \rightarrow f}|^2 \delta^4(P - P' - q) \frac{d^3P'}{2E'} \frac{d^3q}{2\omega_m}, \]  

(4.25)

with now also \( T \rightarrow \infty \), where

\[ \delta^4(P - P' - q) \frac{d^3P'}{2E'} \frac{d^3q}{2\omega_m}. \]  

(4.26)
is Lorentz-invariant. In the rest frame of the decaying baryon $P = (M, 0)$, with $E = M$ the mass of the decaying resonance, and thus expression (4.26) becomes

$$\delta(M - E' - \omega_m)\delta^3(P' + q) \frac{d^3P'}{2E'} \frac{d^3q}{2\omega_m}. \tag{4.27}$$

The differential decay rate is therefore given by

$$\frac{d\Gamma_{i \to f}}{d^3q d^3P'} = \frac{1}{4\pi M} |F_{i \to f}|^2 \delta(M - E' - \omega_m) \delta^3(P' + q) \frac{1}{2E'} \frac{1}{2\omega_m}. \tag{4.28}$$

Now we integrate over $d^3P'$ which yields

$$\frac{d\Gamma_{i \to f}}{d^3q} = \frac{1}{4\pi M} |F_{i \to f}|^2 \delta(M - E' - \omega_m) \frac{1}{2E'} \frac{1}{2\omega_m}. \tag{4.29}$$

Next we go over to spherical coordinates changing the integration measure to

$$d^3q = q^2 d\Omega_q. \tag{4.30}$$

Using the on-shell conditions for the individual final particles

$$E' = \sqrt{M'^2 + \mathbf{P}'^2}, \quad \omega_m = \sqrt{m^2 + \mathbf{q}^2}, \tag{4.31}$$

with $M'$ and $m$ the masses of the final baryon and meson, respectively, allows us to transform to the coordinates $E_f = E' + \omega_m$ with the integration measure

$$|q|d\Omega_q = E_f \omega_m dE_f. \tag{4.32}$$

This leads to the following expression:

$$\frac{d\Gamma_{i \to f}}{dE_f d\Omega_q} = \frac{1}{4\pi M} |F_{i \to f}|^2 \delta(M - E_f) \frac{|q|}{4E_f}. \tag{4.33}$$

After the integration over the energy $E_f$ we are left with

$$\frac{d\Gamma_{i \to f}}{d\Omega_q} = \frac{|q|}{16\pi M^2} |F_{i \to f}|^2. \tag{4.34}$$

At this stage it is necessary to consider the internal quantum numbers of the baryon as well as those of the meson states: The decaying resonance and the final baryon are characterized by the intrinsic spins $\Sigma$ and $\Sigma'$ as well as their projections $M_\Sigma$ and $M_\Sigma'$, respectively. In addition, the baryons are also defined by the isospins $T$ and $T'$ as well as the corresponding projections $M_T$ and $M_{T'}$, respectively. In this work we consider only strong decays, where the involved meson is a pseudoscalar particle, i.e. we are interested in the $\pi$, $\eta$, and $K$-decay channels. Pseudoscalar mesons are defined by an intrinsic spin $\Sigma_m = 0$. With respect to the isospin, the $\pi$ has an isospin
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$T_m = 1$ and corresponding projections $M_{T_m} = 0, \pm 1$, the $\eta$ is defined by $T_m = 0$, and the $K$ has an isospin $T_m = \frac{1}{2}$ with $M_{T_m} = \pm \frac{1}{2}$.

Our interest aims at the calculation of the decay rate, i.e. the decay of baryon resonances to corresponding ground states and pseudoscalar mesons. Therefore, we finally have to average over all spin and isospin projections of the decaying baryon and sum over all spin and isospin projections of the final decay product:

$$
\frac{1}{2\Sigma + 1} \frac{1}{2T + 1} \sum_{M_{\Sigma}, M_{\Sigma'}} \sum_{M_{T}, M_{T'}, M_{T_m}} |F_{i \rightarrow f}|^2. \quad (4.35)
$$

The expression $(4.35)$ is Lorentz-invariant and due to the rotational invariance the remaining angular integration gives just $4\pi$. The final result is then

$$
\Gamma_{i \rightarrow f} = \rho \frac{1}{2\Sigma + 1} \frac{1}{2T + 1} \sum_{M_{\Sigma}, M_{\Sigma'}} \sum_{M_{T}, M_{T'}, M_{T_m}} |F_{i \rightarrow f}|^2, \quad (4.36)
$$

where we call

$$
\rho = \left| \frac{q}{4M^2} \right| \quad (4.37)
$$

the phase-space factor.

Summation and Averaging over the Spin

Assuming we are in the rest frame of the decaying resonance, we make the choice, that the outgoing particles move in $z$-direction, i.e. the meson moves along the positive $z$-axis with three-momentum $q = (0, 0, Q)$, and the final baryon moves along the negative $z$-axis with $P' = -q$. Now we take a look at the total spin quantum numbers: With the choice $q$ in $z$-direction we sum over the spin projections $M_{\Sigma}$ and $M_{\Sigma'}$ to the $z$-axis. Since the decay operator does not depend neither on $\Sigma$ nor on $\Sigma'$, the transition amplitude has the property

$$
F_{i \rightarrow f} \sim \delta_{M_{\Sigma}, M_{\Sigma'}} F_{i \rightarrow f} \quad (4.38)
$$

and furthermore

$$
|F_{i|_{M_{\Sigma}} \rightarrow f|_{M_{\Sigma'}}}| = |F_{i|_{-M_{\Sigma}} \rightarrow f|_{-M_{\Sigma'}}}|. \quad (4.39)
$$

For a decay process, where the final baryon is in the ground state with $\Sigma' = \frac{1}{2}$ the spin summation and averaging in Eq. $(4.36)$ leads to

$$
\frac{2}{2\Sigma + 1} |F_{i|_{M_{\Sigma}} \rightarrow f|_{M_{\Sigma'}}}|^2. \quad (4.40)
$$

$^1$As the pseudoscalar mesons have $\Sigma_m = 0$ no summation over $M_{\Sigma_m}$ emerges.
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Summation and Averaging over the Isospin

Analogously to the spin, we also have to average over all isospin projections of the decaying baryon and to sum over all isospin projections of the final decay products. Differently from the spin, where (due to the fact that all pseudoscalar mesons have an intrinsic spin \( \sum = 0 \)) the summation gets rather simple, the situation is more tricky in case of the isospin. Except for the \( \eta \) all pseudoscalar mesons have an isospin \( T_m \neq 0 \). Clearly, one could directly perform the summation over all isospin projections, thus ending up with a certain number of terms. In order to avoid the resulting computational effort we instead make use of the Wigner-Eckart theorem (see Eq. (C.4) and, e.g., Ref. [87]), which leads to the following replacement in the isospin part of the transition amplitude

\[
\frac{1}{2T+1} \sum_{M_T, M_{T'}, M_{Tm}} |F_{i-f}|^2 = \frac{2T'+1}{2T+1} \frac{|F_{i[T, M_T] \rightarrow f[T', M_{T'}]}(F^m)|^2}{|C^{T'M_{T'} T_{Tm} M_{Tm}}|^2}.
\]

(4.41)

The isospin dependence of the transition amplitude is furnished by the matrix elements of the flavor decay operator \( F^m \) (to be discussed in Chapter 5)

\[
\langle T'M_{T'}|F^m|TM_T\rangle.
\]

(4.42)

It is specified by the flavor index \( m \) referring to a \( \pi^0 \), \( \pi^+ \), or \( \pi^- \) and so on. The advantage is obvious: We need to calculate only one transition amplitude with a flavor operator \( F^m \) corresponding to a \( \pi \), \( \eta \), or \( K \) decay mode; all components in the isospin multiplet are then represented by the same transition amplitude multiplied with the appropriate flavor factor. As an example let us think of the \( N^* \to N\pi \) decay: According to the isospin quantum numbers, possible transitions are \( p \to p\pi^0 \), \( n \to n\pi^0 \), \( p \to n\pi^+ \), and \( n \to p\pi^- \), where \( p \) and \( n \) are abbreviations for the proton \( (T = \frac{1}{2}, M_T = \frac{1}{2}) \) and the neutron \( (T = \frac{1}{2}, M_T = -\frac{1}{2}) \), respectively. For the actual calculation we just use the representative \( p \to p\pi^0 \); by summing and averaging over the isospin components we are finally led to \( 3 \cdot |F_{i-p} \to f_{\pi^0}(F^{\pi^0})|^2 \). A more detailed analysis for all meson decay channels can be found in Appendix C.

4.2 Definition of the Transition Amplitude

The last step is the calculation of the transition amplitude \( F_{i \to f} \) given in Eq. (4.36). Unfortunately, there is no microscopic model available that is able to yield a decay operator in a unique way. In our calculations we will adhere to a simplified model where the meson is created only by one of the three quarks, while the two other quarks are spectators; for the moment the meson is treated as a point-like (fundamental) particle. This is also congruent with the GBE dynamics. A non-relativistic reduction leads to
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the so-called elementary emission model (EEM) (discussed in Chapter 6). In order to find a suitable definition of the decay operator, we start with an interaction Hamiltonian \( H_I \) as provided by the CQMs to describe the corresponding quark-quark-meson (qqm) vertex. Following the formalism as outlined in Ref. [85] we make a perturbation expansion of the \( S \) operator

\[
S = 1 - i \int_{-\infty}^{+\infty} dt_1 H_I(t_1) + (-i)^2 \int_{-\infty}^{+\infty} dt_1 \int_{-\infty}^{t_1} dt_2 H_I(t_1)H_I(t_2) + (-i)^3 \int_{-\infty}^{+\infty} dt_1 \int_{-\infty}^{t_1} dt_2 \int_{-\infty}^{t_2} dt_3 H_I(t_1)H_I(t_2)H_I(t_3) + \ldots ,
\]

which can also be written as

\[
S = T \exp[-i \int_{-\infty}^{+\infty} dt \ H_I(t)],
\]

where \( T \) indicates that the expression is to be evaluated by time-ordering each term in the series expansion for the exponential. We can express \( H_I(t) \) through the interaction Lagrangian \( \mathcal{L}_I(x) \)

\[
H_I(t) = - \int d^3x \mathcal{L}_I(x),
\]

and with a truncation after the first order we obtain

\[
S = T \exp \left( i \int d^4x \mathcal{L}_I(x) \right) \approx 1 + i \int d^4x \mathcal{L}_I(x).
\]

In the decay process \(|f\rangle \neq |i\rangle\), and thus

\[
\langle f | S | i \rangle = i \int d^4x \langle f | \mathcal{L}_I(x) | i \rangle = i \int d^4x \exp[-i(p_i - p_f)x] \langle f | \mathcal{L}_I(0) | i \rangle = i(2\pi)^4 \delta^4(p_i - p_f) \langle f | \mathcal{L}_I(0) | i \rangle.
\]

Now we compare this with Eq. (4.5) and find

\[
F_{i \rightarrow f} = (2\pi)^4 \langle f | \mathcal{L}_I(0) | i \rangle.
\]

### 4.3 Fundamental Quark-Meson Coupling

In the definition of the decay operator in Chapter 5 we shall need the type of coupling between a quark \( q \) and a meson \( m \) in the emission process. Following Ref. [65] we can adopt this coupling either in pseudoscalar or pseudovector form. In the first case the interaction Lagrangian for the meson-quark interaction reads

\[
\mathcal{L}_I^{ps}(x) = -ig_{qqm}\bar{\psi}(x)\gamma_5 \mathcal{F}^m \psi(x)\phi_m(x)
\]
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and in the second case it reads

$$\mathcal{L}_{\mu}^m(x) = -\frac{g_{\text{qqm}}}{2m_1} \bar{\psi}(x) \gamma_\mu \mathcal{F}^m \psi(x) \partial_\mu \phi_m(x)$$ (4.50)

with $g_{\text{qqm}}$ the coupling constant, $\mathcal{F}^m$ the flavor operator for the particular meson channel, and $m_1$ the mass of the single quark. If the involved meson is a $\pi$ or $\eta$, the active quark does not change its strangeness content, i.e. the mass of the active quark is the same in the initial and final state ($m_1 = m'_1$). However, if a $K$ meson couples to the active quark, one has to be aware of the different quark masses $m_u \neq m_s$. Consequently the term $2m_1$ in the denominator of Eq. (4.50) has to be substituted by $(m_1 + m'_1)$ in this case.

For the free quark field satisfying the Dirac equation

$$i\gamma_\mu \partial^\mu - m_1)\psi(x) = 0$$ (4.51)

we define

$$\psi(x) = \frac{1}{(2\pi)^2} \sum_{\sigma_1} \int \frac{d^3p_1}{2p_{10}} \left( e^{-ipx} u(p_1, \sigma_1) a(p_1, \sigma_1) + e^{ipx} v(p_1, \sigma_1) b^\dagger(p_1, \sigma_1) \right),$$

$$\bar{\psi}(x) = \frac{1}{(2\pi)^2} \sum_{\sigma_1} \int \frac{d^3p_1}{2p_{10}} \left( e^{ipx} \bar{u}(p_1, \sigma_1) a(p_1, \sigma_1) + e^{-ipx} \bar{v}(p_1, \sigma_1) b(p_1, \sigma_1) \right),$$ (4.52)

where $u(p, \sigma)$ and $v(p, \sigma)$ are the Dirac spinors with momentum $p$ and spin $\sigma$. The operators $a(a^\dagger)$ and $b(b^\dagger)$ are the annihilation (creation) operators of particles and anti-particles, respectively. As we are only interested in quarks, in the following we will omit the part for the anti-particles. Similarly, for the charge-neutral meson field we get

$$\phi_m(x) = \frac{1}{(2\pi)^2} \int \frac{d^3q}{2\omega_m} \left( e^{iqx} A^\dagger(q) + e^{-iqx} A(q) \right),$$ (4.53)

with operators $A(A^\dagger)$ annihilating (creating) the corresponding particles. With the assumption that only one quark emits a meson, the corresponding initial and final states are given by

$$|i_\text{q}\rangle = a^\dagger(p_1, \sigma_1)|0\rangle,$n
$$|f_\text{qm}\rangle = a^\dagger(p'_1, \sigma'_1)A^\dagger(q)|0\rangle,$$ (4.54)

with $p_1$ ($p'_1$) and $\sigma_1$ ($\sigma'_1$) the momenta and the spins of the single quark, respectively, and $q$ the momentum of the meson. The creation (annihilation) operators satisfy the commutator and anti-commutator relations

$$[A(q), A^\dagger(q')] = 2\omega \delta^3(q - q'),$$
$$\{a(p, \sigma), a^\dagger(p', \sigma')\} = \delta_{\sigma\sigma'} 2p_0 \delta^3(p - p').$$ (4.55)
For the pseudoscalar interaction Lagrangian it follows that
\[
\langle f_{qm}|\mathcal{L}^{ps}_I(0)|i_q\rangle = -\frac{ig_{qgm}}{(2\pi)^{\frac{3}{2}}} \bar{u}(p',\sigma')\gamma_5\mathcal{F}^m u(p_1,\sigma_1),
\] (4.56)

and similarly for the pseudovector case we get
\[
\langle f_{qm}|\mathcal{L}^{pv}_I(0)|i_q\rangle = -\frac{ig_{qgm}}{2m_1} \frac{1}{(2\pi)^{\frac{3}{2}}} \bar{u}(p',\sigma')\gamma_5\gamma^\mu\mathcal{F}^m u(p_1,\sigma_1)q_\mu.
\] (4.57)

For the meson-quark coupling constant we assume the value \( \frac{g_{qgm}}{4\pi} = 0.67 \), which is consistent with the one used in the parametrization of the pseudoscalar GBE CQM. Clearly, formulae (4.56) and (4.57) describe only the quark-meson vertex. For the creation of a meson from a baryon resonance they have to be embedded into the three-quark system (see Chapter 5).

For an isolated quark-meson system the momentum transfer to the meson is given by \( q^\mu = p_1^\mu - p_1'^\mu \) and the pseudoscalar and pseudovector couplings give identical results. To see this, one has to perform a partial integration, whereby one can move the derivative action on the meson field over to the quark fields. One then uses the Dirac equation (4.51) as well as its adjoint to replace the derivative of the quark fields by the corresponding quark masses. One can see, that the pseudoscalar and pseudovector Lagrangians are equivalent provided that the coupling constants have the proper relation, and therefore
\[
\frac{1}{2m_1} \bar{u}(p',\sigma')\gamma_5\gamma^\mu u(p_1,\sigma_1)(p_1 - p_1')_\mu = \bar{u}(p_1,\sigma_1)\gamma_5 u(p_1,\sigma_1).
\] (4.58)

In our actual calculations of decaying baryon resonances, we have to deal with a three-quark system going over into another baryon and a meson. As soon as one requires an overall momentum conservation on the baryon level as well as the respective on-shell conditions of the participating particles one can no longer maintain the condition \( q^\mu = p_1^\mu - p_1'^\mu \) on the quark level. Rather, the momentum transfer to the constituent quark participating in the meson production is
\[
\tilde{q}^\mu = p_1^\mu - p_1'^\mu \neq q^\mu = P^\mu - P'^\mu,
\] (4.59)
i.e. the momentum transfer to the single quark is not identical with the momentum transfer to the baryon as a whole.

The main challenge of Chapter 5 will be to find an appropriate decay operator satisfying the overall momentum conservation within the framework of a Poincaré-invariant theory. There, we will make an ansatz for the decay operator within the point form of RQM.
Chapter 5

Point-Form Calculation of Hadronic Decays

In Chapter 3 we reviewed the basics of RQM, in particular, we discussed the different forms of dynamics according to Dirac [8]. In this thesis all calculations are performed within the point form. First, we will derive the transition amplitude in the point-form formalism. For the concrete calculation we adopt a spectator model assuming both a pseudovector and a pseudoscalar coupling of the active quark to the meson\(^1\). In this respect some useful notations and properties of the point form can be found in Appendix B.6. Then, we will derive the full expression for the total decay width. In this context we shall also discuss the independence of the PFSM calculation on the reference frame.

In Chapter 4 we found the expression for the decay width to be

\[
\Gamma_{i\to f} = \frac{|q|}{4M^2} \frac{1}{2\Sigma + 1} \frac{1}{2T + 1} \sum_{M_S, M_{S'}} \sum_{M_T, M_{T'}} |F_{i\to f}|^2, \tag{5.1}
\]

where the term \(\frac{|q|}{4M^2} = \rho\) is the phase-space factor and \(F_{i\to f}\) the transition amplitude. Both the phase-space factor and the (spin-averaged) transition amplitude are invariant quantities and comply with the overall four-momentum conservation demanded from a Poincaré-invariant theory.

5.1 The Transition Amplitude

The relativistic transition amplitude for the mesonic decay of a baryon resonance being in a state \(|M, V, \Sigma, M_S, T, M_T\rangle\) to a baryon ground state \(|M', V', \Sigma', M_{S'}, T', M_{T'}\rangle\) is defined by the matrix element of the mesonic decay operator \(\hat{D}_{rd}^{\nu}\)

\[
F_{i\to f} = \langle M', V', \Sigma', M_{S'}, T', M_{T'}|\hat{D}_{rd}^{\nu}|M, V, \Sigma, M_S, T, M_T\rangle, \tag{5.2}
\]

\(^1\)In the following we will call this model the point-form spectator model (PFSM).
where the superscript "m" specifies the involved meson. Note that the decay operator $\hat{D}^{m}_{rd}$ is a reduced operator\(^2\), i.e. the four-delta function expressing the overall momentum conservation of the transition amplitude is not included in Eq. (5.2) but already in the definition of the decay width (see Eq. (4.5) and the following in Chapter 4).

For the concrete calculations it is advantageous to choose an appropriate reference frame. In principle, all frames are equivalent (see the discussion in Section 5.5): However, for simplicity we choose analogously to Chapter 4 the rest frame of the decaying baryon, and we assume that the final baryon as well as the meson move along the $z$-axis, where the meson moves along the positive and the final baryon along the negative $z$-direction with $q = (0, 0, Q)$ and $P' = -q$, respectively. The boosts $B(V)$ and $B(V')$ corresponding to the initial and final baryon states $|M, V, \Sigma, M_{\Sigma}, T, M_{T}\rangle$ and $|M', V', \Sigma', M_{\Sigma'}, T', M_{T'}\rangle$ are therefore given as

\[
B(V) = \begin{pmatrix} \cosh \Delta & 0 & 0 & -\sinh \Delta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sinh \Delta & 0 & 0 & \cosh \Delta \end{pmatrix},
\]

where $\Delta$ is called rapidity and

\[
\sinh \Delta = \frac{Q}{M'},
\]

\[
\cosh \Delta = \sqrt{1 + \frac{Q^2}{M'^2}}.
\]

Here, $M'$ is the mass of the final baryon and $Q$ is the absolute value of the momentum transfer by the meson. It is fixed due to the overall four-momentum conservation $P^\mu = P'^\mu + q^\mu$ and the on-shell condition of the meson $g_{\mu\nu}q^\nu = q_0^2 - q^2 = \omega_m^2 - Q^2 = m^2$ with $m$ the mass and $\omega_m$ the energy of the meson, respectively. In this special frame the four-momenta of the initial and final baryons are given by

\[
P^\mu = B(V) \begin{pmatrix} M \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} M \\ 0 \\ 0 \\ 0 \end{pmatrix},
\]

\[
P'^\mu = B(V') \begin{pmatrix} M' \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} M' \cosh \Delta \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \sqrt{M'^2 + Q^2} \\ 0 \\ 0 \\ -Q \end{pmatrix}.
\]

\(^2\)Details concerning reduced operators can be found, e.g., in Ref. [88].
In the following calculations we will frequently use the product $B^{-1}(V') B(V)$, which is given explicitly as

$$
B(V, V') = B^{-1}(V') B(V) = \begin{pmatrix}
\cosh \Delta & 0 & 0 & \sinh \Delta \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
\sinh \Delta & 0 & 0 & \cosh \Delta
\end{pmatrix}.
$$

(5.8)

For the evaluation of Eq. (5.2) we first insert the appropriate identities with respect to the momentum and the velocity states provided by Eqs. (B.36) and (B.42), respectively. It then becomes

$$
F_{i \rightarrow f} = \sum_{\alpha_1 \alpha_2 \alpha_3} \sum_{\alpha_1' \alpha_2' \alpha_3'} \left\{ \prod_{i=1}^{3} \frac{d^3p_i}{2p_{i0}} \frac{d^3p_i'}{2p_{i0}'} \frac{d^3p_i''}{2p_{i0}''} \frac{d^3p_i'''}{2p_{i0}'''} \int \frac{d^3v}{v_0} \frac{d^3k_1}{2\omega_1} \frac{d^3k_2}{2\omega_2} \frac{d^3k_3}{2\omega_3}
\right.
$$

$$
\times \left( M', V', \Sigma', M_{\Sigma'}, T', M_{T'} | v''; k_1', k_2', k_3'; \mu_1', \mu_2', \mu_3' \right)
$$

$$
\times \left( v''; k_1', k_2', k_3'; \mu_1', \mu_2', \mu_3' | p_1', p_2', p_3'; \sigma_1', \sigma_2', \sigma_3' \right)
$$

$$
\times \left( p_1', p_2', p_3'; \sigma_1', \sigma_2', \sigma_3' | \bar{D}_{r_0} | p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 \right)
$$

$$
\times \left( v; k_1, k_2, k_3; \mu_1, \mu_2, \mu_3 | M, V, \Sigma, M_{\Sigma}, T, M_{T} \right).
$$

(5.9)

The overlaps of the velocity and momentum states are expressed by (see Eq. (B.44))

$$
\langle v''; k_1', k_2', k_3'; \mu_1', \mu_2', \mu_3' | p_1', p_2', p_3'; \sigma_1', \sigma_2', \sigma_3' \rangle
$$

$$
= \prod_{j=1}^{3} D_{\sigma_j' \mu_j'}^{\frac{1}{2}} \left( R_W \left( k_j''; B \left( v'' \right) \right) \right) 2p_{j0} \delta^3 \left( p_j'' - p_j \right),
$$

(5.10)

$$
\langle p_1', p_2', p_3'; \sigma_1', \sigma_2', \sigma_3' | v; k_1, k_2, k_3; \mu_1, \mu_2, \mu_3 \rangle
$$

$$
= \prod_{i=1}^{3} D_{\sigma_i \mu_i}^{\frac{1}{2}} \left( R_W \left( k_i; B \left( v \right) \right) \right) 2p_{i0} \delta^3 \left( p_i - p_i' \right),
$$

(5.11)

where the Wigner rotations are given by

$$
R_W \left( k_j''; B \left( v'' \right) \right) = B^{-1} \left( B \left( v'' \right) k_j \right) B \left( v'' \right) B \left( k_j'' \right),
$$

$$
R_W \left( k_i; B \left( v \right) \right) = B^{-1} \left( B \left( v \right) k_i \right) B \left( v \right) B \left( k_i \right),
$$

(5.12)

with the matrices $B \left( v'' \right)$ and $B \left( v \right)$ corresponding to canonical boosts. Exploiting the $\delta$-functions we now integrate over all three-momenta $p_i'$ and $p_i'''$ and obtain
Inserting them into Eq. (5.13) the transition amplitude becomes

\[
F_{i \rightarrow f} = \sum_{\sigma_1^i \sigma_2^f \sigma_3^i} \sum_{\alpha_1^{i2} \alpha_2^{i3} \alpha_3^{i2}} \int \frac{d^3 v}{v_0} \frac{d^3 k_2}{2 \omega_2} \frac{d^3 k_3}{2 \omega_3} \frac{d^3 v''}{v_0''} \frac{d^3 k'_2}{2 \omega_2'} \frac{d^3 k'_3}{2 \omega_3'} \times \left( \frac{\omega_1 + \omega_2 + \omega_3}{2 \omega_1} \right)^3 \left( \frac{\omega_1'' + \omega_2'' + \omega_3''}{2 \omega_1''} \right)^3
\times \langle M', \mathbf{V}', \Sigma', M\Sigma', T', M\Sigma' | v''; \mathbf{k}_1', \mathbf{k}_2', \mathbf{k}_3'; \mu_1', \mu_2', \mu_3' \rangle
\times \left( \prod_{j=1}^3 D_{\sigma_j^i \mu_j^f}^* \left[ RW \left( k_j''; B (v'') \right) \right] \right)
\times \langle v; \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3; \mu_1, \mu_2, \mu_3 | M, \mathbf{V}, \Sigma, M\Sigma, T, M_T \rangle. \tag{5.13}
\]

The velocity-state representations of the baryon eigenstates are given by (see Eq. (B.45))

\[
\langle v; \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3; \mu_1, \mu_2, \mu_3 | M, \mathbf{V}, \Sigma, M\Sigma, T, M_T \rangle = \frac{\sqrt{2}}{M} v_0 \delta^3 (\mathbf{V} - \mathbf{v}) \sqrt{\frac{2 \omega_1 2 \omega_2 2 \omega_3}{(\sum \omega_i)^3}} \Psi_{M\Sigma M\Sigma T M_T} (k_i; \mu_i), \tag{5.14}
\]

\[
\langle M', \mathbf{V}', \Sigma', M\Sigma', T', M\Sigma' | v''; \mathbf{k}_1', \mathbf{k}_2', \mathbf{k}_3'; \mu_1', \mu_2', \mu_3' \rangle = \frac{\sqrt{2}}{M'} v_0'' \delta^3 (\mathbf{V}' - \mathbf{v}'') \sqrt{\frac{2 \omega_1'' 2 \omega_2'' 2 \omega_3''}{(\sum \omega_i'')^3}} \Psi^{*}_{M'\Sigma' M\Sigma' T' M_T} (k_i''; \mu_i''). \tag{5.15}
\]

Inserting them into Eq. (5.13) the transition amplitude \(F_{i \rightarrow f}\) becomes

\[
F_{i \rightarrow f} = \sum_{\sigma_1^i \sigma_2^f \sigma_3^i} \sum_{\alpha_1^{i2} \alpha_2^{i3} \alpha_3^{i2}} \int \frac{d^3 v}{v_0} \frac{d^3 k_2}{2 \omega_2} \frac{d^3 k_3}{2 \omega_3} \frac{d^3 v''}{v_0''} \frac{d^3 k'_2}{2 \omega_2'} \frac{d^3 k'_3}{2 \omega_3'} \times \left( \frac{\omega_1 + \omega_2 + \omega_3}{2 \omega_1} \right)^3 \left( \frac{\omega_1'' + \omega_2'' + \omega_3''}{2 \omega_1''} \right)^3
\times \frac{\sqrt{2}}{M'} v_0'' \delta^3 (\mathbf{V}' - \mathbf{v}'') \sqrt{\frac{2 \omega_1'' 2 \omega_2'' 2 \omega_3''}{(\sum \omega_i'')^3}} \Psi^{*}_{M'\Sigma' M\Sigma' T' M_T} (k_i''; \mu_i'')
\times \left( \prod_{j=1}^3 D_{\sigma_j^i \mu_j^f}^* \left[ RW \left( k_j''; B (v'') \right) \right] \right)
\times \langle v; \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3; \mu_1, \mu_2, \mu_3 | M, \mathbf{V}, \Sigma, M\Sigma, T, M_T \rangle. \tag{5.13}
\]
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\[ x(\hat{p}_1^\alpha, \hat{p}_2^\beta, \hat{p}_3^\gamma; \sigma_1^\alpha, \sigma_2^\beta, \sigma_3^\gamma | \hat{D}_{\text{rd}}^m | p_1, p_2, p_3; \sigma'_1, \sigma'_2, \sigma'_3) \]

\[ \times \frac{3}{2} \sum_{i=1}^{3} D_{\sigma_i}^{1/2} [R_W (k_i; B (v))] \]

\[ \times \sqrt{\frac{2}{M}} v_0 \delta^3 (V - v) \left[ \frac{2\omega_1 2\omega_2 2\omega_3}{(\sum \omega_i)^3} \Psi_{\Sigma M \Sigma E T M_T} (k_i; \mu_i) \right]. \]  

(5.16)

Now we can integrate over the velocities and obtain

\[ F_{i-f} = \sum_{\sigma_1^\alpha, \sigma_2^\beta, \sigma_3^\gamma} \sum_{\mu_1, \mu_2, \mu_3} \int \frac{d^3 k_2 d^3 k_3 d^3 k_4}{2\omega_2 2\omega_3 2\omega_4} \]

\[ \times \left( \omega_1 + \omega_2 + \omega_3 \right)^3 \left( \omega_{1}'' + \omega_{2}'' + \omega_{3}'' \right)^3 \]

\[ \times \sqrt{\frac{2}{M}} \left[ \frac{2\omega_1 2\omega_2 2\omega_3}{(\sum \omega_i)^3} \Psi_{\Sigma M \Sigma E T M_T} (k_i''; \mu_i'') \right] \]

\[ \times \frac{3}{2} \sum_{j=1}^{3} D_{\sigma_j''}^{1/2} [R_W (k_j''; B (V''))] \]

\[ \times (\hat{p}_1^\alpha, \hat{p}_2^\beta, \hat{p}_3^\gamma; \sigma_1^\alpha, \sigma_2^\beta, \sigma_3^\gamma | \hat{D}_{\text{rd}}^m | p_1, p_2, p_3; \sigma'_1, \sigma'_2, \sigma'_3) \]

\[ \times \frac{3}{2} \sum_{i=1}^{3} D_{\sigma_i}^{1/2} [R_W (k_i; B (V))] \]

\[ \times \sqrt{\frac{2}{M}} \left[ \frac{2\omega_1 2\omega_2 2\omega_3}{(\sum \omega_i)^3} \Psi_{\Sigma M \Sigma E T M_T} (k_i; \mu_i) \right]. \]  

(5.17)

5.1.1 The Point-Form Spectator Model

The reduced decay operator in Eq. (5.17) is a priori a general many-body operator. With present means it is not possible to employ such a full decay operator in the calculation of the matrix elements. Therefore one has to make simplifying assumptions. Here we adhere to a decay model where only one of the constituent quarks inside the baryon couples to a (point-like) meson that is produced in the decay process; the other two act as spectators. At the same time the overall momentum is conserved on the baryon level. Such spectator-model decay operators have already been applied in several other studies, such as the electroweak baryon form-factors [46–48, 89] and pionic as well as \(\eta\) decays of light baryons [20].
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For the meson-quark interaction we consider either a pseudoscalar or a pseudovector coupling. The ansatz for both types of coupling has been extensively discussed in Chapter 4 and is provided by the Eqs. (4.56) and (4.57). In case of pseudovector coupling the matrix elements of the spectator-model decay operator between the three-quark states read

\[ \langle p_1''; p_2''; p_3''; \sigma_1'', \sigma_2'', \sigma_3'' | \hat{D}_{rd}^{pv;m} | p_1; p_2; p_3; \sigma_1', \sigma_2', \sigma_3' \rangle = -3N \frac{i g_{qqm}}{m_1 + m_1} \frac{1}{\sqrt{2\pi}} \bar{u}(p_1'', \sigma_1'') \gamma_5 \gamma_\mu \mathcal{F}^m u(p_1', \sigma_1') q_\mu 
\times 2p_{20} \delta^3 \left( p_2 - p_2'' \right) \delta_{\sigma_2'' \sigma_2'}^\mu 2p_{30} \delta^3 \left( p_3 - p_3'' \right) \delta_{\sigma_3'' \sigma_3'}^\mu, \quad (5.18) \]

where \( q_\mu = p_\mu - P_\mu \) is the four-momentum transfer of the emitted meson to the baryon. At the same time the momentum transfer \( \tilde{q} \) to the single quark coupling to the meson is \( \tilde{q}_\mu = p_{1,\mu} - p_{1,\mu}' \). In case of the pseudoscalar coupling the matrix elements of the spectator-model decay operator between the three-quark states are

\[ \langle p_1''; p_2''; p_3''; \sigma_1'', \sigma_2'', \sigma_3'' | \hat{D}_{rd}^{ps;m} | p_1; p_2; p_3; \sigma_1', \sigma_2', \sigma_3' \rangle = -3N \frac{i g_{qqm}}{\sqrt{2\pi}} \bar{u}(p_1'', \sigma_1'') \gamma_5 \mathcal{F}^m u(p_1', \sigma_1') 2p_{20} \delta^3 \left( p_2 - p_2'' \right) \delta_{\sigma_2'' \sigma_2'}^\mu 2p_{30} \delta^3 \left( p_3 - p_3'' \right) \delta_{\sigma_3'' \sigma_3'}^\mu. \quad (5.19) \]

Again the momentum transfer to the single quark coupling to the meson is \( \tilde{q}_\mu = p_{1,\mu} - p_{1,\mu}' \). In both formulae, \( g_{qqm} \) is the meson-quark coupling constant\(^3\), \( m_1 \) and \( m_1' \) are the masses of the active quark concerning the initial and the final state, respectively, and \( \mathcal{F}^m \) represents the flavor operator for the particular decay channels specified by "\( m \)", which is given by a linear combination of the Gell-Mann flavor matrices (see Appendix C.2). The fact that the spectator quarks 2 and 3 do not participate in the meson-quark interaction is expressed by the invariant spectator conditions involving the \( \delta \)-functions of their three-momenta. Note that the factor 3 on the right-hand side of both equations is chosen due to symmetric reasons: We assume that only the first quark is the one emitting the meson, therefore we have to multiply the total result by a factor of 3. In the point form the generators of the Lorentz transformations are kinematical, thus the PFSM decay operators (5.18) and (5.19) maintain their spectator model character in all reference frames. This Lorentz-covariance of the spectator-model operators is a specific property of the point form and does not exist in the other forms of RQM [90].

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\(^3\)The value of the meson-quark coupling constant is chosen consistently with the one used in the GBE CQM. As outlined in Section 2.2.2, the Graz group assumed only one single coupling constant for all octet mesons \( \pi, \eta, \) and \( K \). The actual value is deduced from the pion-quark coupling constant to be \( \frac{g_{\pi qqm}}{4\pi} = 0.67 \).
Finally, in Eqs. (5.18) and (5.19) there also appears a term $\mathcal{N}$, which is called PFSM normalization factor. In the calculation of the electromagnetic properties of baryons it is needed for the reproduction of the proton charge (the electric form factor at zero momentum transfer). For consistency reasons one introduces this factor for all PFSM operators and therefore also for the hadronic decays. The explicit form of $\mathcal{N}$ reads

$$\mathcal{N} = \mathcal{N}_S = \left( \frac{M}{\sum_i \omega_i} \frac{M'}{\sum_i \omega''_i} \right)^{\frac{1}{2}},$$

(5.20)

where $M$ and $M'$ are the baryon masses (eigenvalues of the interacting mass operator) and $\omega_i$ as well as $\omega''_i$ the individual quark energies. The latter ones are functions of the integration variables

$$\omega_i = \sqrt{m^2_i + k^2_i}, \quad \omega''_i = \sqrt{m''^2_i + k''^2_i}, \quad (i = 1, 2, 3).$$

(5.21)

Thus they are indirectly dependent on the momentum transfer $Q$ entering the boosts. The normalization factor in Eq. (5.20) is expressed in a Lorentz-invariant form, which guarantees that the transition amplitudes in the PFSM are manifestly covariant. However, the choice of $\mathcal{N}$ is not unique, and besides the symmetric choice above, one can use other possible PFSM normalization factors, also complying with the requirements of a Poincaré-invariant theory and maintaining the charge normalization. A detailed discussion on the behaviour of the normalization factor appearing in spectator-model operators in the point form has already been given in Ref. [88]. It has been stated that the general form

$$\mathcal{N}(\alpha) = \left( \frac{M}{\sum_i \omega_i} \right)^{3\alpha} \left( \frac{M'}{\sum_i \omega''_i} \right)^{3(1-\alpha)}$$

(5.22)

with $0 \leq \alpha \leq 1$ gives a set of allowed PFSM normalization factors, where the transition amplitudes are covariant for each choice of $\alpha$.

Recently, in Ref. [91] it has been shown that the exponent $\alpha$ can further be constrained by exploiting time reversal invariance, and it has been seen that the actual choice $\alpha = \frac{1}{2}$ in Eq. (5.22) meets the corresponding requirements. However, also the arithmetic combination

$$\mathcal{N}_A = \frac{1}{2} \left[ \left( \frac{M}{\sum_i \omega_i} \right)^3 + \left( \frac{M'}{\sum_i \omega''_i} \right)^3 \right]$$

(5.23)

is allowed. Concerning the electroweak form factors, one qualitatively found out that both choices, Eq. (5.20) and Eq. (5.23), lead to a rather good
description of the experimental data (they lie approximately between the theoretical predictions using $N_S$ and $N_A$, respectively). In this work we will discuss the results obtained with the standard choice of $N$, i.e. the one given in Eq. (5.20). However, beyond the present work it is certainly interesting to take a closer look at the arithmetic choice provided by Eq. (5.23) in future studies.

Many-Body Aspects of the PFSM

Even though we construct a spectator model, both the pseudoscalar and the pseudovector versions of the decay operator actually include also effective many-body contributions involving all quarks. This is evident from Section 4.3, where we already addressed the fact that the momentum transfer to the single quark is not identical to the momentum transfer to the baryon as a whole. Namely, in the rest frame of the decaying baryon, the overall momentum transfer is given by

$$q^\mu = P^\mu - P'^\mu = \left( \begin{array}{c} M - \sqrt{M'^2 + Q^2} \\ 0 \\ 0 \\ Q \end{array} \right),$$

(5.24)

whereas the momentum transfer $\tilde{q}^\mu$ to the single quark is given by

$$\tilde{q}^\mu = \tilde{p}^\mu - \tilde{p}'^\mu = k_1^\mu - B (V')^\mu \nu k_1'^\nu = \left( \begin{array}{c} \omega_1 - \cosh \Delta \omega_1'' + \sinh \Delta k_{1z}'' \\ 0 \\ 0 \\ k_{1z} + \sinh \Delta \omega_1'' - \cosh \Delta k_{1z}'' \end{array} \right).$$

(5.25)

The latter one is uniquely fixed due to the overall momentum conservation and the two spectator conditions. Obviously, the momentum transfers (5.24) and (5.25) are not the same. Consequently, the decay operators do not represent pure one-body operators, rather they effectively include many-body contributions, where also the spectator quarks participate in the decay process.

The Pseudovector Coupling

In the following we show how to proceed in case of a pseudovector coupling, i.e. we insert the expression (5.18) into Eq. (5.17). First we make use of the Kronecker deltas $\delta_{\sigma_1 \sigma_2''}$ and $\delta_{\sigma_3 \sigma_3''}$ and after some rewriting we get
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\[ F_{i-f}^{pw} = \sum_{\sigma_1' \sigma_1''} \sum_{\mu_1' \mu_2' \mu_3'} \int d^3k_2 d^3k_3 d^3k_2'' d^3k_3'' \]

\[ \times \sqrt{\frac{(\omega_1 + \omega_2 + \omega_3)^3 (\omega_2'' + \omega_3'')^3}{2\omega_1 2\omega_2 2\omega_3}} \]

\[ \times \Psi_{M'S'M''T'M'''}^{\lambda' \lambda''} (k''_i; \mu''_i) \frac{D_{\alpha_1' \mu_1'}^{*} D_{\alpha_2' \mu_2'}^{*} D_{\alpha_3' \mu_3'}^{*}}{D_{\alpha_1' \mu_1'}^{*} D_{\alpha_2' \mu_2'}^{*} D_{\alpha_3' \mu_3'}^{*}} [RW (k''_i; B (V'))] \]

\[ \times \mathcal{N} \bar{u} (p''_1, \sigma''_1) \gamma_5 \gamma^\mu \mathcal{F}^\mu u (p_1, \sigma_1) q_\mu D_{\alpha_1' \mu_1'} [RW (k_1; B (V))] \]

\[ \times D_{\alpha_2' \mu_2'}^{\lambda''} [RW (k''_2; B (V'))] D_{\alpha_3' \mu_3'}^{\lambda''} [RW (k''_3; B (V'))] 2p_{20} \delta^3 (p_2 - p'') \]

\[ \times \Psi_{M'S'M''T'M'''} (k_i; \mu_i) \]

(5.26)

with the abbreviation

\[ f^{pw} = -\frac{3i g_{qmm}}{m_1 + m'_1} \frac{1}{\sqrt{2 \pi} M M'} \]

(5.27)

which is independent of the integration variables, but actually depends on the masses \( m_1 \) and \( m'_1 \) of the active quark (before and after meson emission).

Next, we use the general relations of the Wigner \( D \)-functions

\[ \sum_{\alpha = \pm \frac{1}{2}} D_{\alpha \alpha}^{1/2} (A) D_{\alpha \beta'}^{1/2} (B) = D_{\beta \beta'}^{1/2} (A B), \]

(5.28)

\[ D_{\alpha \alpha}^{1/2} (A^{-1}) = D_{\alpha \beta}^{1/2} (A^{-1}) = D_{\alpha \beta}^{1/2} (A), \]

(5.29)

\[ D_{\alpha \beta}^{1/2} (I) = \delta_{\alpha \beta}, \]

(5.30)

and Eq. (5.26) becomes

\[ F_{i-f}^{pw} = \sum_{\sigma_1' \sigma_1''} \sum_{\mu_1' \mu_2' \mu_3'} \int d^3k_2 d^3k_3 d^3k_2'' d^3k_3'' \]

\[ \times \sqrt{\frac{(\omega_1 + \omega_2 + \omega_3)^3 (\omega_2'' + \omega_3'')^3}{2\omega_1 2\omega_2 2\omega_3}} \]

\[ \times \Psi_{M'S'M''T'M'''}^{\lambda' \lambda''} (k''_i; \mu''_i) \frac{D_{\alpha_1' \mu_1'}^{*} D_{\alpha_2' \mu_2'}^{*} D_{\alpha_3' \mu_3'}^{*}}{D_{\alpha_1' \mu_1'}^{*} D_{\alpha_2' \mu_2'}^{*} D_{\alpha_3' \mu_3'}^{*}} [RW (k''_i; B (V'))] \]

\[ \times \mathcal{N} \bar{u} (p''_1, \sigma''_1) \gamma_5 \gamma^\mu \mathcal{F}^\mu u (p_1, \sigma_1) q_\mu D_{\alpha_1' \mu_1'} [RW (k_1; B (V))] \]
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\[ \times D_{\mu_2'^2\mu_2}^2 \left[ R_W^{-1} \left( k_2''; B(V') \right) R_W \left( k_2; B(V) \right) \right] 2p_{200} \delta^3 \left( p_2 - p_2'' \right) \]

\[ \times D_{\mu_1'^2\mu_3}^2 \left[ R_W^{-1} \left( k_3''; B(V') \right) R_W \left( k_3; B(V) \right) \right] 2p_{300} \delta^3 \left( p_3 - p_3'' \right) \]

\[ \times \Psi_{M\Sigma M_{52}T M_{T}} (k_i; \mu_i). \]  

(5.31)

Due to the invariance of the four-delta function \( \delta^4 (p_i - p_i'') \) under Lorentz transformations one can rewrite

\[ 2p_{00} \delta^3 \left( p_i - p_i'' \right) \approx \delta^4 (p_i - p_i'') = \delta^4 \left( B^{-1} (V) p_i - B^{-1} (V, V') k_i'' \right) \]

\[ = \delta^4 \left( k_i - B^{-1} (V, V') k_i'' \right) \approx 2 \omega_i \delta^3 \left( \mathbf{k}_i - B^{-1} (V, V') k_i'' \right) \]  

(5.32)

using the abbreviation (5.8), and one obtains

\[ F_{1-0}^{\mu \nu} = \sum_{\sigma_1'\sigma_2'} \sum_{\mu_1'\mu_2'} \int d^3 k_2 d^3 k_3 d^3 k_2'' d^3 k_3'' \]

\[ \times \frac{1}{\sqrt{\omega_{12}^2 \omega_{13}^2 \sqrt{2 \omega_1 2 \omega_2}}} \sqrt{(\omega_1 + \omega_2 + \omega_3)^3 (\omega_1'' + \omega_2'' + \omega_3'')^3} \]

\[ \times \Psi_{M' \Sigma' M_{52} T' M_{T}} \left( k_i''; \mu_i'' \right) D_{\sigma_1' \mu_1''}^{\frac{2}{3}} \left[ R_W \left( k_i''; B(V') \right) \right] \]

\[ \times f^{\mu \nu} N \bar{u}(p_i'', \sigma_i'') \gamma_5 \gamma^\mu \tau^{\nu} u(p_1, \sigma_1') q_\mu D_{\sigma_1' \mu_1''}^{\frac{2}{3}} \left[ R_W (k_1''; B(V)) \right] \]

\[ \times D_{\mu_1'\mu_3}^2 \left[ R_W^{-1} \left( k_3''; B(V') \right) R_W \left( k_3; B(V) \right) \right] \delta^3 \left( k_3 - B^{-1} (V, V') k_3'' \right) \]

\[ \times \Psi_{M\Sigma M_{52}T M_{T}} (k_i; \mu_i). \]  

(5.33)

Next, we have to exploit the remaining three-delta functions in Eq. (5.33), which results in

\[ F_{1-0}^{\mu \nu} = \sum_{\sigma_1'\sigma_2'} \sum_{\mu_1'\mu_2'} \int d^3 k_2'' d^3 k_3'' \sqrt{\omega_{12}^2 \omega_{13}^2 \sqrt{2 \omega_1 2 \omega_2}} \]

\[ \times \sqrt{(\omega_1 + \omega_2 + \omega_3)^3 (\omega_1'' + \omega_2'' + \omega_3'')^3} \]

\[ \times \Psi_{M' \Sigma' M_{52} T' M_{T}} \left( k_i''; \mu_i'' \right) D_{\sigma_1' \mu_1''}^{\frac{2}{3}} \left[ R_W \left( k_i''; B(V') \right) \right] \]

\[ \times f^{\mu \nu} N \bar{u}(p_i''', \sigma_i''') \gamma_5 \gamma^\mu \tau^{\nu} u(p_1, \sigma_1') q_\mu D_{\sigma_1' \mu_1''}^{\frac{2}{3}} \left[ R_W (k_1''; B(V)) \right] \]

\[ \times D_{\mu_1'\mu_2}^2 \left[ R_W^{-1} \left( k_2''; B(V') \right) R_W \left( k_2; B(V) \right) \right] \]  

(5.34)

(5.35)
The remaining integration variables $k_2^0$ and $k_3^0$ are the three-momenta of the spectator quarks of the residual baryon, and one has to express the unprimed variables concerning the decaying baryon in terms of these variables. Physically, one can imagine the following picture: Both baryon wave functions corresponding to the initial and final baryon states are calculated in their respective rest frames, and they enter the calculation as an input. Through the decay process and thus emission of a meson, the baryon wave functions get connected via the relative boost $B^{-1}(V, V')$; this becomes manifest in the "displacement" of the variables $k_2$ and $k_3$ belonging to the decaying baryon.

Similarly to Eq. (5.32) one can transform the delta-functions in the fifth and sixth line of Eq. (5.33) according to

$$2\omega_i \delta^3(\mathbf{k}_i - B^{-1}(V, V')k''_i) \cong 2\omega''_i \delta^3(\mathbf{k}_i - B(V, V')k_i), \quad (i = 2, 3),$$

where we used the abbreviation (5.8) again. Consequently Eq. (5.33) can also be written as

$$F^p_{1-f} = \sum_{\sigma_1''} \sum_{\mu''_1} \sum_{\mu''_2} \frac{1}{\omega_2 \omega_3} \sqrt{\frac{(\omega_1 + \omega_2 + \omega_3)^3}{(\omega''_1 + \omega''_2 + \omega''_3)^3}} \times
\Psi_{\tilde{M}'\Sigma' M_T T' M_T}^* (\mathbf{k}'_i; \mu''_i) D_{\sigma''_1 \mu''_1} \left[ R_W (k''_i; B(V')) \right]
\times f^{p''} \tilde{N} \bar{u} (p'_1; \sigma''_1 \gamma_5 \gamma^\mu \gamma''_1 \mu''_1 \bar{u} (p_1; \sigma'_1) q_3 \frac{1}{\mu''_2 \mu''_3} \left[ R_W (k''_3; B(V')) \right]
\times D_{\mu''_2 \mu''_3} \left[ R_W^{-1} (k''_2; B(V')) R_W (k_2; B(V)) \right] \delta^3 (k'_2 - B(V, V') k_2)
\times D_3 \frac{1}{\mu''_1 \mu''_3} \left[ R_W^{-1} (k''_3; B(V')) R_W (k_3; B(V)) \right] \delta^3 (k'_3 - B(V, V') k_3)
\times \Psi_{\tilde{M}'\Sigma' M_T T' M_T} (\mathbf{k}_i; \mu_i). \quad (5.36)$$

In this case, the twelve-dimensional integral reduces to the following six-dimensional one.
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\[ F^{pv}_{1-f} = \sum_{\sigma_1} \sum_{\mu_1} \int d^3 k_2 d^3 k_3 \sqrt{\frac{\omega_2' \omega_3'}{\omega_2 \omega_3}} \frac{1}{\sqrt{2 \omega_1' 2 \omega_1}} \]

\[ \times \Psi_{M'\Sigma'M_2'T'M_T} (k_1''; \mu_i) D_{\sigma_1' \mu_1'}^{+} \left[ R_W (k_1''; B (V')) \right] \]

\[ \times \Psi_{M\Sigma'M_2'T'M_T} (k_i; \mu_i), \quad (5.37) \]

and the remaining integration variables \( k_2 \) and \( k_3 \) are the three-momenta of the spectator quarks of the decaying baryon. In this case the primed variables concerning the residual baryon have to be expressed in terms of \( k_2 \) and \( k_3 \). Here, the baryon wave functions are connected via the relative boost \( B (V, V') \), and the variables \( k_2'' \) and \( k_3'' \) belonging to the residual baryon get shifted.

From the technical point of view the two different integrals included in Eqs. (5.34) and (5.37) are well suited for a convergency test of the numerical code: Five out of the six integrations are performed numerically (see Section C.3 of Appendix C). Hence, depending on the chosen integration variables, the sampling points are completely different. If one does not use enough sampling points or has not optimised the numerical method, the numerical results for the transition amplitude may differ. In this thesis we tested both integrations with our code and thereby arrived at reliable convergent results.

In the following we show how to proceed in case of Eq. (5.37)\(^4\): Here, the components of the four-vectors \( k_2'' \) and \( k_3'' \) have to be expressed through the integration variables \( k_2 \) and \( k_3 \), namely

\[ k_2'' = B (V, V') \epsilon^\mu_2 k_2 = \left( m_2^2 + \left( B (V, V') k_2 \right)^2 \right)^{1/2} \frac{B (V, V') k_2}{B (V, V') k_2} \]

\[ = \begin{pmatrix} \cosh \Delta \omega_2 + \sinh \Delta \omega_2 \\ k_{2x} \\ k_{2y} \\ \sinh \Delta \omega_2 + \cosh \Delta \omega_2 \end{pmatrix}, \quad (5.38) \]

\(^4\)The procedure works similarly for the integration over the three-momenta of the spectator quarks of the residual baryon (5.34), but here we do not show this explicitly.
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\[ k_{3}^{0} = B (V, V')_{\mu} k_{3}^{\mu} = \left( \sqrt{m_{3}^{2} + \left( B (V, V') k_{3} \right)^{2}} \right) \]

\[ = \begin{pmatrix} \cosh \Delta \omega_{3} + \sinh \Delta k_{3z} \\ k_{3x} \\ k_{3y} \\ \sinh \Delta \omega_{3} + \cosh \Delta k_{3z} \end{pmatrix} \]  \hspace{1cm} (5.39)

The quark energies \( \omega_{2} \) and \( \omega_{3} \) in Eqs. (5.38) and (5.39) are given by

\[ \omega_{2} = \sqrt{m_{2}^{2} + k_{2}^{2}}, \quad \omega_{3} = \sqrt{m_{3}^{2} + k_{3}^{2}}. \]  \hspace{1cm} (5.40)

As the spectator quarks do not change their mass, it does not matter whether to write \( m_{i} \) or \( m'_{i} \) (for \( i = 2, 3 \)). Eventually, for the active quark we find the expressions (the decaying resonance is at rest)

\[ p_{1}^{\mu} = k_{1}^{\mu} = (\omega_{1}^{*}, k_{1}) = \left( \sqrt{m_{1}^{2} + (-k_{2} - k_{3})^{2}}, -k_{2} - k_{3} \right), \]

\[ p_{1}^{\mu'} = B (V')_{\mu'} k_{1}^{\mu'} = B (V') (\omega_{1}^{*}, k_{1}) \]

\[ = B (V') \left( \sqrt{m_{1}^{2} + (-k_{2}' - k_{3}')^{2}}, -k_{2}' - k_{3}' \right), \]  \hspace{1cm} (5.41)

where the unprimed and primed quark masses in general do not coincide (\( m_{1} \neq m'_{1} \)). Eventually, it is possible to combine the Wigner rotations in Eq. (5.37), and therefore to simplify the corresponding \( D \)-functions, namely

\[ D_{\mu_{2}' \mu_{2}}^{\frac{1}{2}} \left[ R_{W}^{-1} (B (V, V') k_{2}; B (V')) R_{W} (k_{2}; B (V)) \right] \]

\[ = D_{\mu_{2}' \mu_{2}}^{\frac{1}{2}} \left[ R_{W} (k_{2}; B (V, V')) \right] \],

\[ D_{\mu_{3}' \mu_{3}}^{\frac{1}{2}} \left[ R_{W}^{-1} (B (V, V') k_{3}; B (V')) R_{W} (k_{3}; B (V)) \right] \]

\[ = D_{\mu_{3}' \mu_{3}}^{\frac{1}{2}} \left[ R_{W} (k_{3}; B (V, V')) \right]. \]  \hspace{1cm} (5.42)

The transition amplitude \( F_{\mu_{1} \rightarrow \mu}^{\mu_{1}} \) utilising a pseudovector coupling is finally given by the six-dimensional integral

\[ F_{\mu_{1} \rightarrow \mu}^{\mu_{1}} = \sum_{\sigma_{1} \sigma_{1}'} \sum_{\mu_{1} \mu_{1}'} \int d^{3}k_{2} d^{3}k_{3} \sqrt{\frac{\omega_{2} \omega_{3} \sqrt{(\omega_{1} + \omega_{2} + \omega_{3})^{3} (\omega_{1}' + \omega_{2}' + \omega_{3}')^{3}}}{2 \omega_{1} 2 \omega_{3}} \frac{1}{\omega_{2} \omega_{3}}} \]

\[ \times \Psi_{M'} \Sigma_{M_{2}} T^{M_{2}'} \left( k_{1}'; \mu_{1}' \right) D_{\sigma_{1} \mu_{1}}^{\frac{1}{2}} \left[ R_{W} (k_{1}'; B (V')) \right] \]

\[ \times f^{\mu_{1}} N \bar{u} (p_{1}, \sigma_{1}') \gamma_{5} \gamma_{\mu} F \mu_{2} u (p_{1}, \sigma_{1}) q_{\mu} D_{\sigma_{1} \mu_{1}}^{\frac{1}{2}} \left[ R_{W} (k_{1}; B (V)) \right] \]

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\( \times D^{\frac{1}{2}}_{\rho_{\mu} \mu_2} \left[ R_W \left( k_2; B \left( V, V' \right) \right) \right] D^{\frac{1}{2}}_{\rho_{\nu} \mu_3} \left[ R_W \left( k_3; B \left( V, V' \right) \right) \right] \)

\( \times \Psi_{M \Sigma M \Sigma' T M} \left( k_{i}; \mu_i \right) \),

(5.43)

where one has to insert the appropriate variables as prescribed by Eqs. (5.38) - (5.41).

**The Pseudoscalar Coupling**

The calculation of the transition amplitude \( F_{i \rightarrow f}^{ps} \) with a pseudoscalar coupling for the active quark can be performed analogously as in the previous section. One simply has to insert the expression (5.19) into Eq. (5.17) and finally one ends up with

\[
F_{i \rightarrow f}^{ps} = \sum_{\sigma_1' \sigma''} \sum_{\rho_{\mu_2} \rho_{\mu_3}} \int d^3k_2 d^3k_3 \sqrt{\frac{\omega_2 \omega_3}{\omega_2 \omega_3}} \left[ \frac{(\omega_1 + \omega_2 + \omega_3)^3 (\omega''_1 + \omega''_2 + \omega''_3)^3}{2 \omega_1 \omega_2 \omega_3} \right] \times \Psi_{M \Sigma M \Sigma' T M} \left( k_{i}; \mu_i \right)
\]

(5.44)

again using Eqs. (5.39) - (5.41), where

\[
f^{ps} = \frac{-3i g_{qmm}}{2 \pi} \frac{2}{M M'}.
\]

(5.45)

A similar result is obtained, when the integrations over \( k''_2 \) and \( k''_3 \) are left over.

Note that if one inserts \( \bar{q}'' \) (defined in Eq. (5.25)) instead of \( q'' \) (defined in Eq. (5.24)) in the expression (5.43) (representing the transition amplitude for pseudovector coupling), this results in the expression (5.44) representing the transition amplitude for pseudoscalar coupling.

**5.2 Analysis of the Wigner D-Functions**

In the previous section we evaluated the transition amplitudes using a pseudovector and a pseudoscalar coupling of the active quark to the meson, respectively, and we ended up with a six-dimensional integral over the three-momenta of the spectator quarks in the decaying baryon (or alternatively in the residual baryon). The corresponding expressions are given in Eqs. (5.43) and (5.44). For the further evaluation of the integrals it is now important to
simplify the parts of the integrand containing the Wigner D-functions of the 
two spectator quarks together with the spinors of the single quark coupling
to the emitted meson:

\[
\sum_{\sigma'_1 \sigma''_1} D^\pm_{\sigma'_1 \sigma''_1} \left[ R_W \left( k_1^\prime; B \left( V' \right) \right) \right] \bar{u}(p_1^\prime, \sigma''_1) \gamma^\mu u(p_1, \sigma'_1) q_\mu \\
\times D^\pm_{\sigma'_1 \sigma''_1} \left[ R_W \left( k_1; B \left( V \right) \right) \right],
\]  

(5.46)

\[
\sum_{\sigma'_1 \sigma''_1} D^\pm_{\sigma'_1 \sigma''_1} \left[ R_W \left( k_1^\prime; B \left( V' \right) \right) \right] \bar{u}(p_1^\prime, \sigma''_1) \gamma^\mu u(p_1, \sigma'_1) D^\pm_{\sigma'_1 \sigma''_1} \left[ R_W \left( k_1; B \left( V \right) \right) \right],
\]  

(5.47)

corresponding to pseudovector or pseudoscalar coupling. Here we are fo-
cussing on the spin and spatial dependences and neglect the 
avor depen-
dence for the moment.

Concerning the active quark, one uses the intertwining properties of 
matrix elements \( S(\Lambda) \) of the \((\frac{1}{2}, 0) \oplus (0, \frac{1}{2}) \) representation of the Lorentz 
group with respect to the Dirac spinors \( u(p, \sigma) \) with components \( u_\alpha(p, \sigma) \) for \( \alpha = 1, \cdots, 4 \):

\[
S[B(v)] u(k, \mu) = \sum_\beta S_{\alpha \beta}[B(v)] u_\beta(k, \mu) = \sum_\sigma u_\alpha(p, \sigma) D^\pm_{\sigma \mu} [R_W(k, B(v))].
\]  

(5.48)

Consequently, Eq. (5.46) can be rewritten as

\[
\bar{u}(k_1^\prime, \mu_1^\prime) S \left[ B^{-1} \left( V' \right) \right] \gamma^\mu S \left[ B \left( V \right) \right] u(k_1, \mu_1) q_\mu,
\]  

(5.49)

and similarly Eq. (5.47) becomes

\[
\bar{u}(k_1^\prime, \mu_1^\prime) S \left[ B^{-1} \left( V' \right) \right] \gamma^\mu S \left[ B \left( V \right) \right] u(k_1, \mu_1).
\]  

(5.50)

For the specific boost we apply (in case the decaying baryon is at rest) the 
matrices \( S \), which are explicitly given as (see also Ref. [86])

\[
S \left[ B(V') \right] = \exp \left( \frac{1}{2} \Delta \mathbf{n}_\Delta \cdot \mathbf{\alpha} \right) = I_4 \cosh \frac{\Delta}{2} - \alpha_3 \sinh \frac{\Delta}{2},
\]

\[
S \left[ B^{-1}(V') \right] = \exp \left( -\frac{1}{2} \Delta \mathbf{n}_\Delta \cdot \mathbf{\alpha} \right) = I_4 \cosh \frac{\Delta}{2} + \alpha_3 \sinh \frac{\Delta}{2},
\]

\[
S \left[ B(V) \right] = S \left[ B^{-1}(V) \right] = I_4,
\]  

(5.51)

where \( \mathbf{n}_\Delta \) is the unit vector in the direction of the velocity of the final baryon 
and \( \alpha_3 = \gamma_0 \gamma^3 \) is the \((4 \times 4)\) Hermitian matrix defined in Eq. (B.14). Since

\(^5\)More details concerning the connection between Wigner D-functions and the matrices 
\( S \) of boosts can be found, e.g., in Ref. [92].
in our case the boosts are performed only in the z-direction, the expression for the matrix \( S \) is rather simple.

Next we express Eqs. (5.49) and (5.50) using the representation of the four-component quark spinors in terms of \((2 \times 2)\) matrices (the Pauli spin matrices \( \sigma_i \)), i.e. we find the proper terms in the spin space of the active quark corresponding to Eq. (B.22). First, we will deal with the more extensive pseudovector coupling, afterwards we take a closer look at the pseudoscalar coupling.

5.2.1 The Active Quark in Pseudovector Coupling

Inserting the matrices \( S \) given by Eq. (5.51) into the expression (5.49) leads to

\[
\begin{align*}
\bar{u}(k''_1, \mu''_1) & \left( \cosh \frac{\Delta}{2} + \alpha_3 \sinh \frac{\Delta}{2} \right) \gamma_5 \gamma^0 u(k_1, \mu_1) \omega_m \\
- \bar{u}(k''_1, \mu''_1) & \left( \cosh \frac{\Delta}{2} + \alpha_3 \sinh \frac{\Delta}{2} \right) \gamma_5 \gamma^3 u(k_1, \mu_1) Q
\end{align*}
\]

with \( \gamma^\mu q_\mu = \gamma^0 \omega_m - \gamma^3 Q \). According to Eq. (5.51) the Dirac spinors \( \bar{u}(k''_1, \mu''_1) = u^\dagger(k''_1, \mu''_1) \gamma^0 \) and \( u(k_1, \mu_1) \) involve the two-component Pauli spinors (B.24). It is therefore possible to evaluate expression (5.52) further, and one obtains

\[
\begin{align*}
\{ f^0_1 \sigma_x(1) + g^0_1 (k_{1x} \sigma_x(1) + k_{1y} \sigma_y(1)) \} & \mu''_1 \mu_1 \omega_m \\
- \{ f^3_1 \sigma_z(1) + g^3_1 (k_{1x} \sigma_x(1) + k_{1y} \sigma_y(1)) \} & \mu''_1 \mu_1 Q,
\end{align*}
\]

where we used the fact that \( k''_{1x} = k_{1x}, \ k''_{1y} = k_{1y}, \) and \( \sigma_i(1) \) for \( (i = x, y, z) \) are the \((2 \times 2)\) Pauli spin matrices (B.8). Here we introduced the abbreviations

\[
\begin{align*}
 f^0_1 & = \frac{1}{\sqrt{(\omega''_1 + m'_1)(\omega_1 + m_1)}} \left[ - \left[ (\omega''_1 + m'_1) k_{1z} + (\omega_1 + m_1) k''_{1z} \right] \cosh \frac{\Delta}{2} \\
 & + \left[ (\omega''_1 + m'_1) (\omega_1 + m_1) - k_1 \cdot k''_1 + 2 k_{1z} k''_{1z} \right] \sinh \frac{\Delta}{2} \right], \\
 g^0_1 & = \frac{1}{\sqrt{(\omega''_1 + m'_1)(\omega_1 + m_1)}} \left[ - \left( \omega''_1 + m'_1 + \omega_1 + m_1 \right) \cosh \frac{\Delta}{2} \\
 & + \left( k''_{1z} + k_{1z} \right) \sinh \frac{\Delta}{2} \right], \\
 f^3_1 & = \frac{1}{\sqrt{(\omega''_1 + m'_1)(\omega_1 + m_1)}} \left[ - \left( \omega''_1 + m'_1 \right) (\omega_1 + m_1) + k_1 \cdot k''_1 \\
 & - 2 k_{1z} k''_{1z} \cosh \frac{\Delta}{2} + \left[ (\omega''_1 + m'_1) k_{1z} + (\omega_1 + m_1) k''_{1z} \right] \sinh \frac{\Delta}{2} \right],
\end{align*}
\]
\[ g_1^3 = \frac{1}{\sqrt{(\omega''_1 + m'_1)(\omega + m_1)}} \left[ -(k''_{1z} + k_{1z}) \cosh \frac{\Delta}{2} ight. \\
+ \left. (\omega''_1 + m'_1 + \omega + m_1) \sinh \frac{\Delta}{2} \right], \quad (5.54) \]

where \( m_1 \) and \( m'_1 \) represent the masses of the active quark in the decaying and final baryons. The subscripts \( \mu''_1 \mu_1 \) for the terms in curly brackets are used as a shorthand notation for the spin matrices \( \chi^{\dagger}_{\mu_1} \) and \( \chi_{\mu_1} \):

\[ \{\sigma_i(1)\}_{\mu''_1 \mu_1} = \chi^{\dagger}_{\mu_1} \sigma_i(1) \chi_{\mu_1}. \quad (5.55) \]

### 5.2.2 The Active Quark in Pseudoscalar Coupling

In case of pseudoscalar coupling we insert the matrices \( S \) given by Eq. (5.51) into Eq. (5.50) obtaining

\[ \bar{u}(k''_1, \mu'_1) \left( \cosh \frac{\Delta}{2} + \alpha_3 \sinh \frac{\Delta}{2} \right) \gamma_5 u(k_1, \mu_1). \quad (5.56) \]

In the same way as in the previous subsection we finally obtain

\[ \{f_1 \sigma_z(1) + g_1 (k_{1z} \sigma_x(1) + k_{1y} \sigma_y(1))\}_{\mu''_1 \mu_1}, \quad (5.57) \]

where

\[ f_1 = \frac{1}{\sqrt{(\omega''_1 + m'_1)(\omega + m_1)}} \left[ (\omega''_1 + m'_1) k_{1z} - (\omega + m_1) k''_{1z} \right] \cosh \frac{\Delta}{2} \\
+ \left[ (\omega''_1 + m'_1) (\omega + m_1) + k_1 \cdot k''_1 - 2k_{1z} k''_{1z} \right] \sinh \frac{\Delta}{2}, \]

\[ g_1 = \frac{1}{\sqrt{(\omega''_1 + m'_1)(\omega + m_1)}} \left[ (\omega''_1 + m'_1 - \omega - m_1) \cosh \frac{\Delta}{2} \\
- (k''_{1z} + k_{1z}) \sinh \frac{\Delta}{2} \right]. \quad (5.58) \]

### 5.2.3 The Spectator Quarks

As the spectator quarks corresponding to the indices 2 and 3 are not involved in the coupling to the emitted meson, there is no distinction between pseudovector and pseudoscalar coupling. In the spin space of quarks 2 and 3 the Wigner \( D \)-functions can be expressed through \((2 \times 2)\) matrices in the following way

\[ D^{\pm}_{\mu''_1 \mu_1} \left[ R_W(k; B (V, V')) \right] = \{f_i \mathbf{1}_2 + i g_i (k_{1z} \sigma_y(i) - k_{1y} \sigma_x(i))\}_{\mu''_1 \mu_1}, \quad (5.59) \]
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where

\[
\begin{align*}
  f_i &= \frac{(\omega_i + m_i) \cosh \frac{k_i z}{2} + k_i z \sinh \frac{k_i z}{2}}{\sqrt{(\omega_i + m_i)(\omega_i + m_i)}}, \\
  g_i &= \frac{\sinh \frac{k_i z}{2}}{\sqrt{(\omega_i + m_i)(\omega_i + m_i)}}.
\end{align*}
\]

for \( i = 2, 3 \). Here, one wrote out the Wigner rotation (as in Eq. (5.12)) through

\[
R_W (k_i; B (V, V')) = B^{-1} (B (V, V') k_i) B (V, V') B (k_i),
\]

and then reduced the exponential expressions for the three consecutive boosts to the result in Eq. (5.59). More details concerning the calculation of Wigner \( D \)-functions are given, e.g., in Ref. [89].

5.3 Analysis of the Transition Amplitude

Now we can put all ingredients from the previous subsections together and obtain for the transition amplitude the expressions

\[
F_{i-f}^{p_n} = \sum_{\nu_1, \nu_2, \nu_3} \int d^3 k_2 d^3 k_3 \sqrt{\frac{\omega_2 \omega_3}{\omega_2 \omega_3}} \sqrt{\frac{(\omega_1 + \omega_2 + \omega_3)^3 (\omega_1' + \omega_2' + \omega_3')^3}{2\omega_1' 2\omega_3'}} \times \Psi^*_{M' \Sigma'M_{\Sigma'} T'M_{\Sigma'}} (k''_i; \mu_i) F^m f^{p_n} N \\
\times \{ [f_1^i \sigma_z (1) + g_1^i \xi_1] \omega_m - \left[ f_1^i \sigma_z (1) + g_1^i \xi_1 \right] Q \}_{\mu''_i \mu_i} \\
\times \{ f_2 \mu_2 + ig_2 \xi_2 \}_{\mu''_2 \mu_2} \{ f_3 \mu_3 + ig_3 \xi_3 \}_{\mu''_3 \mu_3} \\
\times \Psi_{M \Sigma M_{\Sigma} T M_{T}} (k_i; \mu_i)
\]

and

\[
F_{i-f}^{p_s} = \sum_{\nu_1, \nu_2, \nu_3} \int d^3 k_2 d^3 k_3 \sqrt{\frac{\omega_2 \omega_3}{\omega_2 \omega_3}} \sqrt{\frac{(\omega_1 + \omega_2 + \omega_3)^3 (\omega_1' + \omega_2' + \omega_3')^3}{2\omega_1' 2\omega_3'}} \times \Psi^*_{M' \Sigma'M_{\Sigma'} T'M_{\Sigma'}} (k''_i; \mu_i) F^m f^{p_s} N \{ f_1^i \sigma_z (1) + g_1^i \xi_1 \}_{\mu''_i \mu_i} \\
\times \{ f_2 \mu_2 + ig_2 \xi_2 \}_{\mu''_2 \mu_2} \{ f_3 \mu_3 + ig_3 \xi_3 \}_{\mu''_3 \mu_3} \\
\times \Psi_{M \Sigma M_{\Sigma} T M_{T}} (k_i; \mu_i)
\]
for pseudovector and pseudoscalar coupling, respectively. Here we introduced the further abbreviations

\[ \xi_1 = k_1 x (1) + k_1 y (1), \]
\[ \xi_2 = k_2 x (2) - k_2 y (2), \]
\[ \xi_3 = k_3 x (3) - k_3 y (3). \]  

The last step is to insert the explicit expressions of the initial and final baryon wave functions \( \Psi_{M\Sigma M_S T M_T} (k_i; \mu_i) \) and \( \Psi^*_{M'\Sigma' M'_{S'} T' M'_T} (k'_i; \mu'_i) \) (see Chapter 2 and Appendix A). The integration over the three-momenta \( (k_2, k_3) \) can be replaced by an integration over the Jacobi momenta \( (p, k) \) (see Eqs. (A.59) and (A.60)). The integration measures are equal:

\[ d^3 p d^3 k = d^3 k_2 d^3 k_3. \]

In this work, the final baryons have an intrinsic spin \( \Sigma' = \frac{1}{2} \), where the total orbital angular momentum \( L' = 0 \) and the total spin \( S' = \frac{3}{2} \) satisfying the triangular rule \( |L' - S'| \leq \Sigma' \leq L' + S' \). For the decaying baryons we consider the cases \( 0 \leq L = M_L = \frac{1}{2} \) and/or \( 1 \leq L = 1 \). The internal spin of the decaying baryon can be \( S = \frac{1}{2} \) and/or \( S = \frac{3}{2} \). Consequently, concerning the spatial and spin parts of the basis functions building up the total baryon wave function, we calculate matrix elements of the form

\[ \langle (L'S') \Sigma' M_{\Sigma'} | D_{X|S} | (LS) \Sigma M_{\Sigma} \rangle = \sum_{M_L, M_S} C^{\Sigma M_{\Sigma}}_{LM_L S M_S} \]
\[ \times C^{\Sigma' M_{\Sigma'}}_{LM'_L S' M'_{S'}} \langle (L'M_L | D_X | LM_L ) \langle S'M_S | DS | SM_S \rangle \]  

where \( L (L') \) and \( S (S') \) couple to \( \Sigma (\Sigma') \) with z-projection \( M_{\Sigma} (M_{\Sigma'}) \) (see also Eq. (2.42)). The summation concerns the z-projections \( M_L (M_{L'}) \) and \( M_S (M_{S'}) \), where some Clebsch-Gordon coefficients vanish due to the constraints \( M_L + M_S = M_{\Sigma} \) and \( M_{L'} + M_{S'} = M_{\Sigma'} \). Here we separated the spin part belonging to a spin operator \( D_S \) from the spatial part corresponding to an operator \( D_X \). In Section 4.1 we found that due to symmetry reasons it is sufficient to study only the matrix elements with \( M_{\Sigma} = M_{\Sigma'} = \frac{1}{2} \). Thus, assuming that the final baryon has \( \Sigma' = \frac{1}{2} \) and \( L' = 0 \), \( S' = \frac{1}{2} \) Eq. (5.65) reduces to

\[ \langle (\frac{1}{2}) | D_{X|S} | (LS) \Sigma \frac{1}{2} \rangle \]
\[ = \sum_{M_L, M_S} C_{LM_L S M_S}^{\Sigma \frac{1}{2}} \langle (00) | D_X | LM_L \rangle \langle \frac{1}{2} | D_S | SM_S \rangle \]

\(^{6}\)Of course, it is possible to use also resonance wave functions with even higher total orbital angular momenta, but the associated numerical effort is tremendous.
where the evaluation of the spin matrix elements is extensively discussed in the Appendices A and C.1. For the integration of the spatial part it is necessary to return to the Eqs. (5.62) and (5.63). The total baryon wave functions are composed of sums over a certain number of basis functions. The corresponding expressions for the spatial part of the basis functions in the momentum-space representation are provided in Appendix A.5.2 (see Eqs. (A.45), (A.50) - (A.55)). Now one has to insert these terms into the six-dimensional integral. The resulting integrands are listed in Appendix C.3, where we present the methods for solving the corresponding integrals.

The flavor part of the transition amplitude can be treated independently of the spatial and the spin parts. Both the decaying and the final baryons are each specified by one of the 15 flavor basis functions given in Table A.1. One simply has to evaluate the flavor matrix elements of the flavor operator \( T'|T\rangle |F^m\rangle \), where the flavor basis functions can be given in either one of the configurations 1, 2, or 3. A detailed study of the flavor part is provided in Appendix C.2.

Finally, for the actual calculation of the transition amplitude we have to sum over all basis functions and configurations of the initial and final baryon states (see Eq. (C.1)), and for each resulting term one has to proceed in the way as described above. The computation of the spatial part including a multi-dimensional integration costs a sizeable amount of computer-time. Fortunately, some of the flavor matrix elements vanish from the very beginning and thus it is not necessary to perform the corresponding integrations concerning the spatial part.

### 5.4 The Total Decay Width

The total decay width \( \Gamma_{i\to f} \) given by Eq. (5.1) describes the \( \pi, \eta, \) and \( K \) decays of light and strange baryon resonances. In Chapter 4 we already detailed how to average and sum over the particles involved in the decay process, i.e. Eq. (5.1) can also be written as

\[
\Gamma_{i\to f} = \rho \frac{2}{2\Sigma + 1} \frac{2T' + 1}{2T + 1} \frac{|F_{\Sigma'\Sigma M_T T_{M_T}} - f_{\Sigma'\Sigma M_T T_{M_T}} (F^m)|^2}{|C_{T M_T M_{Tn} M_{Tn}}|^2},
\]

where we choose one representative transition amplitude specified by the initial state \( |M, \Sigma, M_T, T, M_T\rangle \) and the final state \( |M', \Sigma', M_{\Sigma'}, T', M_{T'}\rangle \). Here \( F^m \) is the flavor operator denoting the respective pseudoscalar meson and \( \rho \) denotes the phase-space factor. Following the procedure above we have to insert for the intrinsic spin of the final baryon (ground state) \( \Sigma' = \frac{1}{2} \) with \( z\)-projection \( M_{\Sigma'} = \frac{1}{2} \). Finally we have to insert either Eq. (5.62) or
Eq. (5.63) representing the transition amplitude with a pseudovector and a pseudoscalar coupling, respectively. The numerical results are discussed in Chapter 7 and are listed in Appendix F.

5.5 Different Frames

We stated above, that it is advantageous to choose a concrete reference frame for the calculation of the decay width. For simplicity, we decided to take the rest frame of the decaying baryon, where the final particles move along the z-axis. As we work within a Poincaré-invariant theory in principle all frames are equivalent. However some choices may actually be more suitable for the numerical implementation than others. For the invariant phase-space factor defined in Eqs. (4.36) and (4.37) choosing the rest frame of the decaying baryon simplifies the calculation. In any other frame it would be more complicated.

Concerning the transition amplitude, we work within the point form of RQM, where the boosts are purely kinematical and thus are not affected by the dynamics of the system. Also the PFSM operator is manifestly covariant. Therefore the whole calculation is frame-independent. In the following, we demonstrate the effect when the whole decay process is transferred to a different reference frame, i.e. we apply a boost on the decaying resonance of the form

\[
B (V_\alpha) = \begin{pmatrix}
\cosh \alpha & 0 & 0 & \sinh \alpha \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
\sinh \alpha & 0 & 0 & \cosh \alpha
\end{pmatrix},
\]

(5.68)

which represents a boost in the positive z-direction. Here \( \alpha \) is the rapidity belonging to the velocity \( V_\alpha = (\cosh \alpha, 0, 0, \sinh \alpha) \). This boost has then to be applied on all velocity-dependent quantities originally defined in the rest frame of the decaying baryon\(^7\):

\[
P^\mu \rightarrow P^\mu_\alpha = B (V_\alpha) \begin{pmatrix}
M \\
0 \\
0 \\
0
\end{pmatrix} = \begin{pmatrix}
M \cosh \alpha \\
0 \\
0 \\
M \sinh \alpha
\end{pmatrix},
\]

\[
P'^\mu \rightarrow P'^\mu_\alpha = B (V_\alpha) B (V') \begin{pmatrix}
M' \\
0 \\
0 \\
0
\end{pmatrix} = \begin{pmatrix}
M' \cosh (\alpha - \Delta) \\
0 \\
0 \\
M' \sinh (\alpha - \Delta)
\end{pmatrix},
\]

\( ^7 \)Note that in the rest frame of the decaying baryon \( V = (1, 0, 0, 0) \) and \( B (V) = 1 \).
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\[ q^\mu \rightarrow q'^\mu_{\alpha} = B (V_\alpha) \left( \begin{array}{c} \omega_m \\ 0 \\ 0 \\ Q \end{array} \right) = \left( \begin{array}{c} \omega_m \cosh \alpha + Q \sinh \alpha \\ 0 \\ 0 \\ \omega_m \sinh \alpha + Q \cosh \alpha \end{array} \right) \]. \quad (5.69)

The connection of the \( p_i \) (\( p'_i \)) to the \( k_i \) (\( k'_i \)) is then given by

\[ p'_i = k'_i \rightarrow p'^\mu_{i,\alpha} = B (V_\alpha)^\mu_{\nu} k^\nu_i \]

\[ p'^\mu = B (V')^\mu_{\nu} k'^\nu_{i,\alpha} \rightarrow p'^{\mu\nu}_{i,\alpha} = (B (V_\alpha) B (V'))^\mu_{\nu} k'^\nu_{i,\alpha} \]

for \((i = 1, 2, 3)\). As in the point form the decay operator keeps its spectator-model character in all reference frames, the connection between the primed and unprimed variables is given by

\[ B (V_\alpha)^\mu_{\nu} k^\nu_i = (B (V_\alpha) B (V'))^\mu_{\nu} k'^\nu_i \] \quad (5.71)

for \((i = 2, 3)\), which simply leads to the already known relations (5.39), namely

\[ k'^{\mu\nu}_2 = B (V, V')^\mu_{\nu} k^\nu_2 = B^{-1} (V')^\mu_{\nu} k'^\nu_2 \]
\[ k'^{\mu\nu}_3 = B (V, V')^\mu_{\nu} k^\nu_3 = B^{-1} (V')^\mu_{\nu} k'^\nu_3 \]. \quad (5.72)

In Section 5.2 we had to interpret the Wigner \( D \)-functions concerning the active quark as well as the two spectator quarks. It can be seen immediately, that due to the invariance of the four-delta functions under Lorentz transformations the latter ones are not affected by an additional boost. However, for the active quark the situation is at first sight not so clear: In case of a pseudovector coupling one has the expression

\[ \sum_{\sigma'_1 \sigma''_1} D_{\sigma''_1 \mu_1}^{\frac{1}{2}} \left[ R_W (k'_1; B (V_\alpha) V') \right] \bar{u}(p'^{\mu}_{1,\alpha}, \sigma''_1) \gamma_5 \gamma^\mu u(p_{1,\alpha}, \sigma'_1) q_{\mu,\alpha} \]

\[ \times D_{\sigma'_1 \mu_1}^{\frac{1}{2}} [R_W (k_1; B (V_\alpha))] , \] \quad (5.73)

and using the intertwining properties this leads after some rewriting to

\[ \bar{u}(k'_1, \mu'_1) \gamma_5 \gamma^\mu \left[ \cosh \left( \alpha - \frac{\Delta}{2} \right) + \alpha_3 \sinh \left( \alpha - \frac{\Delta}{2} \right) \right] u(k_1, \mu_1) q_{\mu,\alpha} \] \quad (5.74)

Now we have to insert the \( q_{\mu,\alpha} \) from above, which leads to

\[ \bar{u}(k'_1, \mu'_1) \gamma_5 \left[ \gamma^0 [\omega_m \cosh \alpha + Q \sinh \alpha] - \gamma^3 [\omega_m \sinh \alpha + Q \cosh \alpha] \right] \]

\[ \times \left[ \cosh \left( \alpha - \frac{\Delta}{2} \right) + \alpha_3 \sinh \left( \alpha - \frac{\Delta}{2} \right) \right] u(k_1, \mu_1) q_{\mu,\alpha} \]. \quad (5.75)
Finally one has to combine the terms including the hyperbolic sines and cosines, and it is immediately evident, that the term $\alpha$ concerning the additional rapidity vanishes. Therefore one ends up with an expression, which is identical to Eq. (5.52) found for in the rest frame of the decaying baryon. The same procedure is also true for a pseudoscalar coupling. One can therefore see, that an overall additional boost in z-direction does not change the original transition amplitude. This result is not surprising, as the transition amplitude is considered especially in the point form of RQM.
Chapter 6

Non-Relativistic Limit of the Point-Form Spectator Model

In the previous chapters we discussed the decays of light and strange baryon resonances in the framework of Poincaré-invariant quantum mechanics, especially in its point-form version; in particular, we applied a spectator model for the decay operator. Generally, a fully relativistic calculation is desirable. Hadron reactions can hardly be treated in a non-relativistic approach along with CQMs. Nevertheless, there exist a number of non-relativistic studies in the literature, with some of them including also relativistic corrections. In order to compare with such investigations of strong decays of baryon resonances and above all for demonstrating the sizes of relativistic effects we undertake a non-relativistic limit of our covariant calculations. The non-relativistic reduction of the PFSM will lead us to the so-called elementary-emission model (EEM), which has been frequently used in non-relativistic approaches.

We start out from the PFSM decay operator utilising a pseudovector coupling, with which we calculated the relativistic transition amplitude $F_{i \rightarrow f}$ in Eq. (5.1). In order to demonstrate the principle differences between a non-relativistic theory and a theory adhering to special relativity, we will first give a brief introduction to Galilean relativity, where we closely follow the formalism as outlined in Ref. [6]. Then we continue with the calculation of the non-relativistic limit of the transition amplitude.

6.1 Galilean-Invariant Quantum Mechanics

The principle of relativity states that the laws of physics do not distinguish between different inertial systems. The difference between special relativity and Galilean relativity lies in the respective relations between two inertial systems. Here, Galilean transformations describe the relation between inertial systems and are defined such that they leave Newtons sec-
OND LAW for a free particle unchanged \((m\ddot{x}(t) = 0)\). Classically, the most general coordinate transformations of such type include time translations \((\langle x, t \rangle \rightarrow (x, t + t_0))\), spatial translations \((\langle x, t \rangle \rightarrow (x + x_0, t))\), spatial rotations \((\langle x, t \rangle \rightarrow (Rx, t) \text{ with } R \in SO(3))\), as well as Galilean boosts \((\langle x, t \rangle \rightarrow (x + vt, t))\). These transformations lead to a representation of the Galilean group. For the construction of a Galilean-invariant quantum theory it is necessary to find a unitary ray representation of the group of transformations that relate different inertial systems on the Hilbert space \(H\). This means, that we seek unitary operators, which describe the action of Galilean transformations on the states of the Hilbert space. Here, the true representation of the Galilean group has no reasonable physical interpretation, because the transformation properties of the Hamiltonian \(H\) (the total energy operator) and \(P\) (the total three-momentum operator) resulting from the group structure do not comply with the transformations expected classically (details can be found, e.g., in Ref.\ [6]). However, Bargmann [76] showed that the correct relations between energy and momentum are recovered if one extends the Galilean group by a phase factor. Furthermore, for the physical representation on the Hilbert space it is necessary to replace the \(SO(3)\) by the universal covering group \(SU(2)\). Thus, the unitary ray representation of the Galilean group can be replaced by a single-valued unitary representation of the central extension of the Galilean group. Here, the central extension means that we add a phase and replace \(SO(3)\) by \(SU(2)\).

The central extension of the Galilean group is a Lie group with eleven parameters, thus it possesses eleven infinitesimal Hermitian generators:

\[
\begin{align*}
H & \rightarrow \text{generator of time translations} \\
P & \rightarrow \text{generators of spatial translations} \\
J & \rightarrow \text{generators of rotations} \\
K & \rightarrow \text{generators of Galilean boosts} \\
M & \rightarrow \text{generator of phase transformations} \\
\end{align*}
\]

in the central extension \((6.1)\)

The group transformation properties lead to the following commutation relations for the generators:

\[
\begin{align*}
[J^i, J^j] &= i\varepsilon^{ijk} J^k, & [J^i, K^j] &= i\varepsilon^{ijk} K^k, & [J^i, P^j] &= i\varepsilon^{ijk} P^k, & (6.2) \\
[K^j, H] &= -iP^j, & [K^j, P^k] &= -i\delta_{jk} M. & (6.3)
\end{align*}
\]

Obviously, the Galilean algebra differs from the Poincaré algebra in two points:

- The mass operator acts as a generator.
- The Hamiltonian \(H\) does not appear on the right-hand side of the non-vanishing commutators.
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As a consequence of the second point, the introduction of interaction into the system gets rather simple as compared to a Poincaré-invariant theory: It is possible to include the interaction in the Hamiltonian preserving the commutation relations of the Galilean group without any modification in the other generators. The only condition to the interaction is that it has to commute with the remaining generators.

Finally, we want to make one further remark: In a relativistic theory the action of a particle is invariant under the reparametrization of its world-line. This reparametrization invariance is generated by the constraint $p_\mu p^\mu = m^2$, which is known as mass- or on-shell condition, and it can be viewed as a gauge or redundancy symmetry. Here, a single world-line can be described by an infinite number of different parametrizations. The choice of a time-parameter $\tau = f(x^0, x^1, x^2, x^3)$ corresponds to a gauge-fixing, which amounts to a splitting of space-time into space and time. This leads to a decomposition of the Minkowski space into three-dimensional hypersurfaces of equal time, $\tau = \text{const}$, with time- or light-like normals to the hypersurface, where in this work we use the point form of RQM. In a Galilean theory the time $t$ is default, and such an on-shell condition does not exist.

6.2 The Non-Relativistic Transition Amplitude

In Chapter 3 we found that a baryon can be described by two equivalent sets of states, namely $|E, P, \Sigma, M_\Sigma, T, M_T\rangle$ and $|M, V, \Sigma, M_\Sigma, T, M_T\rangle$. For the calculation of the transition amplitude in Chapter 5 it was specifically convenient to express the four-vector $P^\mu$ in terms of the mass and the velocity (see Eq. (5.2)). However, here we describe the decaying and final baryon states directly by eigenstates characterized by their four-momenta $P^\mu$ and $P'^\mu$, respectively. Thus we start with the transition amplitude\footnote{At this stage, the transition amplitude is still fully relativistic. However, in order to distinguish between the transition amplitude obtained in Chapter 5 and the one to be derived here, we denote it ab initio with the superscript "NR", indicating that we will perform a non-relativistic reduction.}

$$F^\text{NR}_{i \rightarrow f} = \langle P', \Sigma', M_{\Sigma'}, T', M_T' | \hat{D}^\text{nr}_{rd} | P, \Sigma, M_{\Sigma}, T, M_T \rangle.$$ \hspace{1cm} (6.4)

A general four-momentum state can be written as

$$|p\rangle = \sqrt{2p_0}|p\rangle,$$ \hspace{1cm} (6.5)

where $p_0$ is the energy and $p$ the corresponding three-momentum. (For simplicity we left out additional quantum numbers like, e.g, the spin, characterizing a specific state.) In this notation baryon states are normalized in the following way

$$\langle P', \Sigma', M_{\Sigma'}, T', M_T' | P, \Sigma, M_{\Sigma}, T, M_T \rangle$$
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\[ 2P_0 \delta^3(P - P') \delta_{\Sigma \Sigma'} \delta_{M \Sigma M} \delta_{T T'} \delta_{M_T M_{T'}}. \] (6.6)

and Eq. (6.4) becomes

\[ F_{i-j}^{NR} = \sqrt{2E'} \sqrt{2E} \hat{D}_{rd}^{m,NR} |\Phi, \Sigma', M_{\Sigma'}, T', M_{T'} \rangle, \] (6.7)

where \( E = P_0 \) is the energy of the decaying and \( E' = P_0' \) the energy of the final baryon, respectively. Similarly, for the composition of three free quarks with four-momenta \( p_i \) and spin-projections \( \sigma_i = \pm \frac{1}{2} \) the free momentum state can be written as

\[ |p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 \rangle = \sqrt{2p_{10}} \sqrt{2p_{20}} \sqrt{2p_{30}} |p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 \rangle \] (6.8)

where \( p_{i0} \) are the single-quark energies. The corresponding completeness relation is given by the expression

\[ 1 = \sum_{\sigma_1, \sigma_2, \sigma_3} \int d^3p_1 d^3p_2 d^3p_3 |p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 \rangle \langle p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 |. \] (6.9)

In Eq. (6.7) the states of the decaying and final baryons are characterized by the total three-momenta \( P \) and \( P' \), respectively, which in general are not equal to zero. However, the baryon wave functions \( \Psi_{M \Sigma M_T T} (k_i, \mu_i) \) are calculated in their respective rest frames with total three-momentum zero, where the individual internal quark momenta are constrained by \( k_1 + k_2 + k_3 = 0 \). For the following calculations we introduce the completeness relation of the states \( |k_2, k_3, P; \mu_1, \mu_2, \mu_3 \rangle \) describing a free three-quark system with total momentum \( P \), i.e. a free three-quark system transformed from the rest frame with a Galilean boost corresponding to \( P \)

\[ 1 = \sum_{\mu_1, \mu_2, \mu_3} \int d^3k_2 d^3k_3 d^3P |k_2, k_3, P; \mu_1, \mu_2, \mu_3 \rangle \langle k_2, k_3, P; \mu_1, \mu_2, \mu_3 |. \] (6.10)

Here, \( P \) is the total three-momentum of the baryon, and \( k_i \) for \( i = 2, 3 \) are the internal quark momenta of quarks 2 and 3, where quark 1 is defined by the restriction given above. The \( \mu_i \) for \( i = 1, 2, 3 \) are the corresponding spin-projections of the quarks. In a non-relativistic calculation the total three-momenta of the baryons are given by the sums over the individual quark momenta, i.e.

\[ P = \sum_{i=1}^{3} p_i, \quad P' = \sum_{i=1}^{3} p'_i. \] (6.11)

The connection between the three-momenta \( p_i \) of the quarks in an arbitrary frame and the internal three-momenta \( k_i \) in the rest system of the baryon...
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is given by

\[ p_1 = k_1 + m_1 v = \frac{p_1}{M} \approx -k_2 - k_3 + \frac{m_1 p}{m_1 + m_2 + m_3}, \]

\[ p_2 = k_2 + m_2 v = \frac{p_2}{M} \approx k_2 + \frac{m_2 p}{m_1 + m_2 + m_3}, \]

\[ p_3 = k_3 + m_3 v = \frac{p_3}{M} \approx k_3 + \frac{m_3 p}{m_1 + m_2 + m_3}. \] (6.12)

Here we took the non-relativistic limit, where the total mass is approximated according to \( M \approx \sum m_i \) (see also, e.g., Ref. \[93\]). It can be shown easily, that for the integration measure the following relation is true:

\[ d^3 p_1 d^3 p_2 d^3 p_3 = d^3 k_2 d^3 k_3 d^3 p. \] (6.13)

Inserting the completeness relations (6.9) as well as (6.10) into Eq. (6.7) leads to\(^2\)

\[ F_{1-1}^{NR} = \sqrt{2E} \sqrt{2E'} \sum_{\mu_1^0 \mu_2^0 \sigma_1^0} \int d^3 k_2 d^3 k_3 d^3 p d^3 \tilde{P} \nonumber \]

\[ \times \frac{d^3 k_2^0 d^3 p_2^0 d^3 p_3^0 d^3 \tilde{p}_2^0 d^3 \tilde{p}_3^0}{2p_{10}^0 2p_{20}^0 2p_{30}^0 2\tilde{p}_{20}^0 2\tilde{p}_{30}^0} \langle \tilde{P}' , \Sigma' , M_{\Sigma'} , T' , M_{T'} | k_2^0 , k_3^0 | P^{iv} ; \mu_1^0 , \mu_2^0 , \mu_3^0 \rangle \]

\[ \times \langle p_1^0 p_2^0 p_3^0 , \sigma_1^0 \sigma_2^0 \sigma_3^0 | \hat{D}_r^{m_1} | p_1^0 p_2^0 p_3^0 , \sigma_1^0 \sigma_2^0 \sigma_3^0 \rangle \]

\[ \times \langle k_2^0 k_3^0 \tilde{P}^{iv} ; \mu_1^0 \mu_2^0 \mu_3^0 | P , \Sigma , M_{\Sigma} , T , M_{T} \rangle. \] (6.14)

Herein the different expressions represent the momentum-space representation of the final baryon state (the final-baryon wave function)

\[ \langle \tilde{P}' , \Sigma' , M_{\Sigma'} , T' , M_{T'} | k_2^0 , k_3^0 | P^{iv} ; \mu_1^0 , \mu_2^0 , \mu_3^0 \rangle \]

\[ \approx \Psi_{M_{\Sigma} T_{M_{T}}} (k_2^0 ; \mu_1^0) \delta^3 (P^{iv} - \tilde{P}'), \] (6.15)

the momentum-space representation of the initial baryon state (the initial-baryon wave function)

\[ \langle k_2^0 k_3^0 P^{iv} ; \mu_1^0 \mu_2^0 \mu_3^0 | P , \Sigma , M_{\Sigma} , T , M_{T} \rangle \]

\[ \approx \Psi_{M_{\Sigma} T_{M_{T}}} (k_2^0 ; \mu_1^0) \delta^3 (P^{iv} - P), \] (6.16)

\(^2\)Note that the superscripts "\(^iv\)" and "\(^v\)" stand for roman numbers; they must not be confused with velocities.
and the overlaps of free three-quark momentum states

\begin{align}
\langle \mathbf{k}_3 \mathbf{k}_3^\dagger \mathbf{P}^\dagger, \mu_1 \mu_2 \mu_3 | \mathbf{p}_1 \mathbf{p}_2 \mathbf{p}_3, \sigma_1 \sigma_2 \sigma_3 \rangle &= \delta_{\mu_1 \mu_1} \delta_{\mu_2 \mu_2} \delta_{\mu_3 \mu_3} \\
&\times \delta^3 \left( \mathbf{P}^\dagger - \sum \mathbf{p}_i^\dagger \right) \delta^3 \left( \mathbf{p}_2^\dagger - \mathbf{p}_2 \right) \delta^3 \left( \mathbf{p}_3^\dagger - \mathbf{p}_3 \right),
\end{align}

(6.17)

\begin{align}
\langle \mathbf{p}_1^m \mathbf{p}_2^m \mathbf{p}_3^m, \sigma_1^m \sigma_2^m \sigma_3^m | \mathbf{k}_3 \mathbf{k}_3^\dagger \mathbf{P}^\dagger, \mu_1 \mu_2 \mu_3 \rangle &= \delta_{\mu_1 \mu_1} \delta_{\mu_2 \mu_2} \delta_{\mu_3 \mu_3} \\
&\times \delta^3 \left( \mathbf{P}^\dagger - \sum \mathbf{p}_i^m \right) \delta^3 \left( \mathbf{p}_2^m - \mathbf{p}_2 \right) \delta^3 \left( \mathbf{p}_3^m - \mathbf{p}_3 \right).
\end{align}

(6.18)

The $\Psi_{\Sigma^*;M_{\Sigma^*},T_{\Sigma^*};M_{\pi^*}}(\mathbf{k}_3^\dagger; \mu_3)$ in Eq. (6.15) and the $\Psi_{M;M_{\Sigma^*},T_{\Sigma^*};M_{\pi^*}}(\mathbf{k}_3^\dagger; \mu_3)$ in Eq. (6.16) are the rest-frame wave functions. Using Eq. (6.12) we can replace the three-momenta $\mathbf{p}_i^\dagger$ and $\mathbf{p}_i$ by

\begin{align}
\mathbf{p}_i^\dagger &= \mathbf{k}_i^\dagger + \frac{m_i}{m_1^\dagger + m_2 + m_3} \mathbf{P}^\dagger, \\
\mathbf{p}_i &= \mathbf{k}_i + \frac{m_i}{m_1 + m_2 + m_3} \mathbf{P},
\end{align}

(6.19)

for $(i = 2, 3)$. For the matrix element of the decay operator between (free) momentum states we insert the expression for the PFSM operator (5.18) (utilising a pseudovector coupling of the meson to the active quark), namely

\begin{align}
\langle \mathbf{p}_1^m, \mathbf{p}_2^m, \mathbf{p}_3^m; \sigma_1^m, \sigma_2^m, \sigma_3^m | \mathbf{\tilde{D}}_{rd}^{mNR} | \mathbf{p}_1^m, \mathbf{p}_2^m, \mathbf{p}_3^m; \sigma_1^m, \sigma_2^m, \sigma_3^m \rangle \\
&= -3\sqrt{\frac{g_{qmm}}{m_1 + m_2 + m_3}} \frac{1}{\sqrt{2\pi}} \bar{u}(\mathbf{p}_1^m, \sigma_1^m) \gamma_\mu \mathcal{F}^{m} u(\mathbf{p}_1^m, \sigma_1^m) q_\mu \\
&\times 2p_2^m \delta^3 \left( \mathbf{p}_2^m - \mathbf{p}_2 \right) \delta_{\sigma_2^m, \sigma_3^m} 2p_3^m \delta^3 \left( \mathbf{p}_3^m - \mathbf{p}_3 \right) \delta_{\sigma_3^m, \sigma_3^m}.
\end{align}

(6.20)

Although we have not yet performed a non-relativistic reduction of this operator, we indicate it ab initio with a "NR". Within the relativistic calculations presented in Chapter 5 the momentum transfer $\mathbf{q}$ by the emitted meson on the total baryon is not the same as the momentum transfer $\mathbf{q}$ on the single quark (inside the meson). However, in the non-relativistic case these momenta are the same as a consequence of Eq. (6.11), namely

\begin{align}
\mathbf{q} &= \mathbf{P} - \mathbf{P}' = \tilde{\mathbf{q}} = \mathbf{p}_1 - \mathbf{p}_1'.
\end{align}

(6.21)

Here, $\mathbf{P}$ ($\mathbf{P}'$) is the total three-momentum of the decaying (final) baryon, and analogously $\mathbf{p}_1$ ($\mathbf{p}_1'$) is the three-momentum of the active quark in the initial (final) state. Similarly, since only the active quark is involved in the decay process, the energy conservation $E = E' + \omega_m$ on the baryon level is also satisfied on the quark level, namely

\begin{align}
p_{10} = p_{10}' + \omega_m.
\end{align}

(6.22)

This connection has already been used in Ref. [94] for the calculation of $\pi$ and $\eta$ decays in nucleon and $\Delta$ resonances.
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In the following we will use two kinds of non-relativistic approximations. First, we perform a non-relativistic reduction, i.e. we neglect orders of \( (k q)^2 \) and \( (\omega_m)^2 \). In this way one assumes the quark masses \( m_q \) to be large as compared to the absolute value of the three-momentua \( k_q \) and the meson energy \( \omega_m \). The neglect of orders of \( (k q)^2 \) leads to a reduction of the quark energies in the rest-system of the baryon (\( \omega_i \approx m_i \)). Similarly, neglecting orders of \( (\omega_m)^2 \) simplifies the momentum-state representation of the decay operator. Second, we use the non-relativistic relation between the baryon and quark masses, i.e. \( M = p m_i \). We already used this relation above in Eq. (6.12).

For the numerical computation of the transition amplitude one must choose a specific reference frame. Here, in order to comply with Chapter 5 we again take the rest frame of the decaying baryon, where \( P = 0 \), and we define the decay product to move along the z-axis (\( P' = -q, q = (0, 0, Q) \)). Analogously to the previous chapter, we then reduce the multi-dimensional integral (6.14) via the three-delta functions included in the expressions (6.16), (6.18), and (6.20) to a six-dimensional integral over the three-momenta of the spectator quarks of the decaying baryon. One arrives at the expression

\[
F_{i \rightarrow j}^{NR} = \sqrt{2E} \sqrt{2E'} \sum_{\mu, \mu'} \int d^3k_2 d^3k_3 \Psi_{M'\Sigma'M_{\Sigma'T'M'}}^* (k_i'; \mu_i') \times \frac{-3iN}{\sqrt{2p_{10}/2p_{10}'}} \frac{g_{qmm}}{m_1 + \mu'_1} \frac{1}{\sqrt{2\pi}} \bar{u}(p_i', \mu_i') \gamma_5 \gamma_\mu \mathcal{F}_{1m} u(p_1, \mu_1) q_{\mu} \delta_{\mu_2 \mu'_2} \delta_{\mu_3 \mu'_3} \times \Psi_{M\Sigma'M_{\Sigma}T'M} (k_i; \mu_i),
\]

where the non-relativistic reduction of the pieces contained in the second line has still to be performed. Again, the unprimed variables define the quarks of the decaying baryon, whereas the primed ones concern the quarks included in the final baryon. The connection between the \( p_i \) (\( p'_i \)) and the \( k_i \) (\( k'_i \)) is given by

\[
\begin{align*}
 p_1 &= k_1 = -k_2 - k_3, \\
p_2 &= k_2, \\
p_3 &= k_3, \\
p'_1 &= k'_1 - \frac{m'_i q}{m'_1 + m_2 + m_3} = -k'_2 - k'_3 - \frac{m'_1 q}{m'_1 + m_2 + m_3}, \\
p'_2 &= k'_2 - \frac{m_2 q}{m'_1 + m_2 + m_3}, \\
p'_3 &= k'_3 - \frac{m_3 q}{m'_1 + m_2 + m_3}.
\end{align*}
\]

For the spectator quarks 2 and 3 the three-momenta satisfy \( p'_2 = p_2 \) and \( p'_3 = p_3 \), respectively, and accordingly, the primed and unprimed variables...
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correlating the active quark are related by

\[ p_1' = p_1 - q = k_1 - q = -k_2 - k_3 - q, \]
\[ k_1' = -k_2' - k_3' = k_1 - \frac{m_2 + m_3}{m_1' + m_2 + m_3} q = -k_2 - k_3 - \frac{m_2 + m_3}{m_1' + m_2 + m_3} q. \]

(6.25)

In Chapter 5 the connection between the individual quark momenta was provided by the \( (4 \times 4) \) boost matrices (given in Eqs. (5.3) and (5.4)). The transformation properties in Eqs. (6.24) and (6.25) result from a non-relativistic limit of these boost matrices, where one approximates \( \sinh \Delta = \frac{Q}{m_q} \approx \frac{Q}{m_1' + m_2 + m_3} \), and \( \cosh \Delta \approx 1 \). Note that in the non-relativistic calculation of the transition amplitude (6.23) there occur no Wigner \( D \)-functions what makes the spin dependence fairly simple.\(^3\)

Next, we have to evaluate the second line of Eq. (6.23). As a consequence of the non-relativistic reduction we find \( \omega_i \approx m_i \). This also means that the normalization factor (5.22) becomes \( \mathcal{N} = 1 \). Neglecting the flavor as well as constant factors for the moment we have to examine the expression

\[
\frac{1}{m_1 + m_1'} \frac{1}{\sqrt{2p_{10} \sqrt{2p_{10}'} \sqrt{(p_{10} + m_1)(p_{10}' + m_1')}}} \left[ \bar{u}(p_1', \mu_1') \gamma_5 \gamma^0 u(p_1, \mu_1) \omega_m - \bar{u}(p_1', \mu_1') \gamma_5 \gamma^3 u(p_1, \mu_1) Q \right]
\]

(6.26)

Proceeding in the same way as in Section 5.2.1 the analysis of the Dirac spinors leads to

\[
\frac{1}{m_1 + m_1'} \frac{1}{\sqrt{2p_{10} \sqrt{2p_{10}'} \sqrt{(p_{10} + m_1)(p_{10}' + m_1')}}} \times \left\{ -\{ \sigma_z(1) [(p_{10} + m_1) p_{1z} + (p_{10}' + m_1') p_{1z}] + (\sigma_x(1)p_{1z} + \sigma_y(1)p_{1y}) (p_{10} + m_1 + p_{10}' + m_1') \}_{\mu_1' \mu_1} \omega_m \\
+ \{ \sigma_z(1) [(p_{10} + m_1) (p_{10}' + m_1') - p_1^2 - p_{1y}^2 + p_{1z} p_{1z}'] \\
- (\sigma_x(1)p_{1z} + \sigma_y(1)p_{1y}) (p_{1z} + p_{1z}') \}_{\mu_1' \mu_1} Q \right\},
\]

(6.27)

where \( \sigma(1) \) are the Pauli spin matrices relating to the spin of the active quark. Now it remains to find a proper non-relativistic approximation of the active quark energies \( p_{10} \) and \( p_{10}' \). In Eq. (6.22) we found that in the non-relativistic calculation \( \omega_m = p_{10} - p_{10}' \) should be satisfied. However, in our approximation we started with the relativistic transition amplitude corresponding to the relativistic expression for the decay width. Here, for the energy of the meson we still use the relativistic expression, namely

\(^3\)A direct comparison with Section 5.2.3 tells in particular that the terms \( f_i \) and \( g_i \) (see Eq. (5.60)) occurring in the Wigner \( D \)-rotations (5.59) of the spectator quarks 2 and 3 simply reduce to 1 and 0, respectively.

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\[ \omega_m = \sqrt{m_m^2 + q^2}, \] where \( m_m \) is the meson mass and \( q^2 = Q^2 \) is the momentum transfer by the meson constrained by the overall four-momentum conservation and the on-shell condition of the meson. Clearly, inserting the relativistic expressions for the quark energies, we do not arrive at a satisfying solution in this approach, since these energies do not comply with Eq. (6.22). Nevertheless, seeking for a Galilean-invariant transition amplitude we make the following ansatz concerning the energies of the active quark:

\[ p_{10} = m_1 + \frac{\omega_m}{2}, \quad p'_{10} = m'_1 - \frac{\omega_m}{2}. \] (6.28)

This choice seems reasonable, and it effectively leads to the required Galilean-invariant structure. The transition amplitude therefore becomes

\[
F_{i-j}^{NR} = \sqrt{2E} \sqrt{2E'} \sum_{\mu_1, \mu'_1} \int \frac{d^3k_2d^3k_3}{\sqrt{2\pi}} \Psi^*_{M'} \Sigma_{M_mT'T_m} (k_i'; \mu'_i) \\
\times -\frac{3i g_{qjm}}{m_1 + m'_1} \frac{1}{\sqrt{2\pi}} \mathcal{F} m \left\{ \left[ -\omega_m \left( m_1 + m'_1 \right) \frac{1}{2m_1 m_1} \sigma(1) \cdot p_1 \right. \\
+ \left. \left( 1 + \frac{\omega_m}{2m_1} \right) \sigma(1) \cdot q \right] \delta_{\mu_2} \delta_{\mu_3} \right\}_{\mu'_i \mu_i} \\
\times \Psi_{M' \Sigma T' M_T} (k_i; \mu_i). \] (6.29)

In case of \( \pi \) and \( \eta \) decays, the quark masses of the active quarks are the same in the initial and final baryons, i.e. \( m_1 = m'_1 \), and the expression simplifies to

\[
F_{i-j}^{NR} = \sqrt{2E} \sqrt{2E'} \sum_{\mu_1, \mu'_1} \int \frac{d^3k_2d^3k_3}{\sqrt{2\pi}} \Psi^*_{M'} \Sigma_{M_mT'T_m} (k_i'; \mu'_i) \\
\times -\frac{3i g_{qjm}}{2m_1} \frac{\mathcal{F} m}{\sqrt{2\pi}} \left\{ \left[ -\frac{\omega_m}{m_1} \sigma(1) \cdot p_1 + \left( 1 + \frac{\omega_m}{2m_1} \right) \sigma(1) \cdot q \right] \delta_{\mu_2} \delta_{\mu_3} \right\}_{\mu'_i \mu_i} \\
\times \Psi_{M' \Sigma T' M_T} (k_i; \mu_i). \] (6.30)

Using \( p_1 = p'_1 + q \) one can rewrite the Eq. (6.30) and thus obtains

\[
F_{i-j}^{NR} = \sqrt{2E} \sqrt{2E'} \sum_{\mu_1, \mu'_1} \int \frac{d^3k_2d^3k_3}{\sqrt{2\pi}} \Psi^*_{M'} \Sigma_{M_mT'T_m} (k_i'; \mu'_i) \\
\times -\frac{3i g_{qjm}}{2m_1} \frac{\mathcal{F} m}{\sqrt{2\pi}} \left\{ \left[ \sigma(1) \cdot q - \frac{\omega_m}{2m_1} \sigma(1) \cdot (p_1 + p'_1) \right] \delta_{\mu_2} \delta_{\mu_3} \right\}_{\mu'_i \mu_i} \\
\times \Psi_{M' \Sigma T' M_T} (k_i; \mu_i). \] (6.31)

This result is the well-known expression of the transition amplitude in the non-relativistic EEM, as it can be found in several textbooks (see, e.g.,
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Refs. [14,65]). It adheres to the picture where an elementary meson is emitted from a single quark (inside the decaying resonance). This model has mainly been developed for the emission of pions. However, it is applicable for all pseudoscalar mesons, and it is characterized specifically by its simplicity. The expressions proportional to $\sigma(1) \cdot q$ and $\sigma(1) \cdot (p_1 + p'_1)$ are called ”direct” and ”recoil” term, respectively. For a detailed discussion on this kind of model we refer to [10]. In the case a pseudoscalar coupling is assumed, one would only recover the direct term in an analogous non-relativistic reduction.

Finally, it remains to compute the actual decay widths of the baryon resonances utilising the non-relativistic reduction of the transition amplitude. Analogously to Chapter 5 one first has to insert the explicit expressions of the respective baryon wave functions into Eq. (6.29). Then, one has to perform the six-dimensional integration over the three-momenta of the spectator quarks. Here, we adopt the same methods as for the relativistic calculations (see Appendix C.3). The numerical effort in the non-relativistic calculation is by far less extensive than in the relativistic case, and the computations can be performed along with standard methods.
Chapter 7

Results and Discussion

7.1 Covariant Results with Pseudovector Coupling

We first analyze the results obtained with the PFSM decay operator using a pseudovector coupling between the active quark and the emitted meson as defined in Eq. (5.18). The detailed predictions by the GBE and OGE CQMs are summarized in the first section of Appendix F.

7.1.1 Non-Strange Decays of Light Baryons

The $\pi$ and $\eta$ decay modes of nucleon and $\Delta$ resonances have already been studied before in the same framework as done here, see Ref. [20]. We have recalculated the corresponding decay widths whereupon we found a bug in the computer code of the previous work. The correct results are now quoted in Tables F.1 and F.4 of Appendix F. Qualitatively, the observations made in Ref. [20] remain valid although the specific values of the respective decay widths have been changed. Only the resonance $N(1535)$, which played an exceptional role in the previous calculations, has to be reinterpreted; it is now congruent with the overall observation found in the sector of non-strange decays of light baryons [95] (see also the discussion below).

In the following we first summarize the main observations that can be made from the study of $\pi$ and $\eta$ decays of light baryons. Regarding the $\pi$ decay channel almost all results show similar characteristics: the theoretical predictions are always smaller than the experimental data or they reach at most their values. Only for the $N(1710)$ resonance (and also for the $N(1535)$ in case of the OGE CQM) the theoretical value falls into the range of the experimental data. Nowhere the experimental data are exceeded by the theoretical predictions. These findings are independent of the particular CQM. They are very similar for both the GBE and OGE CQMs.

Among other ingredients the reproduction of the decay widths is mainly
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influenced by the overlaps of the resonance and ground-state wave functions as well as their masses. Especially the latter define the available phase space. In general, a decay width to a specific ground state will result the larger the larger is the mass of the decaying resonance. In the comparison of results from different CQMs one has to bear in mind that the spectra are predicted with varying accuracy. Not all theoretical values coincide with experimental data. As a result the theoretical decay widths are afflicted with certain shortcomings in the reproduction of the excitation spectra. In order to get rid of these mass effects one may use experimental masses as input instead of the theoretical ones predicted by the different quark models. This also allows a better estimation of the influences from distinct wave functions (quark model dynamics). We have therefore made an analogous study of decay widths employing experimental masses throughout. The pertinent results are included in the last two columns of the tables in Appendix F. As compared to the results with theoretical masses only slight changes are found for the GBE CQM (for the decay modes considered in this subsection). The reason is that the experimental masses are rather well reproduced by the GBE CQM, see the Tables D.1 and D.2. By contrast, for the OGE CQM the changes are bigger, especially with regard to the decays of the positive-parity resonances $N(1440)$, $N(1710)$, and $\Delta(1600)$. From the comparison of the GBE and OGE CQM calculations with experimental masses one learns that the effects from the wave-function behaviour (GBE versus OGE dynamics) are sizable in some cases. In particular, for the $\frac{3}{2}^+$ state $\Delta(1600)$ the OGE prediction is by far bigger than the GBE prediction; still it is much too small as compared to experiment. Similarly, for the $\frac{1}{2}^-$ states $N(1535)$, $N(1650)$, and $\Delta(1620)$ the $\pi$ decay widths with the OGE wave functions are considerably larger than the ones with the GBE wave functions. Eventually, also in case of the $N(1440)$ the OGE wave function leads to a larger decay width. For all the other $\pi$ decay widths the theoretical predictions by both the GBE and OGE CQMs are of similar magnitudes. Therefore the wave functions play only a minor role in these cases. Indeed, the $N(1520)$, $N(1675)$, $N(1700)$, $\Delta(1232)$, and $\Delta(1700)$ have already been classified as structure-independent before in the context of non-relativistic (or relativized) calculations both along the EEM and the pair-creation model (see Refs. [19,96,97]). Interestingly, the same classification is now manifested in a covariant approach (along the PFSM decay operator).

With respect to the $\eta$ decays of the nucleon and $\Delta$ resonances sizable decay widths occur only for the $N(1535)$ and $N(1650)$. In case of the $N(1535)$ both the GBE and OGE CQMs underestimate the experimental data. On the other hand, for $N(1650)$ the results are much too large. This behaviour, which is common to both types of CQMs, may be connected to the one in the $\pi$ decay channel, where the theoretical prediction of the decay width of $N(1650)$ is unexpectedly small. As for the $\pi$ decays we can
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compare the influences from the GBE and OGE wave functions when using experimental resonance masses instead of theoretical ones. We find that in cases of the $\frac{1}{2}^-$ resonances $N(1535)$ and $N(1650)$ the OGE wave functions again lead to larger results as compared to the GBE wave functions; a sizeable effect is also seen for the $\eta$ decay of the $N(1710)$. For the remaining (structure-independent) resonances $N(1520)$, $N(1675)$, and $N(1700)$ the $\eta$ decay widths are practically the same for both types of wave functions, in close analogy to the $\pi$ decays above.

While the results for the $\eta$ decay widths represent the first ones calculated in a covariant theory (following the point form of RQM), fully relativistic results are available for the $\pi$ decay widths also from an alternative covariant approach, namely the one by the Bonn group solving the Bethe-Salpeter equation. It is now interesting to compare our results with the predictions obtained by the Bonn group for their II CQM. In Table F.25 of Appendix F we have collected the $\pi$ decay widths as given by the GBE, OGE, and II$^1$ CQMs, where in each case the theoretical resonance masses have been used. We note a surprising similarity of all the results - in particular, all predictions underestimate the experimental data - even though the dynamical concepts underlying each CQM are rather different and distinct theories are followed. We work in RQM with a fixed number of degrees of freedom, the Bonn group has employed a field-theoretic formalism. Certainly, both theories are covariant and they both use a spectator-model decay operator. We remark that much the same similarity has been found already before when comparing the elastic electroweak nucleon form factors as calculated by the Graz [98] and Bonn groups [49].

The main new results of this thesis consist in the investigation of all other possible meson decays beyond the $\pi$ and $\eta$ decays of the light baryons. In particular, we have produced first covariant quark-model predictions for

- non-strange decays of strange baryons,
- strange decays of light baryons, and
- strange decays of strange baryons.

We discuss them consecutively in the following sections.

7.1.2 Non-Strange Decays of Strange Baryons

Let us first examine the possible $\pi$ and $\eta$ decay modes of the strange baryons (see Eqs. C.16 and C.30 in Appendix C) with the same PFSM decay operator. The predictions of the GBE and OGE CQMs are collected in Tables F.2, F.3, and F.5. In general, they show a completely different behaviour

\footnote{The values for the II CQM have been taken from Ref. [52]; the figures for the $N(1710)$ and $\Delta(1600)$ resonances were not given there.}
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than the non-relativistic (or relativized) results available hitherto (see, for example, Refs. [53,54]). Basically the characteristic features of the PFSM results, namely to underestimate the experimental data, are also found here.

Regarding the $\Lambda \to \Sigma \pi$ decay widths the GBE CQM, and likewise the OGE CQM, seem to predict the $\Lambda(1405)$ decay rather reasonably. However, this is a result of the wrong theoretical masses produced by both CQMs (like they are wrong for many other CQMs) as seen in Table D.3. In case one substitutes the experimental mass for $\Lambda(1405)$ the decay widths are drastically reduced and they fall far below the experimental data. The situation is quite similar with the $\Lambda(1520)$. At this point let us make a further remark on the quality of the $\Lambda$ spectrum as produced by the GBE and OGE CQMs. With the exception of the $\Lambda(1405)$ the GBE CQM leads to a rather realistic description of the experiments, whereas the theoretical masses produced by the OGE CQM are generally too large. Clearly, this observation should be taken into account when examining the decay widths.

For the other $\Lambda \to \Sigma \pi$ decays the CQM predictions usually underestimate the experimental data or at most reach their magnitudes (notably for the $\Lambda(1690)$). In this regard the characteristics of the $\pi$ decays of the $\Lambda$ resonances are reminiscent of the ones found above for the $\pi$ decays of the nucleon and $\Delta$ resonances. However, the $\Lambda(1670)$ represents a notable exception: here the theoretical predictions overshoot the experimental data significantly. The reason may be due to a considerable mixing of this state with the singlet $\Lambda(1405)$. For the $\Lambda(1800)$ we obtain large $\pi$ decay widths both for the GBE and OGE CQMs. Unfortunately, the PDG does not present an experimental datum for this (partial) decay width but only marks the decay as "seen". A direct comparison with experiment is thus not possible. We remark, however, that the reported total decay width of this resonance is rather large (200-400 MeV) and the percentage of the $K$ decay is about 25-40 \%. One may thus expect a considerable portion for the $\pi$ decay channel.

The influences of distinct quark-model wave functions are again best studied along with the calculations using experimental masses instead of the theoretical ones. Large differences are observed for $\Lambda(1600)$, $\Lambda(1670)$, and $\Lambda(1800)$. For the other cases ($\Lambda(1690)$, $\Lambda(1810)$, and $\Lambda(1830)$) wave-function effects are minor.

The $\Sigma$ resonances have two possible $\pi$ decay modes, namely $\Sigma \to \Sigma \pi$ and $\Sigma \to \Lambda \pi$. Beyond the three- and four-star resonances of the PDG we have included in our investigations also the low-lying two-star resonances, in particular, the $\frac{1}{2}^-$ $\Sigma(1620)$ and the $\frac{1}{2}^+$ $\Sigma(1880)$ states as well as the $\Sigma(1560)$ and $\Sigma(1690)$, with yet unknown spin and parity $J^P$. We assign to the latter two the $J^P$ values $\frac{1}{2}^-$ and $\frac{3}{2}^+$ (cf. the discussion in Section 7.4 below). The experimental data are rather scarce for the pionic $\Sigma$ decays. Several of them are only marked as "seen" by the PDG. Other ones that are in principle
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possible have not even been observed (see the Tables F.2 and F.3).

Concerning the $\Sigma \to \Sigma \pi$ we observe the theoretical predictions to approximately fit the experimental data in case of the $\Sigma(1670)$, $\Sigma(1750)$, and $\Sigma(1775)$ resonances. The $\Sigma(1385) \to \Sigma \pi$ results underestimate the experimental data both for the GBE and OGE CQMs. In the $\Sigma \to \Lambda \pi$ decay channel all the theoretical predictions remain below the available experimental data or at most reach them from below (for the $\Sigma(1670)$). For the $\frac{1}{2}^-$ resonances $\Sigma(1560)$ as well as $\Sigma(1620)$ we find rather large theoretical decay widths. A comparison to experiment, however, is not possible as no data are available in the PDG compilation; only the $\Sigma(1560) \to \Lambda \pi$ is "seen".

At this point we add an important remark with regard to the $\frac{1}{2}^-$ $\Sigma$ spectrum. In this case, for both the GBE and OGE CQMs three theoretical states are quoted. As usual one compares with experimental masses from three- and four-star resonances. For this particular $J^P$ excitation spectrum, however, only one such state is given by the PDG, namely the $\Sigma(1750)$ (see the Figs. 2.1 and 2.2). A priori it is not clear, which one of the theoretical states should be identified with this $\Sigma(1750)$ resonance. By the investigation of the corresponding decay widths we find a natural solution to this problem. Upon examination of the magnitudes of the decay widths pertaining to the three theoretical states it turns out that only the third excitation (in case of the GBE CQM) actually leads to a theoretical prediction whose magnitude is in line with the characteristics of the decay widths of the other baryon resonances in the same flavor multiplet, namely, it does not lead to an overestimation of the experimental value. Moreover, the decay widths of the other two (lower-lying) states fit better to $\Sigma(1560)$ and $\Sigma(1620)$ by the same reasoning; they, however, are experimentally known only as two-star resonances. Thus by including also the two-star resonances we are led to a consistent classification of all the $\frac{1}{2}^-$ $\Sigma$ states into proper flavor multiplets (see Section 7.4 and Appendix E). Thereby it is also implied that the $\Sigma(1560)$ resonance, whose $J^P$ is not yet definitely established by experiment, should be the lowest-lying $\frac{1}{2}^-$ state in the $\Sigma$ excitation spectrum.

As for the previous baryon resonance decays, we have studied the influence of different dynamics (GBE versus OGE) on the quark-model wave functions by calculating the decay widths with experimental instead of theoretical resonance masses. We observe a similar behaviour as for the light baryons above, i.e. the results can essentially be classified by the two categories of structure-dependent and structure-independent resonances, even though the systematics is not so clear as in cases of the nucleon and $\Delta$ resonances. For instance, the $\Sigma(1560)$ (as a structure-dependent resonance) shows a pronounced sensitivity on the wave functions for the $\Sigma \pi$ decay mode,

\footnote{A deeper analysis of the flavor content of the two upper $\frac{1}{2}^-$ $\Sigma$ resonances tells that these states are reversed in order for the OGE CQM relative to the GBE CQM.}
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whereas the $\Sigma(1560) \rightarrow \Lambda\pi$ decay widths are practically the same for both the GBE and OGE CQMs.

Finally, we consider the $\Xi \rightarrow \Xi\pi$ decay widths (see the lower part of Table F.3). Only for the decay width of the $\Xi(1530)$ the PDG gives an experimental datum, which is underestimated by our theoretical predictions. The $\pi$ decay width of the $\Xi(1820)$ is estimated to be small by the PDG, and the ones of $\Xi(1690)$ and $\Xi(1950)$ are only marked as "seen". In all cases, except maybe for $\Xi(1950)$, the theoretical decay widths are rather small and they do not exhibit sensible dependences on the type of CQM wave functions.

Concerning the $\eta$ decays of strange baryons (see Table F.5) experimental data are only available in two cases, namely, for the $\Lambda(1670)$ and $\Sigma(1750)$ resonances. For the $\Lambda(1670) \rightarrow \Lambda\eta$ there is only a OGE CQM prediction, when using theoretical masses, and it turns out too large; the corresponding decay is not possible for the GBE CQM, as the theoretical mass difference between $\Lambda(1670)$ and the $\Lambda(1116)$ ground state is not big enough to allow the emission of an $\eta$ meson (the $\Lambda$ ground state lies slightly too high). However, when using experimental masses, one obtains theoretical results falling into the range of the experimental data for both the GBE and OGE CQMs. The two predictions differ, however, since $\Lambda(1670)$ is to be characterized as a structure-dependent resonance. For the $\eta$ decay of the $\Sigma(1750)$ both the GBE and the OGE predictions remain far below the experimental datum. Again they differ sensibly from each other, since also $\Sigma(1750)$ is structure-dependent. While the other $\eta$ decay widths result rather small and do not show much dependence on the type of CQM wave function, there is the striking case of the $\Lambda(1800)$ resonance, for which we obtain surprisingly big decay widths. For this state we note a remarkable similarity with the $\eta$ decay of the $N(1650)$, which has the same $LS$ structure.

7.1.3 Strange Decays of Light and Strange Baryons

For the strange ($K$) decays of light and strange baryons fully relativistic results from CQMs have not been available so far. In this section we discuss the covariant predictions of the GBE and OGE CQMs for the strange decays of nucleon and $\Delta$ resonances as well as the $\Lambda$, $\Sigma$, and $\Xi$ resonances. The results are presented in Tables F.6 - F.8. Here, we distinguish between strange decays, where the strange-quark content increases upon transition from the initial resonance to the final ground state (i.e. a light quark transforms to a strange one upon emission of a $K$ meson, see Table F.6), and the ones, where the strange-quark content decreases (i.e. a strange quark transforms to a light one upon emission of a $K$ meson Tables F.7 and F.8). Both decay types are discussed in Section C.2 of Appendix C, see the Eqs. C.39 and C.40.

Out of the first category of strange decays (with increasing strangeness
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In the baryon, let us first discuss the $K$ decays of nucleon and $\Delta$ resonances. Experimentally measured are only the $N \rightarrow \Lambda K$ decays, no data are available for the other cases. Except for the decay $N(1710) \rightarrow \Sigma K$ the theoretical decay widths are all rather small for both the GBE and OGE CQMs and remain far below the experimental data. This is true for theoretical as well as experimental masses used, and there is practically no influence from different wave functions. For the $N(1710) \rightarrow \Sigma K$ decay we find a larger width for the GBE CQM and a huge one for the OGE CQM. This is mostly an effect from the theoretical masses, which are too high by about 60 MeV for the GBE and by 150 MeV for the OGE CQMs, respectively (see Figs. 2.1 and 2.2). Surprisingly the decay width remains still rather large in case of the OGE CQM, even when employing experimental masses (and the mass effect is washed out). This result is obviously generated by the specific behaviour in the overlap of the resonance and ground-state wave functions under relativistic boosts.

Next, we take a look at the $N \rightarrow \Xi K$ and $\Sigma \rightarrow \Xi K$ decays. The pertinent results are listed in the lower part of Table F.6. Applying experimental masses, only three decays are possible, namely, the ones of $\Lambda(1830)$, $\Sigma(1880)$, and $\Sigma(1940)$. All of them are not measured by experiment. The theoretical predictions for the decay widths are always pretty small.

Let us now consider the strange decays $\Lambda \rightarrow NK$ and $\Sigma \rightarrow NK$ (with decreasing strangeness in the baryon). All of them are measured for the resonances and for the three- and four-star resonances.

In the $\Lambda \rightarrow NK$ channel two decays result with widths of reasonable magnitudes, the ones of $\Lambda(1520)$ and $\Lambda(1600)$. In case experimental masses are employed, the theoretical decay widths do not exceed the experimental data but approach them from below. There is always a big dependence on the input baryon masses. This is most striking in case of $\Lambda(1405)$. In principle it cannot decay into $NK$. Only, the CQMs predict its mass by far too high, and we may calculate "artificial" decay widths. Their values are rather large. They quickly become small and finally disappear when the resonance mass falls down towards the experimental one. The remaining $\Lambda \rightarrow NK$ decay widths all turn out much too small. This is especially true for the $\Lambda(1800)$ resonance whose experimental $NK$ decay width is almost 100 MeV. The theoretical prediction is at least ten times smaller. Surprisingly we found for the same resonance rather large decay widths in the $\Lambda\eta$ and $\Sigma\pi$ channels; the first one is not measured, the second one has only been "seen". When comparing the results from the calculation using experimental masses, we observe appreciable differences in the cases of the $\Lambda(1600)$, $\Lambda(1690)$, and $\Lambda(1810)$ resonances, which thus show a noticeable structure dependence.

The $\Sigma \rightarrow NK$ decay widths are all too small for the three- and four-star resonances. On the contrary, for the two-star resonance $\Sigma(1620)$ we obtain rather large predictions both for the GBE and OGE CQMs. Unfortunately, no comparison to experiment is possible. Also for the two-star
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resonance $\Sigma(1560)$ both CQMs produce appreciable decay widths. A considerable structure dependence is found in the cases of the $\Sigma(1660)$, $\Sigma(1670)$, $\Sigma(1690)$, $\Sigma(1750)$, and $\Sigma(1880)$ states (as it is seen from the results when using experimental resonance masses).

Finally, we consider the $K$ and $\bar{K}$ decays. No experimental data are quoted by the PDG, even though all decays have been seen and the $\Xi(1820) \rightarrow \Lambda K$ and $\Xi(1820) \rightarrow \Sigma K$ are marked by "large" and "small", respectively. The total decay width of $\Xi(1820)$ ranges between 14 and 39 MeV. The theoretical predictions for both the $\Xi(1820) \rightarrow \Lambda K$ and $\Xi(1820) \rightarrow \Sigma K$ decay widths are of similar magnitudes, with a tendency of the first one being smaller and the second one being larger (contrary to the estimate of the relative magnitudes by the PDG). Otherwise, the $\Lambda K$ and $\Sigma K$ decay widths are rather small, except for $\Xi(1690) \rightarrow \Sigma K$ when theoretical masses are used. For experimental masses as input also this decay width becomes small, with a sizable sensitivity on different wave functions.

In summary, all our results for covariant $K$ decay widths calculated with pseudovector coupling underestimate the experimental data or at most reach their values from below; this is most clearly seen when experimental masses are used as input (and mass effects are thus suppressed). Also our fully relativistic results are rather different from non-relativistic ones (see the discussion below in Section 7.3) or results with relativistic corrections available in the literature [18,53,54].

7.1.4 Different Treatment of Momentum Transfer

In Chapter 5 we defined the PFSM decay operator within the framework of a Poincaré-invariant theory. Due to the requirement of overall momentum conservation as well as the on-mass-shell conditions of the participating particles it has immediately been evident that the momentum transfer $\tilde{q}$ to the active quark is not identical to the momentum transfer $q$ to the baryon as a whole (see Eqs. (5.24) and (5.25)). Accordingly, we learned that the PFSM decay operator represents an effective many-body operator rather than a pure one-body operator. In case of pseudovector coupling the momentum transfer occurs explicitly in the expression of the corresponding decay operator (5.18). In principle, it is not clear, which momentum transfer ($q$ or $\tilde{q}$) is more adequate for the description of the strong baryon resonance decays. Both of them lead to a decay operator, which complies with the principles of RQM, and the actual choice is not unique. It seems reasonable to first insert the $q$, i.e. the momentum transfer to the baryon as a whole. This has led to the results collected in Tables F.1 - F.8 and discussed in the previous subsections. However, we have also performed a calculation with $q$ replaced by $\tilde{q}$. The corresponding results are essentially the same as for the decay operator with pseudoscalar coupling to be discussed in the following subsections.
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7.2 Covariant Results with Pseudoscalar Coupling

The PFSM decay operator can also be constructed with a pseudoscalar coupling between the active quark and the emitted meson. As stated in the previous section, this corresponds exactly to the case with pseudovector coupling using $\vec{q}$ for the momentum transfer in Eq. (5.18). The pertinent results are collected in Section F.2 of Appendix F (see the Tables F.9 - F.16) for the GBE and OGE CQMs in both cases, using theoretical as well as experimental baryon masses as input.

We follow the same procedure as above, i.e. we discuss the non-strange decays of nucleon and $\Delta$ resonances, the non-strange decays of strange baryons, and finally the strange decays in light and strange baryons. However, we will not go into great detail concerning the various individual results for the decay widths. Rather, we will qualitatively compare the results obtained here with the ones pertaining to pseudovector coupling in Section 7.1 above.

7.2.1 Non-Strange Decays of Light Baryons

The covariant predictions for the $\pi$ decay widths of the nucleon and $\Delta$ resonances are quoted in Table F.9. Overall they show a similar behaviour as the results in pseudovector coupling listed in Table F.1. Again the theoretical predictions usually remain below the experimental data; for the $N(1535)$ and $N(1710)$ decays the widths match the experiments. The decay widths for the $N(1520)$, $N(1675)$, and $N(1700)$ as well as the $\Delta(1232)$ and $\Delta(1700)$ resonances are rather similar for the pseudoscalar and pseudovector coupling. We note that exactly these resonances were characterized as structure-independent ones in the previous section.

On the other hand, there are appreciable differences in the decay widths for the other resonances. They are most striking for the Roper resonance $N(1440)$ and the $\Delta(1600)$. As compared to the pseudovector coupling the widths become much smaller in the first case, whereas they become much larger in the second case. The situation for $\Delta(1620)$ is rather similar to $\Delta(1600)$. Still, the predictions do not yet reach the experimental values. Also for the remaining (structure-dependent) resonances $N(1535)$, $N(1650)$, and $N(1710)$ the differences between the results for the two different couplings are considerable. In all of these cases the results for pseudoscalar coupling are much larger as the ones for pseudovector coupling.

Concerning the $\eta$ decays of the nucleon resonances (see Table F.12) vigorous changes are found for the $N(1535)$ and $N(1650)$. The theoretical decay widths for pseudoscalar coupling are much smaller than the ones for pseudovector coupling and they deviate further from the experimental data. For the $N(1710) \rightarrow N\eta$ decay the width is much different from the pseudovector coupling in case of the GBE CQM, whereas the deviation is rather small for
the OGE CQM. As in case of the $\pi$ decays, the different couplings do not significantly affect the predictions for the (structure-independent) $N(1520)$, $N(1675)$, and $N(1700)$ resonances.

### 7.2.2 Non-Strange Decays of Strange Baryons

In analogy to Section 7.1.2 let us first discuss the $\Lambda \to \Sigma\pi$ decays given in Table F.10: We observe that the decay operator with pseudoscalar coupling generally leads to smaller predictions as compared to the pseudovector coupling (see Table F.2). This is in particular advantageous for the $\Lambda(1670)$ decay width, which was too big before and now agrees with experiment. Also the decay width for $\Lambda(1800)$ is now much smaller than for pseudovector coupling. The situation is quite similar for $\Lambda(1810)$. Only the results for the $\Lambda(1520)$, $\Lambda(1690)$, and $\Lambda(1830)$ resonances are much the same for the two types of couplings.

With respect to the pionic decays of the strange baryons $\Lambda \to \Sigma\pi$ and $\Sigma \to \Lambda\pi$ (see Tables F.10 and F.11) we also observe a general shift to smaller decay widths. This causes an overall underestimation of the available experimental data. We refrain from a detailed discussion of the various results for the $\pi$ decay widths of all the $\Sigma$ resonances. In summarizing we only state that the theoretical predictions for the decay widths from pseudovector and pseudoscalar couplings are in general different for the so-called structure-dependent resonances. On the other hand they are alike for the structure-independent resonances. One should, however, keep in mind that in some special cases this classification is not so clear-cut.

For the $\Xi \to \Xi\pi$ decay widths listed in Table F.11 we find predictions comparable to the ones for pseudovector coupling with the $\Xi(1950)$ resonance again rather large. Only the $\Xi(1690) \to \Xi\pi$ gets much reduced; note that this resonance is structure-dependent and much similar to the $N(1440)$.

For the $\eta$ decays of strange baryons (see Table F.13) we find practically the same behaviour as for the $\pi$ decays above. The structure-dependent resonances change a lot when replacing the pseudovector coupling by the pseudoscalar one (see, e.g., the $\Lambda(1670)$ and $\Lambda(1800)$ or the $\Sigma(1750)$ decays). Practically no changes are seen for the $\eta$ decays of the (structure-independent) $\Lambda(1690)$ and $\Lambda(1830)$ as well as $\Sigma(1775)$ and $\Sigma(1940)$ resonances.

### 7.2.3 Strange Decays of Light and Strange Baryons

With respect to the $K$ decays of the light and strange baryons (see the results in Tables F.14 - F.16) we find essentially the same behaviour as for the pseudovector coupling in Section 7.1.3. In particular, the nucleon and $\Delta$ $K$ decay widths are all exceedingly small, again with the notable exception of the $N(1710) \to \Sigma K$ for the OGE CQM. Only for the (structure-dependent)
Sigma(1880) resonance the K decay width is much changed.

For the Lambda -> NK and Sigma -> NK decays the predictions with pseudoscalar coupling are again fairly small, and we observe an overall underestimation of the experimental data. The deviations from the pseudovector results are more pronounced for the Lambda(1600), Lambda(1670), Lambda(1800), and Lambda(1810); the decay widths become considerably smaller for Lambda(1600), whereas they become much larger for Lambda(1670). They remain essentially the same for Lambda(1520), Lambda(1609), and Lambda(1830). An analogous behaviour is seen for the Sigma -> NK decays, with the Sigma(1560), Sigma(1620), Sigma(1660), Sigma(1690), and Sigma(1750) exhibiting larger changes and the Sigma(1670), Sigma(1775), and Sigma(1940) remaining essentially unaltered.

For the K decay widths of the Xi resonances in Table F.16 we observe major reductions of their magnitudes only for the Xi(1690). For the other states Xi(1820) and Xi(1950) the results are comparable for pseudovector and pseudoscalar coupling.

The main results of this section (again) consist in the observation of too small predictions for decay widths. For the PFSM decay operator with pseudoscalar coupling most of the decay widths are further reduced as compared to the pseudovector coupling in the previous Section 7.1. One has seen that some specific resonances are especially sensitive to the kind of coupling (pseudovector versus pseudoscalar). The same resonances have also been found to be sensitive to the kind of dynamics (namely, GBE vs. OGE hyperfine interactions). The reasons for the specific behaviours of certain states can be understood in detail by a closer inspection of the spatial as well as spin-flavor symmetries characterizing these states. Herefore, we refer to the discussion in the last Section 7.4 of this chapter as well as to Appendix E.

7.3 Results for the Non-Relativistic Limit of the PFSM

In Chapter 6 we considered the non-relativistic limit of the PFSM decay operator (for the case of pseudovector coupling). In this way we were led to the traditional EEM. Non-relativistic or relativized decay calculations have already been performed before. No satisfactory explanation of the mesonic decays has been reached. Rather the theoretical predictions were found to scatter around the experimental data, with some of them overshooting and some of them underestimating the measured decay widths. Incidentally, several predictions also matched the experimental values. In any case no systematic description could be found.

Still we found it useful to study the non-relativistic of the PFSM decay operator, the main motivation being to examine the sizes of relativistic effects. We therefore undertook calculations applying the non-relativistic
RESULTS AND DISCUSSION

decay operator given in Eq. (6.29). The corresponding results are presented in Tables F.17 - F.24 of Section F.3 in Appendix F.

7.3.1 Non-Strange Decays of Light Baryons

The non-relativistic results for the $\pi$ decay widths of nucleon and $\Delta$ resonances are listed in Table F.17. Most of the theoretical predictions are too small as compared to experiment. However, we find also huge decay widths, namely, for $N(1535)$ and $N(1650)$. Here, the non-relativistic reduction of the decay operator has an enormous impact on the results. Also for the $\Delta(1600)$ and $\Delta(1620)$ resonances the non-relativistic results are rather large. While the $\Delta(1600) \to N\pi$ decay width coincides with experiment, the $\Delta(1620) \to N\pi$ result now overpredicts the experimental datum by far. For the $N(1440)$ resonance we observe the non-relativistic decay width to be smaller than in the relativistic calculation. The same is true for the $N(1710)$ resonance, where the non-relativistic decay width falls into the range of the experimental data. While a certain influence of the non-relativistic reduction is seen in all cases, the sensitivity on relativistic effects is most pronounced in case of the structure-dependent resonances. This observation is further substantiated by the results for the $\eta$ decay modes (see Table F.20); there especially the results for the $N(1535)$ and $N(1650)$ now drastically overshoot the experimental data.

7.3.2 Non-Strange Decays of Strange Baryons

The non-relativistic results for the $\pi$ and $\eta$ decays of strange baryons are quoted in Tables F.18, F.19, and F.21. Again, we observe that the influences of the non-relativistic reduction are tremendous in some specific cases. For example, let us have a look at the resonances $\Lambda(1670)$ and $\Lambda(1800)$, in which cases the non-relativistic $\pi$ decay widths are much larger than the corresponding relativistic ones. Hence, the non-relativistic results overshoot the experimental data by far. Similarly, for the $\eta$ decays we also observe huge decay widths, above all for the $\Lambda(1800)$ resonance. The $\eta$ decay widths of both states, the $\Lambda(1670)$ and $\Lambda(1800)$ appreciably overestimate the experimental data. Likewise the non-relativistic results for $\Sigma(1750)$ are much enhanced in both the $\pi$ and $\eta$ channels. The general behaviour of the $\pi$ and $\eta$ decays of the strange baryons is quite similar to the one described for the light baryons in the previous section.

7.3.3 Strange Decays of Light and Strange Baryons

Eventually, the non-relativistic $K$ decay widths of the various baryon resonances are collected in Tables F.22 - F.24. The non-relativistic reduction has again sizable influences on certain resonances. For some resonances one finds an enhancement of the non-relativistic decay widths, for others the
RESULTS AND DISCUSSION

non-relativistic results become smaller. The biggest changes are again observed for the structure-dependent resonances, e.g., for Λ(1670) → NK, Λ(1810) → NK, Σ(1560) → NK, and Σ(1620) → NK. As a result the non-relativistic predictions scatter around the experimental data, and no systematics are found like for the covariant results (of which no one overshoots an experimental value).

7.4 Multiplet Structure of the Baryon Resonances

The decay widths as analyzed in the previous section are mainly governed by the overlaps of the resonance and ground-state baryon wave functions. In this way the results provide useful insight into the symmetry structures of the spatial, spin, and flavor parts of the baryon states. We have examined the baryon states as generated by the CQMs in Appendix E. Thereby we found a consistent classification scheme of all experimentally known baryons into flavor multiplets. Within each particular multiplet the members show a congruent behaviour with regard to the predictions of decay widths (see the tables in Section E.3.2 of Appendix E) and connected with that also for the spatial density distributions.

In Appendix E we have shown the various shapes of density distributions in each flavor multiplet. We have also calculated the mixings of flavor multiplets by applying flavor-singlet, -octet, and -decuplet projection operators.

Our classification of the light and strange baryon resonances in flavor multiplets is presented in Tables E.15 - E.17 of Appendix E. There, we have included the three- and four-star resonances as well as some low-lying two-star resonances quoted by the PDG. The flavor multiplet structure found here is in an almost overall agreement with the classification scheme recently published by Guzey and Polyakov [99]. Only for the Λ(1810) we differ: It is a flavor singlet in our case, whereas Guzey and Polyakov classify it as a flavor-octet state.

We compared our classification also with the one provided by the PDG [2]. They tentatively do not include one- and two-star resonances, and their scheme is almost the same as the one by Samios et al. [100] back in 1974. According to the PDG the Σ(1750) should be a flavor-octet state. We, however, find that this state is rather a flavor decuplet, in agreement with Guzey and Polyakov.

In summary, by the classification of the baryon resonances into flavor multiplets one gets a better understanding of the underlying symmetries in the baryon wave functions and thus in the CQMs. One learns that the baryon properties, here in particular the decay widths, are governed by the specific quantum numbers determining the various baryons states.
Chapter 8

Conclusion and Outlook

In this thesis we have presented the first study of the various decay modes of all light and strange baryon resonances in the framework of relativistic quantum mechanics. In particular, we have worked in the point-form version of relativistic quantum mechanics and applied constituent quark models based on a Poincaré-invariant mass operator. The latter includes on top of a realistic confinement of linear form two types of hyperfine interactions based on either Goldstone-boson-exchange or one-gluon-exchange dynamics from Refs. [41] and [19], respectively. The decay mechanism has been designed after a spectator model in point form. It allows to deduce covariant results for amplitudes/widths of all decay modes of the light and strange baryons. Our study completes the investigations of the $\pi$ and $\eta$ decays started along the same line in Ref. [20].

We were interested in the predictions of the relativistic constituent quark models for partial decay widths of the light and strange baryon resonances with masses below 2 GeV in the $\pi$, $\eta$, and $K$ decay channels. By this investigation the qualities of the constituent quark models can be tested together with the theory of the decay mechanism. In particular, one gets insight into the adequacy of the quark-model wave functions as well as the transition operator.

After having reviewed the solution of the eigenvalue problem of the mass operator (by means of the stochastic variational method) and the resulting spectral properties of the Goldstone-boson-exchange as well as one-gluon-exchange constituent quark models we have outlined the approach to relativistic hadron reactions within Poincaré-invariant quantum mechanics. Then we have set up the formalism for the calculation of mesonic decays in the point form. In particular, we have developed a covariant model for the meson emission based on a spectator picture, where the coupling to a single quark is assumed of pseudovector or pseudoscalar type. The non-relativistic limit of this decay model leads to the elementary emission model known from the literature.
CONCLUSION AND OUTLOOK

By applying the point-form spectator model for the decay operator we have calculated the transition matrix elements in a manifestly covariant manner. The predictions directly obtained from the Goldstone-boson-exchange and one-gluon-exchange constituent quark models have been compared to each other and to experiment \([2]\). Whenever possible a comparison has also been made with the relativistic predictions for \(\pi\) decay widths by the instanton-induced constituent quark model available from the Bonn group \([52]\).

With our theoretical results a description of decay widths in agreement with experimental data is achieved only in a few places. In most cases the decay widths turn out to be smaller than the ones experimentally measured. However, with the relativistic decay widths a nice systematics is found, namely the quark model predictions in general do not overshoot the experimental data. This is an essential difference as compared to previous non-relativistic results. Theoretical decay widths from a non-relativistic approach following an elementary emission model (without and/or with relativistic corrections) show no such systematics but rather scatter below and above the experimental data. The fact that covariant decay widths are usually too small or at most reach the experimental data is also prevailing in the results of the Bonn group, which were calculated in a completely different manner from a Bethe-Salpeter approach. This observation may be taken as an indication that the decay operator in a spectator model is still incomplete and further ingredients will be necessary. Beyond doubt, in all instances, relativistic effects have been found to be of utmost importance.

On the other hand, the systematics of the relativistic results also confirm notions that have been gained before from non-relativistic studies. The covariant predictions for decay widths show a typical behaviour for the so-called structure-dependent and structure-independent resonances. Practically in all cases it is congruent with the gross features of non-relativistic results. While in the cases of structure-independent resonances the decay widths are rather insensitive to the dynamical ingredients, the decays of structure-dependent resonances are influenced a lot by the type of quark-model wave functions as well as the assumed quark-meson coupling in the transition operator (pseudovector vs. pseudoscalar). The magnitudes of these influences, however, differ again from the non-relativistic theory.

The characteristics of the results found for the decay widths can be traced back to the structure of the baryon wave functions. In this context, we have provided a detailed analysis of the symmetry properties of the ground-state and resonance wave functions of all known light and strange baryons. In particular, we have examined the symmetries prevailing in the spin, flavor, and spatial parts of the baryon wave functions. Thereby we have found a consistent classification of all the low-lying resonances into flavor multiplets. To some cases of (strange) baryon resonances with yet undetermined intrinsic spin and parity we could attribute specific \(J^P\) values. Our ordering of
CONCLUSION AND OUTLOOK

states into flavor singlets, octets, and decuplets specifies the members of the multiplets in a way similar to the classification scheme recently published in Ref. [99], with the only exception of the $\Lambda(1810)$, which is a singlet (and not an octet) in our case.

By the present work we have reached a first level of fully relativistic investigations of mesonic baryon resonance decays. Many problems still remain open. First of all, the decay operator has been assumed in the simplest form possible. Certainly, the point-form spectator model does not only represent a one-body operator – as it effectively contains also many-body contributions – but explicit many-body parts have still to be implemented. In this context, one should also aim at a derivation of the vertex for meson creation on a microscopic basis. So far the meson emission has been treated as point-like.

Our investigations also hint to possible shortcomings in the baryon resonance wave functions. The constituent quark models employed here rely on configurations of three constituent quarks $\{QQQ\}$ only. Thereby the resonances are described as excited bound states (with zero widths) rather than as realistic resonance states corresponding to poles in the complex energy plane. In future investigations one should certainly take into account effects from additional degrees of freedom beyond $\{QQQ\}$. This could be achieved in different ways, e.g., by effective methods or by setting up an explicit multi-channel relativistic theory that is able to treat in addition such configurations like $\{QQQ\pi\}$, $\{QQQ\eta\}$, $\{QQQK\}$, $\{QQQQ\}$ etc.
CONCLUSION AND OUTLOOK
Appendix A

Analysis of the Baryon Wave Function

In Chapter 2 we found appropriate expressions for the full symmetrized basis functions \( \psi^S_\alpha \) expressed with a specific set of Jacobi coordinates (2.40). Here we want to find explicit expressions of its spatial, spin, and flavor parts ready for use with the SVM. Constructing the particular parts, an important observation is that the full basis function has to satisfy the proper symmetry conditions. Here we first restrict ourselves to the spatial, spin, and flavor parts of \( \psi^{(k)}_\alpha (\xi^{(k)}_k, \eta^{(k)}_k) \), the full basis function of configuration \((k)\) in the Jacobi partition \(k\). This means, that the subsystem of the particles \(p\) and \(q\) takes up a prominent position due to the specific symmetry required between these two particles. Thus, particle \(k\) can be considered as the "different" one (see also the middle graphic of Fig. 2.3 in Chapter 2). It should be noted, that in the light and strange sector at most one particle can be different assuming that the \(u\) and \(d\) quarks are treated to be equal, i.e. the quark masses coincide \(m_u = m_d\).

In addition, we then show, how to transform the different parts of the basis functions into other Jacobi partitions. We will also perform a Fourier transformation of the Jacobi coordinates in configuration space to the associated momenta in momentum space. These are necessary steps towards the calculation of the decay widths.

A.1 The Spatial Part

For the derivation of the spatial part of the full basis function, we express the internal motion of the three-quark system\(^1\) in Jacobi coordinates of partition \(k\) defined in Eq. (2.37). The spatial part of \(\psi^{(k)}_\alpha (\xi^{(k)}_k, \eta^{(k)}_k)\) then

\(^1\)The center-of-mass motion is already separated off.
ANALYSIS OF THE BARYON WAVE FUNCTION

contains functions of the form

\[ \varphi_{(\beta, \gamma, \delta, \nu, n, \lambda, l, M_L)}(\xi_k^{(k)}, \eta_k^{(k)}) = \left( \xi_k^{(k)} \right)^{2n+\lambda} \left( \eta_k^{(k)} \right)^{2n+l} \]

\[ \times \exp \left[ -\beta \left( \xi_k^{(k)} \right)^2 - \delta \left( \eta_k^{(k)} \right)^2 + \gamma \xi_k^{(k)} \cdot \eta_k^{(k)} \right] \]

\[ \times \gamma_{LM_L}^{LM}(\xi_k^{(k)}, \eta_k^{(k)}), \]  

(A.1)

where \( \xi_k^{(k)} \) and \( \eta_k^{(k)} \) are the absolute values and \( \tilde{\xi}_k^{(k)} \) and \( \tilde{\eta}_k^{(k)} \) the directions determining the angles of the set of Jacobi coordinates \((\xi_k^{(k)}, \eta_k^{(k)})\), respectively. The last factor of Eq. (A.1) represents the so-called bipolar spherical harmonics and can further be expressed by means of spherical harmonics \( Y_{lm} \) and Clebsch-Gordan coefficients \( C_{\lambda\mu\nu\lambda_l}^{LM} \), which explicitly results in

\[ \gamma_{LM_L}^{LM}(\xi_k^{(k)}, \eta_k^{(k)}) = \left\{ Y_{\lambda}(\tilde{\xi}_k^{(k)}) \otimes Y_{\lambda}(\tilde{\eta}_k^{(k)}) \right\}_{LM_L} = \sum_{\mu, m} C_{\lambda\mu\nu\lambda_l}^{LM} Y_{\lambda}(\xi_k^{(k)}) Y_{lm}(\eta_k^{(k)}). \]  

(A.2)

In order to specify the numerous parameters of the spatial basis function, we distinguish between continuous and discrete ones. To the former class belong the parameters \( \beta, \gamma, \) and \( \delta \) that may pick up values out of a continuous range, whereas to the latter class lie the parameters \( \nu, n, \lambda, l, L, \) and \( M_L \) that can assume discrete values. All of these parameters play the role of variational parameters in the SVM. It should be mentioned that the parameters \( \beta \) and \( \delta \) have to be positive to guarantee that the basis state can be normalized. Furthermore, a constraint demands that the inequality \( \beta \delta > \frac{2}{\pi^2} \) is satisfied. The parameters \( \nu \) and \( n \) are nonnegative integer numbers, which determine the properties of the basis function at small distances. They may become important in order to find a good description of higher radial excitations. Our interest lies mainly in ground states and low-lying baryon resonances, therefore we postulate them to be zero throughout the investigations done in this work. The orbital angular momenta \( \lambda \) and \( l \) with their respective projections \( \mu \) and \( m \) correspond to the Jacobi coordinates \( \xi_k^{(k)} \) and \( \eta_k^{(k)} \), respectively. Their combination leads to the total orbital angular momentum \( L \) with projection \( M_L \). The parameters \( \lambda \) and \( l \) also determine the parity \( P \) of the basis function

\[ P = (-1)^{\lambda+l}. \]  

(A.3)

Now we want to show the symmetry conditions of the spatial part of the full basis function. First we take a look at the properties of the function \( \varphi_{(\beta, \gamma, \delta, \nu, n, \lambda, l, M_L)}(\xi_k^{(k)}, \eta_k^{(k)}) \) considered for the case that the basis function of configuration \( k \) is expressed in Jacobi coordinates of partition \( k \). With

\[ ^2 \text{Notation according to Ref. [77].} \]
ANALYSIS OF THE BARYON WAVE FUNCTION

respect to the exchange of particles $p$ and $q$ we find

$$
\xi_{k}^{(k)} \rightarrow -\xi_{k}^{(k)} \\
\eta_{k}^{(k)} \rightarrow \eta_{k}^{(k)} \\
\mathcal{Y}^{LM}_{\lambda} (\xi_{k}^{(k)}, \eta_{k}^{(k)}) \rightarrow (-1)^{\lambda} \mathcal{Y}^{LM}_{\lambda} (\xi_{k}^{(k)}, \eta_{k}^{(k)}) \\
\exp(\gamma \xi_{k}^{(k)} \cdot \eta_{k}^{(k)}) \rightarrow \exp(-\gamma \xi_{k}^{(k)} \cdot \eta_{k}^{(k)}). \quad (A.4)
$$

Under the assumption$^{3}$ $\gamma = 0$, the effect of the interchange of particles $p$ and $q$ can be written as

$$
\varphi^{(k)}_{(\beta, \gamma, \delta, \nu, n, \lambda, l, L, M)} (\xi_{k}^{(k)}, \eta_{k}^{(k)}) \rightarrow (-1)^{\lambda} \varphi^{(k)}_{(\beta, \gamma, \delta, \nu, n, \lambda, l, L, M)} (\xi_{k}^{(k)}, \eta_{k}^{(k)}), \quad (A.5)
$$
i.e. the spatial part is symmetric for an even and antisymmetric for an odd $\lambda$. Setting $\gamma = 0$ does not mean any restriction in the flexibility of the test functions, since through the transformation of Jacobi coordinates from $(\xi_{p}^{(k)}, \eta_{p}^{(k)}) \rightarrow (\xi_{k}^{(k)}, \eta_{k}^{(k)})$ and $(\xi_{q}^{(k)}, \eta_{q}^{(k)}) \rightarrow (\xi_{k}^{(k)}, \eta_{k}^{(k)})$ the correlation terms are recovered. Eventually, we have to consider the other two partitions with respect to the exchange of particles $p$ and $q$ (given in the specific configuration $(k)$). In this case the exchange of particles $p$ and $q$ yields

$$
\xi_{p}^{(k)} \rightarrow -\xi_{p}^{(k)} \\
\eta_{p}^{(k)} \rightarrow \eta_{q}^{(k)} \\
\xi_{q}^{(k)} \rightarrow -\xi_{p}^{(k)} \\
\eta_{q}^{(k)} \rightarrow \eta_{p}^{(k)},
$$

which clearly shows no definite symmetry as the coordinates in partition $p$ go over into coordinates of partition $p$ and vice versa. Similarly, the bipolar spherical harmonics act under the exchange in the following way

$$
\mathcal{Y}^{LM}_{\lambda} (\xi_{p}^{(k)}, \eta_{p}^{(k)}) \rightarrow (-1)^{\lambda} \mathcal{Y}^{LM}_{\lambda} (\xi_{q}^{(k)}, \eta_{q}^{(k)}) \\
\exp(\gamma \xi_{p}^{(k)} \cdot \eta_{p}^{(k)}) \rightarrow \exp(-\gamma \xi_{q}^{(k)} \cdot \eta_{q}^{(k)}) \\
\mathcal{Y}^{LM}_{\lambda} (\xi_{q}^{(k)}, \eta_{q}^{(k)}) \rightarrow (-1)^{\lambda} \mathcal{Y}^{LM}_{\lambda} (\xi_{p}^{(k)}, \eta_{p}^{(k)}) \\
\exp(\gamma \xi_{q}^{(k)} \cdot \eta_{q}^{(k)}) \rightarrow \exp(-\gamma \xi_{p}^{(k)} \cdot \eta_{p}^{(k)}). \quad (A.6)
$$

Demanding $\gamma$ to be zero, a wave function with a definite symmetry regarding the exchange of $p$ and $q$ can be constructed by plus-minus combinations

$$
\varphi^{(k)}_{(\beta, \gamma, \delta, \nu, n, \lambda, l, L, M)} (\xi_{p}^{(k)}, \eta_{p}^{(k)}) = \pm \varphi^{(k)}_{(\beta, \gamma, \delta, \nu, n, \lambda, l, L, M)} (\xi_{q}^{(k)}, \eta_{q}^{(k)}). \quad (A.7)
$$

In case of a plus combination the basis function is symmetric for $(-1)^{\lambda} = 1$, i.e. $\{ (-1)^{\lambda} = 1; + \}$, and in case of the minus combination for $(-1)^{\lambda} = -1$, i.e. $\{ (-1)^{\lambda} = -1; - \}$.

$^{3}$The parameter $\gamma$ corresponds to the correlation term in the exponent of the Gaussian.
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Analogously, also an antisymmetric wave function with respect to the exchange of \( p \) and \( q \) can be constructed by the plus-minus combinations given in Eq. (A.7). In that case, for the plus combination one has \((-1)^{\lambda} = -1\), i.e. \( \{( -1)^{\lambda} = -1; + \} \), and for the minus combination \((-1)^{\lambda} = 1\), i.e. \( \{( -1)^{\lambda} = 1; - \} \), respectively.

A.2 The Spin Part

The constituent quarks of our model are spin-\( \frac{1}{2} \) particles. In order to guarantee the proper symmetries of the spin part of the wave function we again consider first the case, where configuration and partition coincide. Given a specific configuration \(( k )\), one first couples the single-particle intrinsic spins of the pair \( p \) and \( q \) (in partition \( k \)) to the spin \( s \) with its projection \( m \):

\[
\chi_{sm}(p, q) = \left\{ \chi_{\frac{1}{2}}(p) \otimes \chi_{\frac{1}{2}}(q) \right\}_{sm},
\]

(A.8)

Then one adds the intrinsic spin of the third particle \( k \)

\[
\chi^{(k)}(s,SM_{S}),k = \left\{ \chi_{sm}(p, q) \otimes \chi_{\frac{1}{2}}(k) \right\}_{SM_{S}}
\]

(A.9)

with \( S \) and \( M_{S} \) the total spin and its projection, respectively. The spinor is denoted by \( \chi_{\frac{1}{2}}(i) \) for particle \( i \) (for \( i = 1, 2, 3 \)). Similarly to the spatial part, the spins \( s \) and \( S \) are discrete variational parameters in the SVM.

For the coupling of three spin-\( \frac{1}{2} \) particles there exist only three possibilities, namely \(( s, S ) = (0, \frac{1}{2} ), (1, -\frac{1}{2} ), \text{and } (1, \frac{1}{2} )\) in accordance with the mixed-antisymmetric, mixed-symmetric and totally symmetric irreducible representations of \( SU(2)_{S} \). Under the exchange of the particles \( p \) and \( q \) we find for the spin wave function \( \chi^{(k)}(s,SM_{S}),k \) a behavior quite similar to the spatial part before, namely

\[
\chi^{(k)}(s,SM_{S}),k \rightarrow (-1)^{s+1} \chi^{(k)}(s,SM_{S}),k
\]

(A.10)

For partitions \( p \) and \( q \) the symmetry properties are given by

\[
\chi^{(k)}(s,SM_{S}),p \rightarrow (-1)^{s+1} \chi^{(k)}(s,SM_{S}),q'
\]

\[
\chi^{(k)}(s,SM_{S}),q \rightarrow (-1)^{s+1} \chi^{(k)}(s,SM_{S}),p'
\]

(A.11)

Again, in order to guarantee a proper symmetry between \( p \) and \( q \) one constructs plus-minus combinations

\[
\chi^{(k)}(s,SM_{S}),p \pm \chi^{(k)}(s,SM_{S}),q
\]

(A.12)

Satisfying the requirements \( \{ (-1)^{s+1} = -1; - \} \) and \( \{ (-1)^{s+1} = 1; + \} \), these combinations are symmetric with respect to the exchange of particles \( p \) and \( q \). For \( \{ (-1)^{s+1} = -1; + \} \) and \( \{ (-1)^{s+1} = 1; - \} \) we obtain antisymmetric combinations.
A.3 The Flavor Part

The flavor of the constituent quarks is properly taken into account by applying $f_1 = u$, $f_2 = d$ and $f_3 = s$ in the flavor wave functions

$$
\phi_F^{(k)} = \sum_{i_1, i_2, i_3 = 1}^{3} \alpha_F^{(i_1 i_2 i_3)} |f_{i_1} f_{i_2} f_{i_3}|,
$$

(A.13)

with $\alpha_F^{(i_1 i_2 i_3)}$ being the coupling coefficients. The superscript $(i_1 i_2 i_3)_k$ denotes a cyclic permutation of $i_1 i_2 i_3$, where $i_k$ is at the first position, i.e. explicitly:

$$(i_1 i_2 i_3)_1 = i_1 i_2 i_3, \quad (i_1 i_2 i_3)_2 = i_2 i_3 i_1, \quad (i_1 i_2 i_3)_3 = i_3 i_1 i_2.$$

(A.14)

In order to describe the wave function of all light and strange baryons in an appropriate way, altogether 15 flavor basis functions are necessary. In Table A.1 we list them for the specific configuration $(k = 1)$. The flavor basis functions of the other two configurations $(k = 2, 3)$ can be obtained by a cyclic permutation. The (discrete) parameters determining the flavor basis functions $\phi_F^{(k)}$ are given by the hypercharge $Y$ and total isospin $T$ with $M_T$ its projection. $P_F$ specifies the symmetry with respect to the exchange of particles $p$ and $q$, where $\phi_F^{(k)}$ is symmetric for $P_F = 0$ and antisymmetric for $P_F = 1$, respectively.

<table>
<thead>
<tr>
<th>$F$</th>
<th>Baryon</th>
<th>$\phi_F^{(1)}$</th>
<th>$Y$</th>
<th>$T, M_T$</th>
<th>$P_F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$p$</td>
<td>$\frac{1}{\sqrt{2}}(uud - udu)$</td>
<td>1</td>
<td>$\frac{1}{2}, \frac{1}{2}$</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>$p$</td>
<td>$-\frac{1}{\sqrt{2}}(uud + udu - 2duu)$</td>
<td>1</td>
<td>$\frac{1}{2}, \frac{1}{2}$</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>$n$</td>
<td>$\frac{1}{\sqrt{6}}(dud - ddu)$</td>
<td>1</td>
<td>$\frac{1}{2}, -\frac{1}{2}$</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>$n$</td>
<td>$\frac{1}{\sqrt{6}}(dud + ddu - 2udd)$</td>
<td>1</td>
<td>$\frac{1}{2}, -\frac{1}{2}$</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>$\Delta^{++}$</td>
<td>uu</td>
<td>1</td>
<td>$\frac{3}{2}, \frac{3}{2}$</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>$\Delta^{+}$</td>
<td>$\frac{1}{\sqrt{3}}(uud + udu + uu)$</td>
<td>1</td>
<td>$\frac{1}{2}, \frac{1}{2}$</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>$\Delta^{0}$</td>
<td>$\frac{1}{\sqrt{3}}(udd + dud + ddu)$</td>
<td>1</td>
<td>$\frac{3}{2}, -\frac{1}{2}$</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>$\Delta^{-}$</td>
<td>ddd</td>
<td>1</td>
<td>$\frac{3}{2}, -\frac{3}{2}$</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>$\Lambda$</td>
<td>$\frac{1}{\sqrt{2}}(sud - sdu)$</td>
<td>0</td>
<td>$0, 0$</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>$\Sigma^{+}$</td>
<td>suu</td>
<td>0</td>
<td>1, 1</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>$\Sigma^{0}$</td>
<td>$\frac{1}{\sqrt{2}}(sud + sdu)$</td>
<td>0</td>
<td>1, 0</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>$\Sigma^{-}$</td>
<td>sdd</td>
<td>0</td>
<td>1, -1</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>$\Xi^{0}$</td>
<td>uss</td>
<td>-1</td>
<td>$\frac{1}{2}, \frac{1}{2}$</td>
<td>0</td>
</tr>
<tr>
<td>14</td>
<td>$\Xi^{-}$</td>
<td>dss</td>
<td>-1</td>
<td>$\frac{1}{2}, -\frac{1}{2}$</td>
<td>0</td>
</tr>
<tr>
<td>15</td>
<td>$\Omega$</td>
<td>sss</td>
<td>-2</td>
<td>0, 0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table A.1: Flavor basis functions for light and strange baryons for the specific configuration $(k = 1)$, with $Y$ the hypercharge, $T$ and $M_T$ the isospin and its projection. $P_F$ specifies the symmetry with respect to the exchange of particles $p$ and $q$, where $\phi_F^{(k)}$ is symmetric for $P_F = 0$ and antisymmetric for $P_F = 1$, respectively.

basis functions $\phi_F^{(k)}$ are given by the hypercharge $Y$ and total isospin $T$ with
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its projection $M_T$. In addition one introduces a parameter $P_F$ in order to specify the symmetry with respect to the exchange of particles $p$ and $q$. The states within each isospin multiplet (e.g., $\Delta^{++}$, $\Delta^+$, $\Delta^0$, $\Delta^-$) are degenerate, thus one only needs to calculate one representative of each multiplet.

The baryons $N$, $\Delta$, and $\Omega$ are made up of constituent quarks with equal masses, respectively. Thus, their flavor basis functions are simply states of the irreducible mixed-symmetric and mixed-antisymmetric octet and totally symmetric decuplet representations of $SU(3)_F$ (see, e.g., Ref. [101]).

In the case of the baryons $\Lambda$, $\Sigma$, and $\Xi$ one of the three constituent quarks has a different mass. In order to construct a totally symmetric basis function $\psi^S_\alpha$, we proceed in a way analogous to the spatial and spin parts above, i.e., we first construct three basis functions corresponding to three different configurations and then add them up. Now, we put the flavor of the "different" quark at position $k$ corresponding to a specific configuration. The other two constituent quarks localized at $p$ and $q$ have the same mass. Then we combine the flavor wave function with the space-spin part of the wave function, which depends on the Jacobi coordinates (containing explicitly the masses of the particles). Here we have to assure that the proper quark-masses for each configuration are employed.

A.3.1 The Role of Different Quark Masses

Since in the baryons $N$, $\Delta$, and $\Omega$ all three quarks have equal masses, each of the quarks can appear at position $k$ in the flavor basis state of configuration $(k)$. Consequently, in this case it is sufficient to employ only zero-type basis functions.

The situation is different in case of $\Lambda$, $\Sigma$, and $\Xi$. Assuming configuration $(k)$ the quark with the different mass is fixed to the position $k$. As a consequence one has to design artificial flavor basis functions starting from the irreducible representations of $SU(3)_F$, complying with this requirement. Strictly speaking, each of the baryons $\Lambda$, $\Sigma$, and $\Xi$ appears in three representations of $SU(3)_F$. This reduces to just one general flavor basis function in each case. If we only used zero-type basis functions, we would have a space-spin part of the basis function given only in the partition where the two equally heavy quarks form a pair. In order not to lose correlation terms, it is necessary to employ also plus and minus basis functions.

\footnote{As an example, the $\Xi^+$ occurs in the two mixed-symmetric and -antisymmetric as well as in the totally symmetric representation of $SU(3)_F$.}

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A.4 Transformation into a Different Partition

For the calculations done in this work we have to exploit matrix elements of the form

$$
\langle \Psi_{XSF}' | D | \Psi_{XSF} \rangle = \sum_{i,j} c_i^* c_j \langle \psi_{\alpha_i}' | D | \psi_{\alpha_j}' \rangle = \sum_{i,j} \sum_{k,p=1}^3 c_i^* c_j \langle \psi_{\alpha_i}'^{(k)} | D | \psi_{\alpha_j}'^{(p)} \rangle,
$$

(A.15)

where $|\Psi_{XSF}'\rangle$ and $|\Psi_{XSF}\rangle$ are the final and initial baryon states, respectively, and $D$ represents a general operator. We can see that the matrix elements of an operator $D$ between two baryon states can be simplified to the calculation of the matrix elements of $D$ between basis states of two (possibly different) configurations ($k$) and ($p$). Clearly, in the sum over all possible configurations there occur matrix elements between basis functions with space-spin parts given in different partitions. These matrix elements can only be calculated, if we transform the basis states into coordinates of the same partition. The transformation of the basis functions leads to linear combinations of basis functions of the same type as the original ones. Finally, one will end up with the calculation of matrix elements between basis functions expressed by coordinates of one single partition.

A.4.1 Transformation of the Spatial Part

Given a configuration ($k$), we consider here only the coordinates and quark masses belonging to this specific configuration. The linear transformation of a pair of Jacobi coordinates can then be written as

$$
\begin{pmatrix}
\xi_{(k)}^i \\
\eta_{(k)}^i
\end{pmatrix} = A_{kq} \begin{pmatrix}
\xi_{(q)}^i \\
\eta_{(q)}^i
\end{pmatrix},
$$

(A.16)

The matrix $A_{kq}$ is defined by

$$
A_{kq} = \begin{pmatrix}
-m_p m_q & -P(kpq) \\
P(kpq) m_q & m_p
\end{pmatrix},
$$

(A.17)

with $P(kpq) = 1$ representing an even and $P(kpq) = -1$ an odd permutation ($kpq$) of (123), and $\mu_k$, $M_k$ being the reduced masses:

$$
\mu_k = \frac{m_p m_q}{m_p + m_q},
$$

$$
M_k = \frac{m_k(m_p + m_q)}{m_k + m_p + m_q}.
$$

(A.18)

The exponent of the Gaussian does not change its form under the transformation into another partition. One can show that

$$
-\beta (\xi_{(k)}^k)^2 - \delta (\eta_{(k)}^k)^2 + \gamma \xi_{(k)}^k \cdot \eta_{(k)}^k = -\beta_{kq} (\xi_{(q)}^{(k)})^2 - \delta_{kq} (\eta_{(q)}^{(k)})^2 + \gamma_{kq} \xi_{(q)}^{(k)} \cdot \eta_{(q)}^{(k)},
$$

(A.19)

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where the new parameters are defined by applying the transformation matrix

\[
\beta_{kq} = [(A_{kq})_{11}]^2 \beta + [(A_{kq})_{21}]^2 \delta - (A_{kq})_{11} (A_{kq})_{21} \gamma,
\]

\[
\delta_{kq} = [(A_{kq})_{12}]^2 \beta + [(A_{kq})_{22}]^2 \delta - (A_{kq})_{12} (A_{kq})_{22} \gamma,
\]

\[
\gamma_{kq} = -2 (A_{kq})_{11} (A_{kq})_{12} \beta - 2 (A_{kq})_{21} (A_{kq})_{22} \delta + [(A_{kq})_{11} (A_{kq})_{22} + (A_{kq})_{12} (A_{kq})_{21}] \gamma.
\]  

(A.20)

Here we can see that a correlation term \( \gamma_{kq} \) appears in the Gaussian even if \( \gamma = 0 \) for the coordinates of the original partition. The other terms of the spatial part of the basis state are transformed by applying the identity [77]

\[
|\xi + \eta\rangle^{2N+L} Y_{LM}(\xi + \eta) = \sum_{n_1 n_2 l_1 l_2} D_{n_L}^{n_1 l_1 n_2 l_2} \xi^{2n_1 + l_1} \eta^{2n_2 + l_2} Y_{l_1 l_2}^{LM}(\xi, \eta). \quad (A.21)
\]

The summation indices are constrained by \( 2n_1 + 2n_2 + l_1 + l_2 = 2N + L \) and the term \( D_{n_L}^{n_1 l_1 n_2 l_2} \) is given by

\[
D_{n_L}^{n_1 l_1 n_2 l_2} = \frac{B_{n_1 l_1} B_{n_2 l_2}}{B_{n_L}} \frac{(2N + L)!}{(2n_1 + l_1)!(2n_2 + l_2)!} C_{l_1 l_2}^{L} \quad (A.22)
\]

with

\[
B_{n_L} = \frac{4\pi(2n + l)!}{2^{n!}(2n + 2l + 1)!}
\]

\[
C_{l_1 l_2}^{L} = \sqrt{\frac{(2l_1 + 1)(2l_2 + 1)}{4\pi(2L + 1)}} C_{l_1 l_2}^{L0}. \quad (A.23)
\]

To avoid \( C_{l_1 l_2}^{L} = 0 \), the indices \( l_1, l_2, \) and \( L \) have to build a triangle further satisfying the demand that \((-1)^{l_1 + l_2 + L} = 1\). The detailed derivation can be found in Ref. [55].

Expressing the complete spatial basis function of partition \( k \) as a linear combination of functions given in partition \( q \) one obtains

\[
(\xi_{k}^{(k)})^{2\nu + \lambda}(\eta_{k}^{(k)})^{2n + l} Y_{LM}^{\nu \lambda}(\xi_{k}^{(k)}, \eta_{k}^{(k)})
= \sum D_{n_L}^{n_1 l_1 n_2 l_2} (A_{kq})_{11}^{2n_1 + l_1} [(A_{kq})_{12}]^{2n_1 + l_1} [(A_{kq})_{21}]^{2n_1 + l_1}
\times [(A_{kq})_{22}]^{2n_1 + \lambda_1} (\xi_{q}^{(k)})^{2\nu + \lambda_1 + 2n_1 + l_1} (\eta_{q}^{(k)})^{2\nu_1 + \lambda + 2n_1 + l_1} + \lambda_1 + 2n_1 + l_1
\times Y_{l_1 l_2}^{\nu \lambda}(\xi_{k}^{(k)}, \eta_{k}^{(k)}) Y_{LM}^{\nu \lambda}(\xi_{q}^{(k)}, \eta_{q}^{(k)})
= \sum B_{n_L}^{n_1 l_1 n_2 l_2} (A_{kq})_{11}^{2n_1 + l_1} [(A_{kq})_{12}]^{2n_1 + l_1} [(A_{kq})_{21}]^{2n_1 + l_1}
\times (\xi_{k}^{(k)})^{2\nu + \lambda} (\eta_{k}^{(k)})^{2n + l} Y_{LM}^{\nu \lambda}(\xi_{k}^{(k)}, \eta_{k}^{(k)}), \quad (A.24)
\]

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where the summation indices have to satisfy the following conditions

\[ 2\nu_1 + \lambda_1 + 2\nu_1' + \lambda_1' = 2\nu + \lambda, \]
\[ 2n_1 + l_1 + 2n_1' + l_1' = 2n + l, \]  
(A.25)

\[ (-1)^{\lambda_1 + \lambda_1'} + 1 = 1, \quad (-1)^{l_1' + l} = 1, \]
\[ (-1)^{\lambda_1 + l_1 + \kappa_1} = 1, \quad (-1)^{\lambda_1' + l_1' + \kappa_1'} = 1. \]  
(A.26)

In addition a triangle has to be built by each of the triplets \((\lambda_1, \lambda_1', \lambda), (l_1, l_1', l), (\lambda_1, l_1, \kappa_1), \) and \((\lambda_1', l_1', \kappa_1').\) These conditions guarantee that the sum in Eq. (A.24) is finite. Using Eq. (A.26) one gets

\[ (-1)^{\lambda + l} = (-1)^{\kappa_1 + \kappa_1'}, \]  
(A.27)

which can be identified with the parity, independent of the coordinates used to describe the state.

A.4.2 Transformation of the Spin Part

For the spin part of a basis function given in configuration \((k)\) one proceeds in the same way as for the spatial part above. Again, the transformation into an other partition \(k\) results in a linear combination of spin basis functions of the same type as the original ones given in partition \(q\). Here the transformation consists just in a recoupling of the spins of the three constituent quarks given in any configuration \((k)\), where the transformation does not depend on the specific configuration \((k)\). It is advantageous to express the spin wave function (A.9) as the overlap of an arbitrary three-particle spin state \(|\chi^{(k)}\rangle_k\) with the spin part of the baryon state \(|\langle(s_1, s_2)SM_S|\rangle_k\)

\[ \chi^{(k)}_{(s_1, s_2), k} = \langle(s_1, s_2)SM_S | \chi^{(k)}\rangle_k. \]  
(A.28)

Then one can easily see that

\[ \langle(s_1, s_2)SM_S | \chi^{(k)}\rangle_k = \sum_{s_1=0,1} \langle(s_1, s_2)SM_S | \chi^{(k)}\rangle_q, \]  
(A.29)

where

\[ \langle(s_1, s_2)SM_S | \chi^{(k)}\rangle_q = P(kq)\sqrt{(2s_1 + 1)(2s_1' + 1)} \left\{ \begin{array}{ccc} \frac{1}{2} & \frac{1}{2} & s_1' \\ \frac{1}{2} & S & s_1 \end{array} \right\}. \]  
(A.30)
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with

\[
P(kq) = \begin{cases} 
(-1)^s, & (kq) = (12), (23), (31) \\
(-1)^{s^1}, & (kq) = (21), (32), (31) 
\end{cases} \tag{A.31}
\]

displaying the phase factor. In Eq. (A.30) there occur the so-called 6j-symbols in the notation \(\{a \ b \ c \ d \ e \ f\}\) (see Ref. [77]).

In this work we will calculate matrix elements of a decay operator \(D\) in a spectator model, where only one of the quarks couples to the generated meson being an external particle, and the other two quarks are spectators. It is possible to choose quark 1 as the active quark. Consequently, we have to transform all spin matrix elements according to partition 1. With respect to the spin, we therefore deal with baryon states \(|(s_1)^2SM_s\rangle\), where quarks 2 and 3 couple to an intermediate spin \(s\), and only then the spin-1 of quark 1 is added to give a total spin \(S\). It is possible to decouple the total spin state using the row of Clebsch-Gordon coefficients:

\[
|(s_1^2SM_s)\rangle = \sum_{\mu_1, m} C^{SM_s}_{sm_{\mu_1}} |sm\rangle |\frac{1}{2}\mu_1\rangle
\]

\[
= \sum_{\mu_1, \mu_2, \mu_3} C^{SM_s}_{sm_{\mu_1}} C^{SM_s}_{\frac{1}{2}\mu_2 \frac{1}{2}\mu_3} |\frac{1}{2}\mu_1\rangle |\frac{1}{2}\mu_2\rangle |\frac{1}{2}\mu_3\rangle. \tag{A.32}
\]

Here we have to sum over all possible z-projections \(\mu_i\) of the spin-\(\frac{1}{2}\) quarks. Clearly, some terms vanish due to the properties of the Clebsch-Gordon coefficients, namely, that the constraints \(\mu_1 + m = M_S\) as well as \(\mu_2 + \mu_3 = m\) have to be fulfilled.

A.5 Transformation of the Baryon Wave Functions to Momentum Space

For the concrete calculation of the matrix elements of the decay operator in this thesis, the baryon wave functions created with the SVM in configuration space need to be transformed to momentum space. This is realized by transforming each individual basis function \(\psi_{\alpha}(k) (\xi^{(k)}_k, \eta^{(k)}_k)\) of the total baryon wave function.

In order to obtain a momentum-space representation of a function \(F(\xi^{(k)}_k, \eta^{(k)}_k) = f_1(\xi^{(k)}_k) \cdot f_2(\eta^{(k)}_k)\) expressed through the specific set of Jacobi coordinates, where the configuration coincides with the partition, one

\[\text{See the discussion in Chapter 5.}\]
has to perform the Fourier transformations\(^6\)

\[
FT (f_1(\xi)) = f_1(p_\xi) = \frac{1}{(2\pi)^2} \int d^3\xi \; f_1(\xi) \; e^{-ip_\xi \cdot \xi},
\]

\[
FT (f_2(\eta)) = f_2(p_\eta) = \frac{1}{(2\pi)^2} \int d^3\xi \; f_2(\eta) \; e^{-ip_\eta \cdot \eta},
\]  \hspace{1cm} (A.33)

where we neglected the indices \(k\) and \((k)\) for simplicity. Here, the coordinates \(p_{\xi_k}\) and \(p_{\eta_k}\) are the conjugate variables in momentum space, i.e. the momenta corresponding to \(\xi_{k}\) and \(\eta_{k}\), respectively. With the assumption \(\gamma = 0\), the spatial basis function (A.1) becomes

\[
\varphi_{(\beta,\gamma,\delta,\nu,n,\lambda_1,LM_{x_1})}(\xi_{k}, \eta_{k}) = (\xi_{k})^{2\nu+\lambda} (\eta_{k})^{2n+l} \times \exp\left[-\beta \left(\xi_{k}\right)^2 - \delta \left(\eta_{k}\right)^2\right] Y_{\lambda l}(\xi_{k}, \eta_{k}).
\]  \hspace{1cm} (A.34)

Now we identify the functions \(f_1(\xi_{k})\) and \(f_2(\eta_{k})\) with the expressions

\[
f_1(\xi_{k}) = (\xi_{k})^{2\nu+\lambda} \exp\left[-\beta \left(\xi_{k}\right)^2\right] Y_{\lambda l}(\xi_{k}),
\]

\[
f_2(\eta_{k}) = (\eta_{k})^{2n+l} \exp\left[-\delta \left(\eta_{k}\right)^2\right] Y_{\lambda l}(\eta_{k}),
\]  \hspace{1cm} (A.35)

and perform the Fourier transformations of Eq. (A.33). The resulting functions have the following form:

\[
f_1(p_{\xi_k}) = \sum_{k_1=0}^{\nu} A_1 \left(p_{\xi_k}\right)^{2k_1+\lambda} \exp\left[-\frac{1}{4\beta} \left(p_{\xi_k}\right)^2\right] Y_{\lambda l}(p_{\xi_k}),
\]

\[
f_2(p_{\eta_k}) = \sum_{k_2=0}^{n} A_2 \left(p_{\eta_k}\right)^{2k_2+l} \exp\left[-\frac{1}{4\delta} \left(p_{\eta_k}\right)^2\right] Y_{\lambda l}(p_{\eta_k}),
\]  \hspace{1cm} (A.36)

with \(A_1\) and \(A_2\) given by

\[
A_1 = (-i)^{\lambda} (-1)^{k_1} \frac{\nu!}{k_1!(\nu-k_1)!} \frac{(2\beta)^{-(\nu+\lambda+k_1+\frac{3}{2})}}{(2(\nu+\lambda+1)-1)!} \frac{(2(\nu+\lambda+1)-1)!}{(2(k_1+\lambda+1) - 1)!},
\]

\[
A_2 = (-i)^{l} (-1)^{k_2} \frac{n!}{k_2!(n-k_2)!} \frac{(2\delta)^{-(n+l+k_2+\frac{3}{2})}}{(2(n+l+1)-1)!} \frac{(2(n+l+1)-1)!}{(2(k_2+l+1) - 1)!},
\]  \hspace{1cm} (A.37)

respectively. Analogously to the configuration space, \(p_{\xi_k}\) and \(p_{\eta_k}\) are the absolute values and \(\tilde{p}_{\xi_k}\) and \(\tilde{p}_{\eta_k}\) represent the angular dependence in the

---

\(^6\)Here we take the symmetric choice for the Fourier transformation.
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set of Jacobi momenta \((p^{(k)}_k, k^{(k)})\). The final expression for the momentum-
space test functions reads:

\[
\varphi^{(k)}_{\beta, \gamma, \delta, \nu, \eta, \lambda, \ell, \Lambda, \Lambda_L} (p^{(k)}_{\xi_k}, p^{(k)}_{\eta_k}) = \sum_{k_l=0}^{\nu} \sum_{k_{\ell}=0}^{n} A_1 A_2 \left( p^{(k)}_{\xi_k} \right)^{2k_1+\lambda} \left( p^{(k)}_{\eta_k} \right)^{2k_2+\ell} \\
\times \exp \left[ -\frac{1}{4\beta} \left( p^{(k)}_{\xi_k} \right)^2 \right] Y_{\lambda\mu} (p^{(k)}_{\xi_k}) \exp \left[ -\frac{1}{4\delta} \left( p^{(k)}_{\eta_k} \right)^2 \right] Y_{\lambda\mu} (p^{(k)}_{\xi_k}). \tag{A.38}
\]

With the assumption \(\nu = n = 0\) the expression (A.38) gets much simpler:

\[
\varphi^{(k)}_{\beta, \gamma, \delta, \nu, \eta, \lambda, \ell, \Lambda, \Lambda_L} (p^{(k)}_{\xi_k}, p^{(k)}_{\eta_k}) = \tilde{c} \left( p^{(k)}_{\xi_k} \right)^{\lambda} \left( p^{(k)}_{\eta_k} \right)^{\ell} \\
\times \exp \left[ -\frac{1}{4\beta} \left( p^{(k)}_{\xi_k} \right)^2 - \frac{1}{4\delta} \left( p^{(k)}_{\eta_k} \right)^2 \right] Y_{\lambda\mu} (p^{(k)}_{\xi_k}, p^{(k)}_{\xi_k}), \tag{A.39}
\]

with

\[
\tilde{c} = (-i)^{\lambda+\ell} (2\beta)^{-\lambda-\frac{3}{2}} (2\delta)^{-\ell-\frac{3}{2}}. \tag{A.40}
\]

One can easily recognize the original form of the spatial wave function in configuration space. Whenever we have to perform calculations in momentum space, we therefore substitute the parameters \(\beta\) and \(\delta\) used by the SVM in configuration space by the corresponding terms

\[
\beta \to \frac{1}{4\beta} \quad \text{and} \quad \delta \to \frac{1}{4\delta} \tag{A.41}
\]

belonging to the momentum-space basis functions. In addition, we have to substitute the linear parameters \(c_i\) of Eq. (2.34) by the product with a term \(\tilde{c}_i\) yielding \(c_i \to \tilde{c}_i c_i\). In Eq. (A.40) the term \((-i)^{\lambda+\ell}\) causes a changing of sign depending on the sum of \(\lambda + \ell\). The complex number \(i = e^{\frac{i\pi}{2}}\) is just a phase factor and can thus be suppressed. However, the proper implementation of the relative sign between different basis functions \(\psi^{S \alpha_i}_{\delta}\) is of utmost importance.

A.5.1 Transformation into a Different Partition

In analogy to the previous section we want to deduce here the rules for the transformation of the spatial part of the basis functions given in the momentum-space representation into a different partition. This is a necessary step for the calculation of various matrix elements between states given in two different partitions.

Here we are interested in the lowest-lying excitations, for which the variational parameters \(n\) and \(\nu\) can well be restricted to zero. Non-vanishing values of \(\nu\) and \(n\) may become important if one considers higher radial excitations.
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Postulating that the parameters $n$ and $\nu$ are set to zero, the spatial wave functions (A.1) and (A.39) (corresponding to the configuration-space and the momentum-space representation, respectively) have the same functional dependence on either the Jacobi coordinates or the associated momenta. Therefore one can apply here all transformation rules deduced in Section A.4, where we again start from the case where all momenta and quark masses are given in the specific configuration $(k)$.

However, for the linear transformation of a pair of Jacobi momenta\footnote{For simplicity we rename the momenta $p^{(k)}_k$ and $k^{(k)}_k$ corresponding to the Jacobi coordinates $\xi^{(k)}_k$ and $n^{(k)}_k$, respectively, in the following way: $p^{(k)}_{\xi_k} = p^{(k)}_k$, $p^{(k)}_{n_k} = k^{(k)}_k$.} $(p^{(k)}_k, k^{(k)}_k)$ one has to find the proper transformation matrix $\hat{A}_{kq}$, which is different from the one used for the transformation of a pair of Jacobi coordinates:

$$\begin{pmatrix} p^{(k)}_k \\ k^{(k)}_k \end{pmatrix} = \hat{A}_{kq} \begin{pmatrix} p^{(k)}_q \\ k^{(k)}_q \end{pmatrix}. \quad (A.42)$$

Analogously to the configuration space, one finds the transformation matrix in momentum space to be

$$\hat{A}_{kq} = \begin{pmatrix} -\frac{\mu_k}{m_p} & -P(kpq)\frac{\mu_q}{m_q} \\ P(kpq) & -\frac{\mu_q}{m_p} \end{pmatrix}. \quad (A.43)$$

This looks quite similar to the transformation matrix (A.17). The connection between $A$ and $\hat{A}$ is actually given by $\hat{A}_{kq} = A_{qk}^T$.

A.5.2 Analysis of the Basis Functions in Momentum Space

For the analysis of the spatial part of the basis functions expressed in the momentum-space representation we rewrite Eq. (A.39) using Eq. (A.2) and obtain

$$\varphi(\beta, \gamma=0, \delta, \lambda, L, M_L)(p, k) = \tilde{c} \, p^L \, k^l \exp \left[ -\frac{1}{4\beta} p^2 - \frac{1}{4\delta} k^2 \right] \times \sum_{\mu, m} C^{LM_L}_{\lambda \mu \lambda \mu} (\hat{\mu}) Y_{\lambda \mu}(\hat{k}),$$

which is the basis function in the original partition. Here, the indices $k$ and $(k)$ specifying the partition and the configuration of the Jacobi momenta $(p^{(k)}_k, k^{(k)}_k)$, respectively, are neglected for simplicity. Using spherical polar coordinates (see Eq. (C.61)) the expression for a basis function with $L = M_L = 0$ simplifies to

$$\varphi(\beta, \gamma=0, \delta, 0, 0, 0, 0)(p, k) = \tilde{c} \, \exp \left[ -\frac{1}{4\beta} p^2 - \frac{1}{4\delta} k^2 \right] Y_{00}(\hat{p}) Y_{00}(\hat{k}), \quad (A.45)$$

where the spherical harmonics $Y_{00}(\hat{p}) = Y_{00}(\hat{k}) = \frac{1}{\sqrt{4\pi}}$. A basis function with $L = 1$ can be constructed from different combinations of $\lambda$ and $l$ satisfying
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the triangular rule $|\lambda - l| \leq L \leq \lambda + l$. Here we consider the cases, where either $(\lambda = 1, l = 0)$ or $(\lambda = 0, l = 1)$. The former case leads to

$$\varphi(\beta, \gamma=0, 0, 1, 0, 1, 0, 1) (p, k) = \tilde{c} p \exp \left[ -\frac{1}{4\beta} p^2 - \frac{1}{4\delta} k^2 \right] \frac{1}{2} \sum_{\mu} C_{1\mu 00}^{1M_L} Y_{1\mu}(\hat{p}) Y_{00}(\hat{k})$$

whereas for the latter case one obtains

$$\varphi(\beta, \gamma=0, 0, 0, 1, 0, 0, 1) (p, k) = \tilde{c} k \exp \left[ -\frac{1}{4\beta} p^2 - \frac{1}{4\delta} k^2 \right] \frac{1}{2} \sum_{m} C_{001m}^{1M_L} Y_{00}(\hat{p}) Y_{1m}(\hat{k})$$

Obviously, the Clebsch-Gordan coefficients are simply 1. The z-projections $M_L$ can be 0, ±1, and the spherical harmonics relating to the Jacobi momentum $p$ are given by

$$Y_{1+1}(\hat{p}) = -\frac{1}{2} \sqrt{\frac{3}{2\pi}} \sin \theta_p e^{i\phi_p} = \sqrt{\frac{3}{4\pi}} \left( -\frac{1}{\sqrt{2\pi}} \left( \cos \phi_p + i \sin \phi_p \right) \right) \sin \theta_p,$$

$$Y_{10}(\hat{p}) = \frac{1}{2} \sqrt{\frac{3}{\pi}} \cos \theta_p,$$

$$Y_{1-1}(\hat{p}) = \frac{1}{2} \sqrt{\frac{3}{2\pi}} e^{-i\phi_p} = \sqrt{\frac{3}{4\pi}} \sin \theta_p \left( \frac{1}{\sqrt{2\pi}} \left( \cos \phi_p - i \sin \phi_p \right) \right) \sin \theta_p;$$

replacing $\hat{p}$ with $\hat{k}$ one obtains the spherical harmonics relating to the momentum $k$. So far we used the spherical polar coordinates defined in Eq. (C.61). It is advantageous to introduce the new set of coordinates

$$p^+ = -\frac{1}{\sqrt{2\pi}} (p_x + ip_y),$$

$$p^0 = p_z,$$

$$p^- = \frac{1}{\sqrt{2\pi}} (p_x - ip_y),$$

$$k^+ = -\frac{1}{\sqrt{2\pi}} (k_x + ik_y),$$

$$k^0 = k_z,$$

$$k^- = \frac{1}{\sqrt{2\pi}} (k_x - ik_y).$$

Then Eq. (A.46) corresponding to $(\lambda = 1, l = 0)$ becomes

$$\varphi(\beta, \gamma=0, 1, 0, 1, 0, 1) (p, k) = \tilde{c} p^+ \exp \left[ -\frac{1}{4\beta} p^2 - \frac{1}{4\delta} k^2 \right] \frac{\sqrt{3}}{4\pi}$$
ANALYSIS OF THE BARYON WAVE FUNCTION

for $L = 1, M_L = +1$,

$$\varphi_{(\beta, \gamma=0,6,1,10)}(p, k) = \tilde{c}_p^0 \exp \left[ -\frac{1}{4\beta} p^2 - \frac{1}{4\delta} k^2 \right] \sqrt{\frac{3}{4\pi}}$$

(A.51)

for $L = 1, M_L = 0$, and

$$\varphi_{(\beta, \gamma=0,6,1,1-1)}(p, k) = \tilde{c}_p^{-} \exp \left[ -\frac{1}{4\beta} p^2 - \frac{1}{4\delta} k^2 \right] \sqrt{\frac{3}{4\pi}}$$

(A.52)

for $L = 1, M_L = -1$. Similarly Eq. (A.47) corresponding to $(\lambda = 0, l = 1)$ gets

$$\varphi_{(\beta, \gamma=0,6,0,1,1+1)}(p, k) = \tilde{c}_k^{+} \exp \left[ -\frac{1}{4\beta} p^2 - \frac{1}{4\delta} k^2 \right] \sqrt{\frac{3}{4\pi}}$$

(A.53)

for $L = 1, M_L = +1$,

$$\varphi_{(\beta, \gamma=0,6,0,1,10)}(p, k) = \tilde{c}_k^{0} \exp \left[ -\frac{1}{4\beta} p^2 - \frac{1}{4\delta} k^2 \right] \sqrt{\frac{3}{4\pi}}$$

(A.54)

for $L = 1, M_L = 0$, and

$$\varphi_{(\beta, \gamma=0,6,0,1,1-1)}(p, k) = \tilde{c}_k^{-} \exp \left[ -\frac{1}{4\beta} p^2 - \frac{1}{4\delta} k^2 \right] \sqrt{\frac{3}{4\pi}}$$

(A.55)

for $L = 1, M_L = -1$.

Clearly, one can proceed in the same way with higher angular momenta $L$. However, the expressions get much more elaborate and for the actual calculations performed in this work we restrict the investigations to baryon wave functions with $L = 0, 1$.

A.6 Nomenclature of the Baryon Wave Function

The total baryon wave function introduced in Chapter 2 and extensively analysed in this appendix is used as an input for the calculation of transition matrix elements with the operator $\hat{D}_{\ell_d}^\mu_r$ (to be discussed in Chapter 5). As mentioned above, this computation is performed in momentum space. The total wave function of a baryon ground state or resonance produced in the center-of-momentum frame $|M, P = 0, \Sigma, M_{\Sigma}, T, M_T \rangle$ is thus represented as

$$\Psi_{M\Sigma M_{\Sigma}TM_T}(p, k; \mu_i)$$

(A.56)

with the normalization

$$\delta_{M'\Sigma} \delta_{M_{\Sigma}'} \delta_{M_T} \delta_{T'} \delta_{M_{\Sigma}TM_T} = \sum_{\mu_1 \mu_2 \mu_3} \int \frac{d^3p d^3k}{2\pi^3}$$

$$\Psi_{M'\Sigma M_{\Sigma}TM_T}^*(p, k; \mu_i) \Psi_{M\Sigma M_{\Sigma}TM_T}(p, k; \mu_i).$$

(A.57)
ANALYSIS OF THE BARYON WAVE FUNCTION

The $\mu_i$ (with $i = 1, 2, 3$) are the spin-projections of the individual quarks. Furthermore, $M$ denotes the mass of the specific baryon resonance, and $\Sigma$, $T$ represent the intrinsic spin and total isospin with corresponding $z$-projections $M_\Sigma$, $M_T$, respectively. In this notation we neglect for simplicity all other internal quantum numbers.

As the baryon wave function is originally expressed in the rest frame, one can introduce alternatively to the Jacobi momenta also internal quark momenta $k_i$ ($i = 1, 2, 3$) with

$$k_1 + k_2 + k_3 = 0. \quad (A.58)$$

If we consider, e.g., the Jacobi momenta of partition 1, the connection to these new variables is given as

$$p = \frac{m_3}{m_2 + m_3} k_2 - \frac{m_2}{m_2 + m_3} k_3,$$
$$k = k_1 = -k_2 - k_3, \quad (A.59)$$

where we neglected the indices specifying the configuration as well as the partition. In the other direction these relations read

$$k_2 = p - \frac{m_2}{m_2 + m_3} k,$$
$$k_3 = -p - \frac{m_3}{m_2 + m_3} k, \quad (A.60)$$

where $k_1$ is fixed due to Eq. (A.58). Consequently, the total baryon wave function can also be written as

$$\Psi_{M\Sigma M_T T M} (k_i; \mu_i) \quad (A.61)$$

with the constraint (A.58). The corresponding normalization condition is provided in Eq. (B.47).
Appendix B

Notations in Relativistic Quantum Mechanics

B.1 Metric of the Minkowski Space-Time

In this thesis we shall use the four-vector notation

\[ x^\mu = (x^0, x^1, x^2, x^3) = (t, x, y, z) \] (B.1)

for contravariant, and

\[ x_\mu = (x_0, x_1, x_2, x_3) = (t, -x) = (t, -x, -y, -z) \] (B.2)

for covariant four-vectors, respectively. The conversion is achieved through the metric tensor

\[ g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \] (B.3)

in the way

\[ x_\mu = g_{\mu\nu} x^\nu \] (B.4)

using the Einstein summation convention. Furthermore, we shall use latin indices to denote the spatial components and zero to denote the time component

\[ x^i = -x_i = (x, y, z), \quad x^0 = x_0 = t. \] (B.5)

The inner (scalar) product is then defined as

\[ x \cdot x = x^2 = x_\mu x^\mu = x_0^2 - \mathbf{x}^2. \] (B.6)

The four-momentum vector of a free particle is defined as

\[ p^\mu = (p^0, p^1, p^2, p^3) = (E, \mathbf{p}) \] ,

where \( E = \sqrt{m^2 + \mathbf{p}^2} \) corresponds to the positive-energy solution of the Dirac equation (see also (B.12)).
B.2 Pauli Spin Matrices

The Pauli spin matrices

\[
\sigma^1 = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\] (B.8)

build a set of \((2 \times 2)\) traceless Hermitian matrices. They do not commute; instead they satisfy the commutation relations

\[
[\sigma^i, \sigma^j] = 2i \epsilon_{ijk} \sigma^k, \quad (B.9)
\]

where \(i, j, k = 1, 2, 3; \epsilon_{123,231,312} = +1; \epsilon_{213,132,321} = -1\). This is known as the Lie algebra of the generators of \(SU(2)\), the group of the special unitary transformations. The \(\epsilon_{ijk}\) are the structure constants of this group. The \(\sigma\)-matrices possess a determinant \(\det \sigma^i = -1\) (for \(i = 1, 2, 3\)), and furthermore for the anti-commutator one has the relation

\[
\{\sigma^i, \sigma^j\} = 2\delta_{ij} \sigma^0, \quad (B.10)
\]

where \(\sigma^0 = 1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\) is the \((2 \times 2)\) unit matrix. The connection to the spin operator \(\hat{S}\) of the two-dimensional representation of \(SU(2)\) is given by \(\hat{S} = \frac{1}{2} \sigma\). Hence, states of spin-\(\frac{1}{2}\) particles are labelled by the eigenvalues of \(\hat{S}_3 = \frac{1}{2} \sigma_3\) and the Casimir operator is given by \(\hat{S}^2 = \frac{1}{2} \sigma^2 = S(S + 1)\), where \(S\) is the maximum eigenvalue of \(\hat{S}_3\). One finally finds the following expressions:

\[
\hat{S}_3 (\frac{1}{2}, \pm \frac{1}{2}) = \pm \frac{1}{2}, \quad \hat{S}_2 (\frac{1}{2}, \pm \frac{1}{2}) = S(S + 1) \frac{1}{2} \pm \frac{1}{2}. \quad (B.11)
\]

B.3 Dirac \(\gamma\)-Matrices

We start with the Dirac equation, which has the form

\[
i \frac{\partial \psi}{\partial t} = H_0 \psi = (\alpha \cdot p + \beta m) \psi \quad (B.12)
\]

with \(\psi\) the wave function, \(H_0\) the free Hamilton (Dirac) operator, and \(\hbar = c = 1\). In order to specify the matrices \(\alpha = (\alpha^1, \alpha^2, \alpha^3)\) and \(\beta\) the following constraints

\[
\beta^2 = (\alpha^i)^2 = 1 \quad \{\alpha^i, \alpha^j\} = 2\delta_{ij} \quad \{\alpha^i, \beta\} = 0
\]

\(\alpha^i = (\alpha^i)^\dagger\) and \(\beta^i = (\beta^i)^\dagger\) (B.13)
have to be fulfilled. Satisfying these requirements one finally obtains \((4 \times 4)\)-dimensional Hermitian matrices given by

\[
\alpha^i = \begin{pmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{pmatrix} \quad \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\]

(B.14)

with \(1\) the \((2 \times 2)\) unit matrix and \(\sigma^i (i = 1, 2, 3)\) the Pauli spin matrices discussed above. Now we can express the Dirac \(\gamma\)-matrices in terms of the traceless matrices \(\alpha^i, \beta\) as

\[
\gamma^0 = \beta \quad \gamma^i = \beta \alpha^i \quad \text{(for } i = 1, 2, 3),
\]

(B.15)

which is known as Pauli realisation\(^1\) and yields

\[
\gamma^0 = \begin{pmatrix} \sigma^0 & 0 \\ 0 & -\sigma^0 \end{pmatrix} \quad \gamma^1 = \begin{pmatrix} 0 & \sigma^1 \\ -\sigma^1 & 0 \end{pmatrix} \\
\gamma^2 = \begin{pmatrix} 0 & \sigma^2 \\ -\sigma^2 & 0 \end{pmatrix} \quad \gamma^3 = \begin{pmatrix} 0 & \sigma^3 \\ -\sigma^3 & 0 \end{pmatrix}
\]

The corresponding contravariant notation reads

\[
\gamma^\mu = (\gamma^0, \gamma^i) = (\gamma^0, \gamma^1, \gamma^2, \gamma^3).
\]

(B.16)

The \(\gamma\)-matrices have the following properties

\[
\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} 1 \quad \text{with } g^{\mu\nu} = \text{diag}(1, -1, -1, -1)
\]

\[
\gamma^0 = (\gamma^0)^\dagger \quad \text{Hermitian}
\]

\[
\gamma^i = -(\gamma^i)^\dagger \quad \text{Anti-Hermitian}
\]

\[
\overline{\gamma^\mu} = \gamma^0 (\gamma^\mu)^\dagger \gamma^0 = \gamma^\mu \quad \text{Dirac-adjoint}
\]

\[
\gamma^5 = i \gamma^0 \gamma^1 \gamma^2 \gamma^3 = \overline{\gamma^3} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{with } (\gamma^5)^2 = -1.
\]

(B.17)

The commutator relation is given as

\[
\frac{i}{2} [\gamma^\mu, \gamma^\nu] = \sigma^{\mu\nu} = i (\gamma^\mu \gamma^\nu - g^{\mu\nu} 1) = -\sigma^{\nu\mu}.
\]

(B.18)

In Eqs. (B.17) and (B.18) the indices \(\mu, \nu\) take the values \(\mu, \nu = 0, 1, 2, 3\). The covariant notation is obtained with the metric tensor \(g_{\mu\nu}\) through the relation

\[
\gamma_\mu = g_{\mu\nu} \gamma^\nu.
\]

(B.19)

Obviously one obtains that

\[
\gamma^0 = \gamma_0 \quad \gamma^i = -\gamma_i \quad (i = 1, 2, 3) \quad \gamma^5 = -\gamma_5.
\]

(B.20)

The frequently used inner product between the \(\gamma\)-matrices and a four-vector \(a_\mu\) is given by

\[
\gamma^\mu a_\mu = \gamma_\mu a^\mu = \gamma^0 a_0 - \gamma_\cdot a.
\]

(B.21)

\(^1\)Another useful realisation is called Weyl’s realisation.
NOTATIONS IN RELATIVISTIC QUANTUM MECHANICS

B.4 Dirac Spinor

With regard to the wave function $\psi$ we want to take a closer look at the Dirac spinor $u(p, \sigma)$ for the positive energy solution of the Dirac equation. It is given as four-vector

$$u(p, \sigma) = \sqrt{E + m} \left( \begin{array}{c} \chi_\sigma \\ \sigma \cdot p \frac{E}{E+m} \chi_\sigma \end{array} \right),$$

(B.22)

where $E = \sqrt{m^2 + p^2}$, and $\chi_\sigma$ is the two-component Pauli spinor. The Pauli spinors are normalised to

$$\chi_\sigma^\dagger \chi_\sigma = 1.$$  

(B.23)

For spin-$\frac{1}{2}$ particles with spin projections $\sigma = \pm \frac{1}{2}$ one has

$$\chi_{\sigma}^\dagger = \left( \begin{array}{c} 1 \\ 0 \end{array} \right) \quad \text{and} \quad \chi_{-\frac{1}{2}} = \left( \begin{array}{c} 0 \\ 1 \end{array} \right).$$

(B.24)

Consequently, for the normalisation of the Dirac spinor $u(p, \sigma)$ one obtains

$$u(p, \sigma') u(p, \sigma) = u^\dagger(p, \sigma') \gamma^0 u(p, \sigma) =$$

$$(E + m) \left( \begin{array}{cc} \chi_\sigma^\dagger & \chi_{\sigma'}^\dagger \frac{\sigma \cdot p}{E+m} \chi_\sigma \end{array} \right) \left( \begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array} \right) \left( \begin{array}{c} \chi_\sigma \\ \frac{\sigma \cdot p}{E+m} \chi_\sigma \end{array} \right) = \delta_{\sigma\sigma'} 2m,$$

(B.25)

where we used the relation

$$\sigma \cdot a \sigma \cdot b = \mathbf{1} a \cdot b + i [a, b].$$

(B.26)

Similarly one obtains $u^\dagger(p, \sigma')u(p, \sigma) = \delta_{\sigma\sigma'} 2E$.

B.5 Gell-Mann Flavor Matrices

The hyperfine interaction specific for the GBE CQM discussed in Chapter 2 as well as the transition amplitudes introduced in Chapters 4 and 5 describing the mesonic decays of baryon resonances are explicitly flavor-dependent. The hyperfine interaction of the GBE CQM between quarks $q$ and $k$ contains operators of the form

$$\chi_q^a \lambda_k^a \quad (a = 1, 2, \ldots, 8)$$

(B.27)

in the flavor space; similarly, the transition operator contains linear combinations of the $\lambda^a$. There are eight independent operators corresponding to
the $SU(3)_F$, which can be written as $(3 \times 3)$ matrices, namely the Gell-Mann matrices, given by

\[ \lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \]

\[ \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \]

\[ \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}. \] (B.28)

It is obvious, that the $SU(3)$ contains the $SU(2)$ as a subgroup, and as in the case of $SU(2)$ these matrices are again traceless and Hermitian. The commutation relations of the matrices $\frac{\lambda^i}{2} (i = 1, 2, \ldots, 8)$ are given by

\[ \left[ \frac{\lambda^i}{2}, \frac{\lambda^j}{2} \right] = i f_{ijk} \left( \frac{\lambda^k}{2} \right) \] (B.29)

with the structure constants $f_{ijk}$ being antisymmetric under interchange of any pair of indices. The anti-commutation relations of the Gell-Mann matrices are given by

\[ \left\{ \frac{1}{2} \lambda^i, \frac{1}{2} \lambda^j \right\} = \frac{1}{3} \delta_{ij} + d_{ijk} \left( \frac{1}{2} \lambda^k \right). \] (B.30)

Here the structure constants $d_{ijk}$ are symmetric with respect to an exchange of indices. The values of the structure constants can be found, e.g., in Ref. [102].

In analogy to a spin state in $SU(2)$ the flavor state $\phi$ is now written as

\[ \phi = \begin{pmatrix} u \\ d \\ s \end{pmatrix}, \] (B.31)

where $u$, $d$, and $s$ denote the flavors up, down, and strange, respectively. In this representation we write a quark flavor state as

\[ u = f_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad d = f_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad s = f_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}. \] (B.32)
NOTATIONS IN RELATIVISTIC QUANTUM MECHANICS

B.6 Point-Form Properties of Baryon States

In any reference frame a baryon is specified by its mass, four-momentum, spin, helicity, and flavor content. We write a general eigenstate of the corresponding operators as \(|E, P, \Sigma, M_\Sigma, T, M_T\rangle\), where \(E = P_0\) is the energy and \(P\) the three-momentum. Furthermore, \(\Sigma\) and \(T\) are the intrinsic spin and isospin, and \(M_\Sigma, M_T\) are the corresponding \(z\)-projections, respectively.

The eigenstate is normalized in the relativistically invariant form

\[
\langle E', P', \Sigma', M_{\Sigma'}, T', M_{T'} | E, P, \Sigma, M_\Sigma, T, M_T \rangle = 2 P_0 \delta^3 (P - P') \delta_{\Sigma \Sigma'} \delta_{M_\Sigma M_{\Sigma'}} \delta_{TT'} \delta_{M_T M_{T'}}. \tag{B.33}
\]

The same baryon state can equivalently be expressed as \(|M, V, \Sigma, M_\Sigma, T, M_T\rangle\) or \(|P^n, \Sigma, M_\Sigma, T, M_T\rangle\). According to (B.33) the normalization of the baryon states can then be expressed as

\[
\langle M', V', \Sigma', M_{\Sigma'}, T', M_{T'} | M, V, \Sigma, M_\Sigma, T, M_T \rangle = 2 MV_0 \delta^3 (MV - M'V') \delta_{\Sigma \Sigma'} \delta_{M_\Sigma M_{\Sigma'}} \delta_{TT'} \delta_{M_T M_{T'}}
= 2 V_0 \frac{M}{M^2} \delta^3 (V - V') \delta_{MM'} \delta_{\Sigma \Sigma'} \delta_{M_\Sigma M_{\Sigma'}} \delta_{TT'} \delta_{M_T M_{T'}}, \tag{B.34}
\]

where \(M\) is the mass, and \(V\) the spatial part of the four-velocity, which is constrained by \(V_\mu V^\mu = 1\). As the eigenstates transform under Lorentz-transformations as a time-like four-vector, there exists a Lorentz-transformation that boosts the four-vector to its own rest frame. This rest-eigenstate transforms then as a spin-\(\Sigma\) irreducible representation under a rotation.

Baryons are considered to be composed of three constituent quarks. The baryon wave function can be given in different basis representations, in particular, with the (reducible) tensor product of one-particle states or the (irreducible) three-particle velocity states. The orthogonality and completeness relations of the free three-particle states are given in a relativistically invariant manner by

\[
\langle p'_1, p'_2, p'_3; \sigma'_1, \sigma'_2, \sigma'_3 | p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 \rangle = 2 p_0 \delta^3 (p'_1 - p_1)^2 p_0 \delta^3 (p'_2 - p_2)^2 p_0 \delta^3 (p'_3 - p_3) \delta_{\sigma_1 \sigma'_1} \delta_{\sigma_2 \sigma'_2} \delta_{\sigma_3 \sigma'_3}, \tag{B.35}
\]

and

\[
I = \sum_{\sigma_1, \sigma_2, \sigma_3} \int \frac{d^3 p_1}{2p_1} \frac{d^3 p_2}{2p_2} \frac{d^3 p_3}{2p_3} |p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 \rangle \langle p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 | p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 |. \tag{B.36}
\]

Since for all three four-momenta \(p_i^n\) the on-mass-shell condition is fulfilled, we have nine independent momentum-type variables.
NOTATIONS IN RELATIVISTIC QUANTUM MECHANICS

The velocity states $|v, k_1, k_2, k_3; \mu_1, \mu_2, \mu_3\rangle$ introduced in Chapter 3 (see Eq. (3.52)) are characterized by their dependence on the momenta $k_i$ and the velocity $v$, which is the absolute value of $v$, the spatial part of the interaction-independent four-velocity

$$v^\mu = \frac{P^\mu_{\text{free}}}{M_{\text{free}}},$$

(B.37)

where the free mass $M_{\text{free}}$ is defined by

$$M_{\text{free}} = \sum_{i=1}^{3} \omega_i = \sum_{i=1}^{3} \sqrt{m_i^2 + k_i^2},$$

(B.38)

and the free four-momentum $P^\mu_{\text{free}}$ is given by

$$P^\mu_{\text{free}} = \sum_{i=1}^{3} p_i^\mu.$$

(B.39)

The $k_i$ satisfy

$$\sum_{i=1}^{3} k_i = 0,$$

and we have again nine independent momentum-like variables. In order to find the completeness and the normalization relations with respect to the velocity states one has to perform a coordinate transformation and take care of the invariant integration measures using the corresponding Jacobideterminant

$$\mathcal{J} \left\{ \frac{\partial (p_1, p_2, p_3)}{\partial (v, k_2, k_3)} \right\} = \frac{2p_{10}2p_{20}2p_{30} (\omega_1 + \omega_2 + \omega_3)^3}{2\omega_12\omega_22\omega_3v_0}.$$

(B.41)

The completeness relation is then given by

$$1 = \sum_{\mu_1, \mu_2, \mu_3} \int \frac{d^3v \, d^3k_2 \, d^3k_3 \, (\omega_1 + \omega_2 + \omega_3)^3}{2\omega_12\omega_22\omega_3v_0} \times |v; k_1, k_2, k_3; \mu_1, \mu_2, \mu_3\rangle \langle v; k_1, k_2, k_3; \mu_1, \mu_2, \mu_3|,$$

(B.42)

and the corresponding orthonormalization condition reads

$$\langle v; k_1, k_2, k_3; \mu_1, \mu_2, \mu_3| v'; k_1', k_2', k_3'; \mu_1', \mu_2', \mu_3' \rangle = \frac{2\omega_12\omega_22\omega_3}{(\omega_1 + \omega_2 + \omega_3)^3} \delta^3 (v - v') \delta^3 (k_2 - k_2') \delta^3 (k_3 - k_3') \delta_{\mu_1 \mu_1'} \delta_{\mu_2 \mu_2'} \delta_{\mu_3 \mu_3'} v_0 \delta^3 (v - v').$$

(B.43)
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The connection between the velocity states and the free three-particle states is evaluated, e.g., in Ref. [89], and it explicitly reads

\[
\langle p'_1, p'_2, p'_3; \sigma'_1, \sigma'_2, \sigma'_3 | v; k_1, k_2, k_3; \mu_1, \mu_2, \mu_3 \rangle = \prod_{i=1}^{3} D_{\sigma'_i \mu_i}^{\frac{1}{2}} [R_W (k_i; B (v))] 2p_0 \delta^3 (p_i - p'_i). \tag{B.44}
\]

The baryon states represented with the velocity states lead to the baryon wave functions in the center-of-momentum frame

\[
\langle v; k_1, k_2, k_3; \mu_1, \mu_2, \mu_3 | M, V, \Sigma, M_\Sigma, T, M_T \rangle = \sqrt{\frac{2}{M}} v_0 \delta^3 (v - V) \sqrt{\frac{2 \omega_1 \omega_2 \omega_3}{(\omega_1 + \omega_2 + \omega_3)^3}} \Psi_{M_\Sigma M_T M_T} (k_i; \mu_i). \tag{B.45}
\]

The normalization of the baryon wave function is then given by

\[
\sum_{\mu_1 \mu_2 \mu_3} \int \frac{d^3v}{v_0} \frac{d^3k_2}{2 \omega_2} \frac{d^3k_3}{2 \omega_3} \frac{(\omega_1 + \omega_2 + \omega_3)^3}{2 \omega_1} \langle M', V', \Sigma', M_\Sigma', T', M_{T'} | v; k_1, k_2, k_3; \mu_1, \mu_2, \mu_3 \rangle \langle v; k_1, k_2, k_3; \mu_1, \mu_2, \mu_3 | M, V, \Sigma, M_\Sigma, T, M_T \rangle = \frac{2V_0}{M^2} \delta^3 (V - V') \delta_{M M'} \delta_{M_\Sigma M_\Sigma'} \delta_{T T'} \delta_{M_T M_{T'}}. \tag{B.46}
\]

The advantage of the velocity-state representation is that the motion of the system as a whole and the internal motion can be separated. The dependence on the internal momentum is given by the center-of-momentum wave function, which is normalized as

\[
\delta_{M M'} \delta_{M_\Sigma M_\Sigma'} \delta_{T T'} \delta_{M_T M_{T'}} = \sum_{\mu_1 \mu_2 \mu_3} \int d^3k_2 d^3k_3 \Psi^*_{M' \Sigma' M_{T'} T_{T'}} (k_i; \mu_i) \Psi_{M \Sigma M_T M_T} (k_i; \mu_i). \tag{B.47}
\]
Appendix C

Calculation of Matrix Elements

In order to obtain physical observables like, e.g., the decay widths, we have to calculate the matrix elements of the corresponding operators as given in Eq. (A.15)

\[
\langle \Psi_{XSF} | D | \Psi_{XSF} \rangle = \sum_{i,j} \sum_{k,p=1}^3 c'_i c_j \langle \psi^{(k)}_{\alpha'_i} | D | \psi^{(p)}_{\alpha_j} \rangle.
\]  

(C.1)

Since the initial and final baryon states are expressed through variational wave functions, this involves a sum over all basis functions and configurations as defined in Chapter 2. The decay operator in the spectator model defined in Chapter 5 has the form

\[
\hat{D}^{m}_{rd} = D(1) \otimes I(2) \otimes I(3) + I(1) \otimes D(2) \otimes I(3) + I(1) \otimes I(2) \otimes D(3),
\]  

(C.2)

when expressed as a tensor product on the three-particle Hilbert space. Here, the superscript \( m \) denotes the specific kind of involved meson, and the subscript indicates that we are concerned with a reduced operator. The three terms on the right-hand side of Eq. (C.2) act on the quarks 1, 2, and 3, respectively, while the other two particles are formally not involved. Due to the symmetry properties of the baryon states it is possible to rewrite Eq. (C.2)

\[
\hat{D}^{m}_{rd} = 3 \ D(1) \otimes I(2) \otimes I(3),
\]  

(C.3)

where we assume that only the first particle takes part in the decay process.

In the following we present some details for calculating the spin- and flavor-dependence as well as the momentum-space integration of the matrix elements.
CALCULATION OF MATRIX ELEMENTS

C.1 Spin Matrix Elements

For the matrix elements of the decay operator in the spin space, a very helpful instrument is provided by the Wigner-Eckart theorem (see, e.g., Ref. [87]): In particular one can write the matrix elements between the baryon spin states defined in Section A.4.2 of Appendix A as

\[
\langle s_0^1 s_2^1 \mid S \mid \mathcal{D}_{jm} \mid \frac{1}{2} S M S \rangle = \frac{(-1)^j}{\sqrt{2S' + 1}} C^{S'M_{S'}}_{S M_{Sjm}} \langle s_0^1 \frac{1}{2} S' \mid \mathcal{D}_j \mid \frac{1}{2} S \rangle
\]

(C.4)

with \( S \) (\( S' \)) the total spin of the initial (final) baryon with projection \( M_S \) (\( M_{S'} \)). Here, the spin matrix elements of \( \mathcal{D}_{jm} \), representing an irreducible tensor operator of rank \( j \), are expressed in terms of reduced matrix elements and Clebsch-Gordan coefficients \( C^{S'M_{S'}}_{S M_{Sjm}} \).

In general, the partitions of the final and initial baryon states do not coincide. Consequently, one has to calculate reduced matrix elements of the form

\[
\langle (s_0^1 \frac{1}{2} S') \mid \mathcal{D}_j \mid (s_0^1 \frac{1}{2} S'' \mid (s_0^1 \frac{1}{2} S') \rangle = \sum_{s'=0}^{1} \sum_{s=0}^{1} \langle (s'' \frac{1}{2} S'') \mid \mathcal{D}_j \mid (s'' \frac{1}{2} S'') \rangle \times \langle (s_0^1 \frac{1}{2} S') \mid \mathcal{D}_j \mid (s_0^1 \frac{1}{2} S') \rangle \delta_{s'' s} \delta_{s''' s''},
\]

where we have used the completeness of spin states two times. In each case such spin states are employed where the spins of two quarks are coupled to an intermediate spin \( s = 0, 1 \) and the third particle, being at a position \( k \) (\( p, q \)) corresponding to partition \( k \) (\( p, q \)) with spin \( j = \frac{1}{2} \), is added to \( s \) to give a total spin \( S \). The evaluation of the overlaps of the spin states of different partitions has already been treated in Section A.4.2 of Appendix A (see Eq. (A.29)). The determination of the central factor in Eq. (C.5), with the decay operator between spin states of the same partition, is explained below. We may thus specify to the case where the total baryon spin is coupled according to partition 1. Consequently the spins of quarks 2 and 3 are first coupled to \( s \) and \( s' \), respectively, and the spin \( j = \frac{1}{2} \) of particle 1 is then added to give the total spin \( S \) (\( S' \)).

Apart from the unit operator \( 1 \) three different types of spin matrix elements involving one to three Pauli spin operators occur. With the abbreviation \( \kappa = 2\kappa + 1 \) we obtain for the partition 1

\[
\langle (s_0^1 \frac{1}{2} S') \mid 1 \mid (s_0^1 \frac{1}{2} S') \rangle = \sqrt{S' S} \delta_{S' S} \delta_{s' s}.
\]

(C.6)

For the spin operator \( \sigma(1)_1 \) of quark 1, which is a tensor operator of rank 1 (denoted by the subscript 1), one gets

\[
\langle (s_0^1 \frac{1}{2} S') \mid \sigma(1)_1 \mid (s_0^1 \frac{1}{2} S) \rangle = \delta_{s' s}(-1)^{S' + s + 3/2} \sqrt{6 \cdot S' S} \left\{ \begin{array}{ccc} \frac{1}{2} & s & S' \\ \frac{1}{2} & 1 & \frac{1}{2} \end{array} \right\}.
\]

(C.7)
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For the tensor product of the two spin operators of the quarks 2 and 3, which is a tensor operator of rank $\kappa$ ($\kappa = 0, 1, 2$), one has

$$\langle (s' \frac{1}{2}) S' | \{ \{\sigma(2) \otimes \sigma(3)\}_{\kappa} (s \frac{1}{2}) S \rangle_1 = 6 (-1)^{S+S'+\kappa+1/2} \sqrt{\kappa \tilde{\kappa}} \tilde{S} \tilde{S}' \tilde{s} \tilde{s}'$$

$$\times \left\{ \begin{array}{ccc} s & \frac{1}{2} & S \\ S' & l & s' \end{array} \right\} \left\{ \begin{array}{ccc} 1 & 1 & l \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{array} \right\}.$$  

(C.8)

Finally, for the tensor product of all three spin operators, which is a tensor operator of rank $\kappa'$ ($\kappa' = 0, 1, 2, 3$), one finds

$$\langle (s' \frac{1}{2}) S' | \{ \{\sigma(2) \otimes \sigma(3)\}_{\kappa} \otimes \sigma(1)\}_{\kappa'} (s \frac{1}{2}) S \rangle_1 = 6 \sqrt{6 \kappa \tilde{\kappa} \tilde{\kappa}' \tilde{S} \tilde{S}' \tilde{s} \tilde{s}'}$$

$$\times \left\{ \begin{array}{ccc} \kappa & 1 & \kappa' \\ \kappa' & s' & S' \\ s & \frac{1}{2} & S \end{array} \right\} \left\{ \begin{array}{ccc} 1 & 1 & \kappa \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{array} \right\}.$$  

(C.9)

The objects in curly brackets represent the 6j- and 9j-symbols (in the notation of Ref. [77]).

In this work, we calculate decay widths, where the final baryon has positive parity and an intrinsic spin $\Sigma' = \frac{1}{2}$, which is built up from the total orbital angular momentum $L'$ and the total spin $S'$ satisfying the triangular rule. The final baryons (of positive parity) are necessarily ground states with $L' = 0$ and $S' = \frac{1}{2}$. As worked out in Section 4.1, it is sufficient to evaluate only transition elements with positive $z$-projection $M_{\Sigma} = M_{\Sigma'} = +\frac{1}{2}$. Therefore, because of $M_{L'} = 0$, we only have to deal with spin projections $M_{S'} = +\frac{1}{2}$ for all final baryon states. Altogether, we thus need 16 different spin matrix elements given by the following expressions:

$$\text{spin}(1) = \langle (s' \frac{1}{2}) S' + \frac{1}{2}\sigma_z(1)(s \frac{1}{2}) S + \frac{1}{2} \rangle$$

$$\text{spin}(2) = \langle (s' \frac{1}{2}) S' + \frac{1}{2}\sigma_z(1)[-\sigma_x(2)\sigma_x(3) - \sigma_y(2)\sigma_y(3)](s \frac{1}{2}) S + \frac{1}{2} \rangle$$

$$\text{spin}(3) = \langle (s' \frac{1}{2}) S' + \frac{1}{2}[\sigma_x(1)\sigma_y(3) - \sigma_y(1)\sigma_x(3)](s \frac{1}{2}) S + \frac{1}{2} \rangle$$

$$\text{spin}(4) = \langle (s' \frac{1}{2}) S' + \frac{1}{2}[\sigma_x(1)\sigma_y(2) - \sigma_y(1)\sigma_x(2)](s \frac{1}{2}) S + \frac{1}{2} \rangle$$

$$\text{spin}(5) = \langle (s' \frac{1}{2}) S' + \frac{1}{2}\sigma^+(1)(s \frac{1}{2}) S - \frac{1}{2} \rangle$$

$$\text{spin}(6) = \langle (s' \frac{1}{2}) S' + \frac{1}{2}\sigma^-(1)(s \frac{1}{2}) S + \frac{1}{2} \rangle$$

$$\text{spin}(7) = \langle (s' \frac{1}{2}) S' + \frac{1}{2}\sigma_z(1)\sigma^+(2)](s \frac{1}{2}) S - \frac{1}{2} \rangle$$

$$\text{spin}(8) = \langle (s' \frac{1}{2}) S' + \frac{1}{2}\sigma_z(1)\sigma^-(2)](s \frac{1}{2}) S + \frac{1}{2} \rangle$$

$$\text{spin}(9) = \langle (s' \frac{1}{2}) S' + \frac{1}{2}\sigma_z(1)\sigma^+(3)](s \frac{1}{2}) S - \frac{1}{2} \rangle$$

$$\text{spin}(10) = \langle (s' \frac{1}{2}) S' + \frac{1}{2}\sigma_z(1)\sigma^-(3)](s \frac{1}{2}) S + \frac{1}{2} \rangle$$

$$\text{spin}(11) = \langle (s' \frac{1}{2}) S' + \frac{1}{2}\sigma^+(1)[-\sigma_x(2)\sigma_x(3) - \sigma_y(2)\sigma_y(3)](s \frac{1}{2}) S - \frac{1}{2} \rangle$$

$$\text{spin}(12) = \langle (s' \frac{1}{2}) S' + \frac{1}{2}\sigma^-(1)[-\sigma_x(2)\sigma_x(3) - \sigma_y(2)\sigma_y(3)](s \frac{1}{2}) S + \frac{1}{2} \rangle$$

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\begin{align*}
\text{spin}(13) &= \langle (s_0^\frac{1}{2}S^0 + \frac{1}{2}i\sigma^x(2)|\sigma_x(1)\sigma_y(3) - \sigma_y(1)\sigma_x(3)) \rangle (s_0^\frac{1}{2}S^0 - \frac{1}{2}) \\
&+ \langle (s_0^\frac{1}{2}S^0 + \frac{1}{2}i\sigma^y(3)|\sigma_x(1)\sigma_y(2) - \sigma_y(1)\sigma_x(2)) \rangle (s_0^\frac{1}{2}S^0 + \frac{1}{2}) \\
\text{spin}(14) &= \langle (s_0^\frac{1}{2}S^0 + \frac{1}{2}i\sigma^z(2)|\sigma_x(1)\sigma_y(3) - \sigma_y(1)\sigma_x(3)) \rangle (s_0^\frac{1}{2}S^0 + \frac{1}{2}) \\
&+ \langle (s_0^\frac{1}{2}S^0 + \frac{1}{2}i\sigma^y(3)|\sigma_x(1)\sigma_y(2) - \sigma_y(1)\sigma_x(2)) \rangle (s_0^\frac{1}{2}S^0 + \frac{1}{2}) \\
\text{spin}(15) &= \langle (s_0^\frac{1}{2}S^0 + \frac{1}{2}i\sigma^x(1)|\sigma_x(2)\sigma_y(3) - \sigma_y(2)\sigma_x(3)) \rangle (s_0^\frac{1}{2}S^0 - \frac{1}{2}) \\
&+ \langle (s_0^\frac{1}{2}S^0 + \frac{1}{2}i\sigma^y(3)|\sigma_x(2)\sigma_y(2) - \sigma_y(2)\sigma_x(2)) \rangle (s_0^\frac{1}{2}S^0 + \frac{1}{2}) \\
\text{spin}(16) &= \langle (s_0^\frac{1}{2}S^0 + \frac{1}{2}i\sigma^z(2)|\sigma_x(2)\sigma_y(3) - \sigma_y(2)\sigma_x(3)) \rangle (s_0^\frac{1}{2}S^0 + \frac{1}{2})
\end{align*}

Here, the initial (final) spin states are specified by \(S\) (\(S'\)) with \(z\)-projections \(M_S\) (\(M_{S'}\)). The \((2 \times 2)\) matrices \(\sigma^+(i)\) and \(\sigma^-(i)\) are defined by

\begin{align*}
\sigma^+(i) &= -\frac{1}{\sqrt{2}} (\sigma_x(i) + i\sigma_y(i)) \\
\sigma^-(i) &= \frac{1}{\sqrt{2}} (\sigma_x(i) - i\sigma_y(i))
\end{align*}

for \(i = 1, 2, 3\). According to Eq. (A.32) the baryon spin states decompose into sums over tensor products of three spin-\(\frac{1}{2}\) states with corresponding Clebsch-Gordon coefficients. It should be noted that the nomenclature \(\text{spin}(i)\) above was chosen according to the implementation of the formulae in the numerical FORTRAN code.

C.2 Flavor Matrix Elements

Proceeding in the same way as above we now show the details for the calculation of the flavor matrix elements of the decay operator.

Every decay operator corresponds to a specific kind of meson produced; the latter can be \(\pi^0, \pi^+, \pi^-, \eta = \eta^8, K^0, \bar{K}^0, K^+, \text{and } K^-\). Thus, the flavor part of the decay operator \(D^m_{\lambda_M}\) is constructed out of the Gell-Mann flavor matrices \(\lambda^i (i = 1, \cdots, 8)\) defined in Appendix B. The explicit structure of the flavor operator \(\mathcal{F}^m\) (introduced in Eqs. (4.49) and (4.50)) for the various mesons of type \(m\) is given as

\begin{align*}
\mathcal{F}^{\pi^0} &= \lambda^3 = \left(\begin{array}{ccc}
1 & 0 & 0 \\
0 & -1 & 0 \\
0 & 0 & 0
\end{array}\right), \\
\mathcal{F}^{\pi^+} &= \frac{1}{\sqrt{2}}(\lambda^1 - i\lambda^2) = \sqrt{2}\left(\begin{array}{ccc}
0 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0
\end{array}\right), \\
\mathcal{F}^{\pi^-} &= \frac{1}{\sqrt{2}}(\lambda^1 + i\lambda^2) = \sqrt{2}\left(\begin{array}{ccc}
0 & 1 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{array}\right), \\
\mathcal{F}^{\eta} &= \lambda^8 = \frac{1}{\sqrt{3}}\left(\begin{array}{ccc}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & -2
\end{array}\right).
\end{align*}
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\[ \mathcal{F}^{K_0} = \frac{1}{\sqrt{2}} (\lambda^6 - i \lambda^7) = \sqrt{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \]

\[ \mathcal{F}^{K_0} = \frac{1}{\sqrt{2}} (\lambda^6 + i \lambda^7) = \sqrt{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \]

\[ \mathcal{F}^{K^+} = \frac{1}{\sqrt{2}} (\lambda^4 - i \lambda^5) = \sqrt{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \]

\[ \mathcal{F}^{K^-} = \frac{1}{\sqrt{2}} (\lambda^4 + i \lambda^5) = \sqrt{2} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \] (C.12)

Consequently, for the decay of a baryon resonance one has to exploit flavor matrix elements of the form

\[ \langle f'_1 f'_2 f'_3 | F^{m}(1) \otimes 1(2) \otimes 1(3) | f_i f_j f_k \rangle = \langle f'_1 | F^{m}(1) | f_i \rangle \delta_{f'_2 f_2} \delta_{f'_3 f_3}, \] (C.13)

where \( |f_i f_j f_k \rangle \) is defined in Eq. (A.13) leading to the flavor wave functions \( \phi^{(1)}_F \) given in Table A.1. Eq. (C.13), where only the first quark participates in the decay process and the two other quarks are not involved, is a consequence of the spectator model defined in Section 5.1.1.

In the following we discuss the various possible transition amplitudes of light and strange baryon resonances with respect to the isospin of the participating particles. The decay width defined in Eq. (4.36) depends on the isospins of the decaying and final baryons as well as the involved meson; there one has to sum and average over the corresponding \( z \)-projections. Using the Wigner-Eckhart theorem given in Eq. (C.4) we substitute the summation and averaging over the isospin according to Eq. (4.41). Thus we only need to calculate one specific transition amplitude for each decay mode, and the final decay width is then given as the squared transition amplitude multiplied by a corresponding numerical factor

\[ \Gamma_{i \rightarrow f} = \frac{|q|}{4\Sigma^2 + 1} \frac{2 T^* + 1}{2 T + 1} \frac{|F_{i1\Sigma M_1 T M_T}^{m} f_{12\Sigma M_2 T M_T}^{m} f_{31\Sigma M_3 T M_T}^{m} |^2}{|C_{T M_T M_T M_T M_T} |^2}. \] (C.14)

Here we present for each decay channel considered in this work all possible transitions. We also specify how the decay width is determined from the flavor part of each decay mode.

C.2.1 \( \pi \)-Meson Decays

The \( \pi \) meson is an isospin triplet with isospin \( T_m = 1 \), and thus the \( \pi^0 \), \( \pi^+ \), and \( \pi^- \) correspond to the \( z \)-projections \( M_T = 0, \pm 1 \), respectively. The
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associated flavor functions are

\[
\begin{align*}
\pi^+ & : \ u\bar{d} \\
\pi^0 & : \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) \\
\pi^- & : \ d\bar{u}.
\end{align*}
\] (C.15)

For the actual calculations the meson mass \(m_\pi\) is taken in concordance with the GBE CQM, where one used the value \(m_\pi = 139\) MeV. Concerning the strong baryon decays the following decay channels are considered:

\[
\begin{align*}
N & \rightarrow N\pi \\
\Delta & \rightarrow N\pi \\
\Lambda & \rightarrow \Sigma\pi \\
\Sigma & \rightarrow \Lambda\pi \\
\Sigma & \rightarrow \Sigma\pi \\
\Xi & \rightarrow \Xi\pi.
\end{align*}
\] (C.16)

For the decay width of \(N \rightarrow N\pi\) the following transitions have to be taken into account:

\[
\begin{align*}
p & \rightarrow p\pi^0 \\
p & \rightarrow n\pi^+ \\
n & \rightarrow n\pi^0 \\
n & \rightarrow p\pi^-.
\end{align*}
\] (C.17)

For the actual calculation of the decay width we pick out the first transition and find

\[
\Gamma_{i\rightarrow f} \propto 3 \cdot |F_{i\rightarrow f}| \langle \mathcal{F}_{\pi^0} \rangle^2.
\] (C.18)

In case of \(\Delta \rightarrow N\pi\) the possible transitions are

\[
\begin{align*}
\Delta^{++} & \rightarrow p\pi^0 \\
\Delta^{+} & \rightarrow p\pi^0 \\
\Delta^{-} & \rightarrow n\pi^+ \\
\Delta^{0} & \rightarrow n\pi^0 \\
\Delta^{0} & \rightarrow p\pi^- \\
\Delta^{-} & \rightarrow n\pi^-,
\end{align*}
\] (C.19)

where we pick out the second one and find

\[
\Gamma_{i\rightarrow f} \propto \frac{3}{2} \cdot |F_{i\rightarrow f}| \langle \mathcal{F}_{\pi^0} \rangle^2.
\] (C.20)
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Next, we consider the \( \pi \) decays in the strange sector. Here we start with the \( \Lambda \to \Sigma \pi \) channel and find the following three transitions

\[
\begin{align*}
\Lambda^0 & \to \Sigma^0 \pi^0 \\
\Lambda^0 & \to \Sigma^+ \pi^- \\
\Lambda^0 & \to \Sigma^- \pi^+. \\
\end{align*}
\]  

(C.21)

Using the first transition we obtain

\[
\Gamma_{i \to f} \propto 3 \cdot |F_{1\Lambda^0 \to 1\Sigma^0}(F\pi^0)|^2. 
\]  

(C.22)

Similarly, for \( \Sigma \to \Lambda \pi \) the possible transitions are

\[
\begin{align*}
\Sigma^0 & \to \Lambda^0 \pi^0 \\
\Sigma^+ & \to \Lambda^0 \pi^+ \\
\Sigma^- & \to \Lambda^0 \pi^-, \\
\end{align*}
\]  

(C.23)

and from the first one we find

\[
\Gamma_{i \to f} \propto 1 \cdot |F_{1\Sigma^0 \to 1\Lambda^0}(F\pi^0)|^2. 
\]  

(C.24)

The \( \Sigma \) baryon resonance also has the decay channel \( \Sigma \to \Sigma \pi \), where the possible transitions read

\[
\begin{align*}
\Sigma^0 & \to \Sigma^0 \pi^0 \\
\Sigma^0 & \to \Sigma^+ \pi^- \\
\Sigma^0 & \to \Sigma^- \pi^+ \\
\Sigma^+ & \to \Sigma^0 \pi^+ \\
\Sigma^+ & \to \Sigma^+ \pi^0 \\
\Sigma^- & \to \Sigma^0 \pi^- \\
\Sigma^- & \to \Sigma^- \pi^0. \\
\end{align*}
\]  

(C.25)

Choosing the transition \( \Sigma^+ \to \Sigma^+ \pi^0 \) we obtain for the decay width

\[
\Gamma_{i \to f} \propto 2 \cdot |F_{1\Sigma^+ \to 1\Sigma^+}(F\pi^0)|^2. 
\]  

(C.26)

Finally the \( \Xi \to \Xi \pi \) decays are specified by

\[
\begin{align*}
\Xi^0 & \to \Xi^0 \pi^0 \\
\Xi^0 & \to \Xi^- \pi^+ \\
\Xi^- & \to \Xi^0 \pi^- \\
\Xi^- & \to \Xi^- \pi^0. \\
\end{align*}
\]  

(C.27)

with the result for the decay width given by

\[
\Gamma_{i \to f} \propto 3 \cdot |F_{1\Xi^0 \to 1\Xi^0}(F\pi^0)|^2. 
\]  

(C.28)

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C.2.2 η-Meson Decays

The η meson is an isospin singlet with $T_\eta = 0$ and consequently zero $z$-projection. Its flavor function is given as

$$\eta : \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s}).$$  \hspace{1cm} (C.29)

Again we employ the same mass as in the GBE CQM, which is $m_\eta = 547$ MeV. Proceeding in the same way as above we consider in case of the η decays the following channels:

\begin{align*}
N &\rightarrow N\eta \\
\Lambda &\rightarrow \Lambda\eta \\
\Sigma &\rightarrow \Sigma\eta \\
\Xi &\rightarrow \Xi\eta \\
\Omega &\rightarrow \Omega\eta.
\end{align*}  \hspace{1cm} (C.30)

Starting with $N \rightarrow N\eta$ the transitions are given by

\begin{align*}
p &\rightarrow p\eta \\
n &\rightarrow n\eta.
\end{align*}  \hspace{1cm} (C.31)

Both transitions lead to the same result for the decay width

$$\Gamma_{i\rightarrow f} \propto 1 \cdot |F_{i\rightarrow f}(\mathcal{F}^\eta)|^2,$$  \hspace{1cm} (C.32)

where we have used the decay of the first type. As both the $\Lambda$ resonance and the η meson are isospin singlets, the only transition for the decay channel $\Lambda \rightarrow \Lambda\eta$ is provided by

$$\Lambda^0 \rightarrow \Lambda^0\eta,$$  \hspace{1cm} (C.33)

and therefore

$$\Gamma_{i\rightarrow f} \propto 1 \cdot |F_{i\rightarrow f}(\mathcal{F}^\eta)|^2.$$  \hspace{1cm} (C.34)

For the $\Sigma \rightarrow \Sigma\eta$ decay channel the possible transitions are

\begin{align*}
\Sigma^0 &\rightarrow \Sigma^0\eta \\
\Sigma^+ &\rightarrow \Sigma^+\eta \\
\Sigma^- &\rightarrow \Sigma^-\eta,
\end{align*}  \hspace{1cm} (C.35)

and again the final decay width can be calculated by

$$\Gamma_{i\rightarrow f} \propto 1 \cdot |F_{i\rightarrow f}(\mathcal{F}^\eta)|^2.$$  \hspace{1cm} (C.36)

The $\Xi$ resonance has an isospin $T_\Xi = \frac{1}{2}$ and for the decay as well as for the corresponding transitions $\Xi \rightarrow \Xi\eta$ we can directly adopt everything from the case $N \rightarrow N\eta$. Therefore we do not list the specific transitions for this case explicitly. Eventually, the $\Omega \rightarrow \Omega\eta$ decay channel is completely analogous to the $\Lambda \rightarrow \Lambda\eta$, and again we do not write up the transitions explicitly.
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C.2.3  $K$-Meson Decays

$K$ mesons have an isospin $T_m = \frac{1}{2}$ and we take again an overall mass $m_K = 494$ MeV in concordance with the GBE CQM. Different from the $\pi$ and $\eta$ mesons we have to distinguish between two isospin multiplets: The first one comprises the $K^+$ and $K^0$ corresponding to $M_{T_{K^+}} = +\frac{1}{2}$ and $M_{T_{K^0}} = -\frac{1}{2}$, respectively, and the flavor functions are given as

\begin{align*}
K^+ : & \quad u\bar{s} \\
K^0 : & \quad d\bar{s}.
\end{align*}

In a decay process involving these $K$ mesons, the strange-quark content has to be balanced in the residual baryon. In case of $K^+$ and $K^0$ it increases.

The second isospin multiplet contains the $\bar{K}^0$ and $K^-$ corresponding to $M_{T_{\bar{K}^0}} = +\frac{1}{2}$ and $M_{T_{K^-}} = -\frac{1}{2}$ with the flavor functions

\begin{align*}
\bar{K}^0 : & \quad \bar{d}s \\
K^- : & \quad \bar{u}s
\end{align*}
respectively. This leads to a decrease of the strange-quark content in the residual baryon relative to the decaying resonance.

In particular, we address here the transitions of the first type

\begin{align*}
N & \rightarrow \Lambda K \\
N & \rightarrow \Sigma K \\
\Delta & \rightarrow \Sigma K \\
\Lambda & \rightarrow \Xi K \\
\Sigma & \rightarrow \Xi K,
\end{align*}

where the strange-quark content increases in the residual baryon, and of the second type

\begin{align*}
\Lambda & \rightarrow NK \\
\Sigma & \rightarrow NK \\
\Xi & \rightarrow \Lambda K \\
\Xi & \rightarrow \Sigma K \\
\Omega & \rightarrow \Xi K,
\end{align*}

with decreasing strange-quark content in the residual baryon.

We start with the decays listed in (C.39) and find for the $N \rightarrow \Lambda K$ decay channel the specific transitions

\begin{align*}
p & \rightarrow \Lambda^0 K^+ \\
n & \rightarrow \Lambda^0 K^0.
\end{align*}
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Using the second one we get for the decay width

$$\Gamma_{i \to f} \propto 1 \cdot |F_{i|n \to f_{\Lambda^0}}(\mathcal{F}^{K^0})|^2. \quad (C.42)$$

For the $N \to \Sigma K$ decay channel the possible transitions are

$$\begin{align*}
p & \to \Sigma^+ K^0 \\
p & \to \Sigma^0 K^+ \\
n & \to \Sigma^0 K^0 \\
n & \to \Sigma^- K^+,
\end{align*} \quad (C.43)$$

where one obtains the decay width

$$\Gamma_{i \to f} \propto \frac{3}{2} \cdot |F_{i|p \to f_{\Sigma^+}}(\mathcal{F}^{K^0})|^2 \quad (C.44)$$

using the $p \to \Sigma^+ K^0$ transition. Next we consider $\Delta \to \Sigma K$ with

$$\begin{align*}
\Delta^{++} & \to \Sigma^+ K^+ \\
\Delta^+ & \to \Sigma^+ K^0 \\
\Delta^+ & \to \Sigma^0 K^+ \\
\Delta^0 & \to \Sigma^0 K^0 \\
\Delta^0 & \to \Sigma^- K^+ \\
\Delta^- & \to \Sigma^- K^0,
\end{align*} \quad (C.45)$$

and from the second transition we deduce the decay width to be

$$\Gamma_{i \to f} \propto 3 \cdot |F_{i|\Delta^+ \to f_{\Sigma^+}}(\mathcal{F}^{K^0})|^2. \quad (C.46)$$

The decay channel $\Lambda \to \Xi K$ includes the transitions

$$\begin{align*}
\Lambda & \to \Xi^0 K^0 \\
\Lambda & \to \Xi^- K^+,
\end{align*} \quad (C.47)$$

and using $\Lambda^0 \to \Xi^0 K^0$ the decay width is determined by

$$\Gamma_{i \to f} \propto 2 \cdot |F_{i|\Lambda^0 \to f_{\Xi^0}}(\mathcal{F}^{K^0})|^2. \quad (C.48)$$

Finally, the decay mode $\Sigma \to \Xi K$ includes the transitions

$$\begin{align*}
\Sigma^+ & \to \Xi^0 K^+ \\
\Sigma^0 & \to \Xi^- K^+ \\
\Sigma^0 & \to \Xi^0 K^0 \\
\Sigma^- & \to \Xi^- K^0.
\end{align*} \quad (C.49)$$
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The corresponding decay width can be obtained by

$$\Gamma_{i\rightarrow f} \propto 2 \cdot |F_{i|\gamma_0 \rightarrow f|\gamma_0} (\mathcal{F}^{K^0})|^2. \quad (C.50)$$

For the decays listed in (C.40) we perform the same considerations and obtain the following results:

\[ \Lambda \rightarrow NK: \]

\[ \Lambda^0 \rightarrow pK^- \]
\[ \Lambda^0 \rightarrow n\bar{K}^0 \quad (C.51) \]

with the decay width given by

$$\Gamma_{i\rightarrow f} \propto 2 \cdot |F_{i|\Lambda^0 \rightarrow f|n} (\mathcal{F}\bar{K}^0)|^2. \quad (C.52)$$

\[ \Sigma \rightarrow NK: \]

\[ \Sigma^+ \rightarrow p\bar{K}^0 \]
\[ \Sigma^0 \rightarrow n\bar{K}^0 \]
\[ \Sigma^- \rightarrow nK^- \quad (C.53) \]

with the decay width given by

$$\Gamma_{i\rightarrow f} \propto 1 \cdot |F_{i|\Sigma^+ \rightarrow f|p} (\mathcal{F}\bar{K}^0)|^2. \quad (C.54)$$

\[ \Xi \rightarrow \Lambda K: \]

\[ \Xi^0 \rightarrow \Lambda^0\bar{K}^0 \]
\[ \Xi^- \rightarrow \Lambda^0K^- \quad (C.55) \]

with the decay width given by

$$\Gamma_{i\rightarrow f} \propto 1 \cdot |F_{i|\Xi^0 \rightarrow f|\Lambda^0} (\mathcal{F}\bar{K}^0)|^2. \quad (C.56)$$

\[ \Xi \rightarrow \Sigma K: \]

\[ \Xi^0 \rightarrow \Sigma^+K^- \]
\[ \Xi^0 \rightarrow \Sigma^0\bar{K}^0 \]
\[ \Xi^- \rightarrow \Sigma^0K^- \]
\[ \Xi^- \rightarrow \Sigma^-\bar{K}^0 \quad (C.57) \]
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with the decay width given by
\[ \Gamma_{i \rightarrow f} \propto 3 \cdot |F_{i,20} \rightarrow f_{j,0} (\mathcal{F}^0)|^2. \]  
(C.58)

\[ \Omega \rightarrow \Xi K: \]
\[ \begin{align*}
\Omega & \rightarrow \Xi^0 K^- \\
\Omega & \rightarrow \Xi^- K^0
\end{align*} \]  
(C.59)

with the decay width given by
\[ \Gamma_{i \rightarrow f} \propto 2 \cdot |F_{i,0} \rightarrow f_{j,0} (\mathcal{F}^0)|^2. \]  
(C.60)

C.3 Numerical and Analytical Integration

In order to evaluate the transition matrix elements in Chapter 5, it is necessary to insert specific completeness relations leading to multidimensional integrals over three-momenta as well as velocities. Finally, it is possible to reduce the calculation to a six-dimensional integration over the three-momenta of the spectator quarks \(k_2\) and \(k_3\) of the decaying baryon resonance, or equivalently the corresponding Jacobi momenta \(\mathbf{p}\) and \(\mathbf{k}\) (see Eqs. (A.59) and (A.60)). Here we take a closer look at the integration over \(d^3p d^3k\); however, the procedure works similarly for \(d^3k_2 d^3k_3\).

First of all we switch over to spherical coordinates:
\[ \begin{align*}
p_x &= p \cos \phi_p \sin \theta_p \\
p_y &= p \sin \phi_p \sin \theta_p \\
p_z &= p^\parallel = p \cos \theta_p \\
k_x &= k \cos \phi_k \sin \theta_k \\
k_y &= k \sin \phi_k \sin \theta_k \\
k_z &= k^\parallel = k \cos \theta_k.
\end{align*} \]  
(C.61)

In addition we substitute for the absolute values \(p\) and \(k\) the new variables \(\rho\) and \(\psi\) by
\[ \begin{align*}
p &= \rho \cos \psi, \\
k &= \rho \sin \psi.
\end{align*} \]  
(C.62)

Thereby the two integrations over \(p\) and \(k\) from 0 to infinity are transformed into an integration over the finite interval \(0 \leq \psi \leq \frac{\pi}{2}\) for the angle \(\psi\) and an integration over the infinite interval \(0 \leq \rho < \infty\) for \(\rho\). The transformation of the integration area is illustrated in Fig. C.1. The integration measure
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![Diagram with integration area transformation](image)

Figure C.1: Transformation of integration area $0 \leq p < \infty$, $0 \leq \rho < \infty$ into $0 \leq \psi \leq \frac{\pi}{2}$, $0 \leq \rho < \infty$.

then looks like

$$d^3p d^3k = \rho^{5} \sin^{2} \psi \cos^{2} \psi \sin \theta_p \sin \theta_k d\rho d\psi d\theta_p d\theta_k d\phi_p d\phi_k,$$

\[ (C.63) \]

where the ranges of the integration are explicitly given by

\[
\begin{align*}
0 & \leq \rho \leq \infty \\
0 & \leq \psi \leq \frac{\pi}{2} \\
0 & \leq \theta_p \leq \pi \\
0 & \leq \theta_k \leq \pi \\
0 & \leq \phi_p \leq 2\pi \\
0 & \leq \phi_k \leq 2\pi.
\end{align*}
\]  
\[ (C.64) \]

In general, the integrands, expressed in terms of these six variables, have a complicated structure, and thus one has to resort to numerical methods. Only the dependence on $\phi_p$ and $\phi_k$ can further be simplified as these variables appear only as arguments of powers of sine and/or cosine. There are terms like, e.g.,

$$p \cdot k = \rho^2 \sin \psi \cos \psi [\cos \theta_p \cos \theta_k + \sin \theta_p \sin \theta_k \cos(\phi_p - \phi_k)]$$

\[ (C.65) \]

with a dependence on $(\phi_p - \phi_k)$. Indeed, considering all possible integrands there appear only cosines with this argument, and it turns out that a transformation of the variables $\phi_p$ and $\phi_k$ to

$$\phi_- = \phi_p - \phi_k$$
$$\phi_+ = \frac{1}{2}(\phi_p + \phi_k)$$

\[ (C.66) \]

with $d\phi_p d\phi_k = d\phi_- d\phi_+$ allows to perform an analytical integration over $\phi_\pm$. One is then left with a five-dimensional integral, which has to be evaluated numerically. For the integration over $\rho$ we use a stepwise Gaussian quadrature over finite intervals stopping as soon as $\rho$ exceeds some limit value.
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beyond which the change of the integral is negligible. The four remaining finite integrations can be solved utilising, e.g., the Gaussian quadrature integration.

C.3.1 Analytical Integration over $\phi_+$

In this section we show for all types of conceivable integrands how to perform the analytic integration over the angle $\phi_+$. First of all we have to rewrite the integrand depending on $\phi_p, \phi_k$ in terms of $\phi_-, \phi_+$ using Eqs. (C.66)

$$f(\rho, \psi, \theta_p, \theta_k, \phi_p, \phi_k) = f(\rho, \psi, \theta_p, \theta_k, \phi_-, \phi_+). \quad (C.67)$$

At the moment we treat $\rho$, $\psi$, $\theta_p$, and $\theta_k$ as constants and consider the double integral

$$2 \int_{0}^{2\pi} d\phi_p \int_{0}^{2\pi} d\phi_k f(\phi_p, \phi_k) = \int_{0}^{2\pi} d\phi_- \int_{\phi_-}^{2\pi-\phi_-} d\phi_+ f(\phi_-, \phi_+)$$

$$= \int_{0}^{2\pi} d\phi_- \int_{\phi_-}^{2\pi-\phi_-} d\phi_+ f(\phi_-, \phi_+) + \int_{0}^{2\pi} d\phi_- \int_{2\pi-\phi_-}^{\phi_-} d\phi_+ f(\phi_-, \phi_+) \quad (C.68)$$

$$= \int_{0}^{2\pi} d\phi_- \int_{\phi_-}^{2\pi-\phi_-} d\phi_+ [f(\phi_-, \phi_+) + f(-\phi_-, \phi_+)].$$

All integrands used in the present work can be rewritten in the form

$$f(\rho, \psi, \theta_p, \theta_k, \phi_-, \phi_+) = \tilde{f}(\rho, \psi, \theta_p, \theta_k, \cos \phi_-) \cdot p_x^{n_1} p_y^{n_3} k_x^{n_3} k_y^{n_4}, \quad (C.69)$$

where $n_i$ are integers. The new function $\tilde{f}$ contains $\phi_-$ only as an argument of the cosine, and thus it does not change when replacing $\phi_-$ by $-\phi_-$. The $x$- and $y$-components of $p$ and $k$ imply both $\phi_-$ and $\phi_+$ according to Eqs. (C.61) and (C.66). The resulting integrals can be written as

$$\mathcal{I}_{n_1, n_2, n_3, n_4}(\rho, \psi, \theta_p, \theta_k) = \int_{0}^{2\pi} d\phi_- \int_{\phi_-}^{2\pi-\phi_-} d\phi_+ \tilde{f}(\rho, \psi, \theta_p, \theta_k, \cos \phi_-)$$

$$\times \left[ p_x^{n_1} p_y^{n_3} k_x^{n_3} k_y^{n_4} \big|_{\phi_-} + p_x^{n_1} p_y^{n_3} k_x^{n_3} k_y^{n_4} \big|_{-\phi_-} \right], \quad (C.70)$$

and they are still functions of $\rho, \psi, \theta_p$, and $\theta_k$ depending on the integers $n_i$ up to a total power of all $x$- and $y$-components of momenta $N = \sum n_i = 4$. Notice that $N$ has to be even since both combined powers $n_1 + n_3$ of the $x$- and $n_2 + n_4$ of the $y$-components, respectively, have to be even for themselves,
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otherwise the integral would vanish. Due to rotational symmetry, in all expressions the \(x\)- and \(y\)-components can be mutually exchanged, i.e.

\[
\mathcal{I}_{n_1,n_2,n_3,n_4}(\rho, \psi, \theta_p, \theta_k) = \mathcal{I}_{n_2,n_1,n_4,n_3}(\rho, \psi, \theta_p, \theta_k).
\]  
(C.71)

The analytical integration over \(\phi_+\) can easily be performed using, e.g., MATHEMATICA and one is then left with integrals over \(\phi_-\) multiplied by corresponding factors \(\alpha(\rho, \psi, \theta_p, \text{ and } \theta_k)\) depending on the \(n_i\)

\[
\mathcal{I}_{n_1,n_2,n_3,n_4}(\rho, \psi, \theta_p, \theta_k) = \alpha(\rho, \psi, \theta_p, \theta_k) \cdot \mathcal{I}_n(\rho, \psi, \theta_p, \theta_k),
\]  
(C.72)

where

\[
\mathcal{I}_n(\rho, \psi, \theta_p, \theta_k) = \int_0^{2\pi} d\phi_- (2\pi - \phi_-) f(\rho, \psi, \theta_p, \theta_k, \cos \phi_-) \cos n\phi_-.
\]  
(C.73)

In the following we will leave out the arguments of all \(\mathcal{I}_{n_1,n_2,n_3,n_4}\) and \(\mathcal{I}_n\) and present the various results of the remaining integrands:

\(\text{N}=0:\)

\[
\mathcal{I}_{0,0,0,0} = 2\mathcal{I}_0
\]  
(C.74)

\(\text{N}=2:\)

\[
\begin{align*}
\mathcal{I}_{0,2,0,0} &= \mathcal{I}_{0,0,2,0} = \rho^2 \cos^2 \psi \sin^2 \theta_p \mathcal{I}_0 \\
\mathcal{I}_{1,0,1,0} &= \mathcal{I}_{0,1,0,1} = \rho^2 \sin \psi \cos \psi \sin \theta_p \sin \theta_k \mathcal{I}_1 \\
\mathcal{I}_{0,0,2,0} &= \mathcal{I}_{0,0,0,2} = \rho^2 \sin^2 \psi \sin^2 \theta_k \mathcal{I}_0
\end{align*}
\]  
(C.75)

\(\text{N}=4:\)

\[
\begin{align*}
\mathcal{I}_{0,4,0,0} &= \mathcal{I}_{0,0,4,0} = \frac{3}{4} \rho^4 \cos^4 \psi \sin^4 \theta_p \mathcal{I}_0 \\
\mathcal{I}_{2,2,0,0} &= \frac{1}{3} \mathcal{I}_{4,0,0,0} = \frac{1}{3} \rho^4 \cos^4 \psi \sin^4 \theta_p \mathcal{I}_0 \\
\mathcal{I}_{3,0,1,0} &= \mathcal{I}_{3,0,0,1} = \frac{3}{4} \rho^4 \cos^3 \psi \sin \sin^3 \theta_p \sin \theta_k \mathcal{I}_1 \\
\mathcal{I}_{1,2,1,0} &= \frac{1}{3} \mathcal{I}_{3,0,1,0} = \frac{1}{3} \rho^4 \cos^3 \psi \sin \sin^3 \theta_p \sin \theta_k \mathcal{I}_1 \\
\mathcal{I}_{2,0,2,0} &= \mathcal{I}_{0,2,0,2} = \frac{1}{4} \rho^4 \cos^2 \psi \sin^2 \psi \sin^2 \theta_p \sin^2 \theta_k \mathcal{I}_0 (2\mathcal{I}_0 + \mathcal{I}_2) \\
\mathcal{I}_{2,0,0,2} &= \mathcal{I}_{0,2,2,0} = \frac{1}{4} \rho^4 \cos^2 \psi \sin^2 \psi \sin^2 \theta_p \sin^2 \theta_k \mathcal{I}_0 (2\mathcal{I}_0 - \mathcal{I}_2) \\
\mathcal{I}_{1,1,1,1} &= \frac{1}{2} (\mathcal{I}_{2,0,2,0} - \mathcal{I}_{2,0,0,2}) = \frac{1}{3} \rho^4 \cos^2 \psi \sin^2 \psi \sin^2 \theta_p \sin^2 \theta_k \mathcal{I}_2 \\
\mathcal{I}_{1,0,3,0} &= \mathcal{I}_{1,0,0,3} = \frac{3}{4} \rho^4 \cos \psi \sin^3 \psi \sin \theta_p \sin^3 \theta_k \mathcal{I}_1 \\
\mathcal{I}_{1,0,1,2} &= \frac{1}{3} \mathcal{I}_{1,0,3,0} = \frac{1}{3} \rho^4 \cos \psi \sin^3 \psi \sin \theta_p \sin^3 \theta_k \mathcal{I}_1 \\
\mathcal{I}_{0,0,4,0} &= \mathcal{I}_{0,0,0,4} = \frac{3}{4} \rho^4 \sin^4 \psi \sin^4 \theta_k \mathcal{I}_0 \\
\mathcal{I}_{0,0,2,2} &= \frac{1}{4} \mathcal{I}_{0,0,4,0} = \frac{1}{4} \rho^4 \sin^4 \psi \sin^4 \theta_k \mathcal{I}_0
\end{align*}
\]  
(C.76)
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Finally, one is left with five-dimensional integrals of the form

$$\int_0^\infty d\rho \int d\psi \int d\theta_p \int d\theta_k \rho^5 \sin^2 \psi \cos^2 \psi \sin \theta_p \sin \theta_k \mathcal{I}_{n_1,n_2,n_3,n_4}(\rho,\psi,\theta_p,\theta_k)$$

$$= \int_0^\infty d\rho \int d\psi \int d\theta_p \int d\theta_k \rho^5 \sin^2 \psi \cos^2 \psi \sin \theta_p \sin \theta_k \alpha(\rho,\psi,\theta_p,\theta_k) \times \int_0^{2\pi} d\phi_- (2\pi - \phi_-) \tilde{f}(\rho,\psi,\theta_p,\theta_k, \cos \phi_-) \cos n\phi_-, \quad (C.77)$$

which are solved with a FORTRAN code utilising the numerical methods mentioned above.

C.3.2 Actual Integrands for the Relativistic Transition Amplitude

The relevant integrands of the spatial basis functions referring to the initial and final baryons are given in Eqs. (5.62) and (5.63), respectively. According to Eq. (5.66), where $L' = M_{L'} = 0$ for the final baryon state, we have to consider different orbital angular momenta $L$ concerning the decaying baryon (see also Eqs. (A.45) and (A.50)-(A.55)). In the following we list the integrands that result from the pseudovector coupling\(^1\) corresponding to Eq. (5.62), which is given by

$$F_{i \rightarrow f}^{\mu\nu} = \sum_{\mu^1\mu^2\mu^3, \mu'^1\mu'^2\mu'^3} \int d^3k_2 d^3k_3 f(\omega_i,\omega''_i) \Psi^*_M \Sigma^* M_T^* M_T' \left( k''_i; \mu''_i \right) F^m f^{\mu\nu} N$$

$$\times \left\{ [f_1^0 \sigma_z(1) + g_0^0 \xi_1] \omega_m - \left[ f_1^3 \sigma_z(1) + g_1^3 \xi_1 \right] Q \right\}_{\mu''_i \mu_i} \times \{ f_2 \textbf{l}_2 + ig_2 \xi_2 \}_{\mu''_i \mu_2} \{ f_3 \textbf{l}_2 + ig_3 \xi_3 \}_{\mu''_i \mu_3}$$

$$\times \Psi_{M \Sigma M_T^* M_T'} \left( \textbf{k}_i; \mu_i \right), \quad (C.78)$$

with

$$f(\omega_i,\omega''_i) = \frac{\omega''_i \omega'_i \sqrt{(\omega_1 + \omega_2 + \omega_3)^3 (\omega'_1 + \omega'_2 + \omega'_3)^3}}{\sqrt{2\omega_1 \omega'_1}}. \quad (C.79)$$

According to Eq. (C.1) the initial and final baryon wave functions are given as the sums over the corresponding basis functions represented in all three configurations (1), (2), and (3). Hence, the individual integrands

\(^1\)The case of pseudoscalar coupling can be obtained by a simple replacement, see the final remark at the end of this section.
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$i, (k) (I_{(LM), i → f}^{pv})^{j, (p)}$ included in Eq. (C.78) are specified by the indices $i$ and $j$ denoting the number of the basis functions of the involved baryons as well as by the configurations $(k)$ and $(p)$ concerning the final and initial state. Each of the integrands depends on the discrete quantum numbers $\lambda$, $l$, and $L$ concerning the decaying baryon (the final baryon wave function is fixed to $\lambda' = l' = L' = 0$). Neglecting the flavor part of the basis functions and leaving out overall constants let us pick out one of the integrands $I_{(LM), i → f}^{pv}$ with fixed indices $i$, $j$, $(k)$, and $(p)$ and discuss the cases for different quantum numbers $\lambda$, $l$, and $L$:

$L = M_L = 0, (\lambda = l = 0)$:

$$I_{(000), i → f}^{pv} = f(\omega_i, \omega_i'') \exp \left[ -\frac{1}{4\beta^2} p''^2 - \frac{1}{4\delta^2} k''^2 \right] \exp \left[ -\frac{1}{4\beta^2} p^2 - \frac{1}{4\delta^2} k^2 \right] \times \left\{ \tilde{f}_1 f_2 f_3 \sigma_z (1) + i \tilde{g}_1 g_2 f_3 \xi_1 \xi_2 + i \tilde{g}_1 f_2 g_3 \xi_1 \xi_3 - \tilde{f}_1 g_2 g_3 \sigma_z (1) \xi_2 \xi_3 \right\} \mu'_\mu_i. \quad (C.80)$$

In this case, both the decaying and the final baryons have zero total orbital angular momentum, and also the subsystem angular momenta $\lambda$ and $l$ (see Section A.5) are both zero.

For a decaying baryon with total orbital angular momentum $L = 1$ the situation is more intricate. According to the triangular rule, $L = 1$ can be built through different combinations of $\lambda$ and $l$, where in this work we restricted the combinations to $(\lambda = 1, l = 0)$ and $(\lambda = 0, l = 1)$. The former one leads to the spatial basis functions specified in Eqs. (A.50) - (A.52) for $M_L = 0, \pm 1$. Accordingly, the latter case corresponds to the basis functions given in Eqs. (A.53) - (A.55). Therefore, we are left with two possible integrands, namely

$L = 1, M_L = 0, \pm 1, (\lambda = 1, l = 0)$:

$$I_{(110), i → f}^{pv} = f(\omega_i, \omega_i'') \exp \left[ -\frac{1}{4\beta^2} p''^2 - \frac{1}{4\delta^2} k''^2 \right] \exp \left[ -\frac{1}{4\beta^2} p^2 - \frac{1}{4\delta^2} k^2 \right] \times \left\{ C_{11S+1/2}^{\Sigma_{1/2}} + C_{11S-1/2}^{\Sigma_{1/2}} \right\} \left( \tilde{g}_1 f_2 f_3 \xi_1 + i \tilde{f}_1 g_2 f_3 \sigma_z (1) \xi_2 + i \tilde{f}_1 f_2 g_3 \sigma_z (1) \xi_3 - \tilde{g}_1 g_2 g_3 \xi_1 \xi_2 \xi_3 \right) \quad (C.81)$$

$L = 1, M_L = 0, \pm 1, (\lambda = 0, l = 1)$:

$$I_{(101), i → f}^{pv} = f(\omega_i, \omega_i'') \exp \left[ -\frac{1}{4\beta^2} p''^2 - \frac{1}{4\delta^2} k''^2 \right] \exp \left[ -\frac{1}{4\beta^2} p^2 - \frac{1}{4\delta^2} k^2 \right]$$
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\[
\times \left\{ C_{11S-\frac{1}{2}}^{\frac{1}{2}} k^+ + C_{1-\frac{1}{2}}^{\frac{1}{2}} k^- \right\} \left( \hat{g}_1 f_2 f_3 \xi_1 + i \hat{f}_1 g_2 f_3 \sigma_z(1) \xi_2 + i \hat{f}_1 f_2 g_3 \sigma_z(1) \xi_3 \\
- \hat{g}_1 g_2 g_3 \xi_2 \xi_3 + C_{10S-\frac{1}{2}}^{\frac{1}{2}} k^0 \left( \hat{f}_1 f_2 f_3 \sigma_z(1) + i \hat{g}_1 g_2 f_3 \xi_2 \xi_3 \\
+ i \hat{g}_1 f_2 g_3 \xi_2 \xi_3 - \hat{f}_1 g_2 g_3 \sigma_z(1) \xi_2 \xi_3 \right) \right\}_{\mu'\mu}.
\]  
(C.82)

Here we introduced the abbreviations

\[
\hat{f}_1 = f_1^0 \omega_m - f_1^3 Q, \\
\hat{g}_1 = g_1^0 \omega_m - g_1^3 Q.
\]  
(C.83)

For the terms \( f_i \) as well as \( g_i \) for \( i = 2, 3 \) and \( \xi_i \) for \( i = 1, 2, 3 \) we refer to Eqs. (5.60) and (5.64) in Chapter 5, respectively; eventually the terms \( f_0^1 \), \( g_0^1 \), \( f_1^3 \), and \( g_1^3 \) are defined in Eqs. (5.54). For simplicity we write down only the basis functions for the case where the configuration coincides with the partition. In Eqs. (C.80) - (C.82) one observes that only certain terms of the original integrand in Eq. (5.62) are kept. The other terms have been left out, since they do not contribute to the integral due to the properties of the sine and cosine (as even or odd functions) in the integration interval.

Let us briefly take a look at the simplest case, i.e. Eq. (C.80). Writing out the \( \xi_i \) by use of Eqs. (5.64) the curly bracket in the second line leads us to

\[
\left\{ \hat{f}_1 f_2 f_3 \sigma_z(1) + i \hat{g}_1 g_2 f_3 \xi_1 \xi_2 + i \hat{g}_1 f_2 g_3 \xi_1 \xi_3 - \hat{f}_1 g_2 g_3 \sigma_z(1) \xi_2 \xi_3 \right\}_{\mu'\mu}
\]
(C.84)

The occurring spin operators are sandwiched between baryon spin states. This leads to expressions like written down in Eqs. (C.10). In particular, we obtain for the case of Eq. (C.84):

\[
\left\{ \hat{f}_1 f_2 f_3 \sigma_z(1) + \hat{g}_1 g_2 f_3 k_{1x} k_{2x} i (\sigma_x(1) \sigma_y(2) - \sigma_y(1) \sigma_x(2)) \\
+ \hat{g}_1 f_2 g_3 k_{1x} k_{3x} i (\sigma_z(1) \sigma_y(3) - \sigma_y(1) \sigma_z(3)) \\
- \hat{f}_1 g_2 g_3 k_{2x} k_{3x} \sigma_z(1) (\sigma_x(2) \sigma_x(3) + \sigma_y(2) \sigma_y(3)) \right\}_{\mu'\mu}
\]
(C.85)
CALCULATION OF MATRIX ELEMENTS

This formula is introduced as it stands into the numerical computer code. The procedure is executed in the same manner for the other two cases, however the expressions are rather lengthy and we refrain from giving them here.

For the integrands of the transition amplitude utilising a pseudoscalar coupling one only has to replace the $\tilde{f}_1 (\tilde{g}_1)$ by $f_1 (g_1)$ defined in Eqs. (5.58).
Appendix D

Spectra of the Baryon Resonances

Baryon states are characterized by their masses, intrinsic spins and parities, where one usually abbreviates the spin and the parity with the notation \( J^P \). In this appendix we list these quantities as resulting from experiment and the GBE as well as the OGE CQMs.

The experimental data of all baryon states addressed in this work are taken from the latest compilation of the PDG [2]. In addition to the phenomenologically confirmed three- and four-star states of the Baryon Summary Table we include also some selective (low-lying) two-star resonances. In order to distinguish the three- and four-star states from the two-star states, we specify the latter ones with "**".

The theoretical results are obtained from the pseudoscalar version of the GBE CQM as parameterized in Ref. [41] and the OGE CQM as parameterized in Ref. [19]. In Figs. 2.1, 2.2, and E.2 the corresponding mass spectra are shown in comparison to experiment.

In Tables D.1 - D.6 we list the results for the various light and strange baryons, i.e. the nucleon, \( \Delta \), \( \Lambda \), \( \Sigma \), \( \Xi \), and \( \Omega \) resonances. In the first column of each table we denote the baryon resonance according to the nomenclature of the PDG. In the second column the pertinent (intrinsic) spin and parity \( J^P \) are specified, in the third column the total orbital angular momentum \( L \) and the total spin \( S \) are listed. The fourth and fifth columns contain the theoretical mass as predicted by the GBE and OGE CQMs. In the last two columns the experimental mass is given together with the measured spin and parity \( J^P \). As we always assume isospin symmetry we quote a mass range for each isospin multiplet. In case the spin and/or parity are not available from experiment, we put a question mark in the last column.

By the investigation of the baryon spectra alone one is not always able to uniquely attribute all theoretical states to the proper phenomenological counterparts. Especially, in cases, where some low-lying resonances are not
SPECTRA OF THE BARYON RESONANCES

Table D.1: Ground state and resonance energy levels in the nucleon spectrum.

<table>
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<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Mass [MeV]</td>
<td></td>
</tr>
<tr>
<td>$N$</td>
<td>$\frac{1}{2}^+$</td>
<td>0, $\frac{1}{2}$</td>
<td>939</td>
<td>939</td>
<td>938 - 940</td>
<td>$\frac{1}{2}^+$</td>
</tr>
<tr>
<td>$N(1440)$</td>
<td>$\frac{1}{2}^+$</td>
<td>0, $\frac{1}{2}$</td>
<td>1459</td>
<td>1577</td>
<td>1420 - 1470</td>
<td>$\frac{1}{2}^+$</td>
</tr>
<tr>
<td>$N(1520)$</td>
<td>$\frac{3}{2}^-$</td>
<td>1, $\frac{1}{2}$</td>
<td>1519</td>
<td>1521</td>
<td>1515 - 1525</td>
<td>$\frac{3}{2}^-$</td>
</tr>
<tr>
<td>$N(1535)$</td>
<td>$\frac{1}{2}^-$</td>
<td>1, $\frac{1}{2}$</td>
<td>1519</td>
<td>1521</td>
<td>1525 - 1545</td>
<td>$\frac{1}{2}^-$</td>
</tr>
<tr>
<td>$N(1650)$</td>
<td>$\frac{1}{2}^-$</td>
<td>1, $\frac{3}{2}$</td>
<td>1647</td>
<td>1690</td>
<td>1645 - 1670</td>
<td>$\frac{1}{2}^-$</td>
</tr>
<tr>
<td>$N(1675)$</td>
<td>$\frac{5}{2}^-$</td>
<td>1, $\frac{3}{2}$</td>
<td>1647</td>
<td>1690</td>
<td>1670 - 1680</td>
<td>$\frac{5}{2}^-$</td>
</tr>
<tr>
<td>$N(1700)$</td>
<td>$\frac{3}{2}^-$</td>
<td>1, $\frac{3}{2}$</td>
<td>1647</td>
<td>1690</td>
<td>1650 - 1750</td>
<td>$\frac{3}{2}^-$</td>
</tr>
<tr>
<td>$N(1710)$</td>
<td>$\frac{1}{2}^+$</td>
<td>0, $\frac{1}{2}$</td>
<td>1776</td>
<td>1859</td>
<td>1680 - 1740</td>
<td>$\frac{1}{2}^+$</td>
</tr>
</tbody>
</table>

firmly established (as three- or four-star states) or their spins and/or parities are not measured, it is not always clear which theoretical state should be identified with which experimentally observed state. For instance, in the $\Xi$ spectrum the spins and parities of the three-star resonances $\Xi(1690)$ and $\Xi(1950)$ are not determined from experiment, and one does not know which theoretical states should be attributed to these resonances. Through studies of further properties of the resonances, such as the mesonic decays done in this work, one can get further hints for the correspondence between theoretical and experimental states. Regarding the above-mentioned excitations $\Xi(1690)$ and $\Xi(1950)$ we find that they are best classified as $\frac{1}{2}^+$ and $\frac{5}{2}^-$ states, respectively. In this way they fit into flavor multiplets of alike baryon resonances with consistent properties (seen by means of decay widths). In the $\Sigma$ spectrum we have also included several low-lying two-star resonances. They are found to correspond to some theoretical states produced by the CQMs and at the same time they are needed to complete certain excited flavor multiplets according to the classification scheme of baryon ground and excited states discussed in Chapter 7 and detailed in Appendix E.
## Spectra of the Baryon Resonances

Table D.2: Ground state and resonance energy levels in the $\Delta$ spectrum.

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>$\Delta$</td>
<td>$\frac{3}{2}^+$</td>
<td>0, $\frac{3}{2}$</td>
<td>1240</td>
<td>1231</td>
<td>1231 − 1233</td>
<td>$\frac{3}{2}^+$</td>
<td></td>
</tr>
<tr>
<td>$\Delta(1600)$</td>
<td>$\frac{3}{2}^+$</td>
<td>0, $\frac{3}{2}$</td>
<td>1718</td>
<td>1854</td>
<td>1550 − 1700</td>
<td>$\frac{3}{2}^+$</td>
<td></td>
</tr>
<tr>
<td>$\Delta(1620)$</td>
<td>$\frac{1}{2}^-$</td>
<td>1, $\frac{1}{2}$</td>
<td>1642</td>
<td>1621</td>
<td>1600 − 1660</td>
<td>$\frac{1}{2}^-$</td>
<td></td>
</tr>
<tr>
<td>$\Delta(1700)$</td>
<td>$\frac{3}{2}^-$</td>
<td>1, $\frac{1}{2}$</td>
<td>1642</td>
<td>1621</td>
<td>1670 − 1750</td>
<td>$\frac{3}{2}^-$</td>
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</table>

Table D.3: Ground state and resonance energy levels in the $\Lambda$ spectrum.

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<tr>
<td>$\Lambda$</td>
<td>$\frac{1}{2}^+$</td>
<td>0, $\frac{1}{2}$</td>
<td>1136</td>
<td>1113</td>
<td>1116</td>
<td>$\frac{1}{2}^+$</td>
<td></td>
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<td>$\Lambda(1405)$</td>
<td>$\frac{1}{2}^-$</td>
<td>1, $\frac{1}{2}$</td>
<td>1556</td>
<td>1628</td>
<td>1402 − 1410</td>
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<td></td>
</tr>
<tr>
<td>$\Lambda(1520)$</td>
<td>$\frac{3}{2}^-$</td>
<td>1, $\frac{1}{2}$</td>
<td>1556</td>
<td>1628</td>
<td>1519 − 1521</td>
<td>$\frac{3}{2}^-$</td>
<td></td>
</tr>
<tr>
<td>$\Lambda(1600)$</td>
<td>$\frac{1}{2}^+$</td>
<td>0, $\frac{1}{2}$</td>
<td>1625</td>
<td>1747</td>
<td>1560 − 1700</td>
<td>$\frac{1}{2}^+$</td>
<td></td>
</tr>
<tr>
<td>$\Lambda(1670)$</td>
<td>$\frac{1}{2}^-$</td>
<td>1, $\frac{1}{2}$</td>
<td>1682</td>
<td>1734</td>
<td>1660 − 1680</td>
<td>$\frac{1}{2}^-$</td>
<td></td>
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<tr>
<td>$\Lambda(1690)$</td>
<td>$\frac{3}{2}^-$</td>
<td>1, $\frac{1}{2}$</td>
<td>1682</td>
<td>1734</td>
<td>1685 − 1695</td>
<td>$\frac{3}{2}^-$</td>
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<td>$\Lambda(1800)$</td>
<td>$\frac{1}{2}^-$</td>
<td>1, $\frac{3}{2}$</td>
<td>1778</td>
<td>1844</td>
<td>1720 − 1850</td>
<td>$\frac{1}{2}^-$</td>
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<tr>
<td>$\Lambda(1810)$</td>
<td>$\frac{1}{2}^+$</td>
<td>0, $\frac{1}{2}$</td>
<td>1799</td>
<td>1957</td>
<td>1750 − 1850</td>
<td>$\frac{1}{2}^+$</td>
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<td>$\Lambda(1830)$</td>
<td>$\frac{5}{2}^-$</td>
<td>1, $\frac{3}{2}$</td>
<td>1778</td>
<td>1844</td>
<td>1810 − 1830</td>
<td>$\frac{5}{2}^-$</td>
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</table>
SPECTRA OF THE BARYON RESONANCES

Table D.4: Ground state and resonance energy levels in the $\Sigma$ spectrum.

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<tr>
<td>$\Sigma$</td>
<td>$1^+ 0, \frac{1}{2}^{-}$</td>
<td>1189 – 1197</td>
<td>$\frac{1}{2}^+$</td>
<td>1180</td>
<td>1213</td>
<td></td>
<td></td>
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<tr>
<td>$\Sigma(1385)$</td>
<td>$\frac{3}{2}^+$</td>
<td>1383 – 1387</td>
<td>$\frac{3}{2}^+$</td>
<td>1389</td>
<td>1373</td>
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<tr>
<td>$\Sigma(1560)**$</td>
<td>$\frac{1}{2}^-$</td>
<td>1546 – 1576</td>
<td>$\frac{1}{2}^-$</td>
<td>1677</td>
<td>1732</td>
<td></td>
<td></td>
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<tr>
<td>$\Sigma(1620)**$</td>
<td>$\frac{3}{2}^-$</td>
<td>1594 – 1643</td>
<td>$\frac{3}{2}^-$</td>
<td>1736</td>
<td>1829</td>
<td></td>
<td></td>
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<tr>
<td>$\Sigma(1660)$</td>
<td>$\frac{1}{2}^+$</td>
<td>1630 – 1690</td>
<td>$\frac{1}{2}^+$</td>
<td>1616</td>
<td>1845</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Sigma(1670)$</td>
<td>$\frac{3}{2}^-$</td>
<td>1665 – 1685</td>
<td>$\frac{3}{2}^-$</td>
<td>1677</td>
<td>1732</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Sigma(1690)**$</td>
<td>$\frac{3}{2}^+$</td>
<td>1670 – 1727</td>
<td>$\frac{3}{2}^+$</td>
<td>1865</td>
<td>1991</td>
<td></td>
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<tr>
<td>$\Sigma(1750)$</td>
<td>$\frac{1}{2}^-$</td>
<td>1730 – 1800</td>
<td>$\frac{1}{2}^-$</td>
<td>1759</td>
<td>1784</td>
<td></td>
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<tr>
<td>$\Sigma(1775)$</td>
<td>$\frac{5}{2}^-$</td>
<td>1770 – 1780</td>
<td>$\frac{5}{2}^-$</td>
<td>1736</td>
<td>1829</td>
<td></td>
<td></td>
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<tr>
<td>$\Sigma(1880)**$</td>
<td>$\frac{1}{2}^+$</td>
<td>1806 – 2025</td>
<td>$\frac{1}{2}^+$</td>
<td>1911</td>
<td>2049</td>
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<tr>
<td>$\Sigma(1940)$</td>
<td>$\frac{3}{2}^-$</td>
<td>1900 – 1950</td>
<td>$\frac{3}{2}^-$</td>
<td>1736</td>
<td>1829</td>
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Table D.5: Ground state and resonance energy levels in the $\Xi$ spectrum.

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<tbody>
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<td>$\Xi$</td>
<td>$\frac{1}{2}^+$</td>
<td>1315 – 1321</td>
<td>$\frac{1}{2}^+$</td>
<td>1348</td>
<td>1346</td>
<td></td>
<td></td>
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<tr>
<td>$\Xi(1530)$</td>
<td>$\frac{3}{2}^+$</td>
<td>1532 – 1535</td>
<td>$\frac{3}{2}^+$</td>
<td>1528</td>
<td>1516</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Xi(1690)$</td>
<td>$\frac{1}{2}^+$</td>
<td>1680 – 1700</td>
<td>$\frac{1}{2}^+$</td>
<td>1805</td>
<td>1975</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Xi(1820)$</td>
<td>$\frac{3}{2}^-$</td>
<td>1818 – 1828</td>
<td>$\frac{3}{2}^-$</td>
<td>1792</td>
<td>1894</td>
<td></td>
<td></td>
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<tr>
<td>$\Xi(1950)$</td>
<td>$\frac{3}{2}^-$</td>
<td>1935 – 1965</td>
<td>$\frac{3}{2}^-$</td>
<td>1881</td>
<td>1993</td>
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Table D.6: $\Omega$ ground state.

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<tr>
<td>$\Omega$</td>
<td>$\frac{3}{2}^+$</td>
<td>1672</td>
<td>$\frac{3}{2}^+$</td>
<td>1656</td>
<td>1661</td>
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Appendix E

Flavor Multiplets of Baryon Resonances

Light and strange baryons are classified according to their flavor symmetry. Within $SU(3)_F$ one has the flavor singlet, octet, and decuplet for light and strange baryons. There is no problem in identifying the members of the ground-state flavor multiplets. They are well known both from theory and experiment. For the excited multiplets, however, one faces difficulties in classifying the proper excited states (resonances). Baryon spectroscopy alone is usually not sufficient for a unique identification of the various resonances. Rather one has to include further properties for grouping the states into the right multiplets. Important criteria for that are the particular symmetries in the space, spin, and flavor degrees of freedom. All together have to produce a symmetric three-quark state. In this context it is essential to investigate the particular flavor symmetry of the various baryon states. Within CQMs this can be achieved by applying appropriate flavor projection operators. The spatial and spin symmetries can be best explored by analyzing such observables as decay widths etc. They provide the necessary additional insight into the structure of the wave functions of ground and excited states so that they can be attributed to the proper flavor multiplets.

In the following we will first discuss the flavor symmetries in the various baryon ground-states and resonances, then we take a closer look at the corresponding probability densities along with results for mesonic decay widths. Finally, we discuss the particular flavor multiplets in detail.

E.1 Flavor Symmetries in Baryon Resonances

Within the concept of CQMs following $SU(3)_F$ the light and strange baryons can be classified according to their flavor symmetry. For the $N$, $\Delta$, and $\Omega$ baryons it is immediately clear to which flavor multiplet they belong: The nucleon is a pure flavor octet, whereas the $\Delta$ and $\Omega$ are pure flavor decuplets.
On the other hand, the Λ baryon is generally a mixture of flavor singlet and octet, whereas the Σ and Ξ baryons are mixtures of flavor octet and decuplet. For the classification of the baryons into the various (excited) multiplets it is important to know the degree of mixing. It can be determined by applying a projection operator on the specific flavor multiplets.

In momentum space the baryon wave functions are normalized by the conditions (A.57) and (B.47), using either the Jacobi momenta $p$ and $k$ or the internal quark momenta $k_2$ and $k_3$ of the quarks 2 and 3, respectively. Starting out from the normalization condition (A.57) the flavor octet, singlet, and decuplet percentages $\alpha^{fm}$ in the Λ, Σ, and Ξ baryons can be calculated by applying the projection operator $P^{fm}$

$$\alpha^{fm} = \sum_{\mu_1 \mu_2 \mu_3} \int d^3p d^3k \Psi^*_M \Sigma \Sigma M_{\Sigma \Sigma} T M_T (p, k; \mu_i) P^{fm} \Psi_M \Sigma \Sigma M_{\Sigma \Sigma} T M_T (p, k; \mu_i).$$

(E.1)

Here, the superscript ”$fm$” denotes the flavor multiplet, i.e. the octet is denoted by ”$8$”, the singlet by ”$1$”, and the decuplet by ”$10$”. Obviously, $\alpha^{fm}$ can range from 0 to 1 corresponding to zero and 100 percent contribution from a specific multiplet.

**Flavor Symmetry of Λ Baryons**

In order to find out the respective singlet and octet percentages in the Λ states, one starts with calculating the singlet percentage $\alpha^1$ by applying the flavor-singlet projection operator $P^1$

$$P^1 = \frac{1}{6} (|uds\rangle - |usd\rangle + |dus\rangle - |sud\rangle - |sdu\rangle)$$

$$\times (|uds\rangle - |usd\rangle + |dus\rangle - |sud\rangle - |sdu\rangle).$$

(E.2)

It is a completely antisymmetric operator containing 36 components. The octet percentage is then given by

$$\alpha^8 = 1 - \alpha^1.$$  (E.3)

**Flavor Symmetries of Σ and Ξ Baryons**

Similarly, in case of the Σ and Ξ baryons one determines the respective octet and decuplet percentages by first applying the flavor-decuplet projection operator $P^{10}$. Here it is sufficient to consider only one member of an isospin multiplet. For the Σ we get along with

$$P^{10} (\Sigma^0) = \frac{1}{6} (|uds\rangle + |usd\rangle + |dus\rangle + |sud\rangle + |sdu\rangle)$$

$$\times (|uds\rangle + |usd\rangle + |dus\rangle + |sud\rangle + |sdu\rangle).$$  (E.4)
FLAVOR MULTIPLETS OF BARYON RESONANCES

In case of the $\Xi$ one uses

$$P_{10}^{10}(\Xi^0) = \frac{1}{3} (|ssu| + |sus| + |uss|)^2 (|ssu| + |sus| + |uss|)^2$$

The octet percentage is then obtained by

$$\alpha^8 = 1 - \alpha^{10}. \quad (E.5)$$

E.2 Probability Densities of Baryon Resonances

For additional insight into the structure of the baryon states we examine the spatial distributions of the probability densities resulting from the baryon wave functions. The baryon wave functions are normalized by

$$\int d^3\xi d^3\eta \Psi^{*}_{M\Sigma M_T T M_T}(\xi, \eta) \Psi_{M\Sigma M_T T M_T}(\xi, \eta) = 1. \quad (E.6)$$

Here, $\xi$ and $\eta$ are the Jacobi coordinates, $M$ is the mass and $\Sigma$ as well as $T$ are the intrinsic spin and isospin with $z$-projections $M_\Sigma$ and $M_T$ (for details of the notation regarding the wave functions see Appendix A). Evidently, the integral in Eq. (E.6) consists of radial and angular parts; one can separate the radial from the angular integration and carry out the latter:

$$\int d\xi d^3\eta \Psi^{*}_{M\Sigma M_T T M_T}(\xi, \Omega, \eta) \Psi_{M\Sigma M_T T M_T}(\xi, \Omega, \eta) = \int d\xi d\eta \rho(\xi, \eta). \quad (E.7)$$

Here, $\rho(\xi, \eta)$ represents the spatial probability density (referring to the regular part of the baryon wave function); the complete angular dependence, which can be reduced to a relative angle between $\xi$ and $\eta$, has been integrated over. The dependence of $\rho(\xi, \eta)$ on $\xi$ and $\eta$ provides insight into the spatial probability distribution of the three-quark system.

E.3 Symmetry Structure of Baryon Wave Functions and Connection to Decay Widths

E.3.1 GBE and OGE Wave Functions

First, it is interesting to see, if there are any qualitative differences between the baryon wave functions from the two different types of CQMs considered in this work (both solved within the SVM). Therefore, we calculated the probability densities pertaining to the GBE and OGE CQMs. We refrain from showing all of them here. Instead, we have plotted the example of the probability densities of the nucleon ground state and its first radial
excitation, the Roper resonance $N(1440)$, in Fig. E.1. Obviously, the two CQMs lead to density distributions with qualitatively congruent shapes. This is true also for all the other baryon states considered in this work. The actual differences between the two CQMs consist only in spatial distributions of the OGE CQM that are narrower around the origin (as is clearly visible from Fig. E.1). This is due to a stronger confinement interaction in the OGE CQM.

In the following we will constrain the investigation of the symmetry structure of the baryon wave functions (by density distributions) to the case of the GBE CQM only. For the OGE CQM one would obtain qualitatively similar results.

E.3.2 Wave Functions and Decay Widths in Baryon Flavor Multiplets

Now we provide a detailed analysis of the symmetry properties of the spatial parts of all the baryon wave functions. This is also an important issue for the understanding of the matrix elements in the calculation of the decay widths. Beyond the ground states we consider also all baryon resonances
Figure E.2: Energy levels of $\Sigma$ and $\Xi$ baryon resonances for the OGE (left) and GBE (right) CQMs as parametrized in Refs. [41] and [19], respectively. The shadowed boxes represent the experimental values with their uncertainties [2].

The grouping of the baryon states into the various flavor multiplets is done according to their $J^P$ and their properties evidenced by the flavor symmetries and the typical spatial behaviour as well as the characteristics of the results for the decay widths. In the tables below we list for the various decay modes the experimentally known partial decay widths as resulting from the PFSM decay operator in pseudovector coupling. In case they are not measured we give the total decay widths instead. In the fourth and fifth columns we give the partial decay widths as resulting from the GBE and OGE CQMs, and in the last two columns we present them as percentages of the corresponding experimental decay widths.

$\frac{1}{2}^+ \text{ Octet Ground States: } N(939), \Sigma(1193), \Xi(1318), \text{ and } \Lambda(1116)$

The octet ground states are characterized by intrinsic spin and parity $J^P = \frac{1}{2}^+$ and they have no strong decays. For the calculations of their wave functions as done in this work the intrinsic spin (total angular momentum) $J$ is constructed from total orbital angular momentum $L = 0$ and total spin $S = \frac{1}{2}$. Due to the constraint of a totally symmetric wave func-
tion (see Eq. (2.34)), the spatial and the spin-flavor parts must have the same symmetries. For the octet ground states, whose spin-flavor parts are more or less totally symmetric\(^1\), this means that also the spatial parts are almost completely symmetric. From Fig. E.3 we thus get the impression of an almost symmetric spatial density distribution for all ground-state octet members.

\( ^{1}\) Octet States: \( N(1440), \Sigma(1660), \Xi(1690), \text{ and } \Lambda(1600) \)

The states \( N(1440), \Sigma(1660), \Xi(1690), \text{ and } \Lambda(1600) \) all behave like the first radial excitation of the \( J^P = \frac{1}{2}^+ \) ground-state octet and thus they have the same total orbital angular momentum \( L = 0 \) and total spin \( S = \frac{1}{2} \). The wave functions have almost the same spin-flavor symmetry properties as the octet ground states. The corresponding probability densities \( \rho(\xi, \eta) \) are shown in Fig. E.4. They have a quarter-ring shaped dip in their \( (\xi, \eta) \)-dependence.

The possible strong decays are listed in Table E.1 together with the experimental data and the theoretical predictions for partial decay widths from the GBE and OGE CQMs.

We note that the \( \Xi(1690) \) resonance is not determined experimentally with respect to its \( J^P \). We identify the first \( \frac{1}{2}^+ \) excitation in the \( \Xi \) spectrum with this state. This is justified by the typical spatial probability distribution of this state as well as the characteristics of its decay widths (congruent with the other ones from the same multiplet, all being rather small). In addition a state with this energy would not fit into any other \( \Xi \) excitation spectrum.

\( ^{1}\) Octet States: \( N(1710) \text{ and } \Sigma(1880) \)

The second excited \( \frac{1}{2}^+ \) states \( N(1710) \) and \( \Sigma(1880) \) with \( L = 0 \) and \( S = \frac{1}{2} \) have a predominantly mixed-symmetric spin-flavor part. Therefore the spatial part is also mainly characterized by a mixed symmetry and it comes along with the typical density distribution visible in Fig. E.5. With respect to the \( \Lambda \) state exhibiting a spin-flavor part with mainly mixed symmetry we note that this can be furnished both with an antisymmetric as well as mixed-symmetric flavor wave function, leading to two different \( \frac{1}{2}^+ \) \( \Lambda \) excitations. It turns out that the lower one, to be identified with the experimentally observed \( \Lambda(1810) \), is mainly a flavor singlet state (with a percentage of 92 \%, see the detailed discussion below). The higher one of these two excitations again fits into this octet. It has a mixed-symmetric flavor wave function and, of course, the mixed-symmetric spatial structure as typically seen in Fig. E.5. It is left out from the figure because in experiment there is no candidate, which it could be identified with. In the \( \Xi \) spectrum we also

\(^1\)For instance, the nucleon ground-state contains only a rather small mixed-symmetric spin-flavor part of about 1 \%. This, however, is very important for a realistic description of the neutron structure [98].
Figure E.3: $\frac{1}{2}^+$ octet baryon ground states $N(939)$, $\Sigma(1193)$, $\Xi(1318)$, and $\Lambda(1116)$. 
Table E.1: Covariant predictions for the partial decay widths (in MeV) as predicted by the GBE and OGE CQMs along the PFSM utilizing a pseudovector coupling in comparison to experiment [2] and given as percentages of the upper and lower bounds of the partial decay widths from experiment. Theoretical baryon masses are used, except for the cases denoted by a *, where experimental baryon masses are employed.

<table>
<thead>
<tr>
<th>Decay</th>
<th>Experiment</th>
<th>Relativistic GBE</th>
<th>OGE</th>
<th>% of Exp. Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N(1440) \rightarrow N\pi$</td>
<td>$(195 \pm 30)_{-55}^{+113}$</td>
<td>30</td>
<td>59</td>
<td>9 – 27</td>
</tr>
<tr>
<td>$\Sigma(1660) \rightarrow \Sigma\pi$</td>
<td>$\Gamma_{tot} = 40 – 200$</td>
<td>10</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td>$\Sigma(1660) \rightarrow \Lambda\pi$</td>
<td>$\Gamma_{tot} = 40 – 200$</td>
<td>8</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>$\Xi(1690) \rightarrow \Xi\pi$</td>
<td>$\Gamma_{tot} &lt; 30$</td>
<td>0.8</td>
<td>1.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.5*</td>
<td>0.5*</td>
<td></td>
</tr>
<tr>
<td>$\Lambda(1600) \rightarrow \Sigma\pi$</td>
<td>$(53 \pm 38)_{-10}^{+60}$</td>
<td>3</td>
<td>33</td>
<td>2 – 60</td>
</tr>
<tr>
<td>$\Sigma(1660) \rightarrow NK$</td>
<td>$(20 \pm 10)_{-6}^{+30}$</td>
<td>0.9</td>
<td>0.9</td>
<td>2 – 23</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.2*</td>
<td>0.5*</td>
<td>2 – 30*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 – 13*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Xi(1690) \rightarrow \Lambda K$</td>
<td>$\Gamma_{tot} &lt; 30$</td>
<td>1.1</td>
<td>1.3</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.5*</td>
<td>0.4*</td>
<td></td>
</tr>
<tr>
<td>$\Xi(1690) \rightarrow \Sigma K$</td>
<td>$\Gamma_{tot} &lt; 30$</td>
<td>9.2</td>
<td>55</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.06*</td>
<td>0.16*</td>
<td></td>
</tr>
<tr>
<td>$\Lambda(1600) \rightarrow NK$</td>
<td>$(33.75 \pm 11.25)_{-15}^{+30}$</td>
<td>15</td>
<td>35</td>
<td>20 – 200</td>
</tr>
<tr>
<td></td>
<td></td>
<td>14*</td>
<td>21*</td>
<td>19 – 187*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>28 – 280*</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure E.4: $\frac{1}{2}^+$ octet baryon states $N(1440)$, $\Sigma(1660)$, $\Xi(1690)$, and $\Lambda(1600)$. 
FLAVOR MULTIPLETS OF BARYON RESONANCES

observe a state with the same $L$ and $S$ quantum numbers, but the mass of this eigenstate is higher than 2 GeV, and we refrain from assigning this state to any experimentally known state.

The corresponding decay widths are shown in Table E.2. A comparison to experiment is only possible for the $N(1710)$. Its theoretical $N\pi$ decay widths are in agreement with the data, for the other decay modes, $N\eta$ and $\Lambda K$, the predictions turn out to be very small. For the $\Sigma(1880)$ the PDG does not give any partial decay widths nor a total decay width. Instead we have quoted a range for the total decay width that can be extracted from other available sources cited in Ref. [2].

Table E.2: Same notation as in Table E.1 but for the $\frac{1}{2}^+$ octet states $N(1710)$ and $\Sigma(1880)$.

<table>
<thead>
<tr>
<th>Decay</th>
<th>Experiment</th>
<th>Relativistic</th>
<th>% of Exp. Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N(1710)$ $\rightarrow N\pi$</td>
<td>(15 ± 5)$^{+30}_{-5}$</td>
<td>19 21</td>
<td>38–380</td>
</tr>
<tr>
<td>$\Sigma(1880)$ $\rightarrow \Lambda\pi$</td>
<td>$\Gamma_{tot} = 30 - 372$</td>
<td>1.7 0.5</td>
<td></td>
</tr>
<tr>
<td>$\Sigma(1880)$ $\rightarrow \Sigma\pi$</td>
<td>$\Gamma_{tot} = 30 - 372$</td>
<td>3.0 3.0</td>
<td></td>
</tr>
<tr>
<td>$N(1710)$ $\rightarrow N\eta$</td>
<td>(6 ± 1)$^{+11}_{-4}$</td>
<td>0.02 0.06</td>
<td>0–2</td>
</tr>
<tr>
<td>$\Sigma(1880)$ $\rightarrow \Sigma\eta$</td>
<td>$\Gamma_{tot} = 30 - 372$</td>
<td>0.01 0.03</td>
<td></td>
</tr>
<tr>
<td>$N(1710)$ $\rightarrow \Lambda K$</td>
<td>(15 ± 10)$^{+37.5}_{-2.5}$</td>
<td>$\approx 0$ $\approx 0$</td>
<td>$\approx 0$ $\approx 0$</td>
</tr>
<tr>
<td>$N(1710)$ $\rightarrow \Sigma K$</td>
<td>$\Gamma_{tot} = 50 - 250$</td>
<td>3.9 389</td>
<td></td>
</tr>
<tr>
<td>$\Sigma(1880)$ $\rightarrow N K$</td>
<td>$\Gamma_{tot} = 30 - 372$</td>
<td>0.02 $\approx 0$</td>
<td></td>
</tr>
<tr>
<td>$\Sigma(1880)$ $\rightarrow \Xi K$</td>
<td>$\Gamma_{tot} = 30 - 372$</td>
<td>0.11 0.06</td>
<td></td>
</tr>
</tbody>
</table>

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Figure E.5: $1^+$ octet baryon states $N(1710)$ and $\Sigma(1880)$. 
\( \frac{1}{2}^- \) **Octet States:** \( N(1535), \Sigma(1560), \) and \( \Lambda(1670) \)

The octet resonances \( N(1535), \Sigma(1560), \) and \( \Lambda(1670) \) have an intrinsic spin and parity \( J^P = \frac{1}{2}^- \), where \( J \) is constructed from \( L = 1 \) and \( S = \frac{1}{2} \). As the CQMs we consider do not contain any tensor (nor spin-orbit) forces, these states are degenerate with the corresponding ones in the \( \frac{3}{2}^- \) excitation spectra. Therefore, the wave functions of the \( N(1520), \Sigma(1670), \) and \( \Lambda(1690) \) states to be discussed (together with \( \Xi(1820) \)) below will also be the same as the ones shown in Fig. E.6. However, the decay widths of the corresponding states in the \( \frac{1}{2}^- \) and \( \frac{3}{2}^- \) spectra are different (compare Tables E.3 and E.5). The theoretical decay widths for \( N(1535) \to N\pi \) as well as \( N(1535) \to N\eta \) remain below the experimental data. On the other hand, the predictions for the \( \Lambda(1670) \to \Sigma\pi \) (and in case of the OGE CQM also for the \( \Lambda(1670) \to \Sigma\eta \)) decay widths considerably exceed the experimental data. The reason might be that the \( \Lambda(1670) \) is not a pure flavor octet but contains an appreciable flavor-singlet component of about 30%.

<table>
<thead>
<tr>
<th>Decay</th>
<th>Experiment</th>
<th>Relativistic</th>
<th>% of Exp.</th>
<th>Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N(1535) \to N\pi )</td>
<td>( (68 \pm 15)^{+14}_{-9} )</td>
<td>25</td>
<td>39</td>
<td>26–57</td>
</tr>
<tr>
<td>( \Sigma(1560) \to \Sigma\pi )</td>
<td>( \Gamma_{tot} = 9 – 109 )</td>
<td>58</td>
<td>102</td>
<td></td>
</tr>
<tr>
<td>( \Sigma(1560) \to \Lambda\pi )</td>
<td>( \Gamma_{tot} = 9 – 109 )</td>
<td>1.6</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>( \Lambda(1670) \to \Sigma\pi )</td>
<td>( (14.0 \pm 5.3)^{+8.3}_{-2.5} )</td>
<td>69</td>
<td>103</td>
<td>250–1113</td>
</tr>
<tr>
<td>( N(1535) \to N\eta )</td>
<td>( (79 \pm 11)^{+15}_{-11} )</td>
<td>27</td>
<td>35</td>
<td>26–47</td>
</tr>
<tr>
<td>( \Lambda(1670) \to \Lambda\eta )</td>
<td>( (6.1 \pm 2.6)^{+3.8}_{-2.5} )</td>
<td>19</td>
<td>152–1900</td>
<td></td>
</tr>
<tr>
<td>( \Sigma(1560) \to NK )</td>
<td>( \Gamma_{tot} = 9 – 109 )</td>
<td>8</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>( \Lambda(1670) \to NK )</td>
<td>( (8.75 \pm 1.75)^{+4.5}_{-2.0} )</td>
<td>0.27</td>
<td>0.03</td>
<td>2–5</td>
</tr>
<tr>
<td>( \Sigma(1560) \to NK )</td>
<td>( \Gamma_{tot} = 9 – 109 )</td>
<td>6*</td>
<td>5*</td>
<td></td>
</tr>
<tr>
<td>( \Lambda(1670) \to NK )</td>
<td>( (8.75 \pm 1.75)^{+4.5}_{-2.0} )</td>
<td>0.39*</td>
<td>0.42*</td>
<td>3–8*</td>
</tr>
</tbody>
</table>
Figure E.6: \( \frac{1}{7} \) octet baryon states \( N(1535), \Sigma(1560), \) and \( \Lambda(1670). \)
FLAVOR MULTIPTS OF BARYON RESONANCES

$\frac{1}{2}^-$ Octet States: $N(1650)$, $\Sigma(1620)$, and $\Lambda(1800)$

The states $N(1650)$, $\Sigma(1620)$, and $\Lambda(1800)$ with $J^P = \frac{1}{2}^-$ and $L = 1$, $S = \frac{3}{2}$ fall into a common flavor octet. They are again degenerate with states of higher intrinsic spins, namely the $N(1650)$ with the $\frac{3}{2}^-$ $N(1700)$ as well as the $\frac{5}{2}^-$ $N(1675)$, the $\Sigma(1620)$ with the $\frac{3}{2}^-$ $\Sigma(1940)$ as well as the $\frac{5}{2}^-$ $\Sigma(1775)$, and the $\Lambda(1800)$ with the $\frac{5}{2}^-$ $\Lambda(1830)$. For a degenerate $\frac{3}{2}^-$ $\Lambda$ state fitting in here there is no suitable candidate known from experiment. The measured $\frac{3}{2}^-$ $\Lambda(1690)$ rather falls into the octet with $N(1520)$ (see the subsection below). As a consequence of the degeneracies the corresponding wave functions of the above-mentioned states are, of course, all the same. The probability densities of the $N(1650)$, $\Sigma(1620)$, and the $\Lambda(1800)$ are shown in Fig. E.7. The possible decay widths are presented in Table E.4. While the predictions for the $N(1650) \rightarrow N\pi$ as well as $N(1650) \rightarrow \Lambda K$ decay widths are much too small, the $N(1650) \rightarrow N\gamma$ decay width is overestimated. Also the $\Lambda(1800) \rightarrow NK$ decay width remains much below the experiment. For the other cases only total decay widths can be quoted, where the data for the $\Lambda(1800)$ stem from the PDG and the other ones are collected from other available sources cited in Ref. [2].

$\frac{3}{2}^-$ Octet States: $N(1520)$, $\Sigma(1670)$, $\Xi(1820)$, and $\Lambda(1690)$

The negative-parity excitations $N(1520)$, $\Sigma(1670)$, $\Xi(1820)$, and $\Lambda(1690)$, specified by the quantum numbers $J^P = \frac{3}{2}^-$ and $L = 1$, $S = \frac{3}{2}$, have a spin-flavor part of predominantly mixed symmetry and fit together in a flavor octet. With the exception of $\Lambda(1690)$, all states represent the lowest $\frac{3}{2}^-$ excitations. In the $\Lambda$ spectrum the lowest $\frac{3}{2}^-$ excitation turns out to be the $\Lambda(1520)$, which is predominantly a flavor singlet. The probability densities are shown in Fig. E.8. As outlined above, for $N$, $\Sigma$, and $\Lambda$, they are the same as for the $J^P = \frac{1}{2}^-$ states $N(1535)$, $\Sigma(1560)$, and $\Lambda(1670)$. The widths of the possible decays are given in Table E.5. The theoretical predictions show the typical behaviour of structure-independent resonances.

$\frac{5}{2}^-$ Octet States: $N(1700)$ and $\Sigma(1940)$

The octet states $N(1700)$ and $\Sigma(1940)$ have an intrinsic spin and parity $J^P = \frac{5}{2}^-$ and $L = 1$, $S = \frac{3}{2}$. Both states represent the lowest radial excitations for the given $L$ and $S$ quantum numbers. Their density distributions are shown in Fig. E.9. As outlined above, they are the same as for the $J^P = \frac{1}{2}^-$ states $N(1650)$ and $\Sigma(1620)$, respectively. The decay widths are listed in Table E.6. The theoretical predictions are all rather small.
Figure E.7: $\frac{1}{3}$ octet baryon states $N(1650)$, $\Sigma(1620)$, and $\Lambda(1800)$. 
Figure E.8: $\frac{3}{2}^-$ octet baryon states $N(1520)$, $\Sigma(1670)$, $\Xi(1820)$, and $\Lambda(1690)$. 

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Figure E.9: $\frac{3}{2}^-$ octet baryon states $N(1700)$ and $\Sigma(1940)$. 
Table E.4: Same notation as in Table E.1 but for the $\frac{1}{2}^-$ octet states $N(1650)$, $\Sigma(1620)$, and $\Lambda(1800)$.

<table>
<thead>
<tr>
<th>Decay Experiment</th>
<th>Relativistic</th>
<th>% of Exp. Width</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GBE</td>
<td>OGE</td>
</tr>
<tr>
<td>$N(1650) \rightarrow N\pi$</td>
<td>$(128 \pm 29)^{+19}_{-12}$</td>
<td>6.3 9.9</td>
</tr>
<tr>
<td>$\Sigma(1620) \rightarrow \Lambda\pi$</td>
<td>$\Gamma_{tot} = 10 - 106$</td>
<td>19 25</td>
</tr>
<tr>
<td>$\Sigma(1620) \rightarrow \Sigma\pi$</td>
<td>$\Gamma_{tot} = 10 - 106$</td>
<td>32 44</td>
</tr>
<tr>
<td>$\Lambda(1800) \rightarrow \Sigma\pi$</td>
<td>$\Gamma_{tot} = 200 - 400$</td>
<td>68 101</td>
</tr>
<tr>
<td>$N(1650) \rightarrow N\eta$</td>
<td>$(11 \pm 6)^{+2}_{-1}$</td>
<td>50 74</td>
</tr>
<tr>
<td>$\Lambda(1800) \rightarrow \Lambda\eta$</td>
<td>$\Gamma_{tot} = 200 - 400$</td>
<td>43 65</td>
</tr>
<tr>
<td></td>
<td>50* 69*</td>
<td>263 - 1250*</td>
</tr>
<tr>
<td>$N(1650) \rightarrow \Lambda K$</td>
<td>$(11.6 \pm 6.6)^{+2.2}_{-0.6}$</td>
<td>0.03 0.12</td>
</tr>
<tr>
<td></td>
<td>0.07* 0.04*</td>
<td>0 - 2*</td>
</tr>
<tr>
<td>$\Sigma(1620) \rightarrow NK$</td>
<td>$\Gamma_{tot} = 10 - 106$</td>
<td>55 55</td>
</tr>
<tr>
<td></td>
<td>57* 58*</td>
<td></td>
</tr>
<tr>
<td>$\Lambda(1800) \rightarrow NK$</td>
<td>$(97.5 \pm 22.5)^{+40}_{-25}$</td>
<td>4.2 6.4</td>
</tr>
<tr>
<td></td>
<td>4.5* 5.5*</td>
<td>3 - 9*</td>
</tr>
</tbody>
</table>

$\frac{5}{2}^-$ Octet States: $N(1675)$, $\Sigma(1775)$, $\Xi(1950)$, and $\Lambda(1830)$

This multiplet contains the flavor octet states $N(1675)$, $\Sigma(1775)$, $\Xi(1950)$, and $\Lambda(1830)$ with $J^P = \frac{5}{2}^-$ and $L = 1, S = \frac{3}{2}$. Here, the $N(1675)$ is degenerate with the $\frac{1}{2}^-$ $N(1650)$ as well as the $\frac{3}{2}^-$ $N(1700)$, the $\Sigma(1775)$ with the $\frac{1}{2}^- \Sigma(1620)$ as well as the $\frac{3}{2}^- \Sigma(1940)$, and the $\Lambda(1830)$ with the $\frac{1}{2}^- \Lambda(1800)$. All of these states represent the first radial excitations with density distributions as seen in Fig. E.10. The corresponding decay widths are listed in Table E.7. The results are typical for structure-independent resonances. The $J^P$ of $\Xi(1950)$ is not yet known from experiment. We classify it into this flavor octet and thus attribute to it $J^P = \frac{5}{2}^-$. Its density distribution is typical for the $\frac{5}{2}^-$ octet, and there is no other candidate that would fit in here.
Figure E.10: \( \frac{5}{2}^- \) octet baryon states \( N(1675), \Sigma(1775), \Xi(1950), \) and \( \Lambda(1830). \)
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Table E.5: Same notation as in Table E.1 but for the $\frac{3}{2}^+$ octet states $N(1520)$, $\Sigma(1670)$, $\Xi(1820)$, and $\Lambda(1690)$.

<table>
<thead>
<tr>
<th>Decay</th>
<th>Experiment</th>
<th>Relativistic Width</th>
<th>% of Exp. Width</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>GBE</td>
<td>OGE</td>
</tr>
<tr>
<td>$N(1520)$ $\rightarrow N\pi$</td>
<td>$\left(69 \pm 6\right)_7^{+7} -_8^{-8}$</td>
<td>21</td>
<td>23</td>
</tr>
<tr>
<td>$\Sigma(1670)$ $\rightarrow \Sigma\pi$</td>
<td>$\left(27 \pm 9\right)_6^{+12} -_6^{-6}$</td>
<td>15</td>
<td>23</td>
</tr>
<tr>
<td>$\Sigma(1670)$ $\rightarrow \Lambda\pi$</td>
<td>$\left(6 \pm 3\right)_1^{+3} -_1^{-3}$</td>
<td>2.5</td>
<td>2.0</td>
</tr>
<tr>
<td>$\Xi(1820)$ $\rightarrow \Xi\pi$</td>
<td>$\Gamma_{tot} = 14 - 39$</td>
<td>0.4</td>
<td>1.6</td>
</tr>
<tr>
<td>$\Lambda(1690)$ $\rightarrow \Sigma\pi$</td>
<td>$\left(18 \pm 6\right)_2^{+4} -_2^{-4}$</td>
<td>19</td>
<td>25</td>
</tr>
<tr>
<td>$N(1520)$ $\rightarrow N\eta$</td>
<td>$\left(0.26 \pm 0.05\right)_1^{+0.03} -_1^{-0.01}$</td>
<td>0.11</td>
<td>0.11</td>
</tr>
<tr>
<td>$\Lambda(1690)$ $\rightarrow \Lambda\eta$</td>
<td>$\Gamma_{tot} = 50 - 70$</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>$\Sigma(1670)$ $\rightarrow NK$</td>
<td>$\left(6.0 \pm 1.8\right)_1^{+2.6} -_1^{-1.4}$</td>
<td>1.1</td>
<td>1.0</td>
</tr>
<tr>
<td>$\Xi(1820)$ $\rightarrow \Lambda\pi$</td>
<td>$\Gamma_{tot} = 14 - 39$</td>
<td>2.7</td>
<td>6.2</td>
</tr>
<tr>
<td>$\Xi(1820)$ $\rightarrow \Sigma\pi$</td>
<td>$\Gamma_{tot} = 14 - 39$</td>
<td>4.1</td>
<td>9.3</td>
</tr>
<tr>
<td>$\Lambda(1690)$ $\rightarrow NK$</td>
<td>$\left(15 \pm 3\right)_2^{+3} -_2^{-3}$</td>
<td>1.2</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Singlet State: $\Lambda(1810)$

As explained above the theoretically calculated $\frac{1}{2}^+$ $\Lambda$ state with quantum numbers $L = 0$ and $S = \frac{1}{2}$ that falls close to the experimentally seen $\Lambda(1810)$ (and is the second excitation above the $\frac{1}{2}^+ \Lambda(1116)$) exhibits a spin-flavor part with mixed symmetry. However, its flavor wave function is almost antisymmetric, with a singlet percentage of 92%. Thus the $\Lambda(1810)$ should be classified as a flavor singlet\(^2\). The spatial part is typical for a mixed symmetry as the probability density shows in Fig. E.11. The decay widths are shown in Table E.8. The $\Lambda(1810) \rightarrow \Sigma\pi$ as well as the $\Lambda(1810) \rightarrow \Lambda\eta$

\(^2\)The $\frac{1}{2}^+$ $\Lambda$ excitation with a mixed-symmetric flavor wave function comes only as the third excitation above the $\Lambda(1116)$ and fits into an octet.
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Table E.6: Same notation as in Table E.1 but for the \( \frac{3}{2}^- \) octet states \( N(1700) \) and \( \Sigma(1940) \).

<table>
<thead>
<tr>
<th>Decay</th>
<th>Experiment</th>
<th>Relativistic Width</th>
<th>% of Exp. Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N(1700) \rightarrow N\pi )</td>
<td>( (10 \pm 5)^{+3}_{-3} )</td>
<td>1.0</td>
<td>1.3</td>
</tr>
<tr>
<td>( \Sigma(1940) \rightarrow \Sigma\pi )</td>
<td>( \Gamma_{tot} = 150 - 300 )</td>
<td>2.2</td>
<td>3.7</td>
</tr>
<tr>
<td>( \Sigma(1940) \rightarrow \Lambda\pi )</td>
<td>( \Gamma_{tot} = 150 - 300 )</td>
<td>0.2</td>
<td>0.4</td>
</tr>
<tr>
<td>( N(1700) \rightarrow N\eta )</td>
<td>( (0 \pm 1)^{+0.5}_{-0.5} )</td>
<td>0.5</td>
<td>0.9</td>
</tr>
<tr>
<td>( \Sigma(1940) \rightarrow \Sigma\eta )</td>
<td>( \Gamma_{tot} = 150 - 300 )</td>
<td>( \approx 0 )</td>
<td>( \approx 0 )</td>
</tr>
<tr>
<td>( N(1700) \rightarrow \Lambda K )</td>
<td>( (1.5 \pm 1.5)^{+1.5} )</td>
<td>( \approx 0 )</td>
<td>0.05</td>
</tr>
<tr>
<td>( N(1700) \rightarrow \Sigma K )</td>
<td>( \Gamma_{tot} = 50 - 150 )</td>
<td>( \approx 0^* )</td>
<td>( \approx 0^* )</td>
</tr>
<tr>
<td>( \Sigma(1940) \rightarrow NK )</td>
<td>( (22 \pm 22)^{+16} )</td>
<td>1.1</td>
<td>1.5</td>
</tr>
<tr>
<td>( \Sigma(1940) \rightarrow \Xi K )</td>
<td>( \Gamma_{tot} = 150 - 300 )</td>
<td>0.01*</td>
<td>0.02*</td>
</tr>
</tbody>
</table>

decay widths are predicted much too small.

\( \frac{1}{2}^- \) Singlet State: \( \Lambda(1405) \)

The \( \Lambda(1405) \) with \( J^P = \frac{1}{2}^- \) constructed from \( L = 1 \) and \( S = \frac{1}{2} \) has a flavor wave function that is predominantly antisymmetric, with a flavor singlet percentage of 71%. The probability density is shown in Fig. E.12. It can only decay via \( \pi \) emission, and the pertinent decay width is underestimated by the CQMs (see Table E.9).

\( \frac{3}{2}^- \) Singlet State: \( \Lambda(1520) \)

For the GBE and OGE CQMs without tensor forces used in this work the \( \Lambda(1520) \) is degenerate with the \( \Lambda(1405) \). Therefore also their density distributions are the same (cf. Figs. E.13 and E.12). The partial widths for both its \( \Sigma\pi \) and \( NK \) decay modes are realistically described by the CQMs, see Table E.10.

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Table E.7: Same notation as in Table E.1 but for the $\frac{5}{2}^-$ octet states $N(1675)$, $\Sigma(1775)$, $\Xi(1950)$, and $\Lambda(1830)$.

<table>
<thead>
<tr>
<th>Decay</th>
<th>Experiment</th>
<th>Relativistic</th>
<th>% of Exp. Width</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>GBE</td>
<td>OGE</td>
</tr>
<tr>
<td>$N(1675)$ $\rightarrow N\pi$</td>
<td>$(60 \pm 8)_{-7}^{+7}$</td>
<td>8.4</td>
<td>10</td>
</tr>
<tr>
<td>$\Sigma(1775)$ $\rightarrow \Sigma\pi$</td>
<td>$(4.2 \pm 1.8)_{-0.3}^{+0.8}$</td>
<td>1.9</td>
<td>3.8</td>
</tr>
<tr>
<td>$\Sigma(1775)$ $\rightarrow \Lambda\pi$</td>
<td>$(20 \pm 4)_{-2}^{+2}$</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>$\Xi(1950)$ $\rightarrow \Xi\pi$</td>
<td>$\Gamma_{tot} = 40 - 80$</td>
<td>14</td>
<td>28</td>
</tr>
<tr>
<td>$\Lambda(1830)$ $\rightarrow \Sigma\pi$</td>
<td>$(52 \pm 19)_{-12}^{+11}$</td>
<td>14</td>
<td>19</td>
</tr>
<tr>
<td>$N(1675)$ $\rightarrow N\eta$</td>
<td>$(0 \pm 0.8)_{-0.15}^{+0.15}$</td>
<td>1.5</td>
<td>2.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.0*</td>
<td>2.0*</td>
</tr>
<tr>
<td>$\Sigma(1775)$ $\rightarrow \Sigma\eta$</td>
<td>$\Gamma_{tot} = 105 - 135$</td>
<td>$\approx 0$</td>
<td>0.05</td>
</tr>
<tr>
<td>$\Xi(1950)$ $\rightarrow \Xi\eta$</td>
<td>$\Gamma_{tot} = 40 - 80$</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td>$\Lambda(1830)$ $\rightarrow \Lambda\eta$</td>
<td>$\Gamma_{tot} = 60 - 110$</td>
<td>0.6</td>
<td>2.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.0*</td>
<td>1.8*</td>
</tr>
<tr>
<td>$N(1675)$ $\rightarrow \Lambda K$</td>
<td>&lt; 1.7</td>
<td>$\approx 0$</td>
<td>$\approx 0$</td>
</tr>
<tr>
<td>$\Sigma(1775)$ $\rightarrow NK$</td>
<td>$(48.0 \pm 3.6)_{-5.6}^{+6.5}$</td>
<td>11</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>13*</td>
<td>12*</td>
</tr>
<tr>
<td>$\Xi(1950)$ $\rightarrow \Lambda K$</td>
<td>$\Gamma_{tot} = 40 - 80$</td>
<td>2.5</td>
<td>4.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.3*</td>
<td>3.6*</td>
</tr>
<tr>
<td>$\Xi(1950)$ $\rightarrow \Sigma K$</td>
<td>$\Gamma_{tot} = 40 - 80$</td>
<td>2.3</td>
<td>4.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.7*</td>
<td>3.6*</td>
</tr>
<tr>
<td>$\Lambda(1830)$ $\rightarrow NK$</td>
<td>$(6.18 \pm 3.33)_{-1.05}^{+1.05}$</td>
<td>0.09</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.11*</td>
<td>0.12*</td>
</tr>
<tr>
<td>$\Lambda(1830)$ $\rightarrow \Xi K$</td>
<td>$\Gamma_{tot} = 60 - 110$</td>
<td>$\approx 0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\approx 0^*$</td>
<td>$\approx 0^*$</td>
</tr>
</tbody>
</table>
FLAVOR MULTIPLETS OF BARYON RESONANCES

Figure E.11: $^{1+}$ singlet baryon state $\Lambda(1810)$.

Figure E.12: $^{1-}$ singlet baryon state $\Lambda(1405)$.

Figure E.13: $^{3-}$ singlet baryon state $\Lambda(1520)$.
FLAVOR MULTIPLETS OF BARYON RESONANCES

Table E.8: Same notation as in Table E.1 but for the $\frac{1}{2}^+$ singlet state $\Lambda(1810)$.

<table>
<thead>
<tr>
<th>Decay</th>
<th>Experiment</th>
<th>Relativistic</th>
<th>% of Exp. Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda(1810)$ → $\Sigma\pi$</td>
<td>(38 ± 23) $^{+40}_{-10}$</td>
<td>3.8</td>
<td>2.1</td>
</tr>
<tr>
<td>$\Lambda(1810)$ → $\Lambda\eta$</td>
<td>$\Gamma_{tot} = 50 – 250$</td>
<td>0.9</td>
<td>$\approx 0$</td>
</tr>
<tr>
<td>$\Lambda(1810)$ → $NK$</td>
<td>(52.5 ± 22.5) $^{+50}_{-20}$</td>
<td>4.1</td>
<td>11.7</td>
</tr>
</tbody>
</table>

Table E.9: Same notation as in Table E.1 but for the $\frac{1}{2}^-$ singlet state $\Lambda(1405)$. In this case we include only the experimental mass as the theoretical one is too unreliable.

<table>
<thead>
<tr>
<th>Decay</th>
<th>Experiment</th>
<th>Relativistic</th>
<th>% of Exp. Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda(1405)$ → $\Sigma\pi$</td>
<td>(50 ± 0) $^{+2}_{-2}$</td>
<td>15*</td>
<td>17*</td>
</tr>
</tbody>
</table>

Table E.10: Same notation as in Table E.1 but for the $\frac{3}{2}^-$ singlet state $\Lambda(1520)$.

<table>
<thead>
<tr>
<th>Decay</th>
<th>Experiment</th>
<th>Relativistic</th>
<th>% of Exp. Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda(1520)$ → $\Sigma\pi$</td>
<td>(6.55 ± 0.16) $^{+0.04}_{-0.04}$</td>
<td>5</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2.8*</td>
</tr>
<tr>
<td>$\Lambda(1520)$ → $NK$</td>
<td>(7.02 ± 0.16) $^{+0.46}_{-0.44}$</td>
<td>12</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6*</td>
</tr>
</tbody>
</table>

$\frac{3}{2}^+$ Decuplet Ground States: $\Delta(1232)$, $\Sigma(1385)$, $\Xi(1530)$, and $\Omega(1672)$

The decuplet ground states $\Delta(1232)$, $\Sigma(1385)$, $\Xi(1530)$, and $\Omega(1672)$ are characterized by $J^P = \frac{3}{2}^+$, where $L = 0$ and $S = \frac{3}{2}$. Their wave functions are
totally symmetric, with symmetric spin, symmetric flavor, and symmetric spatial parts. The density distributions are of the same characteristics as for the ground-state octet, only the spatial extension is broader, see Fig. E.14. The decay widths are given in Table E.11. All of them are too small. The $\Omega(1672)$ ground state has no strong decay. The remaining states have only $\pi$ decay modes. The theoretical predictions all underestimate the experimental values.

Table E.11: Same notation as in Table E.1 but for the $\frac{3}{2}^+$ decuplet states $\Delta(1232)$, $\Sigma(1385)$, and $\Xi(1530)$.

<table>
<thead>
<tr>
<th>Decay</th>
<th>Experiment</th>
<th>Relativistic</th>
<th>% of Exp.</th>
<th>Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta(1232)$ $\rightarrow N\pi$</td>
<td>(118)$^{+2}_{-2}$ 1.2</td>
<td>35</td>
<td>31</td>
<td>29 - 30</td>
</tr>
<tr>
<td>$\Sigma(1385)$ $\rightarrow \Sigma\pi$</td>
<td>(4.2 $\pm$ 0.5)$^{+0.7}_{-0.5}$</td>
<td>3.1</td>
<td>0.5</td>
<td>57 - 97</td>
</tr>
<tr>
<td>$\Sigma(1385)$ $\rightarrow \Lambda\pi$</td>
<td>(31.3 $\pm$ 0.5)$^{+1.4}_{-1.3}$</td>
<td>11</td>
<td>11</td>
<td>30 - 42</td>
</tr>
<tr>
<td>$\Xi(1530)$ $\rightarrow \Xi\pi$</td>
<td>(9.9 $\pm$ 0) $^{+1.7}_{-1.9}$</td>
<td>2.2</td>
<td>1.3</td>
<td>19 - 28</td>
</tr>
</tbody>
</table>

$\frac{3}{2}^+$ Decuplet States: $\Delta(1600)$ and $\Sigma(1690)$

The $\Delta(1600)$ with $J^P = \frac{3}{2}^+$ constructed with $L = 0$ and $S = \frac{3}{2}$ represents the first radial excitations above the $\frac{3}{2}^+$ $\Delta(1232)$ ground state. It is a pure decuplet state. The CQMs yield an additional $\frac{3}{2}^+$ $\Sigma$ state above the $\Sigma(1385)$ decuplet ground state. The only available candidate observed in experiment is the $\Sigma(1690)$. It is a two-star resonance with a yet undetermined $J^P$. It fits naturally into the $\frac{3}{2}^+$ decuplet, where it contains a small admixture of about 1% from the flavor octet. The probability distributions of both the $\Delta(1600)$ and the $\Sigma(1690)$ have the typical shapes of a symmetric spatial structure, see Fig. E.15. The decay widths are collected in Table E.12. The theoretical predictions turn out to be very small. The resonances are found to be structure-dependent. In a non-relativistic calculation the decay widths result much larger. Only $\Delta(1600) \rightarrow N\pi$ is measured, and it is by far underestimated by the covariant results.

$\frac{1}{2}^-$ Decuplet States: $\Delta(1620)$ and $\Sigma(1750)$

The resonances $\Delta(1620)$ and $\Sigma(1750)$ can be identified as $\frac{1}{2}^-$ decuplet states with $L = 1$ and $S = \frac{1}{2}$. Here, the $\Delta$ resonance represents the lowest ex-
Figure E.14: $\frac{3}{2}^+$ decuplet baryon states $\Delta(1232)$, $\Sigma(1385)$, $\Xi(1530)$, and $\Omega(1672)$. 
Figure E.15: $\frac{3}{2}^+$ decuplet baryon states $\Delta(1600)$, and $\Sigma(1690)$. 
Table E.12: Same notation as in Table E.1 but for the $\frac{3}{2}^+$ decuplet states $\Delta(1600)$ and $\Sigma(1690)$.

<table>
<thead>
<tr>
<th>Decay</th>
<th>Experiment</th>
<th>Relativistic Width</th>
<th>% of Exp. Width</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>GBE</td>
<td>OGE</td>
</tr>
<tr>
<td>$\Delta(1600) \rightarrow N\pi$</td>
<td>$(61 \pm 25)_{-10}^{+20}$</td>
<td>0.5</td>
<td>5.1</td>
</tr>
<tr>
<td>$\Sigma(1690) \rightarrow \Lambda\pi$</td>
<td>$\Gamma_{tot} = 15 - 300$</td>
<td>$\approx 0$</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\approx 0^*$</td>
<td>0.6$^*$</td>
</tr>
<tr>
<td>$\Sigma(1690) \rightarrow \Sigma\pi$</td>
<td>$\Gamma_{tot} = 15 - 300$</td>
<td>0.4</td>
<td>2.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.2$^*$</td>
<td>1.1$^*$</td>
</tr>
<tr>
<td>$\Sigma(1690) \rightarrow NK$</td>
<td>$\Gamma_{tot} = 15 - 300$</td>
<td>$\approx 0$</td>
<td>1.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\approx 0^*$</td>
<td>0.8$^*$</td>
</tr>
</tbody>
</table>

Citation with the given quantum numbers and it is a pure decuplet state. The GBE and OGE CQMs produce three $\frac{1}{2}^-$ $\Sigma$ resonances (below 2 GeV). The lower two$^3$ have already been identified as the octet members $\Sigma(1560)$ and $\Sigma(1620)$, see the discussion above. Therefore the $\Sigma(1750)$ must be considered as a decuplet member, even though it has a moderate flavor octet admixture of 6%. The probability densities of $\Delta(1620)$ and $\Sigma(1750)$ are shown in Fig. E.16. The decay widths are collected in Table E.13. Except for the $\Sigma(1750) \rightarrow \Sigma\pi$ decay width in the case of the OGE CQM the theoretical results all underestimate the experimental data.

$\frac{3}{2}^-$ Decuplet State: $\Delta(1700)$

The $\Delta(1700)$ with $J^P = \frac{3}{2}^-$ and $L = 1, S = \frac{1}{2}$ is again a pure decuplet state. For the CQMs considered here it is degenerate with the $J^P = \frac{1}{2}^-$ $\Delta(1620)$. The other members of this decuplet have not been observed. The probability density of $\Delta(1700)$ is shown in Fig. E.17. Due to the degeneracy it is the same as for $\Delta(1620)$ above. The widths of the two possible decay modes are given in Table E.14. The theoretical predictions come out (much) too small.

---

$^3$This is strictly true only for the GBE CQM, while for the OGE CQM the $\Sigma(1620)$ and $\Sigma(1750)$ are reversed in their ordering.
Figure E.16: $\frac{1}{2}^-$ decuplet baryon states $\Delta(1620)$ and $\Sigma(1750)$.

Figure E.17: $\frac{3}{2}^-$ decuplet baryon state $\Delta(1700)$. 
Table E.13: Same notation as in Table E.1 but for the $\frac{1}{2}^-$ decuplet states $\Delta(1620)$ and $\Sigma(1750)$.

<table>
<thead>
<tr>
<th>Decay</th>
<th>Experiment</th>
<th>Relativistic</th>
<th>% of Exp. Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta(1620)$ → $N\pi$</td>
<td>$(36 \pm 7)^{+2}_{-2}$</td>
<td>1.2</td>
<td>2.8</td>
</tr>
<tr>
<td>$\Sigma(1750)$ → $\Sigma\pi$</td>
<td>$(3.6 \pm 3.6)^{+5.6}_{-5.6}$</td>
<td>10</td>
<td>1.0</td>
</tr>
<tr>
<td>$\Sigma(1750)$ → $\Lambda\pi$</td>
<td>$\Gamma_{tot} = 60 - 160$</td>
<td>1.0</td>
<td>2.8</td>
</tr>
<tr>
<td>$\Sigma(1750)$ → $\Sigma\eta$</td>
<td>$(31.5 \pm 18.0)^{+35.5}_{-4.5}$</td>
<td>6.0</td>
<td>2.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.8*</td>
<td>1.4*</td>
</tr>
<tr>
<td>$\Sigma(1750)$ → $NK$</td>
<td>$(22.5 \pm 13.5)^{+28}_{-3}$</td>
<td>≈ 0</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0*</td>
<td>0.6*</td>
</tr>
</tbody>
</table>

Table E.14: Same notation as in Table E.1 but for the $\frac{3}{2}^-$ decuplet state $\Delta(1700)$.

<table>
<thead>
<tr>
<th>Decay</th>
<th>Experiment</th>
<th>Relativistic</th>
<th>% of Exp. Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta(1700)$ → $N\pi$</td>
<td>$(45 \pm 15)^{+20}_{-10}$</td>
<td>3.8</td>
<td>4.1</td>
</tr>
<tr>
<td>$\Delta(1700)$ → $\Sigma K$</td>
<td>$\Gamma_{tot} = 200 - 400$</td>
<td>≈ 0*</td>
<td>≈ 0*</td>
</tr>
</tbody>
</table>

E.4 Classification of Baryons into Flavor Multiplets

By the analysis of the baryon wave functions and the mesonic decay widths in the previous section we have reached a consistent classification of all the light and strange baryon resonances (below 2 GeV) that have so far been observed in experiment. They fit together into flavor multiplets where all the members exhibit the same features with respect to the spatial density distributions and the symmetries of the spin-flavor parts. Thereby one also arrives at a more reliable identification of theoretical states produced by CQMs. Up to now, when dealing with CQMs one usually considered only the experimentally well-known three- and four-star states as compiled by the PDG [2] (cf. the Figs. 2.1 and 2.2 in Chapter 2). By our investigations it turned out that it is reasonable to take into account also the experimentally lesser known (low-lying) two-star resonances. In Fig. E.2 above we have included in particular the $\Sigma(1560)$, $\Sigma(1620)$, $\Sigma(1690)$, and $\Sigma(1880)$
FLAVOR MULTIPLECTS OF BARYON RESONANCES

Table E.15: Classification of baryon ground states and resonances into flavor octets with percentages given as superscripts. The Σ resonances set in boldface represent two-star resonances, for the Σ(1560) and both Ξ resonances in boldface the \( J^P \) is not yet determined from experiment. The last column quotes the denotation of the multiplets according to Guzey and Polyakov [99]. Only the resonances that have been observed in experiment are included.

<table>
<thead>
<tr>
<th>((LS) J^P)</th>
<th>(N(939)^{100})</th>
<th>(\Sigma(1193)^{100})</th>
<th>(\Xi(1318)^{100})</th>
<th>(\Lambda(1116)^{100})</th>
<th>Ref. [99]</th>
</tr>
</thead>
<tbody>
<tr>
<td>((1/2)^+)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>((1/2)^+)</td>
<td>(N(1440)^{100})</td>
<td>(\Sigma(1660)^{100})</td>
<td>(\Xi(1690)^{100})</td>
<td>(\Lambda(1600)^{96})</td>
<td>3</td>
</tr>
<tr>
<td>((1/2)^+)</td>
<td>(N(1710)^{100})</td>
<td>(\Sigma(1880)^{99})</td>
<td></td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>((3/2)^-)</td>
<td>(N(1535)^{100})</td>
<td>(\Sigma(1560)^{94})</td>
<td>(\Lambda(1670)^{72})</td>
<td></td>
<td>9</td>
</tr>
<tr>
<td>((3/2)^-)</td>
<td>(N(1650)^{100})</td>
<td>(\Sigma(1620)^{100})</td>
<td>(\Lambda(1800)^{100})</td>
<td></td>
<td>14</td>
</tr>
<tr>
<td>((3/2)^-)</td>
<td>(N(1520)^{100})</td>
<td>(\Sigma(1670)^{94})</td>
<td>(\Xi(1820)^{97})</td>
<td>(\Lambda(1690)^{72})</td>
<td>8</td>
</tr>
<tr>
<td>((3/2)^-)</td>
<td>(N(1700)^{100})</td>
<td>(\Sigma(1940)^{100})</td>
<td></td>
<td></td>
<td>11</td>
</tr>
<tr>
<td>((3/2)^-)</td>
<td>(N(1675)^{100})</td>
<td>(\Sigma(1775)^{100})</td>
<td>(\Xi(1950)^{100})</td>
<td>(\Lambda(1830)^{100})</td>
<td>12</td>
</tr>
</tbody>
</table>

For the Σ(1620) an Σ(1880) the intrinsic spins and parities are experimentally determined to be \( \frac{1}{2}^- \) and \( \frac{1}{2}^+ \), respectively. For the other Σ(1560) and Σ(1690) two-star resonances \( J^P \) is not known. The same is true for the three-star resonances Ξ(1690) and Ξ(1950). By advocation of these resonances in addition to the well-established three- and four-star resonances one can interpret the theoretical levels generated by the CQMs consistently with the \( SU(3)_F \) multiplets and their inherent symmetries. For instance, in the GBE CQM the three \( \frac{1}{2}^- \) Σ levels (below 2 GeV) are most naturally explained as belonging to the first and second \( \frac{1}{2}^- \) octets (listed in lines four and five of Table E.15) and the first \( \frac{1}{2}^- \) decuplet (listed in line three of Table E.17); see also the \( \frac{1}{2}^- \) Σ spectrum in Fig. E.2.

In addition, it becomes possible to attribute specific \( J^P \) values to experimentally seen states whose intrinsic spin and parity have not yet been determined (reliably). For instance, for the three-star states Ξ(1690) and Ξ(1950) the PDG quotes no definitive \( J^P \). As we have learned in the previous section, we can classify the Ξ(1690) as a \( \frac{3}{2}^- \) octet state and the Ξ(1950) as a \( \frac{1}{2}^- \) octet state; see Table E.15. Similarly the Σ(1560) and the Σ(1690) can be classified as \( \frac{1}{2}^- \) octet and \( \frac{3}{2}^- \) decuplet states, respectively; see Tables E.15 and E.17.

Finally we arrive at the classification of baryon ground states and reso-
FLAVOR MULTIPLETS OF BARYON RESONANCES

Table E.16: Classification of Λ ground and resonance states into flavor singlets with percentages given as superscripts. The last column quotes the denotation of the multiplets according to Guzey and Polyakov [99].

<table>
<thead>
<tr>
<th>(LS)J^P</th>
<th>Ref. [99]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0\frac{1}{2})\frac{1}{2}^+</td>
<td>Λ(1810)^{92}</td>
</tr>
<tr>
<td>(1\frac{1}{2})\frac{1}{2}^-</td>
<td>Λ(1405)^{71}</td>
</tr>
<tr>
<td>(1\frac{1}{2})\frac{3}{2}^-</td>
<td>Λ(1520)^{71}</td>
</tr>
</tbody>
</table>

Table E.17: Classification of baryon ground states and resonances into flavor decuplets with percentages given as superscripts. The resonance set in boldface represents a two-star resonance whose J^P is not yet determined. The last column quotes the denotation of the multiplets according to Guzey and Polyakov [99]. Only the resonances that have been observed in experiment are included.

<table>
<thead>
<tr>
<th>(LS)J^P</th>
<th>Ref. [99]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0\frac{3}{2})\frac{3}{2}^+</td>
<td>Δ(1232)^{100}</td>
</tr>
<tr>
<td>(0\frac{1}{2})\frac{3}{2}^+</td>
<td>Δ(1600)^{100}</td>
</tr>
<tr>
<td>(1\frac{1}{2})\frac{1}{2}^-</td>
<td>Δ(1620)^{100}</td>
</tr>
<tr>
<td>(1\frac{1}{2})\frac{3}{2}^-</td>
<td>Δ(1700)^{100}</td>
</tr>
</tbody>
</table>

nances into flavor multiplets as summarized in Tables E.15–E.17. In some cases, namely for the Λ, Σ, and Ξ resonances, there occur mixtures of flavor multiplets. The percentages of the predominant flavor multiplets are given as superscripts. The values strictly apply for the GBE CQM. They turn out to be very similar, however, for the OGE CQM.

A classification as done here, was recently undertaken by Guzey and Polyakov [99]. They found rather similar flavor multiplets, as quoted also in the last columns of the Tables E.15–E.17. In fact, we agree in all cases except for the Λ(1810), which is a flavor singlet in our classification, whereas it should be a flavor octet according to Guzey and Polyakov.
Appendix F

Decay Widths of Light and Strange Baryon Resonances

Here we list the $\pi$, $\eta$, and $K$ decay widths of the light and strange baryon resonances as predicted by the GBE [35,41] and OGE [19] CQMs. Whenever possible a comparison to phenomenological data as compiled by the PDG [2] is given.

The theoretical predictions listed in the first and second sections are calculated within the framework of RQM utilizing a PFSM decay operator for pseudovector and pseudoscalar couplings, respectively (see Eqs. (5.18) and (5.19)). The corresponding formalism is provided in Chapter 5.

In the third section we present the results obtained with a non-relativistic reduction of the decay operator in the case of pseudovector coupling. The pertinent theoretical background is given in Chapter 6.

Finally, in the fourth section of this appendix a comparison of our relativistic results with the ones available from the Bonn group for their CQM (based on the instanton-induced hyperfine interaction and solved within a Bethe-Salpeter approach) is provided; it concerns only the $\pi$ decays of nucleon and $\Delta$ resonances.

F.1 Relativistic Results with Pseudovector Coupling

The covariant results for decay widths in case of a PFSM operator with pseudovector coupling are collected in Tables F.1 - F.8.

In each table the first and second columns denote the decaying resonance with its intrinsic spin and parity $J^P$, and the third column lists the experimental decay widths with their uncertainties after the PDG [2]. The direct theoretical predictions by the GBE and OGE CQMs are presented in the fourth and fifth columns, respectively. The last two columns contain
Table F.1: Covariant predictions for π decay widths of light baryon resonances as obtained with the GBE and the OGE CQMs along the PFSM in pseudovector coupling. In the last two columns experimental resonance masses (instead of theoretical masses) have been used in the calculation. The experimental data are from the latest compilation of the PDG [2].

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>GBE</td>
<td>OGE</td>
<td>GBE</td>
</tr>
<tr>
<td>N(1440) 1/2^+</td>
<td>(195 ± 30)^+113^-55</td>
<td>30</td>
<td>59</td>
<td>28</td>
</tr>
<tr>
<td>N(1520) 3/2^-</td>
<td>(69 ± 6)^+7^-8</td>
<td>21</td>
<td>23</td>
<td>22</td>
</tr>
<tr>
<td>N(1535) 1/2^-</td>
<td>(68 ± 15)^+14^-9</td>
<td>25</td>
<td>39</td>
<td>24</td>
</tr>
<tr>
<td>N(1650) 1/2^-</td>
<td>(128 ± 29)^+19^-12</td>
<td>6</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>N(1675) 5/2^-</td>
<td>(60 ± 8)^+7^-7</td>
<td>8</td>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td>N(1700) 3/2^-</td>
<td>(10 ± 5)^+3^-3</td>
<td>1.0</td>
<td>1.3</td>
<td>1.1</td>
</tr>
<tr>
<td>N(1710) 1/2^-</td>
<td>(15 ± 5)^+30^-5</td>
<td>19</td>
<td>21</td>
<td>15</td>
</tr>
<tr>
<td>Δ(1232) 3/2^+</td>
<td>(118)^+2^-2</td>
<td>35</td>
<td>31</td>
<td>33</td>
</tr>
<tr>
<td>Δ(1600) 3/2^+</td>
<td>(61 ± 26)^+26^-10</td>
<td>0.5</td>
<td>5.1</td>
<td>0.2</td>
</tr>
<tr>
<td>Δ(1620) 1/2^-</td>
<td>(36 ± 7)^+2^-2</td>
<td>1.2</td>
<td>2.8</td>
<td>1.4</td>
</tr>
<tr>
<td>Δ(1700) 3/2^-</td>
<td>(45 ± 15)^+20^-10</td>
<td>3.8</td>
<td>4.1</td>
<td>4.6</td>
</tr>
</tbody>
</table>

The theoretical predictions in case experimental resonance masses are used instead of the ones reproduced by the CQMs.

In Tables F.1, F.2, and F.3 we list the results for the π decay widths, where Table F.1 comprises the results in the light baryon sector, and Tables F.2 and F.3 the ones in the strange baryon sector. In the same manner we find in Tables F.4 and F.5 the results obtained for the η decay widths. The K decay widths are finally listed in Tables F.6, F.7, and F.8 where in Table F.6 the strange-quark content increases, i.e. a light quark transforms to a strange one upon emission of a K meson. By contrast, in Tables F.7 and F.8 the strange-quark content decreases, i.e. a strange quark transforms to a light one upon emission of a K meson.
Table F.2: Same as Table F.1 but for the $\Sigma \pi$ decay channel.

<table>
<thead>
<tr>
<th>Decay $\to \Sigma \pi$</th>
<th>$J^P$</th>
<th>Experiment [MeV]</th>
<th>Th. Masses</th>
<th>Exp. Masses</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda(1405)$</td>
<td>$^1_2^-$</td>
<td>$(50 \pm 2)$</td>
<td>55</td>
<td>78</td>
</tr>
<tr>
<td>$\Lambda(1520)$</td>
<td>$^3_2^-$</td>
<td>$(6.55 \pm 0.16)^{+0.04}_{-0.04}$</td>
<td>5.3</td>
<td>8.9</td>
</tr>
<tr>
<td>$\Lambda(1600)$</td>
<td>$^1_2^+$</td>
<td>$(53 \pm 38)^{+60}_{-10}$</td>
<td>3</td>
<td>33</td>
</tr>
<tr>
<td>$\Lambda(1670)$</td>
<td>$^1_2^-$</td>
<td>$(14.0 \pm 5.3)^{+8.3}_{-2.5}$</td>
<td>69</td>
<td>103</td>
</tr>
<tr>
<td>$\Lambda(1690)$</td>
<td>$^3_2^-$</td>
<td>$(18 \pm 6)^{+4}_{-2}$</td>
<td>19</td>
<td>25</td>
</tr>
<tr>
<td>$\Lambda(1800)$</td>
<td>$^1_2^-$</td>
<td>$seen$</td>
<td>68</td>
<td>101</td>
</tr>
<tr>
<td>$\Lambda(1810)$</td>
<td>$^1_2^+$</td>
<td>$(38 \pm 23)^{+40}_{-10}$</td>
<td>3.8</td>
<td>2.1</td>
</tr>
<tr>
<td>$\Lambda(1830)$</td>
<td>$^5_2^-$</td>
<td>$(52 \pm 19)^{+11}_{-12}$</td>
<td>14</td>
<td>19</td>
</tr>
<tr>
<td>$\Sigma(1385)$</td>
<td>$^3_2^+$</td>
<td>$(4.2 \pm 0.5)^{+0.7}_{-0.5}$</td>
<td>3.1</td>
<td>0.5</td>
</tr>
<tr>
<td>$\Sigma(1560)$</td>
<td>$^1_2^-$</td>
<td></td>
<td>58</td>
<td>102</td>
</tr>
<tr>
<td>$\Sigma(1620)$</td>
<td>$^1_2^-$</td>
<td></td>
<td>32</td>
<td>44</td>
</tr>
<tr>
<td>$\Sigma(1660)$</td>
<td>$^1_2^+$</td>
<td>$seen$</td>
<td>10</td>
<td>24</td>
</tr>
<tr>
<td>$\Sigma(1670)$</td>
<td>$^3_2^-$</td>
<td>$(27 \pm 9)^{+12}_{-6}$</td>
<td>15</td>
<td>23</td>
</tr>
<tr>
<td>$\Sigma(1690)$</td>
<td>$^3_2^+$</td>
<td></td>
<td>0.4</td>
<td>2.7</td>
</tr>
<tr>
<td>$\Sigma(1750)$</td>
<td>$^3_2^-$</td>
<td>$(3.6 \pm 3.6)^{+5.6}_{-2.5}$</td>
<td>10</td>
<td>0.9</td>
</tr>
<tr>
<td>$\Sigma(1775)$</td>
<td>$^3_2^-$</td>
<td>$(4.2 \pm 1.8)^{+0.8}_{-0.3}$</td>
<td>1.9</td>
<td>3.8</td>
</tr>
<tr>
<td>$\Sigma(1880)$</td>
<td>$^1_2^+$</td>
<td></td>
<td>3.0</td>
<td>3.0</td>
</tr>
<tr>
<td>$\Sigma(1940)$</td>
<td>$^3_2^-$</td>
<td>$seen$</td>
<td>2.2</td>
<td>3.7</td>
</tr>
</tbody>
</table>
### DECAY WIDTHS OF LIGHT AND STRANGE BARYON RESONANCES

Table F.3: Same as Table F.1 but for the $\Lambda\pi$ and $\Xi\pi$ decay channels.

<table>
<thead>
<tr>
<th>Decay</th>
<th>$J^P$</th>
<th>Experiment [MeV]</th>
<th>Th. Masses</th>
<th>Exp. Masses</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>GBE</td>
<td>OGE</td>
</tr>
<tr>
<td>$\to \Lambda\pi$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Sigma(1385)$</td>
<td>$\frac{3}{2}^+$</td>
<td>$(31.3 \pm 0.5)^{+4.4}_{-4.3}$</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>$\Sigma(1560)$</td>
<td>$\frac{1}{2}^-$</td>
<td>seen</td>
<td>1.6</td>
<td>1.5</td>
</tr>
<tr>
<td>$\Sigma(1620)$</td>
<td>$\frac{1}{2}^-$</td>
<td>seen</td>
<td>19</td>
<td>25</td>
</tr>
<tr>
<td>$\Sigma(1660)$</td>
<td>$\frac{1}{2}^+$</td>
<td>seen</td>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>$\Sigma(1670)$</td>
<td>$\frac{3}{2}^-$</td>
<td>$(6 \pm 3)^{+3}_{-1}$</td>
<td>2.5</td>
<td>2.0</td>
</tr>
<tr>
<td>$\Sigma(1690)$</td>
<td>$\frac{3}{2}^+$</td>
<td>seen</td>
<td>$\approx 0$</td>
<td>1.2</td>
</tr>
<tr>
<td>$\Sigma(1750)$</td>
<td>$\frac{1}{2}^-$</td>
<td>seen</td>
<td>1.0</td>
<td>2.8</td>
</tr>
<tr>
<td>$\Sigma(1775)$</td>
<td>$\frac{5}{2}^-$</td>
<td>$(20 \pm 4)^{+3}_{-2}$</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>$\Sigma(1880)$</td>
<td>$\frac{1}{2}^+$</td>
<td>seen</td>
<td>1.7</td>
<td>0.5</td>
</tr>
<tr>
<td>$\Sigma(1940)$</td>
<td>$\frac{3}{2}^-$</td>
<td>seen</td>
<td>0.2</td>
<td>0.4</td>
</tr>
<tr>
<td>$\to \Xi\pi$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Xi(1530)$</td>
<td>$\frac{3}{2}^+$</td>
<td>$(9.9)^{+1.7}_{-1.9}$</td>
<td>2.2</td>
<td>1.3</td>
</tr>
<tr>
<td>$\Xi(1690)$</td>
<td>$\frac{1}{2}^+$</td>
<td>seen</td>
<td>0.8</td>
<td>1.8</td>
</tr>
<tr>
<td>$\Xi(1820)$</td>
<td>$\frac{3}{2}^-$</td>
<td>small</td>
<td>0.4</td>
<td>1.6</td>
</tr>
<tr>
<td>$\Xi(1950)$</td>
<td>$\frac{5}{2}^-$</td>
<td>seen</td>
<td>14</td>
<td>28</td>
</tr>
</tbody>
</table>
DECAY WIDTHS OF LIGHT AND STRANGE BARYON RESONANCES

Table F.4: Covariant predictions for $\eta$ decay widths of light baryon resonances as obtained with the GBE and the OGE CQMs along the PFSM in pseudovector coupling. In the last two columns experimental resonance masses (instead of theoretical masses) have been used in the calculation. The experimental data are from the latest compilation of the PDG [2].

<table>
<thead>
<tr>
<th>Decay $\rightarrow N\eta$</th>
<th>$J^P$</th>
<th>Experiment [MeV]</th>
<th>Th. Masses</th>
<th>Exp. Masses</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N(1520)$</td>
<td>$\frac{3}{2}^-$</td>
<td>$(0.26 \pm 0.05)^{+0.03}_{-0.03}$</td>
<td>0.11 0.11</td>
<td>0.11 0.10</td>
</tr>
<tr>
<td>$N(1535)$</td>
<td>$\frac{1}{2}^-$</td>
<td>$(79 \pm 11)^{+15}_{-11}$</td>
<td>27 35</td>
<td>30 39</td>
</tr>
<tr>
<td>$N(1650)$</td>
<td>$\frac{1}{2}^-$</td>
<td>$(11 \pm 6)^{+3}_{-1}$</td>
<td>50 74</td>
<td>50 69</td>
</tr>
<tr>
<td>$N(1675)$</td>
<td>$\frac{5}{2}^-$</td>
<td>$(0 \pm 0.8)^{+0.15}_{-0}$</td>
<td>1.5 2.4</td>
<td>2.0 2.0</td>
</tr>
<tr>
<td>$N(1700)$</td>
<td>$\frac{3}{2}^-$</td>
<td>$(0 \pm 1)^{+0.5}_{-0.5}$</td>
<td>0.5 0.9</td>
<td>0.9 1.0</td>
</tr>
<tr>
<td>$N(1710)$</td>
<td>$\frac{1}{2}^+$</td>
<td>$(6 \pm 1)^{+11}_{-4}$</td>
<td>0.02 0.06</td>
<td>0.05 0.27</td>
</tr>
</tbody>
</table>

Table F.5: Same as Table F.4 but for the sector of strange baryons.

<table>
<thead>
<tr>
<th>Decay $\rightarrow \Lambda\eta$</th>
<th>$J^P$</th>
<th>Experiment [MeV]</th>
<th>Th. Masses</th>
<th>Exp. Masses</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda(1670)$</td>
<td>$\frac{1}{2}^-$</td>
<td>$(6.1 \pm 2.6)^{+3.8}_{-2.5}$</td>
<td>19 4</td>
<td>6</td>
</tr>
<tr>
<td>$\Lambda(1690)$</td>
<td>$\frac{1}{2}^-$</td>
<td>0.2</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>$\Lambda(1800)$</td>
<td>$\frac{1}{2}^-$</td>
<td>43 65</td>
<td>46 62</td>
<td></td>
</tr>
<tr>
<td>$\Lambda(1810)$</td>
<td>$\frac{1}{2}^+$</td>
<td>0.9</td>
<td>$\approx$ 0</td>
<td>0.7 0.7</td>
</tr>
<tr>
<td>$\Lambda(1830)$</td>
<td>$\frac{1}{2}^-$</td>
<td>0.6 2.2</td>
<td>2.0 1.8</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Decay $\rightarrow \Sigma\eta$</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Sigma(1620)$</td>
<td>$\frac{1}{2}^-$</td>
<td>3.0</td>
<td>3.8</td>
<td></td>
</tr>
<tr>
<td>$\Sigma(1690)$</td>
<td>$\frac{1}{2}^-$</td>
<td>0.03</td>
<td>1.9</td>
<td></td>
</tr>
<tr>
<td>$\Sigma(1750)$</td>
<td>$\frac{1}{2}^-$</td>
<td>$(31.5 \pm 18.0)^{+38.5}_{-4.5}$</td>
<td>6.0 2.1</td>
<td>3.8 1.4</td>
</tr>
<tr>
<td>$\Sigma(1775)$</td>
<td>$\frac{1}{2}^-$</td>
<td>$\approx$ 0</td>
<td>0.05</td>
<td>0.02 0.01</td>
</tr>
<tr>
<td>$\Sigma(1880)$</td>
<td>$\frac{1}{2}^+$</td>
<td>0.01 0.03</td>
<td>$\approx$ 0</td>
<td>0.3</td>
</tr>
<tr>
<td>$\Sigma(1940)$</td>
<td>$\frac{1}{2}^-$</td>
<td>$\approx$ 0</td>
<td>$\approx$ 0</td>
<td>$\approx$ 0 0.02</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Decay $\rightarrow \Xi\eta$</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Xi(1690)$</td>
<td>$\frac{1}{2}^+$</td>
<td></td>
<td>2.5</td>
<td></td>
</tr>
<tr>
<td>$\Xi(1950)$</td>
<td>$\frac{5}{2}^-$</td>
<td></td>
<td>0.09 0.05</td>
<td>0.06</td>
</tr>
</tbody>
</table>

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Table F.6: Covariant predictions for $K$ decay widths as obtained with the GBE and the OGE CQMs along the PFSM in pseudovector coupling. In the last two columns experimental resonance masses (instead of theoretical masses) have been used in the calculation. The experimental data are from the latest compilation of the PDG [2].

<table>
<thead>
<tr>
<th>Decay</th>
<th>$J^P$</th>
<th>Experiment [MeV]</th>
<th>Th. Masses GBE</th>
<th>Exp. Masses GBE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>GBE</td>
<td>OGE</td>
</tr>
<tr>
<td>$\rightarrow \Lambda K$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N(1650)$</td>
<td>$\frac{1}{2}^-$</td>
<td>$(11.6 \pm 6.6)^{+2.2}_{-0.6}$</td>
<td>0.03</td>
<td>0.12</td>
</tr>
<tr>
<td>$N(1675)$</td>
<td>$\frac{1}{2}^-$</td>
<td>&lt; 1.7</td>
<td>$\approx 0$</td>
<td>$\approx 0$</td>
</tr>
<tr>
<td>$N(1700)$</td>
<td>$\frac{3}{2}^-$</td>
<td>$(1.5 \pm 1.5)^{+1.5}_{-1.5}$</td>
<td>$\approx 0$</td>
<td>0.05</td>
</tr>
<tr>
<td>$N(1710)$</td>
<td>$\frac{1}{2}^+$</td>
<td>$(15 \pm 10)^{-37.5}_{+2.5}$</td>
<td>$\approx 0$</td>
<td>$\approx 0$</td>
</tr>
<tr>
<td>$\rightarrow \Sigma K$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N(1700)$</td>
<td>$\frac{3}{2}^-$</td>
<td></td>
<td>$\approx 0$</td>
<td>$\approx 0$</td>
</tr>
<tr>
<td>$N(1710)$</td>
<td>$\frac{1}{2}^+$</td>
<td>3.9</td>
<td>389</td>
<td>0.4</td>
</tr>
<tr>
<td>$\Delta(1600)$</td>
<td>$\frac{3}{2}^+$</td>
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<td>$\approx 0$</td>
<td>0.6</td>
</tr>
<tr>
<td>$\Delta(1700)$</td>
<td>$\frac{3}{2}^-$</td>
<td></td>
<td>$\approx 0$</td>
<td>$\approx 0$</td>
</tr>
<tr>
<td>$\rightarrow \Xi K$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Lambda(1800)$</td>
<td>$\frac{1}{2}^-$</td>
<td></td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>$\Lambda(1810)$</td>
<td>$\frac{1}{2}^+$</td>
<td></td>
<td>1.7</td>
<td></td>
</tr>
<tr>
<td>$\Lambda(1830)$</td>
<td>$\frac{5}{2}^-$</td>
<td></td>
<td>$\approx 0$</td>
<td>$\approx 0$</td>
</tr>
<tr>
<td>$\Sigma(1690)$</td>
<td>$\frac{3}{2}^+$</td>
<td></td>
<td>$\approx 0$</td>
<td>0.16</td>
</tr>
<tr>
<td>$\Sigma(1880)$</td>
<td>$\frac{1}{2}^+$</td>
<td></td>
<td>0.11</td>
<td>0.06</td>
</tr>
<tr>
<td>$\Sigma(1940)$</td>
<td>$\frac{3}{2}^-$</td>
<td></td>
<td>0.01</td>
<td>0.02</td>
</tr>
</tbody>
</table>

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DECAY WIDTHS OF LIGHT AND STRANGE BARYON RESONANCES

### Table F.7: Same as Table F.6 but the decay channel is $NK$.

<table>
<thead>
<tr>
<th>Decay $\rightarrow NK$</th>
<th>$J^P$</th>
<th>Experiment [MeV]</th>
<th>Th. Masses</th>
<th>Exp. Masses</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda(1405)$</td>
<td>$\frac{1}{2}^-$</td>
<td><em>not possible</em></td>
<td>147</td>
<td>127</td>
</tr>
<tr>
<td>$\Lambda(1520)$</td>
<td>$\frac{3}{2}^-$</td>
<td>$(7.02 \pm 0.16)^{+0.46}_{-0.44}$</td>
<td>12</td>
<td>24</td>
</tr>
<tr>
<td>$\Lambda(1600)$</td>
<td>$\frac{1}{2}^+$</td>
<td>$(33.75 \pm 11.25)^{+30}_{-15}$</td>
<td>15</td>
<td>35</td>
</tr>
<tr>
<td>$\Lambda(1670)$</td>
<td>$\frac{1}{2}^-$</td>
<td>$(8.75 \pm 1.75)^{+4.5}_{-2}$</td>
<td>0.27</td>
<td>0.03</td>
</tr>
<tr>
<td>$\Lambda(1690)$</td>
<td>$\frac{3}{2}^-$</td>
<td>$(15 \pm 3)^{+3}_{-2}$</td>
<td>1.2</td>
<td>1.0</td>
</tr>
<tr>
<td>$\Lambda(1800)$</td>
<td>$\frac{1}{2}^-$</td>
<td>$(97.5 \pm 22.5)^{+40}_{-25}$</td>
<td>4.2</td>
<td>6.4</td>
</tr>
<tr>
<td>$\Lambda(1810)$</td>
<td>$\frac{1}{2}^+$</td>
<td>$(52.5 \pm 22.5)^{+50}_{-20}$</td>
<td>4.1</td>
<td>11.7</td>
</tr>
<tr>
<td>$\Lambda(1830)$</td>
<td>$\frac{3}{2}^-$</td>
<td>$(6.18 \pm 3.33)^{+1.05}_{-1.05}$</td>
<td>0.09</td>
<td>0.13</td>
</tr>
<tr>
<td>$\Sigma(1560)$</td>
<td>$\frac{1}{2}^-$</td>
<td></td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>$\Sigma(1620)$</td>
<td>$\frac{1}{2}^-$</td>
<td></td>
<td>55</td>
<td>55</td>
</tr>
<tr>
<td>$\Sigma(1660)$</td>
<td>$\frac{3}{2}^+$</td>
<td>$(20 \pm 10)^{+30}_{-6}$</td>
<td>0.9</td>
<td>0.9</td>
</tr>
<tr>
<td>$\Sigma(1670)$</td>
<td>$\frac{3}{2}^-$</td>
<td>$(6.0 \pm 1.8)^{+2.6}_{-1.4}$</td>
<td>1.1</td>
<td>1.0</td>
</tr>
<tr>
<td>$\Sigma(1690)$</td>
<td>$\frac{3}{2}^+$</td>
<td></td>
<td>$\approx 0$</td>
<td>1.4</td>
</tr>
<tr>
<td>$\Sigma(1750)$</td>
<td>$\frac{1}{2}^-$</td>
<td>$(22.5 \pm 13.5)^{+28}_{-3}$</td>
<td>$\approx 0$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\Sigma(1775)$</td>
<td>$\frac{3}{2}^-$</td>
<td>$(48.0 \pm 3.6)^{+6.5}_{-5.6}$</td>
<td>11</td>
<td>15</td>
</tr>
<tr>
<td>$\Sigma(1880)$</td>
<td>$\frac{1}{2}^+$</td>
<td></td>
<td>0.02</td>
<td>$\approx 0$</td>
</tr>
<tr>
<td>$\Sigma(1940)$</td>
<td>$\frac{3}{2}^-$</td>
<td>$(22 \pm 22)^{+16}$</td>
<td>1.1</td>
<td>1.5</td>
</tr>
</tbody>
</table>
DECAY WIDTHS OF LIGHT AND STRANGE BARYON RESONANCES

Table F.8: Same as Table F.6 but the decay channels are $\Lambda K$ and $\Sigma K$.

<table>
<thead>
<tr>
<th>Decay $\rightarrow \Lambda K$</th>
<th>$J^P$</th>
<th>Experiment [MeV]</th>
<th>Th. Masses GBE</th>
<th>Exp. Masses OGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Xi(1690)$ $\frac{1}{2}^+$</td>
<td>seen</td>
<td>1.1 1.3 0.5 0.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Xi(1820)$ $\frac{3}{2}^-$</td>
<td>large</td>
<td>2.7 6.2 4.5 3.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Xi(1950)$ $\frac{5}{2}^-$</td>
<td>seen</td>
<td>2.5 4.4 4.3 3.6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Decay $\rightarrow \Sigma K$</th>
<th>$J^P$</th>
<th>Experiment [MeV]</th>
<th>Th. Masses GBE</th>
<th>Exp. Masses OGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Xi(1690)$ $\frac{1}{2}^+$</td>
<td>seen</td>
<td>9 55 0.06 0.16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Xi(1820)$ $\frac{3}{2}^-$</td>
<td>small</td>
<td>4.1 9.3 5.1 4.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Xi(1950)$ $\frac{5}{2}^-$</td>
<td>seen</td>
<td>2.3 4.3 3.7 3.6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

F.2 Relativistic Results with Pseudoscalar Coupling

Analogously to the previous section, the covariant results for decay widths in case of a PFSM operator with pseudoscalar coupling are collected in Tables F.9 - F.16. The notation is the same as before.
Table F.9: Covariant predictions for $\pi$ decay widths of light baryon resonances as obtained with the GBE and the OGE CQMs along the PFSM in pseudoscalar coupling. In the last two columns experimental resonance masses (instead of theoretical masses) have been used in the calculation. The experimental data are from the latest compilation of the PDG [2].

<table>
<thead>
<tr>
<th>Decay $\rightarrow N\pi$ $J^P$</th>
<th>Experiment [MeV]</th>
<th>Th. Masses</th>
<th>Exp. Masses</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$GBE$ $OGE$ $GBE$ $OGE$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N(1440)$ $\frac{1}{2}^+$</td>
<td>$(195 \pm 30)^{+113}_{-55}$</td>
<td>1.6</td>
<td>0.8</td>
</tr>
<tr>
<td>$N(1520)$ $\frac{3}{2}^-$</td>
<td>$(69 \pm 6)^{+7}_{-8}$</td>
<td>22</td>
<td>28</td>
</tr>
<tr>
<td>$N(1535)$ $\frac{1}{2}^-$</td>
<td>$(68 \pm 15)^{+14}_{-9}$</td>
<td>58</td>
<td>69</td>
</tr>
<tr>
<td>$N(1650)$ $\frac{1}{2}^-$</td>
<td>$(128 \pm 29)^{+14}_{-12}$</td>
<td>20</td>
<td>27</td>
</tr>
<tr>
<td>$N(1675)$ $\frac{3}{2}^-$</td>
<td>$(60 \pm 8)^{+7}_{-7}$</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>$N(1700)$ $\frac{3}{2}^-$</td>
<td>$(10 \pm 5)^{+3}_{-3}$</td>
<td>1.1</td>
<td>1.7</td>
</tr>
<tr>
<td>$N(1710)$ $\frac{1}{2}^+$</td>
<td>$(15 \pm 5)^{+30}_{-5}$</td>
<td>35</td>
<td>61</td>
</tr>
<tr>
<td>$\Delta(1232)$ $\frac{3}{2}^+$</td>
<td>$(118)^{+2}_{-2}$</td>
<td>48</td>
<td>44</td>
</tr>
<tr>
<td>$\Delta(1600)$ $\frac{3}{2}^+$</td>
<td>$(61 \pm 26)^{+25}_{-10}$</td>
<td>23</td>
<td>17</td>
</tr>
<tr>
<td>$\Delta(1620)$ $\frac{1}{2}^-$</td>
<td>$(36 \pm 7)^{+1}_{-2}$</td>
<td>14</td>
<td>17</td>
</tr>
<tr>
<td>$\Delta(1700)$ $\frac{3}{2}^-$</td>
<td>$(45 \pm 15)^{+20}_{-10}$</td>
<td>2.8</td>
<td>3.6</td>
</tr>
</tbody>
</table>
Table F.10: Same as Table F.9 but for the $\Sigma\pi$ decay channel.

<table>
<thead>
<tr>
<th>Decays</th>
<th>(J^P)</th>
<th>Experiment [MeV]</th>
<th>Th. Masses</th>
<th>Exp. Masses</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Lambda(1405))</td>
<td>(\frac{1}{2}^-)</td>
<td>((50 \pm 2))</td>
<td>2.7</td>
<td>4.2</td>
</tr>
<tr>
<td>(\Lambda(1520))</td>
<td>(\frac{3}{2}^-)</td>
<td>((6.55 \pm 0.16)_{-0.04}^{+0.04})</td>
<td>5.8</td>
<td>10.3</td>
</tr>
<tr>
<td>(\Lambda(1600))</td>
<td>(\frac{1}{2}^+)</td>
<td>((53 \pm 38)_{-10}^{+60})</td>
<td>1.8</td>
<td>1.2</td>
</tr>
<tr>
<td>(\Lambda(1670))</td>
<td>(\frac{1}{2}^-)</td>
<td>((14.0 \pm 5.3)_{-2.5}^{+8.3})</td>
<td>26</td>
<td>34</td>
</tr>
<tr>
<td>(\Lambda(1690))</td>
<td>(\frac{3}{2}^-)</td>
<td>((18 \pm 6)_{-2}^{+4})</td>
<td>19</td>
<td>27</td>
</tr>
<tr>
<td>(\Lambda(1800))</td>
<td>(\frac{1}{2}^-)</td>
<td>(seen)</td>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td>(\Lambda(1810))</td>
<td>(\frac{1}{2}^+)</td>
<td>((38 \pm 23)_{-10}^{+40})</td>
<td>0.4</td>
<td>1.5</td>
</tr>
<tr>
<td>(\Lambda(1830))</td>
<td>(\frac{3}{2}^-)</td>
<td>((52 \pm 19)_{-12}^{+11})</td>
<td>10</td>
<td>18</td>
</tr>
<tr>
<td>(\Sigma(1385))</td>
<td>(\frac{3}{2}^+)</td>
<td>((4.2 \pm 0.5)_{-0.5}^{+0.7})</td>
<td>3.3</td>
<td>0.6</td>
</tr>
<tr>
<td>(\Sigma(1560))</td>
<td>(\frac{1}{2}^-)</td>
<td></td>
<td>21</td>
<td>36</td>
</tr>
<tr>
<td>(\Sigma(1620))</td>
<td>(\frac{1}{2}^-)</td>
<td></td>
<td>1.1</td>
<td>2.7</td>
</tr>
<tr>
<td>(\Sigma(1660))</td>
<td>(\frac{1}{2}^+)</td>
<td>(seen)</td>
<td>0.01</td>
<td>0.5</td>
</tr>
<tr>
<td>(\Sigma(1670))</td>
<td>(\frac{3}{2}^-)</td>
<td>((27 \pm 9)_{-6}^{+12})</td>
<td>14</td>
<td>24</td>
</tr>
<tr>
<td>(\Sigma(1750))</td>
<td>(\frac{1}{2}^-)</td>
<td>((3.6 \pm 3.6)_{-6}^{+5.6})</td>
<td>8</td>
<td>1.3</td>
</tr>
<tr>
<td>(\Sigma(1690))</td>
<td>(\frac{3}{2}^+)</td>
<td></td>
<td>7.5</td>
<td>2.8</td>
</tr>
<tr>
<td>(\Sigma(1775))</td>
<td>(\frac{3}{2}^-)</td>
<td>((4.2 \pm 1.8)_{-0.3}^{+0.8})</td>
<td>1.6</td>
<td>3.4</td>
</tr>
<tr>
<td>(\Sigma(1880))</td>
<td>(\frac{1}{2}^+)</td>
<td></td>
<td>14</td>
<td>26</td>
</tr>
<tr>
<td>(\Sigma(1940))</td>
<td>(\frac{3}{2}^-)</td>
<td>(seen)</td>
<td>1.9</td>
<td>3.7</td>
</tr>
</tbody>
</table>
Table F.11: Same as Table F.9 but for the $\Lambda \pi$ and $\Xi \pi$ decay channels.

<table>
<thead>
<tr>
<th>Decay</th>
<th>$J^P$</th>
<th>Experiment [MeV]</th>
<th>Th. Masses</th>
<th>Exp. Masses</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>GBE</td>
<td>OGE</td>
</tr>
<tr>
<td>$\rightarrow \Lambda \pi$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Sigma(1385)$</td>
<td>$^3^+ \frac{\pi}{2}$</td>
<td>$(31.3 \pm 0.5)^{+4.4}_{-4.3}$</td>
<td>12</td>
<td>15</td>
</tr>
<tr>
<td>$\Sigma(1560)$</td>
<td>$^1^- \frac{\pi}{2}$</td>
<td>seen</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>$\Sigma(1620)$</td>
<td>$^1^- \frac{\pi}{2}$</td>
<td></td>
<td>11</td>
<td>19</td>
</tr>
<tr>
<td>$\Sigma(1660)$</td>
<td>$^1^+ \frac{\pi}{2}$</td>
<td>seen</td>
<td>0.4</td>
<td>2.2</td>
</tr>
<tr>
<td>$\Sigma(1670)$</td>
<td>$^3^+ \frac{\pi}{2}$</td>
<td>$(6 \pm 3)^{+3}_{-1}$</td>
<td>2.3</td>
<td>2.1</td>
</tr>
<tr>
<td>$\Sigma(1690)$</td>
<td>$^3^+ \frac{\pi}{2}$</td>
<td></td>
<td>11</td>
<td>9</td>
</tr>
<tr>
<td>$\Sigma(1750)$</td>
<td>$^1^- \frac{\pi}{2}$</td>
<td>seen</td>
<td>2.6</td>
<td>8.6</td>
</tr>
<tr>
<td>$\Sigma(1775)$</td>
<td>$^5^- \frac{\pi}{2}$</td>
<td>$(20 \pm 4)^{+3}_{-2}$</td>
<td>5</td>
<td>9</td>
</tr>
<tr>
<td>$\Sigma(1880)$</td>
<td>$^1^+ \frac{\pi}{2}$</td>
<td></td>
<td>4.0</td>
<td>1.4</td>
</tr>
<tr>
<td>$\Sigma(1940)$</td>
<td>$^3^- \frac{\pi}{2}$</td>
<td>seen</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>$\rightarrow \Xi \pi$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Xi(1530)$</td>
<td>$^3^+ \frac{\pi}{2}$</td>
<td>$(9.9)^{+1.7}_{-1.9}$</td>
<td>2.1</td>
<td>1.6</td>
</tr>
<tr>
<td>$\Xi(1690)$</td>
<td>$^1^+ \frac{\pi}{2}$</td>
<td>seen</td>
<td>$\approx 0$</td>
<td>0.01</td>
</tr>
<tr>
<td>$\Xi(1820)$</td>
<td>$^3^- \frac{\pi}{2}$</td>
<td>small</td>
<td>0.4</td>
<td>1.6</td>
</tr>
<tr>
<td>$\Xi(1950)$</td>
<td>$^5^- \frac{\pi}{2}$</td>
<td>seen</td>
<td>11</td>
<td>25</td>
</tr>
</tbody>
</table>
### Decay Widths of Light and Strange Baryon Resonances

Table F.12: Covariant predictions for $\eta$ decay widths of light baryon resonances as obtained with the GBE and the OGE CQMs along the PFSM in pseudoscalar coupling. In the last two columns experimental resonance masses (instead of theoretical masses) have been used in the calculation. The experimental data are from the latest compilation of the PDG [2].

<table>
<thead>
<tr>
<th>Decay $\rightarrow N\eta$</th>
<th>$J^P$</th>
<th>Experiment [MeV]</th>
<th>Th. Masses GBE</th>
<th>Exp. Masses GBE</th>
<th>Exp. Masses OGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N(1520)$ $\frac{3}{2}^-$</td>
<td>(0.26 ± 0.05)$^{+0.03}_{-0.03}$</td>
<td>0.1 0.1 0.1 0.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N(1535)$ $\frac{1}{2}^-$</td>
<td>(79 ± 11)$^{+15}_{-11}$</td>
<td>0.1 0.2 0.3 0.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N(1650)$ $\frac{1}{2}^-$</td>
<td>(11 ± 6)$^{+2}_{-1}$</td>
<td>1.2 1.9 1.3 1.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N(1675)$ $\frac{5}{2}^-$</td>
<td>(0 ± 0.8)$^{+0.15}_{-0.1}$</td>
<td>1.2 2.2 1.6 1.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N(1700)$ $\frac{3}{2}^-$</td>
<td>(0 ± 1)$^{+0.5}_{-0.5}$</td>
<td>0.5 0.9 0.8 1.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N(1710)$ $\frac{1}{2}^+$</td>
<td>(6 ± 1)$^{+11}_{-4}$</td>
<td>1.1 2.9 0.4 0.3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table F.13: Same as Table F.12 but for the sector of strange baryons.

<table>
<thead>
<tr>
<th>Decay $\rightarrow \Lambda\eta$</th>
<th>$J^P$</th>
<th>Experiment [MeV]</th>
<th>Th. Masses GBE</th>
<th>Exp. Masses GBE</th>
<th>Exp. Masses OGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda(1670)$ $\frac{1}{2}^-$</td>
<td>(6.1 ± 2.6)$^{+3.8}_{-2.5}$</td>
<td>$\approx 0$ $\approx 0$ $\approx 0$ $\approx 0$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Lambda(1690)$ $\frac{1}{2}^-$</td>
<td></td>
<td>0.17 0.01 0.02</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Lambda(1800)$ $\frac{1}{2}^-$</td>
<td></td>
<td>0.8 3.6 1.8 1.8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Lambda(1810)$ $\frac{1}{2}^+$</td>
<td></td>
<td>0.5 1.1 0.3 0.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Lambda(1830)$ $\frac{3}{2}^-$</td>
<td></td>
<td>0.4 2.1 1.5 1.7</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Decay $\rightarrow \Sigma\eta$</th>
<th>$J^P$</th>
<th>Experiment [MeV]</th>
<th>Th. Masses GBE</th>
<th>Exp. Masses GBE</th>
<th>Exp. Masses OGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Sigma(1620)$ $\frac{1}{2}^-$</td>
<td></td>
<td>$\approx 0$ 0.13</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Sigma(1690)$ $\frac{1}{2}^+$</td>
<td></td>
<td>3.1 1.6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Sigma(1750)$ $\frac{1}{2}^-$</td>
<td>(31.5 ± 18.0)$^{+38.5}_{-4.5}$</td>
<td>0.05 0.01 $\approx 0$ $\approx 0$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Sigma(1775)$ $\frac{3}{2}^+$</td>
<td></td>
<td>$\approx 0$ 0.95 0.02 0.02</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Sigma(1880)$ $\frac{1}{2}^-$</td>
<td></td>
<td>0.5 1.5 0.2 0.07</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Sigma(1940)$ $\frac{3}{2}^+$</td>
<td></td>
<td>$\approx 0$ $\approx 0$ $\approx 0$ $\approx 0$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Decay $\rightarrow \Xi\eta$</th>
<th>$J^P$</th>
<th>Experiment [MeV]</th>
<th>Th. Masses GBE</th>
<th>Exp. Masses GBE</th>
<th>Exp. Masses OGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Sigma(1690)$ $\frac{3}{2}^+$</td>
<td></td>
<td>$\approx 0$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Sigma(1950)$ $\frac{1}{2}^-$</td>
<td></td>
<td>0.04 0.02 0.03</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
DECAY WIDTHS OF LIGHT AND STRANGE BARYON RESONANCES

Table F.14: Covariant predictions for $K$ decay widths as obtained with the GBE and the OGE CQMs along the PFSM in pseudoscalar coupling. In the last two columns experimental resonance masses (instead of theoretical masses) have been used in the calculation. The experimental data are from the latest compilation of the PDG [2].

<table>
<thead>
<tr>
<th>Decay</th>
<th>$J^P$</th>
<th>Experiment [MeV]</th>
<th>Th. Masses</th>
<th>Exp. Masses</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\to \Lambda K$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N(1650)$</td>
<td>$\frac{3}{2}^-$</td>
<td>$(11.6 \pm 6.6)^{+2.2}_{-0.6}$</td>
<td>$\approx 0$</td>
<td>$0.03$ $\approx 0$ $\approx 0$</td>
</tr>
<tr>
<td>$N(1675)$</td>
<td>$\frac{5}{2}^-$</td>
<td>$&lt; 1.7$</td>
<td>$\approx 0$</td>
<td>$\approx 0$ $\approx 0$ $\approx 0$</td>
</tr>
<tr>
<td>$N(1700)$</td>
<td>$\frac{3}{2}^-$</td>
<td>$(1.5 \pm 1.5)^{+1.5}_{-1}$</td>
<td>$\approx 0$</td>
<td>$0.09$ $0.07$ $0.10$</td>
</tr>
<tr>
<td>$N(1710)$</td>
<td>$\frac{1}{2}^+$</td>
<td>$(15 \pm 10)^{+37.5}_{-2.5}$</td>
<td>$0.02$</td>
<td>$\approx 0$ $0.01$ $\approx 0$</td>
</tr>
<tr>
<td>$\to \Sigma K$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N(1700)$</td>
<td>$\frac{3}{2}^-$</td>
<td></td>
<td>$\approx 0$</td>
<td>$\approx 0$</td>
</tr>
<tr>
<td>$N(1710)$</td>
<td>$\frac{1}{2}^+$</td>
<td></td>
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<td>$322$ $0.02$ $24$</td>
</tr>
<tr>
<td>$\Delta(1600)$</td>
<td>$\frac{3}{2}^+$</td>
<td></td>
<td>$1.3$</td>
<td>$4.4$</td>
</tr>
<tr>
<td>$\Delta(1700)$</td>
<td>$\frac{3}{2}^-$</td>
<td></td>
<td>$\approx 0$</td>
<td>$\approx 0$</td>
</tr>
<tr>
<td>$\to \Xi K$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Lambda(1800)$</td>
<td>$\frac{1}{2}^-$</td>
<td></td>
<td></td>
<td>$1.1$</td>
</tr>
<tr>
<td>$\Lambda(1810)$</td>
<td>$\frac{3}{2}^+$</td>
<td></td>
<td></td>
<td>$0.7$</td>
</tr>
<tr>
<td>$\Lambda(1830)$</td>
<td>$\frac{5}{2}^-$</td>
<td></td>
<td>$\approx 0$</td>
<td>$\approx 0$ $\approx 0$</td>
</tr>
<tr>
<td>$\Sigma(1690)$</td>
<td>$\frac{3}{2}^+$</td>
<td></td>
<td>$0.1$</td>
<td>$1.4$</td>
</tr>
<tr>
<td>$\Sigma(1880)$</td>
<td>$\frac{1}{2}^+$</td>
<td></td>
<td>$0.02$</td>
<td>$1.0$ $0.02$ $\approx 0$</td>
</tr>
<tr>
<td>$\Sigma(1940)$</td>
<td>$\frac{3}{2}^-$</td>
<td></td>
<td>$0.02$</td>
<td>$0.03$</td>
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</tbody>
</table>
Table F.15: Same as Table F.14 but the decay channel is $NK$.

<table>
<thead>
<tr>
<th>Decay $\rightarrow NK$</th>
<th>$J^P$</th>
<th>Experiment [MeV]</th>
<th>Th. Masses</th>
<th>Exp. Masses</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda(1405)$</td>
<td>$\frac{1}{2}^-$</td>
<td>$not possible$</td>
<td>3.3</td>
<td>8.0</td>
</tr>
<tr>
<td>$\Lambda(1520)$</td>
<td>$\frac{3}{2}^-$</td>
<td>$(7.02 \pm 0.16)^{+0.46}_{-0.44}$</td>
<td>16</td>
<td>38</td>
</tr>
<tr>
<td>$\Lambda(1600)$</td>
<td>$\frac{1}{2}^+$</td>
<td>$(33.75 \pm 11.25)^{+30}_{-15}$</td>
<td>1.1</td>
<td>0.01</td>
</tr>
<tr>
<td>$\Lambda(1670)$</td>
<td>$\frac{1}{2}^-$</td>
<td>$(8.75 \pm 1.75)^{+4.5}_{-2}$</td>
<td>6.9</td>
<td>9.3</td>
</tr>
<tr>
<td>$\Lambda(1690)$</td>
<td>$\frac{2}{2}^-$</td>
<td>$(15 \pm 3)^{+3}_{-2}$</td>
<td>1.0</td>
<td>1.2</td>
</tr>
<tr>
<td>$\Lambda(1800)$</td>
<td>$\frac{1}{2}^-$</td>
<td>$(97.5 \pm 22.5)^{+40}_{-25}$</td>
<td>2.1</td>
<td>3.8</td>
</tr>
<tr>
<td>$\Lambda(1810)$</td>
<td>$\frac{1}{2}^+$</td>
<td>$(52.5 \pm 22.5)^{+50}_{-20}$</td>
<td>7.0</td>
<td>36</td>
</tr>
<tr>
<td>$\Lambda(1830)$</td>
<td>$\frac{3}{2}^-$</td>
<td>$(6.18 \pm 3.33)^{+1.05}_{-1.05}$</td>
<td>0.08</td>
<td>0.15</td>
</tr>
<tr>
<td>$\Sigma(1560)$</td>
<td>$\frac{1}{2}^-$</td>
<td></td>
<td>0.06</td>
<td>0.11</td>
</tr>
<tr>
<td>$\Sigma(1620)$</td>
<td>$\frac{1}{2}^-$</td>
<td></td>
<td>6.2</td>
<td>7.8</td>
</tr>
<tr>
<td>$\Sigma(1660)$</td>
<td>$\frac{1}{2}^+$</td>
<td>$(20 \pm 10)^{+30}_{-6}$</td>
<td>0.02</td>
<td>0.10</td>
</tr>
<tr>
<td>$\Sigma(1670)$</td>
<td>$\frac{3}{2}^-$</td>
<td>$(6.0 \pm 1.8)^{+2.6}_{-1.4}$</td>
<td>1.1</td>
<td>1.3</td>
</tr>
<tr>
<td>$\Sigma(1690)$</td>
<td>$\frac{3}{2}^+$</td>
<td></td>
<td>7.1</td>
<td>3.6</td>
</tr>
<tr>
<td>$\Sigma(1750)$</td>
<td>$\frac{1}{2}^-$</td>
<td>$(22.5 \pm 13.5)^{+28}_{-3}$</td>
<td>2.3</td>
<td>3.1</td>
</tr>
<tr>
<td>$\Sigma(1775)$</td>
<td>$\frac{3}{2}^-$</td>
<td>$(48.0 \pm 3.6)^{+6.5}_{-5.6}$</td>
<td>10</td>
<td>17</td>
</tr>
<tr>
<td>$\Sigma(1880)$</td>
<td>$\frac{1}{2}^+$</td>
<td></td>
<td>0.04</td>
<td>0.02</td>
</tr>
<tr>
<td>$\Sigma(1940)$</td>
<td>$\frac{3}{2}^-$</td>
<td>$(22 \pm 22)^{+16}_{-1}$</td>
<td>1.0</td>
<td>1.7</td>
</tr>
</tbody>
</table>
Table F.16: Same as Table F.14 but the decay channels are $\Lambda K$ and $\Sigma K$.

<table>
<thead>
<tr>
<th>Decay</th>
<th>$J^P$</th>
<th>Experiment</th>
<th>[MeV]</th>
<th>Th. Masses</th>
<th>Exp. Masses</th>
</tr>
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<tbody>
<tr>
<td></td>
<td></td>
<td>GBE</td>
<td>OGE</td>
<td>GBE</td>
<td>OGE</td>
</tr>
<tr>
<td>→ $\Lambda K$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Xi(1690)$</td>
<td>$\frac{1}{2}^+$</td>
<td>seen</td>
<td>$\approx 0$</td>
<td>0.11</td>
<td>$\approx 0$</td>
</tr>
<tr>
<td>$\Xi(1820)$</td>
<td>$\frac{3}{2}^-$</td>
<td>large</td>
<td>2.7</td>
<td>7.5</td>
<td>4.3</td>
</tr>
<tr>
<td>$\Xi(1950)$</td>
<td>$\frac{5}{2}^-$</td>
<td>seen</td>
<td>2.1</td>
<td>5.0</td>
<td>3.4</td>
</tr>
<tr>
<td>→ $\Sigma K$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Xi(1690)$</td>
<td>$\frac{1}{2}^+$</td>
<td>seen</td>
<td>1.1</td>
<td>1.8</td>
<td>$\approx 0$</td>
</tr>
<tr>
<td>$\Xi(1820)$</td>
<td>$\frac{3}{2}^-$</td>
<td>small</td>
<td>4.5</td>
<td>11.5</td>
<td>5.4</td>
</tr>
<tr>
<td>$\Xi(1950)$</td>
<td>$\frac{5}{2}^-$</td>
<td>seen</td>
<td>1.9</td>
<td>4.6</td>
<td>2.9</td>
</tr>
</tbody>
</table>

F.3 Non-Relativistic Results

The results for decays widths as calculated with the non-relativistic reduction of the decay operator given in Chapter 6 are summarized in Tables F.17 - F.24, where the notation is again as before.
Table F.17: Non-relativistic predictions for $\pi$ decay widths of light baryon resonances as obtained with the GBE and the OGE CQMs. In the last two columns experimental resonance masses (instead of theoretical masses) have been used in the calculation. The experimental data are from the latest compilation of the PDG [2].

<table>
<thead>
<tr>
<th>Decay $\rightarrow N\pi$</th>
<th>$J^P$</th>
<th>Experiment [MeV]</th>
<th>Th. Masses</th>
<th>Exp. Masses</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>GBE</td>
<td>OGE</td>
<td>GBE</td>
</tr>
<tr>
<td>$N(1440)$</td>
<td>$\frac{1}{2}^+$</td>
<td>(195 ± 30)$^{+13}_{-55}$</td>
<td>7</td>
<td>27</td>
</tr>
<tr>
<td>$N(1520)$</td>
<td>$\frac{3}{2}^-$</td>
<td>(69 ± 6)$^{+7}_{-8}$</td>
<td>38</td>
<td>37</td>
</tr>
<tr>
<td>$N(1535)$</td>
<td>$\frac{1}{2}^-$</td>
<td>(68 ± 15)$^{+14}_{-9}$</td>
<td>559</td>
<td>1183</td>
</tr>
<tr>
<td>$N(1650)$</td>
<td>$\frac{1}{2}^-$</td>
<td>(128 ± 29)$^{+19}_{-12}$</td>
<td>157</td>
<td>352</td>
</tr>
<tr>
<td>$N(1675)$</td>
<td>$\frac{5}{2}^-$</td>
<td>(60 ± 8)$^{+7}_{-7}$</td>
<td>13</td>
<td>16</td>
</tr>
<tr>
<td>$N(1700)$</td>
<td>$\frac{3}{2}^-$</td>
<td>(10 ± 5)$^{+3}_{-3}$</td>
<td>2.2</td>
<td>2.7</td>
</tr>
<tr>
<td>$N(1710)$</td>
<td>$\frac{1}{2}^+$</td>
<td>(15 ± 5)$^{+30}_{-5}$</td>
<td>8.2</td>
<td>5.9</td>
</tr>
<tr>
<td>$\Delta(1232)$</td>
<td>$\frac{3}{2}^+$</td>
<td>(118)$^{+2}_{-2}$</td>
<td>89</td>
<td>85</td>
</tr>
<tr>
<td>$\Delta(1600)$</td>
<td>$\frac{3}{2}^+$</td>
<td>(61 ± 26)$^{+25}_{-10}$</td>
<td>93</td>
<td>86</td>
</tr>
<tr>
<td>$\Delta(1620)$</td>
<td>$\frac{1}{2}^-$</td>
<td>(36 ± 7)$^{+2}_{-2}$</td>
<td>76</td>
<td>177</td>
</tr>
<tr>
<td>$\Delta(1700)$</td>
<td>$\frac{3}{2}^-$</td>
<td>(45 ± 15)$^{+20}_{-10}$</td>
<td>10</td>
<td>9</td>
</tr>
</tbody>
</table>
Table F.18: Same as Table F.17 but for the $\Sigma\pi$ decay channel.

<table>
<thead>
<tr>
<th>Decay $\rightarrow \Sigma\pi$</th>
<th>$J^P$</th>
<th>Experiment [MeV]</th>
<th>Th. Masses</th>
<th>Exp. Masses</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda(1405)$</td>
<td>$\frac{1}{2}^-$</td>
<td>(50 ± 2)</td>
<td>320</td>
<td>611</td>
</tr>
<tr>
<td>$\Lambda(1520)$</td>
<td>$\frac{3}{2}^-$</td>
<td>(6.55 ± 0.16)$^{+0.04}_{-0.04}$</td>
<td>4.5</td>
<td>8.0</td>
</tr>
<tr>
<td>$\Lambda(1600)$</td>
<td>$\frac{1}{2}^+$</td>
<td>(53 ± 38)$^{+60}_{-10}$</td>
<td>1.6</td>
<td>34</td>
</tr>
<tr>
<td>$\Lambda(1670)$</td>
<td>$\frac{1}{2}^-$</td>
<td>(14.0 ± 5.3)$^{+8.3}_{-2.5}$</td>
<td>620</td>
<td>1272</td>
</tr>
<tr>
<td>$\Lambda(1690)$</td>
<td>$\frac{3}{2}^-$</td>
<td>(18 ± 6)$^{+4}_{-2}$</td>
<td>24</td>
<td>28</td>
</tr>
<tr>
<td>$\Lambda(1800)$</td>
<td>$\frac{1}{2}^-$</td>
<td>seen</td>
<td>473</td>
<td>1175</td>
</tr>
<tr>
<td>$\Lambda(1810)$</td>
<td>$\frac{1}{2}^+$</td>
<td>(38 ± 23)$^{+40}_{-10}$</td>
<td>55</td>
<td>150</td>
</tr>
<tr>
<td>$\Lambda(1830)$</td>
<td>$\frac{5}{2}^-$</td>
<td>(52 ± 19)$^{+11}_{-12}$</td>
<td>16</td>
<td>24</td>
</tr>
<tr>
<td>$\Sigma(1385)$</td>
<td>$\frac{3}{2}^+$</td>
<td>(4.2 ± 0.5)$^{+0.7}_{-0.5}$</td>
<td>6.5</td>
<td>1.1</td>
</tr>
<tr>
<td>$\Sigma(1560)$</td>
<td>$\frac{1}{2}^-$</td>
<td></td>
<td>480</td>
<td>1249</td>
</tr>
<tr>
<td>$\Sigma(1620)$</td>
<td>$\frac{3}{2}^-$</td>
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<td>135</td>
<td>312</td>
</tr>
<tr>
<td>$\Sigma(1660)$</td>
<td>$\frac{1}{2}^+$</td>
<td>seen</td>
<td>2.1</td>
<td>15</td>
</tr>
<tr>
<td>$\Sigma(1670)$</td>
<td>$\frac{3}{2}^-$</td>
<td>(27 ± 9)$^{+12}_{-6}$</td>
<td>21</td>
<td>32</td>
</tr>
<tr>
<td>$\Sigma(1690)$</td>
<td>$\frac{3}{2}^+$</td>
<td></td>
<td>29</td>
<td>2.3</td>
</tr>
<tr>
<td>$\Sigma(1750)$</td>
<td>$\frac{1}{2}^-$</td>
<td>(3.6 ± 3.6)$^{+5.6}_{-3.6}$</td>
<td>116</td>
<td>34</td>
</tr>
<tr>
<td>$\Sigma(1775)$</td>
<td>$\frac{3}{2}^-$</td>
<td>(4.2 ± 1.8)$^{+0.8}_{-0.3}$</td>
<td>2.9</td>
<td>6.9</td>
</tr>
<tr>
<td>$\Sigma(1880)$</td>
<td>$\frac{1}{2}^+$</td>
<td></td>
<td>1.2</td>
<td>12.2</td>
</tr>
<tr>
<td>$\Sigma(1940)$</td>
<td>$\frac{3}{2}^-$</td>
<td>seen</td>
<td>0.5</td>
<td>1.1</td>
</tr>
</tbody>
</table>
Table F.19: Same as Table F.17 but for the $\Lambda\pi$ and $\Xi\pi$ decay channels.

<table>
<thead>
<tr>
<th>Decay</th>
<th>$J^P$</th>
<th>Experiment [MeV]</th>
<th>Th. Masses</th>
<th>Exp. Masses</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>GBE</td>
<td>OGE</td>
</tr>
<tr>
<td>$\to \Lambda\pi$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Sigma(1385)$</td>
<td>$3^+$</td>
<td>$(31.3 \pm 0.5)^{+4.4}_{-4.3}$</td>
<td>25</td>
<td>28</td>
</tr>
<tr>
<td>$\Sigma(1560)$</td>
<td>$1^-$</td>
<td>$\text{seen}$</td>
<td>43</td>
<td>67</td>
</tr>
<tr>
<td>$\Sigma(1620)$</td>
<td>$1^-$</td>
<td>$\text{seen}$</td>
<td>160</td>
<td>422</td>
</tr>
<tr>
<td>$\Sigma(1660)$</td>
<td>$1^+$</td>
<td>$\text{seen}$</td>
<td>6.2</td>
<td>0.02</td>
</tr>
<tr>
<td>$\Sigma(1670)$</td>
<td>$3^-$</td>
<td>$(6 \pm 3)^{+3}_{-1}$</td>
<td>5.5</td>
<td>5.1</td>
</tr>
<tr>
<td>$\Sigma(1690)$</td>
<td>$3^+$</td>
<td>$\text{seen}$</td>
<td>35</td>
<td>24</td>
</tr>
<tr>
<td>$\Sigma(1750)$</td>
<td>$1^-$</td>
<td>$\text{seen}$</td>
<td>18</td>
<td>105</td>
</tr>
<tr>
<td>$\Sigma(1775)$</td>
<td>$5^-$</td>
<td>$(20 \pm 4)^{+3}_{-2}$</td>
<td>10</td>
<td>21</td>
</tr>
<tr>
<td>$\Sigma(1880)$</td>
<td>$1^+$</td>
<td>$\text{seen}$</td>
<td>4.3</td>
<td>1.5</td>
</tr>
<tr>
<td>$\Sigma(1940)$</td>
<td>$3^-$</td>
<td>$\text{seen}$</td>
<td>1.7</td>
<td>3.5</td>
</tr>
<tr>
<td>$\to \Xi\pi$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Xi(1530)$</td>
<td>$3^+$</td>
<td>$(9.9)^{+1.7}_{-1.9}$</td>
<td>4.4</td>
<td>3.0</td>
</tr>
<tr>
<td>$\Xi(1690)$</td>
<td>$1^+$</td>
<td>$\text{seen}$</td>
<td>0.2</td>
<td>1.2</td>
</tr>
<tr>
<td>$\Xi(1820)$</td>
<td>$3^-$</td>
<td>$\text{small}$</td>
<td>0.3</td>
<td>1.4</td>
</tr>
<tr>
<td>$\Xi(1950)$</td>
<td>$5^-$</td>
<td>$\text{seen}$</td>
<td>19</td>
<td>41</td>
</tr>
</tbody>
</table>
Table F.20: Non-relativistic predictions for $\eta$ decay widths of light baryon resonances as obtained with the GBE and the OGE CQMs. In the last two columns experimental resonance masses (instead of theoretical masses) have been used in the calculation. The experimental data are from the latest compilation of the PDG [2].

<table>
<thead>
<tr>
<th>Decay $\to N\eta$</th>
<th>$J^P$</th>
<th>Experiment [MeV]</th>
<th>Th. Masses</th>
<th>Exp. Masses</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>GBE OGE</td>
<td>GBE OGE</td>
</tr>
<tr>
<td>$N(1520)$</td>
<td>$\frac{3}{2}^-$</td>
<td>$(0.26 \pm 0.05)^{+0.03}_{-0.03}$</td>
<td>0.04 0.04 0.05 0.04</td>
<td></td>
</tr>
<tr>
<td>$N(1535)$</td>
<td>$\frac{1}{2}^-$</td>
<td>$(79 \pm 11)^{+15}_{-11}$</td>
<td>127 236 155 283</td>
<td></td>
</tr>
<tr>
<td>$N(1650)$</td>
<td>$\frac{1}{2}^-$</td>
<td>$(11 \pm 6)^{+\frac{2}{1}}_{-\frac{1}{1}}$</td>
<td>283 623 286 543</td>
<td></td>
</tr>
<tr>
<td>$N(1675)$</td>
<td>$\frac{5}{2}^-$</td>
<td>$(0 \pm 0.8)^{+0.15}_{-0.15}$</td>
<td>1.1 1.8 1.6 1.5</td>
<td></td>
</tr>
<tr>
<td>$N(1700)$</td>
<td>$\frac{3}{2}^-$</td>
<td>$(0 \pm 1)^{+0.5}_{-0.5}$</td>
<td>0.2 0.3 0.4 0.3</td>
<td></td>
</tr>
<tr>
<td>$N(1710)$</td>
<td>$\frac{1}{2}^+$</td>
<td>$(6 \pm 1)^{+11}_{-4}$</td>
<td>2.9 9.3 2.2 4.6</td>
<td></td>
</tr>
</tbody>
</table>

Table F.21: Same as Table F.20 but for the sector of strange baryons.

<table>
<thead>
<tr>
<th>Decay $\to \Lambda\eta$</th>
<th>$J^P$</th>
<th>Experiment [MeV]</th>
<th>Th. Masses</th>
<th>Exp. Masses</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>GBE OGE</td>
<td>GBE OGE</td>
</tr>
<tr>
<td>$\Lambda(1670)$</td>
<td>$\frac{1}{2}^-$</td>
<td>$(6.1 \pm 2.6)^{+3.8}_{-2.5}$</td>
<td>151 22 44</td>
<td></td>
</tr>
<tr>
<td>$\Lambda(1690)$</td>
<td>$\frac{1}{2}^-$</td>
<td>0.08 $\approx \approx 0$</td>
<td>$\approx \approx 0$</td>
<td></td>
</tr>
<tr>
<td>$\Lambda(1800)$</td>
<td>$\frac{1}{2}^-$</td>
<td>223 624 264 526</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Lambda(1810)$</td>
<td>$\frac{1}{2}^+$</td>
<td>2.8 6.3 3.9 2.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Lambda(1830)$</td>
<td>$\frac{1}{2}^+$</td>
<td>0.4 1.6 1.6 1.3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Decay $\to \Sigma\eta$</th>
<th>$J^P$</th>
<th>Experiment [MeV]</th>
<th>Th. Masses</th>
<th>Exp. Masses</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>GBE OGE</td>
<td>GBE OGE</td>
</tr>
<tr>
<td>$\Sigma(1620)$</td>
<td>$\frac{1}{2}^-$</td>
<td>4.7 1.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Sigma(1690)$</td>
<td>$\frac{1}{2}^-$</td>
<td>4.4 0.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Sigma(1750)$</td>
<td>$\frac{1}{2}^-$</td>
<td>$(31.5 \pm 18.0)^{+38.5}_{-4.5}$</td>
<td>25 10 14 6</td>
<td></td>
</tr>
<tr>
<td>$\Sigma(1775)$</td>
<td>$\frac{1}{2}^-$</td>
<td>$\approx \approx 0$</td>
<td>$\approx \approx 0$</td>
<td></td>
</tr>
<tr>
<td>$\Sigma(1880)$</td>
<td>$\frac{1}{2}^-$</td>
<td>$\approx \approx 0$</td>
<td>$\approx \approx 0$</td>
<td></td>
</tr>
<tr>
<td>$\Sigma(1940)$</td>
<td>$\frac{1}{2}^-$</td>
<td>$\approx \approx 0$</td>
<td>$\approx \approx 0$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Decay $\to \Xi\eta$</th>
<th>$J^P$</th>
<th>Experiment [MeV]</th>
<th>Th. Masses</th>
<th>Exp. Masses</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>GBE OGE</td>
<td>GBE OGE</td>
</tr>
<tr>
<td>$\Xi(1690)$</td>
<td>$\frac{1}{2}^+$</td>
<td>0.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Xi(1950)$</td>
<td>$\frac{3}{2}^-$</td>
<td>0.14 0.08 0.09</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table F.22: Non-relativistic predictions for \( K \) decay widths as obtained with the GBE and the OGE CQMs. In the last two columns experimental resonance masses (instead of theoretical masses) have been used in the calculation. The experimental data are from the latest compilation of the PDG [2].

<table>
<thead>
<tr>
<th>Decay</th>
<th>( J^P )</th>
<th>Experiment [MeV]</th>
<th>Th. Masses</th>
<th>Exp. Masses</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Lambda K ) &amp;</td>
<td></td>
<td>GBE</td>
<td>OGE</td>
<td>GBE</td>
</tr>
<tr>
<td>( N(1650) ) &amp; ( \frac{1}{2}^- ) &amp; ((11.6 \pm 6.6)^{+2.2}_{-0.6}) &amp; 0.06 &amp; 0.09 &amp; 0.94 &amp; 0.05</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( N(1675) ) &amp; ( \frac{5}{2}^- ) &amp; &lt; 1.7 &amp; \approx 0 &amp; \approx 0 &amp; \approx 0 &amp; \approx 0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( N(1700) ) &amp; ( \frac{3}{2}^- ) &amp; ((1.5 \pm 1.5)^{+1.5}_{-1.1}) &amp; \approx 0 &amp; \approx 0 &amp; \approx 0 &amp; \approx 0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( N(1710) ) &amp; ( \frac{1}{2}^+ ) &amp; ((15 \pm 10)^{+37.5}_{-25.0}) &amp; 0.07 &amp; \approx 0 &amp; 0.04 &amp; \approx 0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Sigma K ) &amp;</td>
<td></td>
<td>GBE</td>
<td>OGE</td>
<td>GBE</td>
</tr>
<tr>
<td>( N(1700) ) &amp; ( \frac{3}{2}^- ) &amp;</td>
<td>0.04 &amp; 0.12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( N(1710) ) &amp; ( \frac{1}{2}^+ ) &amp;</td>
<td>2.8 &amp; 739 &amp; 0.3 &amp; 42</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta(1600) ) &amp; ( \frac{3}{2}^+ ) &amp;</td>
<td>1.1 &amp; 1.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta(1700) ) &amp; ( \frac{3}{2}^- ) &amp;</td>
<td>0.03 &amp; 0.14</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Xi K ) &amp;</td>
<td></td>
<td>GBE</td>
<td>OGE</td>
<td>GBE</td>
</tr>
<tr>
<td>( \Lambda(1800) ) &amp; ( \frac{1}{2}^- ) &amp;</td>
<td>&amp; 33</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Lambda(1810) ) &amp; ( \frac{3}{2}^+ ) &amp;</td>
<td>&amp; 3.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Lambda(1830) ) &amp; ( \frac{5}{2}^- ) &amp;</td>
<td>&amp; 0.3 &amp; 0.1 &amp; 0.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Sigma(1690) ) &amp; ( \frac{3}{2}^+ ) &amp;</td>
<td>&amp; 0.06 &amp; 0.36</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Sigma(1880) ) &amp; ( \frac{1}{2}^+ ) &amp;</td>
<td>&amp; 0.02 &amp; 0.04 &amp; 0.02 &amp; \approx 0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Sigma(1940) ) &amp; ( \frac{3}{2}^- ) &amp;</td>
<td>&amp; 0.01 &amp; \approx 0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Table F.23:** Same as Table F.22 but the decay channel is $NK$.

<table>
<thead>
<tr>
<th>Decay $\rightarrow NK$</th>
<th>$J^P$</th>
<th>Experiment [MeV]</th>
<th>Th. Masses</th>
<th>Exp. Masses</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda(1405)$</td>
<td>$\frac{1}{2}^-$</td>
<td>$not possible$</td>
<td>1217</td>
<td>2024</td>
</tr>
<tr>
<td>$\Lambda(1520)$</td>
<td>$\frac{3}{2}^-$</td>
<td>$(7.02 \pm 0.16)^{+0.46}_{-0.44}$</td>
<td>23</td>
<td>63</td>
</tr>
<tr>
<td>$\Lambda(1600)$</td>
<td>$\frac{1}{2}^+$</td>
<td>$(33.75 \pm 11.25)^{+30}_{-15}$</td>
<td>4.1</td>
<td>23</td>
</tr>
<tr>
<td>$\Lambda(1670)$</td>
<td>$\frac{1}{2}^-$</td>
<td>$(8.75 \pm 1.75)^{+4.5}_{-2}$</td>
<td>45</td>
<td>86</td>
</tr>
<tr>
<td>$\Lambda(1690)$</td>
<td>$\frac{3}{2}^-$</td>
<td>$(15 \pm 3)^{+3}_{-2}$</td>
<td>4.2</td>
<td>6.5</td>
</tr>
<tr>
<td>$\Lambda(1800)$</td>
<td>$\frac{1}{2}^-$</td>
<td>$(97.5 \pm 22.5)^{+40}_{-25}$</td>
<td>3.1</td>
<td>8.6</td>
</tr>
<tr>
<td>$\Lambda(1810)$</td>
<td>$\frac{1}{2}^+$</td>
<td>$(52.5 \pm 22.5)^{+50}_{-20}$</td>
<td>23</td>
<td>44</td>
</tr>
<tr>
<td>$\Lambda(1830)$</td>
<td>$\frac{5}{2}^-$</td>
<td>$(6.18 \pm 3.33)^{+1.05}_{-1.05}$</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$\Sigma(1560)$</td>
<td>$\frac{1}{2}^-$</td>
<td></td>
<td>57</td>
<td>69</td>
</tr>
<tr>
<td>$\Sigma(1620)$</td>
<td>$\frac{1}{2}^-$</td>
<td></td>
<td>545</td>
<td>951</td>
</tr>
<tr>
<td>$\Sigma(1660)$</td>
<td>$\frac{1}{2}^+$</td>
<td>$(20 \pm 10)^{+30}_{-6}$</td>
<td>0.4</td>
<td>$\approx$ 0</td>
</tr>
<tr>
<td>$\Sigma(1670)$</td>
<td>$\frac{3}{2}^-$</td>
<td>$(6.0 \pm 1.8)^{+2.6}_{-1.4}$</td>
<td>1.9</td>
<td>2.0</td>
</tr>
<tr>
<td>$\Sigma(1690)$</td>
<td>$\frac{3}{2}^+$</td>
<td></td>
<td>26</td>
<td>11</td>
</tr>
<tr>
<td>$\Sigma(1750)$</td>
<td>$\frac{1}{2}^-$</td>
<td>$(22.5 \pm 13.5)^{+28}_{-3}$</td>
<td>10</td>
<td>48</td>
</tr>
<tr>
<td>$\Sigma(1775)$</td>
<td>$\frac{3}{2}^-$</td>
<td>$(48.0 \pm 3.6)^{+6.5}_{-6.6}$</td>
<td>20</td>
<td>41</td>
</tr>
<tr>
<td>$\Sigma(1880)$</td>
<td>$\frac{1}{2}^+$</td>
<td></td>
<td>5.4</td>
<td>13</td>
</tr>
<tr>
<td>$\Sigma(1940)$</td>
<td>$\frac{3}{2}^-$</td>
<td>$(22 \pm 22)^{+16}$</td>
<td>3.3</td>
<td>6.8</td>
</tr>
</tbody>
</table>
Table F.24: Same as Table F.22 but the decay channels are $\Lambda K$ and $\Sigma K$.

<table>
<thead>
<tr>
<th>Decay</th>
<th>$J^P$</th>
<th>Experiment</th>
<th>Kind $[\text{MeV}]$</th>
<th>Th. Masses $\text{GBE}$</th>
<th>Exp. Masses $\text{OGE}$</th>
<th>GBE</th>
<th>OGE</th>
<th>GBE</th>
<th>OGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rightarrow \Lambda K$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Xi(1690) , \frac{1}{2}^+$</td>
<td>seen</td>
<td>0.4</td>
<td>0.5</td>
<td>0.2</td>
<td>0.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Xi(1820) , \frac{3}{2}^-$</td>
<td>large</td>
<td>6</td>
<td>19</td>
<td>10</td>
<td>11</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Xi(1950) , \frac{5}{2}^-$</td>
<td>seen</td>
<td>4</td>
<td>11</td>
<td>8</td>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rightarrow \Sigma K$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Xi(1690) , \frac{1}{2}^+$</td>
<td>seen</td>
<td>0.06</td>
<td>47</td>
<td>$\approx 0$</td>
<td>0.09</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Xi(1820) , \frac{3}{2}^-$</td>
<td>small</td>
<td>10</td>
<td>31</td>
<td>12</td>
<td>17</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Xi(1950) , \frac{5}{2}^-$</td>
<td>seen</td>
<td>4</td>
<td>9</td>
<td>6</td>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

F.4 Comparison with Results by the Bonn Group

Relativistic results for decay widths of a CQM have also been obtained by the Bonn group [52]. They have used their CQM based on instanton-induced dynamics within the context of the Bethe-Salpeter equation [38, 39]. Since they have also employed a spectator-model decay operator, a comparison with our results is meaningful and appears to be interesting, even though it is limited to $\pi$ decays in the light-baryon sector.

In Table F.25 we compare the decay widths as resulting from the GBE and OGE CQMs, employing the PFSM decay operator in pseudovector coupling, with the decay widths obtained from the II CQM. Note that the Bonn group makes no predictions for the $N(1710) \rightarrow N\pi$ and $\Delta(1600) \rightarrow N\pi$ decays.
Table F.25: Covariant predictions for $\pi$ decay widths of light baryon resonances as obtained with the GBE and the OGE CQMs along the PFSM in pseudovector coupling in comparison to a relativistic calculation for the II CQM along the Bethe-Salpeter approach [52]. In all cases the theoretical resonance masses as predicted by the various CQMs have been used in the calculation. The experimental data are from the latest compilation of the PDG [2].

<table>
<thead>
<tr>
<th>Decay $\to N\pi$</th>
<th>$J^P$</th>
<th>Experiment [MeV]</th>
<th>Relativistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N(1440)$</td>
<td>$\frac{1}{2}^+$</td>
<td>$(195 \pm 30)^{+113}_{-58}$</td>
<td>30 59 38</td>
</tr>
<tr>
<td>$N(1520)$</td>
<td>$\frac{3}{2}^-$</td>
<td>$(69 \pm 6)^{+7}_{-8}$</td>
<td>21 23 38</td>
</tr>
<tr>
<td>$N(1535)$</td>
<td>$\frac{1}{2}^-$</td>
<td>$(68 \pm 15)^{+14}_{-9}$</td>
<td>25 39 33</td>
</tr>
<tr>
<td>$N(1650)$</td>
<td>$\frac{1}{2}^-$</td>
<td>$(128 \pm 29)^{+19}_{-12}$</td>
<td>6 10 3</td>
</tr>
<tr>
<td>$N(1675)$</td>
<td>$\frac{3}{2}^-$</td>
<td>$(60 \pm 8)^{+7}_{-7}$</td>
<td>8 10 4</td>
</tr>
<tr>
<td>$N(1700)$</td>
<td>$\frac{3}{2}^+$</td>
<td>$(10 \pm 5)^{+3}_{-3}$</td>
<td>1.0 1.3 0.1</td>
</tr>
<tr>
<td>$N(1710)$</td>
<td>$\frac{3}{2}^+$</td>
<td>$(15 \pm 5)^{+30}_{-5}$</td>
<td>19 21</td>
</tr>
<tr>
<td>$\Delta(1232)$</td>
<td>$\frac{3}{2}^+$</td>
<td>$(118)^{+2}_{-2}$</td>
<td>35 31 62</td>
</tr>
<tr>
<td>$\Delta(1600)$</td>
<td>$\frac{3}{2}^+$</td>
<td>$(61 \pm 26)^{+25}_{-10}$</td>
<td>0.5 5.1</td>
</tr>
<tr>
<td>$\Delta(1620)$</td>
<td>$\frac{1}{2}^-$</td>
<td>$(36 \pm 7)^{+2}_{-2}$</td>
<td>1.2 2.8 4</td>
</tr>
<tr>
<td>$\Delta(1700)$</td>
<td>$\frac{3}{2}^-$</td>
<td>$(45 \pm 15)^{+20}_{-10}$</td>
<td>3.8 4.1 2</td>
</tr>
</tbody>
</table>
DECAY WIDTHS OF LIGHT AND STRANGE BARYON RESONANCES
Acknowledgments

A lot of people supported and helped me throughout the past years. Many thanks to all of you!

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