Twistor-Inspired Methods for Collider Physics
(Codename: BlackHat)

Carola F. Berger
CTP, MIT

Graz – Nov. 19th, 2008
Outline

- Introduction
  - Why NLO? Why not (yet) NLO?
  - On-shell recursion relations at tree level
- On-shell bootstrap at NLO (or: how to pull an amplitude out of a (black) hat)
- BlackHat – first results
- Summary, open questions, and outlook
It’s 2008!

Outline

Introduction

● It’s 2008!
● The LHC
● The Standard Model of Particle Physics
● Perturbation Theory
● SUSY Search: Jets + MET
● SUSY Search: Beware!
● The LHC Wishlists
● Why NLO?
● Why Not Yet NLO?
● A Better Way?
● A Better Way? The Twistor Revolution
● Twistor-space
● Tree Level
● Proof at Tree-Level

BlackHat

Summary and
It’s 2008!
The LHC

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Summary and Outlook

Carola F. Berger
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CERN/ATLAS
The LHC
The LHC

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Summary and Outlook

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The Standard Model of Particle Physics
The Standard Model of Particle Physics
Perturbation Theory

Collisions described by relativistic quantum field theory

Computation of the cross section (∝ probability) of a process via perturbation theory - expansion in the coupling constant

Feynman graphs:

\[ WQ = \left( \frac{q}{\bar{q}} \right) \text{tree graph (LO)} + \text{loop (NLO)} + \ldots \]

\[ WQ = \left( \frac{q}{\bar{q}} \right)^2 \]

Zeit

Raum

tree graph (LO) loop (NLO)
**SUSY Search: Jets + MET**

**Pythia:**

![Pythia Plot](image1)

- SUSY (LM4)
- $t\bar{t}$
- QCD
- $W \rightarrow e\nu$
- $WZ$ (incl)

$P = 1.10 \times 10^{-6}$

$P < 0.002$

**ALPGEN:**

![ALPGEN Plot](image2)

- SUSY (LM4)
- $t\bar{t}$
- $W_{\text{soft}}$ (Alpgen)
- QCD
- $Z_{\text{soft}}$ (Alpgen)
- $WZ$ (incl)

$P = 7.60 \times 10^{-3}$

$P = 0.340136$

Only LO scattering amplitudes. Pythia mostly $2 \rightarrow 2$
SUSY Search: Beware!

EUROPEAN ORGANISATION FOR NUCLEAR RESEARCH

EXPERIMENTAL OBSERVATION OF EVENTS WITH LARGE MISSING TRANSVERSE ENERGY
ACCOMPANIED BY A JET OR A PHOTON(S) IN pp COLLISIONS
AT $\sqrt{s} = 540$ GeV

UA1 Collaboration, CERN, Geneva, Switzerland

Abstract

We report the observation of five events in which a missing transverse energy larger than 40 GeV is associated with a narrow hadronic jet and of two similar events with a neutral electromagnetic cluster (either one or more closely spaced photons). We cannot find an explanation for such events in terms of backgrounds or within the expectations of the Standard Model.
IS SUPERSYMMETRY FOUND? ∗

John ELLIS
CERN, Geneva, Switzerland

and

Marc SHER 1
University of California, Irvine, CA, USA

Received 22 August 1984

Monojet events seen recently by the UA1 collaboration at the CERN pp collider may be due to squarks or gluinos with masses O(40) GeV. The thinness of the observed jets favours the squark interpretation. In this case, we predict that sleptons should have masses between 20 and 30 GeV and that the photino should have a mass between 5 and 10 GeV. Such masses
### The (In)Famous Experimenters’ Wishlists

<table>
<thead>
<tr>
<th>process wanted at NLO ((V \in {Z, W, \gamma}))</th>
<th>background to</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (pp \rightarrow VV + ) jet</td>
<td>(t\bar{t}H), new physics</td>
</tr>
<tr>
<td>2. (pp \rightarrow H + 2) jets</td>
<td>(H) production by vector boson fusion (VBF)</td>
</tr>
<tr>
<td>3. (pp \rightarrow t\bar{t}b\bar{b})</td>
<td>(t\bar{t}H)</td>
</tr>
<tr>
<td>4. (pp \rightarrow t\bar{t} + 2) jets</td>
<td>(t\bar{t}H)</td>
</tr>
<tr>
<td>5. (pp \rightarrow V\bar{V}b\bar{b})</td>
<td>VBF (\rightarrow H \rightarrow VV), (t\bar{t}H), new physics</td>
</tr>
<tr>
<td>6. (pp \rightarrow V\bar{V} + 2) jets</td>
<td>VBF (\rightarrow H \rightarrow VV)</td>
</tr>
<tr>
<td>7. (pp \rightarrow V + 3) jets</td>
<td>new physics</td>
</tr>
<tr>
<td>8. (pp \rightarrow VVV)</td>
<td>SUSY trilepton</td>
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<tr>
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<th>Diboson</th>
<th>Triboson</th>
<th>Heavy flavor</th>
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<tr>
<td>$W + \leq 5j$</td>
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<td>$tt + \leq 3j$</td>
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<td>$tt + \gamma + \leq 2j$</td>
</tr>
<tr>
<td>$W + c\bar{c} + \leq 3j$</td>
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<td>$tt + W + \leq 2j$</td>
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<td>$Z \gamma \gamma + \leq 3j$</td>
<td>$tt + Z + \leq 2j$</td>
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<tr>
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<td>$ZZZ + \leq 3j$</td>
<td>$ttb + \leq 2j$</td>
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### Run II Monte Carlo Workshop 2001

- Single boson
- Diboson
- Triboson
- Heavy flavor

**Table Notes:**
- $j$ represents jets.
- $\gamma$ represents photons.
- $H$ represents Higgs.
- $t$ represents top quarks.
- $b$ represents bottom quarks.

### Summary and Outlook
Why NLO?

Pythia:

\[ \text{plot with histograms and axes} \]

\[ p = 1.10 \cdot 10^{-14} \]

\[ \bar{p} < 0.002 \]

ALPGEN:

\[ \text{plot with histograms and axes} \]

\[ p = 7.60 \cdot 10^{-14} \]

\[ \bar{p} = 0.340136 \]
**Why NLO?**

\[ p_T \text{-distribution of the leading jet in } W\bar{b}\bar{b} \text{ events} \]

LO dashed, NLO solid

\[ \sigma/dp_T(b,\text{max}) \text{ [pb/GeV]} \]

- \( pp \rightarrow e^+\nu_e \bar{b}\bar{b} + X \)
- \( pp \rightarrow e^-\bar{\nu}_e \bar{b}\bar{b} + X \)

\[ W^+jj/W^+\bar{b}\bar{b} \]

\[ W^-jj/W^-\bar{b}\bar{b} \]
Why NLO? Large corrections


\[ \sigma(pp \rightarrow H+X) \ [pb] \]

\[ \sqrt{s} = 14 \text{ TeV} \]

\[ M_H \ [\text{GeV}] \]

\[ \begin{array}{c}
\text{LO} \\
\text{NLO} \\
\text{NNLO}
\end{array} \]
Why Not Yet NLO?

General strategy of “conventional” loop calculation:
1. draw all possible Feynman diagrams (topological task)
2. particles in given diagram (combinatorical task)
3. translate diagrams into formulae via Feynman rules (database look-up)
4. contract Lorentz indices, take traces (algebraic manipulation)
5. reduce to known Master integrals (algebraic manipulation)
6. cancel IR and UV singularities (algebraic manipulation)
7. translate output into computer code (programming)
8. run program (wait, drink coffee)
Why Not Yet NLO?

General strategy of “conventional” loop calculation:
1. draw all possible Feynman diagrams (topological task)
   [1-loop 6-gluon amplitude: 1,034 graphs, 8-gluon: 3,017,490]
2. particles in given diagram (combinatorial task)
3. translate diagrams into formulae via Feynman rules (database look-up) [Lorentz-structure, e.g. 3-gluon vertex has 6 terms]
4. contract Lorentz indices, take traces (algebraic manipulation) [more proliferation of terms]
5. reduce to known Master integrals (algebraic manipulation) [many terms with spurious singularities that should cancel]
6. cancel IR and UV singularities (algebraic manipulation) [if done numerically - unstable]
7. translate output into computer code (programming) [ouch]
8. run program (wait, drink coffee) [not enough coffee in the universe]
A Better Way?

On-shell recursion relations at tree level – many applications

- SUSY - processes with massless fermions
  - Luo, Wen
- QCD - QCD is supersymmetric at tree level
- Massive scalars and fermions
  - Badger, Glover, Khoze, Svrcek; Forde, Kosower; Schwinn, Weinzierl; Ferrario, Rodrigo, Talavera
- Massive vector bosons
  - Bern, Forde, Kosower, Mastrolia; Badger, Glover, Khoze
- Higgs (top loop integrated out)
  - Badger, Dixon, Glover, Khoze
- Gravity
  - Bedford, Brandhuber, Spence, Travaglini; Cachazo, Svrcek; Bjerrum-Bohr, Dunbar, Ita, Perkins, Risager
- Extra dimensions
- Incorporation into Monte Carlos
  - Conley, Peskin, Wizansky

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Twistor-Inspired Methods for Collider Physics - 14/38
A Better Way? The Twistor Revolution

The end result of a calculation is often a lot simpler than intermediate expressions, especially when expressed in the “right” variables.

Instead of four-momenta and polarization vectors use spinors (∼ “square-root” of momenta).

Maximally Helicity Violating (MHV) amplitudes:

\[ A(1^+ \ldots i^- \ldots j^- \ldots n^+) = \frac{\langle ij \rangle^4}{\langle 1 \ 2 \rangle \langle 2 \ 3 \rangle \ldots \langle n - 1 \ n \rangle \langle n \ 1 \rangle} \]

Parke, Taylor (1986)

Witten (2003) discovered even more structure of various amplitudes upon transforming to twistor-space (= Fourier transform in the positive chirality spinors (0,1/2) rep.).
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BlackHat

Summary and
Twistor-space

\[ \int dt \exp \{ -i\omega t \} \]
An \( n \)-particle scattering amplitude is represented by a curve of degree \( d \) and genus \( g \) in twistor-space

\[
d = q - 1 + l, \quad g \leq l.
\]

\( q \) number of negative helicity gluons, \( l \) number of loops.
On-Shell Recursion Relations at Tree Level

Complex continue (shift) spinors and momenta:

\[ p_i \rightarrow p_i(z) \quad p_j \rightarrow p_j(z) \]
\[ p_i + p_j \rightarrow p_i + p_j \]

Momentum conservation is maintained, momenta on-shell \((p_i(z)^2 = p_j(z)^2 = 0)\).

\[ A_n = \sum_{l, m} A_L \cdot A_R \]

\[ p^2 = 0 \]

Britto, Cachazo, Feng
Proof at Tree-Level

Propagators and thus amplitudes are now functions of the complex parameter:

\[
\frac{1}{P_{l...j...m}^2} \rightarrow \frac{1}{P^2_{l...j...m}(z)}
\]

\[
A(z) = \sum_{l,m} \sum_h A^h_L(z) \frac{1}{P^2_{l...j...m}(z)} A_R^{-h}(z)
\]
Proof at Tree-Level

Propagators and thus amplitudes are now functions of the complex parameter:

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\frac{1}{P_{l\ldots j\ldots m}} \rightarrow \frac{1}{P_{l\ldots j\ldots m}(z)}
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\]

If \( A(z \rightarrow \infty) \rightarrow 0 \) - Cauchy’s theorem

\[
\frac{1}{2\pi i} \oint_C \frac{dz}{z} A(z) = 0
\]

\[
A(0) = - \sum_{\text{poles } \alpha} \text{Res}_{z=z_\alpha} \frac{A(z)}{z}
\]

\[
= \sum_{\text{poles } \alpha} \sum_{h} A^h_L(z_\alpha) \frac{1}{P^2_{l\ldots j\ldots m}} A^{-h}_R(z_\alpha)
\]

Britto, Cachazo, Feng, Witten
QCD at One Loop - A Disaster?

Branch cuts (with spurious singularities)
QCD at One Loop - A Disaster?

- Branch cuts (with spurious singularities)
- Double poles, ‘unreal poles’ and nonstandard factorizations
QCD at One Loop - A Disaster?

- Branch cuts (with spurious singularities)
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  \[ A(z \to \infty) \neq 0 \]
What is BlackHat?
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- **http://www.blackhat.com** – Black Hat Briefings and Trainings: "The Black Hat Briefings are a series of highly technical information security conferences..."

- **http://www.blackhat.org** – Good Guys Still Wear Black: "Blackhat.org is dedicated to the exploits and collection of eccentricities that is me."

- Wikipedia: "A black hat is a person who compromises the security of a computer system without permission from an authorized party, typically with malicious intent."
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- **BlackHat bootstrap**: see this talk
The BlackHat Bootstrap

Split up amplitude into cut parts and a rational remainder:

\[ A(z) = C(z) + R(z) \]

Cut parts – all logarithms, dilogs, and associated constants. Rational parts – ratios of momentum invariants only. Compute cut parts via unitarity methods, rational remainder via on-shell recursion relations.

Method can be straightforwardly translated into computer code (code-name: BlackHat).

\( C \) and \( R \) factorize independently. But: \( C \) and \( R \) talk to each other via behavior at \( z \to \infty \) and spurious singularities! Need to keep this in mind when constructing recursion relations for \( R \).

CFB, Bern, Dixon, Forde, Kosower + Febres-Cordero, Ita, Maitre
Black Cuts

Any $n$-leg one-loop amplitude expressible in terms of scalar box, triangle and bubble integrals:

$$\mathcal{A} = c_4 I_4 + c_3 I_3 + c_2 I_2 + \text{rational}$$

We know the integrals, the task is to determine the coefficients

Bern, Dixon, Dunbar, Kosower
Unitarity

Unitarity ⇒ optical theorem:

\[ S^\dagger S = 1 \quad \Rightarrow \quad S = 1 + iT \]

\[ -i \left( T - T^\dagger \right) = T^\dagger T \]

\[ 2 \Im M(a \rightarrow b) = \sum_f \int d\text{PS}_f M^*(b \rightarrow f) M(a \rightarrow f) \]

\[ \int d\text{LIPS}(-l_1, l_2) A^\text{tree}(-l_1, m_1, .., m_2, l_2) A^\text{tree}(-l_2, m_2+1, .., m_1-1, l_1) \]
Generalized Unitarity

\[ c_4 I_4 = c_4 \int \frac{1}{d^4 l} \frac{1}{l^2 (l - K_1)^2 (l - K_2)^2 (l - K_3)^2} \]

\[ \frac{1}{P^2 + i\epsilon} = \frac{1}{P^2} + i\delta^+(P^2) \]

**Box integrals have unique leading singularity \( \Rightarrow \) generalized unitarity**

\[ c_4 \Delta_{LS} I_4 = \int d^4 l \delta^+(l^2) \delta^+((l - K_1)^2) \times \delta^+((l - K_2)^2) \delta^+((l - K_3)^2) \times A_{\text{tree}}^{1} \times A_{\text{tree}}^{2} \times A_{\text{tree}}^{3} \times A_{\text{tree}}^{4} \]

\[ c_4 = A_{\text{tree}}^{1} \times A_{\text{tree}}^{2} \times A_{\text{tree}}^{3} \times A_{\text{tree}}^{4} \]

**Tree graphs on shell**

Trees “recycled” into loops

**References**

Britto, Cachazo, Feng
Generalized Unitarity contd.

Triangle coefficients from triple cuts, bubble coefficients from double cuts.

But life’s not so simple – “leakage” from higher-point integrals into lower point ones because integrals are not fully localized any more.

However, the singularity structures are unique – need procedure to disentangle coefficients:

- Holomorphic anomaly – algebraic but non-linear manipulations instead of integrals

- Clever parametrization of integral – read off coefficients directly

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Disentangling Coefficients

Parametrization of loop momenta (schematically):

\[ l_{\mu}^n = \alpha_1 K_1^\mu + \alpha_2 K_2^\mu + \alpha_3 t K_3(K_1, K_2)^\mu + \frac{\alpha_4}{t} K_4(K_1, K_2)^\mu \]

Triple cut then gives:

\[ C_3 = \sum_i b_i \xi(t - t_i) + \sum_{j=-3}^{3} c_j t^j \]

\[ l_i^2(t) \sim \xi_i(t - t_i). \]

Boxes have extra poles in \( t \) from propagators that go on-shell.

But we know the boxes, so subtract them off.
Disentangling Coefficients contd.

Triangle contributions after subtraction of boxes:

\[ T_3 = \sum_{j=-3}^{3} c_j t^j \]

\(c_0\) is the triangle coefficient, extract via discrete Fourier transform

\[ c_0 = \frac{1}{7} \sum_{j=0}^{6} T_3 \left( t_0 e^{2\pi ij/7} \right) \]

BlackHat: CFB, Bern, Dixon, Febres-Cordero, Forde, Ita, Kosower, Maitre
Black Recursion

\[ A(z) = C(z) + R(z) \]

\[ A(0) = C(0) + \text{Inf } A - \sum_{\text{poles } \alpha} \text{Res}_{z=z_\alpha} \frac{R(z)}{z} \]

\[ = C(0) + \text{Inf } A + \sum_{\text{configs}} A_L \frac{1}{P^2} A_R \]

\[ \sum_{\text{configs}} = \{ R + \sum_{\text{configs}} R + \text{Inf } \} \]

Loops “recycled” into loops
(ignoring slight subtleties with spurious singularities)

CFB, Bern, Dixon, Forde, Kosower
Unknown Contributions

Introduction

BlackHat
- QCD at One Loop - A Disaster?
- What is BlackHat?
- The BlackHat Bootstrap
- Black Cuts
- Unitarity
- Generalized Unitarity
- Disentangling Coefficients
- Black Recursion
- The Bootstrap Formalism
- Example
- All-multiplicity Solutions
- Sample Numerical Results

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Summary and Outlook

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The Bootstrap Formalism

\[ A(0) = C(0) + R_{\text{recurs}}^{[j,l]} + \text{Inf}_{[j,l]} \left[ C + R_{\text{recurs}}^{\{a,b\}} \right] \]

- Passes all nontrivial checks
- In some cases recursion can be solved \( \Rightarrow \) analytical all-multiplicity formulae
- Working algorithm for all other (massless) configurations
- With new discrete Fourier transform for cut parts numerically very stable

CFB, Bern, Dixon, Forde, Kosower
Example: $A_6(1^-, 2^-, 3^-, 4^+, 5^+, 6^+)$

$$X(1, 2, 3, 4, 5, 6) \equiv X(3, 2, 1, 6, 5, 4)$$

$$\tilde{C}_6(1^-, 2^-, 3^-, 4^+, 5^+, 6^+) = \frac{1}{3c_F} A_{6;1}^{\text{flip} 1}(1^-, 2^-, 3^-, 4^+, 5^+, 6^+) + \frac{2}{9} A_{6}^{\text{tree}}(1^-, 2^-, 3^-, 4^+, 5^+, 6^+) + \hat{C}_6^a + \hat{C}_6^a$$

$$\hat{R}_6 = \hat{R}_6^a + \hat{R}_6^a$$
All-multiplicity Solutions

CFB, Bern, Dixon, Forde, Kosower
Sample Numerical Results

Timing (dual CPU 2+ GHz processor):

- ++ -- - -- 279ms
- -- ++ + + 157ms
- -- + + ++ 53.9ms
- + + + ++ 4.94ms

BlackHat: CFB, Bern, Dixon, Febres-Cordero, Forde, Ita, Kosower, Maitre
Sample Numerical Results - $V + \text{Jets}$

$A(\bar{q}, g, g, g, q, \bar{\ell}, \ell)$

![Graph showing numerical results]

BlackHat: CFB, Bern, Dixon, Febres-Cordero, Forde, Ita, Kosower, Maitre
Summary

✔ All-multiplicity formulae for some one-loop gluon amplitudes
   Forde, Kosower; CFB, Bern, Dixon, Forde, Kosower

✔ Some all-multiplicity results for parts of Higgs plus gluons (and fermion pair) at NNLO (effective theory - top loop integrated out)
   CFB, Del Duca, Dixon

All of the above ≪ ∞ pages

✔ Working algorithm for all other configurations of one-loop gluon amplitudes!
   CFB, Bern, Dixon, Forde, Kosower

✔ Working code for massless gluon, fermion, V 1-loop amplitudes, numerically very stable. Massive partons under testing.
   Complexity does not increase much when adding extra legs!

   BlackHat Collaboration: CFB, Bern, Dixon, Febres-Cordero, Forde, Ita, Kosower, Maitre
To-Do List

- Implement massive partons
- Interface with NLO Monte Carlo to get cross sections (e.g. AMEGIC++ \textsuperscript{Gleisberg, Krauss})
- Attack the wishlists...
- Better understand structure of 1-loop gauge theory amplitudes
- Generalize to multiloop amplitudes
- ...