Vacuum stability and the mass of the Higgs boson

Holger Gies

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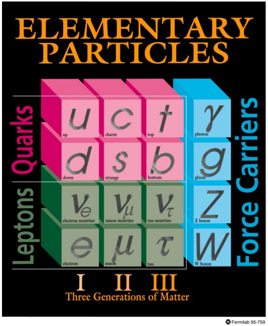
& C. Gneiting, R. Sondenheimer, PRD 89 (2014) 045012, [arXiv:1308.5075],

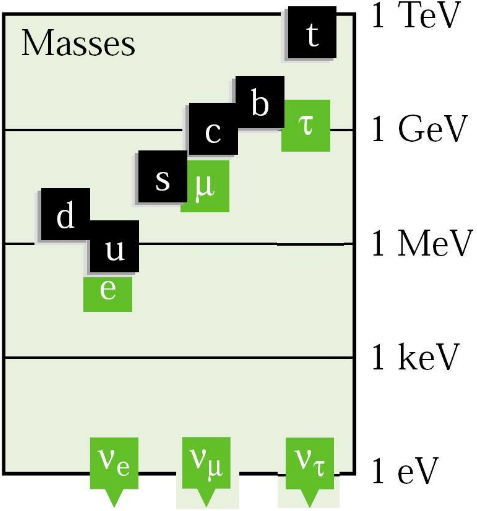
& R. Sondenheimer, EPJ C75 (2015) 2, 68, [arXiv:1407.8124],

& A. Eichhorn, J. Jaeckel, T. Plehn, M.M. Scherer, R. Sondenheimer, JHEP 1504 (2015) 022, [arXiv:1501.02812]

Seminar, KFU Graz, 28.08.2015

Standard model particle spectrum: pre-LHC





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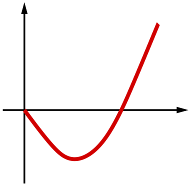
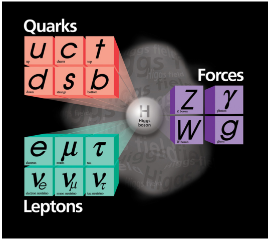
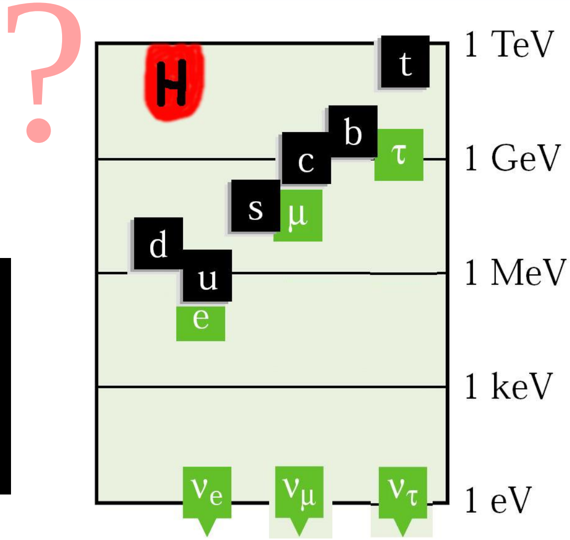


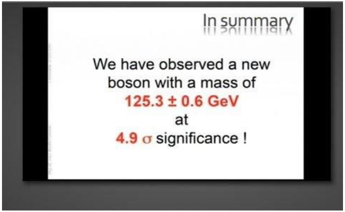




(ANDERSON'62; BROUT,ENGLERT'64; HIGGS'64; GURALNIK,HAGEN,KIBBLE'64)

Search for the Higgs boson

▷ 4 Jul. 2012
ATLAS & CMS
@CERN



▷ 14 Mar 2013, CERN press release:

"... the new particle is looking more and more like a Higgs boson ..."

$$\text{CMS'12}: \quad 125.3 \pm 0.4(stat) \pm 0.5(sys) GeV,$$

$$\text{ATLAS'12}: \quad 126.0 \pm 0.4(stat) \pm 0.4(sys) GeV$$


JOGHURT MILD
Ehrmann
Bioghurt
PUR
JOGHURT MILD
Ehrmann
Bioghurt
PUR

Numbers matter

standard model

best before:

$$\Lambda = M_{\text{Planck}} \; ?$$

Validity range of the standard model

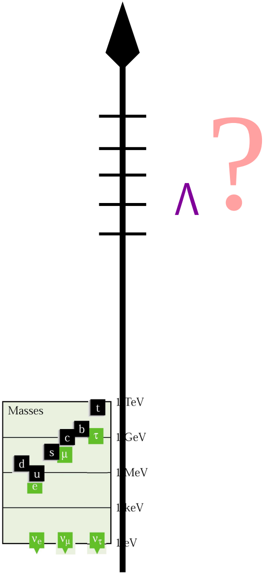
▷ Λ:

- UV cutoff

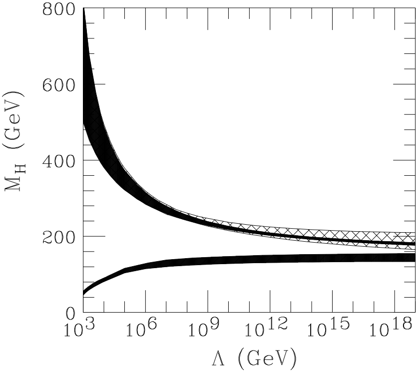
  SM as effective theory

- scale of maximum UV extension

- scale of new physics:
  $\Lambda_{NP} \leq \Lambda$



Higgs boson mass and maximum validity scale



(HAMBYE,RIESSELMANN'97)

SM: e.g. (KRIVE,LINDE'76; MAIANI,PARISI,PETRONZIO'78; KRASNIKOV'78; POLITZER, WOLFRAM'78; HUNG'79; LINDNER'85; WETTERICH'87; SHER'88; FORT,JONES,STEPHENSON, EINHORN'93; ALTARELLI,ISIDORI'94; SCHREMPP,WIMMER'96; . . .)

BSM: e.g., (CABBIBO, MAIANI,PARISI,PETRONZIO'79; ESPINOSA,QUIROS'91; . . .)

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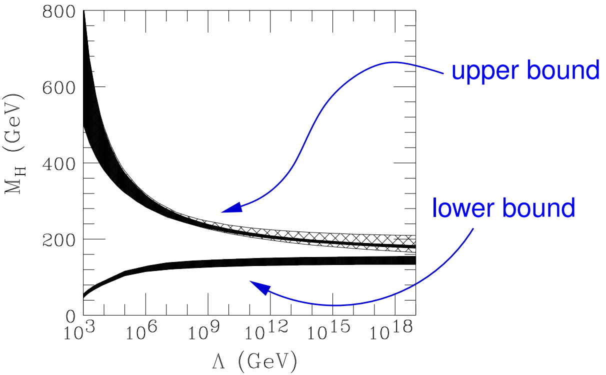

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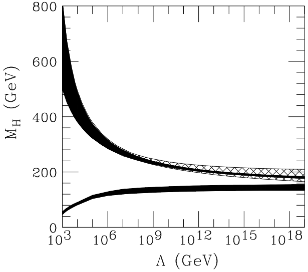
Upper bound of Higgs boson mass



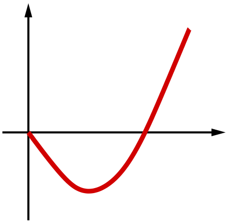
▷ triviality bound

= perturbativity bound

= unitarity bound

▷ Higgs boson mass

$$m_H^2 \sim v^2\, U''_{\text{eff}}(\phi = v), \qquad U''_{\text{eff}}(\phi = v) \sim \lambda_R$$



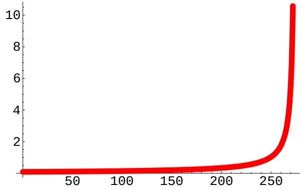
$\Longrightarrow$ large Higgs boson masses: scalar interactions dominate ($\lambda \gtrsim 1$)

Upper bound of Higgs boson mass

▷ triviality: perturbation theory predicts its own failure (LANDAU'55)

(GELL-MANN, LOW'54)

$$\frac{1}{\lambda_R} - \frac{1}{\lambda_\Lambda} = \beta_0 \ln \frac{\Lambda}{v}, \quad \beta_0 > 0$$



▷ λ_R and v fixed:

$$\Longrightarrow \quad \Lambda_{\text{sing}} \quad \simeq \quad v \exp\left(\frac{1}{\beta_0 \lambda_R}\right)$$

Landau pole singularity

▷ in practice: "perturbativity criterion", e.g.

$$\lambda_{\Lambda_{\text{pert}}} \leq \frac{\pi}{4} \qquad \Longrightarrow \Lambda = \Lambda_{\text{pert}}$$

Upper bound of Higgs boson mass

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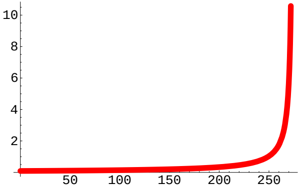


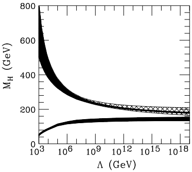
▷ Triviality

$$\lim \frac{\Lambda}{v} \to \infty : \qquad \Longrightarrow \qquad \lambda_R \to 0$$

▷ in simple models: nonperturbative evidence from lattice simulations

(LUESCHER,WEISZ'88; HASENFRATZ,JANSEN,LANG,NEUHAUS,YONEYAMA'87; WOLFF'11; BUIVIDOVICH'11; WEISZ,WOLFF'12, ...)

Lower bound of Higgs boson mass



▷ vacuum stability / meta-stability bound

▷ effective potential á la Coleman Weinberg:

$$U_{\text{eff}}(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{2}\lambda(\phi)\phi^4$$

▷ e.g., $\lambda(\phi)$ from “RG-improved” perturbation theory:

$$\partial_t \lambda = \frac{3}{4\pi^2}\left(-h_t^4 + h_t^2\lambda + \frac{1}{16}\left[2g^4 + (g^2 + g'^2)^2\right] - \frac{1}{4}\lambda(3g^2 + g'^2) + \lambda^2\right)$$

Lower bound of Higgs boson mass

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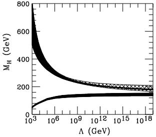

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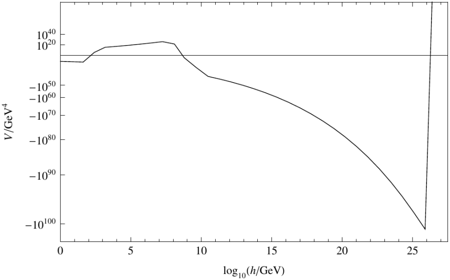
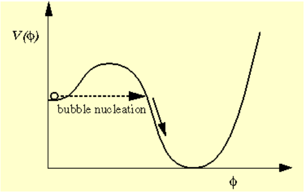


(GABRIELLI ET AL.'13)

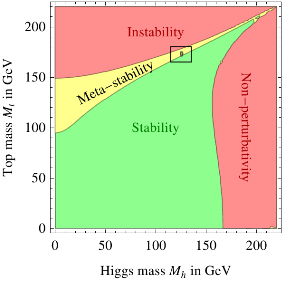
Lower Bound of Higgs boson mass

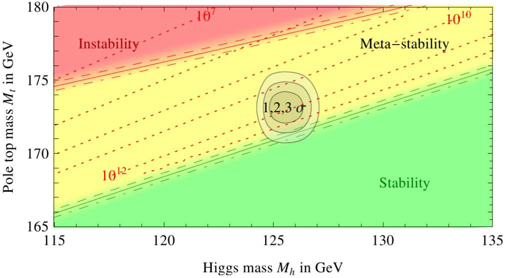
▷ meta-stability:

tunneling time > age of universe



▷ "Near critical" standard model: (BUTTAZZO ET AL.'13)



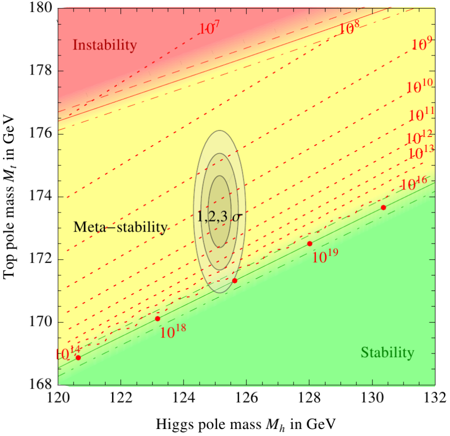


NNLO calculation (DEGRASSI ET AL.'12)

earlier calculations, e.g., (ISIDORI,RIDOLFI,STRUMIA'01)

Lower Bound of Higgs boson mass

▷ "Near critical" standard model: (BUTTAZZO ET AL.'13: UPDATE V4)



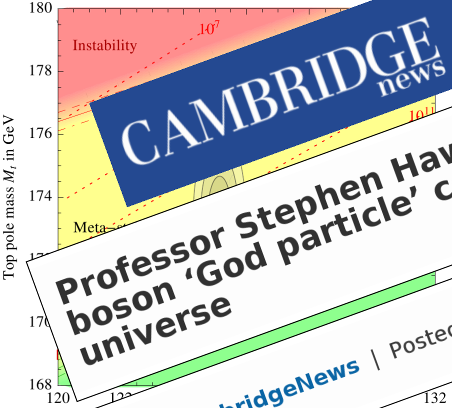
"Stability" seems to prefer

$m_H \nearrow \quad \simeq 130\text{GeV}$

or $m_{top} \searrow \quad \simeq 171\text{GeV}$

Lower Bound of Higgs boson

▷ "Near critical" standard model:



CAMBRIDGE news

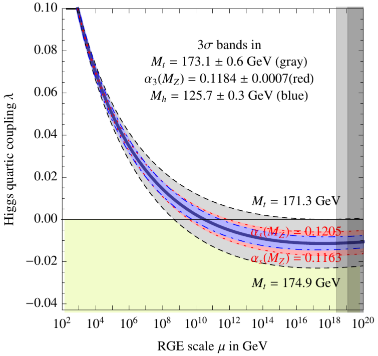
Professor Stephen Hawking says the Higgs boson 'God particle' could destroy the universe

By CambridgeNews | Posted: September 09, 2014

"The Higgs potential has the worrisome feature that it might become metastable at energies above 100bn gig-electron-volts,"

Lower bound of Higgs boson mass

▷ Reasons for concern?



α_s dependence

top mass dependence

(DEGRASSI ET AL.'12)

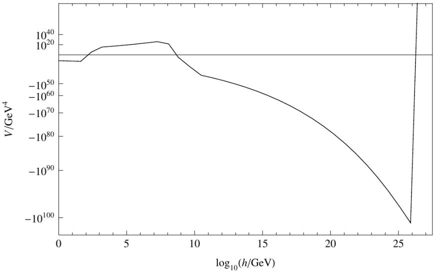
▷ larger error of top pole mass $\times 3$? (ALEKHIN,DJOUADI,MOCH'12)

compared to the "MC fit parameter" determined by Tevatron

▷ Decay of the false vacuum (COLEMAN'77)

- warning time $\leq 10^{-21}$s
- "*. . . the expansion of the bubble is a clean sweep . . .*"

2nd thoughts on the lower bound

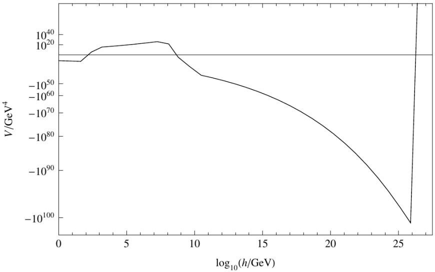


- ▷ True minimum of $U_{\mathrm{eff}}(\phi)$ at

  $\phi \sim 10^{25}\mathrm{GeV} > M_{\mathrm{Pl}}$?

- ▷ UV$\rightarrow$IR RG flow?

2nd thoughts on the lower bound



▷ True minimum of $U_{\mathrm{eff}}(\phi)$ at

$$\phi \sim 10^{25}\mathrm{GeV} > M_{\mathrm{Pl}} \qquad ?$$

▷ UV→IR RG flow?

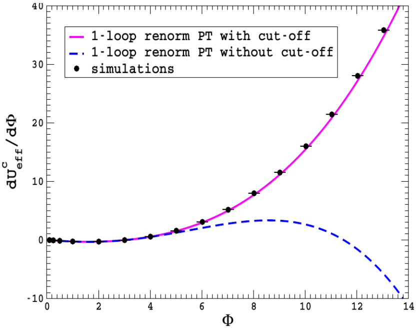
▷ simple top-Higgs Yukawa model:

lattice simulation

vs. 1-loop PT with cutoff

vs. 1-loop "Λ-removed" PT

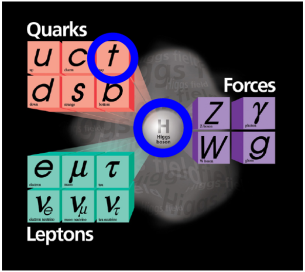
(HOLLAND,KUTI'03; HOLLAND'04)



Criticism: too few scales? (EINHORN,JONES'07)

Top-Higgs Yukawa toy model

(Holland, Kuti'03; Branchina, Faivre'05)



Z_2 symmetric model

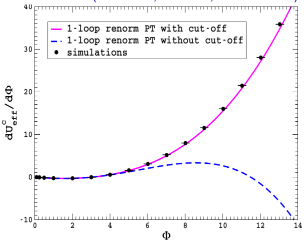
$$S = \int \frac{1}{2}(\partial\phi)^2 + U(\phi) + \bar{\psi} i \partial\!\!\!/ \psi + ih\phi\bar{\psi}\psi$$

- includes relevant top quark + Higgs field (+ largest Yukawa coupling)
- discrete symmetry breaking $\rightarrow$ no Goldstone bosons (as in SM)
- avoids intricate questions arising from gauge symmetry

(Frohlich, Morchio, Strocchi'81; Lang, Rebbi, Virasoro'81; Jersak, Lang, Neuhaus, Vones'85; Maas '12)

Top-Higgs Yukawa models

(HOLLAND, KUTI'03; HOLLAND'04)



▷ generating functional:

$$Z[J] = \int_\Lambda \mathcal{D}\phi\mathcal{D}\bar{\psi}\mathcal{D}\psi\, e^{-S[\phi,\bar{\psi},\psi]+\int J\phi} = \int_\Lambda \mathcal{D}\phi\; e^{-S_B[\phi]-S_{F,\Lambda}[\phi]+\int J\phi}$$

▷ top-induced effective potential

$$U_F(\phi) = -\frac{1}{2\Omega}\ln\frac{\det_\Lambda(-\partial^2+h_t^2\phi^\dagger\phi)}{\det_\Lambda(-\partial^2)},$$

▷ CAVE: cutoff Λ / regularization dependent

Top-induced effective potential

▷ exact results for fermion determinants for homogeneous ϕ

▷ e.g., sharp cutoff:

$$U_{\mathrm{F},t}(\phi) = \underbrace{-\frac{\Lambda^2}{8\pi^2} h_t^2 |\phi|^2}_{<0 \text{ (mass-like term)}} + \frac{1}{16\pi^2} \underbrace{\left[h_t^4 |\phi|^4 \ln\left(1 + \frac{\Lambda^2}{h_t^2 |\phi|^2}\right) + h_t^2 |\phi|^2 \Lambda^2 - \Lambda^4 \ln\left(1 + \frac{h_t^2|\phi|^2}{\Lambda^2}\right)\right]}_{>0 \text{ (interaction part)}}$$

▷ mass-like term: contributes to χSB $\Longrightarrow$ $v \simeq 246\mathrm{GeV}$

▷ interaction part: strictly positive

$\Longrightarrow$ cannot induce instability for any finite Λ

(HG,Sondenheimer'14)

“Rederiving” the instability

▷ try to send $\Lambda \to \infty$:

$$U_{\mathrm{F},t}(\phi) = -\frac{\Lambda^2}{8\pi^2} h_t^2 |\phi|^2 + \frac{1}{16\pi^2} h_t^4 |\phi|^4 \left[\ln \frac{\Lambda^2}{h_t^2 |\phi|^2} + \text{const.} + \mathcal{O}\left(\frac{h_t^2 |\phi|^2}{\Lambda^2} \right) \right]$$

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▷ renormalization: trade $(\Lambda, m_\Lambda, \lambda_\Lambda)$ for (μ, v, λ_v)

$$U_{\mathrm{F},t}(\phi) \overset{?}{\to} -\frac{1}{16\pi^2} h_t^4 |\phi|^4 \left(\ln \frac{h_t^2 |\phi|^2}{\mu^2} + \text{const.} \right)$$

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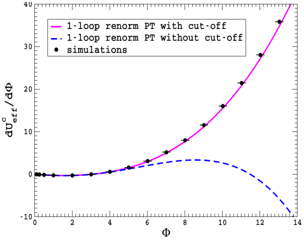

(Holland, Kuti'03; Holland'04)

▷ “instability” occurs beyond $\frac{h_t^2|\phi|^2}{\Lambda^2} > 1$

(HG, Sondenheimer'14)

▷ similar problems for other reg's

▷ implicit renormalization conditions would violate unitarity

(Branchina, Faivre'05; Gneiting'05)

Contradiction?

$$\partial_t \lambda = -\frac{3}{4\pi^2} h_t^4 + \dots$$

$$\Longrightarrow \lambda \searrow \quad < 0$$

$$\Updownarrow$$

interaction part of

$$U_{\mathrm{F},t}(\phi) > 0$$

strictly positive

Contradiction?

$$\partial_t \lambda = -\frac{3}{4\pi^2} h_t^4 + \ldots$$

$$\Longrightarrow \lambda \searrow \quad < 0$$

$$\Longleftrightarrow$$

$$U_{\text{eff}}(\phi) \sim \frac{1}{2}\lambda(\phi)\phi^4$$

$$\Updownarrow$$

$$\Longleftrightarrow$$

interaction part of

$$U_{\mathrm{F},t}(\phi) > 0$$

strictly positive

Contradiction?

$$\partial_t \lambda = -\frac{3}{4\pi^2} h_t^4 + \dots$$

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$\Leftrightarrow$

$$\cancel{U_{\mathrm{eff}}(\phi) = \frac{1}{2}\lambda(\phi)\phi^4}$$

$\Updownarrow$

$\Leftrightarrow$

interaction part of

$$U_{\mathrm{F},t}(\phi) > 0$$

strictly positive

CAVE: $\mathcal{O}\left(\frac{h_t^2|\phi|^2}{\Lambda^2}\right)$-terms

for finite Λ

Summary, Part I

- no in-/meta-stability from top (fermion) fluctuations

  . . . if cutoff Λ is kept finite but arbitrary

- no in-/meta-stability at all ?

$$U_{\text{eff}}(\phi) = U_\Lambda(\phi) + U_\text{B}(\phi) + U_\text{F}(\phi)$$

Summary, Part I

- no in-/meta-stability from top (fermion) fluctuations

  ...if cutoff Λ is kept finite but arbitrary

- no in-/meta-stability at all ?

$$U_{\rm eff}(\phi) = \underbrace{U_\Lambda(\phi)}_{\text{arbitrary}} + \underbrace{U_{\rm B}(\phi) + U_{\rm F}(\phi)}_{\text{generically stable}}$$

...in-/meta-stabilities from the bare action/UV completion

- lower Higgs mass bounds?

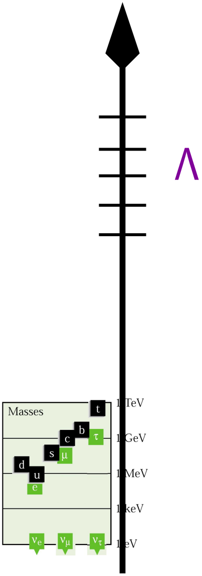
$\Longrightarrow$ nonperturbative methods recommended if not needed

▷ extensive lattice simulations: (Fodor,Holland,Kuti,Nogradi,Schroeder'07) (Gerhold,Jansen'07'09'10)

▷ constraining 4th generations: (Gerhold,Jansen,Kallarackal'10; Bulava,Jansen,Nagy'13)

▷ implications for dark matter models: (Eichhorn, Scherer'14)

Higgs boson mass bounds as a UV to IR mapping



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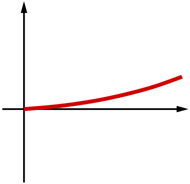

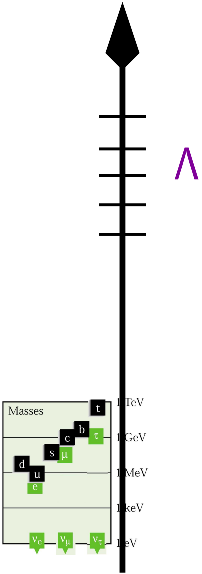

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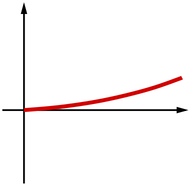

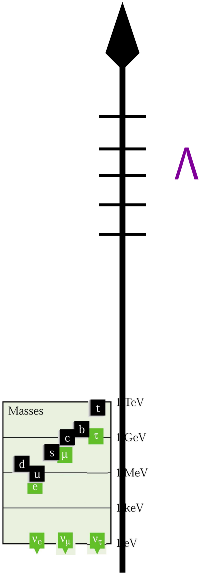

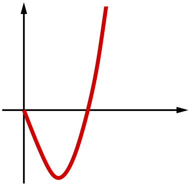

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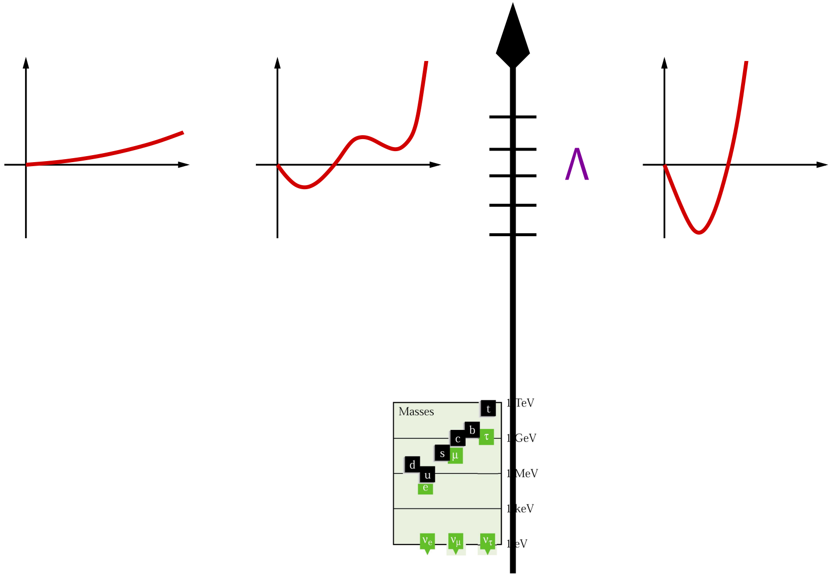
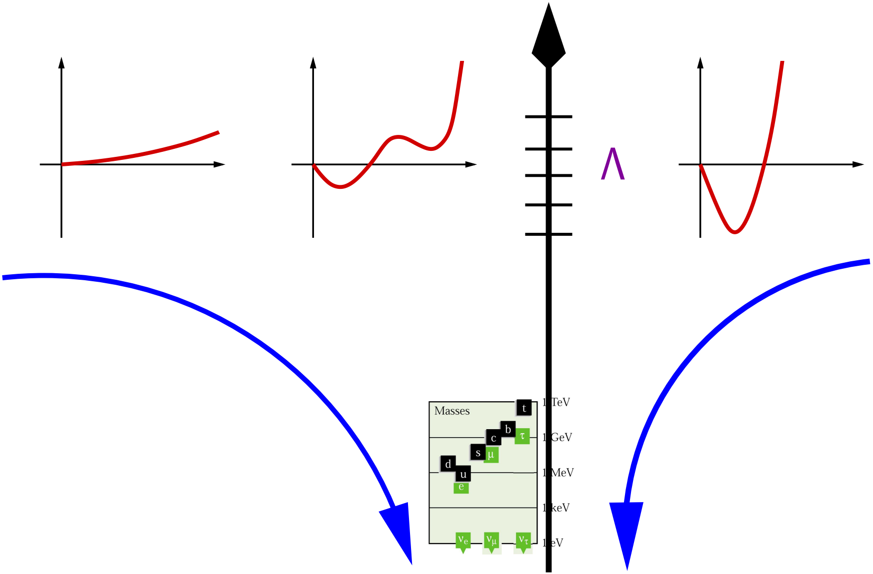
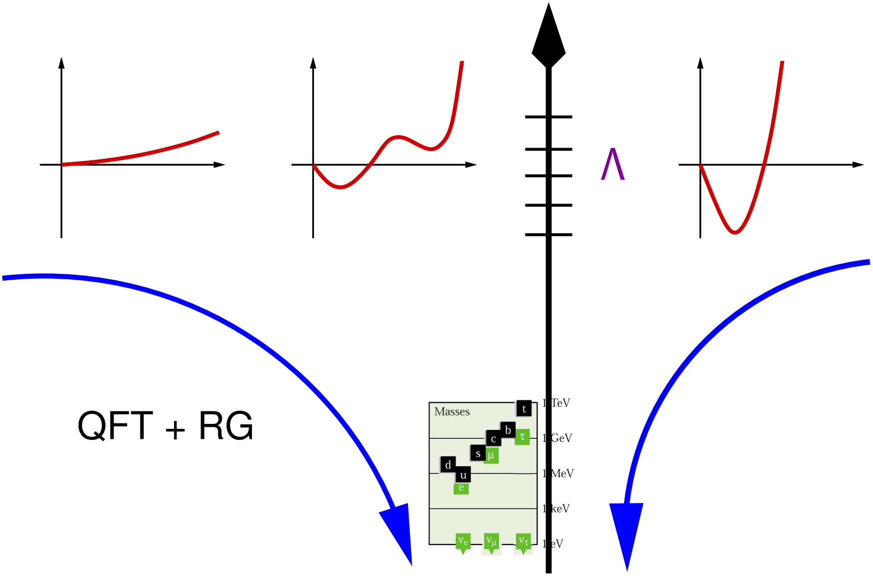


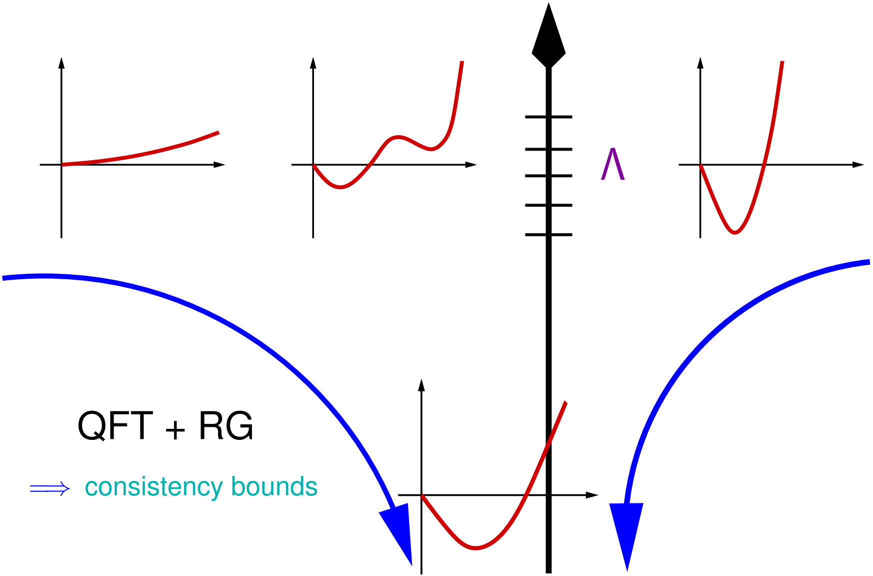
Higgs boson mass bounds as a UV to IR mapping



Higgs boson mass bounds as a UV to IR mapping



Higgs boson mass bounds as a UV to IR mapping



QFT + RG

$\Longrightarrow$ consistency bounds

Higgs boson mass bounds as a UV to IR mapping

▷ microscopic action at cutoff Λ:

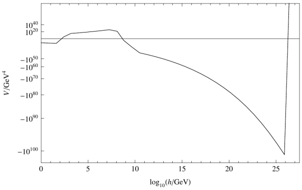
$$S_\Lambda = S_\Lambda(m_\Lambda^2, \lambda_\Lambda, \lambda_{6,\Lambda}, \dots, h_\Lambda, \dots)$$

$\Longrightarrow$ RG: mapping to IR observables

$$\stackrel{\text{RG}}{\Longrightarrow} \quad v \simeq 246\text{GeV}, \;\; m_{\text{top}} \simeq 173\text{GeV}, \;\; m_{\text{H}} = m_{\text{H}}[S_\Lambda]$$

▷ By contrast: "RG-improved" PT

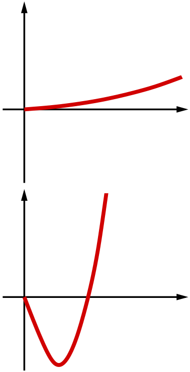
$$\text{fix: } v, m_{\text{top}} \;\; m_{\text{H}} \quad \stackrel{\text{upwards RG}}{\Longrightarrow} \quad U_{\text{eff}}$$



Simple Example: mean-field theory

▷ MFT $\hat{=}$ large-N_f limit $\hat{=}$ pure top loop:

(HG, Gneiting, Sondenheimer'13)



$$U_{\mathrm{MF}}(\phi) = U_\Lambda(\phi) - \ln \det_{\mathrm{reg}}(i\partial\!\!\!/ + ih_\Lambda \phi)$$

▷ UV bare potential, e.g,

$$U_\Lambda(\phi) = \frac{1}{2} m_\Lambda^2 \phi^2 + \frac{1}{8}\lambda_\Lambda \phi^4, \quad \lambda_\Lambda \geq 0$$

▷ trade: $m_\Lambda, h_\Lambda \quad \Longleftrightarrow \quad v, m_{\mathrm{top}}$

▷ Higgs boson mass:

$$m_{\mathrm{H}}^2(\Lambda, \lambda_\Lambda) \;=\; \frac{m_{\mathrm{top}}^4}{4\pi^2 v^2}\left[2\ln\left(1 + \frac{\Lambda^2}{m_{\mathrm{top}}^2}\right) - \frac{3\Lambda^4 + 2m_{\mathrm{top}}^2\Lambda^2}{(\Lambda^2 + m_{\mathrm{top}}^2)^2}\right] + v^2\lambda_\Lambda$$

Simple Example: mean-field theory

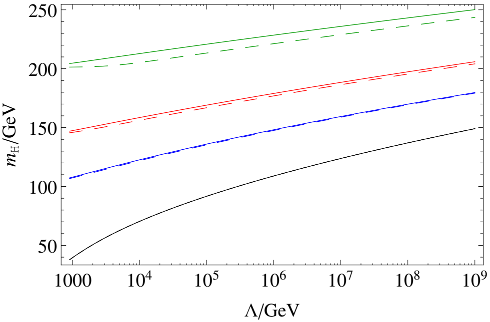
▷ Higgs boson mass bound (HG,GNEITING,SONDENHEIMER'13)

$$m_{\mathrm{H}}^2(\Lambda, \lambda_\Lambda) = \frac{m_{\mathrm{top}}^4}{4\pi^2 v^2}\left[2\ln\left(1+\frac{\Lambda^2}{m_{\mathrm{top}}^2}\right) - \frac{3\Lambda^4 + 2m_{\mathrm{top}}^2\Lambda^2}{(\Lambda^2+m_{\mathrm{top}}^2)^2}\right] + v^2\lambda_\Lambda$$

requirement: well defined path integral: $\lambda_\Lambda \geq 0$ for ϕ^4 theory

cf. lattice (HOLLAND'04; FODOR,HOLLAND,KUTI,NOGRADI,SCHROEDER'07; GERHOLD,JANSEN'07'09'10)

▷ extended mean field $\hat{=}$ NLO $1/N_{\mathrm{f}}$ expansion:



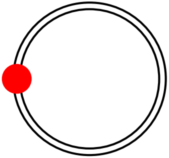
$$\lambda_\Lambda = \begin{cases} 0 \\ 1/6 \\ 1/3 \\ 2/3 \end{cases}$$

Nonperturbative tool: functional RG

▷ RG flow equation:

$$\partial_t \Gamma_k \equiv k\partial_k \Gamma_k = \frac{1}{2}\,\mathrm{Tr}\,\partial_t R_k \left(\Gamma_k^{(2)} + R_k\right)^{-1}$$

$$=$$

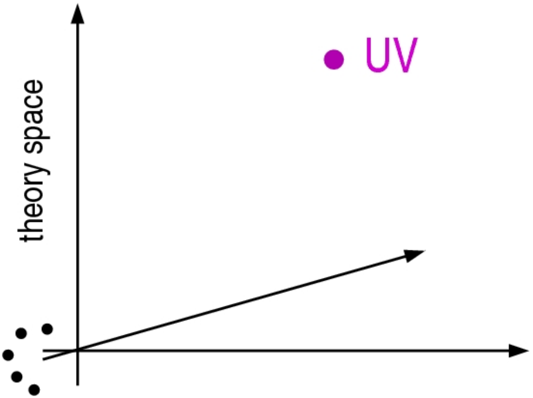


(WETTERICH'93)

RG Flow Equation

$$\partial_t \Gamma_k \;=\; \frac{1}{2}\,\mathrm{Tr}\;\partial_t R_k(\Gamma_k^{(2)} + R_k)^{-1}$$

▷ RG trajectory: $\Gamma_{k=\Lambda} = S_\Lambda = \int \frac{1}{2}(\partial\phi)^2 + U_\Lambda(\phi) + \ldots$



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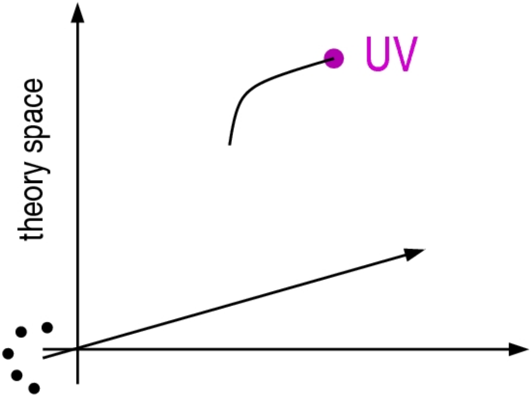

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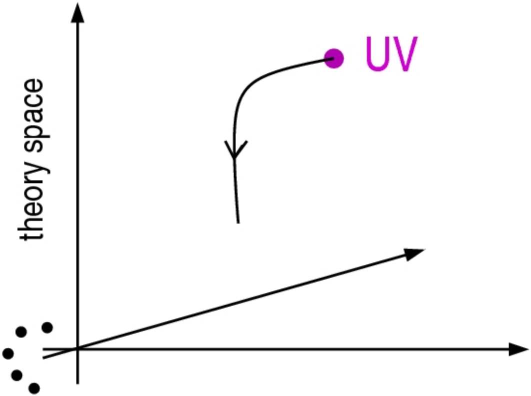
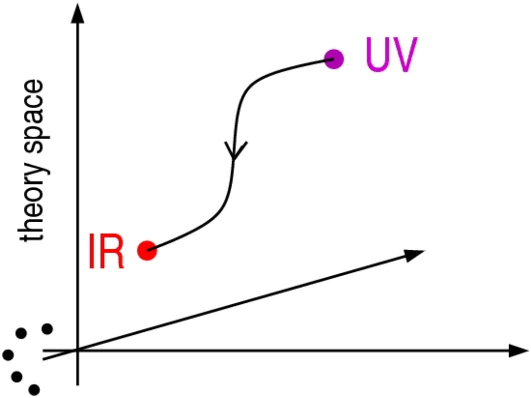


RG Flow Equation

$$\partial_t \Gamma_k = \frac{1}{2} \,\mathrm{Tr}\, \partial_t R_k (\Gamma_k^{(2)} + R_k)^{-1}$$

▷ RG trajectory: $\Gamma_{k\to 0} = \Gamma = \int U_{\text{eff}} + \ldots$

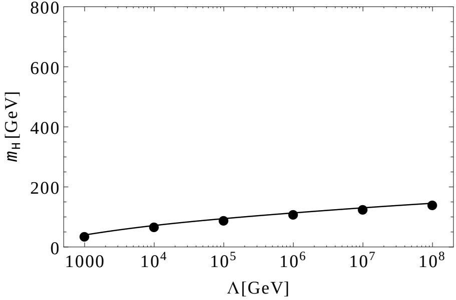


Higgs boson mass bounds from functional RG

▷ Top-Higgs toy model (HG, Gneiting, Sondenheimer'13)

▷ Systematic derivative expansion:

$$\Gamma_k = \int d^d x \left(\frac{Z_{\phi k}}{2} \partial_\mu \phi \partial^\mu \phi + U_k(\phi^2) + Z_{\psi k} \bar{\psi} i \partial\!\!\!/ \psi + i h_k \phi \bar{\psi} \psi \right)$$



$\lambda_\Lambda = 0$

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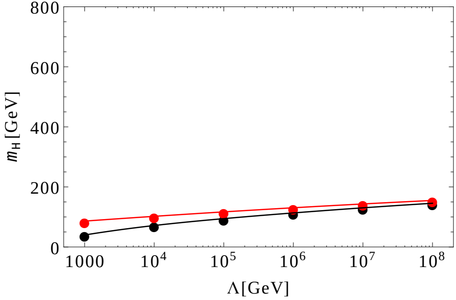

$\lambda_\Lambda = 0$
$\lambda_\Lambda = 0.1$

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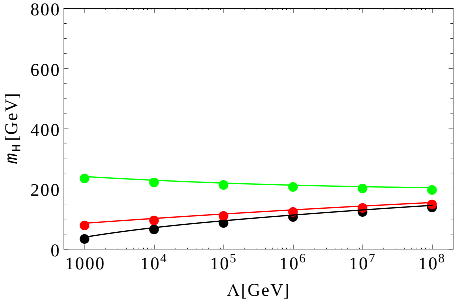

$\lambda_\Lambda = 0$
$\lambda_\Lambda = 0.1$
$\lambda_\Lambda = 1$

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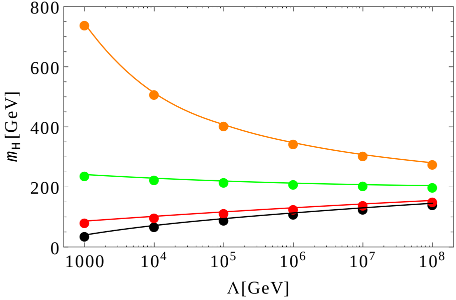

$\lambda_\Lambda = 0$
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$\lambda_\Lambda = 10$

Higgs boson mass bounds from functional RG

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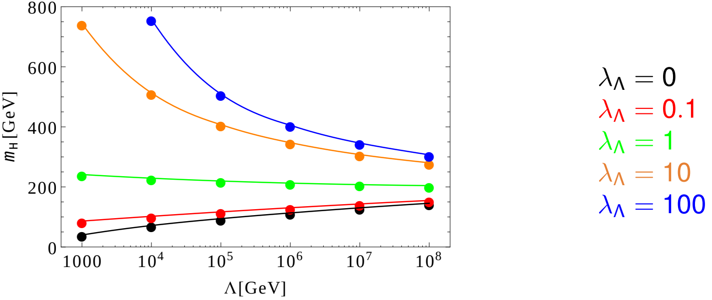

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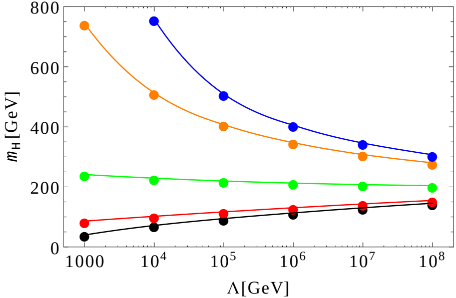


$\lambda_\Lambda = 0$
$\lambda_\Lambda = 0.1$
$\lambda_\Lambda = 1$
$\lambda_\Lambda = 10$
$\lambda_\Lambda = 100$

$\Longrightarrow$ "conventional" lower bound for $\lambda_\Lambda = 0$ ($\simeq$ mean-field result)

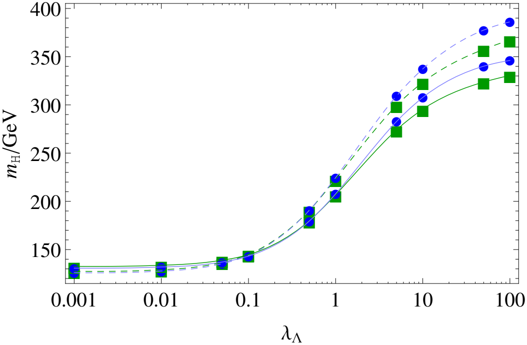
agreement with lattice (HOLLAND'04; FODOR,HOLLAND,KUTI,NOGRADI,SCHROEDER'07; GERHOLD,JANSEN'07'09'10)

The IR window for the Higgs boson mass

▷ $m_{\mathrm{H}} \sim v\lambda_{\mathrm{R}}$ (WETTERICH'87)

mapping: $\lambda_\Lambda \quad \to \lambda_{\mathrm{R}}$ not surjective on $\mathbb{R}_+$

▷ e.g. for ϕ^4 bare potential, fix $\Lambda = 10^7\,\mathrm{GeV}$ (HG,GNEITING,SONDENHEIMER'13)



convergence check of

- derivative expansion
  ΔNLO / LO $\sim$ 10%
  @ strong coupling
- U_{eff} solver
  (polynom. exp.)

General microscopic actions

▷ S_Λ is a priori unconstrained. Consider, e.g., (HG, Gneiting, Sondenheimer'13)

$$U_\Lambda = \frac{\lambda_{1\Lambda}}{2}\phi^2 + \frac{\lambda_{2\Lambda}}{8}\phi^4 + \frac{\lambda_{3\Lambda}}{48}\phi^6$$

▷ for $\lambda_{3\Lambda} > 0$ we can choose $\lambda_{2\Lambda} < 0$:

General microscopic actions

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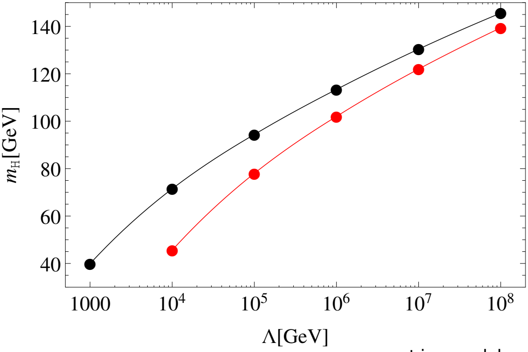


$\lambda_{3\Lambda} = 0,\ \lambda_{2\Lambda} = 0$

$\lambda_{3\Lambda} = 3,\ \lambda_{2\Lambda} = -0.08$

▷ lower bound relaxed

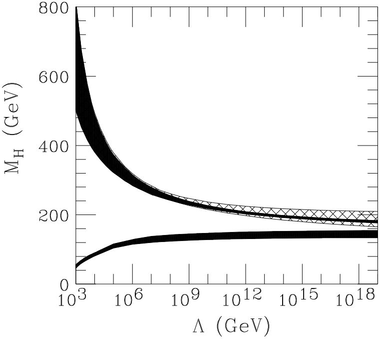
$\Longrightarrow$ consistency bound
$<$ "conventional" bound

string models with $\lambda < 0$: (Hebecker,Knochel,Weigand'13)

Renormalizable field theories

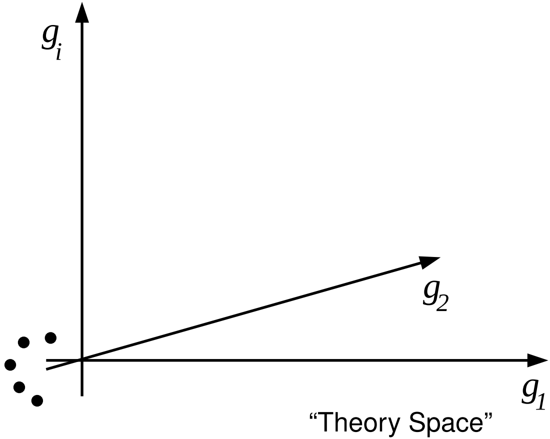
▷ seeming contradiction with common wisdom . . . ?

". . . observables are determined by renormalizable operators . . . "

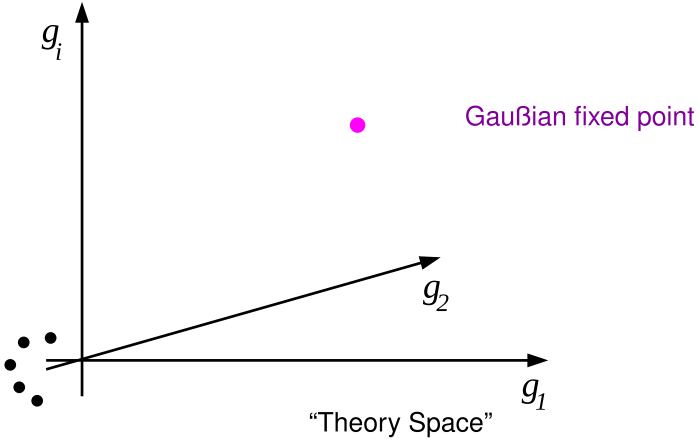


▷ y-axis: m_H observable ✓, x-axis: Λ ?

RG mechanism for "lowering" the lower bound

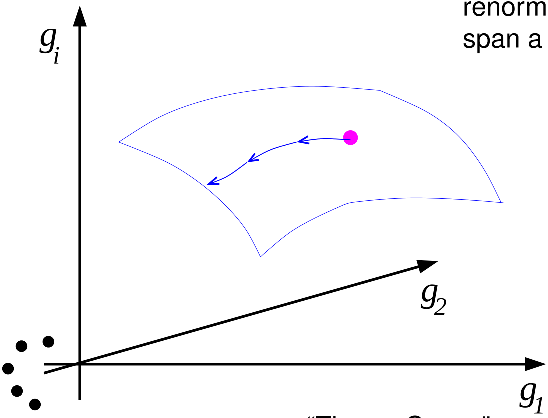


RG mechanism for “lowering” the lower bound



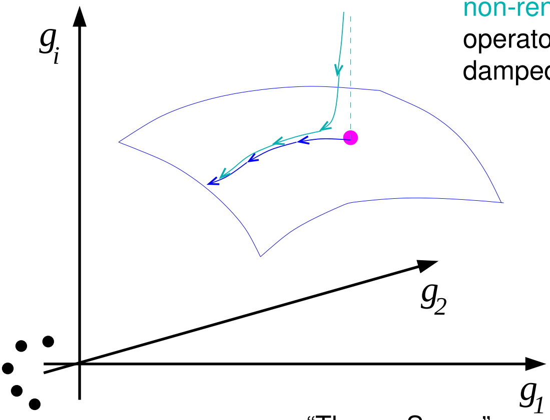
RG mechanism for “lowering” the lower bound

renormalizable operators
span a hyperplane



“Theory Space”

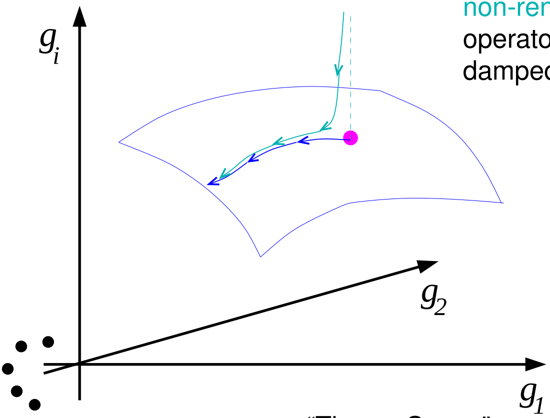
RG mechanism for "lowering" the lower bound



"Theory Space"

non-renormalizable operators are exponentially damped towards the IR flow

RG mechanism for “lowering” the lower bound



“Theory Space”

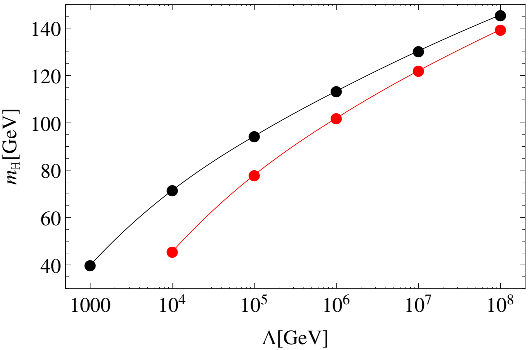
non-renormalizable operators are exponentially damped towards the IR flow

BUT: RG “time” (scales) is needed for this damping

Consistency bounds from generalized bare actions

▷ e.g., (HG,Gneiting,Sondenheimer'13)

$$U_\Lambda = \frac{\lambda_{1\Lambda}}{2}\phi^2 + \frac{\lambda_{2\Lambda}}{8}\phi^4 + \frac{\lambda_{3\Lambda}}{48}\phi^6$$



$\lambda_{3\Lambda} = 0,\ \lambda_{2\Lambda} = 0$

$\lambda_{3\Lambda} = 3,\ \lambda_{2\Lambda} = -0.08$

$\Longrightarrow$ consistency bound $\simeq$ shifted Λ axis

Towards the standard model

▷ chiral Yukawa model: (HG,Sondenheimer'14)

$$S = \int \Big[\partial_\mu \phi^\dagger \partial^\mu \phi + U(\phi^\dagger \phi) + \bar{t} i \partial\!\!\!/ t + \bar{b} i \partial\!\!\!/ b$$
$$+ i h_b (\bar{\psi}_L \phi b_R + \bar{b}_R \phi^\dagger \psi_L) + i h_t (\bar{\psi}_L \phi_{\mathcal{C}} t_R + \bar{t}_R \phi_{\mathcal{C}}^\dagger \psi_L) \Big]$$

$$\phi = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_4 + i\phi_3 \end{pmatrix} \quad \psi_L = \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$

▷ enforce decoupling of Goldstone bosons ($m_G = 0$)

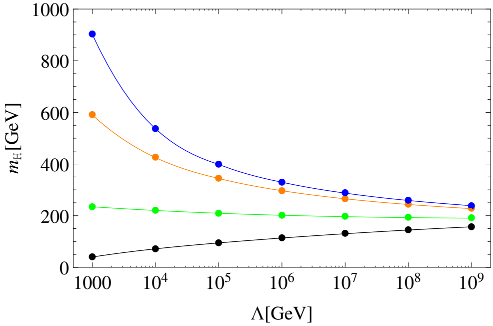
$$\frac{k^2}{k^2 + m_G^2} \quad \to \quad \frac{k^2}{k^2 + m_G^2 + g v_k^2}$$

▷ choose "gauge boson" masses $g v_k^2 = (80.4\,\mathrm{GeV})^2$

cf. lattice model (Gerhold,Jansen'07'09'10)

Conventional lower Higgs boson mass bound

▷ for ϕ^4-type bare potentials: (HG,SONDENHEIMER'14)



FRG:
NLO derivative expansion

$\lambda_{2\Lambda} = 0$
$\lambda_{2\Lambda} = 1$
$\lambda_{2\Lambda} = 10$
$\lambda_{2\Lambda} = 100$

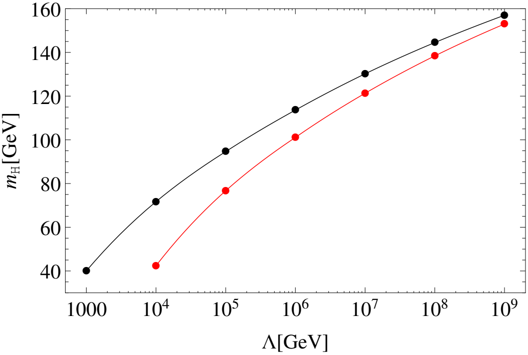
$\Longrightarrow$ lower bound close to simple toy model:

. . . bottom quark has little quantitative influence

"Lowering" the lower Higgs boson mass bound

▷ generalized bare potential with $\lambda_{6,\Lambda}(\phi^\dagger\phi)^3$ interaction:

(HG,Sondenheimer'14)



$\lambda_{3\Lambda} = 0,\ \lambda_{2\Lambda} = 0$

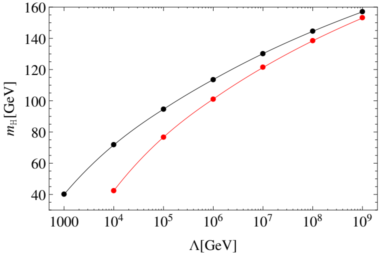
$\lambda_{3\Lambda} = 3,\ \lambda_{2\Lambda} = -0.1$

$\Longrightarrow$ same RG mechanism at work

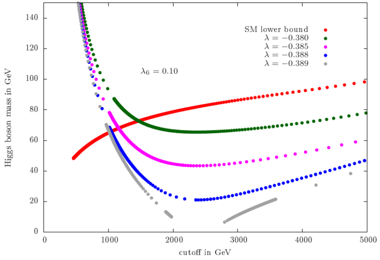
"Lowering" the lower Higgs boson mass bound

▷ generalized bare potential with $\lambda_{6,\Lambda}(\phi^\dagger\phi)^3$ interaction

▷ comparison with lattice data:



(HG,SONDENHEIMER'13,'14)



(HEDGE,JANSEN,LIN,NAGY'13)

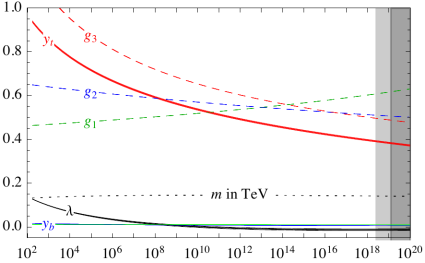
(CHU,JANSEN,KNIPPSCHILD,LIN,NAGY'15)

$\Longrightarrow$ RG mechanism confirmed

A “serious” toy model

(EICHHORN,HG,JAECKEL,PLEHN,SCHERER,SONDENHEIMER'15)

▷ Standard Model vs.



(BUTTAZZO ET AL.'13)

$\Longrightarrow$ toy model satisfies SM constraints at low energies:

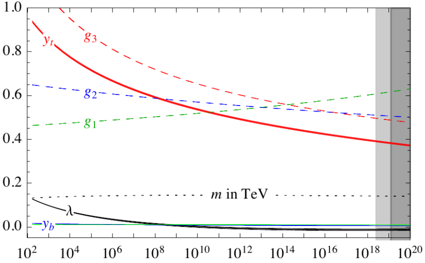
$$m_{\mathrm{H}} \simeq 125\mathrm{GeV}, \quad m_{\mathrm{top}}(m_{\mathrm{top}}) \simeq 164\mathrm{GeV}, \quad \alpha_{\mathrm{s}}(M_Z) \simeq 0.1184$$

$\Longrightarrow$ naive instability scale: $\Lambda_{\mathrm{I}} \simeq 10^{10}$

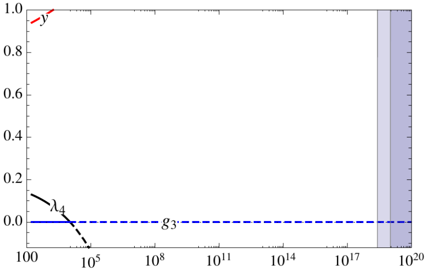
A “serious” toy model

(EICHHORN,HG,JAECKEL,PLEHN,SCHERER,SONDENHEIMER'15)

▷ Standard Model vs. ▷ Z_2 model



(BUTTAZZO ET AL.'13)



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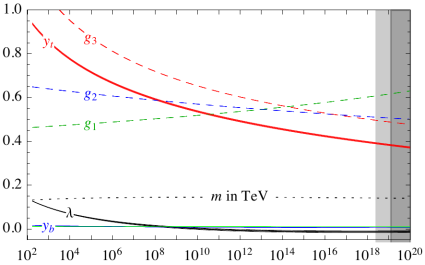
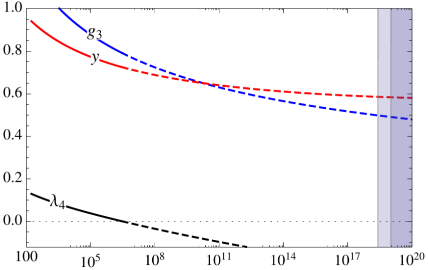
▷ Standard Model vs. ▷ $Z_2 \otimes SU(3)$ model





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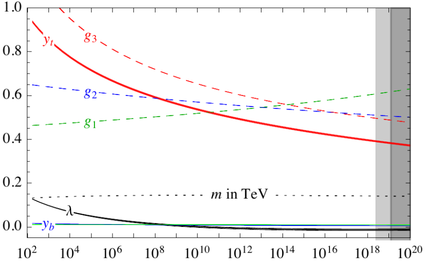
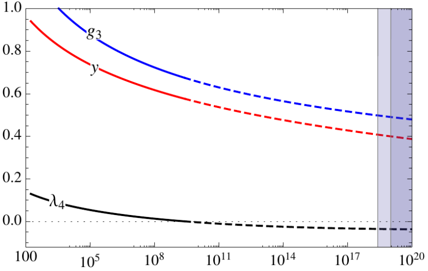
A "serious" toy model

(Eichhorn,HG,Jaeckel,Plehn,Scherer,Sondenheimer'15)

▷ Standard Model vs. ▷ $Z_2 \otimes SU(3)$ + fiducial EW



(Buttazzo et al.'13)



$\Longrightarrow$ toy model satisfies SM constraints at low energies:

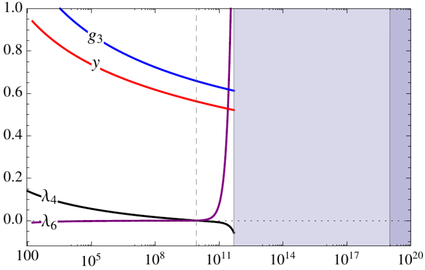
$$m_{\mathrm{H}} \simeq 125\mathrm{GeV}, \quad m_{\mathrm{top}}(m_{\mathrm{top}}) \simeq 164\mathrm{GeV}, \quad \alpha_{\mathrm{s}}(M_Z) \simeq 0.1184$$

$\Longrightarrow$ naive instability scale: $\Lambda_{\mathrm{I}} \simeq 10^{10}$

Stable flows with higher dimensional operators

(Eichhorn, HG, Jaeckel, Plehn, Scherer, Sondenheimer'15)

$+\phi^6$ term:

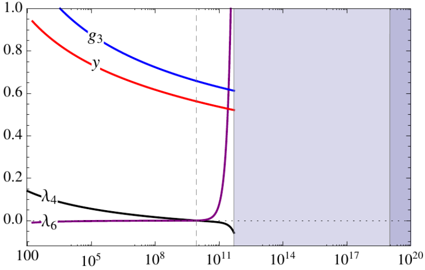


$\Longrightarrow$ cutoff scale $\Lambda \simeq 10^2 \times \Lambda_{\rm I} >$ naive instability scale

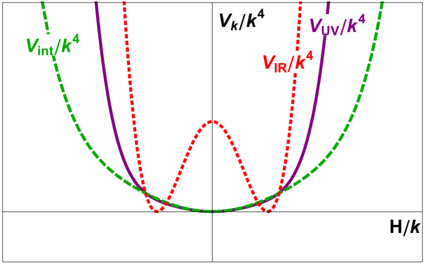
Stable flows with higher dimensional operators

(Eichhorn, HG, Jaeckel, Plehn, Scherer, Sondenheimer'15)

$+\phi^6$ term:



RG flow of the potential:



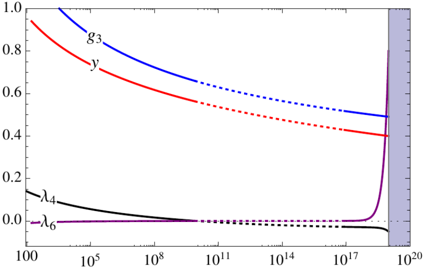
$\Longrightarrow$ cutoff scale $\Lambda \simeq 10^2 \times \Lambda_\mathrm{I} >$ naive instability scale

$\Longrightarrow$ stable potential on all scales (UV $\to$ IR)

Pseudo-stable flows with higher dimensional operators

(EICHHORN,HG,JAECKEL,PLEHN,SCHERER,SONDENHEIMER'15)

$+\phi^6$ term:

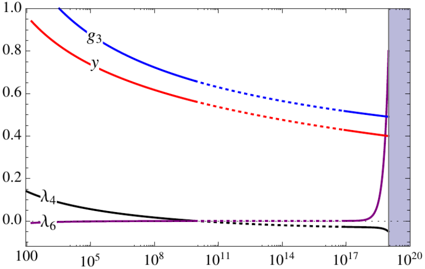


$\Longrightarrow$ cutoff scale $\Lambda \simeq 10^{10} \times \Lambda_{\mathrm{I}} \gg$ naive instability scale

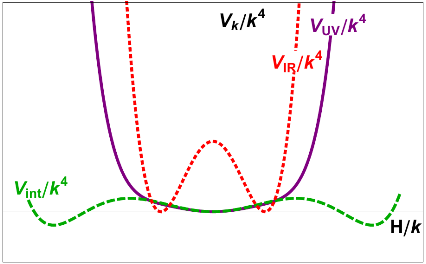
Pseudo-stable flows with higher dimensional operators

(Eichhorn,HG,Jaeckel,Plehn,Scherer,Sondenheimer'15)

$+\phi^6$ term:



RG flow of the potential:



$\Longrightarrow$ cutoff scale $\Lambda \simeq 10^{10} \times \Lambda_I \gg$ naive instability scale

$\Longrightarrow$ stable potential on UV and IR scales

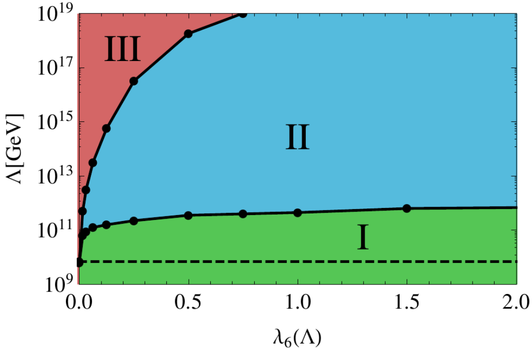
$\Longrightarrow$ meta-stable potential at intermediate scales

PE artifact?
convergence
radius?

Stability analysis of the effective potential

(Eichhorn,HG,Jaeckel,Plehn,Scherer,Sondenheimer'15)

▷ function RG analysis (polynomially expanded potential)



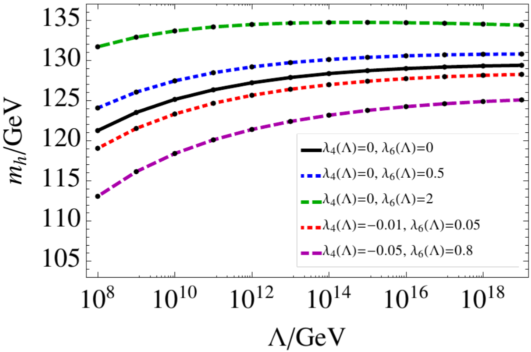
I stable on all scales

II pseudo-stable (UV- and IR-stable, meta-stable inbetween)

III meta-stable UV potential (IR fate ?)

Higgs mass consistency bounds

(Eichhorn,HG,Jaeckel,Plehn,Scherer,Sondenheimer'15)



$\Longrightarrow$ $\Delta m_{\mathrm{H}} \simeq 1$ GeV (in stable region)

$\Longrightarrow$ $\Delta m_{\mathrm{H}} \simeq 5$ GeV (in pseudo-stable region)

measured Higgs mass could be within consistency bounds!

Summary, Part II

- Bounds on the Higgs boson mass (or any other physical IR observable) arise from a mapping

$$S_{\text{micro}} \quad \rightarrow \quad \mathcal{O}_{\text{phys}}$$

...provided by the RG

- For "effective quantum field theories" (with a cutoff Λ):

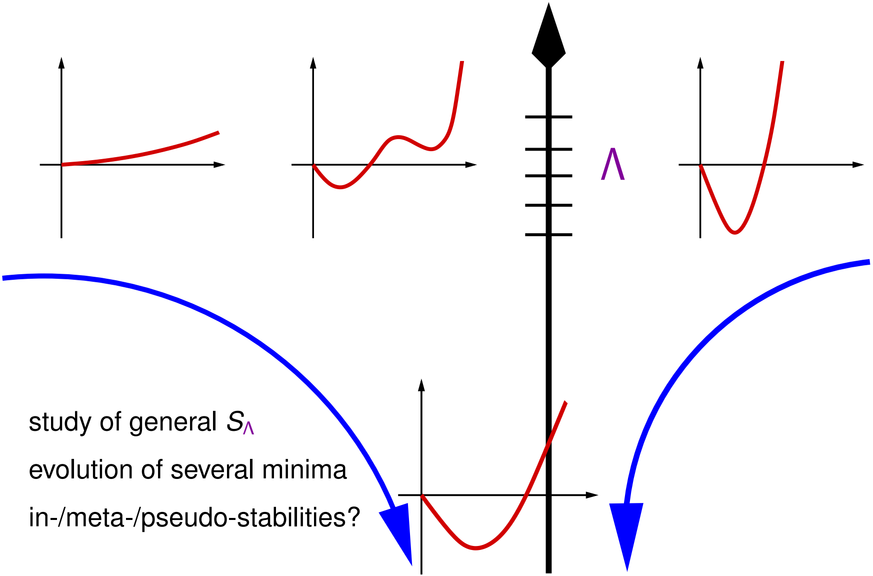
$$\text{bounds on } \mathcal{O}_{\text{phys}} = f[S_\Lambda]$$

...full S_Λ not just the "renormalizable" operators

- "lowering" the conventional lower Higgs boson mass bound is possible

...without in-/meta-stable vacuum

TODO list



study of general S_Λ

evolution of several minima

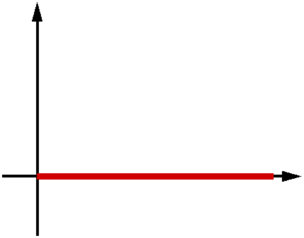
in-/meta-/pseudo-stabilities?

Implications

- if $m_\mathrm{H} <$ conventional lower bound:
  - new physics at lower scales
  - first constraints on underlying UV completion

- if m_H exactly on the convenional lower bound:
  (e.g. if $m_\mathrm{top} \simeq 171\mathrm{GeV}$) …“criticality”
  - underlying UV completion has to explain absence of higher dimensional operators

▷ flat potential



Candidates

- standard model + asymptotically safe gravity (WEINBERG'76; REUTER'96)

  gravity fluctuations induces a UV fixed point $\lambda_* \simeq 0$ (PERCACCI ET AL'03'09)

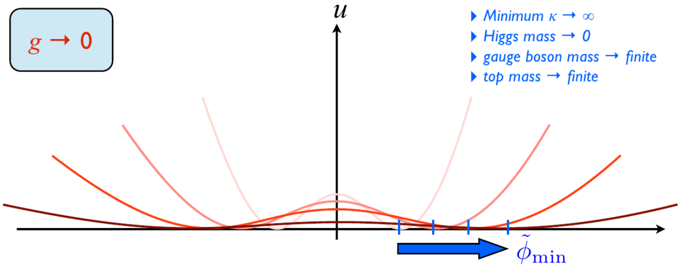
  $\Longrightarrow$ m_{H} put onto conventional lower bound (WETTERICH,SHAPOSHNIKOV'10)

  (BEZRUKOV,KALMYKOV,KNIEHL,SHAPOSHNIKOV'12)

- Asymptotically safe gauged Higgs Yukawa model

  (HG,RECHENBERGER,SCHERER,ZAMBELLI'13)

  $\Longrightarrow$ line of fixed points approaching flat potential with $v/k \to \infty$



Summary, Part III

- Numbers matter

  . . . m_{top}, m_{H}

- QFT is more than a collection of recipes

  . . . new insight from new tools

- vacuum stability: no reason for concern

  . . . so far . . .