Vacuum stability
and the mass of the Higgs boson

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C. Gneiting, R. Sondenheimer, PRD 89 (2014) 045012, [arXiv:1308.5075],

Seminar, KFU Graz, 28.08.2015
Standard model particle spectrum: pre-LHC
Standard model particle spectrum

Quarks:
- u, c, t
- d, s, b

Leptons:
- e, μ, τ
- ν_e, ν_μ, ν_τ

Forces:
- Z
- γ
- W
- g

(Anderson’62; Brout, Englert’64; Higgs’64; Guralnik, Hagen, Kibble’64)
Search for the Higgs boson

วาดเนื้อหา:

4 Jul. 2012
ATLAS & CMS
@CERN

14 Mar 2013, CERN press release:
“... the new particle is looking more and more like a Higgs boson ...”

CMS’12: $125.3 \pm 0.4\,(stat) \pm 0.5\,(sys)\,GeV$
ATLAS’12: $126.0 \pm 0.4\,(stat) \pm 0.4\,(sys)\,GeV$
Numbers matter

standard model

best before:

\[ \Lambda = M_{\text{Planck}} \ ? \]
Validity range of the standard model

\( \Lambda: \)

- UV cutoff
  
  SM as effective theory

- scale of maximum UV extension

- scale of new physics:
  \( \Lambda_{NP} \leq \Lambda \)
Higgs boson mass and maximum validity scale

SM: e.g. (KRIVE, LINDE’76; MAIANI, PARISI, PETRONZIO’78; KRASNIKOV’78; POLITZER, WOLFRAM’78; HUNG’79; LINDNER’85; WETTERICH’87; SHER’88; FORT, JONES, STEPHENSON, EINHORN’93; ALTARELLI, ISIDORI’94; SCHREMP, WIMMER’96; . . .)

BSM: e.g., (CABBIBO, MAIANI, PARISI, PETRONZIO’79; ESPINOSA, QUIROS’91; . . .)
Higgs boson mass and maximum validity scale

\[ M_H \text{ (GeV)} \]
\[ \Lambda \text{ (GeV)} \]

**SM**: e.g., (Krive, Linde'76; Maiani, Parisi, Petronzio'78; Krasnikov'78; Politzer, Wolfram'78; Hung'79; Lindner'85; Wetterich'87; Sher'88; Fort, Jones, Stephenson, Einhorn'93; Altarelli, Isidori'94; Schrempp, Wimmer'96; ...)

**BSM**: e.g., (Cabbibo, Maiani, Parisi, Petronzio'79; Espinosa, Quiros'91; ...)

(Hambye, RiesseLMann'97)
Upper bound of Higgs boson mass

- triviality bound
  - = perturbativity bound
  - = unitarity bound

- Higgs boson mass

\[ m_H^2 \sim v^2 U''_{\text{eff}}(\phi = v), \quad U''_{\text{eff}}(\phi = v) \sim \lambda_R \]

\[ \Rightarrow \] large Higgs boson masses: scalar interactions dominate (\( \lambda \gtrsim 1 \))
Upper bound of Higgs boson mass

➦ triviality: perturbation theory predicts its own failure

\[ \frac{1}{\lambda_R} - \frac{1}{\lambda_\Lambda} = \beta_0 \ln \frac{\Lambda}{v}, \quad \beta_0 > 0 \]

➦ \( \lambda_R \) and \( v \) fixed:

\[ \Rightarrow \quad \Lambda_{\text{sing}} \approx v \exp \left( \frac{1}{\beta_0 \lambda_R} \right) \]

Landau pole singularity

➦ in practice: “perturbativity criterion”, e.g.

\[ \lambda_{\Lambda_{\text{pert}}} \leq \frac{\pi}{4} \quad \Rightarrow \quad \Lambda = \Lambda_{\text{pert}} \]
Upper bound of Higgs boson mass

- triviality: perturbation theory predicts its own failure (Landau'55) (Gell-Mann, Low'54)

\[
\frac{1}{\lambda_R} - \frac{1}{\lambda_\Lambda} = \beta_0 \ln \frac{\Lambda}{v}, \quad \beta_0 > 0
\]

- Triviality

\[
\lim_{\frac{\Lambda}{v} \to \infty} : \quad \implies \lambda_R \to 0
\]

- in simple models: nonperturbative evidence from lattice simulations (Luescher, Weisz'88; Hasenfratz, Jansen, Lang, Neuhaus, Yoneyama'87; Wolff'11; Buividovich'11; Weisz, Wolff'12, . . .)
Lower bound of Higgs boson mass

- vacuum stability / meta-stability bound
- effective potential à la Coleman Weinberg:

\[ U_{\text{eff}}(\phi) = -\frac{1}{2} \mu^2 \phi^2 + \frac{1}{2} \lambda(\phi) \phi^4 \]

- e.g., \( \lambda(\phi) \) from “RG-improved” perturbation theory:

\[ \partial_t \lambda = \frac{3}{4\pi^2} \left( -h_i^4 + h_i^2 \lambda + \frac{1}{16} [2g^4 + (g^2 + g'^2)^2] - \frac{1}{4} \lambda(3g^2 + g'^2) + \lambda^2 \right) \]
Lower bound of Higgs boson mass

▷ effective potential à la Coleman Weinberg:

\[ U_{\text{eff}}(\phi) = -\frac{1}{2} \mu^2 \phi^2 + \frac{1}{2} \lambda(\phi) \phi^4 \]

▷ e.g., \( \lambda(\phi) \) from “RG-improved” perturbation theory:

\[ \partial_t \lambda = \frac{3}{4\pi^2} \left( -h_i^4 + h_t^2 \lambda + \frac{1}{16} \left[ 2g^4 + (g^2 + g'^2)^2 \right] - \frac{1}{4} \lambda(3g^2 + g'^2) + \lambda^2 \right) \]
Lower Bound of Higgs boson mass

▶ meta-stability:
tunneling time > age of universe

▶ “Near critical” standard model: \textbf{(Buttazzo et al.'13)}

NNLO calculation \textbf{(Degrassi et al.'12)}

earlier calculations, e.g., \textbf{(Isidori,Ridolfi,Strumia'01)}
Lower Bound of Higgs boson mass

“Near critical” standard model: (Buttazzo et al.’13: Update v4)

“Stability” seems to prefer

\[ m_H \nearrow \simeq 130 \text{GeV} \]

or \[ m_{\text{top}} \searrow \simeq 171 \text{GeV} \]
Lower Bound of Higgs boson mass

“Near critical” standard model: (Buttazzo et al. '13: UPDATE)

"Stability" seems to prefer $m_{H} \uparrow \approx 130$ GeV or $m_{t} \downarrow \approx 171$ GeV

“The Higgs potential has the worrisome feature that it might become metastable at energies above 100bn gig-electron-volts,”

Professor Stephen Hawking says the Higgs boson ‘God particle’ could destroy the universe

By CambridgeNews | Posted: September 09, 2014
Lower bound of Higgs boson mass

- Reasons for concern?
  - $\alpha_s$ dependence
  - Top mass dependence
    - (Degrassi et al.’12)
  - Larger error of top pole mass $\times 3$?
    - (Alekhin, Djouadi, Moch’12)
      - Compared to the “MC fit parameter” determined by Tevatron
  - Decay of the false vacuum
    - Warning time $\leq 10^{-21}$ s
    - “… the expansion of the bubble is a clean sweep …”
    - (Coleman’77)
2nd thoughts on the lower bound

- True minimum of $U_{\text{eff}}(\phi)$ at

  $\phi \sim 10^{25}\,\text{GeV} > M_{\text{Pl}}$ ?

- UV$\rightarrow$IR RG flow?
2nd thoughts on the lower bound

True minimum of $U_{\text{eff}}(\phi)$ at

$\phi \sim 10^{25}\text{GeV} > M_{\text{Pl}}$ ?

UV→IR RG flow?

simple top-Higgs Yukawa model:

lattice simulation
vs. 1-loop PT with cutoff
vs. 1-loop "Λ-removed" PT

(HOLLAND, KUTI'03; HOLLAND'04)

Criticism: too few scales? (EINHORN, JONES'07)
Top-Higgs Yukawa toy model

$Z_2$ symmetric model

$$S = \int \frac{1}{2} (\partial \phi)^2 + U(\phi) + \bar{\psi} i\gamma^5 \psi + ih\phi \bar{\psi} \psi$$

- includes relevant top quark + Higgs field (+ largest Yukawa coupling)

- discrete symmetry breaking $\rightarrow$ no Goldstone bosons (as in SM)

- avoids intricate questions arising from gauge symmetry

(Holland, Kuti'03; Branchina, Faivre'05; Frohlich, Morchio, Strocchi'81; Lang, Rebbi, Virasoro'81; Jersak, Lang, Neuhaus, Vones'85; Maas '12)
Top-Higgs Yukawa models

▷ generating functional:

\[ Z[J] = \int_\Lambda \mathcal{D}\phi \mathcal{D}\bar{\psi} \mathcal{D}\psi \, e^{-S[\phi, \bar{\psi}, \psi] + \int J\phi} = \int_\Lambda \mathcal{D}\phi \, e^{-S_B[\phi] - S_{F,\Lambda}[\phi] + \int J\phi} \]

▷ top-induced effective potential

\[ U_F(\phi) = -\frac{1}{2\Omega} \ln \left( \frac{\det_\Lambda(-\partial^2 + h_i^2 \phi^\dagger \phi)}{\det_\Lambda(-\partial^2)} \right), \]

▷ CAVE: cutoff \( \Lambda \) / regularization dependent
Top-induced effective potential

- exact results for fermion determinants for homogeneous $\phi$
- e.g., sharp cutoff:

\[
U_{F,t}(\phi) = -\frac{\Lambda^2}{8\pi^2} h_t^2 |\phi|^2 \\
\leq 0 \quad \text{(mass-like term)}
\]

\[
+ \frac{1}{16\pi^2} \left[ h_t^4 |\phi|^4 \ln \left(1 + \frac{\Lambda^2}{h_t^2 |\phi|^2}\right) + h_t^2 |\phi|^2 \Lambda^2 - \Lambda^4 \ln \left(1 + \frac{h_t^2 |\phi|^2}{\Lambda^2}\right) \right]
\]

\geq 0 \quad \text{(interaction part)}

- mass-like term: contributes to $\chi_{SB} \implies \nu \simeq 246\text{GeV}$

- interaction part: strictly positive

\implies \text{cannot induce instability for any finite } \Lambda

(HG, SONDENHEIMER’14)
“Rederiving” the instability

try to send $\Lambda \to \infty$:

$$U_{F,t}(\phi) = -\frac{\Lambda^2}{8\pi^2} h_t^2 |\phi|^2 + \frac{1}{16\pi^2} h_t^4 |\phi|^4 \left[ \ln \frac{\Lambda^2}{h_t^2 |\phi|^2} + \text{const.} + O \left( \frac{h_t^2 |\phi|^2}{\Lambda^2} \right) \right]$$
“Rederiving” the instability

▷ try to send \( \Lambda \to \infty \):

\[
U_{F,t}(\phi) = -\frac{\Lambda^2}{8\pi^2} h_t^2 |\phi|^2 + \frac{1}{16\pi^2} h_t^4 |\phi|^4 \left[ \ln \frac{\Lambda^2}{h_t^2 |\phi|^2} + \text{const.} + \mathcal{O} \left( \frac{h_t^2 |\phi|^2}{\Lambda^2} \right) \right]
\]

▷ renormalization: trade \((\Lambda, m_\Lambda, \lambda_\Lambda)\) for \((\mu, \nu, \lambda_\nu)\)

\[
U_{F,t}(\phi) \to -\frac{1}{16\pi^2} h_t^4 |\phi|^4 \left( \ln \frac{h_t^2 |\phi|^2}{\mu^2} + \text{const.} \right)
\]
“Rederiving” the instability

▷ try to send $\Lambda \to \infty$:

$$U_{F,t}(\phi) = -\frac{\Lambda^2}{8\pi^2} h_t^2 |\phi|^2 + \frac{1}{16\pi^2} h_t^4 |\phi|^4 \left[ \ln \frac{\Lambda^2}{h_t^2 |\phi|^2} + \text{const.} + \mathcal{O}\left(\frac{h_t^2 |\phi|^2}{\Lambda^2}\right) \right]$$

▷ renormalization: trade $(\Lambda, m_\Lambda, \lambda_\Lambda)$ for $(\mu, \nu, \lambda_\nu)$

$$U_{F,t}(\phi) \rightarrow \frac{1}{16\pi^2} h_t^4 |\phi|^4 \left( \ln \frac{h_t^2 |\phi|^2}{\mu^2} + \text{const.} \right)$$

▷ “instability” occurs beyond $\frac{h_t^2 |\phi|^2}{\Lambda^2} > 1$

(Holland, Kuti’03; Holland’04)

▷ similar problems for other reg’s

▷ implicit renormalization conditions would violate unitarity

(Branchina, Faiivre’05; Gneiting’05)
Contradiction?

\[ \partial_t \lambda = -\frac{3}{4\pi^2} h_t^4 + \ldots \]

\[ \Rightarrow \lambda \downarrow < 0 \]

interaction part of \( U_{F,t}(\phi) > 0 \)

strictly positive
Contradiction?

\[ \partial_t \lambda = -\frac{3}{4\pi^2} h_t^4 + \ldots \]

\[ \implies \lambda \downarrow < 0 \]

interaction part of

\[ U_{F,t}(\phi) > 0 \]

strictly positive

\[ U_{\text{eff}}(\phi) \sim \frac{1}{2} \lambda(\phi) \phi^4 \]
Contradiction?

\[ \partial_t \lambda = -\frac{3}{4\pi^2} h_t^4 + \ldots \]

\[ \implies \lambda \downarrow < 0 \]

interaction part of \( U_{F,t}(\phi) > 0 \) strictly positive

CAVE: \( \mathcal{O} \left( \frac{h_t^2|\phi|^2}{\Lambda^2} \right) \)-terms for finite \( \Lambda \)
Summary, Part I

- no in-/meta-stability from top (fermion) fluctuations
  \[ \cdots \text{if cutoff } \Lambda \text{ is kept finite but arbitrary} \]
- no in-/meta-stability at all ?

\[ U_{\text{eff}}(\phi) = U_{\Lambda}(\phi) + U_{B}(\phi) + U_{F}(\phi) \]
Summary, Part I

• no in-/meta-stability from top (fermion) fluctuations
  ... if cutoff $\Lambda$ is kept finite but arbitrary

• no in-/meta-stability at all?

$$U_{\text{eff}}(\phi) = \underbrace{U_\Lambda(\phi)}_{\text{arbitrary}} + \underbrace{U_B(\phi) + U_F(\phi)}_{\text{generically stable}}$$

... in-/meta-stabilities from the bare action/UV completion

• lower Higgs mass bounds?

$\implies$ nonperturbative methods recommended if not needed

▶ extensive lattice simulations:
  (Fodor,Holland,Kuti,Nogradi,Schroeder’07)
  (Gerhold,Jansen’07’09’10)

▶ constraining 4th generations:
  (Gerhold,Jansen,Kallarackal’10; Bulava,Jansen,Nagy’13)

▶ implications for dark matter models:
  (Eichhorn, Scherer’14)
Higgs boson mass bounds as a UV to IR mapping
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QFT + RG
Higgs boson mass bounds as a UV to IR mapping

$\Lambda_{\text{QFT + RG}} \Rightarrow \text{consistency bounds}$
Higgs boson mass bounds as a UV to IR mapping

- microscopic action at cutoff $\Lambda$:

$$S_{\Lambda} = S_{\Lambda}(m_{\Lambda}^2, \lambda_{\Lambda}, \lambda_{6,\Lambda}, \ldots, h_{\Lambda}, \ldots)$$

$$\Rightarrow$$ RG: mapping to IR observables

$$v \simeq 246\,\text{GeV}, \quad m_{\text{top}} \simeq 173\,\text{GeV}, \quad m_H = m_H[S_{\Lambda}]$$

- By contrast: “RG-improved” PT

fix: $v, m_{\text{top}} , m_H \quad \uparrow \quad \text{upwards RG} \quad U_{\text{eff}}$
Simple Example: mean-field theory

- MFT $\overset{\triangle}{=} \text{large-}N_f \text{ limit} \overset{\triangle}{=} \text{pure top loop}:
  
  $$U_{\text{MF}}(\phi) = U_{\Lambda}(\phi) - \ln \det_{\text{reg}}(i\partial + ih_{\Lambda}\phi)$$

- UV bare potential, e.g,
  
  $$U_{\Lambda}(\phi) = \frac{1}{2} m_{\Lambda}^2 \phi^2 + \frac{1}{8} \lambda_{\Lambda} \phi^4, \quad \lambda_{\Lambda} \geq 0$$

- trade: $m_{\Lambda}, h_{\Lambda} \iff v, m_{\text{top}}$

- Higgs boson mass:
  
  $$m_{H}^2(\Lambda, \lambda_{\Lambda}) = \frac{m_{\text{top}}^4}{4\pi^2 v^2} \left[ 2 \ln \left( 1 + \frac{\Lambda^2}{m_{\text{top}}^2} \right) - \frac{3\Lambda^4 + 2m_{\text{top}}^2\Lambda^2}{(\Lambda^2 + m_{\text{top}}^2)^2} \right] + v^2 \lambda_{\Lambda}$$
Simple Example: mean-field theory

▷ Higgs boson mass bound

\[
m_H^2(\Lambda, \lambda_\Lambda) = \frac{m_{\text{top}}^4}{4\pi^2 v^2} \left[ 2 \ln \left(1 + \frac{\Lambda^2}{m_{\text{top}}^2}\right) - \frac{3\Lambda^4 + 2m_{\text{top}}^2\Lambda^2}{(\Lambda^2 + m_{\text{top}}^2)^2}\right] + v^2 \lambda_\Lambda
\]

requirement: well defined path integral: \( \lambda_\Lambda \geq 0 \) for \( \phi^4 \) theory

cf. lattice \((\text{HOLLAND’04; FODOR,HOLLAND,KUTI,NOGRAĐI,}\text{SCHROEDER’07; GERHOLD,JANSEN’07’09’10})\)

▷ extended mean field \( \hat{=} \) NLO \( 1/N_f \) expansion:

\[
\lambda_\Lambda = \begin{cases} 
0 \\
1/6 \\
1/3 \\
2/3 
\end{cases}
\]
Nonperturbative tool: functional RG

IR: $k \rightarrow 0$ \hspace{1cm} UV: $k \rightarrow \Lambda$

 RG flow equation:

$$ \partial_t \Gamma_k \equiv k \partial_k \Gamma_k = \frac{1}{2} \Tr \partial_t R_k (\Gamma^{(2)}_k + R_k)^{-1} $$

(WETTERICH'93)
RG Flow Equation

\[ \partial_t \Gamma_k = \frac{1}{2} \text{Tr} \partial_t R_k (\Gamma_k^{(2)} + R_k)^{-1} \]

▷ RG trajectory:

\[ \Gamma_{k=\Lambda} = S_\Lambda = \int \frac{1}{2} (\partial \phi)^2 + U_\Lambda(\phi) + \ldots \]

theory space

\[ \bullet \text{ UV} \]
RG Flow Equation

\[ \partial_t \Gamma_k = \frac{1}{2} \text{Tr} \left( \partial_t R_k (\Gamma_k^{(2)} + R_k)^{-1} \right) \]

▷ RG trajectory:
RG Flow Equation

\[ \partial_t \Gamma_k = \frac{1}{2} \text{Tr} \partial_t R_k (\Gamma_k^{(2)} + R_k)^{-1} \]

RG trajectory:
RG Flow Equation

$$\partial_t \Gamma_k = \frac{1}{2} \text{Tr} \partial_t R_k (\Gamma_k^{(2)} + R_k)^{-1}$$

RG trajectory:

$$\Gamma_{k \to 0} = \Gamma = \int U_{\text{eff}} + \ldots$$
Higgs boson mass bounds from functional RG

- Top-Higgs toy model

- Systematic derivative expansion:

\[ \Gamma_k = \int d^d x \left( \frac{Z_{\phi k}}{2} \partial_\mu \phi \partial^\mu \phi + U_k(\phi^2) + Z_{\psi k} \bar{\psi} i \partial \psi + i h_k \phi \bar{\psi} \psi \right) \]

\[ \lambda_\Lambda = 0 \]
Top-Higgs toy model

Systematic derivative expansion:

$$\Gamma_k = \int d^d x \left( \frac{Z_{\phi k}}{2} \partial_\mu \phi \partial^\mu \phi + U_k(\phi^2) + Z_{\psi k} \bar{\psi} i \gamma_5 \psi + i h_k \phi \bar{\psi} \psi \right)$$

\[\lambda_\Lambda = 0\]
\[\lambda_\Lambda = 0.1\]
Higgs boson mass bounds from functional RG

- Top-Higgs toy model

- Systematic derivative expansion:

\[
\Gamma_k = \int d^d x \left( \frac{Z_{\phi k}}{2} \partial_{\mu} \phi \partial^{\mu} \phi + U_k(\phi^2) + Z_{\psi k} \bar{\psi} i \gamma_{\mu} \gamma_5 \psi + i h_k \phi \bar{\psi} \psi \right)
\]

\[
\lambda_\Lambda = 0 \\
\lambda_\Lambda = 0.1 \\
\lambda_\Lambda = 1
\]
Higgs boson mass bounds from functional RG

▷ Top-Higgs toy model

▷ Systematic derivative expansion:

\[
\Gamma_k = \int d^d x \left( \frac{Z_{\phi k}}{2} \partial_\mu \phi \partial^\mu \phi + U_k(\phi^2) + Z_{\psi k} \bar{\psi} i \gamma_\mu \psi + i h_k \phi \bar{\psi} \psi \right)
\]

\[
\lambda_\Lambda = 0
\]
\[
\lambda_\Lambda = 0.1
\]
\[
\lambda_\Lambda = 1
\]
\[
\lambda_\Lambda = 10
\]
Higgs boson mass bounds from functional RG

- Top-Higgs toy model
- Systematic derivative expansion:

\[ \Gamma_k = \int d^d x \left( \frac{Z_{\phi k}}{2} \partial_\mu \phi \partial^\mu \phi + U_k(\phi^2) + Z_{\psi k} \bar{\psi} i \phi \psi + ih_k \phi \bar{\psi} \psi \right) \]
Higgs boson mass bounds from functional RG

- Top-Higgs toy model
- Systematic derivative expansion:

\[ \Gamma_k = \int d^d x \left( \frac{Z\phi_k}{2} \partial_\mu \phi \partial^\mu \phi + U_k(\phi^2) + Z\psi_k \bar{\psi} i \partial \psi + i h_k \phi \bar{\psi} \psi \right) \]

\[ \lambda_\Lambda = 0 \]
\[ \lambda_\Lambda = 0.1 \]
\[ \lambda_\Lambda = 1 \]
\[ \lambda_\Lambda = 10 \]
\[ \lambda_\Lambda = 100 \]

\[ \Rightarrow \text{"conventional" lower bound for } \lambda_\Lambda = 0 \text{ (\(\approx\) mean-field result)} \]

agreement with lattice (HOLLAND’04; FODOR,HOLLAND,KUTI,NOGRADE,SCHROEDER’07; GERHOLD,JANSEN’07’09’10)
The IR window for the Higgs boson mass

$m_H \sim \nu \lambda_R$  

(Wetterich’87)

mapping: $\lambda_\Lambda \rightarrow \lambda_R$ not surjective on $\mathbb{R}_+$

(e.g. for $\phi^4$ bare potential, fix $\Lambda = 10^7$ GeV)  

(HG, Gneiting, Sondenheimer’13)

convergence check of

- derivative expansion $\Delta \text{NLO} / \text{LO} \sim 10\% @$ strong coupling
- $U_{\text{eff}}$ solver (polynom. exp.)
General microscopic actions

▷ $S_{\Lambda}$ is a priori unconstrained. Consider, e.g., (HG, Gneiting, Sondenheimer’13)

$$U_{\Lambda} = \frac{\lambda_{1\Lambda}}{2} \phi^2 + \frac{\lambda_{2\Lambda}}{8} \phi^4 + \frac{\lambda_{3\Lambda}}{48} \phi^6$$

▷ for $\lambda_{3\Lambda} > 0$ we can choose $\lambda_{2\Lambda} < 0$: 
General microscopic actions

▶ $S_\Lambda$ is a priori unconstrained. Consider, e.g., (HG, Gneiting, Sondheimer’13)

$$U_\Lambda = \frac{\lambda_{1\Lambda}}{2} \phi^2 + \frac{\lambda_{2\Lambda}}{8} \phi^4 + \frac{\lambda_{3\Lambda}}{48} \phi^6$$

▶ for $\lambda_{3\Lambda} > 0$ we can choose $\lambda_{2\Lambda} < 0$:

\[
\begin{align*}
\lambda_{3\Lambda} &= 0, \lambda_{2\Lambda} = 0 \\
\lambda_{3\Lambda} &= 3, \lambda_{2\Lambda} = -0.08
\end{align*}
\]

▶ lower bound relaxed

$$\quad \implies \text{consistency bound} < \text{“conventional” bound}$$

string models with $\lambda < 0$: (Hebecker, Knochel, Weigand’13)
Renormalizable field theories

▶ seeming contradiction with common wisdom . . . ?

“. . . observables are determined by renormalizable operators . . .”

▶ y-axis: $m_H$ observable ✓, x-axis: $\Lambda$ ?
RG mechanism for “lowering” the lower bound

"Theory Space"
RG mechanism for “lowering” the lower bound

“Theory Space”

Gaußian fixed point
RG mechanism for “lowering” the lower bound

renormalizable operators span a hyperplane

“Theory Space”
RG mechanism for “lowering” the lower bound

non-renormalizable operators are exponentially damped towards the IR flow

“Theory Space”
RG mechanism for “lowering” the lower bound

non-renormalizable operators are exponentially damped towards the IR flow

BUT: RG “time” (scales) is needed for this damping

"Theory Space"
Consistency bounds from generalized bare actions

\[ U_\Lambda = \frac{\lambda_1 \Lambda}{2} \phi^2 + \frac{\lambda_2 \Lambda}{8} \phi^4 + \frac{\lambda_3 \Lambda}{48} \phi^6 \]

\( \lambda_3 \Lambda = 0, \lambda_2 \Lambda = 0 \)

\( \lambda_3 \Lambda = 3, \lambda_2 \Lambda = -0.08 \)

\[ \iff \text{ consistency bound } \Rightarrow \text{ shifted } \Lambda \text{ axis} \]
Towards the standard model

- chiral Yukawa model:

\[
S = \int \left[ \partial_{\mu} \phi^{\dagger} \partial^{\mu} \phi + U(\phi^{\dagger} \phi) + \bar{t} i \gamma_i t + \bar{b} i \gamma_i b \\
+ i h_b (\bar{\psi}_L \phi_R + \bar{b}_R \phi^{\dagger}_L \psi_L) + i h_t (\bar{\psi}_L \phi_C t_R + \bar{t}_R \phi^{\dagger}_C \psi_L) \right]
\]

\[
\phi = \begin{pmatrix} \phi_1 + i \phi_2 \\ \phi_4 + i \phi_3 \end{pmatrix} \quad \psi_L = \begin{pmatrix} t_L \\ b_L \end{pmatrix}
\]

- enforce decoupling of Goldstone bosons \((m_G = 0)\)

\[
\frac{k^2}{k^2 + m^2_G} \rightarrow \frac{k^2}{k^2 + m^2_G + g v_k^2}
\]

- choose “gauge boson” masses \(g v_k^2 = (80.4 \text{ GeV})^2\)

cf. lattice model (Gerhold, Jansen'07'09'10)
Conventional lower Higgs boson mass bound

for $\phi^4$-type bare potentials:

\[ \lambda_2 \Lambda = 0 \]
\[ \lambda_2 \Lambda = 1 \]
\[ \lambda_2 \Lambda = 10 \]
\[ \lambda_2 \Lambda = 100 \]

FRG: NLO derivative expansion

lower bound close to simple toy model:

...bottom quark has little quantitative influence
“Lowering” the lower Higgs boson mass bound

generalized bare potential with $\lambda_6,\Lambda (\phi^\dagger \phi)^3$ interaction:

$\lambda_3,\Lambda = 0$, $\lambda_2,\Lambda = 0$

$\lambda_3,\Lambda = 3$, $\lambda_2,\Lambda = -0.1$

$\Rightarrow$ same RG mechanism at work
“Lowering” the lower Higgs boson mass bound

- generalized bare potential with $\lambda_6, \Lambda (\phi^\dagger \phi)^3$ interaction

- comparison with lattice data:

RG mechanism confirmed
A “serious” toy model

(Eichhorn, HG, Jaeckel, Plehn, Scherer, Sondenheimer’15)

Standard Model vs.

\[
m_H \approx 125\text{GeV}, \quad m_{\text{top}}(m_{\text{top}}) \approx 164\text{GeV}, \quad \alpha_s(M_Z) \approx 0.1184
\]

\[\Lambda \approx 10^{10}\]

(Buttazzo et al.’13)
A “serious” toy model

Standard Model vs. Z\(_2\) model

(Eichhorn, HG, Jaeckel, Plehn, Scherer, Sondenheimer’15)

\(~\Rightarrow\) toy model satisfies SM constraints at low energies:

\[ m_H \simeq 125\text{GeV}, \quad m_{\text{top}}(m_{\text{top}}) \simeq 164\text{GeV}, \quad \alpha_s(M_Z) \simeq 0.1184 \]

\(~\Rightarrow\) naive instability scale: \(\Lambda_1 \simeq 10^{10}\)
A “serious” toy model

Standard Model vs. $\mathbb{Z}_2 \otimes SU(3)$ model

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naive instability scale: $\Lambda \approx 10^{10}$
A “serious” toy model

(Eichhorn, HG, Jaeckel, Plehn, Scherer, Sondenheimer ’15)

→ Standard Model vs. Z$_2 \otimes$SU(3) + fiducial EW

$Z_2 \otimes SU(3)$ + fiducial EW

(Buttazzo et al.’13)

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→ naive instability scale: $\Lambda_1 \simeq 10^{10}$
Stable flows with higher dimensional operators

\[ + \phi^6 \text{ term:} \]

\[ \Rightarrow \text{cutoff scale } \Lambda \approx 10^2 \times \Lambda_1 > \text{naive instability scale} \]
Stable flows with higher dimensional operators

\( + \phi^6 \) term:

RG flow of the potential:

\[ \Lambda \simeq 10^2 \times \Lambda_i > \text{naive instability scale} \]

\[ \text{stable potential on all scales } (\text{UV} \rightarrow \text{IR}) \]
Pseudo-stable flows with higher dimensional operators

$\phi^6$ term:

$\Lambda \approx 10^{10} \times \Lambda_I \gg$ naive instability scale
Pseudo-stable flows with higher dimensional operators

\( + \phi^6 \) term:

RG flow of the potential:

\[ c_{\text{cutoff}} \approx 10^{10} \times \Lambda_I \gg \text{naive instability scale} \]

\[ \implies \text{stable potential on UV and IR scales} \]

\[ \implies \text{meta-stable potential at intermediate scales} \]

\( \text{PE artifact? convergence radius?} \)
Stability analysis of the effective potential

(Eichhorn, HG, Jaeckel, Plehn, Scherer, Sondheimer '15)

- function RG analysis (polynomially expanded potential)

I stable on all scales
II pseudo-stable (UV- and IR-stable, meta-stable inbetween)
III meta-stable UV potential (IR fate?)
Higgs mass consistency bounds

\[ \Delta m_H \simeq 1 \text{ GeV (in stable region)} \]

\[ \Delta m_H \simeq 5 \text{ GeV (in pseudo-stable region)} \]

measured Higgs mass could be within consistency bounds!
Summary, Part II

- Bounds on the Higgs boson mass (or any other physical IR observable) arise from a mapping

\[ S_{\text{micro}} \rightarrow \mathcal{O}_{\text{phys}} \]

... provided by the RG

- For “effective quantum field theories” (with a cutoff \( \Lambda \)):

bounds on \( \mathcal{O}_{\text{phys}} = f[S_{\Lambda}] \)

... full \( S_{\Lambda} \) not just the “renormalizable” operators

- “Lowering” the conventional lower Higgs boson mass bound is possible

... without in-/meta-stable vacuum
study of general $S_\wedge$
evolution of several minima
in-/meta-/pseudo-stabilities?
Implications

• if $m_H <$ conventional lower bound:
  • new physics at lower scales
  • first constraints on underlying UV completion

• if $m_H$ exactly on the conventional lower bound:
  (e.g. if $m_{\text{top}} \simeq 171\text{GeV}$) . . . “criticality”
  • underlying UV completion has to explain absence of higher dimensional operators

▷ flat potential
Candidates

- standard model + asymptotically safe gravity grav. fluctuations induces a UV fixed point $\lambda_\ast \simeq 0$ ([Percacci et al.’03’09])

$\implies m_H$ put onto conventional lower bound ([Wetterich, Shapiroshnikov’10])

- Asymptotically safe gauged Higgs Yukawa model
  ([HG, Rechenberger, Scherer, Zambelli’13])

$\implies$ line of fixed points approaching flat potential with $v/k \to \infty$

$g \to 0$

- $\min k \to \infty$
- Higgs mass $\to 0$
- gauge boson mass $\to$ finite
- top mass $\to$ finite

$\phi_{\min}$
• Numbers matter
  \( m_{\text{top}}, m_H \)

• QFT is more than a collection of recipes
  \( \ldots \) new insight from new tools

• vacuum stability: no reason for concern
  \( \ldots \) so far \( \ldots \)