Electromagnetic Probes in Heavy-Ion Collisions I
Foundations

Hendrik van Hees
Goethe University Frankfurt and FIAS

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Outline

1. Plan of the lectures
2. Electromagnetic Probes: Phenomenology
3. QCD and its (“accidental”) symmetries
4. Strongly interacting matter: QCD/hadronic models at finite $T, \mu$
5. References
Plan of the Lectures

- **Lecture I: Fundamentals**
  - symmetries and conservation laws in (quantum) field theory
  - QCD, chiral symmetry, and the relation with electromagnetic probes
  - radiation from a transparent thermal source (McLerran-Toimela formula)

- **Lecture II: Phenomenology from SIS to LHC energies**
  - transport and hydrodynamics
  - collective flow
  - effective hadronic models for vector mesons
  - dileptons at SIS (HADES), SPS (NA60), RHIC (STAR, PHENIX), FAIR, LHC
  - direct photons at RHIC (STAR, PHENIX) and LHC (ALICE)
Why Electromagnetic Probes?

- $\gamma, \ell^\pm$: only e. m. interactions
- reflect whole “history” of collision
- chance to see chiral symm. rest. directly?

Fig. by A. Drees (from [RW00])

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Vacuum Baseline: $e^+e^- \rightarrow$ hadrons

\[ R := \frac{\sigma_{e^+e^-\rightarrow \text{hadrons}}}{\sigma_{e^+e^-\rightarrow \mu^+\mu^-}} \]

- Probes all hadrons with quantum numbers of $\gamma^*$
- $R_{QM} = N_c \sum_{f=u,d,s} Q_f^2 = 3 \times [(2/3)^2 + (-1/3)^2 + (-1/3)^2] = 2$
- Our aim $pp \rightarrow \ell^+\ell^-$, $pA \rightarrow \ell^+\ell^-$, $AA \rightarrow \ell^+\ell^-$ ($\ell = e, \mu$)
The CERES findings: Dilepton enhancement

- **pp (pBe):** “elementary reactions”; baseline (mandatory to understand first!)
- **pA:** “cold nuclear matter effects”; next step (important as baseline for other observables like “$J/\psi$ suppression”)
- **AA:** “medium effects”; hope to learn something about in-medium properties of vector mesons, fundamental QCD properties
The CERES findings: Dilepton enhancement

Pb-Au 158 AGeV

$\sigma/\sigma_{geo} \approx 28\%$

$<dN_{ch}/d\eta>=245$

$2.1<\eta<2.65$

$p_t>0.2$ GeV/$c$

$\Theta_{ee}>35$ mrad

combined 95/96 data

$<dN_{ee}/d\Omega_{ee}>/\langle N_{ch} \rangle$ (100 MeV/$c^2$)$^{-1}$
The standard model in a nutshell: particles and forces

Quantum Chromodynamics: QCD

- Theory for strong interactions: QCD

\[ \mathcal{L}_{\text{QCD}} = -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{\psi} (i\not{D} - \hat{M}) \psi \]

- non-Abelian gauge group SU(3)\text{color}
  - each quark: color triplet
  - covariant derivative: \( D_\mu = \partial_\mu + ig \hat{T}_a A^a_\mu \) (\( a \in \{1, \ldots, 8\} \))
  - field-strength tensor \( F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - gf^{a}_{bc} A^b_\mu A^c_\nu \)
  - group structure constants: \([\hat{T}^a, \hat{T}^b] = if^{a}_{bc} \hat{T}^b \hat{T}^c, \hat{T}^a = (\hat{T}^a)^\dagger \in \mathbb{C}^{3 \times 3}\)

- Particle content:
  - \( \psi \): Quarks with flavor \((u,d;c,s;t,b)\) (mass eigenstates!)
  - \( \hat{M} = \text{diag}(m_u, m_d, m_s, \ldots) \) = current quark masses
  - \( A^a_\mu \): gluons, gauge bosons of SU(3)\text{color}

- Symmetries
  - fundamental building block: local SU(3)\text{color} symmetry
  - in light-quark sector: approximate chiral symmetry (\( \hat{M} \to 0 \))
  - dilatation symmetry (scale invariance for \( \hat{M} \to 0 \))
Features of QCD

- asymptotically free: at large momentum transfers $\alpha_s = 4\pi g_s^2 \to 0$
- running from renormalization group (due to self-interactions of gluons!): Nobel prize 2004 for Gross & Wilczek, Politzer (1973)

![](image.png)

- quarks and gluons confined in hadrons
- theoretically not fully understood (nonperturbative phenomenon!)
- need of effective hadronic models at low energies: (Chiral) symmetry!
Chiral Symmetry of (massless) QCD

- Consider only light $u,d$ quarks
- iso-spin 1/2 doublet: $\psi = \begin{pmatrix} u \\ d \end{pmatrix} = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$
- NB: $\psi$ has three “indices”: Dirac spinor, color, flavor iso-spin!
- $\gamma$ matrices: \( \{ \gamma_\mu, \gamma_5 \} = 2\eta_{\mu\nu} \mathbb{1}, \gamma_5 := i\gamma_0\gamma_1\gamma_2\gamma_3, \gamma_5\gamma_\mu = -\gamma_\mu\gamma_5, \gamma_5^\dagger = \gamma_5, \gamma_5^2 = \mathbb{1} \)
- Diracology of left and right-handed components
  \[
  \psi_L = \frac{\mathbb{1} - \gamma_5}{2} \psi = P_L \psi, \quad \psi_R = \frac{\mathbb{1} + \gamma_5}{2} \psi = P_R \psi,
  \]
  \[
  P_{L/R}^2 = P_{L/R}, \quad P_R P_L = P_L P_R = 0, \quad P_{L/R} \gamma_5 = \gamma_5 P_{L/R} = \mp P_{L/R}
  \]
  \[
  P_{L/R} \gamma_\mu = \gamma_\mu P_{L/R}, \quad \overline{P_L \psi} = \overline{\psi} P_R, \quad \overline{P_R \psi} = \overline{\psi} P_L
  \]
  \[
  \overline{\psi} \gamma_\mu \psi = \overline{\psi}_L \gamma_\mu \psi_L + \overline{\psi}_R \gamma_\mu \psi_R, \quad \overline{\psi} \psi = \overline{\psi}_L \psi_R + \overline{\psi}_R \psi_L
  \]
- $\overline{\psi} := \psi^\dagger \gamma_0, \overline{\gamma_5 \psi} = \psi^\dagger \gamma_5^\dagger \gamma_0 = -\overline{\psi} \gamma_5$
- in the massless limit ($m_u = m_d = 0$)
  \[
  \mathcal{L}_{u,d} = \overline{\psi} i \slashed{D} \psi = \overline{\psi}_L i \slashed{D} \psi_L + \overline{\psi}_R i \slashed{D} \psi_R
  \]
Chiral Symmetry

- in the massless limit \((m_u = m_d = 0)\)
- a lot of global chiral symmetries:
  - change of independent phases for left and right components:
    \[
    \psi_L(x) \rightarrow \exp(i\phi_L)\psi_L(x), \quad \psi_R(x) \rightarrow \exp(i\phi_R)\psi_R(x)
    \]
  - symmetry group \(U(1)_L \times U(1)_R\)
  - independent “iso-spin rotations”
    \[
    \psi_L(x) \rightarrow \exp(i\vec{\alpha}_L \cdot \vec{T})\psi_L(x), \quad \psi_R(x) \rightarrow \exp(i\vec{\alpha}_R \cdot \vec{T})\psi_R(x)
    \]
  - \(\vec{T} = \vec{\tau}/2, \vec{\tau}: \) Pauli matrices; symmetry group \(SU(2)_L \times SU(2)_R\)
  - alternative notation scalar-pseudoscalar phases/iso-spin rotations
    \[
    \psi \rightarrow \exp(i\phi_s)\psi, \quad \psi \rightarrow \exp(i\gamma_5\phi_a)\psi
    \]
    \[
    \psi \rightarrow \exp(i\vec{\alpha}_V \cdot \vec{T})\psi, \quad \psi \rightarrow \exp(i\gamma_5\vec{\alpha}_A \cdot \vec{T})\psi
    \]
- \(U(1)_s\) and \(SU(2)_V\) are subgroups that are symmetries even if \(m_u = m_d \neq 0\) ⇒ Heisenberg’s iso-spin symmetry!
Currents: relation to mesons

- based on [Koc97, Sch03, Din11]
- Noether: each global symmetry leads to a conserved quantity
- from chiral symmetries

\[ j_s^\mu = \overline{\psi} \gamma^\mu \psi, \quad j_a^\mu = \overline{\psi} \gamma^\mu \gamma_5 \psi \]
\[ j_V^\mu = \overline{\psi} \gamma^\mu \gamma_T \psi, \quad j_A^\mu = \overline{\psi} \gamma^\mu \gamma_5 \gamma_T \psi \]

- Link to mesons: Build Lorentz-invariant objects with corresponding quantum numbers
  - \( \sigma \): \( \overline{\psi} \psi \) (scalar and iso-scalar)
  - \( \pi \)'s: \( i \overline{\psi} \gamma_5 \psi \) (pseudoscalar and iso-vector)
  - \( \rho \)'s: \( \overline{\psi} \gamma_\mu \gamma_T \psi \) (vector and iso-vector)
  - \( a_1 \)'s: \( \overline{\psi} \gamma_\mu \gamma_5 \gamma_T \psi \) (axialvector and iso-axialvector)
- in nature: \( \sigma \) and \( \pi \)'s; \( \rho \)'s and \( a_1 \)'s do not have same mass!
- reason: QCD ground state not symmetric under pseudoscalar and pseudovector trafos since \( \langle \text{vac} | \overline{\psi} \psi | \text{vac} \rangle \neq 0 \)
Spontaneous symmetry breaking

- spontaneously broken symmetry: ground state not symmetric
- vacuum necessarily degenerate
- vacuum invariant under scalar and vector transformations: $U(1)_L \times U(1)_R$ broken to $U(1)_S$; $SU(2)_L \times SU(2)_R$ broken to $SU(2)_V$
- for each broken symmetry massless scalar Goldstone boson
- there are three pions which are very light compared to other hadrons (finite masses due to explicit breaking through $m_u, m_d$!)
- but no pseudoscalar isoscalar light particle! ($m_\eta \simeq 548$ MeV)
- reason: $U(1)_a$ anomaly
  - axialscalar symmetry does not survive quantization!
  - good for explanation of correct decay rate for $\pi_0 \rightarrow \gamma\gamma$
  - axialscalar current not conserved $\partial_{\mu} j_{a}^{\mu} = \frac{3}{8} \alpha_s \varepsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma}$
- explicit breaking due to quark masses
  - can be treated perturbatively $\Rightarrow$ chiral perturbation theory
  - axial-vector current only approximately conserved $\Rightarrow$ PCAC
  - a lot of low-energy properties of hadrons derivable
The minimal linear $\sigma$ model

- Chiral symmetry realized by $\text{SO}(4)$: meson fields $\phi \in \mathbb{R}^4$
- Describes $\sigma$ and pions ($\pi^\pm$, $\pi^0$)

$$\mathcal{L}_{\chi_{\text{limit}}} = \frac{1}{2} (\partial_\mu \phi)(\partial^\mu \phi) - V(\phi) = \frac{1}{2} (\partial_\mu \phi)(\partial^\mu \phi) - \frac{\lambda}{4} (\phi^2 - f^2_\pi)^2$$

- Spontaneous symmetry breaking: mexican-hat potential

- Doesn’t cost energy to excite field in direction of the rim
  $\Rightarrow$ massless Nambu-Goldstone bosons (pions)
- Vacuum expectation value $\langle \phi^0 \rangle = f_\pi \neq 0$
- Symmetry spontaneously broken from $\text{SO}(4)$ to $\text{SO}(3)_V$
- Particle spectrum: 4 field-degrees of freedom $\Rightarrow$ vacuum inv. 3-dim $\text{SO}(3)$
  $\Rightarrow$ 3 massless pions $\Rightarrow 4 - 3 = 1$ massive $\sigma$
Explicit symmetry breaking

- explicit $\chi$-symmetry breaking due to $m_{\text{quark}}$: $m_\pi \simeq 140$ MeV
- Gell-Mann-Oakes-Renner relation: $m_\pi^2 f_\pi^2 = -m \langle \bar{q}q \rangle$
- vector (isospin) symmetry only fulfilled for $m_u = m_d$
- in reality: $m_u \simeq 1.7$-$3.3$ MeV, $m_d \simeq 4.1$-$3.3$ MeV
- isospin symmetry as strongly broken as $\chi$ symmetry!
Most accurate experiment related to $\chi_{SB}$

- weak decay $\tau \rightarrow \nu_\tau + n \cdot \pi$
- weak interactions: Quantum-Flavor Dynamics (QFD)
- QFD = Glashow-Salam-Weinberg model (Nobel 1979) + Higgs, Englert (Nobel 2013) et al
- charged currents $\propto j^\mu_V - j^\mu_A$
- $n$ even: must go through vector current
- $n$ odd: must go through axialvector current

Data: ALEPH at LEP
Phenomenology from Chiral Symmetry

- Use (approximate) chiral symmetry to build effective models
- Ward identities
  - PCAC: \( \langle 0 | \partial_{\mu} j_{A \mu}^{k} | \pi^{j}(\vec{k}) \rangle = i F_{\pi}^{2} m_{\pi}^{2} \delta^{kj} \)
  - \( m_{\pi}^{2} F_{\pi}^{2} = -(m_u + m_d) \langle 0 | \bar{u}u | 0 \rangle \) (Gell-Mann-Oakes-Renner relation)
- Spontaneous breaking causes splitting of chiral partners:

  \[ \text{qq-excitations of the QCD vacuum} \]

  \[ \text{P-S, V-A splitting in the physical vacuum} \]

\[ \begin{array}{c}
\pi (140) \\
\rho (770) \\
a_1 (1260) \\
f_1 (1285) \\
f_0 (400-1200) \\
f_1 (1420) \\
\phi (1020) \\
\omega (782) \\
\end{array} \]
Finite Temperature/Density: Idealized theory picture

- partition sum: \( Z(V, T, \mu_q, \Phi) = \text{Tr}\{\exp[-(H[\Phi] - \mu_q N)/T]\} \)

\[ Z[V, T, \mu, \Phi] \]

Dynamical quantities

- off equilibrium: derivation of BUU,...

Thermodyn. potentials
bulk properties
lattice QCD

analytic continuation

\[ T, \mu \rightarrow 0 \]

vacuum

Real Time

Imag. Time

[CSHY85, Lv87, LeB96, KG06]
Finite Temperature

- Asymptotic freedom
  - quark condensate melts at high enough temperatures/densities
- all bulk properties from partition sum:

\[ Z(V, T, \mu_q) = \text{Tr}\{\exp[-(H - \mu_qN)/T]\} \]

- Free energy: \( \Omega = -\frac{T}{V} \ln Z = -P \)
- Quark condensate: \( \langle \overline{\psi} q \psi_q \rangle_{T, \mu_q} = \frac{V}{T} \frac{\partial P}{\partial m_q} \)
- Lattice QCD (at \( \mu_q = 0 \))
  - chiral symmetry \( \Leftrightarrow \langle \overline{\psi} \psi \rangle \)
  - deconfinement transition \( \Leftrightarrow \) Polyakov Loop \( \text{tr}\left\langle P \exp(i \int_0^\beta d\tau A^0) \right\rangle \)
  - Chiral symmetry restoration and deconfinement transition at same \( T_c \)
in the medium: vector-axialvector currents mix
due to thermal pions
possible mechanism for $\chi_{SR}$!
in low-density/temperature approximation: model independent
see [DEI90a, DEI90b, UBW02, SYZ96, SYZ97]
The QCD Phase Diagram

Quark-Gluon Plasma \(<\bar{q}q>=<qq>=0\)

Hadron Gas \(<\bar{q}q>\neq 0\)

Color SupCon \(<qq>\neq 0\)

Heavy-Ion Expts

SIS

AGS

SPS

RHIC

Nuclei

SupCon

Em. Probes in HICs I

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What can we learn from em. probes in heavy-ion collisions?

- only **penetrating probe**
  - leptons and photons leave **hot and dense fireball** unaffected
  - they are produced during the **entire fireball evolution**
  - dileptons provide information on **in-medium spectral properties of hadrons**

- theoretical challenge
  - need an understanding of **QCD medium** at all stages of its evolution
    ⇒ **transport models, hydrodynamics**
  - need to identify **all sources of dileptons and photons**
  - **perturbative QCD** not applicable
    ⇒ **non-perturbative QCD, effective hadronic models**
  - evaluate **dilepton and photon rates** ⇒ **QFT at finite \( T \) and \( \mu_B \)**
- photon and dilepton thermal emission rates given by same electromagnetic-current-correlation function \((J_\mu = \sum_f Q_f \overline{\psi}_f \gamma_\mu \psi_f)\)

- McLerran-Toimela formula

\[
\Pi_{\mu \nu}^{\lesssim}(q) = \int d^4 x \exp(i q \cdot x) \langle J_\mu(0) J_\nu(x) \rangle_T = -2n_B(q_0) \text{Im} \Pi_{\mu \nu}^{\text{ret}}(q)
\]

\[
q_0 \frac{dN_\gamma}{d^4 x d^3 \vec{q}} = -\frac{\alpha_{\text{em}}}{2\pi^2} g^{\mu \nu} \text{Im} \Pi_{\mu \nu}^{\text{ret}}(q, u) \bigg|_{q_0=|\vec{q}|} f_B(p \cdot u)
\]

\[
\frac{dN_{e^+ e^-}}{d^4 x d^4 k} = -g^{\mu \nu} \frac{\alpha^2}{3q^2 \pi^3} \text{Im} \Pi_{\mu \nu}^{\text{ret}}(q, u) \bigg|_{q^2=M_{e^+ e^-}^2} f_B(p \cdot u)
\]

- manifestly Lorentz covariant (dependent on four-velocity of fluid cell, \(u\))

- to lowest order in \(\alpha\): \(4\pi \alpha \Pi_{\mu \nu} \simeq \Sigma^{(\gamma)}_{\mu \nu}\)

- derivable from underlying thermodynamic potential, \(\Omega\)!
Vector Mesons and chiral symmetry

- **vector** and axial-vector mesons ↔ respective current correlators
  \[
  \Pi^\mu_\nu_{V/A}(p) := \int d^4x \exp(ipx) \left\langle J^\nu_{V/A}(0) J^\mu_{V/A}(x) \right\rangle_{\text{ret}}
  \]

- Ward-Takahashi Identities of \( \chi \) symmetry \( \Rightarrow \) Weinberg-sum rules
  \[
  f_\pi^2 = -\int_0^\infty \frac{dp_0^2}{\pi p_0^2} \left[ \text{Im} \Pi_V(p_0, 0) - \text{Im} \Pi_A(p_0, 0) \right]
  \]

- spectral functions of vector (e.g. \( \rho \)) and axial vector (e.g. \( a_1 \)) directly related to order parameter of chiral symmetry!
at high enough temperatures and or densities: melting of $\langle \bar{q}q \rangle$

$\Rightarrow$ spontaneous breaking of chiral symmetry suspended

$\Rightarrow$ chiral phase transition; chiral-symmetry restoration ($\chi_{SR}$)

which scenario is right? microscopic mechanisms behind $\chi_{SR}$?
http://dx.doi.org/10.1016/0370-1573(85)90136-X

http://dx.doi.org/10.1016/0370-2693(90)90138-V

http://dx.doi.org/10.1016/0370-2693(83)91595-2

http://scipp.ucsc.edu/~dine/ph222/goldstone_lecture.pdf


http://arxiv.org/abs/nucl-th/0409054


http://dx.doi.org/10.1016/0370-2693(96)00802-7

http://link.aps.org/abstract/PRD/v56/p05605
http://dx.doi.org/10.1103/PhysRevLett.88.042002