The Confinement Problem: Critical discussion of Tomboulis's approach

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work done in collaboration with

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(arXiv:0711.4930 [hep-th], arXiv:0803.3019 [hep-th])

Christian Lang:

and many more years of productive work!

Overview

1. What is Confinement?
2. Simplifications
3. String Tensions
4. Electric and Magnetic Fluxes
5. From Lattice to Continuum
6. General Strategy

7. Migdal-Kadanoff Aproximate RG
8. Tomboulis's Idea
9. Tomboulis's Decimation (MKT RG)
10. Tomboulis's Approach: More Details
11. Tomboulis's Approach: Holes
12. Chances to Fix the Holes

1. What is quark confinement?

Empirically no particles with e/3 electric charge exist.

Confinement in QCD: hot issue end of 70's, early 80's.

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Possible formulations:

(1) "There are no colored states."

Not appropriate:

- what is color empirically?
- 'screening' of color.

Complementarity Higgs - Confinement

Electroweak theory: Higgs mechanism as confinement

('t Hooft 1979, Fröhlich, Morchio Strocchi 1980)

(2) “Scattering of hadrons produces only hadrons”

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(3) “QCD has only superselections sectors with integer electric charge”

Simple Z_2 Higgs model: deconfinement transition characterized by appearance of new sectors (Fredenhagen and Marcu 1983)

Still too hard in QCD.

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Idea: Probe the glue!

Compromise between requirements:

- physically sensible
- tractable

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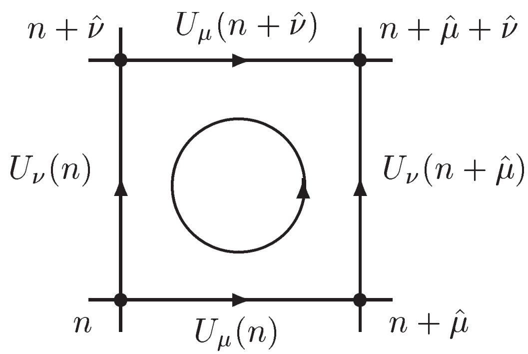
Glue forming flux tube $\longrightarrow$ linearly rising potential.

To create flux tube: either

- heavy external sources (Wilson 1974, Polyakov 1978)
- nontrivial topology (e.g. torus) ('t Hooft 1979)

Lattice Yang-Mills theory:



$$U \in SU(N)$$

$$U_{\mu\nu} = U_\mu(n) U_\nu(n+\hat{\mu}) U_\mu(n+\hat{\nu})^\dagger U_\nu(n)^\dagger$$

Wilson's action:

$$S[U] = \frac{2}{g^2} \sum_{n\in\Lambda} \sum_{\mu<\nu} \mathrm{Re}\,\mathrm{tr}[\mathbb{1} - U_{\mu\nu}(n)]$$

Expectation values:

$$\langle A \rangle = \frac{1}{Z} \int d[U] A([U]) e^{-S}$$

Reflection positivity

Lattice time symmetric, $t = 0$ on or between lattice lines.

ϑ time reflection + complex conjugation.

$A_+[U]$ depends only on U times ≥ 0, $\vartheta A_+[U] = A_+[\vartheta U]$ only on times ≤ 0.

Fact:

$$\boxed{\langle A_+ \vartheta A_+ \rangle \geq 0}$$

$\approx$ equivalent to

$$\exists \textit{ positive transfer matrix } \mathcal{T} = e^{-H}$$

on $\mathcal{H} = L_2([U])$

Explicitly

Temporal gauge: $U_4(n) = \mathbb{1} \forall n$

$$\mathcal{T}([U]_{\rm sp}, [V]_{\rm sp}) = \exp\{(S_{\rm sp}[U] + S_{\rm sp}[V] + S_{\rm t}[U,V])\}$$

where

$$S_{\rm sp}[U] = \frac{1}{g^2} \sum_{n\in\Lambda_{\rm sp}} \sum_{1\le\mu<\nu\le 3} {\rm Re}\,{\rm tr}[\mathbb{1} - U_{\mu\nu}(n)]$$

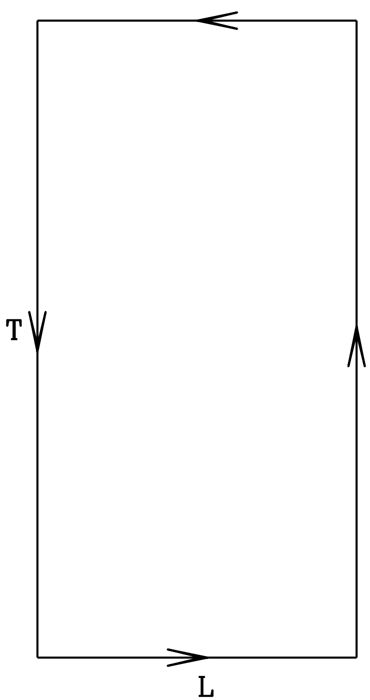
and

$$S_{\rm t}[U,V] = \frac{2}{g^2} \sum_{n\in\Lambda_{\rm sp}} \sum_{1\le\mu\le 3} {\rm Re}\,{\rm tr}[\mathbb{1} - U_\mu(n) V_\mu(n)^\dagger]$$

3. Three string tensions

(1) Wilson 1974:

Minimal energy in a $q\bar{q}$ state $\psi_{q\bar{q}}(L)$ with thin flux line



$$W(C_{LT}) = \operatorname{tr} \prod_{(n,\mu)\in C_{LT}} U_\mu(n)$$

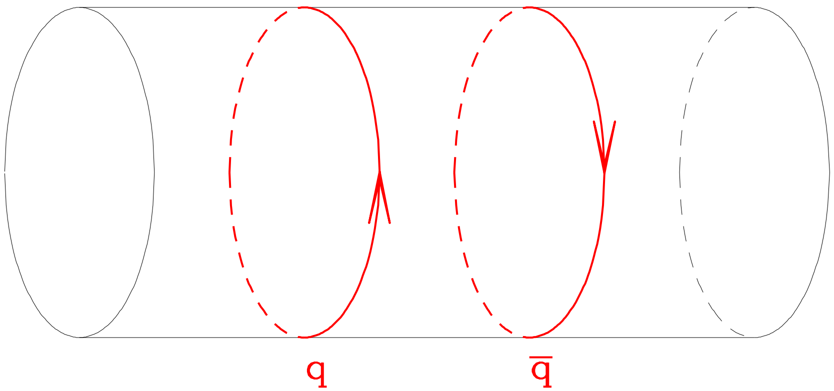
$$= \left(\psi_{q\bar{q}}(L), e^{-TH}\psi_{q\bar{q}}(L)\right)$$

$$\sigma_W = -\lim_{L\to\infty}\lim_{T\to\infty}\frac{1}{LT}\log\langle W(C_{LT})\rangle$$

(2) Polyakov 1978:

Free energy per distance of heavy $q\bar{q}$ pair

Periodic b.c. in time (size β)



$$P(n) = \mathrm{tr} \prod_{t=0}^{\beta-1} U_4(n+t)$$

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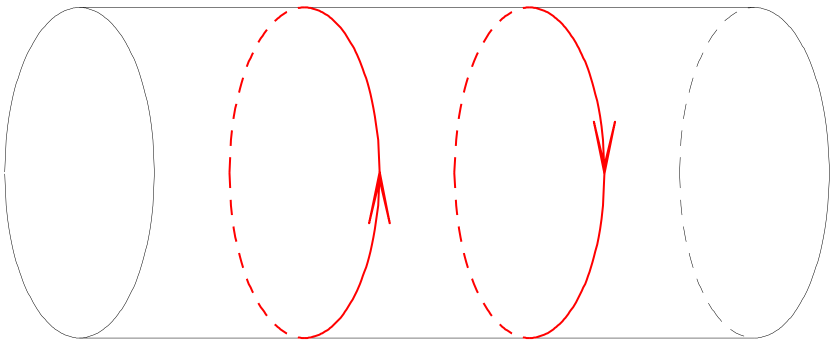

$$P(n) = \mathrm{tr} \prod_{t=0}^{\beta-1} U_4(n+t)$$

$$\langle P(0) P(x)^* \rangle = \frac{\mathrm{tr}\, Q_{q(0)\bar{q}(x)} e^{-\beta H}}{\mathrm{tr}\, Q_0 e^{-\beta H}}$$

$Q_{q(0)\bar{q}(x)}$ projection on subspace with heavy quarks,
Q_0 projection on subspace with no heavy quarks.

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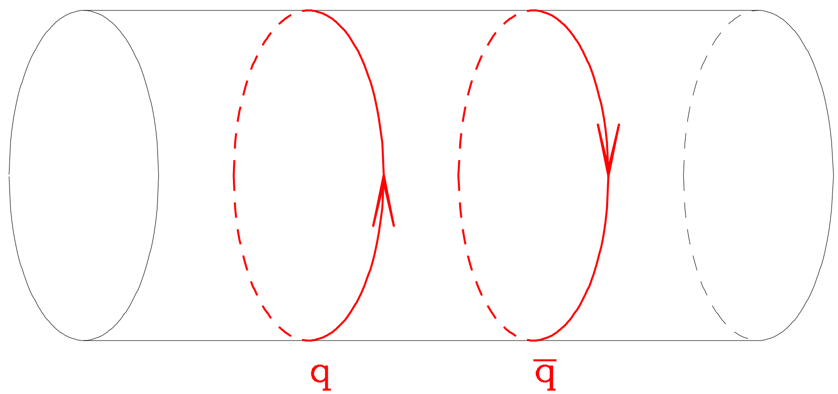

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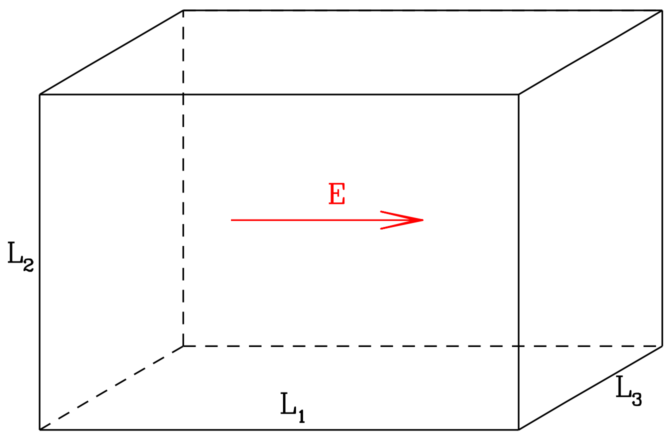
$$\sigma_P(\beta) = - \lim_{|x| \to \infty} \frac{1}{\beta |x|} \log \langle P(0)P(x)^* \rangle$$

(3)'t Hooft 1979:

Free energy/length of (central) color electric flux in torus



$$\omega \in C(SU(N)) \cong Z_N$$

$$E_k(n)U_k(n) = \omega U_k(n)$$

$$E_k(n)U_{k'}(n') = U_{k'}(n')$$

$$\text{for } (n,k) \neq (n',k')$$

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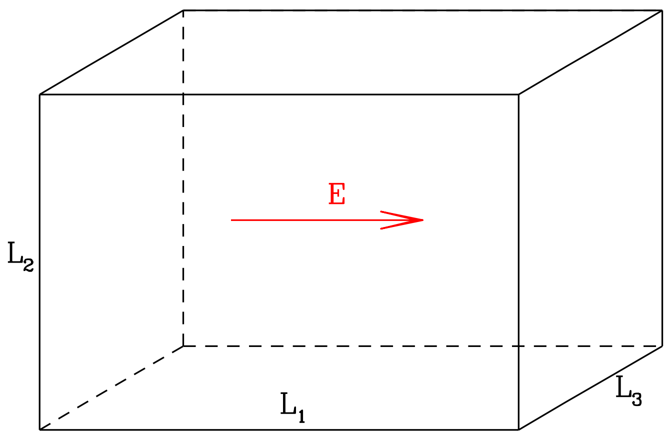

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Flux $\Phi_1$in 1 direction $\prod_{n_2,n_3=1}^{L} E_1(n_1\hat{1} + n_2\hat{2} + n_3\hat{3})$

Independent of n_3, spectrum in $\widehat{C}(SU(N)) \cong Z_N$.

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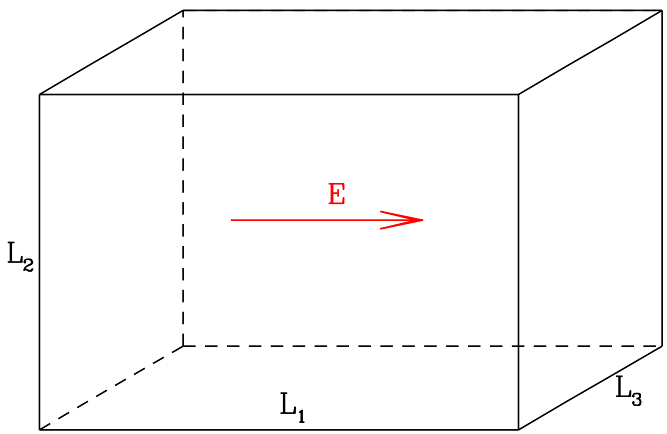

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$$\sigma_{tH} = -\lim_{L_2,L_3\to\infty} \lim_{L_1\to\infty} \frac{1}{\beta L_1}(\log \operatorname{tr} Q_\omega e^{-\beta H} - \log \operatorname{tr} Q_0 e^{-\beta H})$$

Q_ω spectral projector for $\omega \in Z_N$

How are string tensions related?

$$\sigma_W \geq \sigma_P\big|_{\beta=\infty} \geq \sigma_{tH}\big|_{\beta=\infty}$$

first inequality trivial, second inequality uses RP in space direction (Borgs&Seiler 1983). More generally

$$\sigma_P(\beta) \geq \sigma_{tH}(\beta)$$

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Tomboulis&Yaffe 1985, Kovacs&Tomboulis 2001:

$$\log\langle W(C_{LT})\rangle \leq \frac{LT}{\beta L_1} \log \frac{\mathrm{tr}\, Q_\omega \exp(-\beta H)}{\mathrm{tr} Q_0 \exp(-\beta H)}$$

(Wilson loop in (23) plane, torus $L_1 \times L_2 \times L_3 \times \beta$).

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Confinement in 't Hooft's sense sufficient!

Conjecture: $\sigma_W = \sigma_P\Big|_{\beta=\infty} = \sigma_{tH}\Big|_{\beta=\infty}$.

4. *Electric and magnetic fluxes*

’t Hooft 1979: Torus $L_1 \times L_2 \times L_3 \times \beta$

$$F_\omega^{\mathrm{el}} \equiv -\frac{1}{\beta} \log \frac{\mathrm{tr}\, Q_\omega \exp(-\beta H)}{\mathrm{tr} Q_0 \exp(-\beta H)}$$

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Central magnetic flux ω in direction 1:

$$F_\omega^{\mathrm{mag}} \equiv -\frac{1}{\beta} \log \frac{Z_\omega}{Z_1}$$

Z_ω: $U_{23}(n) \mapsto \omega U_{23}(n)$ for fixed n_2, n_3 ('vortex sheet').

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Fourier transform on $C(SU(N)) = Z_N$:

$$\widehat{Z}_\omega \equiv \tfrac{1}{N} \sum_{\omega' \in Z_N} \tfrac{\omega}{\omega'} Z_{\omega'} = {\rm tr}\, Q_\omega \exp(-\beta H)$$

Electric-magnetic duality

$\beta \to L_4$

Spreading of magnetic flux $\iff$ electric confinement.

$$\frac{Z_\omega}{Z_1} \geq \exp\left(-cL_2L_3e^{-\alpha L_1 L_4}\right) \iff \frac{\widehat{Z}_\omega}{\widehat{Z}_1} \leq c' L_2 L_3 e^{-\alpha L_1 L_4}$$

$$\sigma_{tH} \geq \alpha.$$

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(always L_2, L_3 transverse directions, $L_1 L_4 \gg \log(L_2 L_3)$.)

Magnetic and electric confinement cannot coexist

Third possibility

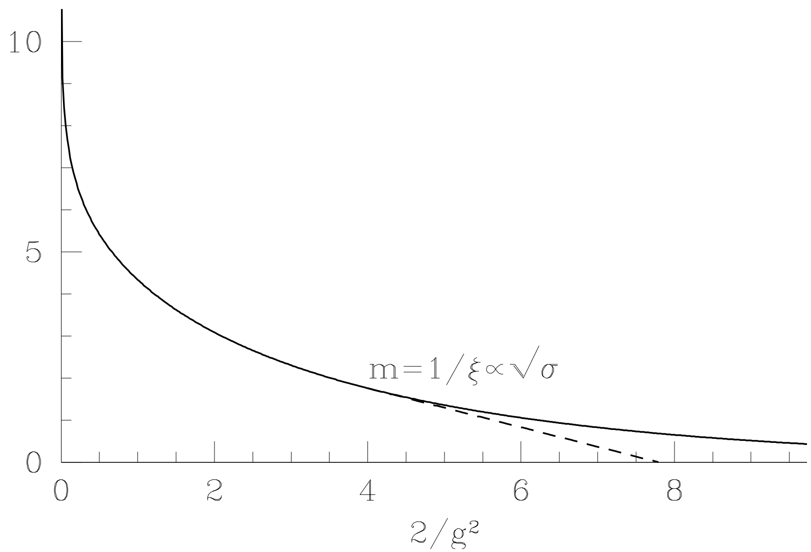
Coulomb phase:

$$\frac{Z_\omega}{Z_1} \sim \exp\left(-\frac{cL_2L_3}{L_1L_4}\right)$$

$$\frac{\widehat{Z}_\omega}{\widehat{Z}_1} \sim \exp\left(-\frac{cL_2L_3}{L_1L_4}\right)$$

This happens for $U(1)$ in 4D (Guth 1980, Fröhlich and Spencer 1982)

5. From Lattice to Continuum



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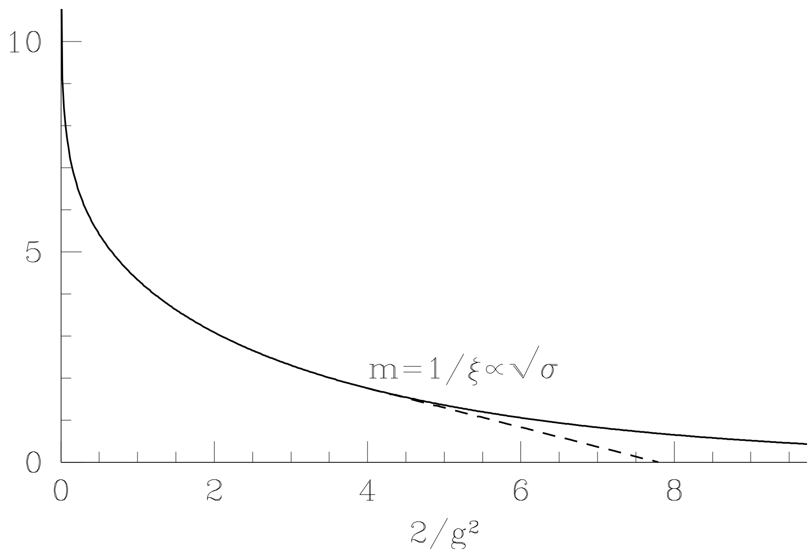

Procedure:

- Take dynamically generated scale ξ as unit
- Rescale observables accordingly
- Drive system to critical point $\xi \to \infty$

Belief: Only one scale

$$\lim_{\xi \to \infty} \frac{1}{\xi\sqrt{\sigma}} \neq 0, \infty$$

Continuum (if it exists): massive, confining.

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Millennium problem (Clay Institute):

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Real problem: Coexistence of asymptotic freedom and confinement.

More modest (Tomboulis): $\sigma > 0 \, \forall g^2 > 0$

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What if?

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Confining, massive continuum limit would probably exist.

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Confining, massive continuum limit would probably exist.

Presumably not asymptotically free, because not

$$g^2 \to 0 \quad \text{for} \quad \xi \to \infty \, .$$

6. General Strategy

Proof (or disproof?) of more modest problem

$$\sigma > 0 \quad \forall \beta < \infty$$

probably requires Rigorous Renormalization Group.

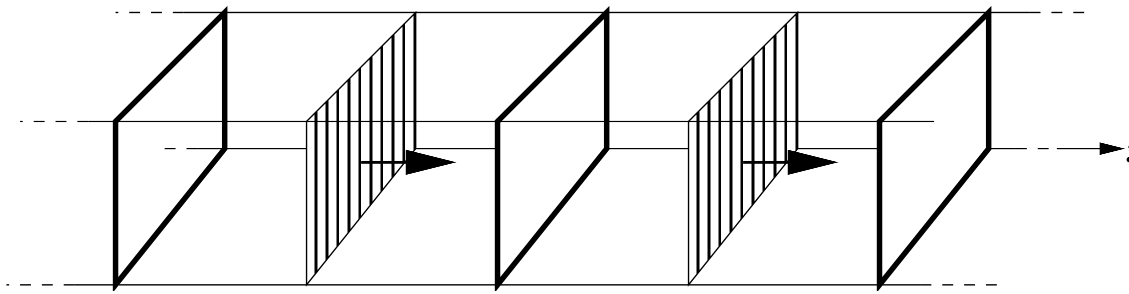
- Integrate out short distance fluctuations step by step
- Follow evolution of relevant ‘operators’s, control errors
- Do this for twisted and untwisted partition function

Note: Involves *nonlocal* actions

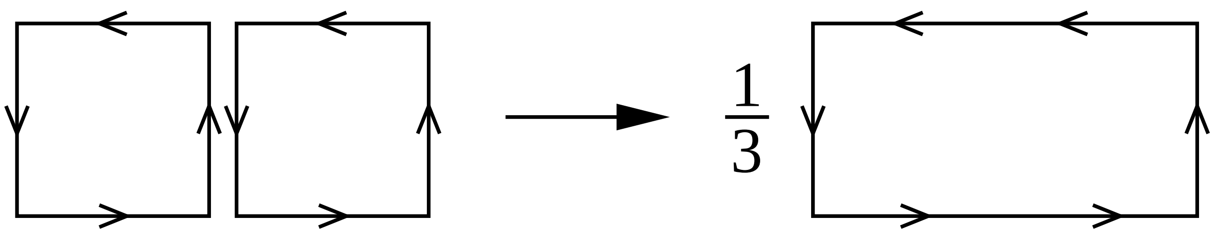
Balaban 1983-1989: shows the way (maybe).

7. Migdal-Kadanoff RG

Idea: ‘potential moving’ in two directions.



Then integrate out:



$(N = 3)$

Plaquette coupling $\rightarrow$ new plaquette coupling

Analytically:

Character expansion of plaquette coupling

$$f(U) \equiv \exp A_p(U) = F_0 \left[1 + \sum_{j \neq 0} (2j+1) c_j(\beta) \chi_j(U) \right] .$$

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MKRG (decimation): alternate raising coupling function to power $b^2 = 4$ and normalized Fourier coefficients c_j to power $b^2 = 4$.

Concretely

$$f^{(n)}(U) \mapsto \frac{f^{(n)}(U)^4}{\int f^{(n)}(U)^4 dU} \equiv g^{(n)}(U)$$

$$g^{(n)}(U) = 1 + \sum_{j>0} (2j+1) c_j(n) \chi_j(U)$$

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In MKRG:

no structural difference between abelian/non-abelian models.

8. Tomboulis's Charming Idea (1983)

$SU(2)$ Yang-Mills on lattice, want to show spreading of central magnetic flux in $L_1 \times L_2 \times L_3 \times L_4$ torus:

$$\frac{Z_{-1}}{Z_1} \geq \exp\left[-cL_2L_3e^{-\alpha L_1 L_4}\right] \quad \text{for} \quad L_1L_4 \gg \log(L_2L_3)$$

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where Z_{-1} has twisted b.c. in the (23)-direction.
This would follow from

$$\frac{Z_{-1}}{Z_1} \geq \frac{Z_{-1}^{MKT}(n)}{Z_1^{MKT}(n)}$$

$Z_1^{MKT}(n)$, $Z_{-1}^{MKT}(n)$ partition functions under n-fold iteration of the (modified) Migdal-Kadanoff decimation.

Assume: Inequality holds and MKT iteration eventually leads to strong coupling regime $\Longrightarrow$

- spreading of magnetic flux
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Question: True for MKT RG in $D = 4$? Depends on a parameter $r = 1 - \epsilon$.

9. Tomboulis's Decimation

Modify MKRG by introducing parameter r:

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MKT decimation: alternate raising coupling function to power $b^2 = 4$ and Fourier coefficients c_j to power $b^2 r = 4r$.

Concretely

$$f^{(n)}(U) \mapsto \frac{f^{(n)}(U)^4}{\int f^{(n)}(U)^4 dU} \equiv g^{(n)}(U)$$

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Note: Introducing $r < 1$ has same effect as increasing dimension (deconfining transition possible – see below).

10. A Few Details:

Decimated lattices $\Lambda^{(n)}$

$$Z_\omega(\Lambda^{(n)}, \{c_j(n)\}) \equiv \int d[U] \prod_p \left[\sum_j (2j+1) c_j(n) \chi_j(\omega_p U_p) \right]$$

($\omega_p = -1$ on 'vortex sheet', $\omega = 1$ else.).

$$F_0(n) \equiv \int dU [f^{(n-1)}(U)]^4 \,.$$

$$f^{(n)}(U) = 1 + \sum_{j>0} (2j+1) c_j(n)^4 \chi_j(U) \,.$$

Note: $c_j(n) \geq 0 \;\forall\; j, n$ ('weak reflection positivity').

Upper and Lower Bounds

$$Z_1(\Lambda^{(n)}, \{c_j(n)\}) \leq Z_1(\Lambda^{(n-1)}, \{c_j(n-1)\})$$

$$Z_1(\Lambda^{(n-1)}, \{c_j^L(n-1)\}) \leq F_0(n)^{|\Lambda^{(n)}|} Z_1(\Lambda^{(n)}, \{c_j(n)\})$$

where $c_j^L(n-1) \equiv 0$ for $j > 0$ and $c_0^L(n-1) \equiv c_0(n-1)$.

Reason: Weak Reflection Positivity (RP)

Analogous bounds for

$$Z^+(\Lambda^{(n)}, \{c_j(n)\}) \equiv Z_1(\Lambda^{(n)}, \{c_j(n)\}) + Z_{-1}(\Lambda^{(n)}, \{c_j(n)\})$$

(funny combination chosen to have RP).

Interpolation

$$Z_1(\Lambda) = \left[\prod_{m=1}^{n} \tilde{F}_0(m, t, \alpha(m))^{|\Lambda^{(m)}|} \right] Z_1(\Lambda^{(n)}, \{\alpha c_j(n)\}) \quad (*)\,,$$

$$\tilde{F}_0(m, t_m, \alpha) \equiv F_0(\{c_j(n)\})^{h_t(\alpha)}\,,$$

where

$$h_t(\alpha) = \exp\left[-\tfrac{t(1-\alpha)}{\alpha}\right]$$

Given $t_m > 0$, $\alpha(t_m) \equiv \alpha_m$ chosen s. t. $(*)$ true $\forall m$.
Same procedure for

$$Z^+(\Lambda) \equiv Z_1(\Lambda) + Z_{-1}(\Lambda) \quad \longrightarrow \alpha^+(t_m)$$

Choose (see next slide) at each step t_m, t_m^+ such that

$$h(\alpha(t_m), t_m) = h(\alpha^+(t_m^+), t_m^+)$$

Then fairly easy to see:

$$\exists\, 0 < \alpha(n), \alpha^+(n) \quad \text{such that}$$

$$\frac{Z^+(\Lambda, \{c_j(0)\})}{Z_1(\Lambda, \{c_j(0)\})} = \frac{Z^+(\Lambda^{(n)}, \{\alpha^+(n) c_j(n)\})}{Z_1(\Lambda^{(n)}, \{\alpha(n) c_j(n)\})}$$

More on Interpolation

$$|\alpha^+(n) - \alpha(n)| = O\left(\frac{1}{|\Lambda^{(n)}|}\right)$$

T. insists on keeping $\alpha < 1 - \delta\,, \alpha^+ < 1 - \delta^+$ uniformly.
Necessitates $r < 1$.

Needed to ensure

$$\frac{\partial \alpha^{(m)}}{\partial t} \geq \eta_1(\delta) > 0, \quad -\frac{dh(\alpha^{(m)},t)}{dt} \geq \eta_2(\delta) > 0$$

as well as

$$\frac{\partial \alpha^{+(m)}}{\partial t} \geq \eta_1^+(\delta^+) > 0, \quad -\frac{dh(\alpha^{+(m)},t)}{dt} \geq \eta_2^+(\delta^+) > 0$$

Bounds (uniform in n, volume) allow to solve

$$h(\alpha(t_m), t_m) = h(\alpha^+(t_m^+), t_m^+)$$

Crucial Claim

Existence of common interpolation parameter α^* for both twisted and untwisted partition functions?

Crucial point in T.'s argument:

$$\exists\, 0 < \alpha^*(n) \leq 1 - \delta \quad \text{such that}$$

$$\frac{Z_{-1}(\Lambda, \{c_j(0)\})}{Z_1(\Lambda, \{c_j(0)\})} = \frac{Z_{-1}(\Lambda^{(n)}, \{\alpha^*(n) c_j(n)\})}{Z_1(\Lambda^{(n)}, \{\alpha^*(n) c_j(n)\})}$$

This would suffice to prove $\sigma_{tH} > 0\ \forall\beta$.

11. Problems

(1) The parameter r*:*

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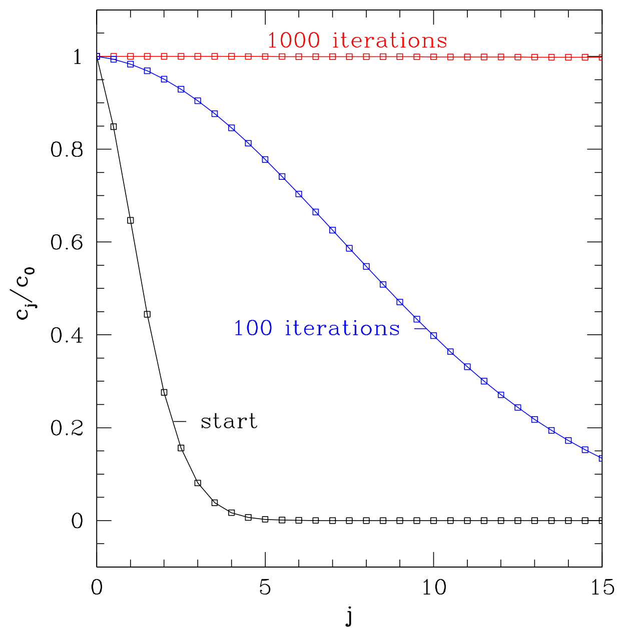
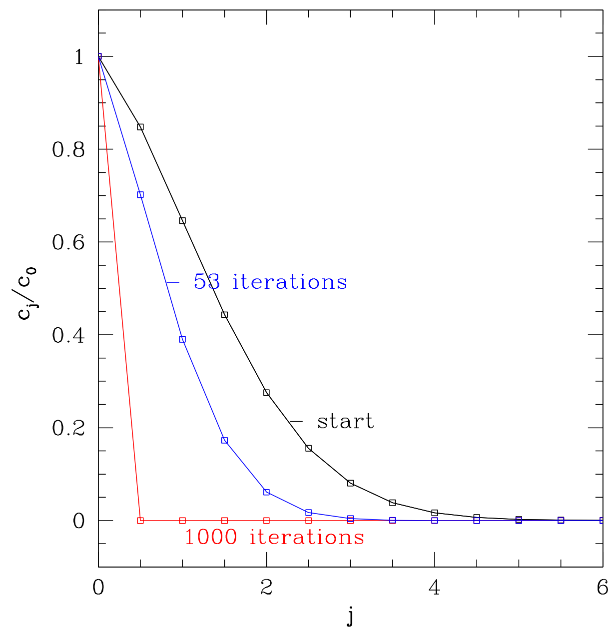


$\beta = 4.80$



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Evolution of c_j/c_0 under MKT RG with $r = 0.9$.

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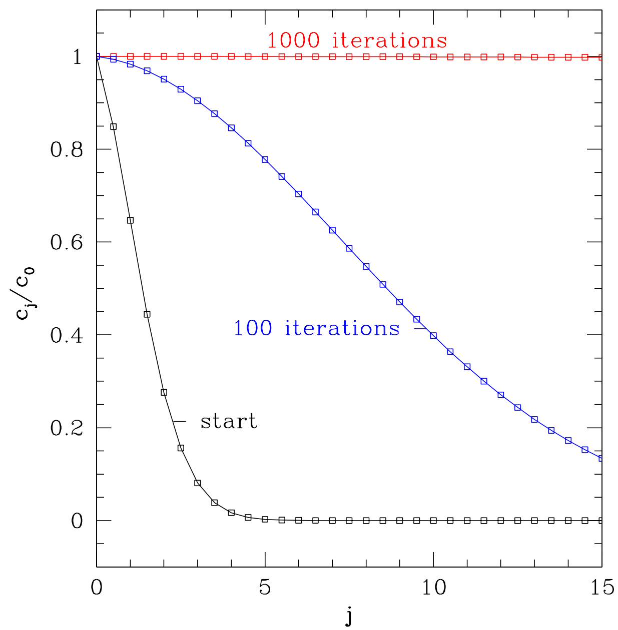

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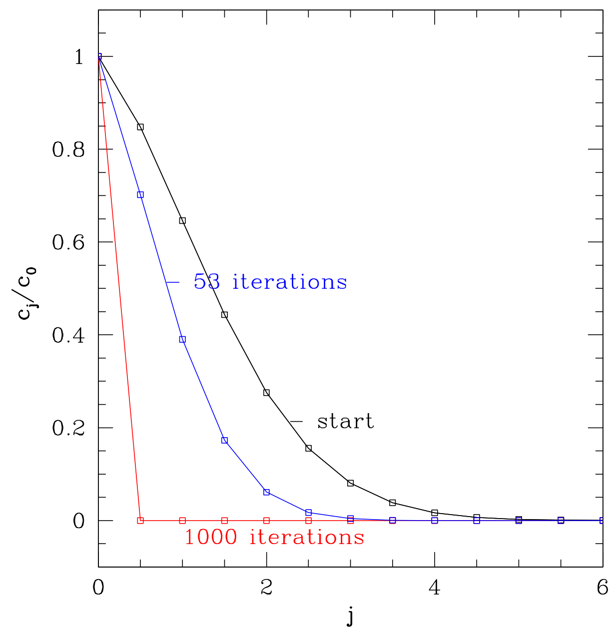

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Evolution of c_j/c_0 under MKT RG with $r = 0.9$.

In MKTRG:

no structural difference between abelian/nonabelian models

(2) Existence of common interpolation parameter α^* for both twisted and untwisted partition functions?

Crucial point in T.'s argument:

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$$\frac{Z_{-1}(\Lambda,\{c_j\})}{Z_1(\Lambda,\{c_j\})} = \frac{Z_{-1}(\Lambda^{(n)},\{\alpha^*(n)c_j(n)\})}{Z_1(\Lambda^{(n)},\{\alpha^*(n)c_j(n)\})}$$

Argument based on implicit function theorem:

Flawed.

T. introduces function $\Psi(\lambda, t)$ in terms of interpolated partition functions and requires

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Counterexample (thesis of Takuya Kanazawa):

$$\Psi(\lambda, t) \equiv e^{-t} - 1 + 2\lambda$$

$t(\lambda) = -\log(1 - 2\lambda)$ singular at $t = 1/2$. More is needed.

In Detail

(Appendix C)

Assume $\exists t = t_I$ s.t. $\alpha(t_I) > \alpha^+(t_I)$. Then t_0 s.t.

$$h(\alpha^+(t_I, t_I) = h(\alpha(t_0), t_0)$$

exists t derivatives bounded away from 0.

$$\Phi_n^+(\alpha) \equiv \frac{1}{|\Lambda^{(n)}| \log F_0(n)} \log Z^+ \left(\alpha\{c_j(n)\}\right)$$

$$\Psi(\lambda, t) \equiv h(\alpha(t), t) + (1-\lambda)\Phi_n^+(\alpha^+(t_I)) + \lambda\Phi_n^+(\alpha(t)) - \Phi_{n-1}^+(\alpha(t_I))$$

$\Psi(\lambda, t) = 0$ true for $\lambda = 0, t = t_0$ by definition of α^+.

$\Psi(1, t) = 0$ gives desired common $\alpha = \alpha^+$.

Existence?

(3) The Fundamental Issue

Fact: In $D = 4$ MKRG as well as MKTRG show no structural difference between

- $SU(N)$ and $U(N)$
- abelian or nonabelian gauge groups

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- $SU(N)$ and $U(N)$
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Consequence: Comparison argument with MK RG has to fail for $U(1)$.

Any such argument that does not explicitly make use of nonabelian (semisimple) nature has to fail.

12. Chances to fix the holes?

- Choice of r very subtle. Have to ensure interpolation parameters α^* bounded away from 1 and convergence of MKT to strong coupling FP.
(r dependent on n as suggested verbally by T.?)

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