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Infrared Behavior of Three-Point Functions in Landau Gauge Yang-Mills Theory

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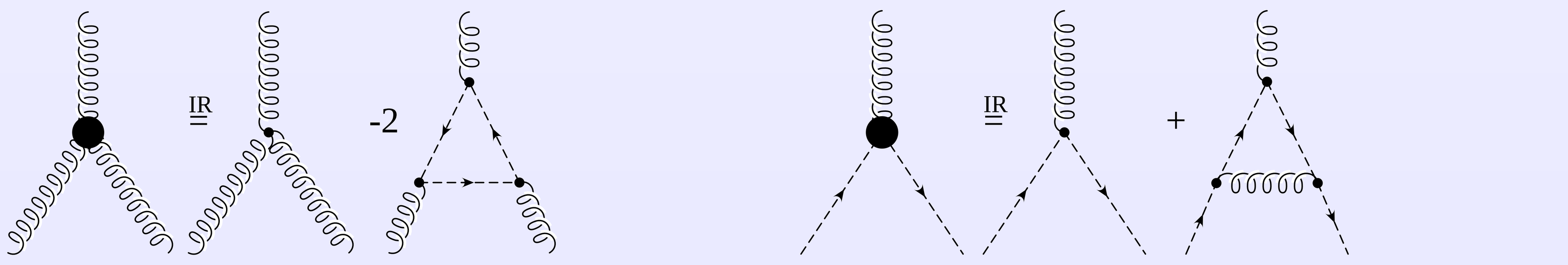
Infrared Behavior of Green Functions

The non-perturbative aspects of Yang-Mills theory are encoded in the infrared (IR) behavior of its Green functions. For very small momenta below the intrinsic scale Λ_{QCD} they can be described by power laws. This scaling solution [1] is in accordance with the scenarios for confinement of Gribov-Zwanziger and Kugo-Ojima [2]. The general expression for the so-called IR exponent of an arbitrary Green function with m gluon and $2n$ ghost legs under the assumption that all momenta go to zero uniformly is [3]

$$\delta_{2n,m} = (n-m)\kappa + (1-n)\left(\frac{d}{2}-2\right).$$

This formula is valid in $d = 2, 3, 4$ dimensions for the corresponding values of κ. As it is clear that we can have more than one independent momentum for vertex functions, the question arises what happens when only one of these goes to zero? Interestingly it turns out that additional divergences can occur that do not change the uniform solution [4] (see also the talk by R. Alkofer).

For the calculation of the three-point vertices we used the known power laws for the propagators and the following truncations for the Dyson-Schwinger equations (DSEs), which is motivated by ghost dominance in the IR:



For the dressed ghost-gluon vertex we used the bare one, justified by a simple argument of Taylor [5], which is supported by lattice simulations and the DSE solution [6]. The truncation considers only the first order of the skeleton expansion.

Three-Point Integrals

The ghost-triangle diagram of the three-gluon vertex is decomposed in the IR as

$$\Gamma^{gh-\Delta}_{\mu\nu\rho}(p_1,p_2,p_3) = \sum_{i=1}^{10} E_i(p_1,p_2,p_3)\tau^i_{\mu\nu\rho}(p_1,p_2,p_3).$$

The applied tensor decomposition [7] reveals that only ten instead of the expected fourteen scalar functions E_i and tensors $\tau^i_{\mu\nu\rho}$ are necessary. The former consist of massless three-point integrals,

$$\int \frac{d^dq}{(2\pi)^d} \frac{1}{((q+p_1)^2)^{\nu_1}((q-p_2)^2)^{\nu_2}(q^2)^{\nu_3}},$$

where ν_1, ν_2 and ν_3 are non-integer numbers. We employed the Negative Dimensions Integration Method (NDIM) which yields a full analytic solution in terms of Appell's series F_4. Using several different analytic continuations we can calculate the momentum dependence of the diagram. A further advantage of NDIM is that the results are valid for arbitrary values of d. As the variables of the Appell's series are p_1^2/p_3^2 and p_2^2/p_3^2 we plot the scalars E_i from the tensor decomposition as functions of p_1^2 and p_2^2 with p_3^2 fixed. There are some distinct kinematic configurations which are indicated in fig. 1. We plot the Euclidean region, which is determined by momentum conservation and given by the red area in fig. 1. The ghost-gluon vertex can be decomposed similarly into two parts.

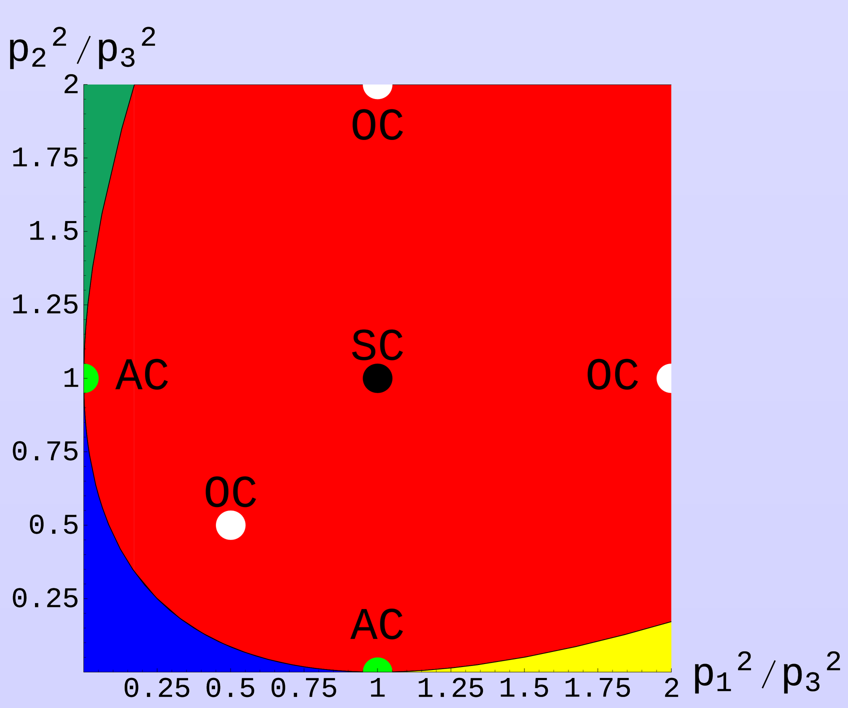


Fig. 1:
Symmetric configuration (SC): all momenta squared equal.
Orthogonal configuration (OC): two momenta squared equal, orthogonal to each other.
Asymmetric configuration (AC): two momenta squared equal, third momentum orthogonal.

Three-Gluon Vertex

The complete kinematic dependence for each single scalar function E_i can be calculated. Examples for a part of the Euclidean region are shown in fig. 2.

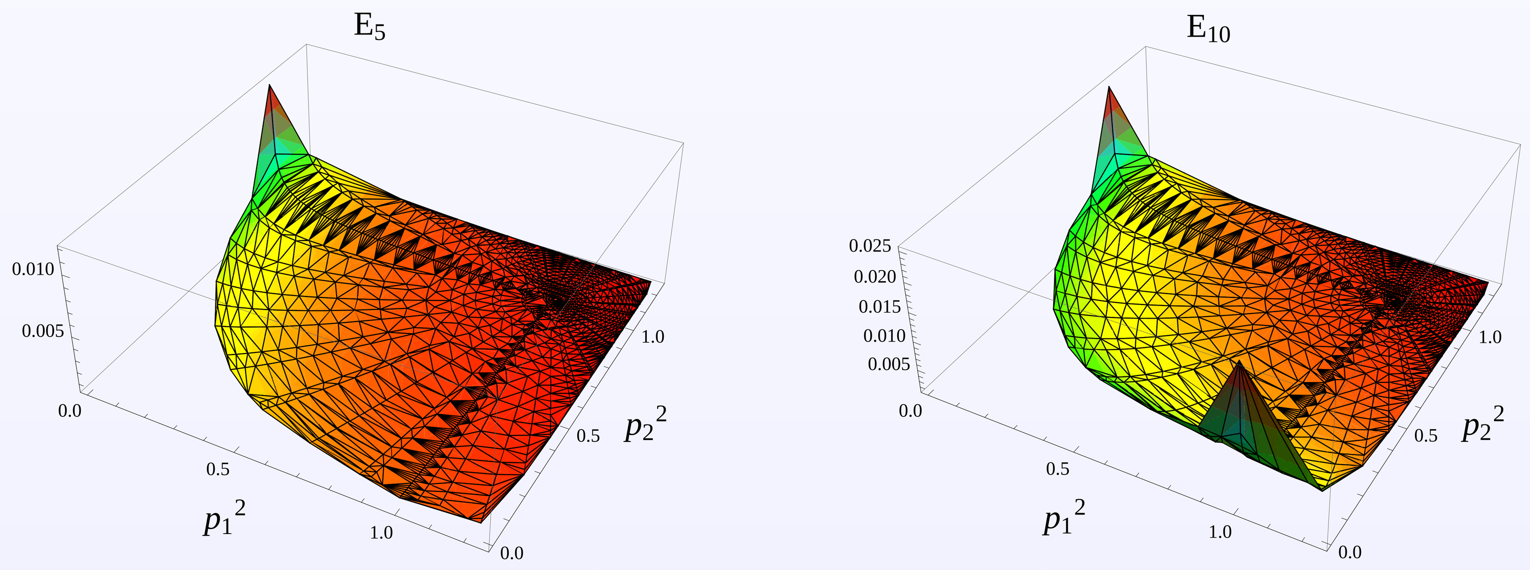


Fig. 2: *The two scalar functions E_5 and E_{10} of the three-gluon vertex with $p_3^2 = 1$. The kinematic singularities at the asymmetric points clearly dominate the structure.*

The behavior when one of the three momenta gets small compared to the others, i.e. the region around the asymmetric points $(0,1)$, $(1,0)$ and (∞,∞), is of special interest. This case corresponds to the emission/absorption of a soft gluon. We calculated the scalar functions along the lines from the symmetric point to the asymmetric ones. Near the endpoints we extracted the exponents in two, three and four dimensions and indeed found kinematic singularities, which are in agreement with the power counting analysis in ref. [4].

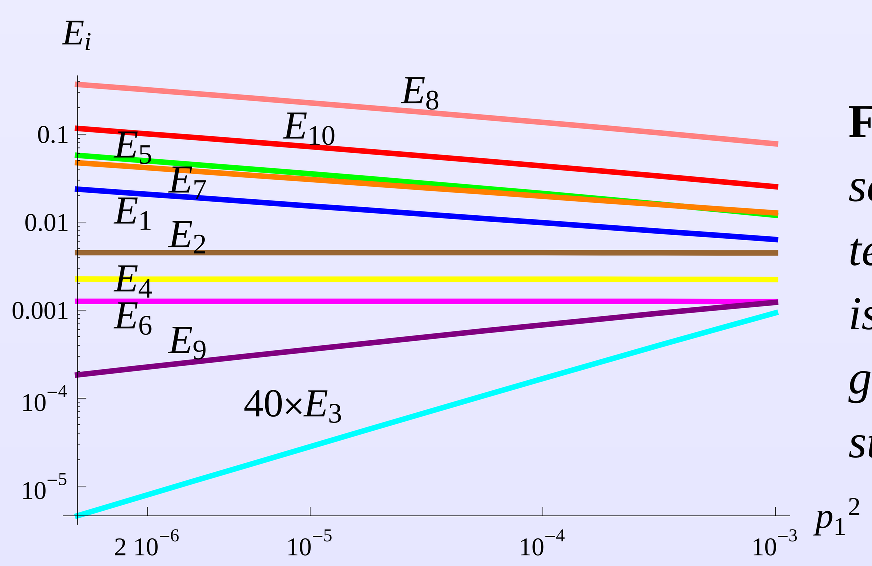


Fig. 3: *A double-logarithmic plot of the scalar functions E_i of the three-gluon vertex when p_1^2 becomes small and $p_3^2 = p_2^2$ is held fixed. Three cases can be distinguished: Five scalar functions diverge, three stay constant, two vanish.*

Tensor	p_1 soft
$\tau_1 = p_{1_\mu}p_{1_\nu}p_{1_\rho}/p_1^2$	$\mathbf{1-2\kappa}+\frac{d-4}{2}$
$\tau_2 = p_{2_\mu}p_{2_\nu}p_{2_\rho}/p_2^2$	0
$\tau_3 = (p_{1_\mu}p_{1_\nu}p_{2_\rho}+p_{1_\mu}p_{2_\nu}p_{1_\rho}+p_{2_\mu}p_{1_\nu}p_{1_\rho})/p_1^2$	$2-2\kappa+\frac{d-4}{2}$
$\tau_4 = (p_{1_\mu}p_{2_\nu}p_{2_\rho}+p_{2_\mu}p_{1_\nu}p_{2_\rho}+p_{2_\mu}p_{2_\nu}p_{1_\rho})/p_2^2$	0
$\tau_5 = g_{\mu\nu}p_{1_\rho}+g_{\mu\rho}p_{1_\nu}+g_{\nu\rho}p_{1_\mu}$	$\mathbf{1-2\kappa}+\frac{d-4}{2}$
$\tau_6 = g_{\mu\nu}p_{2_\rho}+g_{\mu\rho}p_{2_\nu}+g_{\nu\rho}p_{2_\mu}$	0
$\tau_7 = p_{1_\mu}p_{1_\nu}p_{2_\rho}/p_1^2$	$\mathbf{1-2\kappa}+\frac{d-4}{2}$
$\tau_8 = p_{1_\mu}p_{2_\nu}p_{2_\rho}/p_2^2$	$\mathbf{1-2\kappa}+\frac{d-4}{2}$
$\tau_9 = (p_{1_\mu}p_{2_\nu}(p_2-p_1)_\rho+(p_2-p_1)_\mu p_{1_\nu}p_{2_\rho})/(p_1p_2)$	$3/2-2\kappa+\frac{d-4}{2}$
$\tau_{10} = g_{\nu\rho}p_{1_\mu}-g_{\mu\nu}p_{2_\rho}$	$\mathbf{1-2\kappa}+\frac{d-4}{2}$

Table 1: *The IR exponents for the scalar functions corresponding to the indicated tensors of the three-gluon vertex when p_1^2 becomes soft .*

Ghost-Gluon Vertex

To verify the self-consistency of the assumption of a bare ghost-gluon vertex we calculated its momentum dependence for uniform scaling and the case of only one small momentum. The results for the former clearly support the bare version, whereas those for the latter show that the structure of the ghost-gluon vertex is richer than expected and features even a divergence $(1-2\kappa)$ for the longitudinal scalar function when the gluon momentum vanishes. However, this is no contradiction to Taylor's argument because the momentum of the tensor, which is itself soft, leads to the expected suppression.

References

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