Infrared Behavior Of Three-Point Functions in Landau Gauge Yang-Mills Theory

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SIC!QFT
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3 Summary
Quantum Chromodynamics

- QCD describes the interaction of quarks and gluons.
- In a given gauge one has UV and IR phenomena.
- UV: Well described by perturbation theory due to asymptotic freedom.
- IR: Non-perturbative methods (models, lattice, ERGE, Dyson-Schwinger equations (DSEs)), special phenomena (dynamical chiral symmetry breaking and confinement).
- A first step in understanding confinement is to consider only ghosts and gluons, i.e. Yang-Mills (YM) theory.
Propagators in the IR

\[ D_{\mu\nu}(p) = \left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \frac{Z(p)}{p^2}, \quad D_G(p) = -\frac{G(p)}{p^2} \]

Dressing functions in the IR: power-like behavior $\rightarrow$ IR exponents


- Gluon: \( Z(p)^{IR} \propto (p^2)^{2\kappa} \)
- Ghost: \( G(p)^{IR} \propto (p^2)^{-\kappa} \)
- \( \kappa = 0.59 \ldots \) [Zwanziger, Phys.Rev.D65, ’02; Lerche, v. Smekal, Phys.Rev.D65, ’02]
- Dominant contribution for the gluon propagator: ghost loop
## Vertices in the Infrared

### Ghost-Gluon Vertex

Transversality of the gluon propagator $\rightarrow$ **bare in the IR**, i.e. IR exponent 0.

Landau gauge!!!

### General Vertices

\[
\rho_{m,2n} = (n - m)\kappa + (1 - n)\left(\frac{d}{2} - 2\right)
\]

- Ghost contributions dominate.
- Same qualitative behavior in 2, 3 and 4 dimensions ($\kappa$ different!).
- Valid for only **one momentum scale**.
Vertices in the Infrared II

\[ \Gamma_{\mu\nu\rho...}(p_1, p_2, p_3, \ldots) = \sum_{i=0}^{n} H_i(p_1, p_2, p_3, \ldots) \tau_{\mu\nu\rho...}^{(i)} \]

\[ H_{iR}^j(p_1, p_2, p_3, \ldots) \rightarrow \sum_{j=1}^{n-1} (q_j^2) \delta_{i,j} H_{iR}^j \left( \frac{p_1^2}{q_j^2}, \frac{p_2^2}{q_j^2}, \ldots \right) \]

- Different tensors can behave differently \( \rightarrow \) power counting provides the lowest possible IR exponent \( \rightarrow \) Kai Schwenzer’s talk.

- **Structure** for the dressing functions as a function of the **momentum scales** is possible \( \rightarrow \) explicit calculations necessary (tensor decomposition, solution of integrals).
Three-Gluon Vertex: Dyson-Schwinger Equation

Many terms, . . .
Three-Gluon Vertex: Dyson-Schwinger Equation

Many terms, ...

...but power counting arguments show that the ghost triangle is the dominant integral (lowest order of skeleton expansion).
Three-gluon Vertex: Ghost Triangle

Dominant diagram for three-gluon vertex:

\[
\alpha \int \frac{d^d q}{(2\pi)^d} \frac{(q + p_1)_\mu (q - p_2)_\rho q_\nu}{((q + p_1)^2)^{\kappa+1}((q - p_2)^2)^{\kappa+1}(q^2)^{\kappa+1}}
\]

Non-integer powers \(\rightarrow\) no standard techniques applicable

Negative Dimension Integration Method (NDIM)

- Allow \(d < 0\).
- All values for the exponents allowed.
- Results are hypergeometric series.

\(\rightarrow\) Appell’s function \(F_4\)
Excursion: Hypergeometric Series

Generalization of the geometric series

\[ \sum_{n=0}^{\infty} z^n = 1 + z + z^2 + \ldots. \]
## Excursion: Hypergeometric Series

### Generalization of the geometric series

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\sum_{n=0}^{\infty} z^n = 1 + z + z^2 + \ldots.
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### Pochhammer symbol

\[
(a, n) := \frac{\Gamma(a+n)}{\Gamma(a)}
\]
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Example: Gaussian hypergeometric series
\[ F(a, b; c; z) = 2F_1(a, b; c; z) := \sum_{n=0}^{\infty} \frac{(a,n)(b,n)}{(c,n)} \frac{z^n}{n!} \]
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Generalized hypergeometric series
\[ pF_q(a_1, \ldots, a_p; b_1, \ldots, b_q; z) := \sum_{n=0}^{\infty} \frac{(a_1)_n \ldots (a_p)_n}{(b_1)_n \ldots (b_q)_n} \frac{z^n}{n!} \]
Excursion: Hypergeometric Series

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Appell’s series \( F_4 \)

- Two-dimensional hypergeometric series
\[ F_4(a, b; c, d; x, y) := \sum_{m,n=0}^{\infty} \frac{(a,m+n)(b,m+n)}{(c,m)(d,n)} \frac{x^m y^n}{m! n!} \]
Massless Three-Point Integral

- 4 Appell’s series with $x = \frac{p_1^2}{p_3^2}$, $y = \frac{p_2^2}{p_3^2}$.
- Prefactors depend also on squared momenta.
- Assuming one momentum scale $\rightarrow$ expected power law.
Massless Three-Point Integral

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Region of convergence of standard series representation.
Massless Three-Point Integral

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Momentum conservation
$\rightarrow$ Euclidean region
$\rightarrow$ analytic continuation
Massless Three-Point Integral

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- Assuming one momentum scale $\rightarrow$ expected power law.

- Symmetric configuration: all squared momenta equal
- Orthogonal configuration: two momenta orthogonal, equal abs. value
- Asymmetric configuration: one momentum vanishing, soft gluon
Ghost Triangle: Kinematic Structure

Tensor decomposition à la Davydychev: 10 tensors (instead of 14) are relevant for the ghost triangle.

\[ \Gamma^{IR}_{\mu\nu\rho}(p_1, p_2, p_3) = \sum_{i=0}^{10} H^R_i(p_1^2, p_2^2, p_3^2) \tau^{(i)}_{\mu\nu\rho}(p_1, p_2) \]

Weak singularities, but important for integration over all momenta.
Ghost Triangle: Kinematic Structure

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\[ p_1^\nu p_1^\rho p_2^\mu + p_1^\mu p_1^\rho p_2^\nu + p_1^\nu p_1^\rho p_2^\nu \]

\[ p_1^\mu g^\nu_\rho - p_2^\rho g^\mu^\nu \]

Weak singularities, but important for integration over all momenta.
Ghost Triangle: Asymmetric Point

Line from symmetric to asymmetric point \((0, 1)\).
Ghost Triangle: Asymmetric Point

Approach to asymmetric point \((0, 1)\).
Ghost Triangle: Asymmetric Point

Approach to asymmetric point \((0, 1)\).
Ghost Triangle: Exponents

Limit imposed by power counting: \(1 - 2\kappa\)

<table>
<thead>
<tr>
<th>Tensor</th>
<th>(p_1) soft</th>
<th>(p_2) soft</th>
</tr>
</thead>
<tbody>
<tr>
<td>(p_1_\mu p_1_\nu p_1_\rho / p_1^2)</td>
<td>(1 - 2\kappa)</td>
<td>(0)</td>
</tr>
<tr>
<td>(p_2_\mu p_2_\nu p_2_\rho / p_2^2)</td>
<td>(0)</td>
<td>(1 - 2\kappa)</td>
</tr>
<tr>
<td>((p_1_\mu p_1_\nu p_2_\rho + p_1_\mu p_2_\nu p_1_\rho + p_2_\mu p_1_\nu p_1_\rho) / p_1^2)</td>
<td>(2 - 2\kappa)</td>
<td>(0)</td>
</tr>
<tr>
<td>((p_1_\mu p_2_\nu p_2_\rho + p_2_\mu p_1_\nu p_2_\rho + p_2_\mu p_2_\nu p_1_\rho) / p_2^2)</td>
<td>(0)</td>
<td>(2 - 2\kappa)</td>
</tr>
<tr>
<td>(g_{\mu\nu} p_1_\rho + g_{\mu\rho} p_1_\nu + g_{\nu\rho} p_1_\mu)</td>
<td>(1 - 2\kappa)</td>
<td>(0)</td>
</tr>
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<td>(g_{\mu\nu} p_2_\rho + g_{\mu\rho} p_2_\nu + g_{\nu\rho} p_2_\mu)</td>
<td>(0)</td>
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<td>(p_1_\mu p_1_\nu p_2_\rho / p_1^2)</td>
<td>(1 - 2\kappa)</td>
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</tr>
<tr>
<td>(p_1_\mu p_2_\nu p_2_\rho / p_2^2)</td>
<td>(1 - 2\kappa)</td>
<td>(1 - 2\kappa)</td>
</tr>
<tr>
<td>((p_1_\mu p_2_\nu (p_2 - p_1)<em>\rho + (p_2 - p_1)</em>\mu p_1_\nu p_2_\rho) / (p_1 p_2))</td>
<td>(3/2 - 2\kappa)</td>
<td>(3/2 - 2\kappa)</td>
</tr>
<tr>
<td>(g_{\nu\rho} p_1_\mu - g_{\mu\nu} p_2_\rho)</td>
<td>(1 - 2\kappa)</td>
<td>(1 - 2\kappa)</td>
</tr>
</tbody>
</table>
Ghost-Gluon Vertex

\[ \text{Ghost-Gluon Vertex} \]

\[ = \text{Diagram 1} + \text{Diagram 2} - \text{Diagram 3} + \text{Diagram 4} + \frac{1}{2} \]

Solution has singularities in single integrals which cancel.

Soft gluon: singularity \( \approx -0.19 \) for dressing function, but ghost-gluon vertex itself finite.

All other combinations tensor/soft momentum vanish.

Self-consistency shown.
Ghost-Gluon Vertex

- Only two tensors → calculation simpler?
- Solution has singularities in single integrals which cancel.
- Soft gluon: singularity \((1 - 2\kappa \approx -0.19)\) for dressing function, but ghost-gluon vertex itself finite.
- All other combinations tensor/soft momentum vanish.
- Self-consistency shown.
Summary

Assuming a fixed point with divergent ghost and vanishing gluon propagators, three-point functions in the IR ...

- ... obey a simple power law for one momentum scale and
- ... have additional singularities for two different momentum scales.
- The ghost triangle has 10 tensors, which fulfill the restrictions from power counting.
- The ghost-gluon vertex self-consistently becomes trivial.