High-Finesse Optical Quantum Gates for Electron Spins in Artificial Molecules

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A doped semiconductor double-quantum-dot molecule is proposed as a qubit realization. The quantum information is encoded in the electron spin, thus benefiting from the long relevant decoherence times; the enhanced flexibility of the molecular structure allows one to map the spin degrees of freedom onto the orbital ones and vice versa and opens the possibility for high-finesse (conditional and unconditional) quantum gates by means of stimulated Raman adiabatic passages.

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Quantum bits or qubits are the building block for future quantum computers [1]. The requirements for such quantum hardware are manifold: first, qubits should consist of at least two long-lived states, usually referred to as 0 and 1; second, it should be possible to modify the state of a single qubit unconditionally or dependent on the setting of a second qubit (one- and two-qubit quantum gates); finally, one should be able to measure the final qubit states. Evidently, the main challenge in identifying physical systems as qubits is to bridge between the complementary requirements of long quantum memory and fast quantum gates: while the first point requires excitations well protected from environment, the latter one calls for strong and well-controllable interaction channels between the qubits and the external control.

To overcome this difficulty, in their seminal work Cirac and Zoller proposed to pursue a mixed strategy in which the quantum information is stored in metastable atomic states and the light coupling to additional auxiliary states is used to perform the quantum gates [2]. The recent progress in the fabrication and control of semiconductor quantum dots [3], sometimes referred to as “artificial atoms,” suggested that similar schemes could be identified also in the technologically more promising solid state: indeed, in Refs. [4,5] optical excitations (excitons) in artificial atoms were proposed as qubits, with Coulomb interactions between the optically excited electrons and holes providing a means to perform conditional quantum gates. However, it was soon realized that the radiative lifetime of excitons (∼ns) is too short to comply with the exceptional requirements for quantum memory. Around the same time different work proposed spin of excess electrons as a viable quantum memory [6,7] and estimated lifetimes of the order of microseconds. Apparently, a combination of such long spin memory with ultrafast optical gating provides a likely candidate for the first proof-of-principle solid-state quantum computer, in particular, in view of the superb standards of present-day sample growth and coherent-carrier control. In turn, a diversity of strategies for such a mixed approach was put forward, e.g., based on cavity quantum electrodynamics [7], charged excitons [8,9], or RKKY interactions [10]. Yet, the shortcomings of these proposals are either lacking strategies for performing conditional or unconditional gates, or possible environment losses during gating.

It is the purpose of this Letter to propose a quantum computation scheme based on long spin memory and ultrashort optical quantum gates which does not suffer from radiative losses during gating. Most importantly and in contrast to all existing proposals based on electron spin in quantum dots, we consider a vertically coupled double dot (“artificial molecule”) as the building block for a single qubit. The quantum hardware then consists of laterally arranged quantum-dot molecules (e.g., through seeded growth [11]) which can be individually addressed through frequency selective laser pulses. Besides, in order for the qubits to be correctly defined, the interdot tunneling in the lateral directions has to be suppressed [6]. Our central observation concerns the fact that these artificial molecules host besides the spin-degenerate electron state (used as the qubit) further long-lived auxiliary states which can be exploited during gating to map the information stored in the spin degrees of freedom onto the orbital ones and vice versa, and optically switch on and off qubit-qubit interactions on an ultrashort time scale. As shown below, with these strategies it becomes possible to perform all quantum gates efficiently by means of stimulated Raman adiabatic passage (STIRAP) [12] and to hereby suppress environment losses during gating.

1. Qubit Identification.—We start by considering two vertically coupled quantum dots (see Fig. 1) inside a field-effect structure. The electric field in the growth direction has two consequences: first, it transfers a single excess electron from a nearby n-type reservoir to the artificial molecule, where further charging is suppressed because of the Coulomb blockade; second, it enhances the electron localization in one of the two dots (labeled as large, $L$, as compared to the small one, $S$). Although in the following we are not too specific about the details of this quantum-dot molecule (model calculations are presented...
pose to perform all quantum gates solely by means of external laser pulses. The resulting optical selection rules are sketched in Fig. 1.

The notation in Eq. (1), the negatively charged-exciton state $(\alpha|S_{x} = -\frac{1}{2}\rangle + \beta|S_{x} = +\frac{1}{2}\rangle)$, is ideal for quantum control in the solid state [14,15].

Unconditional gates. — Recently, Kis and Renzoni [16] extended this original STIRAP level scheme to an additional long-lived auxiliary state (here $|2\rangle$) and showed that within the resulting model it becomes possible to perform generic quantum gates. For the sake of clarity, let us briefly rephrase the main steps of this control within the present scheme: suppose that initially the system wave function is $|\Psi\rangle = |L\rangle \otimes (|S_{x} = -\frac{1}{2}\rangle + \beta|S_{x} = +\frac{1}{2}\rangle)$. Next, the quantum-dot structure is subject to a first STIRAP process, consisting of a sequence of three laser pulses: the first one (Stokes pulse) couples the states $2$ and $3$; the second ones (pump pulses) affect the $0$-$3$ and $1$-$3$ transitions, with Rabi frequencies $\Omega_{2,1}(t) = \Omega(t) \cos \chi$ and $\Omega_{1,1}(t) = \Omega(t) \exp(i\eta) \sin \chi$, respectively [16] (here $\Omega(t)$ is the envelope and $\chi$ and $\eta$ are phase factors): such selective coupling can be achieved by the above-mentioned selection of the light polarizations and frequencies. Incidentally, with a specific choice of the laser parameters ($\chi = -\pi/2$ and $\eta = 0$) this first sequence maps the information stored in the electron spin onto the orbital degrees of freedom:

$$|\Psi\rangle \rightarrow (\alpha|L\rangle + \beta|S\rangle) \otimes |S_{x} = -\frac{1}{2}\rangle$$

as discussed in the following, this possibility is crucial to the implementation of the conditional dynamics within the present scheme. The single-qubit rotation is completed by a second, reversed STIRAP process (pump pulses before the Stokes one): any unitary transformation of the SU$_2$ group can be performed through an appropriate choice of $\chi$ and $\eta$. Note that the different energies of the involved electron states, Eq. (1), result in additional dynamic phase factors, which should be incorporated into the quantum algorithm.

Conditional gates. — The conditional (controlled) dynamics can be implemented within the present scheme by exploiting the electrostatic interaction changes resulting from intermediate population of $2$. As an illustrative example, let us consider a controlled-NOT gate in a structure consisting of two quantum-dot molecules, Fig. 2, where the system is initially in state $|1\rangle \otimes |0\rangle_{c}$, with $c$ and $t$ denoting the control and target qubit, respectively. At the beginning the information of both qubits is encoded.

2. Quantum gates. — As a major improvement, we propose to perform all quantum gates solely by means of STIRAP processes. This technique was originally developed in the field of atomic physics [12] as an optimal quantum control strategy to channel the system between two long-lived states (here $|0\rangle$ and $|1\rangle$) through optical coupling to an interconnecting state (here $|3\rangle$): to avoid radiative environment losses of $3$, one exploits the renormalized radiation-matter states (trapped state) for the transfer process, which is achieved by slowly (adiabatically) varying the exciting laser fields and keeping the population of state $3$ negligible throughout. As a further advantage, such control does not require a detailed knowledge of the system parameters (i.e., oscillator strengths) and therefore is of very robust nature, thus rendering this scheme ideal for quantum control in the solid state [14,15].

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in the respective electron spins: a first STIRAP process applied to the control qubit then maps the $|S_x = \pm \frac{1}{2}\rangle$ component onto the orbital degrees of freedom, Fig. 2(b), independent of the target qubit setting. In what follows, we exploit the fact that this modified charge distribution exerts a potential change on the target qubit and leads to different transition frequencies. Thus, in the next step, Fig. 2(c), the double STIRAP pulse sequence discussed above is applied to the target qubit with the modified laser frequencies; apparently, this procedure rotates the target qubit dependent on the control qubit setting. Finally, the quantum information of the control qubit is mapped back to the spin degrees of freedom, Fig. 2(d).

By now the strength of our present proposal should have become obvious: its ability to map the quantum information between spin and orbital degrees of freedom. On the one hand, this allows for a high-finesse gating through stimulated Raman adiabatic passage. On the other hand, it becomes possible to turn on selectively qubit-qubit interactions only during gating; this inter-qubit control emerges naturally for the double-dot structure under investigation without requiring additional switching of external electric or magnetic fields and appears advantageous over related proposals [5,8] where a compromise between optical and interdot coupling had to be taken. Thus, the present scheme fully benefits from the long spin coherence and the ultrafast optical gating.

3. Double-dot structure.—As a final step, we comment on the possibility to design a quantum-dot structure with the desired level scheme of Fig. 1. Quite generally, the relevant features for the implementation of such a scheme are (i) single-electron wave functions sufficiently localized in either dot (to minimize environment losses during gating and to maximize electrostatic potential changes), (ii) a charged-exciton state with the hole delocalized over the double-dot structure (such that all transitions between 0-3, 1-3, and 2-3 acquire comparable oscillator strengths), (iii) energetically well separated transition frequencies $\omega_0$ and $\omega_2$ (in order to energetically resolve the 0-3 and 2-3 transitions, which are induced by optical fields with the same polarization). Model calculations were performed to demonstrate that such manifold requirements can indeed be simultaneously fulfilled. We adopt the framework presented in Ref. [17] where we calculated single- and few-particle states for prototypical GaAs/AlGaAs double-dot structures within the envelope-function and effective-mass approximations, assuming a prototypical confinement potential which is double-well-like along $z$ and parabolic in the in-plane directions. In addition, we consider a slight asymmetry in the double-dot structure (well widths of $l_x = 3.5 \text{ nm}$ and $l_y = 3.6 \text{ nm}$, respectively, and an interdot distance $d = 7 \text{ nm}$), an applied electric field, and a charged-exciton state with light-hole character (to enhance the interdot tunneling of holes; alternatively, it might be advantageous to use type-II quantum dots where the hole is only Coulomb bound and its wave function becomes strongly delocalized).

Results of our calculations are shown in Fig. 3. Panel (a) shows the carrier distributions along $z$; the two lower plots represent the single-electron densities $\rho^e_i(z) = |\phi_i^e(z)|^2$; the small overlap between $\phi_0^e$ and $\phi_2^e$ allows one to almost completely suppress environment losses due to phonon-assisted tunneling during gating, as discussed in more detail by Pazý et al. [15], where lifetimes of the order of nanoseconds were estimated; in the upper part of panel (a) we report the electron (light gray) and hole (dark gray) densities $\rho^{e,h}_i(z)$ of the interconnecting charged-exciton state 3, with $\rho^{e,h}_i(z) = \int dxdy\langle 3|\hat{\psi}_{e,h}^i(r)\hat{\psi}_{e,h}(r)|3\rangle$ and $\hat{\psi}_{e,h}(r)$ the field operator for electrons and holes, respectively. While the use of light fields with linear polarizations allows the coupling of the charged exciton with electron states sharing the orbital state but with opposite spins, the present overlap of $\rho^{h}(z)$ with both $\rho_0^e(z)$ and $\rho_2^e(z)$ ensures comparable oscillator strengths to transitions to states where electrons are localized in opposite dots (irrespective of their spin orientations). This indeed can be seen in Fig. 3(b) where we plot the absorption spectra associated to the three initial states, with the transitions 0-3 (1-3) and 2-3 indicated by the shaded regions. The additional peaks with larger oscillator strength are attributed to additional intradot transitions; however, the energetic separation

![Figure 2](image-url)
In conclusion, we have proposed a novel semiconductor-based implementation scheme for quantum information processing, where the qubit is identified with the spin of an excess electron in a vertically coupled double-dot structure. By use of further auxiliary states it becomes possible to perform all quantum gates by means of stimulated Raman adiabatic passage and to hereby almost completely suppress environment losses during gating. In addition, an efficient mechanism for turning on and off qubit-qubit interactions, as requested for conditional quantum gates, has been proposed. We think that our present work constitutes an important step forward for the implementation of a first proof-of-principle solid-state quantum computer and could open the possibility for highly efficient quantum gates.

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