The effects of Talbot and Lau

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Abstract

While the Lau effect was discovered by E. Lau in 1948, the Talbot effect was initially observed by H. Talbot in 1836. In recent years this effect, a diffraction effect in the near field, became increasingly important and applied in several fields of physics, in particular in quantum optics, where both effects are needed to realize the manufacturing of interferometers. After a short overview of Talbots discovery, the input of Lord Rayleigh is mentioned. He found the first mathematical relation between the grating structure, the wavelength of the light and the so called Talbot distance. The specifications of diffraction in the near-field region are explained, and the Kirchhoff diffraction integral is introduced. First, the form of the Fresnel approximation is shown yielding in the final diffraction integral. Furthermore the calculation of the intensity distribution is derived. Moreover it is indicated that the operation field of the Talbot effect is restricted to the use of point-like sources illustrating the importance of the Lau effect. Lau studied diffraction effects using two gratins instead of one. As a consequence self-imaging using a three-dimensional incoherent source became possible. Finally a short overview of the fields of application using the Lau effect and the Talbot effect (even apart from interferometer) as well as a short description of Montgomerys classification concerning self-imaging objects is given.

I. Introduction

The Kapitza-Dirac-Talbot-Lau interferometer (Gerlich et al.) is used to observe interference of molecules which have nearly the size of a small virus. Therefore they consist of up to 430 atoms. This experiments is made to demonstrate the limit between macrocosm and microcosm through measuring the De Broglie wavelength of the molecules. The Kapitza-Dirac-Talbot-Lau interferometer is, among others, based on the Talbot and the Lau effect. The Talbot effect describes the self-imaging of a grating at a certain distance from the grating itself. The Lau effect gives a description of self-imaging too, but Laus experimental setup was different as Talbots, because Lau used a not-point-like source. Both effects were actually mentioned first in the 19th and 20th century, respectively. The effects are nowadays often needed to realize experiments in modern physics. H. Talbot observed white light falling through a grating. First, the grating was positioned in the focus of the used convex lens yielding to an image of the grating on the screen. Talbot enlarged the distance between the lens and the grating. Nevertheless the colours of the "strips" changed, the image of the grating stayed sharp. This change of colours indicates a dependence of the wavelength. Lord Rayleigh realized this dependency and found the first relation between the distance among the first self-imaging and the original grating, and the wavelength of light

\[ L_T = \frac{d^2}{\lambda}, \] (1)

where the distance \( L_T \) denotes the Talbot length, \( d \) is the period of the diffraction grating, and \( \lambda \) the wavelength of light.

II. Main Body

The Talbot effect therefore describes the self-imaging of a diffraction grating. The grating is illuminated with monochromatic, plane waves of any kind out of a point-like source. The self-images occur in integer multiples of the Talbot length

\[ L_T = n \cdot \frac{2d^2}{\lambda}, \] (2)
where \( n \) is an integer.

Since this effect takes place in the optical near-field, the wave curvature has to be considered. The diffraction grating and the screen are positioned within a small distance. The interference pattern is depending on the square of the wavelength as well as of the distance, respectively. As a result the Fresnel approximation has to be used to get the wave function within the near-field out of the Kirchhoff diffraction integral. The Kirchhoff diffraction integral is given by

\[
\psi_p = \frac{a_Q k_0}{2\pi i} \int dS \frac{e^{ik_0(d + d_1)}}{d + d_1} \cos \Theta + \cos \Theta' \frac{x}{2},
\]

where \( \psi_p \) denotes the wave function, \( a_Q \) the amplitude of the source, \( k_0 \) the wave vector, \( f_s \) the aperture function, and \( \cos \Theta + \cos \Theta' / 2 \) the slope factor. The intensity \( I \) is described by

\[
I = |\psi_p|^2.
\]

Considering the quadratic terms in \( x \) and \( y \) in a Taylor expansion of the light paths \( d \) and \( d_1 \) yields to

\[
d \approx L(1 + \frac{x^2 + y^2 + x'^2 + y'^2 - 2(xx' + yy')}{2L^2} + ...)
\]

\[
d_1 \approx L_1 \left(1 + \frac{x^2 + y^2}{2L_1^2} + ...\right),
\]

where \( L \) is the distance between the grating and the screen.

The diffraction integral is therefore given by

\[
\psi_p = \frac{a_Q k_0}{2\pi i} \frac{e^{ik_0(L + L_1 + y^2)/(2L)}}{LL_1} \int dx dy f_S(x, y)
\cdot e^{ik_0((x^2 + y^2 - 2(xx' + yy')/(2L)) + ((x^2 + y^2)/(2L_1)))}. 
\]

(7)

With \( L' = (1/L) + (1/L_1) \) and \( \vec{K} = k_0/(L \cdot \vec{P}) \) the diffraction integral in near-field-approximation can be written as

\[
\psi_p = \frac{a_Q k_0}{2\pi i} \frac{e^{ik_0(L + L_1 + y^2)/(2L)}}{LL_1} \int dx dy f_S(x, y)
\cdot e^{ik_0((x^2 + y^2)/L_1')} e^{-i\vec{K} \cdot \vec{r}}.
\]

(8)

Furthermore, this equation is applied for the description of the Talbot effect as well. It is impossible to give an exact solution of the Fresnel integral, hence, the intensity distribution of the Talbot plane can just be calculated by a numerical approximation. The self-imaging effect is therefore reduced to interferences of partial beams. The operation field of the Talbot effect is limited by the fact that the source has to be point-like. The solution of this problem gave E. Lau in 1948 with a study about effects of diffraction using two gratings, one after another. The diffraction gratings had the same period and parallel openings. Waves from a three-dimensional incoherent, (therefore not point-like) source fell onto two identical gratings. If positioned in a certain distance from each other, an image of the first grating can be observed after the second one. Because of the spatial expanded source, each point of the first grating presents an independent point-source which illuminates the second grating. Each point-source creates a Talbot-picture on the screen. The mean of all intensity distributions created from a point within the opening of the grating, gives the total intensity of the opening. These are figured up to become the intensity distribution seen on the screen. As mentioned above, the Talbot and the Lau effect are used to realize a Kapitza-Dirac-Talbot-Lau interferometer (Gerlich et al.). With the interferometer it is possible to verify the wave-nature of atoms and molecules. Even before the application of the Kapitza-Dirac-Talbot-Lau interferometer, the Talbot an Lau effect were used within interferometric experiment. The first Talbot-Lau Interferometer was used to verify De Broglies relation was built in 1994 by Clauser and Li.
were done of W. D. Montgomery in 1967 by defining a class of self-imaging objects. He found the requirements for a plane object to self-image without use of lenses when hit by a plane, monochromatic wave under a right angle. Based on the studies of Montgomery both, the Talbot and the Lau effect, are applied in many optical issues, exemplarily in the measurements of refraction indices using the Talbot effect (Bhattacharya) or the measuring of focal distances (Nakano, Murata and Singh et al.)