Excited hadrons in $n_f = 2$ QCD

Christian B. Lang

Inst. f. Physik, FB Theoretische Physik
Universität Graz

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Collaborators in this project:
G. Engel, C. Gattringer, L. Ya Glozman, C. Hagen,
M. Limmer, D. Mohler, A. Schäfer
Chirally Improved Dirac operator

General ansatz for fermion action:

\[ D_{mn} = \sum_{\alpha=1}^{16} \Gamma_{\alpha} \sum_{p \in \mathcal{P}_{m,n}^\alpha} c_{p}^{\alpha} \prod_{l \in p} U_l \delta_{n,m+p} \]

Insert the ansatz in the Ginsparg-Wilson-equation, truncate the length of the contributions (to, e.g., 4) and compare the coefficients!

Leads to a set of (e.g. 50) algebraic equations, which can be solved (norm minimization).
Study with 2 dynamical CI fermions

- Chirally improved fermions ($D_{CI}$), $n_f = 2$ light quarks
- $1 \times$ Stout smearing
- Lüscher-Weisz gauge action
- Hybrid Monte Carlo simulation
  - Hasenbusch mass preconditioning, 2 pseudofermions
  - Chronological inverter (min residue extrap.)
  - Mixed precision inverter (cf. Dürr et al., PRD79, 014501)
  - Each unit MD time between 70 and 100 steps
- Here: Three ensembles for $16^3 \times 32$:

<table>
<thead>
<tr>
<th>set</th>
<th>$\beta_{LW}$</th>
<th>$m_0$</th>
<th>$t_{MD}$</th>
<th>config's</th>
<th>$a[fm]$</th>
<th>$m_\pi[MeV]$</th>
<th>$m_{AWI}[MeV]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4.70</td>
<td>-0.050</td>
<td>600</td>
<td>100</td>
<td>0.151(2)</td>
<td>525(7)</td>
<td>42.8(4)</td>
</tr>
<tr>
<td>B</td>
<td>4.65</td>
<td>-0.060</td>
<td>1200</td>
<td>200</td>
<td>0.150(1)</td>
<td>470(4)</td>
<td>34.1(2)</td>
</tr>
<tr>
<td>C</td>
<td>4.58</td>
<td>-0.077</td>
<td>1200</td>
<td>200</td>
<td>0.144(1)</td>
<td>322(5)</td>
<td>15.3(4)</td>
</tr>
</tbody>
</table>

- Eigenvalues close to circle but wider distribution
- $Z_A/Z_V \approx 1.07$
(cf., Michael, Lüscher/Wolff; recently: Blossier et al.)

- Each channel: Several interpolators $O_i$
- Compute all cross-correlations

\[
C(t)_{ij} = \langle O_i(t) \overline{O}_j(0) \rangle = \sum_n \langle 0 | O_i | n \rangle \langle n | O_j^\dagger | 0 \rangle e^{-t E_n}.
\]

- Solve the generalized eigenvalue problem:

\[
C(t) \bar{u}^{(n)}(t) = \lambda^{(n)}(t) C(t_0) \bar{u}^{(n)}(t)
\]

Then

\[
\lambda^{(n)}(t) \propto e^{-t E_n} (1 + \mathcal{O} (e^{-t \Delta E_n})).
\]

- The eigenvectors are “fingerprints” of the state
Hadron operators are built on spatially 3x HYP smeared gauge configurations.

We need several interpolators!

- Different Dirac structure
- Quark sources:
  - Point
  - Wall
  - (Spatially) smeared, covariant
  - Laplacian eigenvectors (Distillation method, Peardon et al.)
  - Derivative sources
Examples for mesons:

\( \bar{u}_s \Gamma d_s \)

where \( u_s, d_s \) denote smeared and/or derivative smeared quarks.

E.g. for \( \pi \):

\[
\begin{align*}
\bar{u}_n \gamma_5 d_n, \quad & \bar{u}_n \gamma_5 d_w, \quad \bar{u}_w \gamma_5 d_w, \\
\bar{u}_n \gamma_t \gamma_5 d_n, \quad & \bar{u}_n \gamma_t \gamma_5 d_w, \quad \bar{u}_w \gamma_t \gamma_5 d_w, \\
\bar{u}_{\partial_i} \gamma_i \gamma_5 d_n, \quad & \bar{u}_{\partial_i} \gamma_i \gamma_5 d_w, \quad \bar{u}_{\partial_i} \gamma_i \gamma_t \gamma_5 d_w, \\
\bar{u}_{\partial_i} \gamma_5 d_{\partial_i}, \quad & \bar{u}_{\partial_i} \gamma_t \gamma_5 d_{\partial_i}
\end{align*}
\]

Examples for \( N \) (and \( \Sigma, \Xi \)):

\[
N^{(i)} = \epsilon_{abc} \Gamma_1^{(i)} u_a \left( u_b^T \Gamma_2^{(i)} d_c - d_b^T \Gamma_2^{(i)} u_c \right)
\]

with the choices

<table>
<thead>
<tr>
<th>( i )</th>
<th>( \Gamma_1^{(i)} )</th>
<th>( \Gamma_2^{(i)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>( C \gamma_5 )</td>
</tr>
<tr>
<td>2</td>
<td>( \gamma_5 )</td>
<td>( C )</td>
</tr>
<tr>
<td>3</td>
<td>( i )</td>
<td>( C \gamma_4 \gamma_5 )</td>
</tr>
</tbody>
</table>

projected to parity (and spin)
Example: Angular momentum content of $\rho$

<table>
<thead>
<tr>
<th>Type</th>
<th>Interpolator:</th>
<th>Chiral classification:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vector</td>
<td>$O_{\rho}^V = \bar{u}(x)\gamma^i d(x)$</td>
<td>$(0, 1) \oplus (1, 0)$</td>
</tr>
<tr>
<td>Pseudotensor</td>
<td>$O_{\rho}^T = \bar{u}(x)\gamma_4 \gamma^i d(x)$</td>
<td>$(\frac{1}{2}, \frac{1}{2})b$</td>
</tr>
</tbody>
</table>

- If both operators couple to $\rho$: Chiral symmetry is broken (for $a \to 0$ only $V$ couples: pert. th. RG)
- Unitarily equivalent to angular momentum $\{^{2S+1}L_J\}$ basis in CMS: $|^{3}S_1\rangle, |^{3}D_1\rangle$

Smearing size defines infrared scale: Allows study of IR scale dependence of Vector vs. Tensor contribution and thus S-wave and D-wave admixture.
Contribution of lattice operator $i$ to eigenstate $(n)$ 

$$a_i^{(n)} = \langle 0 | O_i | n \rangle$$, eigenvectors $\vec{u}^{(n)}$.

Then 

$$\frac{C(t)_{ij} u_j^{(n)}}{C(t)_{kj} u_j^{(n)}} \sim \frac{a_i^{(n)}}{a_k^{(n)}}$$

gives the ratio of coupling of different lattice operators to the eigenstate.

$R \rightarrow 0$: only $(0, 1) \otimes (1, 0)$, $S/D = \sqrt{2}$

large $R$: mainly S-wave

Glozman et al.,

PRL103(09)121601;

ArXiv:0909.2939
$0^{-+} : \pi^\pm(140), \pi^\pm(1300)$

Choosing the cosh-fit interval on basis of eigenvalues: Excited pion state, including partially quenched data (open symbols)
$0^{++}: a_0(980), a_0(1450)$

...compared to $\pi\eta_2$ channel (mass of $\eta_2$ estimated)... compatible with Jansen et al., arXiv:0906.4720

...tetraquark state?
1−−: $\rho^\pm (770), \rho^\pm (1450)$

- No decay yet (p-wave)
- More contamination with higher excitations, thus $t_0 = 2$ is preferable. Optimal combination chosen for each data set.
- 2nd excitation $\rho(1720)$ signal is seen for some combinations of interpolators
- Exotic (1−+) channel is too noisy.
$0^- : K(490)$ and $K(1460)$

Kaon ($0^{-+}$)

GS: (4) $t=4-15$; 1E: [A:(4,8,11,14,17)$t=2-6$; B:(4,7,10,14,17)$t=3-6$; C:(4,8,11,14,17)$t=3-6$]

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$\frac{1}{2}^+: N(940), N(1440) P_{11}, N(1710) P_{11}$

Two excitations (higher one vague), too high up! Roper?

Quenched results (Burch et al., PRD 74 (2006) 014504);
cf., Mahbub arXiv:0905.3616
Excited $\Delta$ clearly seen, but too high (squeezed?)
$M_\Omega$ is used to fix strange (only valence) quark mass.
Conclusion

- Excellent ground state masses for (in lattice units) small lattices and all mesons: $0^{-+}, 1^{--}, 1^{+-}, 1^{++}, 2^{--}, 2^{--}, 2^{++}$
- $a_0$ behaves unlike other mesons
- Weak signals for excited states towards smaller pion masses
- Ground state baryons fine, excited state baryons too high: volume squeezing ($Lm_\pi = 6.36, 5.63, 3.66$)?
- Axial charge for N, Σ and Ξ see ArXiv 0910.4190.
- Ongoing: enhancing statistics and parameter sets; other approaches with more interpolators (cf. distillation method)?