Renormalisation group and gravitation

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Lecture I

introduction

- **physics of classical gravity**

  Einstein's theory $\quad G_N = 6.7 \times 10^{-11} \frac{m^3}{\mathrm{kg}\, s^2}$

  classical action

$$S_{\mathrm{EH}} = \frac{1}{16\pi G_N} \int \sqrt{\det g}(-R(g_{\mu\nu}) + 2\Lambda)$$

  valid on length scales $\quad \sim 10^{-2} - 10^{28}\,\mathrm{cm}$

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  short distances: gravity not tested below $10^{-2} cm$

  ample space for 'new' physics:

  e.g. extra dimensions

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  'new' physics:

  dark matter / dark energy

  large distance modifications of gravity

  extra dimensions

introduction

- **physics of classical gravity**

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- **physics of quantum gravity**

  Planck length $\ell_{\mathrm{Pl}} = \left(\frac{\hbar G_N}{c^3}\right)^{1/2} \approx 10^{-33}\,\mathrm{cm}$

  Planck mass $M_{\mathrm{Pl}} \approx 10^{19}\mathrm{GeV}$

  Planck time $t_{\mathrm{Pl}} \approx 10^{-44}\,\mathrm{s}$

  Planck temperature $T_{\mathrm{Pl}} \approx 10^{32}\,\mathrm{K}$

  expect **quantum modifications** at energy scales $E \approx M_{\mathrm{Pl}}$

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- **physics of quantum gravity**

  path integral quantisation

$$\int [dg_{\mu\nu}]_{\mathrm{ren.}} \exp\left(-S[g_{\mu\nu}] + \mathrm{sources}\right)$$

perturbation theory

- **structure of UV divergences**

gravity: $[g_{\mu\nu}] = 0\,, \quad [\mathrm{Ricci}] = 2\,, \quad [G_N] = 2 - d$

effective expansion parameter: $g_{\mathrm{eff}} \equiv G_N\, p^2 \sim \frac{p^2}{M_{\mathrm{Pl}}^2}$

N-loop Feynman diagram $\sim \int dp\, p^{A\,-[G]N}$

$[G] > 0:$ superrenormalisable

$[G] = 0:$ renormalisable

$[G] < 0:$ **dangerous** interactions

perturbation theory

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- **perturbative non-renormalisability**

  gravity with matter interactions

  pure gravity (Goroff-Sagnotti term)

perturbation theory

- **effective theory for gravity** (Donoghue '94)

  quantum corrections computable for energies $E^2/M_{\rm Pl}^2 \ll 1$

  knowledge of UV completion not required

perturbation theory

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  1-loop 'running coupling'

$$G(r) = G_0 \left(1 - \frac{167}{30\pi} \frac{G_0 \hbar}{r^2}\right)$$

perturbation theory

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- **higher derivative gravity I** (Stelle '77)

  R^2 gravity perturbatively renormalisable

  unitarity issues at high energies

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- **higher derivative gravity I** (Stelle '77)

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- **higher derivative gravity II** (Gomis, Weinberg '96)

  all higher derivative operators

  gravity 'weakly' perturbatively renormalisable

  no unitarity issues at high energies

renormalisation group

- **running couplings**

  quantum fluctuations modify interactions

  couplings depend on eg. energy or distance

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- **asymptotic freedom of YM theory**

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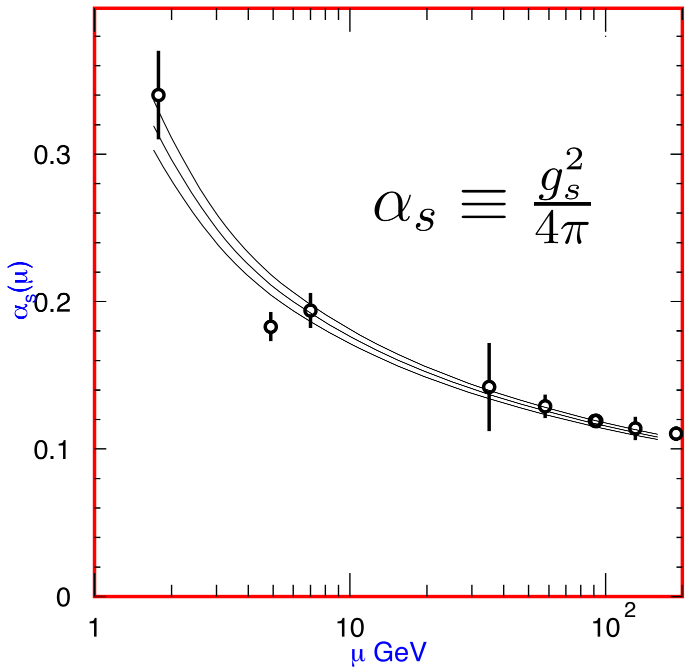
- **asymptotic freedom of YM theory**

  running coupling

  (taken from PDG)



renormalisation group

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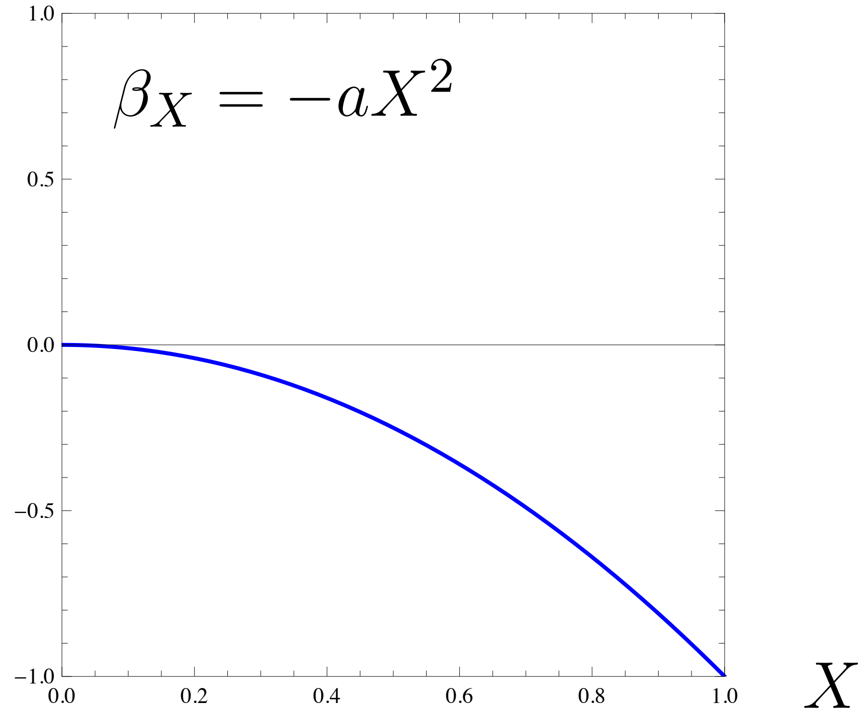
- **asymptotic freedom of YM theory**

  coupling $X = g_s^2/(4\pi)$

  $$\beta_X \equiv \frac{dX}{d\ln\mu}$$

  trivial UV fixed point

  $$X_* = 0$$



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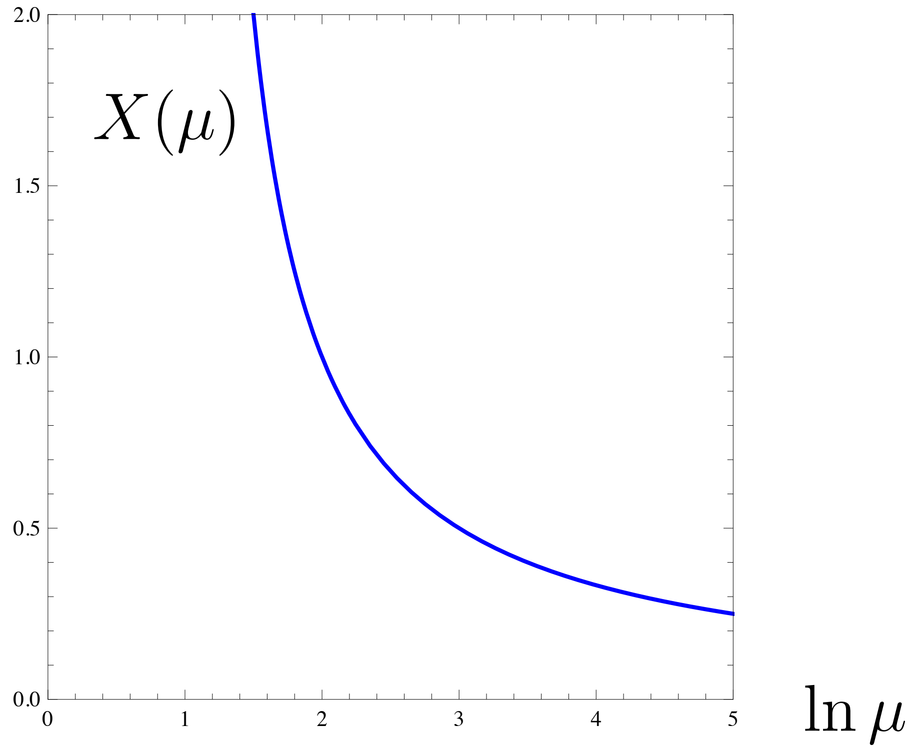
  IR Landau pole

  $$\mu \approx \Lambda_{\rm QCD}$$



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- $U(1)$ **theory**

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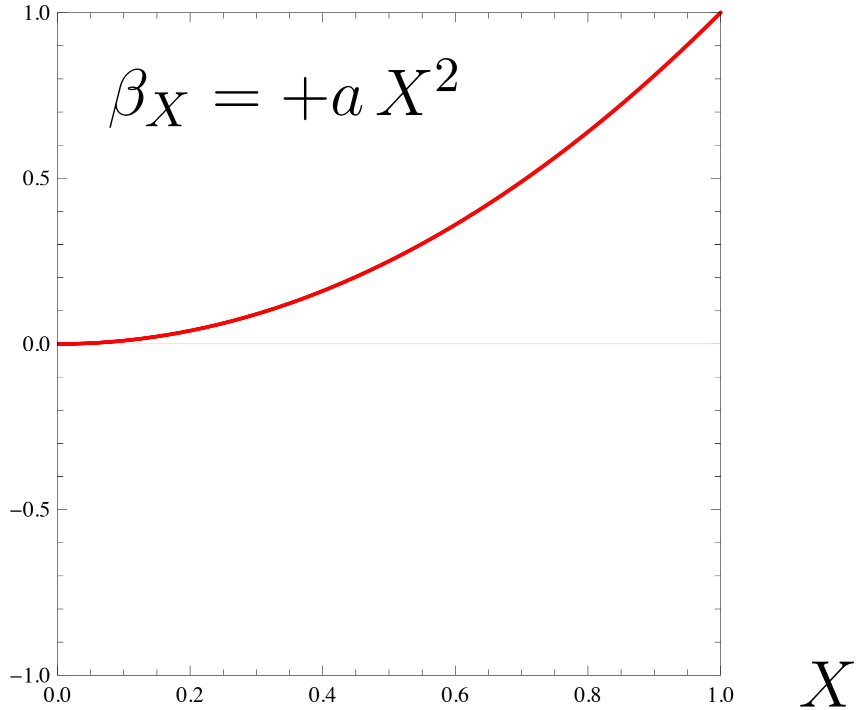

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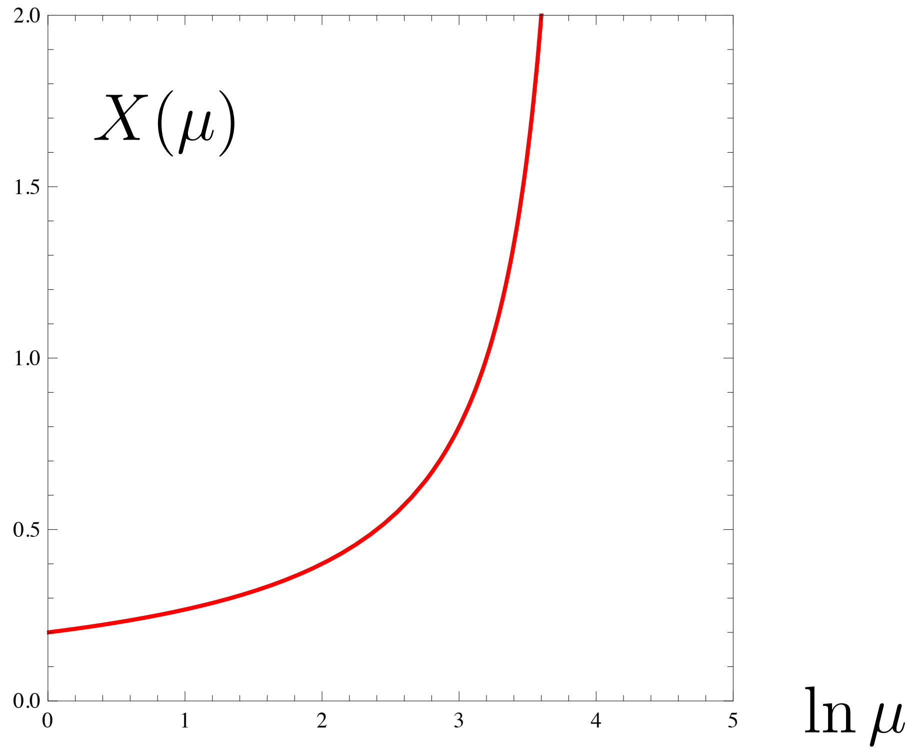

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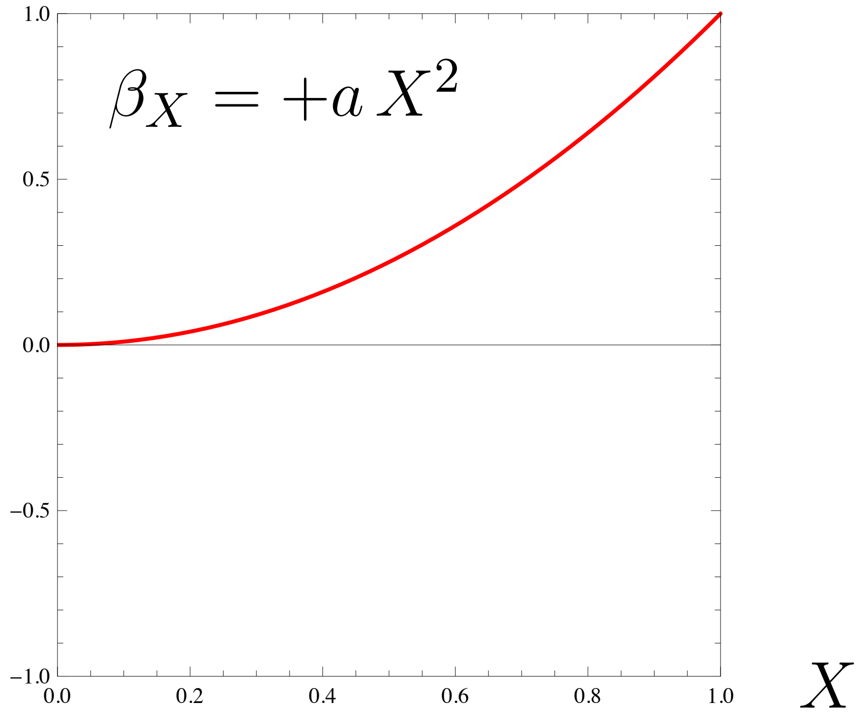

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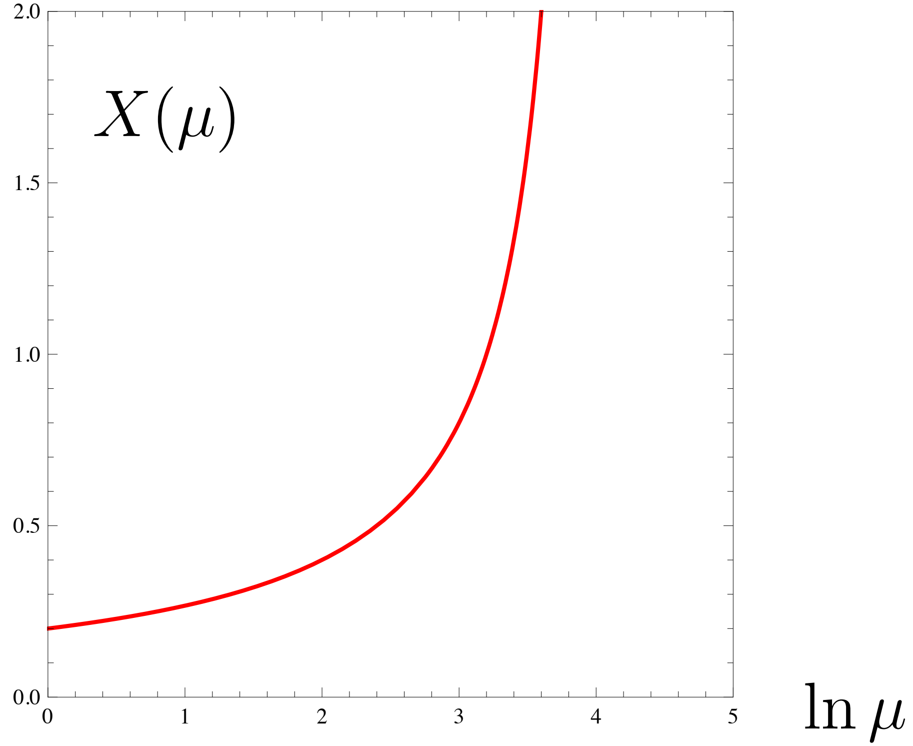

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- **gravitation**

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  replace

$$G_N \to G(\mu)$$

$$g_{\rm eff} = G_N\,\mu^2 \to g(\mu) \equiv G(\mu)\,\mu^2$$

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  coupling $X = G_N \mu^2$

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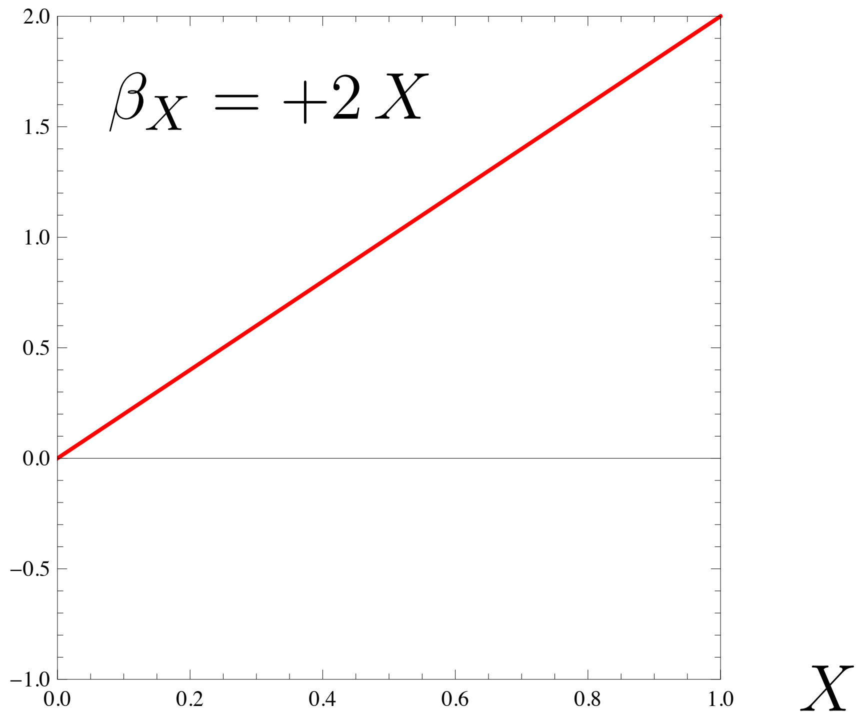
  trivial IR fixed point

  no Landau pole



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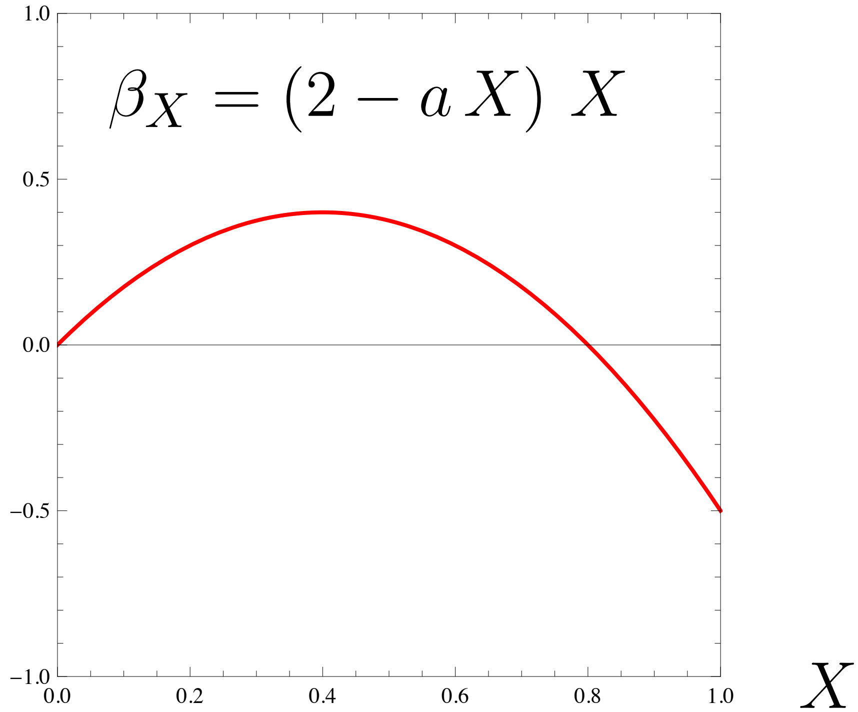
  non-trivial UV fixed point

  $$X_* \neq 0$$



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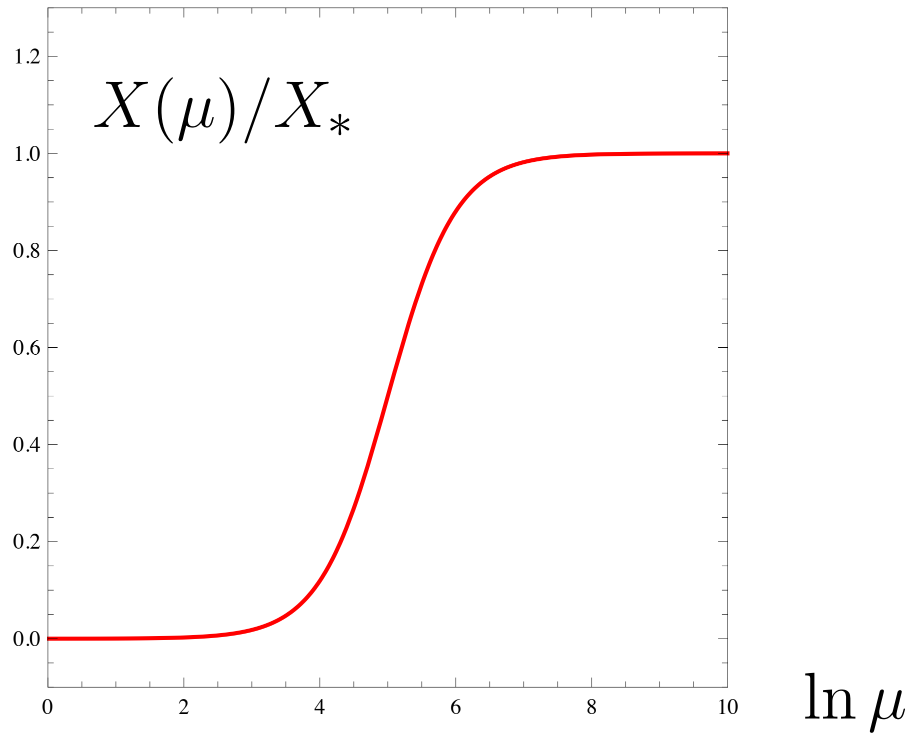
  cross-over from

  UV to IR fixed point



renormalising the un-renormalisable

- **running couplings**

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- **asymptotically safe gravity** (Weinberg '79)

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  non-trivial UV fixed point for gravity

  well-defined continuum limit

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  well-defined continuum limit

  critical trajectory

    stable, marginal, unstable directions

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  predictive power

  finite number of unstable directions

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- **non-gravitational example**

  Gross-Neveu models in $D > 2$ (Gawedzki, Kupiainen, '85)