Properties of gauge orbits

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Why gauge-fixing?
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• Questions requiring gauge-fixing
  • Properties of elementary particles – gluons, Higgs,…
  • Gauge-dependent mechanisms, e.g. confinement
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• Supporting methods beyond the lattice, e.g. DSEs,…

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  • Cold, dense quark matter
  • Disparate scales, like in the standard model
  • Non-equilibrium, scattering cross-sections
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- To access domains like
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  - Disparate scales, like in the standard model
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- For simplification, like in hadron spectroscopy
- Requires (not yet achieved) non-perturbative control
Basics of gauge-fixing

• Important basic quantity: Correlation functions

• Propagators and vertices

• Gauge-dependent in general
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• Local gauge conditions sufficient in perturbation theory

• Landau gauge: $\partial^\mu A^a_\mu = 0$
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• Important basic quantity: **Correlation functions**
  
  • Propagators and vertices
  
  • Gauge-dependent in general

• Requires gauge-fixing to determine

• Local gauge conditions sufficient in perturbation theory
  
  • Landau gauge: \( \partial^\mu A_\mu^a = 0 \)

  • Equivalent: Condition on a correlation function, the gluon propagator: \( p^\mu p^\nu D^{ab}_{\mu\nu} = 0 \)
Non-perturbative gauge-fixing [Gribov 1978, Singer 1978]

- Beyond perturbation theory local conditions insufficient
  - Gribov-Singer ambiguity due to Gribov copies
  - Requires a well-defined and method-independent resolution
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- If two gauges are different they differ at least in one correlation function
Non-perturbative gauge-fixing

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- Correlation functions contain all information
- If two gauges are different they differ at least in one correlation function
- Gauges can be specified by imposing conditions on the correlation functions

[Maas 2008, 2009]
Gauge (re)construction [Maas, 2009]

- Basic building blocks to (re)construct a gauge
Gauge (re)construction  [Maas, 2009]

- **Basic building blocks to (re)construct a gauge**

- **Gluon propagator** $D_{\mu\nu}^{ab}$  [Zwanziger, 1990s+2000s, many others]

- **Total trace** $( - ) \int d^d p D_{\mu\mu}^{aa}$

- **Connected to the fundamental modular domain**
Gauge (re)construction

• Basic building blocks to (re)construct a gauge

• **Gluon propagator** $D_{\mu \nu}^{ab}$
  - **Total trace** $(\; - \;) \int d^d p \, D_{\mu \mu}^{aa}$
  - Connected to the fundamental modular domain

• **Ghost propagator** $D_{G}^{ab}$
  - **B-parameter** $B=\lim_{p \to 0} p^2 D_{G}^{aa}(p)/\mu^2 D_{G}^{aa}(\mu)$
  - Generates a one-parameter family of correlation functions in the continuum
  - **Assume:** Positive only in the 1st Gribov region
Distribution in trD-B-space [Maas 2009, unpublished]

- Projection of the first Gribov region
- Positive Faddeev-Popov operator

3d, 26^3, beta=3.47, 39 copies per configuration
Distribution in $trD-B$-space

$V=(5.7 \text{ fm})^3$

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- Projection of the first Gribov region
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Distribution in $\text{trD-B-space}$ [Maas 2009, unpublished]

- $V=(5.7 \text{ fm})^3$
- $3d, 26^3, \beta=3.47, 39$ copies per configuration

- Projection of the first Gribov region
- Positive Faddeev-Popov operator
- Uncorrelated for different Gribov copies
Constructing gauges – independent of a method

[Maas 2009]

• Select a permitted (set of) constraint(s)
Constructing gauges - independent of a method

[Maas 2009]

- Select a permitted (set of) constraint(s)
- If a complete specification: Done
Constructing gauges – independent of a method

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- Select a permitted (set of) constraint(s)
  - If a complete specification: Done
  - If not: There exist Gribov copies degenerate in the constraints
    - Select randomly among degenerate Gribov copies
    - Resulting correlation functions will be averages over all other possible constraints
Possible gauges [Maas 2009, unpublished]

- **Minimal Landau gauge:**
  No further constraint
Possible gauges [Maas 2009, unpublished]

\[ b = G(0.280 \text{ GeV})/G(\infty \text{ GeV}) \text{ for } V = (4.4 \text{ fm})^3 \]

Average: 2.97

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Possible gauges

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Constructing gauges – independent of a method

[Maas 2009]

- Select a permitted (set of) constraint(s)
  - If a complete specification: Done
  - If not: There exist Gribov copies degenerate in the constraints
    - Select randomly among degenerate Gribov copies
    - Resulting correlation functions will be averages over all other possible constraints
  - Always: Inside first Gribov region
  - Can be implemented in all methods
Possible gauges

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Possible gauges

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- **Absolute Landau gauge:**
  Require minimal $|\text{tr}D|$
Possible gauges

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- **Absolute Landau gauge:**
  Require minimal $|\text{trD}|$

- **MaxB gauge:**
  Require maximum $B$
Possible gauges

- **Minimal Landau gauge:**
  No further constraint

- **Absolute Landau gauge:**
  Require minimal $|\text{tr}D|$ 

- **MaxB gauge:**
  Require maximum B

- **Others possible**
  - Minimize B or trD, combined constraints, averages,...
Obstacles...

- **What are permitted constraints?**
  - Requires knowledge of all Gribov copies: Gribov problem
Severity of the Gribov problem

Number of Gribov copies at $a \approx 0.22 \text{ fm}$

- Number of Gribov copies rises strongly with volume
Severity of the Gribov problem

- Number of Gribov copies rises strongly with volume...
- ...but also with discretization!

[Maas 2009, Mehta et al. 2009]
Obstacles...

- **What are permitted constraints?**
  - Requires knowledge of all Gribov copies: Gribov problem
  - Requires eg to know permitted range of B or trD value
Permitted corridors

\[ \text{trD corridor} \]

\[ \text{Maas 2009, unpublished} \]

\[ \text{3d, } a=0.22 \text{ fm} \]

- \textbf{trD: (small) range scales strongly with discretization}
Permitted corridors  

- trD: (small) range scales strongly with discretization
- B: Opens up with volume

[Maas 2009, unpublished]
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- **How many constraints are possible?**
  - Unknown – but no hints for more than just one
  - Known constraints are related to free renormalization conditions – relevance?
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- Unspecified constraints: Outside lattice gauge theory knowledge of undetermined averages required
Ghosts and gluons (in 3d) [Maas 2009, unpublished]

- **Ghosts strongly dependent up to 1 GeV**

  Bornyakov et al. 2009

[Compare: e.g. Bornyakov et al. 2009]
Ghosts and gluons (in 3d) [Maas 2009, unpublished]

- Ghosts strongly dependent up to 1 GeV
- Gluons not very sensitive beyond 100 MeV

[Compare: e.g. Bornyakov et al. 2009]
**Coupling and matter** [Maas 2009, unpublished]

- Derived quantities inherit dependencies

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*Effective coupling*

![Graph showing the relationship between effective coupling (\(\alpha/p\)) and momentum (p) for 3d, L=4.4 fm, a=0.17 fm.](image)
Coupling and matter [Maas 2009, unpublished]

Effective coupling

- Derived quantities inherit dependencies
- Matter fields weakly influenced

Scalar - Confinement phase

Renormalized mass is 1 at 1.5
- Minimal Landau gauge
- MaxB Landau gauge
- Absolute Landau gauge

\[ 1/(p^2 + 1^2) \]

3d, \( L=4.4 \text{ fm}, a=0.17 \text{ fm} \)

4d, confinement: beta=2.0, kappa=0.25, lambda=0.5
Summary

Read more: arXiv: 0808.3047, 0810.1987, 0907.5185
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• **Gauge-fixing is necessary for many questions and practical applications**

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- **Gauge-fixing is necessary for many questions and practical applications**

- **Understanding gauge-fixing is required to have control in these cases**

- **Beyond perturbation theory effects up-to the characteristic scale (1 GeV for QCD) in some quantities**

Read more: arXiv: 0808.3047, 0810.1987, 0907.5185
Summary

- Gauge-fixing is necessary for many questions and practical applications
- Understanding gauge-fixing is required to have control in these cases
- Beyond perturbation theory effects up-to the characteristic scale (1 GeV for QCD) in some quantities
- Well-defined non-perturbative gauges possible
  - Construction based on correlation functions
  - Can provide a well-defined framework
  - Decoupling vs. scaling is possibly just a gauge choice?

Read more: arXiv: 0808.3047, 0810.1987, 0907.5185