Towards Higgs Sector Spectroscopy

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TRIUMF
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What is non-perturbative?
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- Strong interactions are non-perturbative
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  - But not always: Asymptotic freedom
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  • Bound states, phase transitions,…
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- Weak interactions can be non-perturbative
  - QED is weakly interacting, but has non-perturbative features like atoms, molecules, matter with phase structure,…
  - Bound states, phase transitions,…
- Are there (relevant) non-perturbative effects in the weak interactions and the Higgs?
Overview

• The Standard model and the Higgs
Overview

• The Standard model and the Higgs
• Standard approach to the Higgs
Overview

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• Full non-perturbative treatment
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• Particle/bound-state duality
• Consequences
  • Spectroscopy and excited states
• Summary
The Standard Model
Particles

• The standard model (until now) describes the physics accessible in accelerator-based experiments
Particles

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• Contains two kinds of particles
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• Force particles
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    - Photon
Particles

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• Matter
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• Force particles
  • Photon, gluon
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- Higgs is a bit of both, but more like matter
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    • 6 flavors of leptons

  • Force particles
    • Photon, gluon, W- and Z-boson

• Higgs is a bit of both, but more like matter
  • Possibly recently observed
The properties of the particles

- The properties of the particles are different
The properties of the particles

- The properties of the particles are different

Quarks: Fermions

Masses:
Up: 2-3 MeV
Down: 4-6 MeV
Strange: 80-130 MeV
Charm: 1270(10) MeV
Bottom: 4190(200) MeV
Top: 172000(1500) MeV
The properties of the particles

- The properties of the particles are different

Quarks: Fermions
- u, c, t
- d, s, b

Leptons: Fermions
- \( \nu_e, \nu_\mu, \nu_\tau \)
- e, \( \mu \), \( \tau \)

Masses:
- Up: 2-3 MeV
- Down: 4-6 MeV
- Strange: 80-130 MeV
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- Top: 172000(1500) MeV

- Electron: 0.5 MeV
- Muon: 106 MeV
- Tauon: 1777 MeV

Neutrinos:
- Masses < 0.3 eV
- Mass hierarchy unknown
- Masses are different
The properties of the particles

- The properties of the particles are different

<table>
<thead>
<tr>
<th>Quarks: Fermions</th>
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**Photon:** Massless boson
The properties of the particles

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\(\gamma\) Photon: Massless boson
\(g\) Gluon: Massless boson
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\(\gamma\) Photon: Massless boson
\(g\) Gluon: Massless boson

Weak gauge bosons

\(\text{W} \quad \text{Z}\)
- W: 80375(23) MeV
- Z: 91188(2) MeV
The properties of the particles

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Quarks: Fermions
- u
- c
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Leptons: Fermions
- $\nu_e$
- $\nu_\mu$
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- e
- $\mu$
- $\tau$

Higgs: Boson
- $h$

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- Neutrinos:
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Masses are different

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- $\gamma$

Gluons: Massless bosons
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Weak gauge bosons
- $W$
- $Z$

Higgs: Boson
- Mass: 125000 MeV?

Sectors

\[ \text{u, c, t, g, h} \]
\[ \text{d, s, b, W} \]
\[ \text{\(\nu_e\), \(\nu_\mu\), \(\nu_\tau\), Z} \]
\[ \text{e, \(\mu\), \(\tau\), \(\gamma\)} \]
Sectors

- The force particles mediate the forces between the matter particles
• The force particles mediate the forces between the matter particles

• Each force particle can be associated with a particular force or sector of the standard model
Sectors

- Particles can be grouped according to the forces
Sectors

- Particles can be grouped according to the forces
  - Electromagnetic sector
Sectors

- Particles can be grouped according to the forces
  - Electromagnetic sector
  - The strong sector

- Electromagnetic sector
- Strong sector

Particles: $\nu_e, \nu_\mu, \nu_\tau$
Particles can be grouped according to the forces:

- **Electromagnetic sector**
- **The strong sector**
- **The weak sector**
## Sectors

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<thead>
<tr>
<th>Strong sector</th>
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Sectors

- Particles can be grouped according to the forces
  - Electromagnetic sector
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  - The weak sector
  - The Higgs sector – actually 13 different interactions
The Higgs sector
The Higgs effect

- The Higgs is assumed to create much of the mass
The Higgs effect

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- Mechanism: “Higgs condenses”
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- Mechanism: “Higgs condenses and the particles are slowed (gain mass)”
The Higgs effect

- The Higgs is assumed to create much of the mass
- Mechanism: “Higgs condenses and the particles are slowed (gain mass)”
- Higgs only particle which has a static mass
The elusiveness of the Higgs
The elusiveness of the Higgs

- Higgs couples to a particle proportional to its mass
The elusiveness of the Higgs

- Higgs couples to a particle proportional to its mass

[ATLAS & CMS, '11+'12 data]

CMS Preliminary
\[ \bar{s} = 7 \text{ TeV}, L = 5.1 \text{ fb}^{-1} \]
\[ \bar{s} = 8 \text{ TeV}, L = 5.3 \text{ fb}^{-1} \]

\[ \Sigma \text{ weights / } 2 \text{ GeV} \]

\[ m_{\gamma \gamma} \text{ (GeV)} \]
The elusiveness of the Higgs

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- Counting experiment – no theory input

[ATLAS & CMS, '11+'12 data]
The elusiveness of the Higgs

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- Possibly observed at 125 GeV – Assume to be right
The elusiveness of the Higgs

- Higgs couples to a particle proportional to its mass
- Counting experiment – no theory input
- Possibly observed at 125 GeV – Assume to be right
  - Required $\sim 10$ fb$^{-1}$ to observe
The trouble with the Higgs

• Why is there a problem?
The trouble with the Higgs

- Why is there a problem?
- Higgs mass is very sensitive to quantum corrections
  - Standard model coupling constants fine-tuned by 14 orders of magnitude (!) for a 125 GeV Higgs mass
  - So-called hierarchy problem
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The trouble with the Higgs

[Introduction: Morrissey et al. '09]

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The trouble with the Higgs

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- Consistency of a theory with Higgs is unproven
  - Theory could be only valid with a cutoff (trivial)
- The coupling of the Higgs to matter is arbitrary
  - Only description of the masses of quarks and leptons
- Triggered many proposals for alternatives
  - Supersymmetry, Technicolor,...
The basic task

- Describe the Higgs sector of the standard model
The basic task

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• 12+ orders of magnitude of scales
  • Neutrino mass to electroweak scale
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• Strongly-interacting chiral gauge theory
The basic task

- Describe the Higgs sector of the standard model
- 12+ orders of magnitude of scales
  - Neutrino mass to electroweak scale
- Chiral theory
- Strong interactions of the quarks – QCD
- Strongly-interacting chiral gauge theory
- Simplify: Just weak gauge bosons and the Higgs
The task at hand

• Describe W and the Higgs
The task at hand

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  - Higgs sector alone well understood [Callaway, PR'88]
    - So-called sigma model
    - Only with a cutoff well-defined
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  • W sector alone more complicated \[\text{Maas, PR'13}\]
    • Yang-Mills theory
    • Gauge theory
    • Strongly interacting
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  [Lang et al., Münster et al., ALPHA collaboration, Wittig et al., Jansen et al., Rummukainen et al.]
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    • Gauge theory
    • Strongly interacting

• Next stop: Higgs+W sector [Lang et al., Münster et al., ALPHA collaboration, Wittig et al., Jansen et al., Rummukainen et al.]

• Also investigations of Higgs+Yukawa [Gerhold et al. PLB’11]
Standard description
Standard description: Perturbation theory
The Higgs sector as a gauge theory

- The Higgs sector is a gauge theory
The Higgs sector as a gauge theory

- The Higgs sector is a gauge theory

\[ L = -\frac{1}{4} W^a_{\mu\nu} W^{\mu\nu}_a \]

\[ W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu \]

- WS

\[ W^a_\mu \]
The Higgs sector as a gauge theory

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\[ L = -\frac{1}{4} W^a_{\mu \nu} W^{\mu \nu}_a \]

\[ W^a_{\mu \nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g f^{a}_{b c} W^b_{\mu} W^c_\nu \]

- Ws

- Coupling \( g \) and some numbers \( f^{abc} \)
The Higgs sector as a gauge theory

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- Ws

\[ W^a_\mu \]

- No QED: Ws and Zs are degenerate

- Coupling \( g \) and some numbers \( f^{abc} \)
The Higgs sector as a gauge theory

- The Higgs sector is a gauge theory

\[ L = -\frac{1}{4} W^a_{\mu\nu} W^a_{\mu\nu} + (D^i_{\mu} h^i) + D^i_{ik} h_k \]

\[ W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + gf^{a}_{bc} W^b_{\mu} W^c_{\nu} \]

\[ D^i_{\mu} = \delta^i_{\mu} \partial_\mu \]

- Ws \quad W^a_\mu

- Higgs \quad h_i

- No QED: Ws and Zs are degenerate

- Coupling \( g \) and some numbers \( f^{abc} \)
The Higgs sector as a gauge theory

- The Higgs sector is a gauge theory

\[ L = -\frac{1}{4} W^a_{\mu\nu} W^{\mu\nu}_a + \left(D^{ij}_\mu h^j\right) + D^\mu_{ik} h_k \]

\[ W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g f^{a}_{bc} W^b_\mu W^c_\nu \]

\[ D^{ij}_\mu = \delta^{ij} \partial_\mu - ig W^a_\mu t^{ij}_a \]

- Ws

\[ W^a_\mu \]

- Higgs

\[ h_i \]

- No QED: Ws and Zs are degenerate

- Coupling \( g \) and some numbers \( f^{abc} \) and \( t^{ij}_a \)
The Higgs sector as a gauge theory

- The Higgs sector is a gauge theory

\[
L = -\frac{1}{4} W^a_{\mu \nu} W^{\mu \nu}_a + (D^i j h^j)^+ D^i k h_k + \lambda (h^a h^+_a - \nu^2)^2
\]

- Ws

\[
W^a_{\mu \nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g f^{a}_{\mu} W^b_\mu W^c_\nu
\]

- Higgs

\[
D^i j = \delta^i j \partial_\mu - i g W^a_\mu t^i^a
\]

- No QED: Ws and Zs are degenerate

- Couplings g, \nu, \lambda and some numbers \(f^{abc}\) and \(t^i^a\)
Symmetries

\[ L = - \frac{1}{4} W^a_{\mu \nu} W^a_{\mu \nu} + (D^i_j h^j) + D^i_{ik} h_k + \lambda (h^a h^+_a - v^2)^2 \]

\[ W^a_{\mu \nu} = \partial^a_{\mu} W^{a}_{\nu} - \partial^a_{\nu} W^{a}_{\mu} + g f^a_{bc} W^b_{\mu} W^c_{\nu} \]

\[ D^i_{ij} = \delta^i_j \partial^i_{\mu} - ig W^a_{\mu} t^a_{ij} \]
Symmetries

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L = -\frac{1}{4} W^a_{\mu \nu} W_{a \mu \nu} + (D^j_{\mu} h^j) + D_{ik}^u h_k + \lambda (h^a h_a^+ - \nu^2)^2
\]

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W^a_{\mu \nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + gf_{bc}^a W^b_\mu W^c_\nu
\]

\[
D_{ij}^\mu = \delta_{ij} \partial_\mu - ig W^a_\mu t_{ij}^a
\]

• Local SU(2) gauge symmetry

  • Invariant under arbitrary gauge transformations \( \phi^a(x) \)

\[
W^a_\mu \rightarrow W^a_\mu + (\delta_b^a \partial_\mu - g f_{bc}^a W^c_\mu) \phi^b
\]

\[
h_i \rightarrow h_i + g t_{ij}^a \phi^a h_j
\]
Symmetries

\[
L = -\frac{1}{4} W^a_{\mu \nu} W^a_{\mu \nu} + (D^i_j h^j) + D^i_k h^k + \lambda (h^a h^a + - v^2)^2
\]

\[
W^a_{\mu \nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + gf^{a}_{bc} W^b_\mu W^c_\nu
\]

\[
D^i_j = \delta^{ij} \partial_\mu - ig W^a_\mu t^i_a
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- Local SU(2) gauge symmetry
  - Invariant under arbitrary gauge transformations \( \phi^a(x) \)
    \[
    W^a_\mu \rightarrow W^a_\mu + (\delta^a_b \partial_\mu - gf^{a}_{bc} W^c_\mu) \phi^b
    \]
    \[
    h_i \rightarrow h_i + g t^i_j \phi^a h_j
    \]
- Global SU(2) Higgs flavor symmetry
  - Acts as right-transformation on the Higgs field only
    \[
    W^a_\mu \rightarrow W^a_\mu
    \]
    \[
    h_i \rightarrow h_i + a^i_j h_j + b^i_j h_j^*
    \]
Elementary states

\[ L = -\frac{1}{4} W^a_{\mu \nu} W^\mu_\nu + (D^i_j h^j) + D^\mu_{ik} h_k + \lambda (h^a h^a + - \nu^2)^2 \]

\[ W^a_{\mu \nu} = \partial_{\mu} W^a_\nu - \partial_{\nu} W^a_\mu + gf^{a}_{bc} W^b_\mu W^c_\nu \]

\[ D^i_j = \delta^i_j \partial_{\mu} - igW^a_\mu t^{ij}_a \]
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• Higgs field is not gauge-invariant
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\]

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• Higgs field is not gauge-invariant

• Irrespective of phase

[Caudy & Greensite PRD 2008
Maas, EPJC 2011
Philipsen et al. NPB 1996]
Elementary states

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- Higgs field is not gauge-invariant
  - Irrespective of phase
  - Neither is the Higgs expectation value
  - Same applies to the W bosons
  - Higgs pole mass is not renormalization-group invariant
Composite states

• Only bound states and cross sections
gauge-invariant

[Fröhlich et al. PLB 80,
't Hooft ASIB 80,
Bank et al. NPB 79]
Composite states

- Only bound states and cross sections gauge-invariant
  - Higgs-Higgs
Composite states

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• Higgs-Higgs, W-W
Composite states

- Only bound states and cross sections gauge-invariant
- Higgs-Higgs, W-W, Higgs-Higgs-W etc.

[Fröhlich et al. PLB 80, 't Hooft ASIB 80, Bank et al. NPB 79]
Composite states

• Only bound states and cross sections gauge-invariant
  • Higgs-Higgs, W-W, Higgs-Higgs-W etc.

• Applies also to full standard model
  • Also fermions, except for right-handed neutrinos
Classical analysis

\[ L = -\frac{1}{4} W_{\mu \nu}^a W_{a \mu \nu} + (D_{\mu}^{ij} h^j)^+ + D_{ik}^{\mu} h_k + \lambda (h^a h_a^+ - \nu^2)^2 \]
Classical analysis

\[ L = \lambda \left( h^a h^+_a - \nu^2 \right)^2 \]

- Classical analysis of the Higgs sector

[Bohm et al. 2001]
Classical analysis

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- Experiments decides
  - Higgs mass is tachyonic

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Classical minima

- Classical analysis of the Higgs sector
Classical analysis

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Higgs potential

- Shape depends on parameters
- Experiments decides
  - Higgs mass is tachyonic
- Classical minimum
- Global gauge choice

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- Non-zero condensate shifts Higgs mass to an ordinary mass.
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[Bohm et al. 2001]
Classical analysis

\[ L = \lambda \left( h^a h^+_a - v^2 \right)^2 \]

- Classical analysis of the Higgs sector
- Non-zero condensate shifts Higgs mass to an ordinary mass
- Perform perturbative expansion around the classical vacuum

Shape depends on parameters

Experiments decides
- Higgs mass is tachyonic

Classical minimum

Global gauge choice

[Bohm et al. 2001]
Standard approach

• Minimize action classically
  • Yields \( hh^+ = v^2 \) - Higgs vev
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$$h(x) = \begin{bmatrix} \phi^1(x) + i \phi^2(x) \\ v + \eta(x) + i \phi^3(x) \end{bmatrix} \Rightarrow \langle h \rangle = \begin{bmatrix} 0 \\ v \end{bmatrix}$$

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- $\eta$ mass depends at tree-level on $v$
- Perform perturbation theory
Implications of global transformation

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  • Local symmetry intact and cannot be broken

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- Consequence: Symmetry in charge space not manifest (hidden)
  - Complicated charge tensor structures
  - Symmetry expressed in STIs/WTIs
Masses of the particles

- Higgs mass is well-defined
  - Very sensitive to parameters
  - Hierarchy problem
Masses of the particles

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  • Only possible for SU(2)

[Bohm et al. 2001]
Masses of the particles

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- Fermions acquire masses due to Yukawa coupling to the Higgs
  - Neglected here
Challenges and problems

• Approach works very well in perturbation theory
  • Experimental data well described
  • Quantum corrections to classical physics small
• Only self-consistent for a light Higgs (<0.5-1 TeV)
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• Non-perturbative origin?

[Bohm et al. 2001]
Beyond perturbation theory
Non-aligned gauges

• Explicit charge direction inconvenient beyond perturbation theory
Non-aligned gauges

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    - Aligned Landau gauges also possible
Higgs vev

\[ \langle \theta \rangle \text{ [a.u.]} \]

- 'Higgs phase'
- 'Confinement phase'

\[ 0 \quad 0.05 \quad 0.1 \quad 0.15 \quad 0.2 \quad 0.25 \quad 0.3 \]

\[ \frac{1}{L} \text{ [a.u.]} \]
Differentiating phases

• How to distinguish phases?

[Maas, EPJC'11, '12]
Differentiating phases

- How to distinguish phases?
- Relative orientation $\langle \int hdx \int hdy \rangle$
  - $\int hdx$ is the magnetization
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Phase diagram (sketch)

\[
\langle (\theta)^2 \rangle \quad [\text{a.u.}]
\]

\[
\int hdx \quad \int hdy
\]

\[
\int \quad \text{g(Classical gauge coupling)}
\]

\[
f(\text{Classical Higgs mass})
\]

\[
g(\text{Classical gauge coupling})
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Phase diagram (sketch)

Higgs phase

Confinement phase

1st order or crossover

Coulomb gauge

Landau gauge

$g$ (Classical gauge coupling)

$\frac{1}{L}$ [a.u.]
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- No classical input – full non-perturbative calculation
  - Higgs mass dynamically non-tachyonic in a full calculation?
  - How is the Higgs mass determined?

[Maas, EPJC'11, '12]
Gauge-dependent correlators

• Gauge-dependent correlators are different
Gauge-dependent correlators

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  • Can have multiplicative and additive renormalizations
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    - Nielsen identities only guarantee gauge-parameter-independency but not gauge-invariance

[Nielsen NPB'75]
Gauge-dependent correlators

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    • Simpler to perform in momentum space
  • Pole masses not necessarily physical
    • Scheme- or scale-dependency
    • Nielsen identities only guarantee gauge-parameter-independency but not gauge-invariance
      [Nielsen NPB'75]
  • Not necessarily positive semi-definite
    • Gauge-dependent states are not part of the physical Hilbert space
Masses from Euclidean propagators

- No exact results on time-like momenta
- Masses can be inferred from Fourier transform
- Long-time behavior relevant
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Gluons

- Positivity violations observed for gluons in Yang-Mills theory
- Precise structure not yet known
- Similar results in 4d [Bowman et al. PRD'07]
- Expensive calculations
Setup

\[ L = - \frac{1}{4} W_{\mu \nu}^a W_{\alpha}^{\mu \nu} + (D_{\mu}^i h_j^j)^+ D_{ik}^\mu h_k + \lambda (h_a^a h_a^+ - \nu^2)^2 \]

- Tree-level setup Higgs+W
Setup

\[ L = -W^a_{\mu \nu} W^{\mu \nu}_a/4 + (D^j_\mu h^j)^+ D^\mu_{ik} h_k + \lambda (h^a h^+_a - v^2)^2 \]

- Tree-level setup Higgs+W
  - Aligned gauge
    - W mass: 80.375 GeV (sets the scale)
    - Higgs mass: 157.6 GeV, Higgs vev: 246 GeV
Setup

\[ L = -W^a_{\mu \nu} W^a_{\mu \nu} / 4 + (D^{ij}_\mu h^j)^+ D^i_{ik} h^i_k + \lambda (h^a h^{a\dagger} - v^2)^2 \]

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    - W mass, Higgs vev both zero
    - Higgs mass: 78.8i GeV
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- Translate both to tree-level parameters
  - v=246 GeV
  - \(\lambda=0.0513\) (4!\(\lambda=1.23\))
  - g=0.641 (\(\alpha=0.0327\))
Setup

\[ L = -W_{\mu \nu}^a V_{\nu}^\mu W_{a}^{\mu \nu} / 4 + (D^{ij}_\mu h^j)^+ D^\mu_{ik} h_k + \lambda (h^a h^+_a - v^2)^2 \]

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- **Quite perturbative**
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- Quite perturbative
- Non-perturbative: Simulate Higgs+W [Maas EPJC'11, PR'13]
Renormalization scheme with

\[ D(\mu) = \frac{1}{\left(\mu^2 + (80.375 \, \text{GeV})^2\right)} \quad \wedge \mu = 80.375 \, \text{GeV} \]
W boson

- Renormalization scheme with
  \[ D(\mu) = \frac{1}{\mu^2 + (80.375 \text{ GeV})^2} \land \mu = 80.375 \text{ GeV} \]

- Massive-like propagator

- Dynamical mass generation
W boson

Fit type | Mass  | Remark
---------|-------|--------
Screening mass | 79.1(2) GeV | RG dependent
**W boson**

### Schwinger function

![Schwinger function graph](image)

<table>
<thead>
<tr>
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[Maas, EPJC 2011, Maas unpublished, $24^4$, $\beta=2.3$, $\kappa=0.32$, $\lambda=1$]

Slides left: 1/16
# W boson

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### Schwinger function

- **Lattice data**
- **Tree-level particle fit**
- **Stable particle fit**

### W propagator

- **D(p) [GeV^-2]**
  - **10^3**
  - **10^4**
  - **10^5**

<table>
<thead>
<tr>
<th>t [GeV^-1]</th>
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<tr>
<td>0.005</td>
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- **24^a, β=2.3, κ=0.32 λ=1**
W boson

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W boson

**Schwinger function**

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Screening mass | $79.1(2)$ GeV | RG dependent
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Configuration space | $79(4)$ GeV |
**W boson**

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<tr>
<td>Simple pole</td>
<td>71.2(2) GeV</td>
<td>Fit in momentum space</td>
</tr>
<tr>
<td>With cut</td>
<td>71.2(2) GeV</td>
<td></td>
</tr>
<tr>
<td>Unstable</td>
<td>71.8(1) GeV</td>
<td>Width: 2.1(4) GeV</td>
</tr>
<tr>
<td>Configuration space</td>
<td>79(4) GeV</td>
<td></td>
</tr>
</tbody>
</table>
Higgs boson

- Renormalization scheme with

\[ D(\mu) = D^{tl}(\mu) \]
\[ D(\mu)' = D^{tl}(\mu)' \]
\[ D^{tl}(p) = \frac{1}{(p^2 + (157.6 \text{ GeV})^2)} \]
\[ \mu = 157.6 \text{ GeV} \]
Higgs boson

- Normal propagator – normal mass
Higgs boson

Schwinger function

Higgs propagator

Fit type | Pole mass | Remark
---|---|---
Screening mass | 158.4(1) GeV | RG dependent
Higgs boson

Screening mass

Simple pole

Fit type

Pole mass

Remark

158.4(1) GeV

RG dependent

158.9(5) GeV

RG dependent

Maas, EPJC 2011
Maas unpublished
24ε, β=2.3, κ=0.32 λ=1
**Higgs boson**

**Schwinger function**

- Lattice data
- Tree-level particle fit
- Stable particle fit

**Higgs propagator**

- $D_H(p)$ vs. $p$ for various fits.

<table>
<thead>
<tr>
<th>Fit type</th>
<th>Pole mass</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Screening mass</td>
<td>158.4(1) GeV</td>
<td>RG dependent</td>
</tr>
<tr>
<td>Simple pole</td>
<td>158.9(5) GeV</td>
<td>RG dependent</td>
</tr>
<tr>
<td>With cut</td>
<td>158.9(5) GeV</td>
<td></td>
</tr>
</tbody>
</table>
Higgs boson

Fit type | Pole mass | Remark
--- | --- | ---
Screening mass | 158.4(1) GeV | RG dependent
Simple pole | 158.9(5) GeV | RG dependent
With cut | 158.9(5) GeV | 
Unstable | 158.5(9) GeV | Width 0.9(8) GeV
Different renormalization scheme with mass 90 GeV

<table>
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<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>Screening mass</td>
<td>95.0(1) GeV</td>
<td>RG dependent</td>
</tr>
<tr>
<td>Simple pole</td>
<td>87.8(3) GeV</td>
<td>RG dependent</td>
</tr>
<tr>
<td>With cut</td>
<td>80.1(2) GeV</td>
<td></td>
</tr>
<tr>
<td>Unstable</td>
<td>88.1(5) GeV</td>
<td>Width 1.6+2.0−0.7 GeV</td>
</tr>
</tbody>
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Bound states
Bound states & Consequences
Bound states

• Described by composite operators
  • E.g. Higgsonium $h^+ (x) h(x) = O(x)$
Bound states

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• Masses of ground and excited states obtained from exponential decays
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  • Bound state propagation described by propagators of composite operators, e.g.
    $\langle O^+ (x) O(y) \rangle$

• Masses of ground and excited states obtained from exponential decays

• Physical states with positive semi-definite propagators
  • Excited states appear as change of slope
  • Decays and widths complicated to extract
Higgsonium

- Simplest $0^{++}$ bound state $h^+ (x) h(x)$
Higgsonium

- Simpelst $0^{++}$ bound state $h^+ (x) h(x)$
- Same quantum numbers as the Higgs
- No weak or flavor charge
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- Same quantum numbers as the Higgs
  - No weak or flavor charge
- Mass is about 154 GeV

Lattice data

Fit with lowest mass $154.3^{+1.1}_{-4.9}$ GeV

[Maas unpublished, PoS'11, $24^4$, $\beta=2.3$, $\kappa=0.32$ $\lambda=1$]
Mass relation - Higgs

- Higgsonium: 154 GeV,
- Higgs at tree-level: 159 GeV
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    \[ \langle (h^+ h)(x)(h^+ h)(y) \rangle \]

[Fröhlich et al. PLB 80 Maas'12]
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Mass relation - W

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  \[ \langle (h^+ D_\mu h)(x)(h^+ D_\mu h)(y) \rangle \]
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  • At tree-level same resonances in cross section
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Consequences I – W and Higgs

- Similar relations hold for fermions
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• Precise results require bound-state-bound-state scattering

[Fröhlich et al. PLB 80 Maas'12]
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- Triviality problem remains
Consequences III – Excited states [Maas PoS’11, unpublished]

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- Complete spectrum?

[Maas PoS'11, unpublished]
Low-lying spectrum

PRELIMINARY
Low-lying spectrum

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Low-lying spectrum

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  - Pseudoscalars, higher-spin states,...
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• Requires full understanding of bound state dynamics
Experimental consideration

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- On-resonance production at ILC?
Strategy

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  • Time-like: Not accessible on lattice
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Outlook

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