Formulation of electroweak pion decays in functional methods

Walid Ahmed Mian


2017, 29th Nov.
Motivation

- System of binary neutron stars mergers: Source of gravitational waves (recently measured Abbott et al. PRL 119, 161101 (2017))
- Measurement shows the inner structure of neutron star mergers (Graduate Days)

- Electroweak interactions play an important role
- Consider QCD + electroweak interactions non-perturbatively
Full resolution of electroweak interactions is complicated

- decay captures the main features

Look at the $\pi^{\pm}$-decay

Electroweak interactions approximated by 4-Fermi-interaction

Electroweak interactions violate parity

$$\mathcal{L}_{4\text{-Fermi}} = g_w \left\{ \left[ \bar{\psi}_\nu \gamma^\mu \psi_e^L \right] \left[ \bar{\psi}_u \gamma^\mu \psi_d^L \right] + \left[ \bar{\psi}_e \gamma^\mu \psi_\nu^L \right] \left[ \bar{\psi}_d \gamma^\mu \psi_u^L \right] \right\}$$

- $\psi^L$: Left-handed fermion fields

http://hyperphysics.phy-astr.gsu.edu/hbase/particles/proton.html
**β-decay**

- Full resolution of electroweak interactions is complicated
- β-decay captures the main features
- Look at the $\pi^\pm$-decay
- Electroweak interactions approximated by 4-Fermi-interaction
- Electroweak interactions violate parity

The 4-Fermi interaction is given by:

$$
\mathcal{L}_{4\text{-Fermi}} = g_w \left\{ \left[ \bar{\psi}_\nu \gamma^\mu \psi_e^L \right] \left[ \bar{\psi}_u \gamma^\mu \psi_d^L \right] + \left[ \bar{\psi}_e \gamma^\mu \psi_\nu^L \right] \left[ \bar{\psi}_d \gamma^\mu \psi_u^L \right] \right\}
$$

- $\psi^L$: Left-handed fermion fields
Functional Methods

- Very different energy scales + parity violation
  ⇒ Lattice calculation unfeasible
- Functional approaches (Dyson-Schwinger-Equations (DSEs), Bethe-Salpeter-Equations (BSEs), Functional Renormalization Group (FRG), ...) suitable methods
  1. Continuum, covariant and non-perturbative formulation
  2. High and low energy scales accessible at the same time
- Drawback: Infinite tower of coupled, nonlinear integral equations
- Need truncations
- Correlation functions in Minkowski-space: Access to dynamical observables
- Mass and decay-width of the particle: Need poles of the propagator in Minkowski-space
- Extend method to complex momenta
Resonances in the 2nd Riemann sheet

- Resonances: Pole in the 2nd Riemann sheet
  (Haag, Local Quantum Physics Fields, Particles, Algebras)
- Minkowski-space
  \[ s = P^2 \]
- Euclidean-space
  \[ P^2 \rightarrow -P^2 = s_E \]

\[ z = \pm \text{Im}(\sqrt{x + iy}) = \pm \sqrt{\rho} \sin\left(\frac{\phi}{2}\right) \]
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\[
M < \frac{\Gamma}{2} \\
M > \frac{\Gamma}{2}
\]

\[
s = (M^2 - \frac{\Gamma^2}{4}) - iM\Gamma
\]
Resonances: Pole in the 2nd Riemann sheet

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- Minkowski-space
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DSEs for the Propagators

\[-1 = -1 + \]

\[\Rightarrow \text{Charge conservation} \Rightarrow \text{Vanishing contribution from 4-Fermi interactions}\]

\[\Rightarrow \text{No Influence on the propagator level}\]
BSEs

- BSEs: Bound state equations derived from DSEs and evaluated on the pole.
- Total momenta
  \[ P = p_1 - p_2 \]
- At the pole \( M_{\text{Pole}} \)
  \[ \Gamma^{(4)} \propto \frac{\psi \bar{\psi}}{P^2 + M_{\text{Pole}}^2} \]
- \( \psi \): Bethe-Salpeter-Amplitude
  \[ \psi = \Gamma^{(3)} \bigg|_{\text{Pole}} \]
BSEs

\[ \Gamma^{(4)} = \Gamma^{(4)} + \Gamma^{(4)} + \Gamma^{(4)} + \Gamma^{(4)} \]
BSEs

\[ \Gamma(4) = \pi \]

\[ \Gamma^{(4)} \]

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BSEs

\[ \pi \rightarrow \pi + \Gamma^{(4)} \]

\[ \Gamma^{(4)} + \Gamma^{(4)} + \Gamma^{(4)} \]

\[ u \quad d \quad u \quad d \quad u \quad d \]

\[ W. A. Mian \]
BSEs

\[ \pi = \pi + \pi + \pi \]

\[ \Gamma^{(4)} \]

\[ \nu \]

\[ e \]

\[ d \]

\[ u \]

\[ d \]

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\[ d \]

\[ u \]
\[ \Gamma(4) = \pi + \pi \]

\[ u \quad \longrightarrow \quad \nu \quad \longrightarrow \quad e \quad \longrightarrow \quad d \]

\[ d \]

\[ u \quad \longrightarrow \quad \nu \quad \longrightarrow \quad e \quad \longrightarrow \quad d \]

\[ u \]
Possibility of pion decay in electron and neutrino: Additional contribution to the Bethe-Salpeter-Amplitude

Pure QCD: $M_{\text{Pole}}$ of pion real in Minkowski-space (stable particle)

QCD + electroweak interaction + light leptons: Open decay channel for pion $\Rightarrow$ Searching for poles in the 2nd Riemann sheet
Generating functional of the correlation functions

\[ Z[J] = \int D\varphi \exp \left[ -S[\varphi] + \int J \cdot \varphi \right] \]

Idea: Integrate the fluctuations step by step (scale k)

\[ Z_k[J] = \int D\varphi \exp \left[ -S[\varphi] - \Delta S_k[\varphi] + \int J \cdot \varphi \right] \]

\[ \Delta S_k[\varphi] = \frac{1}{2} \int \varphi R_k \varphi \]

Conditions on the Regulator (momenta p)

\[ \lim_{k^2 \to 0} R_k(p) = 0, \quad \lim_{k^2 \to \Lambda^2 \to \infty} R_k(p) \to \infty, \quad \lim_{\frac{p^2}{k^2} \to 0} R_k(p) > 0 \]
Wetterich equation

\[ \partial_k \Gamma_k [\phi] = \frac{1}{2} \, \text{Str} \left[ \frac{\partial_k R_k}{\Gamma_k^{(2)} [\phi] + R_k} \right] \]

\[ \lim_{k \to \Lambda} \Gamma_k = S, \]

\[ \lim_{k \to 0} \Gamma_k = \Gamma \]

- Interpolation between the microscopic action \( S \) in the UV at \( k = \Lambda \) and the full effective action \( \Gamma \) in the IR at \( k = 0 \)
Unified description of fundamental and composite degrees of freedom

(H. Gies and C. Wetterich, PRD 65, 065001 (2002))

Composite fields depends on scale $k$

E.g.: Transform from fundamental fermions (up and down quark) to bound states (pion and sigma) at each scale

\[
\partial_k \pi_k = \partial_k A \left( \overline{\psi} i \gamma_5 \tau \psi \right) \\
\partial_k \sigma_k = \partial_k A \left( \overline{\psi} \psi \right)
\]
Real Time FRG

Bosonic Propagator:

\[ \Pi(p^2) = \frac{1}{p^2 + m^2 + R^B_k(p^2)} \]

- Additional unphysical poles in the propagators due to the regulator
- Modify regulator by additional \( \Delta M_r \) term:

  Shifting poles outside

\[ \Pi(p^2) = \frac{1}{p^2 + m^2 + \tilde{R}^B_k(p^2) + \Delta M_r^2(k)} \]

\[ x = \frac{p^2}{k^2}, \quad M^2 = m^2 + \Delta M_r^2 \]

J. M. Pawlowski and N. Strodthoff, PRD 92, 094009 (2015)
Bosonic Propagator:

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Flow Equation

\[ \partial_k = \partial_k + \partial_k \]

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\[
\psi_{\pi,ud} = \tau \gamma_5 \left( i f_1 \mathbb{1} + f_2 \slashed{P} + f_3 \slashed{k} + \frac{i}{2} f_4 [\slashed{P}, \slashed{k}] \right)
\]

- \( P \): Total momenta of the fermions
- \( k \): Relative momenta of the fermions

Pion Bethe-Salpeter-Amplitude for the lepton decay

\[
\psi_{\pi,e\nu} = \tau \gamma_5 \left( g_2 \slashed{P} + g_3 \slashed{k} \right)
\]

Only left-handed fermions contribute to the 4-Fermi-interaction:
Vanishing \( g_1 \) and \( g_4 \)
Low-Energy Effective Model: Quark-Meson Model

- Testing ground: Quark-Meson model for 2 degenerate quark flavors (q) at \( \Lambda = 950 \text{ MeV} \)

\[
\Gamma_k = \int \left\{ \frac{1}{2} Z_{\sigma,k} (\partial_\mu \sigma)^2 + \frac{1}{2} Z_{\pi,k} (\partial_\mu \bar{\pi})^2 + Z_{q,k} \bar{q} \phi q + V_k[\rho] - c \sigma \\
+ h_k \bar{q} \left( i \gamma_5 \vec{\tau} \cdot \vec{\pi} + \frac{1}{2} \sigma \mathbb{I} \right) q + \lambda_k \left[ \frac{1}{4} (\bar{q}q)^2 - (\bar{q} \gamma_5 \vec{\pi} q)^2 \right] \right\}
\]

\[
V_k[\rho] = \sum_{i=1}^{N} \frac{V_{k,i}}{i!} (\rho - \kappa)^i
\]

- Considering renormalized quantities

\[
\overline{V}_{k,i} = \frac{v_{k,i}}{Z_{\pi,k}^i}, \quad \overline{h}_k = \frac{h_k}{\sqrt{Z_{\pi,k} Z_{q,k}}},
\]

\[
\overline{m}_{\pi,k} = \frac{m_{\pi,k}}{\sqrt{Z_{\pi,k}}}, \quad \overline{m}_{\sigma,k} = \frac{m_{\sigma,k}}{\sqrt{Z_{\sigma,k}}}, \quad \overline{m}_{q,k} = \frac{m_{q,k}}{Z_{q,k}}
\]
## Low-Energy Effective Model: Quark-Meson Model

- **Different level of truncations**

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W. A. Mian

EW pion decay in FM

2017, 29th Nov. 17 / 25
Set the initial values to get the right masses in the IR
Truncate the meson potential at $N=8$ for all cases.
Flow of the Meson Masses

![Graphs showing the flow of meson masses](image_url)
Quark Mass

\[ \tilde{m}_q(p^2 = 1 \text{MeV}^2) \text{ [MeV]} \]

\[ k \text{ [MeV]} \]

- LPA
- LPA+Y
- LPA+Y+DH
- LPA+MY
- LPA+MY+DH
- Full
- Full+DH

\[ m_q \text{ [MeV]} \]

\[ p^2 \text{ [MeV}^2] \]
Yukawa Coupling

\[ \tilde{h}(p^2=1\text{MeV}^2) \]

\[ \tilde{h}_k \]

\( k \ [\text{MeV}] \)

\( p^2 \ [\text{MeV}^2] \)
Wavefunction renormalization Quark

\[ Z_{q, k} \]

\[ p^2 \text{ [MeV}^2]\]

Full
Full+DH

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EW pion decay in FM
2017, 29th Nov.
Poles of the Propagators from Real Time FRG

Spectral function $\rho$
preliminary — without backcoupling

$\rho_{\psi,Z}$
$\rho_{\psi,M}$
$\rho_{\pi}*10^3$
$\rho_{\sigma}*10^3$

Frequency [MeV]
Consider QCD and electroweak interactions non-perturbative

Goal:

1. $\beta$-decay in neutron stars
2. Dynamical decay process in functional approach

Requirement: Real time calculation

Building numerical setup in functional methods for real time calculation

Access to mass and decay-width of the particle

Self-consistent backcoupling at the level of the Pion

Developed all components for the dynamical decay process in FRG and BSEs and checked for the Quark-Meson model

Next step: Investigation of the dynamical pion decay in both methods
Consider QCD and electroweak interactions non-perturbative

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Thank you for your attention.